Evaluating Catastrophe Risk Transfer Alternatives Through Dynamic Financial Analysis

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EVALUATING CATASTROPHE RISK TRANSFER ALTERNATIVES THROUGH DYNAMIC FINANCIAL ANALYSIS

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1. INTRODUCTION AND ABSTRACT

Companies subject to significant catastrophe losses are continually evaluating their appetite and tolerance for risk. A sound process of linking catastrophe models to financial output is critical to understanding the implications of risk management strategies. Therefore, dynamic financial models are playing an increasingly important role in evaluating risk transfer strategies. In this paper, we will use the example of Butterfly Insurance Company to illustrate the hurdles involved when incorporating catastrophe modeling in DFA and various methods of presenting results. We will also demonstrate how the buyer of a catastrophe reinsurance structure optimally selects the attachment and the limit using either a heuristic approach or a cost of capital approach.

2. ASSUMPTIONS FOR BUTTERFLY INSURANCE COMPANY

Our case study insurer is a Florida Homeowners writer with approximately \$120M in gross written premium and \$134 million in surplus. The model developed for this fictional company centers on the catastrophe risk to the insurer. While financial statements are included in the model, profitability statistics such as those found on an Income Statement or Balance Sheet are not typically the focus of an insurer when deciding on a catastrophe retention. The catastrophe exposure was modeled using the Catalyst catastrophe modeling software. Margins used in pricing catastrophe layers in question are based on market knowledge at the time of the writing of this paper. Also in place is a 50% quote share with a 35% ceding commission. The catastrophe reinsurance structure inures to the benefit of the quota share reinsurance. Because this company writes Florida residential business, the Florida Hurricane Catastrophe Fund (FHCF) applies. This is a semi-mandatory state catastrophe reinsurer. While we will not discuss the FHCF at length, listed below are a few features that need to be considered for accurate modeling. In particular, the FHCF excludes coverage D (loss of use and additional living expenses) and covers a maximum of 5% ALAE. Because of this, losses need to be segregated into coverage D, ALAE, and all other. It is assumed that the company purchases 90% coverage and this coverage inures to the benefit of the other catastrophe reinsurance. Table 1 shows the income statements for each of the retentions that we will examine in this paper. The incurred loss scenario shown assumes the expected non-catastrophe loss ratio (30%) and no catastrophe losses. Listed below the income statement are assumptions for loss and loss adjustment expenses and reinsurance structures underlying the model.

Table 1

Butterfly Insurance Company
Statutory Income Statement

	$25M Retention	$50M Retention	$75M Retention
Gross Written Premium	$ 123,600	$ 123,600	$ 123,600
Ceded Written Premium	(84,885)	(81,335)	(78,523)
Net Written Premium	38,715	42,265	45,077
Gross Earned Premium	$ 122,115	$ 122,115	$ 122,115
Ceded Earned Premium	(84,143)	(80,593)	(77,780)
Net Earned Premium	37,972	41,522	44,335
DEDUCTIONS:			
Net Losses and LAE Incurred	18,928	18,928	18,928
Acquisition Costs	21,012	21,012	21,012
Premium Taxes, Licenses and Fees	2,957	2,957	2,957
Other Underwriting Expenses Incurred	9,888	9,888	9,888
Ceding Commision Expense Recovery	(13,550)	(14,793)	(15,777)
Total Loss & Expenses Incurred	39,234	37,992	37,007
Net Underwriting Gain (Loss)	(1,262)	3,530	7,327
Net Investment Income Earned	8,862	8,982	9,077
Realized Capital Gains (Loss)	0	0	0
Other Income	0	0	0
Net Income before dividends and taxes	7,600	12,512	16,404
Dividends to Policyholders	0	0	0
Federal Income Taxes Incurred	3,285	5,005	6,367
Net Income	4,315	7,508	10,037
Surplus BOY	134,321	134,321	134,321
Net Income (Loss)	4,315	7,508	10,037
Surplus EOY	$ 138,636	$ 141,829	$ 144,359
Ratio Analysis:			
Non-Cat Net Loss and LAE Ratio	48.2%	44.1%	41.3%
Net Loss and LAE Ratio	49.8%	45.6%	42.7%
Underwriting Expense Ratio	52.5%	45.1%	40.1%
Combined Ratio	102.3%	90.7%	82.8%

Loss & LAE Ratio Assumptions

Mean Non-Cat Loss & ALAE Ratio (% of GEP)	30.0%
Coefficient of Variation	11.18%
Non-Cat ULAE (% of GEP)	0.50%
Cat ULAE (% of Cat Loss & ALAE)	3.00%
Cat ALAE (% of Cat Loss)	10.00%

Reinsurance Assumptions

Catastrophe Reinsurance	Initial Lower Limit	Initial Upper Limit	Rate on Line	Reinstatements	Reinsurer's Participation
FHCF	94,345	412,382	4.5%		90.0%
CAT Excess Layer 1 (25M retention)	25,000	100,000	23.2%	1	100.0%
CAT Excess Layer 1 (50M retention)	50,000	100,000	20.7%	1	100.0%
CAT Excess Layer 1 (75M retention)	75,000	100,000	18.8%	1	100.0%
Cat Excess Layer 2	100,000	200,000	7.2%	1	100.0%
Cat Excess Layer 3	200,000	300,000	2.9%	1	100.0%
Cat Excess Layer 4	300,000	500,000	2.9%	1	100.0%

Quota Share Reinsurance	
Reinsurer's Participation	50.0%
Ceding Commission	35.0%

3. WHY FOCUS ON CATASTROPHES

This paper will only consider catastrophe reinsurance; all other forms of reinsurance and risk management are ignored. This may seem like a narrow-minded approach, and, particularly, an approach that flies in the face of traditional DFA, which attempts to consider every risk element at the same time. For many companies, however, catastrophe risk completely dominates all other risks associated with the company. In the example that we are presenting, the non-cat losses are assumed to have a mean loss ratio of 30%, and a coefficient of variation of 11%. With a subject premium of $124M the standard deviation from the non-cat losses alone would be $4M. However, the standard deviation of a single gross catastrophe loss is $121M. Even after the application of catastrophe reinsurance the net catastrophe losses are still likely to cause almost all of the variation in financial results. For example, with a $25M retention the standard deviation of a single retained losses is still $10M. The variation of the total net retained loss is much higher due to multiple events, reinstatement premiums, reinstatement limits, and losses over the reinsurance limit. The domination of catastrophe risk over all other risk is a common phenomenon for a Florida homeowners writer such as Butterfly Insurance Company.

While this paper will only consider the risk of catastrophes, the methods employed can easily be extended to include non-catastrophe variation. For a general discussion of DFA models and other business risk see "Dynamic Financial Models" by the CAS subcommittee on Valuation and Financial Analysis [1].

4. BASICS OF CATASTROPHE MODELING AND DFA

Since the early 1990's there has been a revolution in the measurement of catastrophe risk. No longer do companies look solely at the last few decades of experience to gauge their risk.

Today the current exposure location and building costs are applied to sophisticated meteorological and seismic models to evaluate the risk of losses of all sizes. Berger et al gives a history of the development of these models [2]. There are several models commercially available (Catalyst, AIR, EQECAT, and IRAS are examples). The amount of output given will vary greatly from company to company, model to model, and will often depend on your access to the model. A model retained in-house, allows you to get the greatest amount of detail, while using third party software may limit the data set available to you.

The ultimate goal of the modeling will be to consider the net retained catastrophe losses (for the ceding company), and to measure the total ceded losses (to price the reinsurance). To do this we will want to derive frequency and severity curves for the catastrophic events.

The general forms of catastrophe outputs in increasing order of desirability are:

a) Probable Maximum Loss (PML): The probable maximum loss is the reciprocal of one minus the cumulative distribution function. For example, if a certain event represents the 99th percentile of the largest events for a year, then it is also the 100 year PML. A list of PML's will generally only give you a few data points for probabilities of the largest single occurrence in a year, though annual aggregate PML's are also possible. Since this is not sufficient information to accurately compute net retained catastrophe losses, in this case the best solution is to fit a more fully known curve from a company which contains similar risks to the limited data set given.

b) X simulated years: Here you are presented with, say, 10,000 years of simulated events. You may have the largest event for each year, the sum of all the events, or all the events for each year listed individually. If you have the first or the second, you will still need to transform the data in order to get the aggregate ceded and retained losses. If you have the complete listing of the individual events, then you still have the problem of only a very small number of sample simulations from which to interpolate critical values, like the 99th or 99.6th percentiles.

c) Simulated events, and associated probabilities: Here you are given a set of possible independent events (hurricanes, for example) with the probability of each event's occurrence, and the loss amount resulting for that event. If you are given the result of every event in the model's database, then you are in the best situation because you have all available data from the model. While 10,000 years of simulated events will give you only 100 events above the 100yr PML, an event set with 10,000 events and associated probabilities may have a much finer detail for large events in that 99th percentile.

Table 2

PML Table
for Butterfly Insurance

Return Time	Loss (000's)
2	266
3	2,418
4	9,982
5	21,327
10	105,523
20	259,234
25	297,024
50	451,931
100	626,631
250	836,521
500	1,049,959
1,000	1,206,906
10,000	1,886,538

In all cases, it is important to know the underlying distributions of the data available. For example, if you have an event set with 10,000 events, and accompanying losses and probabilities, it is important to understand if the events are meant to be individual binomial probabilities, Poisson processes, or some other distribution. Our sample company, Butterfly Insurance, comes with a catastrophe curve of 100 events. The "event rate" is the expected frequency of that event (Poisson distributed). The "normalized

Table 3

Simplified Catastrophe Curve
for Butterfly Insurance Company

	Event Rate	Normalized Probability	Cat Loss	Coverage D		Event Rate	Normalized Probability	Cat Loss	Coverage D
1	1.520E-01	1.946E-01	178,502	26%	51	5.416E-03	6.934E-03	51,707,787	4%
2	6.155E-02	7.879E-02	390,485	21%	52	4.164E-03	5.331E-03	56,085,706	3%
3	4.089E-02	5.234E-02	590,055	22%	53	6.535E-03	8.366E-03	60,246,726	3%
4	2.566E-02	3.285E-02	794,572	17%	54	6.188E-03	7.921E-03	65,163,476	3%
5	1.816E-02	2.325E-02	1,004,241	14%	55	5.722E-03	7.325E-03	70,282,382	3%
6	2.128E-02	2.725E-02	1,195,978	16%	56	5.119E-03	6.553E-03	76,214,506	3%
7	1.685E-02	2.157E-02	1,397,269	18%	57	3.502E-03	4.484E-03	82,092,842	3%
8	9.467E-03	1.212E-02	1,607,075	12%	58	3.926E-03	5.026E-03	88,361,550	3%
9	9.231E-03	1.182E-02	1,796,857	11%	59	5.184E-03	6.637E-03	96,503,098	3%
10	8.636E-03	1.106E-02	2,019,902	11%	60	6.144E-03	7.866E-03	102,599,443	3%
11	7.458E-03	9.548E-03	2,223,565	11%	61	4.795E-03	6.139E-03	112,147,264	2%
12	6.433E-03	8.235E-03	2,392,359	11%	62	6.203E-03	7.941E-03	120,759,695	3%
13	4.176E-03	5.346E-03	2,630,005	11%	63	7.157E-03	9.162E-03	130,058,168	3%
14	7.462E-03	9.552E-03	2,823,358	9%	64	5.443E-03	6.968E-03	139,934,948	3%
15	1.046E-02	1.339E-02	3,059,936	8%	65	4.895E-03	6.266E-03	152,103,474	3%
16	8.209E-03	1.051E-02	3,274,813	7%	66	4.821E-03	6.172E-03	162,714,089	3%
17	6.033E-03	7.723E-03	3,588,905	6%	67	4.186E-03	5.359E-03	176,917,196	3%
18	8.712E-03	1.115E-02	3,894,129	9%	68	4.480E-03	5.735E-03	191,529,679	3%
19	8.552E-03	1.095E-02	4,238,908	7%	69	4.498E-03	5.758E-03	206,254,266	2%
20	7.528E-03	9.636E-03	4,571,688	7%	70	5.392E-03	6.902E-03	221,703,770	3%
21	7.865E-03	1.007E-02	4,984,386	9%	71	5.322E-03	6.813E-03	240,667,973	3%
22	7.794E-03	9.978E-03	5,333,774	8%	72	4.573E-03	5.854E-03	260,038,757	3%
23	6.014E-03	7.699E-03	5,777,618	6%	73	2.859E-03	3.659E-03	280,827,933	3%
24	6.093E-03	7.800E-03	6,262,585	6%	74	4.608E-03	5.899E-03	302,852,442	3%
25	6.912E-03	8.848E-03	6,795,264	6%	75	3.865E-03	4.948E-03	324,138,441	3%
26	7.442E-03	9.527E-03	7,304,943	6%	76	2.268E-03	2.903E-03	351,215,455	3%
27	6.647E-03	8.510E-03	7,918,808	5%	77	2.812E-03	3.600E-03	380,944,918	3%
28	5.721E-03	7.324E-03	8,530,132	6%	78	3.201E-03	4.098E-03	409,922,431	3%
29	7.538E-03	9.650E-03	9,309,805	7%	79	2.554E-03	3.269E-03	439,735,914	3%
30	7.294E-03	9.337E-03	10,085,144	6%	80	3.016E-03	3.861E-03	476,197,173	3%
31	6.605E-03	8.455E-03	10,843,286	4%	81	3.015E-03	3.859E-03	513,859,633	3%
32	8.004E-03	1.025E-02	11,692,004	6%	82	2.404E-03	3.077E-03	554,983,227	3%
33	4.914E-03	6.291E-03	12,728,313	6%	83	1.739E-03	2.226E-03	600,634,629	3%
34	4.004E-03	5.125E-03	13,711,989	5%	84	1.333E-03	1.707E-03	651,101,592	3%
35	6.191E-03	7.926E-03	14,806,624	5%	85	1.107E-03	1.417E-03	703,592,671	3%
36	7.852E-03	1.005E-02	16,039,499	5%	86	1.743E-03	2.232E-03	757,360,647	4%
37	5.128E-03	6.564E-03	17,385,348	4%	87	1.179E-03	1.509E-03	827,054,722	3%
38	4.904E-03	6.278E-03	18,740,026	5%	88	9.167E-04	1.173E-03	887,856,598	4%
39	6.857E-03	8.779E-03	20,458,132	4%	89	6.126E-04	7.843E-04	962,007,576	4%
40	7.405E-03	9.479E-03	21,951,996	4%	90	4.246E-04	5.435E-04	1,028,220,582	4%
41	6.792E-03	8.695E-03	23,864,749	4%	91	4.974E-04	6.368E-04	1,112,049,662	4%
42	3.951E-03	5.057E-03	26,067,076	5%	92	4.136E-04	5.294E-04	1,205,495,330	4%
43	5.003E-03	6.404E-03	27,652,466	3%	93	2.547E-04	3.260E-04	1,303,775,508	4%
44	4.502E-03	5.763E-03	29,892,537	3%	94	2.346E-04	3.004E-04	1,412,856,253	4%
45	5.251E-03	6.723E-03	32,599,499	4%	95	2.791E-04	3.573E-04	1,539,692,090	4%
46	5.513E-03	7.058E-03	35,119,089	4%	96	6.422E-05	8.221E-05	1,653,803,890	4%
47	6.754E-03	8.647E-03	38,179,107	4%	97	1.090E-04	1.396E-04	1,793,341,280	5%
48	4.288E-03	5.489E-03	41,250,272	4%	98	2.465E-05	3.156E-05	1,929,968,396	4%
49	7.239E-03	9.267E-03	44,426,313	4%	99	1.486E-05	1.902E-05	2,102,537,408	4%
50	6.714E-03	8.595E-03	48,236,955	3%	100	1.503E-05	1.924E-05	2,299,939,611	3%

Lambda = 0.78115 AAL = 39,957,508

probability" will be used later. "Cat Loss" is the loss from that particular event, and the coverage D percentage is given so that recoveries from the FHCF can be correctly computed.

It is convenient to use a discrete distribution for several reasons. First, it can be important to capture the relationship between Butterfly Insurance's cat loss and the industry cat loss since the industry cat loss will affect the availability and price of future reinsurance. A large industry loss will also impact the amount of available coverage under the FHCF in the subsequent season. In addition, a discrete distribution allows one to easily maintain the relationship between the event loss and the associated coverage D loss. This is important because the FHCF does not allow for recoveries on coverage D losses.

5. CONVERSION OF INDIVIDUAL EVENTS TO FREQUENCY AND SEVERITY

If catastrophe data is presented in the form of many thousand independent catastrophe events, it is clearly impossible to simulate from the direct data - each simulated year would require several thousand random draws to create the catastrophe events. To make the computations manageable it is necessary to convert to separate frequency and severity curves. If data is presented in the form of simulated years, it is still useful to make this conversion, since it allows "bootstrapping" from the limited data given, to a richer dataset. To make this conversion we use the fact that a sum of independent Poisson variables is Poisson where the sum of the lambdas is equal to the mean (again lambda). Similarly, a sum of binomials is approximately Poisson with lambda equal to the sum of the binomial probabilities. In our case the Poisson lambda is the sum of the rates shown in the second and seventh columns of Table 3. The severity distribution is discrete with probabilities equal to the Poisson frequencies re-normalized to one. This is shown in the third and eighth columns of Table 3. An outline of a proof of the equivalency of this model to the original model is shown in the appendix.

6. ESTIMATION METHODS AND ADAPTIVE SAMPLING

In this particular example, the annual frequency of catastrophe events is 0.78115. This means that in 46% of the simulated years there will be no catastrophe at all, in 82% of years there will be one or fewer, and in 96% of years there will be two or fewer. Of those events that occur, half will be relatively small: under $3,000,000. Since we are concerned with extreme events, for example the risk of ruin at the 99.6th percentile (250 year return time), a traditional Monte Carlo simulation spends about half of the computing time on events that produce the exact same answer, and almost all of the computing time on events which have no chance of impacting the critical upper percentiles. To alleviate this situation we can supplement Monte Carlo simulation with a heuristic narrowing of the scope of the simulation.

One alternative to Monte Carlo simulation is to numerically convolute the probability distribution through the fast Fourier transform (see [3]), or the Heckmen-Meyers algorithm (see [4]). This is faster than simulation, however, it requires that the claim severities be identically distributed. Because catastrophe contracts contain a mixture of aggregate and occurrence limits (generally expressed as a certain number of reinstatements, or multiples of the original limit), the nth catastrophe may have a different net severity distribution depending on the preceding catastrophes. In addition, since the FHCF contains special limits (on coverage D and ALAE) it is not clear how to adapt those algorithms to this situation.

The method proposed to increase the efficiency of a simulation is over-sampling the areas of interest, and under-sampling areas that require fewer simulations to converge to the true distribution. In this example, we will break the catastrophe curve into two layers, claims over

$50M, and claims under $50M. We then perform separate simulations for each of the frequency combinations with a probability greater than 1 in 100,000. The probability of frequency combinations outside this range has been placed in the combination of three events over $50 million and three events under $50 million (in italics). From a practical perspective, six or more catastrophes in a year in one state is bad enough, there is no need to generate scenarios worse than that. The case of no catastrophes need only be considered once, and the cases of one or two catastrophes can be handled by exhausting the 100 or 2500 combinations of events in our dataset. For the remainder, we perform 5,000 iterations for all the combinations with a probability greater than 1 in 1,000, and 1000 iterations for all those with a probability less than 1 in 1,000. Using this "adaptive sampling" we can generate a greater accuracy with 56,000 iterations than a Monte Carlo simulation with 1,000,000 iterations.

Table 4

		Number of Events over $50,000,000					
	Probability	0	1	2	3	4	5
Number	0	4.579E-01	7.108E-02	5.517E-03	2.855E-04	1.108E-05	3.439E-07
of	1	2.866E-01	4.449E-02	3.453E-03	1.787E-04	6.934E-06	2.153E-07
Events	2	8.969E-02	1.392E-02	1.081E-03	5.592E-05	2.170E-06	6.737E-08
under	3	1.871E-02	2.905E-03	2.255E-04	*3.919E-05*	4.528E-07	1.406E-08
$50M	4	2.928E-03	4.546E-04	3.528E-05	1.826E-06	7.085E-08	2.200E-09
	5	3.666E-04	5.690E-05	4.417E-06	2.285E-07	8.869E-09	2.754E-10
	6	3.824E-05	5.936E-06	4.608E-07	2.384E-08	9.252E-10	2.872E-11
	7	3.419E-06	5.308E-07	4.120E-08	2.132E-09	8.273E-11	2.568E-12

		Number of Events over $50,000,000					
	Iterations	0	1	2	3	4	5
Number	0	1	50	2,500	1,000	1,000	0
of	1	50	2,500	5,000	1,000	0	0
Events	2	2,500	5,000	5,000	1,000	0	0
under	3	5,000	5,000	1,000	1,000	0	0
$50M	4	5,000	1,000	1,000	0	0	0
	5	1,000	1,000	0	0	0	0
	6	1,000	0	0	0	0	0
	7	0	0	0	0	0	0

7. SELECTION OF THE CATASTROPHE RETENTION AND LIMIT

There are several common methods employed for selecting and evaluating reinsurance structures.

a) Rules of thumb / Management decision criteria

   These are probably the most prevalent of the selection techniques. A possible rule would be for a company to purchase catastrophe reinsurance down to half of its surplus, or its 10 year PML.

b) Regulatory or rating requirements

   A. M. Best requires that companies be able to withstand their 250 year earthquake PML, and their 100 year hurricane PML. In this example the limit will be set at the 250 year PML.

c) Risk of ruin / Expected policyholder deficit

   The previous examples are very similar to a risk of ruin standard. In our case the capital required to support the catastrophe risk will be defined as the 99.6th percentile aggregate retained catastrophe loss with reinstatement premiums ($86 million in the case of a $25 million retention).

d) Standard deviation

   While most companies do not set explicit standard deviation targets they will want to keep the standard deviation (or some other statistical measure of the variability) of their results to a minimum.

e) Cost of capital needed to retain the risk

   When deciding whether to retain or cede a particular risk the company needs to consider the cost of retaining that risk. The company should purchase reinsurance wherever the cost of reinsurance is cheaper than the cost of holding (or obtaining) sufficient capital to absorb a potential loss.

8. SELECTION OF THE CATASTROPHE RETENTION AND LIMIT – HEURISTIC APPROACH

Typical DFA statistics such as loss ratio, combined ratio, and ROE focus on the profitability of the entire company. The presentation of these statistics tends to focus on a "reasonable range of results." Presentation of the range can take the form of confidence intervals (90% range of results for the Net Loss & ALAE Ratio[Exhibit 1]), risk/reward plots (the probability that the company combined ratio is greater than X% versus the mean combined ratio for each alternative strategy [Exhibit 2]), or cumulative distribution functions [Exhibit 3] to name a few. However, these profitability statistics combined with the described presentation methods do not address the real concerns of the company in terms of optimizing their catastrophe reinsurance retention.

The traditional presentation of catastrophe exposure has revolved around the probable maximum loss. A common question is "What is my 100 or 250-year PML?" In terms of probability, these PML's convert to the 99th and the 99.6th percentiles respectively. The 90% confidence interval of the net loss & ALAE ratio, as shown in Exhibit 1, excludes these points. The upper bound of a 90% confidence interval translates to only the 20-year PML; a far smaller event than the 100 or 250 year events that catastrophe reinsurance is protecting against. Also, from this graph, one might assume that the $75M retention, which produces the lowest mean net loss ratio, would be the optimal retention. However, we will show in future exhibits that the $25M retention is actually optimal given this company's risk and financial constraints.

Exhibit 2 shows a risk/reward plot of the combined ratio. This also fails to address catastrophe exposure sufficiently since it ignores the severity of a large PML event. From a risk/reward plot, one can determine that the probability that the combined ratio exceeds a 120% combined ratio under the $25M retention structure is 23.8%. However, it does not address the extent to which 120% is surpassed in the large PML events. A 100 year event may produce a combined ratio of 150% (survivable) or 500% (instant death).

A cumulative distribution function does address the large PML values in that it plots all points on the curve up to the 100th percentile, as shown in Exhibit 3. The net loss & ALAE ratio associated with the 100 year event is found on the graph, however, the scale makes it impossible to see the difference between the three retentions.

Profitability statistics and graphs such as those described above are appropriate to show the relative position of the company after the buying decision has been made, but what is needed to optimally select a retention is a comparison of cost incurred and protection provided when events occur.

Exhibit 1

Net Loss & ALAE Ratio
Statistical Summary
Calendar Year 2001
($000)

	$25M Retention	$50M Retention	$75M Retention
90% Confidence Interval			
High	122.2%	134.4%	143.6%
Mean	72.1%	69.7%	67.0%
Low	40.9%	37.9%	35.4%

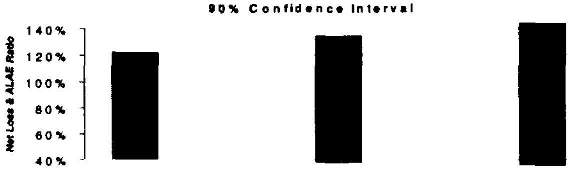


Exhibit 2

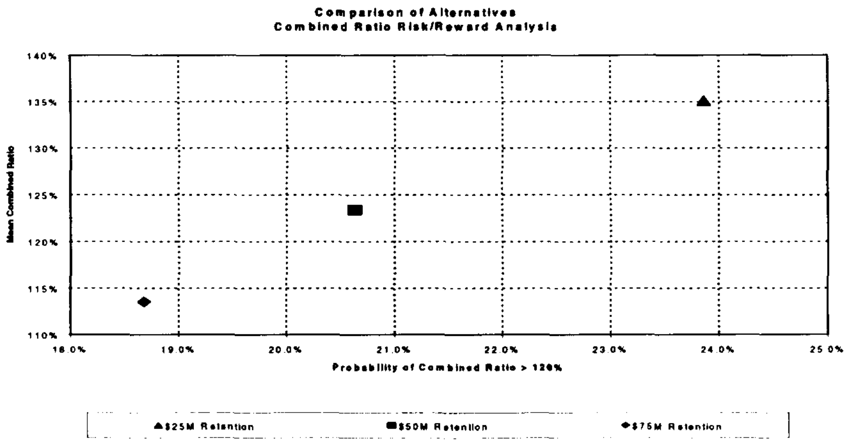
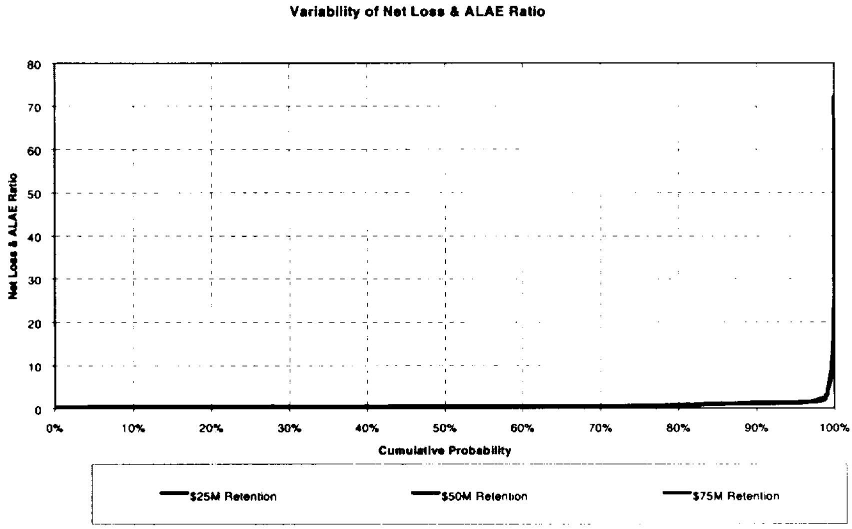


Exhibit 3



As an alternative to DFA, one could choose to stick to the widely accepted method of presenting results in the form of a PML curve. Net retained catastrophe losses and reinstatement premiums would be calculated on individual loss events and ranked by their associated return times. However, this method also fails to properly address the company's true catastrophe exposure. By calculating retained loss and reinstatement premiums for individual loss events only, the impact of multiple events on ceded loss and reinstatement premium is ignored. In general, the results produced by the financial model improve on the "PML curve" type analysis by considering annual aggregate results.

In the following discussion, we will present several statistics and methods for presenting results that attempt to correct for the shortcomings of the methods listed above.

Net Cost/Benefit of Reinsurance Contract

Catastrophe reinsurers charge a high margin for coverage (typically somewhere between 1.5 and 5 times the expected loss to the layer). This is reasonable given the low probability of loss, but high potential severity. This means that on an expected value basis, the company's profitability will be better the less cat reinsurance they buy. However, they may be taking on risk that they cannot handle. The purpose of the net cost/benefit statistic is to provide a cost versus coverage comparison between varying retentions and limits. Reducing cost also reduces benefit. The cedant needs to decide the retention where the marginal benefit of the reduced cost is outweighed by the marginal increase in retained losses. When we are considering net results the smaller the standard deviation the better (the more stable the results), however, because we are measuring recoveries, a larger standard deviations means larger potential benefit to the reinsured.

Exhibit 4

Net Benefit / (Cost) of Catastrophe Reinsurance Program prior to Quota Share

Statistical Summary-Annual Aggregate Loss

($000)

	25M Retention	50M Retention	75M Retention
Minimum (in 20,000 Years)	(46,171)	(39,931)	(33,838)
100 Year Return Time	610,041	573,908	541,151
150 Year Return Time	681,883	669,361	636,610
200 Year Return Time	681,883	669,361	640,966
250 Year Return Time	681,883	669,361	645,742
500 Year Return Time	687,422	669,361	656,549
1000 Year Return Time	719,719	680,173	656,549
Maximum (in 20,000 Years)	1,156,883	1,119,361	1,081,549
Standard Deviation	110,868	102,591	94,693
99% Range of Outcomes			
99.5 Percentile	681,883	669,361	640,966
Mean	(13,768)	(12,099)	(10,029)
0.5 Percentile	(46,171)	(39,931)	(33,838)

99% Range of Outcomes

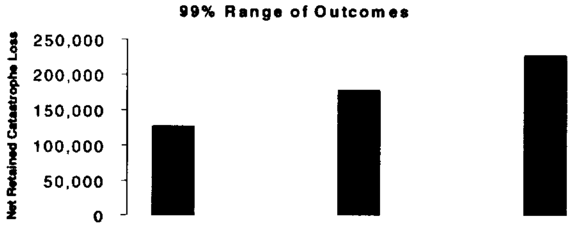
Net Retained Catastrophes

All companies must retain a portion of their catastrophe exposure, but only the largest companies can retain all of it. The question usually revolves around how much volatility the company can absorb. Obviously, the lower the retention, the lower the variability of the net results.

Exhibit 5

Net Retained Catastrophe Loss prior to Quota Share
Statistical Summary - Annual Aggregate Loss
($000)

	25M Retention	50M Retention	75M Retention
Minimum (in 20,000 Years)	0	0	0
100 Year Return Time	55,171	101,222	150,000
150 Year Return Time	74,225	120,185	163,929
200 Year Return Time	127,251	178,001	227,945
250 Year Return Time	173,570	222,730	272,186
500 Year Return Time	410,748	460,748	507,788
1000 Year Return Time	505,759	550,316	598,367
Maximum (in 20,000 Years)	2,652,434	2,702,434	2,752,434
Standard Deviation	**36,847**	**43,173**	**48,925**
99% Range of Outcomes			
99.5th Percentile	**127,251**	**178,001**	**227,945**
Mean	**9,532**	**14,086**	**16,939**
0.5th Percentile	-	-	-



Net Loss & ALAE to Prior Year End Surplus

Many companies define their risk appetite for retaining loss in terms of the ratio of net loss & ALAE to surplus. Our example company uses the rule of thumb that net loss and ALAE (after cat and quota share reinsurance) must be less than or equal to one-half of surplus at the 100 year return time. According to this criteria, the $25M retention is the optimal retention for this company.

Exhibit 6

Net Loss & ALAE to Prior Year End Surplus
Statistical Summary-Annual Aggregate Loss
($000)

	25M Retention	50M Retention	75M Retention
Minimum (in 20,000 Years)	8.9%	9.6%	9.3%
100 Year Return Time	52.7%	72.0%	91.1%
150 Year Return Time	99.2%	115.9%	132.0%
200 Year Return Time	130.2%	148.2%	165.4%
250 Year Return Time	155.2%	172.2%	189.6%
500 Year Return Time	239.5%	258.7%	274.6%
1000 Year Return Time	275.8%	293.8%	312.0%
Maximum (in 20,000 Years)	162.6%	179.4%	197.8%
Standard Deviation	19.0%	21.3%	23.6%
99% Range of Outcomes			
99.5 Percentile	130.2%	148.2%	165.4%
Mean	17.9%	19.6%	20.9%
0.5 Percentile	10.4%	10.4%	10.3%

99% Range of Outcomes

9. SELECTION OF THE CATASTROPHE RETENTION AND LIMIT – COST OF CAPITAL APPROACH

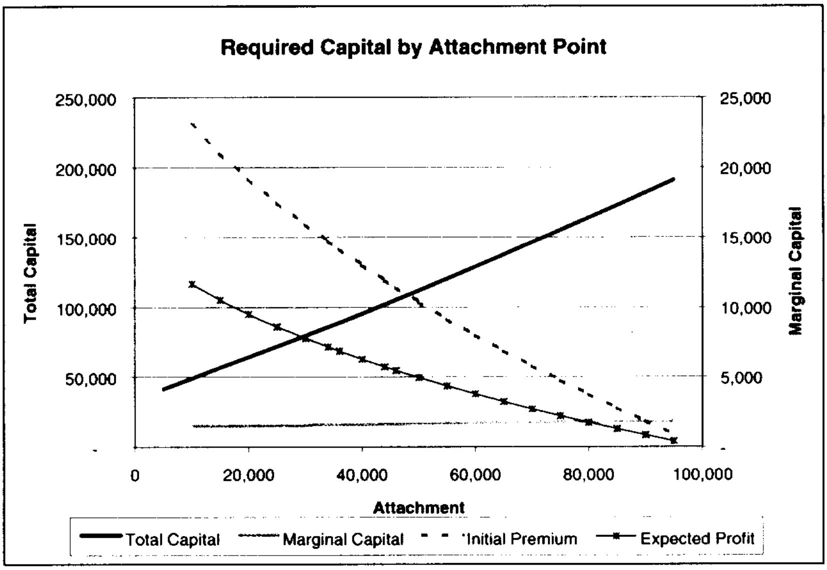
The previous discussions have focused on subjective criteria for evaluating the attachment point for the catastrophe structure. While most companies make their decisions on this basis, an alternative method which considers the cost of capital can give more concrete results. We have already determined that the point where the catastrophe reinsurance runs out will be the single event 250 yr PML. We will also put the risk of ruin on the aggregate net retained losses (for the purposes of calculating required capital) at the 99.6^{th} percentile.

For this method, the company must decide what cost of capital to apply to catastrophe risk. There is agreement that it will be a higher cost of capital than the standard insurance risk, due to the higher variability, but how much? There are several methods for determining the cost of capital on a specific investment, none of them entirely satisfactory. The two most popular methods are the capital asset pricing model (CAPM), and to base the required return on an investment's variance.

CAPM predicts that the required return on an investment is generated from its correlation with the "market," usually defined as a broad stock market index. Since catastrophe risk is not correlated with the stock market performance, CAPM would require only the risk free rate of return for this investment. Since the funds supporting catastrophe risk are available for investment in risk free securities, no return at all would be required to support the catastrophe risk (ignoring taxes). This does not correspond with intuition or experience.

If we base the required return on variance, then the catastrophes limited to \$100M have a variance of about $\$1.1 \times 10^{15}$. The stock market gives returns of around 15.3% with a variance of around 2.6% (S&P total returns 1972-2000). If we assume that \$150M of capital is needed to support this layer, then this amount invested in the stock market would have a variance of about $\$5.85 \times 10^{14}$ of the dollars invested. This implies a return on capital supporting catastrophe risk of 29%. The variance, and the exact cost of capital, would change depending on the layer considered, but, for simplicity, 29% is the required return that we will assume.

Exhibit 7

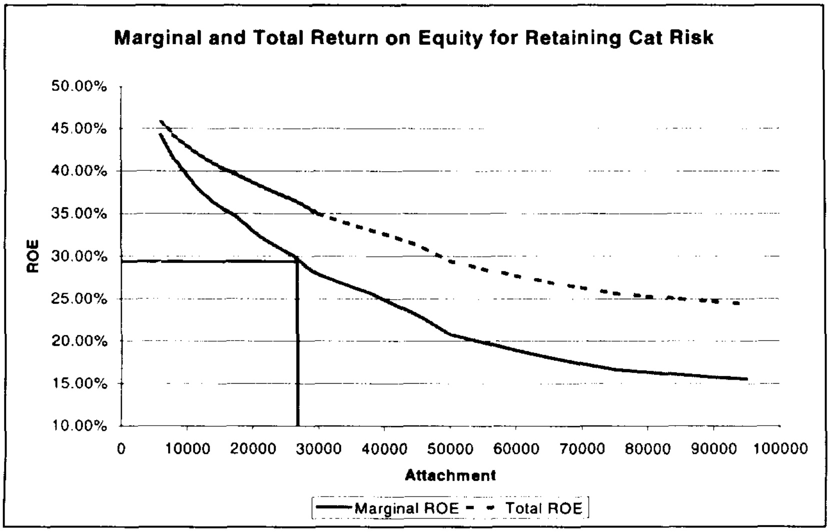


We have now fixed on a 29% cost for the capital used to support the catastrophe reinsurance. Let's also presume that the company will not consider a catastrophe retention lower than \$5 million or higher than \$100 million (essentially the attachment of the FHCF). Since this is the only layer under consideration at this point, we will only consider the capital needed to retain the losses under \$100 million per occurrence. Additional capital may be needed to cover losses above the catastrophe insurance, or non-catastrophe losses, but those will not impact the decision in this layer.

Consider Exhibit 7. First we have plotted the 99.6th percentile net retained loss including reinstatement premiums (labeled Total Capital, and Marginal Capital), and the cost of reinsurance (premium minus average recovery, labeled Expected Profit). The marginal return on capital by retaining a layer of cat risk is:

$$\text{Marginal Return on Capital at Risk} = \frac{\Delta\text{Cat Premium} - \Delta\text{Cat Recoveries}}{\Delta\text{Required Capital}}$$

Exhibit 8



The second graph (Exhibit 8) shows the marginal and total return on capital at risk for the retained amounts under $100 million per occurrence. In this case the optimal attachment is around $27 million. To accommodate our natural bias towards multiples of $5million, we will select an attachment point of $25 million.

10. SUMMARY

Catastrophe loss is the major risk facing many insurance companies. To better understand the management strategies used to control this risk, companies are turning to financial models. The standard outputs and analysis of DFA models are not helpful in selecting catastrophe retentions, and serve only to complicate the decision making. Specialized output exhibits are necessary to illustrate the extreme risks associated with catastrophes. A particularly valuable method for determining the optimal retention considers the return on allocated capital to support potential catastrophe losses.

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[1] Subcommittee on Dynamic Financial Models of the CAS Committee on Valuation and Financial Analysis, "Dynamic Financial Models of Property/Casualty Insurers" 1995 Casualty Actuarial Forum, Vol: Fall, pp 93 - 127

[2] Burger, George, et al, "Incorporating a Hurricane Model into Property Ratemaking" 1996 Casualty Actuarial Forum, Vol: Winter, pp 129 - 190

[3] Wang, Shaun, "Aggregation of Correlated Risk Portfolios: Models and Algorithms" Proceedings of the Casualty Actuarial Society, Volume LXXXV, 1998, pp 848 – 939

[4] Heckman, P. E. and G. G. Meyers, "The Calculation of Aggregate loss Distributions from Claim Severity and Claim Count Distributions," Proceedings of the Casualty Actuarial Society, Volume LXX, 1983, pp 22-61

APPENDIX

OUTLINE OF EQUIVALENCE OF THE MODEL PRESENTED IN SECTION FIVE

We begin with a large number of independent binomial random variables (not necessarily identically distributed), with probabilities $\{p_1, p_2, \ldots, p_n\}$ (or expected frequencies in the case of Poisson events), and associated loss values $\{x_1, x_2, \ldots, x_n\}$. Define $\lambda = \sum p_i$ and consider an alternative model where the frequency of non-zero events is distributed Poisson with parameter λ, and severities are chosen based on the weighted average of p_i. To show that these two models are equivalent, consider the probability of a specific outcome:

$$P(X_1 = x_{i_1}, X_2 = x_{i_2}, \ldots, X_k = x_{i_k}) = \frac{p_{i_1} \times p_{i_2} \times \ldots \times p_{i_k}}{(1-p_{i_1})(1-p_{i_2})\ldots(1-p_{i_k})} \prod (1-p_i)$$

$$\text{since } e^{-\left(\sum p_i\right)} \approx \prod (1-p_i) \text{ and } (1-p_{i_1})(1-p_{i_2})\ldots(1-p_{i_k}) \approx 1 \text{ we have}$$

$$\approx \frac{e^{-\lambda}\lambda^k}{k!} \frac{p_{i_1} \times p_{i_2} \times \ldots \times p_{i_k}}{(\sum p_i)^k} k!$$

$$= P(\text{freq} = k) \frac{p_{i_1} \times p_{i_2} \times \ldots \times p_{i_k}}{(\sum p_i)^k} k!$$

$$= P(\text{freq} = k)\, P(\text{severities} = x_{i_1}, x_{i_2}, \ldots, x_{i_k})$$

In the case of initially Poisson events we begin with:

$$P(X_1 = x_{i_1}, X_2 = x_{i_2}, \ldots, X_k = x_{i_k}) = \frac{\left(p_{i_1} e^{-p_{i1}}\right) \times \left(p_{i_2} e^{-p_{i2}}\right) \times \ldots \times \left(p_{i_k} e^{-p_{ik}}\right)}{(e^{-p_{i1}})(e^{-p_{i2}})\ldots(e^{-p_{ik}})} \prod e^{-p_i}$$

and the result follows more directly.