Exposure Rating Loss Layers: Unifying the Property Perspective of Severity with the Liability Perspective of Frequency

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Abstract

For problems such as rating excess of loss reinsurance and estimating deductible credits, actuaries frequently employ exposure rating factors. In the context of property insurance this takes the form of loss tables such as the Lloyds scale or Salzmann tables. These tables display the fraction of loss cost retained for layers expressed as fractions of insured value, or policy limit. In the liability insurance context, Increased Limits Factors (ILFs) or Excess Loss Factors (ELFs) tables are expressed in terms of actual dollar amounts for attachment points and limits. Implicit in the property tables is the assumption that an increase in policy limit or insured value corresponds to a proportional scale factor increase in the claim severity random variable, but other than the change in scale the distribution of claim sizes remains the same and any increase or decrease in loss cost per exposure is frequency based. Without a special adjustment to the loss cost or premium rate, the implied loss frequency is the same for the larger policy. Implicit in the liability tables is the assumption that larger policies produce the same distribution of claim severity. In summary, the property perspective generally assumes that all the extra exposure shows up as larger claims, and the liability perspective generally assumes that all the extra exposure shows up as more claims. This paper shows how both perspectives for claim severity, and additional considerations of frequency changes may easily be incorporated into a unified model. Additionally, such a unified approach allows for a compromise where increasing exposure for a given policy or risk may be partially reflected in the scale of claim size and partially in the frequency.

A Generic Example of Property Exposure Rating of a Loss Layer

A typical property exposure rating scale might look like:

Fraction of Policy Limit	Fraction of Retained Loss Cost
200%	100%
100%	99%
90%	98%
75%	95%
50%	80%
10%	40%
5%	25%
0%	0%

Note: Losses in excess of the main policy limit occur due to multiple coverage limits, such as personal property and business interruption, or extra contractual obligations, etc.

Suppose an actuary is reviewing two new property risks for reinsurance cost. One risk is a small store covered by a business owners policy (BOP), with a property limit of $300,000. The other is a large industrial warehouse structure with extensive sprinklers and other loss control devices, which is covered by a general commercial fire policy valued at $2 million. The actuary's company has a property per risk reinsurance treaty for its BOP exposures which covers losses of $850,000 excess of $150,000. The company also requires that facultative reinsurance certificates be purchased for all property risks in excess of $1 million.

To estimate the loss cost ceded to the BOP per risk treaty, for the newly insured store, an actuary performs the following exposure rating analysis. The attachment point for the treaty is 50% of the policy limit. This means that the company expects to retain 80% of ground up expected losses (due to the first $150,000 retained layer). The reinsurance limit plus attachment point of the treaty exceeds the maximum loss level of 200% of policy limit. So the reinsurance layer and primary layer together cover 100% of the loss cost. The expected percentage of losses ceded to the reinsurance layer is 100% - 80% = 20%. The company premium rate for BOP policies is $2 per $1,000 of limit. Ignoring expense adjustments and ceding commissions, $120 of the $600 of direct premium are ceded to the treaty.

Now consider the case of the facultative coverage on the warehouse. Since $1 million is also 50% of the limit for the warehouse, the actuary gets the same cession percentage of 20%. If the base rate is the same, $800 of the $4,000 of direct premium on the warehouse will be ceded to the facultative certificate.

Is this reasonable? Probably not. Whereas a fire or other peril might easily destroy the store, it is unlikely that the entire warehouse would be destroyed in a single event. If the reduced loss cost per exposure unit for the larger building is reflected entirely in the rate, this is equivalent to reducing the frequency. This is also probably not reasonable. The warehouse likely has constant movement of stock by small vehicles and cranes. It probably experiences more frequent small to medium size losses.

A Generic Example of Liability Exposure Rating of a Loss Layer

A typical table of liability increased limits factors might look like:

Occurence Limit	Increased Limits Factor
50,000,000	6.125
10,000,000	3.625
5,000,000	3.000
2,000,000	2.250
500,000	1.500
200,000	1.200
100,000	1.000

Suppose an actuary is reviewing two new general liability policies for reinsurance cost. One policy covers a small 1,500 square foot "mom and pop" corner store with $200,000 of sales per year. The other covers a 150,000 square foot discount retail superstore with $20,000,000 of sales per year. Each policy has an occurrence limit of $2 million dollars.

The actuary's company has a reinsurance treaty covering occurrence losses of $1.8 million excess of $200k. From the table above we can see that the rate for $2 million limits is 2.25 times the base rate and the rate for $200,000 is 1.2 times the base rate. So the ceded rate for the reinsurance layer should be 2.25 – 1.2 = 1.05 times the base rate. If the company has a base rate of $1 per $1,000 of sales, then the small store policy should cede $210 of the $450 of direct premium, and the superstore policy should cede $21,000 of the $45,000 of direct premium.

Is this reasonable? Probably not. The larger store will almost certainly experience a higher frequency of claims. However, it is also very likely to experience larger claims, i.e. a different severity distribution. Potential plaintiffs and their lawyers will probably view the larger store as a deep pockets defendant. As such, they will be more willing to pursue larger claims, and less likely to settle for smaller amounts. Juries are also more willing to award larger claims against such a defendant.

A Unified Model

Assume that loss cost per exposure is constant for policies with different magnitudes of exposure. Let E1, S1, and F1 be the exposure, average severity, and average frequency for a policy. Similarly E2, S2, and F2 are the same parameters for a larger risk of like kind. The property perspective is:

$S2 = S1 \times (E2 / E1)$ and $F2 = F1$.

The liability perspective is:

$S2 = S1$ and $F2 = F1 \times (E2 / E1)$.

Now introduce a new parameter, A, and suppose that:

$S2 = S1 \times (E2/E1) ^\wedge A$, and

$F2 = F1 \times (E2/E1) ^\wedge (1 - A)$.

Notice that the property perspective corresponds to $A = 1$, and the liability perspective corresponds to $A = 0$. Values for A between 0 and 1 represent a compromise between these two perspectives.

Now, suppose that we relax the assumption that loss costs per exposure are constant for policies with different magnitudes of exposure. We can do this by introducing a second parameter, B, and restating our equations as:

$S2 = S1 \times (E2/E1)^{A}$, and

$F2 = F1 \times (E2/E1)^{B}$

Note that $A + B$ is not necessarily equal to 1. When $A + B = 1$, loss costs per exposure are constant and
$B = 1 \quad A$.

If L1 and L2 are the expected losses for each policy, then:

$L2 = L1 \times (E2/E1)^{(A + B)}$.

Generic Examples of Exposure Rating a Loss Layer Using the Unified Model

First, we reconsider the property example. Instead of $A = 1$ and $B = 0$, we believe $A = 0.8$ and $B = 0.1$ are more appropriate values. Thus:

$E2 / E1 =$ \$2 million / \$300k $= 6.67$,

$S2 / S1 = (6.67)^{.8} = 4.56$, and

$F2 / F1 = (6.67)^{.1} = 1.21$.

Assume our calculation for the BOP policy on the small store was correct, but the calculation for the warehouse policy must be modified. We need to adjust the loss scale table by multiplying the percentages of policy limit by the factor $4.56 / 6.67 = 0.68$.

This produces an adjusted table of:

Fraction of Policy Limit	Fraction of Retained Loss Cost
135%	100%
68%	99%
61%	98%
51%	95%
34%	80%
7%	40%
3%	25%
0%	0%

This new table suggests a premium cession rate of only 5%. Now consider the situation for total expected losses. Since (6.67) ^ (0.8 + 0.1) = 5.52, we should adjust our direct premium by a factor of 5.52 / 6.67 = 0.83. So the direct premium should be $3,320 instead of $4,000, and the ceded premium should be $166 instead of $800. Notice that in this case our rate for the policy holder has dropped 17%, and almost all of this lower rate is compensated for by decreased reinsurance costs!

Now we reconsider the liability example. Instead of A = 0 and B = 1, we believe A = 0.15 and B = 0.9 are more appropriate values. Thus,

E2 / E1 = $20 million / $200k = 100, and

S2 / S1 = (100) ^ 0.15 = 2.00, and

F2 / F1 = (100) ^ 0.9 = 63.10 .

Assume our calculation for the general liability policy on the corner store was correct, but the calculation for the superstore policy must be modified. We should adjust the increased limits factor table by multiplying the occurrence limits by the factor 2.00 from above.

This produces an adjusted table of:

Occurence Limit	Increased Limits Factor
100,000,000	6.125
20,000,000	3.625
10,000,000	3.000
4,000,000	2.250
1,000,000	1.500
400,000	1.200
200,000	1.000

We can interpolate (using Factor1 + (Z ^ ½) x (Factor2 − Factor1), where Z = ($2,000,000 - $1,000,000)/($4,000,000-$1,000,000), and the square root interpolation is just a rough estimate.) to get a factor of 1.930 for a \$2 million occurrence limit. Now our cession rate should be:

0.93 / 1.93 = 48.2 %.

Whereas before our cession rate was:

\$21,000/\$45,000 = 46.7 %.

Our total expected losses should be adjusted by a factor of :

100 ^ (0.9 + 0.15) = 125.9 .

So the direct premium should be \$56,655 instead of \$45,000, and the ceded premium should be \$27,308 instead of \$21,000.

Estimating Parameters

Taking logarithms allows the equations for frequency and severity to be restated in a linear form:

$$Ln(S2) - Ln(S1) = A\,(Ln(E2) - Ln(E1))$$

$$Ln(F2) - Ln(F1) = B\,(Ln(E2) - Ln(E1))$$

Data may be collected for both claim severity and frequency by exposure size of policy. A and B can then be estimated as the slope estimates from regressions of the logarithm of claim severity and the logarithm of claim frequency, respectively, against the logarithm of exposure by policy. This also automatically generates the scaling factor for expected losses per unit of exposure for a policy as A + B, without any other special data analysis.

Conclusion

Both the standard property and liability methods of exposure rating loss layers correspond to special cases of a more general exposure rating method. The difference is whether higher exposure for a policy is assumed to reflect increased claim severity, as in the property case, or increased claim frequency, as in the liability case. The parameters of the general method encompass additional cases, which may more accurately fit actual loss exposure for different layers of losses. Estimation of the parameters is easily accomplished by regression of logarithms of historical data for claim severity, claim frequency, and exposure by policy. An additional benefit is that once these parameters are estimated they also reflect an estimate of the way in which expected losses per exposure change for policies of different exposure sizes. The parameters may be used in a fairly straightforward way to adjust ILF's, ELF's, loss scales, and rates per exposure for different policies by exposure size.

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