Risk Load and the Default Rate of Surplus

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Abstract

One of the biggest challenges facing the securitization of insurance risk is the translation of pricing techniques between the insurance and capital market worlds. At their heart the two worlds share similar purposes: assigning prices to uncertain future cash flow patterns. While the purposes are similar, historically the techniques and terminology have been somewhat disjoint. Widespread securitization of insurance results will require a manageable model to understand and price insurance risk in a capital market context. Ideally this model would also produce prices which were comparable to other securities available in the capital market. This paper introduces an insurance pricing model that translates aspects of corporate bond pricing – specifically default risk – to an insurance framework. It jointly addresses two favorite topics of casualty actuaries – allocated surplus and risk load, is well suited to DFA applications, and has been fully implemented in an Excel workbook that will be posted on the CAS Website.

Biography

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Risk Load and the Default Rate of Surplus

1. Introduction

One of the biggest challenges facing the securitization of insurance risk is the translation of pricing techniques between the insurance and capital market worlds. At their heart the two worlds share similar purposes: assigning prices to uncertain future cash flow patterns. While the purposes are similar, historically the techniques and terminology have been somewhat disjoint. Widespread securitization of insurance results will require a manageable model to understand and price insurance risk in a capital market context. Ideally this model would also produce prices which were comparable to other securities available in the capital market.

This paper introduces an insurance pricing model that translates aspects of corporate bond pricing – specifically default risk – to an insurance framework. It jointly addresses two favorite topics of casualty actuaries – allocated surplus and risk load, is well suited to DFA applications, and has been fully implemented in an Excel workbook that will be posted on the CAS Website.

The remainder of this paper is organized as follows. Section 2 discusses bond default risk and its applications to an insurance portfolio. Section 3 addresses the determination of the default loss rate on surplus. Section 4 explains the calculation of the required yield on surplus. Section 5 explains the interdependent pricing model and discusses examples of its application. Section 6 compares the new method with current risk load and surplus methods. Section 7 discusses conclusions and areas for further research.

2. The Pricing of Default Risk

Debt instruments such as corporate bonds are priced at yield rates that offer a spread above treasury (default-free) known as the yield premium. Yield premium is strongly related to the bond's rating as given by one of the bond rating agencies (e.g. Moody's, Standard & Poor's, Fitch, Duff & Phelps). The bond rating reflects many qualitative and quantitative factors but one of the most important is default risk.

Moody's defines default as "any missed or delayed disbursement of interest and/or principal" [11]. They distinguish between default rate (frequency of default per issuer) and recovery rate once default has occurred (severity of loss as a percent of par value). The product of these two is known as "default loss rate," the expected loss due to default expressed as a percentage of par value.

The yield premium should at least compensate a purchaser for the expected default loss rate. The remaining "excess" or "true" yield premium above the default loss rate is the actual reward for bearing the uncertainty of defaults. If the yield premium were not more than the expected loss rate, investors would be entering into at best a break-even proposition. If for example a class of high-yield bonds averaged a 4% annual loss rate, and the yield premium above default-free was 4%, then on average investors in this class of bonds would be earning the default-free rate while still facing the possibility of defaults.

The true yield premium depends in part on the default severity potential. At one extreme this class of bonds may have a 100% chance of defaulting 4% of par value. In this case the true yield premium would not be much at all, because there is virtual certainty as to the outcome. At the other extreme it may have a 4% chance of defaulting 100% of par, in which case the true yield premium would need to be substantially higher – but how much higher?

The answer to this question within the capital markets is investor-specific. It is based on how willing an investor is to expose different amounts of her wealth to possible default. The actual market prices are balances between the pricing opinions of parties with differing amounts of wealth, experience and information. Each individual investor determines his own "ideal" or "theoretically correct" price then decides where and by how much to compromise in the market.

How can these concepts be applied to insurance? It is not so much the market price that will inspire the new approach as the concept of pricing based on the exposure of wealth to the risk of default. Supporting surplus acts something like a "perpetual bond," one which never formally reaches maturity (the principal is never due back). The surplus provider expects a certain compensatory periodic yield as a reward for bearing the risk of supporting the insurance portfolio. From the surplus provider's standpoint this risk is the risk of default: if the insurance portfolio's results deteriorate the surplus funds will be called upon to fulfill claims.

3. The Default Loss Rate on Surplus

Consider a portfolio of single-year property policies[1] all incepting on the same date. The company commits an amount of surplus for the year in support of this portfolio. From the company perspective this "committed surplus" *CS* is assumed put in a fund *F*. Denote the fund value at time t as F_t; the fund's initial value is F_0 at time 0 (policy inception).

[1] For simplicity, a single year of a short-tail line is used. It eliminates the need to connect pricing risk loads with prior year reserve fluctuations. However, the model could be applied to any deviation in returns supported by surplus.

Also going into this fund is some portion of the premium P, consisting in its entirety of expected losses $E(L)$, expenses, and risk load R. Expenses are assumed known and paid for outside the fund, making the amount contributed to the fund from premium $E(L) + R$.

Assume the losses are completely paid at the end of the year (time t = T, the "terminal" time). The fund has a stochastic terminal amount F_T depending on the amount of losses paid. This fund has similar financial performance characteristics to a single-period, zero-coupon bond. With a zero coupon bond, principal is put up at time t = 0, and principal plus interest is returned at time t = T. The yield Y on a single-period, zero-coupon bond is

$$Y = F_T / \text{Principal} - 1 \tag{3.1}$$

In the insurance case, the principal amount is the committed surplus CS. The initial fund F_0 is invested in a default-free security earning a known rate r_{df}, so the terminal fund amount is the initial fund F_0 accumulated at the default-free interest rate less any loss payments. The loss payments are stochastic, so the expected yield $E(Y)$ on CS is

$$E(Y) = E(F_T) / CS - 1 \tag{3.2}$$

$$= R / CS \tag{3.3}$$

The terminal amount F_T for a given outcome i with loss L_i is

$$F_T \text{ for outcome } i = F_0 \text{ x } (1 + r_{df}) - L_i$$

$$= [CS + E(L) + R] \text{ x } (1 + r_{df}) - L_i \tag{3.4}$$

Over all possible outcomes[2], the expected value of F_T, $E(F_T)$, is

$$E(F_T) = [CS + E(L) + R] \text{ x } (1 + r_{df}) - \Sigma_i (p_i \text{ x } L_i), \tag{3.5}$$

where p_i = the probability for outcome i, and Σ_i = the sum over all outcomes. The "needed surplus" for outcome i, NS_i, is

$$NS_i = \text{Max } [L_i - E(L) - R , 0] \tag{3.6}$$

[2] This represents a discretization of the loss distribution. This approach was originally developed for use in property catastrophe management (the particular modeling software produced output files with discrete outcomes) but has broader applications.

NS_i is the amount of surplus needed to make up the shortfall (if any) between the premium contribution and the loss amount. The expected value of NS over all outcomes is

$$E(NS) = \Sigma_i \{ p_i \times \text{Max} [L_i - E(L) - R , 0] \} \quad (3.7)$$

An outcome i with loss amount greater than the premium contribution $E(L) + R$ would draw NS_i out of the available CS in order to make the loss payment. This represents a partial (or total) default of the CS. The expected loss rate from default on CS would then be

$$\text{Expected Loss Rate on } CS = E(NS) / CS \quad (3.8)$$

We will call the expected loss rate on committed surplus the "surplus loss rate" or SLR. Given a target SLR and $E(NS)$, the committed surplus CS can be determined using a rearrangement of equation 3.8:

$$CS = E(NS) / SLR \quad (3.9)$$

4. Required Yield on Committed Surplus

So far the model appears to be driven by expected loss rate on committed surplus. However, expected value on its own cannot effectively distinguish between different default severity levels. Solely using expected value, one could not distinguish between a 100% chance of a 1% default and a 1% chance of 100% default, or even a ½% chance of a 200% default.

Surplus tiers (see [8]) provide a framework for addressing severity. The *TYP* on a given incremental amount of surplus is assumed to be an increasing and non-linear function of the portfolio's committed surplus[3]. Consider the example surplus tiers presented in Table 1:

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[3] The TYP could be modeled as a continuous function of CS. The tiers are a discrete simplification to aid in the understanding and development of the concept. They also allow the introduction of the fixed TYP "capacity charge" which would be difficult to model in a continuous functional form.

Table 1 – Example of Surplus Tiers		
Tier #	**Surplus Range**	***TYP***
1	\$0 - \$1,000	25% of *SLR*
2	\$1,000 - \$2,000	50% of *SLR*
3	\$2,000 - \$3,000	100% of *SLR*

Assume \$2,000 of surplus supports this portfolio. The first \$1,000 of surplus is “Tier 1”, the next \$1,000 (“\$1,000 xs \$1,000”) is “Tier 2,” and the next \$1,000 (“\$1,000 xs \$2,000”) is “Tier 3.” If the *SLR*s are 5% for Tier 1 and 3% for Tier 2, the required *TYP*s would be

$$TYP \text{ for Tier 1} = 25\% * 5\% = 1.25\%$$

$$TYP \text{ for Tier 2} = 50\% * 3\% = 1.50\%$$

4.1. *<u>Losses Beyond the CS</u>*

Tier 3 includes losses beyond the *CS* amount of \$2,000 – default beyond the principal. Unlike the bond world, there is a chance of “default” beyond the “principal” in our insurance world. This makes plain one of the strongest criticisms of surplus allocation (e.g. [1]): a company’s exposure to losses from a given portfolio does not stop at the allocated surplus amount. All of a company’s surplus is available to pay any claim. This means our *CS* bond could have defaults that are multiples of its principal. How does the model handle this? And what does it mean?

The model handles it by extending the tiers beyond 100% of *CS*. Its meaning may not be immediately apparent but upon reflection a satisfying interpretation can be found. Default beyond a portfolio’s *CS* amount either means it is dipping into another portfolio’s *CS* or into the common pool of “free” surplus available to any portfolio in the company. To respond properly to this dipping into the common pool, the model needs to require ever-higher *TYP* as the tiers increase. Consider an extension of the previous example with two sub-portfolios of a company, each with \$2,000 of *CS*, and a free pool of \$1,000 to be shared between them, as shown in Table 2:

Table 2 – Tier Example with Two Competing Sub-Portfolios					
Tier #	**PF #1 Loss Rate**	**PF #2 Loss Rate**	***TYP* Requirement**	**PF #1 *TYP***	**PF #2 *TYP***
1	5.00%	3.00%	25% of *SLR*	1.25%	0.75%
2	2.00%	2.00%	50% of *SLR*	1.00%	1.00%
3	0.00%	2.00%	100% of *SLR*	0.00%	2.00%
TOTAL	7.00%	7.00%		2.25%	3.75%

Both Portfolio #1 and #2 have total *CS* loss rates of 7.00%, but PF #2 has a more skewed needed surplus distribution, requiring some of the common pool surplus. Because of the increasing cost of surplus exposure as the tier increases, PF #2 has a higher Total *TYP* of 3.75% vs 2.25% for PF #1.

In the interdependent pricing model presented in Section 5, the calculation of required yield for a given tier begins with the restatement of the *NS* distribution as a percentage of *CS*. Using reinsurance terminology, the lower bound of the tier is the “retention” and the width of the tier is the “limit.” As an example, the tier from 0% to 25% of *CS* would have a retention of 0% and a limit of 25%. The dollar loss to that tier for outcome *i*, $TierLoss_i\$$, would be

$$TierLoss_i\$ = CS \text{ x } \mathrm{Min}[limit, \mathrm{Max}(NS_i - retention, 0)] \qquad (4.1)$$

The expected value of *TierLoss$* over all outcomes, E(*TierLoss*), divided by the tier limit (in $) gives the *SLR* for the tier.

4.2. Calculation of TYP

The *TYP* can be calculated in a number of ways, including:

- As a percentage of the expected loss rate of the tier – a “variable rate”;
- As a fixed additive increment assessed whenever a tier is breached to any degree – a “capacity charge”;
- As a combination of fixed and variable.

The previous examples used only variable *TYP*s. The combination approach gives the most flexibility. Using this approach, *TYP* would be calculated as

$$TYP = Fixed + Variable\ \% \times \mathrm{E}(TierLoss) \tag{4.2}$$

The tier layers and *TYP* values used for demonstration purposes in the remainder of the paper are presented in Table 3:

Table 3 – Tier Layers and True Yield Premiums				
Tier #	**Lower Bound as % *CS***	**Upper Bound as % *CS***	**Variable *TYP* Rate (applied to Tier *SLR*)**	**Fixed *TYP***
1	0%	25%	0.10	0.1%
2	25%	50%	0.25	0.1%
3	50%	75%	0.50	0.1%
4	75%	100%	0.75	0.1%
5	100%	200%	1.00	0.1%
6	200%	400%	2.00	0.1%
7	400%	Maximum	4.00	0.1%

To demonstrate the calculations, assume for outcome *i* the needed surplus NS_i = 60% of *CS*. Table 4 shows the example calculations (using the tier definitions from Table 3):

Table 4 – *TYP*'s for Outcome *i*				
Tier #	**Tier *SLR* as % *CS***	**Variable *TYP***	**Fixed *TYP***	**Total *TYP***
1	25%	2.50%	0.10%	2.60%
2	25%	6.25%	0.10%	6.35%
3	10%	5.00%	0.10%	5.10%

This completes the basic translation of an insurance portfolio being supported by committed surplus to the bond default pricing framework. An interdependent pricing model is now introduced which puts these ideas into practice.

5. The Interdependent Pricing Model

The interdependent pricing model is presented on Exhibits 1-12. Detailed descriptions of the calculations can be found in Appendix A. The Microsoft Excel workbook **PRICEDEF.XLS** which produced Exhibits 1-12 will be posted to the download library of the CAS Website (www.casact.org).

To explore the performance of the model we will begin with a base case then introduce variations to the base assumptions.

5.1. Base Case (Exhibit 1)

For the given loss distribution, tier definitions and yield premiums, the base case solves for the risk load and committed surplus corresponding to:

- A maximum surplus loss rate *SLR* of 2.00%, selected for demonstration purposes. When the *SLR* is at or below this "safety standard" the portfolio is said to be in "loss rate balance."
- Expected yield = Required yield. When this condition holds the portfolio is said to be in **"**yield balance."

The important values are summarized in Table 5 below:

Table 5 – Base Case	
Item	Base Case Value
[1] Committed Surplus *CS*	$3,600
[2] Risk Load *R* as % of E(*L*)	6.15%
[9] *SLR*	2.00%
[11] Expected Yield on *CS*	7.23%
[13] Required Yield on *CS*	7.23%
[14] True Yield Premium *TYP*	0.23%

Note that [14] *TYP* = [11] Expected Yield on *CS* - [4] Default Free Rate - [9] *SLR*.

5.2. *Case 2: More Skewed Loss Distribution, Same CS and Risk Load % (Exhibit 2)*

How will the model respond when the loss distribution is more skewed but the *CS* and Risk Load *R* as a % of E(*L*) remain the same? Case 2 features a more skewed loss ratio distribution with a slightly larger expected value and much larger variance. The important values are summarized in Table 6 below:

Table 6 – More Skewed / Same *CS* and Risk Load %		
Item	Base Case Value	More Skewed Value
[1] Committed Surplus *CS*	$3,600	$3,600
[2] Risk Load *R* as % of E(*L*)	6.15%	6.15%
[9] *SLR*	2.00%	**2.58%**
[11] Expected Yield on *CS*	7.23%	**7.33%**
[13] Required Yield on *CS*	7.23%	**7.89%**
[14] True Yield Premium *TYP*	0.23%	**0.31%**

The *SLR* increases since the same amount of surplus is supporting a more skewed portfolio. The expected yield increases slightly (from 7.23% to 7.33% - due to the higher E(*L*) and hence risk load $), but the required yield increases substantially from 7.23% to 7.89%. The deeper penetration into the *CS* by the more skewed *NS* distribution produces a higher required *TYP* of 0.31%. This portfolio is now out of both yield and loss rate balance.

5.3. *Case 3: More Skewed Loss Distribution, Same CS, In Yield Balance (Exhibit 3)*

Case 3 determines the risk load required to restore yield balance while keeping *CS* fixed. The important values are summarized in Table 7 below:

Table 7 – More Skewed / Same *CS* / In Yield Balance		
Item	Base Case Value	More Skewed Value
[1] Committed Surplus *CS*	$3,600	$3,600
[2] Risk Load *R* as % of E(*L*)	6.15%	**8.18%**
[9] *SLR*	2.00%	**2.46%**
[11] Expected Yield on *CS*	7.23%	**7.76%**
[13] Required Yield on *CS*	7.23%	**7.76%**
[14] True Yield Premium *TYP*	0.23%	**0.30%**

A higher risk load (8.18% vs 6.15%) is required to restore yield balance given the more skewed *NS* distribution. The yield balance point (7.76%) is higher than in the base case, and the *TYP* is also higher (0.30% vs 0.23%).

5.4. *Case 4: More Skewed Loss Distribution, In Yield and Loss Rate Balance (Exhibit 4)*

Case 4 now adds additional surplus to Case 3 to completely restore the base case yield and loss rate balance conditions. The important values are summarized in Table 8 below:

Table 8 – More Skewed / In Yield and Loss Rate Balance		
Item	Base Case Value	More Skewed Value
[1] Committed Surplus *CS*	$3,600	**$4,445**
[2] Risk Load *R* as % of E(*L*)	6.15%	**8.10%**
[9] *SLR*	2.00%	2.00%
[11] Expected Yield on *CS*	7.23%	7.22%
[13] Required Yield on *CS*	7.23%	7.22%
[14] True Yield Premium *TYP*	0.23%	0.22%

It took $845 in additional *CS* to restore loss rate balance. The yield balance point and *TYP* are now essentially equivalent to the base case values.

5.5. *Case 5: Less Skewed Loss Distribution, Same CS and Risk Load % (Exhibit 5)*

Similar to Case 2, Case 5 examines how the model responds when the loss distribution is less skewed but the *CS* and Risk Load as a % of E(*L*) remain the same. The less skewed loss ratio distribution has a slightly smaller expected value and smaller variance. The important values are summarized in Table 9 below:

Table 9 – Less Skewed / Same *CS* and Risk Load %		
Item	Base Case Value	Less Skewed Value
[1] Committed Surplus *CS*	$3,600	$3,600
[2] Risk Load *R* as % of E(*L*)	6.15%	6.15%
[9] *SLR*	2.00%	**1.53%**
[11] Expected Yield on *CS*	7.23%	**7.14%**
[13] Required Yield on *CS*	7.23%	**6.71%**
[14] True Yield Premium *TYP*	0.23%	**0.18%**

The *SLR* is much lower (1.53% vs 2.00%) because the *CS* is not being used as much by the less skewed loss distribution. The expected yield is actually slightly lower, reflecting the fact that the Risk Load $ is lower because E(*L*) is lower. This slight decrease is more than offset by the large decrease in required yield (6.71% vs 7.23%) – again reflecting the decreased use of the surplus. The *TYP* of 0.18% is also lower than Base Case.

5.6. Case 6: Less Skewed Loss Distribution, In Yield and Loss Rate Balance (Exhibit 6)

Exhibit 6 shows the new *CS* and Risk Load % needed to restore both yield and loss rate balance with the less skewed loss ratio distribution. The important values are summarized in Table 10 below:

Table 10 – Less Skewed / In Yield and Loss Rate Balance		
Item	Base Case Value	Less Skewed Value
[1] Committed Surplus *CS*	$3,600	**$2,955**
[2] Risk Load *R* as % of E(*L*)	6.15%	**4.54%**
[9] *SLR*	2.00%	2.00%
[11] Expected Yield on *CS*	7.23%	7.22%
[13] Required Yield on *CS*	7.23%	7.22%
[14] True Yield Premium *TYP*	0.23%	0.22%

$605 less in *CS* and a lower Risk Load % (4.54% vs 6.15%) restores both yield and loss rate balance. The *TYP* and yield balance point are nearly equal to the Base Case values.

5.7. *Case 7: New Risk Added Resulting in More Skewed Loss Distribution, No Additional CS (Exhibit 7)*

One of the critiques of marginal-surplus-based techniques (discussed in Section 6 below) is their inability to handle situations where total surplus is fixed. Exhibit 7 shows how the model handles this situation. A new risk is added resulting in a more skewed loss ratio distribution and larger subject premium[4]. The important values are summarized in Table 11 below:

Table 11 – New Risk / More Skewed / Same *CS* / In Yield Balance

Item	Base Case	With New Risk	New Risk
[1] Committed Surplus *CS*	$3,600	**$3,600**	**$0**
[2] Risk Load *R* as % of E(*L*)	6.15%	**8.18%**	**16.17%**
[9] *SLR*	2.00%	**2.95%**	-
[11] Expected Yield on *CS*	7.23%	**8.31%**	-
[13] Required Yield on *CS*	7.23%	**8.31%**	-
[14] True Yield Premium *TYP*	0.23%	**0.36%**	-

The addition of the new risk without any additional surplus results in a higher overall risk load % (8.18% vs 6.15%), higher *SLR* (2.95% vs 2.00%), a higher yield balance point (8.31% vs 7.23%), and higher *TYP* (0.36% vs 0.23%). The model is now out of loss rate balance. This is the impact of writing a new risk on a portfolio with fixed total *CS*. The new risk has no marginal surplus associated with its addition, yet the method still comes up with a risk load of 16.17% of its expected loss, much higher than the rest of the portfolio.

5.8. *Case 8: New Risk Added Resulting in More Skewed Loss Distribution, In Loss Rate and Yield Balance (Exhibit 8)*

Exhibit 8 shows the addition of the same new risk as in Exhibit 7 but this time allowing additional *CS* to restore Loss Rate balance. The important values are summarized in Table 12 below:

[4] Subject premium is used as a scaling mechanism to convert loss ratios to loss dollars. It is not meant to include risk load, which would introduce recursion.

Table 12 – New Risk / More Skewed / In Loss Rate and Yield Balance			
Item	Base Case Value	With New Risk	New Risk
[1] Committed Surplus *CS*	$3,600	**$5,325**	**$1,725**
[2] Risk Load *R* as % of E(*L*)	6.15%	**8.10%**	**15.78%**
[9] *SLR*	2.00%	2.00%	**-**
[11] Expected Yield on *CS*	7.23%	7.23%	**-**
[13] Required Yield on *CS*	7.23%	7.23%	**-**
[14] True Yield Premium *TYP*	0.23%	0.23%	**-**

Writing the new risk while maintaining loss rate balance requires $1,725 in additional surplus. The resulting risk load for the new risk (15.78%) is slightly lower than in Case 7 with no additional *CS*.

5.9. <u>Case 9: New Risk Added Resulting in Less Skewed Loss Distribution, In Loss Rate and Yield Balance (Exhibit 9)</u>

Exhibit 9 shows the addition of a new risk resulting in a less skewed overall loss distribution, keeping the same *CS* and both Yield and Loss Rate balance. The important values are summarized in Table 13 below:

Table 13 – New Risk / Less Skewed / Same *CS* / In Loss Rate and Yield Balance			
Item	Base Case Value	With New Risk	New Risk
[1] Committed Surplus *CS*	$3,600	**$3,600**	**-**
[2] Risk Load *R* as % of E(*L*)	6.15%	**4.54%**	**-5.96%**
[9] *SLR*	2.00%	1.97%	**-**
[11] Expected Yield on *CS*	7.23%	7.19%	**-**
[13] Required Yield on *CS*	7.23%	7.19%	**-**
[14] True Yield Premium *TYP*	0.23%	0.22%	**-**

The "less skewed" loss distribution is the same one used for Cases 5 and 6. Because of the reduced skewness, the new risk can actually be written at a negative risk load. Stone [12] mentioned the same possibility when discussing *d*, the pricing differential based on capacity considerations – his equivalent to risk load:

> It is highly probable that the value of *d* will be negative for those risks which add substantial capacity to the portfolio, while *d* is likely to be highly positive for most of the familiar Capacity Risks. [12, p.241]

Stone considered those risks that decreased the portfolio ratio of standard deviation to mean (the coefficient of variation or "Exposure Ratio" to Stone) to be "capacity creators." The new risk in this example would certainly qualify as a capacity creator.

5.10. Case 10: Base Case With CS = Maximum Needed Surplus (Exhibit 10)

Case 10 considers what might be termed the "ruin theory" *CS*, set equal to the maximum needed surplus. The important values are summarized in Table 14 below:

Table 14 – Base Case with *CS* = Maximum Needed Surplus

Item	Base Case Value	*CS*=Max(*NS*)
[1] Committed Surplus *CS*	$3,600	**$530**
[2] Risk Load *R* as % of E(*L*)	6.15%	**7.13%**
[9] *SLR*	2.00%	**13.12%**
[11] Expected Yield on *CS*	7.23%	**21.49%**
[13] Required Yield on *CS*	7.23%	**21.49%**
[14] True Yield Premium *TYP*	0.23%	**3.37%**

With the lower surplus amount ($530 vs $3,600) the *SLR* is substantially worse (13.12% vs 2.00%). Using capital market loss rate standards from Moody's [11, p.22], it is doubtful this supporting surplus would even receive a CCC rating which is usually reserved for bonds on the border between speculative grade and outright default. In order to restore yield balance slightly more risk load is needed than in the Base Case (7.13% vs 6.15%), but the yield balance point is quite high at 21.49%. The *TYP* is almost 15 times as high as in the base case. These figures fall far outside those typically seen in the capital market debt community.

5.11. Case 11: Base Case With Twice the CS (Exhibit 11)

One may legitimately ask what is stopping the allocation of too much or too little surplus? Case 11 addresses the issue of too much surplus by doubling the base case *CS*. The important values are summarized in Table 15 below:

Table 15 – Base Case with Twice the *CS*		
Item	Base Case Value	Twice the *CS*
[1] Committed Surplus *CS*	$3,600	**$7,200**
[2] Risk Load *R* as % of E(*L*)	6.15%	6.17%
[9] *SLR*	2.00%	**1.00%**
[11] Expected Yield on *CS*	7.23%	**6.12%**
[13] Required Yield on *CS*	7.23%	**6.12%**
[14] True Yield Premium *TYP*	0.23%	**0.12%**

Several encouraging responses to doubled *CS* are noted:

- The *SLR* is halved to **1.00%**;
- The Expected Yield is reduced to **6.12%**;
- The Required Yield is similarly reduced to **6.12%**; and
- The *TYP* is halved to **0.12%**.

The Risk Load % increases slightly to 6.17%. Intuitively it seems it should be equivalent to the Base Case value. This would be true if there were no fixed *TYP* component. In both cases item [33] Expected Loss $ to Tier is the same – $72.00. However, because of [36] Yield Premium - Fixed, [37] Total Yield % on Tier for Case 10 is 4.50%, which is higher than half the 8.90% value for the Base Case. If the Fixed Yield Premium were set to 0, the Case 11 Risk Load % would equal that of the Base Case.

5.12. <u>Case 12: Base Case With Half the CS (Exhibit 12)</u>

Case 12 addresses the issue of too little surplus by halving the base case *CS*. The important values are summarized in Table 16 below:

Table 16 – Base Case with Half the *CS*		
Item	Base Case Value	Half the *CS*
[1] Committed Surplus *CS*	$3,600	**$1,800**
[2] Risk Load *R* as % of E(*L*)	6.15%	6.15%
[9] *SLR*	2.00%	**4.00%**
[11] Expected Yield on *CS*	7.23%	**9.46%**
[13] Required Yield on *CS*	7.23%	**9.46%**
[14] True Yield Premium *TYP*	0.23%	**0.46%**

Similar to Case 11, the model responds appropriately to half the *CS*:

- The Risk Load % remains the same at 6.15%;
- The *SLR* is doubled to **4.00%**;
- The Expected Yield is increased to **9.46%**;
- The Required Yield is also increased to **9.46%**; and
- The *TYP* is doubled to **0.46%**.

5.13. Summary of Performance Characteristics

Table 17 summarizes the response of the interdependent pricing model to the various changes in Cases 2-12.

Table 17 – Summary of Model Responses

	Change from Base Case			***Model Response***			
Case	**Loss Distrib.**	***CS***	**Risk Load %**	***SLR***	**Expected Yield**	**Required Yield**	***TYP***
2	More Skewed	Same	Same	Higher	Higher	Higher	Higher
3	More Skewed	Same	Higher	Higher	Higher	Higher	Higher
4	More Skewed	Higher	Higher	Same	Same	Same	Same
5	Less Skewed	Same	Same	Lower	Lower	Lower	Lower
6	Less Skewed	Lower	Lower	Same	Same	Same	Same
7	Larger Volume, More Skewed	Same	Higher	Higher	Higher	Higher	Higher
8	Larger Volume, More Skewed	Higher	Higher	Same	Same	Same	Same
9	Larger Volume, Less Skewed	Same	Lower	Same	Same	Same	Same
10	Same	Much Lower	Higher	Much Higher	Much Higher	Much Higher	Much Higher
11	Same	Doubled	Same	Halved	Lower	Lower	Halved
12	Same	Halved	Same	Doubled	Higher	Higher	Doubled

6. Comparison with Other Risk Load Approaches

This section discusses how the new approach relates to the use of variance, ruin thresholds, additional surplus and CAPM in the determination of risk loads.

6.1. Variance

Variance gained prominence as a financial risk measure from the work of Markowitz [9], who equated more variance in the return distribution with more riskiness. However, Markowitz only advocated variance as a relative risk measure for portfolio optimization, not an absolute measure for individual security pricing. In fact, his first choice for a risk measure was semi-variance (only considering downside risk), but he settled on variance due to greater mathematical tractability[5]. This compromise worked well for portfolio optimization, but in cases where the return distribution is skewed, the distinction between upside and downside risk is critical for proper pricing. Variance ignores this distinction.

Using variance for price determination – having risk load be directly proportional to variance – involves an implicit transformation of the return distribution. Individual risk pricing involves the conversion of a given distribution of returns into a value – the "price." This conversion can be thought of as a transform of any deviation from the mean $y = x - \mu$ into a value for that deviation $v(y)$. Variance = $E[(x - \mu)(x - \mu)] = E(y^2)$ = the expected value of the transformed deviation y where the *implicit* transform $v(y) = y^2$. Variance as a risk measure effectively prices deviations from the mean at the square of their value. This implicit transform may not be appropriate. Companies may want to explicitly price the costs of different degrees of exposure to their capital without having to rely on variance's implicit squaring transform.

The interdependent pricing model offers just such an alternative: using the needed surplus distribution and surplus tier framework to *explicitly* transform the distribution of results. This has several advantages:

- It makes the implicit transform explicit and gives companies a framework for quantifying their risk opinions;
- It grounds the pricing process in familiar capital market terms: default loss rate and true yield premium;
- It only deals with downside risk; and
- It is well suited to DFA applications.

[5] This point is made beautifully in Clarkson [3].

6.2 Ruin Thresholds

Ruin theory focuses on the theoretical ruin threshold of the insurance company – the point where surplus hits zero – and the change in the ruin threshold from the addition of a new policy. However, sole focus on the change in a selected percentile of the return distribution (say the 99th) ignores what Philbrick calls "gradations of solvency" which "are not easily handled in ruin theory" [12, p.60]. By reflecting the full impact of the return distribution on surplus the new approach may be considered an application of "impairment theory." Using the entire return distribution instead of a single percentile allows information about the "gradations of solvency" to be reflected in the pricing process.

6.3 Additional Surplus

Meyers [11], Kreps [7] and Bingham [2] all assume the addition of a risk to a portfolio requires additional surplus. What would these methods do if total surplus were considered fixed? They would either produce an undefined risk load (since marginal surplus = 0) or force the user to model the situation as if additional surplus were added even though that may not be an option – for example in areas of high property catastrophe exposure concentration. These methods seem at odds with the old underwriting adage, "We will write any risk for the right price" by apparently requiring "and allocated surplus amount" be appended at the end.

The new approach can handle both fixed and changing total surplus situations equally well – see Cases 7 and 8.

6.4 CAPM

Feldblum [5] suggested property casualty insurance pricing be more closely tied to capital market pricing practices. Feldblum shows how to employ the principles of CAPM to the pricing of different lines of business by estimating underwriting "betas" that reflect the correlation between each line's returns and the return of a "market" insurance portfolio. However, the empirical adequacy of CAPM as an explanatory security pricing model has been called into question (e.g. Fama and French [4]), as has its viability as an insurance pricing practice (Kozik [6]).

Despite these concerns, Feldblum's effort to integrate financial market practices into insurance should not only be applauded but extended. The new approach does just that, adapting capital market debt practices to insurance. Default loss rate is an acknowledged and accepted proxy from the bond community for safety of investment. Ruin theory may be considered the actuarial equivalent of Value at Risk (VAR). Surplus loss rate is the actuarial equivalent of bond default loss rate.

7. Conclusions and Further Research

It is hoped the reader has found this new model informative and interesting. It provides an integrated solution to the risk load and surplus allocation problem, performs well in circumstances where other models falter, and further unifies actuarial pricing methods with those in the capital markets.

Clearly there are areas for further research associated with the ideas in this paper, including:

- Viable functional forms to represent *TYP* as a continuous function of *CS*;
- Extension of the model to surplus supporting reserves;
- Comparison with variance and ruin theory based methods;
- Actual or implicit *TYP* distributions derivable from capital market pricing information;
- Time – Bond default rates are time-dependent. The longer a bond's term, the more the chance of it defaulting.

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[12] Philbrick, Stephen W., Discussion of “Risk Loads for Insurers,” *PCAS* LXXVIII, 1991, p. 56.

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Appendix A

Description of the Interdependent Pricing Model

Each worksheet in the Excel workbook represents a complete, standalone version of the interdependent pricing model. The model begins with the following inputs:

Loss Ratio distribution[6] (Items [16] Probability and [18] Loss Ratio on Exhibit 1)

[24] Tier Limits and [25] Tier Retentions, both expressed as % of *CS*

[35] Yield Premium – Variable

[36] Yield Premium – Fixed

[1] Committed Surplus *CS*

[2] Risk Load as % of Expected Loss E(*L*) = *R%*

[3] Subject Premium[7]

[4] Default-free Interest Rate

The following values are then calculated:

[5] Risk Load $ = *R$* = [2] * [6] Expected Loss E(*L*)

[6] Expected Loss E(*L*) =

SumProduct([16] Probability , [18] Loss L_i)

[7] Initial Fund F_0 = [1] *CS* + [5] *R$* + [6] E(*L*)

[20] Fund Contribution from Premium = [5] + [6]

[21] Needed Surplus distribution NS_i

= Max(0, [18] L_i – [20] Fund Contribution from Premium).

[8] E(NS_i) = SumProduct([16] Probability , [21] NS_i)

[9] Surplus Loss Rate *SLR* = [8] E(NS_i) / [1] *CS*

[22] Terminal Fund Value F_T

= [6] F_0 x (1.00 + [4] Default-free Interest Rate) - [19] Loss L_i

[10] Expected Value of F_T = E(F_T)

= SumProduct([16] Probability , [22] F_T)

[11] Expected Yield on *CS* = [10] E(F_T) / [1] *CS* – 1.00

---

[6] A loss dollar distribution could also be used – see the following footnote.

[7] This is purely used for creation of the loss dollar distribution to facilitate skewing or scale changes due to the introduction of a new risk. It is not meant to be collected premium including the risk load being calculated, which would introduce recursion.

These calculations follow the formulas in Section 3. The most important results are [9] Surplus Loss Rate *SLR* and [11] Expected Yield on *CS*. Now the model uses the surplus tiers to derive [13] Required Yield on *CS* and [14] True Yield Premium:

[26] – [32] NS_i for each tier

= Max(0, Min([21] – [1] x [25], [1] x [24])

= the smaller of the loss to the layer and the limit of the layer

[33] Expected Loss $ to Tier

= SumProduct([16] Probability, NS_i for each tier)

[34] Expected Loss % to Tier = [33] / [1] *CS* / [24] Tier Limit

[37] Total Yield % on Tier =

[36] Yield Premium – Fixed +

[34] Expected Loss % to Tier x [35] Yield Premium – Variable

[38] Total Yield $ on Tier = [37] x [1] *CS* x [24] Limit

[12] Total Yield $ Over All Tiers = Sum([38]) over all tiers

[13] Required Yield on *CS* = [12] / [1] *CS*

[14] True Yield Premium = [13] - [9] *SLR* – [4] Default-free Interest Rate

Exhibit 1
BASE CASE

Inputs:			
[1]	Consumed Surplus	**3,600.00**	*Input*
[2]	Risk Load as % of E[L]	**6.15%**	*Input*
[3]	Subject Premium	**1,000.00**	*Input*
[4]	Default-Free Rate	**5.00%**	*Input*

Calculated Values:

[5]	Risk Load $	43.05	*=[2] * [6]*	[10]	Expected Value of F(T)	3,860.20	*=E{ [22] }*
[6]	Expected Loss	700.00	*=E{ [19] }*	[11]	Expected Yield on CS	7.23%	*=[10] / [1] - 1.00*
[7]	Initial Fund F(0)	4,343.05	*=[1] +[5] +[6]*	[12]	Total Yield $ Over All Tiers	80.10	*=Sum{ [38] }*
[8]	Expected Value of NS(i)	72.00	*=E{ [21] }*	[13]	Required Yield on CS	7.23%	*=[12] / [1] + [4]*
[9]	Surplus Loss Rate	2.00%	*=[8] / [1]*	[14]	True Yield Premium	0.23%	*=[13] - [9] - [4]*

[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]
Outcome # i	Prob f(i) *Input*	Cum Prob F(i) *=Cum Sum of [16]*	Loss Ratio *Input*	Loss L(i) *=[3] * [18]*	Fund Contribution from Prem *=[5] + [6]*	NS(i) *=Max(0, [19] - [20])*	F(T) *=[7] * (1.00 + [4]) - [19]*	NS(i) as % CS *=[21] / [1]*
1	**2.00%**	2.0%	**35.00%**	350.00	743.05	-	4,210.20	0.00%
2	**4.00%**	6.0%	**40.00%**	400.00	743.05	-	4,160.20	0.00%
3	**6.00%**	12.0%	**45.00%**	450.00	743.05	-	4,110.20	0.00%
4	**10.00%**	22.0%	**50.00%**	500.00	743.05	-	4,060.20	0.00%
5	**11.00%**	33.0%	**55.00%**	550.00	743.05	-	4,010.20	0.00%
6	**12.00%**	45.0%	**60.00%**	600.00	743.05	-	3,960.20	0.00%
7	**10.00%**	55.0%	**65.00%**	650.00	743.05	-	3,910.20	0.00%
8	**9.00%**	64.0%	**70.00%**	700.00	743.05	-	3,860.20	0.00%
9	**6.00%**	70.0%	**75.00%**	750.00	743.05	6.95	3,810.20	0.19%
10	**5.00%**	75.0%	**80.00%**	800.00	743.05	56.95	3,760.20	1.58%
11	**4.00%**	79.0%	**85.00%**	850.00	743.05	106.95	3,710.20	2.97%
12	**4.00%**	83.0%	**90.00%**	900.00	743.05	156.95	3,660.20	4.36%
13	**3.00%**	86.0%	**95.00%**	950.00	743.05	206.95	3,610.20	5.75%
14	**3.00%**	89.0%	**100.00%**	1,000.00	743.05	256.95	3,560.20	7.14%
15	**3.00%**	92.0%	**105.00%**	1,050.00	743.05	306.95	3,510.20	8.53%
16	**2.00%**	94.0%	**110.00%**	1,100.00	743.05	356.95	3,460.20	9.92%
17	**2.00%**	96.0%	**115.00%**	1,150.00	743.05	406.95	3,410.20	11.30%
18	**2.00%**	98.0%	**120.00%**	1,200.00	743.05	456.95	3,360.20	12.69%
19	**1.00%**	99.0%	**125.00%**	1,250.00	743.05	506.95	3,310.20	14.08%
20	**1.00%**	100.0%	**130.00%**	1,300.00	743.05	556.95	3,260.20	15.47%

Exhibit 1
BASE CASE

		Tier Definitions as % of CS							
	Tier #	1	2	3	4	5	6	7	
[24]	Limit	25.0%	25.0%	25.0%	25.0%	100.0%	200.0%	99999.0%	*Input*
[25]	Retention	0.0%	25.0%	50.0%	75.0%	100.0%	200.0%	400.0%	*Input*

[15]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	
Outcome # i	Tier 1 NS(i)	Tier 2 NS(i)	Tier 3 NS(i)	Tier 4 NS(i)	Tier 5 NS(i)	Tier 6 NS(i)	Tier 7 NS(i)	*Tier NS(i) = MAX(0, MIN([21] - [25] * [1], [24] * [1]))*
1	-	-	-	-	-	-	-	
2	-	-	-	-	-	-	-	
3	-	-	-	-	-	-	-	
4	-	-	-	-	-	-	-	
5	-	-	-	-	-	-	-	
6	-	-	-	-	-	-	-	
7	-	-	-	-	-	-	-	
8	-	-	-	-	-	-	-	
9	6.95	-	-	-	-	-	-	
10	56.95	-	-	-	-	-	-	
11	106.95	-	-	-	-	-	-	
12	156.95	-	-	-	-	-	-	
13	206.95	-	-	-	-	-	-	
14	256.95	-	-	-	-	-	-	
15	306.95	-	-	-	-	-	-	
16	356.95	-	-	-	-	-	-	
17	406.95	-	-	-	-	-	-	
18	456.95	-	-	-	-	-	-	
19	506.95	-	-	-	-	-	-	
20	556.95	-	-	-	-	-	-	

[33]	Expected Loss $ to Tier	72.00	-	-	-	-	-	-	*=E{ Tier NS(i) }*
[34]	Expected Loss % to Tier	8.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[33] / [1] / [24]*
[35]	Yield Prem - Variable	0.10	0.25	0.50	0.75	1.00	2.00	4.00	*Input*
[36]	Yield Prem - Fixed	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	*Input*
[37]	Total Yield % on Tier	8.90%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[34] *(1+[35]) + [36]*
[38]	Total Yield $ on Tier	80.10	-	-	-	-	-	-	*=[37] * [1] * [24]*

Exhibit 2

CASE 2 -- More Skewed Loss Distribution, Same CS and Risk Load %

Inputs:			
[1]	Consumed Surplus	**3,600.00**	*Input*
[2]	Risk Load as % of E[L]	**6.15%**	*Input*
[3]	Subject Premium	**1,000.00**	*Input*
[4]	Default-Free Rate	**5.00%**	*Input*

Calculated Values:							
[5]	Risk Load $	44.99	*=[2] * [6]*	**[10]**	Expected Value of F(T)	3,863.81	*=E{ [22] }*
[6]	Expected Loss	731.50	*=E{ [19] }*	**[11]**	**Expected Yield on CS**	**7.33%**	***=[10] / [1] - 1.00***
[7]	Initial Fund F(0)	4,376.49	*=[1] +[5] +[6]*	**[12]**	Total Yield $ Over All Tiers	104.19	*=Sum{ [38] }*
[8]	Expected Value of NS(i)	93.05	*=E{ [21] }*	**[13]**	**Required Yield on CS**	**7.89%**	***=[12] / [1] + [4]***
[9]	**Surplus Loss Rate**	**2.58%**	***=[8] / [1]***	**[14]**	**True Yield Premium**	**0.31%**	***=[13] - [9] - [4]***

[15]	**[16]**	**[17]**	**[18]**	**[19]**	**[20]**	**[21]**	**[22]**	**[23]**
Outcome # i	**Prob f(i)** *Input*	**Cum Prob F(i)** *=Cum Sum of [16]*	**Loss Ratio** *Input*	**Loss L(i)** *=[3] * [18]*	**Fund Contribution from Prem** *=[5] + [6]*	**NS(i)** *=Max(0, [19] - [20])*	**F(T)** *=[7] * (1.00 + [4]) - [19]*	**NS(i) as % CS** *=[21] / [1]*
1	**2.00%**	2.0%	**35.00%**	350.00	776.49	-	4,245.31	0.00%
2	**4.00%**	6.0%	**40.00%**	400.00	776.49	-	4,195.31	0.00%
3	**6.00%**	12.0%	**45.00%**	450.00	776.49	-	4,145.31	0.00%
4	**10.00%**	22.0%	**50.00%**	500.00	776.49	-	4,095.31	0.00%
5	**11.00%**	33.0%	**55.00%**	550.00	776.49	-	4,045.31	0.00%
6	**12.00%**	45.0%	**60.00%**	600.00	776.49	-	3,995.31	0.00%
7	**10.00%**	55.0%	**65.00%**	650.00	776.49	-	3,945.31	0.00%
8	**9.00%**	64.0%	**70.00%**	700.00	776.49	-	3,895.31	0.00%
9	**6.00%**	70.0%	**75.00%**	750.00	776.49	-	3,845.31	0.00%
10	**5.00%**	75.0%	**80.00%**	800.00	776.49	23.51	3,795.31	0.65%
11	**4.00%**	79.0%	**85.00%**	850.00	776.49	73.51	3,745.31	2.04%
12	**4.00%**	83.0%	**90.00%**	900.00	776.49	123.51	3,695.31	3.43%
13	**3.00%**	86.0%	**100.00%**	1,000.00	776.49	223.51	3,595.31	6.21%
14	**3.00%**	89.0%	**110.00%**	1,100.00	776.49	323.51	3,495.31	8.99%
15	**3.00%**	92.0%	**120.00%**	1,200.00	776.49	423.51	3,395.31	11.76%
16	**2.00%**	94.0%	**130.00%**	1,300.00	776.49	523.51	3,295.31	14.54%
17	**2.00%**	96.0%	**140.00%**	1,400.00	776.49	623.51	3,195.31	17.32%
18	**2.00%**	98.0%	**150.00%**	1,500.00	776.49	723.51	3,095.31	20.10%
19	**1.00%**	99.0%	**160.00%**	1,600.00	776.49	823.51	2,995.31	22.88%
20	**1.00%**	100.0%	**170.00%**	1,700.00	776.49	923.51	2,895.31	25.65%

Exhibit 2

CASE 2 -- More Skewed Loss Distribution, Same CS and Risk Load %

		Tier Definitions as % of CS							
	Tier #	1	2	3	4	5	6	7	
[24]	Limit	25.0%	25.0%	25.0%	25.0%	100.0%	200.0%	99999.0%	*Input*
[25]	Retention	0.0%	25.0%	50.0%	75.0%	100.0%	200.0%	400.0%	*Input*

[15]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	
Outcome # i	Tier 1 NS(i)	Tier 2 NS(i)	Tier 3 NS(i)	Tier 4 NS(i)	Tier 5 NS(i)	Tier 6 NS(i)	Tier 7 NS(i)	*Tier NS(i) = MAX(0, MIN([21] - [25] * [1], [24] * [1]))*
1	-	-	-	-	-	-	-	
2	-	-	-	-	-	-	-	
3	-	-	-	-	-	-	-	
4	-	-	-	-	-	-	-	
5	-	-	-	-	-	-	-	
6	-	-	-	-	-	-	-	
7	-	-	-	-	-	-	-	
8	-	-	-	-	-	-	-	
9	-	-	-	-	-	-	-	
10	23.51	-	-	-	-	-	-	
11	73.51	-	-	-	-	-	-	
12	123.51	-	-	-	-	-	-	
13	223.51	-	-	-	-	-	-	
14	323.51	-	-	-	-	-	-	
15	423.51	-	-	-	-	-	-	
16	523.51	-	-	-	-	-	-	
17	623.51	-	-	-	-	-	-	
18	723.51	-	-	-	-	-	-	
19	823.51	-	-	-	-	-	-	
20	900.00	23.51	-	-	-	-	-	

[33]	Expected Loss $ to Tier	92.82	0.24	-	-	-	-	-	*=E{ Tier NS(i) }*
[34]	Expected Loss % to Tier	10.31%	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[33] / [1] / [24]*
[35]	Yield Prem - Variable	**0.10**	**0.25**	**0.50**	**0.75**	**1.00**	**2.00**	**4.00**	*Input*
[36]	Yield Prem - Fixed	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	*Input*
[37]	Total Yield % on Tier	11.44%	0.13%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[34] *(1+[35]) + [36]*
[38]	Total Yield $ on Tier	103.00	1.19	-	-	-	-	-	*=[37] * [1] * [24]*

Exhibit 3

CASE 3 -- More Skewed Loss Distribution, Same CS, In Yield Balance

Inputs:			
[1]	Consumed Surplus	3,600.00	*Input*
[2]	**Risk Load as % of E[L]**	8.18%	*Input*
[3]	Subject Premium	1,000.00	*Input*
[4]	Default-Free Rate	5.00%	*Input*

Calculated Values:

[5]	Risk Load $	59.84	*=[2] * [6]*	[10]	Expected Value of F(T)	3,879.40	*=E{ [22] }*
[6]	Expected Loss	731.50	*=E{ [19] }*	[11]	**Expected Yield on CS**	**7.76%**	***=[10] / [1] - 1.00***
[7]	Initial Fund F(0)	4,391.34	*=[1] +[5] +[6]*	[12]	Total Yield $ Over All Tiers	99.27	*=Sum{ [38] }*
[8]	Expected Value of NS(i)	88.60	*=E{ [21] }*	[13]	**Required Yield on CS**	**7.76%**	***=[12] / [1] + [4]***
[9]	**Surplus Loss Rate**	**2.46%**	***=[8] / [1]***	[14]	**True Yield Premium**	**0.30%**	***=[13] - [9] - [4]***

[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]
Outcome # i	Prob f(i) *Input*	Cum Prob F(i) *=Cum Sum of [16]*	Loss Ratio *Input*	Loss L(i) *=[3] * [18]*	Fund Contribution from Prem *=[5] + [6]*	NS(i) *=Max(0, [19] - [20])*	F(T) *=[7] * (1.00 + [4]) - [19]*	NS(i) as % CS *=[21] / [1]*
1	**2.00%**	2.0%	**35.00%**	350.00	791.34	-	4,260.90	0.00%
2	**4.00%**	6.0%	**40.00%**	400.00	791.34	-	4,210.90	0.00%
3	**6.00%**	12.0%	**45.00%**	450.00	791.34	-	4,160.90	0.00%
4	**10.00%**	22.0%	**50.00%**	500.00	791.34	-	4,110.90	0.00%
5	**11.00%**	33.0%	**55.00%**	550.00	791.34	-	4,060.90	0.00%
6	**12.00%**	45.0%	**60.00%**	600.00	791.34	-	4,010.90	0.00%
7	**10.00%**	55.0%	**65.00%**	650.00	791.34	-	3,960.90	0.00%
8	**9.00%**	64.0%	**70.00%**	700.00	791.34	-	3,910.90	0.00%
9	**6.00%**	70.0%	**75.00%**	750.00	791.34	-	3,860.90	0.00%
10	**5.00%**	75.0%	**80.00%**	800.00	791.34	8.66	3,810.90	0.24%
11	**4.00%**	79.0%	**85.00%**	850.00	791.34	58.66	3,760.90	1.63%
12	**4.00%**	83.0%	**90.00%**	900.00	791.34	108.66	3,710.90	3.02%
13	**3.00%**	86.0%	**100.00%**	1,000.00	791.34	208.66	3,610.90	5.80%
14	**3.00%**	89.0%	**110.00%**	1,100.00	791.34	308.66	3,510.90	8.57%
15	**3.00%**	92.0%	**120.00%**	1,200.00	791.34	408.66	3,410.90	11.35%
16	**2.00%**	94.0%	**130.00%**	1,300.00	791.34	508.66	3,310.90	14.13%
17	**2.00%**	96.0%	**140.00%**	1,400.00	791.34	608.66	3,210.90	16.91%
18	**2.00%**	98.0%	**150.00%**	1,500.00	791.34	708.66	3,110.90	19.69%
19	**1.00%**	99.0%	**160.00%**	1,600.00	791.34	808.66	3,010.90	22.46%
20	**1.00%**	100.0%	**170.00%**	1,700.00	791.34	908.66	2,910.90	25.24%

Exhibit 3
CASE 3 -- More Skewed Loss Distribution, Same CS, In Yield Balance

		Tier Definitions as % of CS							
	Tier #	1	2	3	4	5	6	7	
[24]	Limit	25.0%	25.0%	25.0%	25.0%	100.0%	200.0%	99999.0%	*Input*
[25]	Retention	0.0%	25.0%	50.0%	75.0%	100.0%	200.0%	400.0%	*Input*

[15]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	
Outcome # i	Tier 1 NS(i)	Tier 2 NS(i)	Tier 3 NS(i)	Tier 4 NS(i)	Tier 5 NS(i)	Tier 6 NS(i)	Tier 7 NS(i)	*Tier NS(i) = MAX(0, MIN([21] - [25] * [1], [24] * [1]))*
1	-	-	-	-	-	-	-	
2	-	-	-	-	-	-	-	
3	-	-	-	-	-	-	-	
4	-	-	-	-	-	-	-	
5	-	-	-	-	-	-	-	
6	-	-	-	-	-	-	-	
7	-	-	-	-	-	-	-	
8	-	-	-	-	-	-	-	
9	-	-	-	-	-	-	-	
10	8.66	-	-	-	-	-	-	
11	58.66	-	-	-	-	-	-	
12	108.66	-	-	-	-	-	-	
13	208.66	-	-	-	-	-	-	
14	308.66	-	-	-	-	-	-	
15	408.66	-	-	-	-	-	-	
16	508.66	-	-	-	-	-	-	
17	608.66	-	-	-	-	-	-	
18	708.66	-	-	-	-	-	-	
19	808.66	-	-	-	-	-	-	
20	900.00	8.66	-	-	-	-	-	

[33]	Expected Loss $ to Tier	88.51	0.09	-	-	-	-	-	*=E{ Tier NS(i) }*
[34]	Expected Loss % to Tier	9.83%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[33] / [1] / [24]*
[35]	Yield Prem - Variable	**0.10**	**0.25**	**0.50**	**0.75**	**1.00**	**2.00**	**4.00**	*Input*
[36]	Yield Prem - Fixed	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	*Input*
[37]	Total Yield % on Tier	10.92%	0.11%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[34] *(1+[35]) + [36]*
[38]	Total Yield $ on Tier	98.26	1.01	-	-	-	-	-	*=[37] * [1] * [24]*

Exhibit 4

CASE 4 -- More Skewed Loss Distribution, In Yield and Loss Rate Balance

Inputs:			
[1]	**Committed Surplus**	**4,445.00**	*Input*
[2]	**Risk Load as % of E[L]**	**8.10%**	*Input*
[3]	Subject Premium	**1,000.00**	*Input*
[4]	Default-Free Rate	**5.00%**	*Input*

Calculated Values:							
[5]	Risk Load $	59.25	*=[2] * [6]*	[10]	Expected Value of F(T)	4,766.04	*=E{ [22] }*
[6]	Expected Loss	731.50	*=E{ [19] }*	[11]	**Expected Yield on CS**	**7.22%**	***=[10] / [1] - 1.00***
[7]	Initial Fund F(0)	5,235.75	*=[1] +[5] +[6]*	[12]	Total Yield $ Over All Tiers	98.76	*=Sum{ [38] }*
[8]	Expected Value of NS(i)	88.77	*=E{ [21] }*	[13]	**Required Yield on CS**	**7.22%**	***=[12] / [1] + [4]***
[9]	**Surplus Loss Rate**	**2.00%**	***=[8] / [1]***	[14]	**True Yield Premium**	**0.22%**	***=[13] - [9] - [4]***

[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]
Outcome # i	**Prob f(i)** *Input*	**Cum Prob F(i)** *=Cum Sum of [16]*	**Loss Ratio** *Input*	**Loss L(i)** *=[3] * [18]*	**Fund Contribution from Prem** *=[5] + [6]*	**NS(i)** *=Max(0, [19] - [20])*	**F(T)** *=[7] * (1.00 + [4]) - [19]*	**NS(i) as % CS** *=[21] / [1]*
1	**2.00%**	2.0%	**35.00%**	350.00	790.75	-	5,147.54	0.00%
2	**4.00%**	6.0%	**40.00%**	400.00	790.75	-	5,097.54	0.00%
3	**6.00%**	12.0%	**45.00%**	450.00	790.75	-	5,047.54	0.00%
4	**10.00%**	22.0%	**50.00%**	500.00	790.75	-	4,997.54	0.00%
5	**11.00%**	33.0%	**55.00%**	550.00	790.75	-	4,947.54	0.00%
6	**12.00%**	45.0%	**60.00%**	600.00	790.75	-	4,897.54	0.00%
7	**10.00%**	55.0%	**65.00%**	650.00	790.75	-	4,847.54	0.00%
8	**9.00%**	64.0%	**70.00%**	700.00	790.75	-	4,797.54	0.00%
9	**6.00%**	70.0%	**75.00%**	750.00	790.75	-	4,747.54	0.00%
10	**5.00%**	75.0%	**80.00%**	800.00	790.75	9.25	4,697.54	0.21%
11	**4.00%**	79.0%	**85.00%**	850.00	790.75	59.25	4,647.54	1.33%
12	**4.00%**	83.0%	**90.00%**	900.00	790.75	109.25	4,597.54	2.46%
13	**3.00%**	86.0%	**100.00%**	1,000.00	790.75	209.25	4,497.54	4.71%
14	**3.00%**	89.0%	**110.00%**	1,100.00	790.75	309.25	4,397.54	6.96%
15	**3.00%**	92.0%	**120.00%**	1,200.00	790.75	409.25	4,297.54	9.21%
16	**2.00%**	94.0%	**130.00%**	1,300.00	790.75	509.25	4,197.54	11.46%
17	**2.00%**	96.0%	**140.00%**	1,400.00	790.75	609.25	4,097.54	13.71%
18	**2.00%**	98.0%	**150.00%**	1,500.00	790.75	709.25	3,997.54	15.96%
19	**1.00%**	99.0%	**160.00%**	1,600.00	790.75	809.25	3,897.54	18.21%
20	**1.00%**	100.0%	**170.00%**	1,700.00	790.75	909.25	3,797.54	20.46%

Exhibit 4
CASE 4 -- More Skewed Loss Distribution, In Yield and Loss Rate Balance

		Tier Definitions as % of CS							
	Tier #	1	2	3	4	5	6	7	
[24]	Limit	25.0%	25.0%	25.0%	25.0%	100.0%	200.0%	99999.0%	*Input*
[25]	Retention	0.0%	25.0%	50.0%	75.0%	100.0%	200.0%	400.0%	*Input*

[15]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	
Outcome # i	Tier 1 NS(i)	Tier 2 NS(i)	Tier 3 NS(i)	Tier 4 NS(i)	Tier 5 NS(i)	Tier 6 NS(i)	Tier 7 NS(i)	*Tier NS(i) = MAX(0, MIN([21] - [25] * [1], [24] * [1]))*
1	-	-	-	-	-	-	-	
2	-	-	-	-	-	-	-	
3	-	-	-	-	-	-	-	
4	-	-	-	-	-	-	-	
5	-	-	-	-	-	-	-	
6	-	-	-	-	-	-	-	
7	-	-	-	-	-	-	-	
8	-	-	-	-	-	-	-	
9	-	-	-	-	-	-	-	
10	9.25	-	-	-	-	-	-	
11	59.25	-	-	-	-	-	-	
12	109.25	-	-	-	-	-	-	
13	209.25	-	-	-	-	-	-	
14	309.25	-	-	-	-	-	-	
15	409.25	-	-	-	-	-	-	
16	509.25	-	-	-	-	-	-	
17	609.25	-	-	-	-	-	-	
18	709.25	-	-	-	-	-	-	
19	809.25	-	-	-	-	-	-	
20	909.25	-	-	-	-	-	-	

[33]	Expected Loss $ to Tier	88.77	-	-	-	-	-	-	*=E{ Tier NS(i) }*
[34]	Expected Loss % to Tier	7.99%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[33] / [1] / [24]*
[35]	Yield Prem - Variable	0.10	0.25	0.50	0.75	1.00	2.00	4.00	*Input*
[36]	Yield Prem - Fixed	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	*Input*
[37]	Total Yield % on Tier	8.89%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[34] *(1+[35]) + [36]*
[38]	Total Yield $ on Tier	98.76	-	-	-	-	-	-	*=[37] * [1] * [24]*

Exhibit 5

CASE 5 -- Less Skewed Loss Distribution, Same CS and Risk Load %

Inputs:			
[1]	Committed Surplus	**3,600.00**	*Input*
[2]	Risk Load as % of E[L]	**6.15%**	*Input*
[3]	Subject Premium	**1,000.00**	*Input*
[4]	Default-Free Rate	**5.00%**	*Input*

Calculated Values:							
[5]	Risk Load $	41.37	*=[2] * [6]*	[10]	Expected Value of F(T)	3,857.08	*=E{ [22] }*
[6]	Expected Loss	672.75	*=E{ [19] }*	[11]	**Expected Yield on CS**	**7.14%**	***=[10] / [1] - 1.00***
[7]	Initial Fund F(0)	4,314.12	*=[1] +[5] +[6]*	[12]	Total Yield $ Over All Tiers	61.58	*=Sum{ [38] }*
[8]	Expected Value of NS(i)	55.17	*=E{ [21] }*	[13]	**Required Yield on CS**	**6.71%**	***=[12] / [1] + [4]***
[9]	**Surplus Loss Rate**	**1.53%**	***=[8] / [1]***	[14]	**True Yield Premium**	**0.18%**	***=[13] - [9] - [4]***

[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]
Outcome # i	**Prob f(i)** *Input*	**Cum Prob F(i)** *=Cum Sum of [16]*	**Loss Ratio** *Input*	**Loss L(i)** *=[3] * [18]*	**Fund Contribution from Prem** *=[5] + [6]*	**NS(i)** *=Max(0, [19] - [20])*	**F(T)** *=[7] * (1.00 + [4]) - [19]*	**NS(i) as % CS** *=[21] / [1]*
1	**2.00%**	2.0%	**35.00%**	350.00	714.12	-	4,179.83	0.00%
2	**4.00%**	6.0%	**40.00%**	400.00	714.12	-	4,129.83	0.00%
3	**6.00%**	12.0%	**45.00%**	450.00	714.12	-	4,079.83	0.00%
4	**10.00%**	22.0%	**50.00%**	500.00	714.12	-	4,029.83	0.00%
5	**11.00%**	33.0%	**55.00%**	550.00	714.12	-	3,979.83	0.00%
6	**12.00%**	45.0%	**60.00%**	600.00	714.12	-	3,929.83	0.00%
7	**10.00%**	55.0%	**65.00%**	650.00	714.12	-	3,879.83	0.00%
8	**9.00%**	64.0%	**70.00%**	700.00	714.12	-	3,829.83	0.00%
9	**6.00%**	70.0%	**75.00%**	750.00	714.12	35.88	3,779.83	1.00%
10	**5.00%**	75.0%	**80.00%**	800.00	714.12	85.88	3,729.83	2.39%
11	**4.00%**	79.0%	**82.50%**	825.00	714.12	110.88	3,704.83	3.08%
12	**4.00%**	83.0%	**85.00%**	850.00	714.12	135.88	3,679.83	3.77%
13	**3.00%**	86.0%	**87.50%**	875.00	714.12	160.88	3,654.83	4.47%
14	**3.00%**	89.0%	**90.00%**	900.00	714.12	185.88	3,629.83	5.16%
15	**3.00%**	92.0%	**92.50%**	925.00	714.12	210.88	3,604.83	5.86%
16	**2.00%**	94.0%	**95.00%**	950.00	714.12	235.88	3,579.83	6.55%
17	**2.00%**	96.0%	**97.50%**	975.00	714.12	260.88	3,554.83	7.25%
18	**2.00%**	98.0%	**100.00%**	1,000.00	714.12	285.88	3,529.83	7.94%
19	**1.00%**	99.0%	**102.50%**	1,025.00	714.12	310.88	3,504.83	8.64%
20	**1.00%**	100.0%	**105.00%**	1,050.00	714.12	335.88	3,479.83	9.33%

Exhibit 5
CASE 5 -- Less Skewed Loss Distribution, Same CS and Risk Load %

		Tier Definitions as % of CS							
	Tier #	1	2	3	4	5	6	7	
[24]	Limit	25.0%	25.0%	25.0%	25.0%	100.0%	200.0%	99999.0%	*Input*
[25]	Retention	0.0%	25.0%	50.0%	75.0%	100.0%	200.0%	400.0%	*Input*

[15]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	
Outcome # i	Tier 1 NS(i)	Tier 2 NS(i)	Tier 3 NS(i)	Tier 4 NS(i)	Tier 5 NS(i)	Tier 6 NS(i)	Tier 7 NS(i)	*Tier NS(i) = MAX(0, MIN([21] - [25] * [1], [24] * [1]))*
1	-	-	-	-	-	-	-	
2	-	-	-	-	-	-	-	
3	-	-	-	-	-	-	-	
4	-	-	-	-	-	-	-	
5	-	-	-	-	-	-	-	
6	-	-	-	-	-	-	-	
7	-	-	-	-	-	-	-	
8	-	-	-	-	-	-	-	
9	35.88	-	-	-	-	-	-	
10	85.88	-	-	-	-	-	-	
11	110.88	-	-	-	-	-	-	
12	135.88	-	-	-	-	-	-	
13	160.88	-	-	-	-	-	-	
14	185.88	-	-	-	-	-	-	
15	210.88	-	-	-	-	-	-	
16	235.88	-	-	-	-	-	-	
17	260.88	-	-	-	-	-	-	
18	285.88	-	-	-	-	-	-	
19	310.88	-	-	-	-	-	-	
20	335.88	-	-	-	-	-	-	

[33]	Expected Loss $ to Tier	55.17	-	-	-	-	-	-	*=E{ Tier NS(i) }*
[34]	Expected Loss % to Tier	6.13%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[33] / [1] / [24]*
[35]	Yield Prem - Variable	0.10	0.25	0.50	0.75	1.00	2.00	4.00	*Input*
[36]	Yield Prem - Fixed	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	*Input*
[37]	Total Yield % on Tier	6.84%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[34] *(1+[35]) + [36]*
[38]	Total Yield $ on Tier	61.58	-	-	-	-	-	-	*=[37] * [1] * [24]*

Exhibit 6

CASE 6 -- Less Skewed Loss Distribution, In Yield and Loss Rate Balance

Inputs:

[1]	**Committed Surplus**	**2,955.00**	*Input*
[2]	**Risk Load as % of E[L]**	**4.54%**	*Input*
[3]	Subject Premium	**1,000.00**	*Input*
[4]	Default-Free Rate	**5.00%**	*Input*

Calculated Values:

[5]	Risk Load $	30.54	*=[2] * [6]*	[10]	Expected Value of F(T)	3,168.46	*=E{ [22] }*
[6]	Expected Loss	672.75	*=E{ [19] }*	[11]	**Expected Yield on CS**	**7.22%**	***=[10] / [1] - 1.00***
[7]	Initial Fund F(0)	3,658.29	*=[1] +[5] +[6]*	[12]	Total Yield $ Over All Tiers	65.71	*=Sum{ [38] }*
[8]	Expected Value of NS(i)	59.06	*=E{ [21] }*	[13]	**Required Yield on CS**	**7.22%**	***=[12] / [1] + [4]***
[9]	**Surplus Loss Rate**	**2.00%**	***=[8] / [1]***	[14]	**True Yield Premium**	**0.22%**	***=[13] - [9] - [4]***

[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]
Outcome # i	Prob f(i) *Input*	Cum Prob F(i) *=Cum Sum of [16]*	Loss Ratio *Input*	Loss L(i) *=[3] * [18]*	Fund Contribution from Prem *=[5] + [6]*	NS(i) *=Max(0, [19] - [20])*	F(T) *=[7] * (1.00 + [4]) - [19]*	NS(i) as % CS *=[21] / [1]*
1	**2.00%**	2.0%	**35.00%**	350.00	703.29	-	3,491.21	0.00%
2	**4.00%**	6.0%	**40.00%**	400.00	703.29	-	3,441.21	0.00%
3	**6.00%**	12.0%	**45.00%**	450.00	703.29	-	3,391.21	0.00%
4	**10.00%**	22.0%	**50.00%**	500.00	703.29	-	3,341.21	0.00%
5	**11.00%**	33.0%	**55.00%**	550.00	703.29	-	3,291.21	0.00%
6	**12.00%**	45.0%	**60.00%**	600.00	703.29	-	3,241.21	0.00%
7	**10.00%**	55.0%	**65.00%**	650.00	703.29	-	3,191.21	0.00%
8	**9.00%**	64.0%	**70.00%**	700.00	703.29	-	3,141.21	0.00%
9	**6.00%**	70.0%	**75.00%**	750.00	703.29	46.71	3,091.21	1.58%
10	**5.00%**	75.0%	**80.00%**	800.00	703.29	96.71	3,041.21	3.27%
11	**4.00%**	79.0%	**82.50%**	825.00	703.29	121.71	3,016.21	4.12%
12	**4.00%**	83.0%	**85.00%**	850.00	703.29	146.71	2,991.21	4.96%
13	**3.00%**	86.0%	**87.50%**	875.00	703.29	171.71	2,966.21	5.81%
14	**3.00%**	89.0%	**90.00%**	900.00	703.29	196.71	2,941.21	6.66%
15	**3.00%**	92.0%	**92.50%**	925.00	703.29	221.71	2,916.21	7.50%
16	**2.00%**	94.0%	**95.00%**	950.00	703.29	246.71	2,891.21	8.35%
17	**2.00%**	96.0%	**97.50%**	975.00	703.29	271.71	2,866.21	9.19%
18	**2.00%**	98.0%	**100.00%**	1,000.00	703.29	296.71	2,841.21	10.04%
19	**1.00%**	99.0%	**102.50%**	1,025.00	703.29	321.71	2,816.21	10.89%
20	**1.00%**	100.0%	**105.00%**	1,050.00	703.29	346.71	2,791.21	11.73%

Exhibit 6

CASE 6 -- Less Skewed Loss Distribution, In Yield and Loss Rate Balance

		Tier Definitions as % of CS							
	Tier #	1	2	3	4	5	6	7	
[24]	Limit	25.0%	25.0%	25.0%	25.0%	100.0%	200.0%	99999.0%	*Input*
[25]	Retention	0.0%	25.0%	50.0%	75.0%	100.0%	200.0%	400.0%	*Input*

[15]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	
Outcome # i	Tier 1 NS(i)	Tier 2 NS(i)	Tier 3 NS(i)	Tier 4 NS(i)	Tier 5 NS(i)	Tier 6 NS(i)	Tier 7 NS(i)	*Tier NS(i) = MAX(0, MIN([21] - [25] * [1], [24] * [1]))*
1	-	-	-	-	-	-	-	
2	-	-	-	-	-	-	-	
3	-	-	-	-	-	-	-	
4	-	-	-	-	-	-	-	
5	-	-	-	-	-	-	-	
6	-	-	-	-	-	-	-	
7	-	-	-	-	-	-	-	
8	-	-	-	-	-	-	-	
9	46.71	-	-	-	-	-	-	
10	96.71	-	-	-	-	-	-	
11	121.71	-	-	-	-	-	-	
12	146.71	-	-	-	-	-	-	
13	171.71	-	-	-	-	-	-	
14	196.71	-	-	-	-	-	-	
15	221.71	-	-	-	-	-	-	
16	246.71	-	-	-	-	-	-	
17	271.71	-	-	-	-	-	-	
18	296.71	-	-	-	-	-	-	
19	321.71	-	-	-	-	-	-	
20	346.71	-	-	-	-	-	-	

[33]	Expected Loss $ to Tier	59.06	-	-	-	-	-	-	*=E{ Tier NS(i) }*
[34]	Expected Loss % to Tier	8.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[33] / [1] / [24]*
[35]	Yield Prem - Variable	**0.10**	**0.25**	**0.50**	**0.75**	**1.00**	**2.00**	**4.00**	*Input*
[36]	Yield Prem - Fixed	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	*Input*
[37]	Total Yield % on Tier	8.89%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[34] *(1+[35]) + [36]*
[38]	Total Yield $ on Tier	65.71	-	-	-	-	-	-	*=[37] * [1] * [24]*

Exhibit 7

CASE 7 -- New Risk Resulting in More Skewed Loss Distribution, Same CS

Inputs:				New Risk
[1]	**Committed Surplus**	**3,600.00**	*Input*	-
[2]	**Risk Load as % of E[L]**	**8.18%**	*Input*	16.17%
[3]	Subject Premium	**1,200.00**	*Input*	
[4]	Default-Free Rate	**5.00%**	*Input*	

Calculated Values:							
[5]	Risk Load $	71.80	*=[2] * [6]*	[10]	Expected Value of F(T)	3,899.28	*=E{ [22] }*
[6]	Expected Loss	877.80	*=E{ [19] }*	[11]	**Expected Yield on CS**	**8.31%**	***=[10] / [1] - 1.00***
[7]	Initial Fund F(0)	4,549.60	*=[1] +[5] +[6]*	[12]	Total Yield $ Over All Tiers	119.14	*=Sum{ [38] }*
[8]	Expected Value of NS(i)	106.32	*=E{ [21] }*	[13]	**Required Yield on CS**	**8.31%**	***=[12] / [1] + [4]***
[9]	**Surplus Loss Rate**	**2.95%**	***=[8] / [1]***	[14]	**True Yield Premium**	**0.36%**	***=[13] - [9] - [4]***

[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]
Outcome # i	Prob f(i) *Input*	Cum Prob F(i) *=Cum Sum of [16]*	Loss Ratio *Input*	Loss L(i) *=[3] * [18]*	Fund Contribution from Prem *=[5] + [6]*	NS(i) *=Max(0, [19] - [20])*	F(T) *=[7] * (1.00 + [4]) - [19]*	NS(i) as % CS *=[21] / [1]*
1	**2.00%**	2.0%	**35.00%**	420.00	949.60	-	4,357.08	0.00%
2	**4.00%**	6.0%	**40.00%**	480.00	949.60	-	4,297.08	0.00%
3	**6.00%**	12.0%	**45.00%**	540.00	949.60	-	4,237.08	0.00%
4	**10.00%**	22.0%	**50.00%**	600.00	949.60	-	4,177.08	0.00%
5	**11.00%**	33.0%	**55.00%**	660.00	949.60	-	4,117.08	0.00%
6	**12.00%**	45.0%	**60.00%**	720.00	949.60	-	4,057.08	0.00%
7	**10.00%**	55.0%	**65.00%**	780.00	949.60	-	3,997.08	0.00%
8	**9.00%**	64.0%	**70.00%**	840.00	949.60	-	3,937.08	0.00%
9	**6.00%**	70.0%	**75.00%**	900.00	949.60	-	3,877.08	0.00%
10	**5.00%**	75.0%	**80.00%**	960.00	949.60	10.40	3,817.08	0.29%
11	**4.00%**	79.0%	**85.00%**	1,020.00	949.60	70.40	3,757.08	1.96%
12	**4.00%**	83.0%	**90.00%**	1,080.00	949.60	130.40	3,697.08	3.62%
13	**3.00%**	86.0%	**100.00%**	1,200.00	949.60	250.40	3,577.08	6.96%
14	**3.00%**	89.0%	**110.00%**	1,320.00	949.60	370.40	3,457.08	10.29%
15	**3.00%**	92.0%	**120.00%**	1,440.00	949.60	490.40	3,337.08	13.62%
16	**2.00%**	94.0%	**130.00%**	1,560.00	949.60	610.40	3,217.08	16.96%
17	**2.00%**	96.0%	**140.00%**	1,680.00	949.60	730.40	3,097.08	20.29%
18	**2.00%**	98.0%	**150.00%**	1,800.00	949.60	850.40	2,977.08	23.62%
19	**1.00%**	99.0%	**160.00%**	1,920.00	949.60	970.40	2,857.08	26.96%
20	**1.00%**	100.0%	**170.00%**	2,040.00	949.60	1,090.40	2,737.08	30.29%

Exhibit 7
CASE 7 -- New Risk Resulting in More Skewed Loss Distribution, Same CS

		Tier Definitions as % of CS							
	Tier #	1	2	3	4	5	6	7	
[24]	Limit	25.0%	25.0%	25.0%	25.0%	100.0%	200.0%	99999.0%	*Input*
[25]	Retention	0.0%	25.0%	50.0%	75.0%	100.0%	200.0%	400.0%	*Input*

[15]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	
Outcome # i	Tier 1 NS(i)	Tier 2 NS(i)	Tier 3 NS(i)	Tier 4 NS(i)	Tier 5 NS(i)	Tier 6 NS(i)	Tier 7 NS(i)	*Tier NS(i) = MAX(0, MIN([21] - [25] * [1], [24] * [1]))*
1	-	-	-	-	-	-	-	
2	-	-	-	-	-	-	-	
3	-	-	-	-	-	-	-	
4	-	-	-	-	-	-	-	
5	-	-	-	-	-	-	-	
6	-	-	-	-	-	-	-	
7	-	-	-	-	-	-	-	
8	-	-	-	-	-	-	-	
9	-	-	-	-	-	-	-	
10	10.40	-	-	-	-	-	-	
11	70.40	-	-	-	-	-	-	
12	130.40	-	-	-	-	-	-	
13	250.40	-	-	-	-	-	-	
14	370.40	-	-	-	-	-	-	
15	490.40	-	-	-	-	-	-	
16	610.40	-	-	-	-	-	-	
17	730.40	-	-	-	-	-	-	
18	850.40	-	-	-	-	-	-	
19	900.00	70.40	-	-	-	-	-	
20	900.00	190.40	-	-	-	-	-	

[33]	Expected Loss $ to Tier	103.71	2.61	-	-	-	-	-	*=E{ Tier NS(i) }*
[34]	Expected Loss % to Tier	11.52%	0.29%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[33] / [1] / [24]*
[35]	Yield Prem - Variable	**0.10**	**0.25**	**0.50**	**0.75**	**1.00**	**2.00**	**4.00**	*Input*
[36]	Yield Prem - Fixed	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	*Input*
[37]	Total Yield % on Tier	12.78%	0.46%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[34] *(1+[35]) + [36]*
[38]	Total Yield $ on Tier	114.98	4.16	-	-	-	-	-	*=[37] * [1] * [24]*

Exhibit 8

CASE 8 -- New Risk, More Skewed Distribution, In Yield and Loss Rate Balance

	Inputs:			New Risk
[1]	**Committed Surplus**	5,325.00	*Input*	1,725.00
[2]	**Risk Load as % of E[L]**	8.10%	*Input*	15.78%
[3]	Subject Premium	1,200.00	*Input*	
[4]	Default-Free Rate	5.00%	*Input*	

Calculated Values:

[5]	Risk Load $	71.10	*=[2] * [6]*	[10]	Expected Value of F(T)	5,709.80	*=E{ [22] }*
[6]	Expected Loss	877.80	*=E{ [19] }*	[11]	**Expected Yield on CS**	**7.23%**	***=[10] / [1] - 1.00***
[7]	Initial Fund F(0)	6,273.90	*=[1] +[5] +[6]*	[12]	Total Yield $ Over All Tiers	118.51	*=Sum{ [38] }*
[8]	Expected Value of NS(i)	106.53	*=E{ [21] }*	[13]	**Required Yield on CS**	**7.23%**	***=[12] / [1] + [4]***
[9]	**Surplus Loss Rate**	**2.00%**	***=[8] / [1]***	[14]	**True Yield Premium**	**0.23%**	***=[13] - [9] - [4]***

[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]
Outcome # i	Prob f(i) *Input*	Cum Prob F(i) *=Cum Sum of [16]*	Loss Ratio *Input*	Loss L(i) *=[3] * [18]*	Fund Contribution from Prem *=[5] + [6]*	NS(i) *=Max(0, [19] - [20])*	F(T) *=[7] * (1.00 + [4]) - [19]*	NS(i) as % CS *=[21] / [1]*
1	2.00%	2.0%	35.00%	420.00	948.90	-	6,167.60	0.00%
2	4.00%	6.0%	40.00%	480.00	948.90	-	6,107.60	0.00%
3	6.00%	12.0%	45.00%	540.00	948.90	-	6,047.60	0.00%
4	10.00%	22.0%	50.00%	600.00	948.90	-	5,987.60	0.00%
5	11.00%	33.0%	55.00%	660.00	948.90	-	5,927.60	0.00%
6	12.00%	45.0%	60.00%	720.00	948.90	-	5,867.60	0.00%
7	10.00%	55.0%	65.00%	780.00	948.90	-	5,807.60	0.00%
8	9.00%	64.0%	70.00%	840.00	948.90	-	5,747.60	0.00%
9	6.00%	70.0%	75.00%	900.00	948.90	-	5,687.60	0.00%
10	5.00%	75.0%	80.00%	960.00	948.90	11.10	5,627.60	0.21%
11	4.00%	79.0%	85.00%	1,020.00	948.90	71.10	5,567.60	1.34%
12	4.00%	83.0%	90.00%	1,080.00	948.90	131.10	5,507.60	2.46%
13	3.00%	86.0%	100.00%	1,200.00	948.90	251.10	5,387.60	4.72%
14	3.00%	89.0%	110.00%	1,320.00	948.90	371.10	5,267.60	6.97%
15	3.00%	92.0%	120.00%	1,440.00	948.90	491.10	5,147.60	9.22%
16	2.00%	94.0%	130.00%	1,560.00	948.90	611.10	5,027.60	11.48%
17	2.00%	96.0%	140.00%	1,680.00	948.90	731.10	4,907.60	13.73%
18	2.00%	98.0%	150.00%	1,800.00	948.90	851.10	4,787.60	15.98%
19	1.00%	99.0%	160.00%	1,920.00	948.90	971.10	4,667.60	18.24%
20	1.00%	100.0%	170.00%	2,040.00	948.90	1,091.10	4,547.60	20.49%

Exhibit 8

CASE 8 -- New Risk, More Skewed Distribution, In Yield and Loss Rate Balance

		Tier Definitions as % of CS							
	Tier #	1	2	3	4	5	6	7	
[24]	Limit	25.0%	25.0%	25.0%	25.0%	100.0%	200.0%	99999.0%	*Input*
[25]	Retention	0.0%	25.0%	50.0%	75.0%	100.0%	200.0%	400.0%	*Input*

[15]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	
Outcome # i	Tier 1 NS(i)	Tier 2 NS(i)	Tier 3 NS(i)	Tier 4 NS(i)	Tier 5 NS(i)	Tier 6 NS(i)	Tier 7 NS(i)	*Tier NS(i) = MAX(0, MIN([21] - [25] * [1], [24] * [1]))*
1	-	-	-	-	-	-	-	
2	-	-	-	-	-	-	-	
3	-	-	-	-	-	-	-	
4	-	-	-	-	-	-	-	
5	-	-	-	-	-	-	-	
6	-	-	-	-	-	-	-	
7	-	-	-	-	-	-	-	
8	-	-	-	-	-	-	-	
9	-	-	-	-	-	-	-	
10	11.10	-	-	-	-	-	-	
11	71.10	-	-	-	-	-	-	
12	131.10	-	-	-	-	-	-	
13	251.10	-	-	-	-	-	-	
14	371.10	-	-	-	-	-	-	
15	491.10	-	-	-	-	-	-	
16	611.10	-	-	-	-	-	-	
17	731.10	-	-	-	-	-	-	
18	851.10	-	-	-	-	-	-	
19	971.10	-	-	-	-	-	-	
20	1,091.10	-	-	-	-	-	-	

[33]	Expected Loss $ to Tier	106.53	-	-	-	-	-	-	*=E{ Tier NS(i) }*
[34]	Expected Loss % to Tier	8.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[33] / [1] / [24]*
[35]	Yield Prem - Variable	**0.10**	**0.25**	**0.50**	**0.75**	**1.00**	**2.00**	**4.00**	*Input*
[36]	Yield Prem - Fixed	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	*Input*
[37]	Total Yield % on Tier	8.90%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[34] *(1+[35]) + [36]*
[38]	Total Yield $ on Tier	118.51	-	-	-	-	-	-	*=[37] * [1] * [24]*

Exhibit 9

CASE 9 -- New Risk, Less Skewed, Same CS, In Yield and Loss Rate Balance

	Inputs:			New Risk
[1]	**Committed Surplus**	3,600.00	*Input*	-
[2]	**Risk Load as % of E[L]**	4.54%	*Input*	-5.96%
[3]	Subject Premium	1,200.00	*Input*	
[4]	Default-Free Rate	5.00%	*Input*	

Calculated Values:

[5]	Risk Load $	36.65	*=[2] * [6]*	[10]	Expected Value of F(T)	3,858.85	*=E{ [22] }*
[6]	Expected Loss	807.30	*=E{ [19] }*	[11]	**Expected Yield on CS**	**7.19%**	***=[10] / [1] - 1.00***
[7]	Initial Fund F(0)	4,443.95	*=[1] +[5] +[6]*	[12]	Total Yield $ Over All Tiers	78.87	*=Sum{ [38] }*
[8]	Expected Value of NS(i)	70.88	*=E{ [21] }*	[13]	**Required Yield on CS**	**7.19%**	***=[12] / [1] + [4]***
[9]	**Surplus Loss Rate**	**1.97%**	***=[8] / [1]***	[14]	**True Yield Premium**	**0.22%**	***=[13] - [9] - [4]***

[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]
Outcome # i	Prob f(i) *Input*	Cum Prob F(i) *=Cum Sum of [16]*	Loss Ratio *Input*	Loss L(i) *=[3] * [18]*	Fund Contribution from Prem *=[5] + [6]*	NS(i) *=Max(0, [19] - [20])*	F(T) *=[7] * (1.00 + [4]) - [19]*	NS(i) as % CS *=[21] / [1]*
1	**2.00%**	2.0%	**35.00%**	420.00	843.95	-	4,246.15	0.00%
2	**4.00%**	6.0%	**40.00%**	480.00	843.95	-	4,186.15	0.00%
3	**6.00%**	12.0%	**45.00%**	540.00	843.95	-	4,126.15	0.00%
4	**10.00%**	22.0%	**50.00%**	600.00	843.95	-	4,066.15	0.00%
5	**11.00%**	33.0%	**55.00%**	660.00	843.95	-	4,006.15	0.00%
6	**12.00%**	45.0%	**60.00%**	720.00	843.95	-	3,946.15	0.00%
7	**10.00%**	55.0%	**65.00%**	780.00	843.95	-	3,886.15	0.00%
8	**9.00%**	64.0%	**70.00%**	840.00	843.95	-	3,826.15	0.00%
9	**6.00%**	70.0%	**75.00%**	900.00	843.95	56.05	3,766.15	1.56%
10	**5.00%**	75.0%	**80.00%**	960.00	843.95	116.05	3,706.15	3.22%
11	**4.00%**	79.0%	**82.50%**	990.00	843.95	146.05	3,676.15	4.06%
12	**4.00%**	83.0%	**85.00%**	1,020.00	843.95	176.05	3,646.15	4.89%
13	**3.00%**	86.0%	**87.50%**	1,050.00	843.95	206.05	3,616.15	5.72%
14	**3.00%**	89.0%	**90.00%**	1,080.00	843.95	236.05	3,586.15	6.56%
15	**3.00%**	92.0%	**92.50%**	1,110.00	843.95	266.05	3,556.15	7.39%
16	**2.00%**	94.0%	**95.00%**	1,140.00	843.95	296.05	3,526.15	8.22%
17	**2.00%**	96.0%	**97.50%**	1,170.00	843.95	326.05	3,496.15	9.06%
18	**2.00%**	98.0%	**100.00%**	1,200.00	843.95	356.05	3,466.15	9.89%
19	**1.00%**	99.0%	**102.50%**	1,230.00	843.95	386.05	3,436.15	10.72%
20	**1.00%**	100.0%	**105.00%**	1,260.00	843.95	416.05	3,406.15	11.56%

Exhibit 9

CASE 9 -- New Risk, Less Skewed, Same CS, In Yield and Loss Rate Balance

		Tier Definitions as % of CS							
	Tier #	1	2	3	4	5	6	7	
[24]	Limit	25.0%	25.0%	25.0%	25.0%	100.0%	200.0%	99999.0%	*Input*
[25]	Retention	0.0%	25.0%	50.0%	75.0%	100.0%	200.0%	400.0%	*Input*

[15]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	
Outcome # i	Tier 1 NS(i)	Tier 2 NS(i)	Tier 3 NS(i)	Tier 4 NS(i)	Tier 5 NS(i)	Tier 6 NS(i)	Tier 7 NS(i)	*Tier NS(i) = MAX(0, MIN([21] - [25] * [1], [24] * [1]))*
1	-	-	-	-	-	-	-	
2	-	-	-	-	-	-	-	
3	-	-	-	-	-	-	-	
4	-	-	-	-	-	-	-	
5	-	-	-	-	-	-	-	
6	-	-	-	-	-	-	-	
7	-	-	-	-	-	-	-	
8	-	-	-	-	-	-	-	
9	56.05	-	-	-	-	-	-	
10	116.05	-	-	-	-	-	-	
11	146.05	-	-	-	-	-	-	
12	176.05	-	-	-	-	-	-	
13	206.05	-	-	-	-	-	-	
14	236.05	-	-	-	-	-	-	
15	266.05	-	-	-	-	-	-	
16	296.05	-	-	-	-	-	-	
17	326.05	-	-	-	-	-	-	
18	356.05	-	-	-	-	-	-	
19	386.05	-	-	-	-	-	-	
20	416.05	-	-	-	-	-	-	

[33]	Expected Loss $ to Tier	70.88	-	-	-	-	-	-	*=E{ Tier NS(i) }*
[34]	Expected Loss % to Tier	7.88%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[33] / [1] / [24]*
[35]	Yield Prem - Variable	**0.10**	**0.25**	**0.50**	**0.75**	**1.00**	**2.00**	**4.00**	*Input*
[36]	Yield Prem - Fixed	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	*Input*
[37]	Total Yield % on Tier	8.76%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[34] *(1+[35]) + [36]*
[38]	Total Yield $ on Tier	78.87	-	-	-	-	-	-	*=[37] * [1] * [24]*

Exhibit 10

CASE 10 -- Base Case with CS = Max(NS(i)), In Yield Balance

Inputs:

[1]	**Committed Surplus**	**530.00**	*Input*
[2]	**Risk Load as % of E[L]**	**7.13%**	*Input*
[3]	Subject Premium	**1,000.00**	*Input*
[4]	Default-Free Rate	**5.00%**	*Input*

Calculated Values:

[5]	Risk Load $	49.91	*=[2] * [6]*	[10]	Expected Value of F(T)	643.91	*=E{ [22] }*
[6]	Expected Loss	700.00	*=E{ [19] }*	[11]	**Expected Yield on CS**	**21.49%**	***=[10] / [1] - 1.00***
[7]	Initial Fund F(0)	1,279.91	*=[1] +[5] +[6]*	[12]	Total Yield $ Over All Tiers	87.38	*=Sum{ [38] }*
[8]	Expected Value of NS(i)	69.53	*=E{ [21] }*	[13]	**Required Yield on CS**	**21.49%**	***=[12] / [1] + [4]***
[9]	**Surplus Loss Rate**	**13.12%**	***=[8] / [1]***	[14]	**True Yield Premium**	**3.37%**	***=[13] - [9] - [4]***

[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]
Outcome # i	Prob f(i) *Input*	Cum Prob F(i) *=Cum Sum of [16]*	Loss Ratio *Input*	Loss L(i) *=[3] * [18]*	Fund Contribution from Prem *=[5] + [6]*	NS(i) *=Max(0, [19] - [20])*	F(T) *=[7] * (1.00 + [4]) - [19]*	NS(i) as % CS *=[21] / [1]*
1	**2.00%**	2.0%	**35.00%**	350.00	749.91	-	993.91	0.00%
2	**4.00%**	6.0%	**40.00%**	400.00	749.91	-	943.91	0.00%
3	**6.00%**	12.0%	**45.00%**	450.00	749.91	-	893.91	0.00%
4	**10.00%**	22.0%	**50.00%**	500.00	749.91	-	843.91	0.00%
5	**11.00%**	33.0%	**55.00%**	550.00	749.91	-	793.91	0.00%
6	**12.00%**	45.0%	**60.00%**	600.00	749.91	-	743.91	0.00%
7	**10.00%**	55.0%	**65.00%**	650.00	749.91	-	693.91	0.00%
8	**9.00%**	64.0%	**70.00%**	700.00	749.91	-	643.91	0.00%
9	**6.00%**	70.0%	**75.00%**	750.00	749.91	0.09	593.91	0.02%
10	**5.00%**	75.0%	**80.00%**	800.00	749.91	50.09	543.91	9.45%
11	**4.00%**	79.0%	**85.00%**	850.00	749.91	100.09	493.91	18.88%
12	**4.00%**	83.0%	**90.00%**	900.00	749.91	150.09	443.91	28.32%
13	**3.00%**	86.0%	**95.00%**	950.00	749.91	200.09	393.91	37.75%
14	**3.00%**	89.0%	**100.00%**	1,000.00	749.91	250.09	343.91	47.19%
15	**3.00%**	92.0%	**105.00%**	1,050.00	749.91	300.09	293.91	56.62%
16	**2.00%**	94.0%	**110.00%**	1,100.00	749.91	350.09	243.91	66.05%
17	**2.00%**	96.0%	**115.00%**	1,150.00	749.91	400.09	193.91	75.49%
18	**2.00%**	98.0%	**120.00%**	1,200.00	749.91	450.09	143.91	84.92%
19	**1.00%**	99.0%	**125.00%**	1,250.00	749.91	500.09	93.91	94.36%
20	**1.00%**	100.0%	**130.00%**	1,300.00	749.91	550.09	43.91	103.79%

Exhibit 10
CASE 10 -- Base Case with CS = Max(NS(i)), In Yield Balance

		Tier Definitions as % of CS							
	Tier #	1	2	3	4	5	6	7	
[24]	Limit	25.0%	25.0%	25.0%	25.0%	100.0%	200.0%	99999.0%	*Input*
[25]	Retention	0.0%	25.0%	50.0%	75.0%	100.0%	200.0%	400.0%	*Input*

[15]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	
Outcome # i	Tier 1 NS(i)	Tier 2 NS(i)	Tier 3 NS(i)	Tier 4 NS(i)	Tier 5 NS(i)	Tier 6 NS(i)	Tier 7 NS(i)	*Tier NS(i) = MAX(0, MIN([21] - [25] * [1], [24] * [1]))*
1	-	-	-	-	-	-	-	
2	-	-	-	-	-	-	-	
3	-	-	-	-	-	-	-	
4	-	-	-	-	-	-	-	
5	-	-	-	-	-	-	-	
6	-	-	-	-	-	-	-	
7	-	-	-	-	-	-	-	
8	-	-	-	-	-	-	-	
9	0.09	-	-	-	-	-	-	
10	50.09	-	-	-	-	-	-	
11	100.09	-	-	-	-	-	-	
12	132.50	17.59	-	-	-	-	-	
13	132.50	67.59	-	-	-	-	-	
14	132.50	117.59	-	-	-	-	-	
15	132.50	132.50	35.09	-	-	-	-	
16	132.50	132.50	85.09	-	-	-	-	
17	132.50	132.50	132.50	2.59	-	-	-	
18	132.50	132.50	132.50	52.59	-	-	-	
19	132.50	132.50	132.50	102.59	-	-	-	
20	132.50	132.50	132.50	132.50	20.09	-	-	

[33]	Expected Loss $ to Tier	34.34	20.83	10.70	3.45	0.20	-	-	*=E{ Tier NS(i) }*
[34]	Expected Loss % to Tier	25.92%	15.72%	8.08%	2.61%	0.04%	0.00%	0.00%	*=[33] / [1] / [24]*
[35]	Yield Prem - Variable	0.10	0.25	0.50	0.75	1.00	2.00	4.00	*Input*
[36]	Yield Prem - Fixed	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	*Input*
[37]	Total Yield % on Tier	28.61%	19.75%	12.22%	4.66%	0.18%	0.00%	0.00%	*=[34] *(1+[35]) + [36]*
[38]	Total Yield $ on Tier	37.90	26.18	16.19	6.18	0.93	-	-	*=[37] * [1] * [24]*

Exhibit 11
CASE 11 -- Base Case with Twice the CS

Inputs:			
[1]	**Committed Surplus**	**7,200.00**	*Input*
[2]	Risk Load as % of E[L]	**6.17%**	*Input*
[3]	Subject Premium	**1,000.00**	*Input*
[4]	Default-Free Rate	**5.00%**	*Input*

Calculated Values:							
[5]	Risk Load $	43.19	*=[2] * [6]*	[10]	Expected Value of F(T)	7,640.35	*=E{ [22] }*
[6]	Expected Loss	700.00	*=E{ [19] }*	[11]	**Expected Yield on CS**	**6.12%**	***=[10] / [1] - 1.00***
[7]	Initial Fund F(0)	7,943.19	*=[1] +[5] +[6]*	[12]	Total Yield $ Over All Tiers	80.95	*=Sum{ [38] }*
[8]	Expected Value of NS(i)	71.95	*=E{ [21] }*	[13]	**Required Yield on CS**	**6.12%**	***=[12] / [1] + [4]***
[9]	**Surplus Loss Rate**	**1.00%**	***=[8] / [1]***	[14]	**True Yield Premium**	**0.12%**	***=[13] - [9] - [4]***

[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]
Outcome # i	**Prob f(i)** *Input*	**Cum Prob F(i)** *=Cum Sum of [16]*	**Loss Ratio** *Input*	**Loss L(i)** *=[3] * [18]*	**Fund Contribution from Prem** *=[5] + [6]*	**NS(i)** *=Max(0, [19] - [20])*	**F(T)** *=[7] * (1.00 + [4]) - [19]*	**NS(i) as % CS** *=[21] / [1]*
1	**2.00%**	2.0%	**35.00%**	350.00	743.19	-	7,990.35	0.00%
2	**4.00%**	6.0%	**40.00%**	400.00	743.19	-	7,940.35	0.00%
3	**6.00%**	12.0%	**45.00%**	450.00	743.19	-	7,890.35	0.00%
4	**10.00%**	22.0%	**50.00%**	500.00	743.19	-	7,840.35	0.00%
5	**11.00%**	33.0%	**55.00%**	550.00	743.19	-	7,790.35	0.00%
6	**12.00%**	45.0%	**60.00%**	600.00	743.19	-	7,740.35	0.00%
7	**10.00%**	55.0%	**65.00%**	650.00	743.19	-	7,690.35	0.00%
8	**9.00%**	64.0%	**70.00%**	700.00	743.19	-	7,640.35	0.00%
9	**6.00%**	70.0%	**75.00%**	750.00	743.19	6.81	7,590.35	0.09%
10	**5.00%**	75.0%	**80.00%**	800.00	743.19	56.81	7,540.35	0.79%
11	**4.00%**	79.0%	**85.00%**	850.00	743.19	106.81	7,490.35	1.48%
12	**4.00%**	83.0%	**90.00%**	900.00	743.19	156.81	7,440.35	2.18%
13	**3.00%**	86.0%	**95.00%**	950.00	743.19	206.81	7,390.35	2.87%
14	**3.00%**	89.0%	**100.00%**	1,000.00	743.19	256.81	7,340.35	3.57%
15	**3.00%**	92.0%	**105.00%**	1,050.00	743.19	306.81	7,290.35	4.26%
16	**2.00%**	94.0%	**110.00%**	1,100.00	743.19	356.81	7,240.35	4.96%
17	**2.00%**	96.0%	**115.00%**	1,150.00	743.19	406.81	7,190.35	5.65%
18	**2.00%**	98.0%	**120.00%**	1,200.00	743.19	456.81	7,140.35	6.34%
19	**1.00%**	99.0%	**125.00%**	1,250.00	743.19	506.81	7,090.35	7.04%
20	**1.00%**	100.0%	**130.00%**	1,300.00	743.19	556.81	7,040.35	7.73%

Exhibit 11
CASE 11 -- Base Case with Twice the CS

		Tier Definitions as % of CS							
	Tier #	1	2	3	4	5	6	7	
[24]	Limit	25.0%	25.0%	25.0%	25.0%	100.0%	200.0%	99999.0%	*Input*
[25]	Retention	0.0%	25.0%	50.0%	75.0%	100.0%	200.0%	400.0%	*Input*

[15]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	
Outcome # i	Tier 1 NS(i)	Tier 2 NS(i)	Tier 3 NS(i)	Tier 4 NS(i)	Tier 5 NS(i)	Tier 6 NS(i)	Tier 7 NS(i)	*Tier NS(i) = MAX(0, MIN([21] - [25] * [1], [24] * [1]))*
1	-	-	-	-	-	-	-	
2	-	-	-	-	-	-	-	
3	-	-	-	-	-	-	-	
4	-	-	-	-	-	-	-	
5	-	-	-	-	-	-	-	
6	-	-	-	-	-	-	-	
7	-	-	-	-	-	-	-	
8	-	-	-	-	-	-	-	
9	6.81	-	-	-	-	-	-	
10	56.81	-	-	-	-	-	-	
11	106.81	-	-	-	-	-	-	
12	156.81	-	-	-	-	-	-	
13	206.81	-	-	-	-	-	-	
14	256.81	-	-	-	-	-	-	
15	306.81	-	-	-	-	-	-	
16	356.81	-	-	-	-	-	-	
17	406.81	-	-	-	-	-	-	
18	456.81	-	-	-	-	-	-	
19	506.81	-	-	-	-	-	-	
20	556.81	-	-	-	-	-	-	

[33]	Expected Loss $ to Tier	71.95	-	-	-	-	-	-	*=E{ Tier NS(i) }*
[34]	Expected Loss % to Tier	4.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[33] / [1] / [24]*
[35]	Yield Prem - Variable	**0.10**	**0.25**	**0.50**	**0.75**	**1.00**	**2.00**	**4.00**	*Input*
[36]	Yield Prem - Fixed	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	*Input*
[37]	Total Yield % on Tier	4.50%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[34] *(1+[35]) + [36]*
[38]	Total Yield $ on Tier	80.95	-	-	-	-	-	-	*=[37] * [1] * [24]*

Exhibit 12
CASE 12 -- Base Case with Half the CS

Inputs:			
[1]	**Committed Surplus**	**1,800.00**	*Input*
[2]	Risk Load as % of E[L]	**6.15%**	*Input*
[3]	Subject Premium	**1,000.00**	*Input*
[4]	Default-Free Rate	**5.00%**	*Input*

Calculated Values:							
[5]	Risk Load $	43.05	*=[2] * [6]*	[10]	Expected Value of F(T)	1,970.20	*=E{ [22] }*
[6]	Expected Loss	700.00	*=E{ [19] }*	[11]	**Expected Yield on CS**	**9.46%**	***=[10] / [1] - 1.00***
[7]	Initial Fund F(0)	2,543.05	*=[1] +[5] +[6]*	[12]	Total Yield $ Over All Tiers	80.37	*=Sum{ [38] }*
[8]	Expected Value of NS(i)	72.00	*=E{ [21] }*	[13]	**Required Yield on CS**	**9.46%**	***=[12] / [1] + [4]***
[9]	**Surplus Loss Rate**	**4.00%**	***=[8] / [1]***	[14]	**True Yield Premium**	**0.46%**	***=[13] - [9] - [4]***

[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]
Outcome # i	**Prob f(i)** *Input*	**Cum Prob F(i)** *=Cum Sum of [16]*	**Loss Ratio** *Input*	**Loss L(i)** *=[3] * [18]*	**Fund Contribution from Prem** *=[5] + [6]*	**NS(i)** *=Max(0, [19] - [20])*	**F(T)** *=[7] * (1.00 + [4]) - [19]*	**NS(i) as % CS** *=[21] / [1]*
1	**2.00%**	2.0%	**35.00%**	350.00	743.05	-	2,320.20	0.00%
2	**4.00%**	6.0%	**40.00%**	400.00	743.05	-	2,270.20	0.00%
3	**6.00%**	12.0%	**45.00%**	450.00	743.05	-	2,220.20	0.00%
4	**10.00%**	22.0%	**50.00%**	500.00	743.05	-	2,170.20	0.00%
5	**11.00%**	33.0%	**55.00%**	550.00	743.05	-	2,120.20	0.00%
6	**12.00%**	45.0%	**60.00%**	600.00	743.05	-	2,070.20	0.00%
7	**10.00%**	55.0%	**65.00%**	650.00	743.05	-	2,020.20	0.00%
8	**9.00%**	64.0%	**70.00%**	700.00	743.05	-	1,970.20	0.00%
9	**6.00%**	70.0%	**75.00%**	750.00	743.05	6.95	1,920.20	0.39%
10	**5.00%**	75.0%	**80.00%**	800.00	743.05	56.95	1,870.20	3.16%
11	**4.00%**	79.0%	**85.00%**	850.00	743.05	106.95	1,820.20	5.94%
12	**4.00%**	83.0%	**90.00%**	900.00	743.05	156.95	1,770.20	8.72%
13	**3.00%**	86.0%	**95.00%**	950.00	743.05	206.95	1,720.20	11.50%
14	**3.00%**	89.0%	**100.00%**	1,000.00	743.05	256.95	1,670.20	14.28%
15	**3.00%**	92.0%	**105.00%**	1,050.00	743.05	306.95	1,620.20	17.05%
16	**2.00%**	94.0%	**110.00%**	1,100.00	743.05	356.95	1,570.20	19.83%
17	**2.00%**	96.0%	**115.00%**	1,150.00	743.05	406.95	1,520.20	22.61%
18	**2.00%**	98.0%	**120.00%**	1,200.00	743.05	456.95	1,470.20	25.39%
19	**1.00%**	99.0%	**125.00%**	1,250.00	743.05	506.95	1,420.20	28.16%
20	**1.00%**	100.0%	**130.00%**	1,300.00	743.05	556.95	1,370.20	30.94%

Exhibit 12

CASE 12 -- Base Case with Half the CS

		Tier Definitions as % of CS							
	Tier #	1	2	3	4	5	6	7	
[24]	Limit	25.0%	25.0%	25.0%	25.0%	100.0%	200.0%	99999.0%	*Input*
[25]	Retention	0.0%	25.0%	50.0%	75.0%	100.0%	200.0%	400.0%	*Input*

[15]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	
Outcome # i	Tier 1 NS(i)	Tier 2 NS(i)	Tier 3 NS(i)	Tier 4 NS(i)	Tier 5 NS(i)	Tier 6 NS(i)	Tier 7 NS(i)	*Tier NS(i) = MAX(0, MIN([21] - [25] * [1], [24] * [1]))*
1	-	-	-	-	-	-	-	
2	-	-	-	-	-	-	-	
3	-	-	-	-	-	-	-	
4	-	-	-	-	-	-	-	
5	-	-	-	-	-	-	-	
6	-	-	-	-	-	-	-	
7	-	-	-	-	-	-	-	
8	-	-	-	-	-	-	-	
9	6.95	-	-	-	-	-	-	
10	56.95	-	-	-	-	-	-	
11	106.95	-	-	-	-	-	-	
12	156.95	-	-	-	-	-	-	
13	206.95	-	-	-	-	-	-	
14	256.95	-	-	-	-	-	-	
15	306.95	-	-	-	-	-	-	
16	356.95	-	-	-	-	-	-	
17	406.95	-	-	-	-	-	-	
18	450.00	6.95	-	-	-	-	-	
19	450.00	56.95	-	-	-	-	-	
20	450.00	106.95	-	-	-	-	-	

[33]	Expected Loss $ to Tier	70.22	1.78	-	-	-	-	-	*=E{ Tier NS(i) }*
[34]	Expected Loss % to Tier	15.61%	0.40%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[33] / [1] / [24]*
[35]	Yield Prem - Variable	**0.10**	**0.25**	**0.50**	**0.75**	**1.00**	**2.00**	**4.00**	*Input*
[36]	Yield Prem - Fixed	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	**0.10%**	*Input*
[37]	Total Yield % on Tier	17.27%	0.59%	0.00%	0.00%	0.00%	0.00%	0.00%	*=[34] *(1+[35]) + [36]*
[38]	Total Yield $ on Tier	77.70	2.67	-	-	-	-	-	*=[37] * [1] * [24]*