A METHOD FOR RISK QUANTIFICATION

FOR SURPLUS REQUIREMENTS

by Anthony Iafrate

BIOGRAPHY:

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ABSTRACT:

Required surplus and return on equity (ROE) are of concern to the owners of insurance entities. A comparison of actual to required surplus provides a measure of the security afforded to policyholders. Calculation of ROE for individual segments of an operation can be used to evaluate relative performance. In either case, surplus must be correctly allocated to each business segment. Historically these allocations have been based on rules of thumb relating premium to surplus.

This paper presents a model for determining the appropriate premium to surplus ratio for a business segment, given the insurer's risk appetite. A regression based procedure for parameter estimation is outlined. The estimation procedure is illustrated using sample data.

1. INTRODUCTION

Financial stability and expected profits are associated with the degree of leverage of an enterprise. The Capital Asset Pricing Model (CAPM), for example, contemplates a proportional relationship between incremental assumptions of risk and incremental expected returns. The ongoing contraction in the U.S. banking system is evidence of what may result when **expected** profitability is increased without due consideration of the risks assumed.

The notion that increased returns can be garnered by assuming additional risk is very basic to financial theory. A logical argument can be made that regulations designed to insure solvency will impact profitability. A goal of this study is to make the link between required surplus, probability of ruin and expected returns explicit. The study also puts forward a method for estimating the necessary variables.

Section 2 presents a probabilistic approach for determining the surplus required to support premium writings for each segment of the insurance operation. The surplus requirement is calculated by ensuring that the sum of premiums and surplus will be sufficient to pay losses and expenses in a specified percentage of possible outcomes. The criterion is equivalent to that used in ruin theory; however, a method for applying it to business segments is developed.

The focus will be on insurance liabilities; an extension to include asset risk is not conceptually difficult. In its broadest form, the model could provide an estimate of the "risk based capital" associated with a given probability of ruin. Along these lines Ang and Lai [1] present a pricing model which includes the impact of both asset and liability risk.

After estimating the required surplus, one can also estimate the return it is expected to earn. By using required surplus given the ruin probability, rather than available surplus, a true estimate of underwriting profitability is made.

Owners of the enterprise have a keen interest in this estimate because they can opt to own a portfolio of securities which closely matches the insurer's asset portfolio. If the incremental benefit derived from owning an insurance operation is not sufficient to cover the incremental risk assumed, then there is no incentive to own it.

The model can be used to measure the cost associated with reducing the probability of insolvency. As the probability of insolvency is reduced, the surplus required to write a given premium volume increases. As a result, the loss and expense ratio must be reduced (by increasing the premium per exposure) in order to maintain the previous return to surplus. An example is used to illustrate this dynamic.

As a matter of interest, the model also measures the diversification benefit accruing to multiple line underwriters.

In addition, the model has implications for constrained profit maximization models such as that discussed by Brubaker [3] and the more general problem of utility maximization.

Section 3 suggests a method for estimating the parameters of the model. Of course there are several ways to perform the estimation. The one presented is somewhat involved but it has some advantages which may offset the extra effort.

The estimation method attempts to quantify both underwriting and reserving risk. Underwriting risk relates to errors in estimating the **expected** pure premium for an exposure portfolio. Reserve risk differs since it relates to estimation error based on partial data. Reserve risk involves multiple years of exposures but should be less on a per exposure basis than underwriting risk. The relative importance of the two depends on loss ratio volatility and loss emergence and payment patterns.

The method involves regressing ultimate loss ratios on estimated loss ratios at 12 months maturity. The motivation is that the various errors are correlated (which leads to greater volatility than an assumption of independence) and the regression model produces a measure of this.

In Section 4 the presentation of sections 2 and 3 are applied to annual statement data for two large U.S. insurers. Some of the pros and cons of the estimation method are also discussed.

2. THE MODEL

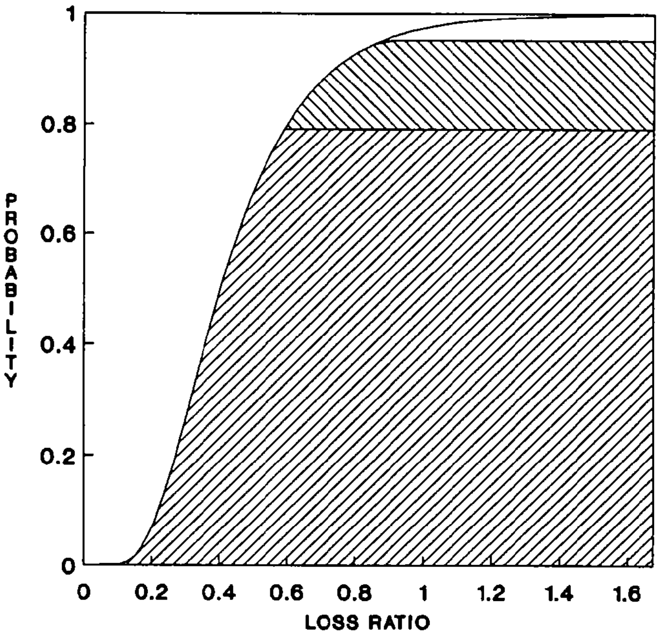
Allocation Criterion

Suppose an insurance operation desires to operate in a manner such that available surplus is adequate in Y proportion of all possible cases. This is referred to as the Yth confidence level. Y is also referred to as probability of ruin in the literature.

Figure 1 is a display of the lognormal probability distribution for a loss ratio which has a mean of 45% and standard deviation of 22.5%. The height of the region with northeast hatching represents the level of losses which premiums less overhead and commissions can cover. The height of the region with northwest hatching represents the surplus required to support the business at the 95% confidence level.

In the model Y is selected by management but it also is a measure of the insurance operation's financial security. Regulators could benefit from the ability to estimate Y values in their efforts to screen out troubled insurers. While the impact of asset risk is not treated in this paper, it would be included in an effective regulatory model.

FIGURE 1
Required Premium/Surplus Ratio



Y could be estimated using annual statement data. The estimate could be compared to a bench mark. The author believes this approach has advantages over the "risk based capital" proposals currently under consideration. First, since *Y* is defined in terms of ruin probability it provides a direct measure of the quantity of interest. Secondly, the meaning of *Y* is clearly stated, while the meaning of "risk based capital" is not. Thirdly, proper calculation of *Y* includes the correlation between all balance sheet items, rewarding firms with diversified asset and liability portfolios.

There are practical problems in producing a *Y* estimate. The ten years of data provided in the annual statement are not likely to suffice, so that old annual statements or other collateral sources are required.

Another problem is that probability distribution of surplus is a complicated function. For example, one might hypothesize that loss ratios are lognormally distributed while certain asset values are normally distributed. If the probability distribution of surplus is a combination of these, it would be difficult to determine analytically. This problem might be circumvented using numerical methods.

Model Development

An insurance portfolio is divided into N ($i=1,\dots,N$) segments each with the following characteristics:

LR_i represents the ultimate, discounted loss ratio for the ith segment of the operation.

$G_i(X)=Prob(LR_i \leq X)$ represents the cumulative distribution function of segment i. Let $m_i = E[LR_i]$ and $v_i^2 = Var(LR_i)$.

a_i represents the proportion of premiums earned in segment *i*.

It will be convenient to use matrix notation; bold type will denote matrices as follows:

$\mathbf{LR}$, $\mathbf{m}$ and $\mathbf{a}$ are N x 1 with *i*th component LR_i, m_i and a_i, respectively.

$\mathbf{v}$ is N x N with *ij*th component $v_{ij} = COV(LR_i, LR_j)$ and $v_{ii} = v_i^2$.

The model can incorporate loss adjustment expenses proportionate to losses or to premiums. That is, they may be included in the loss ratio or the expense ratio. Consider a one segment operation. Since N = 1, we will dispense with subscripts until the multi-segment operation is discussed. Operation at confidence level Y will require funds of C **per unit of premium**, where

(1) $$C=G^{-1}(Y).$$

Segment *i* need not produce 1 unit of gross revenue per premium unit on a present value basis due the timing of receipts. It will be convenient to treat (discounted) expenses as a constant proportion of premiums and deduct them from gross revenue. Let R represent the present value of premiums less expenses. As a result, supplemental funding of S is required per unit of premium, where

$$S=C-E[R]. \tag{2}$$

Since S, the additional funding, is a present value which, by definition, does not come from premiums, it must be supplied by the insurance entity. Therefore, interpret S to be the surplus per premium unit allocated to this segment. Therefore, the premium to surplus ratio for confidence level Y can be written as 1/S.

A Numerical Example

Assume LR is distributed lognormally. For a given confidence level we can associate a premium to surplus ratio with each (m,v^2) combination. The Table below displays coefficients of variation (cv's) rather than variances. This is a matter of convenience related to the lognormal distribution.

Table 2.1

Indicated Premium to Surplus Ratios
Loss Ratio Distribution: Lognormal
Confidence Level = .9999
E[R] = 1.0

CV	m = 0.90	m = 0.80	m = 0.70
0.5	0.41	0.48	0.59
0.4	0.58	0.70	0.89
0.3	0.89	1.12	1.52
0.2	1.60	2.25	3.80
0.1	4.60	12.15	*

* indicates a negative value.

As expected, for a given expected loss ratio, m, the indicated premium to surplus ratio increases as the cv decreases. In addition, the premium to surplus ratio can vary substantially depending on the segment's volatility.

It is also possible to express the premium to surplus ratio analytically. For the lognormal distribution the expression is

$$S^{-1}=\frac{1}{\exp(\Phi^{-1}(Y)\ln(cv^2+1)^{\frac{1}{2}}+\ln(m)-\frac{1}{2}\ln(cv^2+1))-E[R]} \tag{3}$$

where $\Phi^{-1}(Y)$ is the Yth percentile of the standard normal distribution.

As the table illustrates, it is possible for the indicated premium to surplus ratio to be negative. This occurs when the operation requires an amount **less than** collected premium to produce confidence level Y. In effect, each piece of this business written increases capacity (reminiscent of the loaf of bread which became larger as it was eaten).

The premium to surplus ratio calculation performed for a mono-segment operation is useful because (i) it determines how much (homogeneous) business could be written, given the surplus level and desired confidence level. (ii) The procedure can be used to determine the probability of ruin[1] given earned premiums and surplus in addition to the loss ratio distribution.

ROE Calculation

Ignoring income taxes, the return on equity can be written as

$$ROE=\frac{R-LR}{S} \tag{4}$$

Assume that credit worthiness and loss experience are independent so that $E[R|LR]=E[R]$. Then return on equity (ROE) is linear in LR, and $G(x)$

[1]Ignoring asset risk.

implicitly defines a probability distribution for ROE over the interval $(-\infty, \frac{R}{S}]$. It also follows that $E[ROE]=\frac{E[R]-m}{S}$ and

$VAR(ROE)=\left(\frac{V}{S}\right)^2$.

Consider a sample calculation:

Sea of Tranquility Re reinsures a portfolio of flood insurance risks. Historical data indicates that the average loss and LAE ratio, after discount, is 45% with a coefficient of variation of .50. The net present value of premiums less overhead and commissions is 60% of writings. Assume the lognormal distribution suitably model the loss ratio.[2] Using equation (3) the required premium to surplus ratio is 1.64 at the 99% confidence level.

The resulting mean ROE is 24.6% (.246 = 1.64 X (.60 - .45)). The c.v. for the ROE random variable is 1.5 (1.5 = (.5 X .45 X 1.64)/.246).

Now suppose that the regulatory body restricts operations so that probability of ruin can not exceed 0.1% (Y = .999). Continuing with the previous assumptions, the premium to surplus ratio falls to .94 with ROE falling to 14%. The ROE cv is still 1.50. To re-establish the prior expected ROE, a 37.8% rate increase is required; however, a byproduct of the rate increase is a reduction in the ROE c.v. to 1.255. (The ROE cv falls even though ROE variance is unchanged because the mean is increased.) Therefore, a rate increase somewhat less than 37.8% is needed to maintain the insurer's previous level of well-being.

The accompanying Table 2.2 illustrates the effect of various rate level changes.

[2] One might question the appropriateness of the lognormal assumption for such a risk. It is used in keeping with the development of the previous example.

Table 2.2

Tranquility Re
Rate Level Impact on ROE Distribution

Expected Loss Ratio	Rate Level Change	Expected ROE	ROE C.V.
45%	+ 0.0%	14.0%	1.50
40	+12.5	17.0	1.33
35	+28.6	21.5	1.17
30	+50.0	29.3	1.00
25	+80.0	45.9	0.83

The example ignores a change in demand in response to the rate change. Several observations could be made here: (1) Since the coverage is more secure, it becomes more desirable from the purchaser's standpoint. (2) Since the analysis is on a per premium unit basis, changing demand will distort our results only to the extent that adverse selection comes into play.

Multi-Segment Analysis

The multi-segment operation considers the covariance of segment results. Let the subscript *A* denote aggregate statistics for the portfolio of segments. Then

(5) $$m_A = \sum_{i=1}^{N} a_i m_i$$

and

(6) $$LR_A = \sum_{i=1}^{N} a_i LR_i$$

are aggregate expected and actual loss ratios, respectively. Also define $G_A(x)$ as the cumulative distribution function of LR_A. Once these are determined, the multi-segment operation can be treated as if it were a single segment. So that

(7) $$S_A = C_A - E[R_A]$$

where $C_A = G_A^{-1}(Y)$.

However, this analysis allocates surplus to each segment in order to evaluate relative performance based on ROE.

If the individual segments were viewed separately and weighed together without considering their correlation, the quantity $\tilde{S}_A$ would be calculated as

$$\tilde{S}_A = \sum_{i=1}^{N} a_i S_i = C_A - E[R_A]. \tag{8}$$

The relationship between $\tilde{S}_A$ and S_A is expressed as a ratio, Q, where

$$Q = \frac{\sum_{i=1}^{N} a_i S_i}{S_A} = \frac{\sum_{i=1}^{N} a_i (C_i - E[R_i])}{C_A - E[R_A]}. \tag{9}$$

The premium to surplus ratio for segment i becomes Q/S_i. In this method surplus is distributed to each segment in proportion to its need on a stand alone basis. The constant of proportionality, Q, is a measure of diversification benefit. For $Q>1$ there is a benefit, in terms of reduced surplus requirements, to writing multiple segments simultaneously.

A Digression on ROE Maximization

Brubaker [3] describes a constrained profit maximization model for insurers. His constraint is a maximal premium to surplus ratio which varies by segment. Since covariance is not considered, the constraint is linear. As a result, Brubaker mentions that probability of ruin constraints will lead writing only the segment with the highest ROE.

If segment covariance is accounted for, the constraint typically will not be linear. In terms of our presentation, the production constraint could be stated as

$$Y \geq G_A^{-1}(\Sigma a_i(LR_i - E[R_i]))$$

Generally, this defines a convex set of production possibilities. And the typical ROE maximizing production point will include more than one business segment.

Risk Equalization

ROE's by segment are not directly comparable unless the level of risk undertaken per premium unit is equalized. Variance is commonly used to measure risk in financial theory. But the allocation method described here does not equalize ROE **variance** across segments.

In a decision theoretic framework it may be desirable to weigh favorable and unfavorable deviations differently. Since the confidence level measures uncertainty in only one tail of the distribution (where the undesirable deviations reside), it provides a more accurate representation of a segment's riskiness. Consider two insurance contracts to be written for the same premium but with the loss ratio distributions shown in Table 2.3 on the following page.

While each contract has the same mean and variance, B will require more surplus than A if the operation is to perform at the 99% confidence level.

For the purposes of this study, risk is defined to be the probability that a segment will fail, i.e. run out of capital while paying losses. According to this definition, the allocation method presented above equalizes risk across segments. Therefore, the ROE's are comparable.

Table 2.3
Probabilities

Loss Ratio	Contract A	Contract B
0.00	.01	.00
0.49	.00	.98
0.50	.98	.00
1.00	.01	.02
Mean	.5	.5
Variance	.005	.005
99% Conf. Level	.5	1.00

Interest Rates and Surplus

So far some basic concepts have been used without definition. The interest rate that should be used to discount losses is the risk free market rate for a security whose duration matches the loss payout profile. Any return in excess of the risk free rate will entail bearing some risk in addition to that assumed from insureds. Since the ROE relates to the performance of an insurance entity and not an investment banker, the return to asset risk should be excluded.

Clearly an owner of an insurance entity is interested in overall ROE. However, if it is presumed that the owner could duplicate the insurer's asset portfolio on his own, why would he assume additional (underwriting) risk by owning an insurer? What is the return on this risk assumption? That is the question we must answer.

Some might argue that the rate used in loss discounting should be less than the risk free rate. This case is presented by Butsic [5] for use in evaluating loss liabilities and his arguments are correct for his stated purpose. The key to Butsic's discussion is that there is risk entailed in loss reserves. The approach taken here is to allocate surplus to cover loss reserve risk rather than to overstate their present value to provide a safety margin.

Again, from the point of view of the insurance operation owner, statutory surplus is not a particularly meaningful concept. GAAP equity is much closer to the true value of capital tied up in the operation. The term surplus is used throughout this paper in deference to the widespread use of "premium to surplus" as a measure of leverage. Insurers are held accountable by rate regulators as well as stockholders. As a practical matter, our "*S*" may represent different quantities for different audiences.

3. REGRESSION ESTIMATES

Background

Implementation of the model will require four inputs.

(A) The *a* vector must be determined. A prospective ROE calculation could be based on recent history or management's forecast. In a retrospective calculation the elements are known quantities.

(B) The ***m*** vector must be estimated. One alternative is to at least partially base ***m*** on the profit and expense provisions in the rates - these could be adjusted for perceived adequacy. Another approach, which is detailed in the next sections, uses historical loss ratios discounted and projected to ultimate to calculate ***m***.

(C) The ***v*** matrix must be estimated. Table 1 showed how dependent the indicated premium to surplus ratio is on loss ratio volatility. This makes proper estimation of ***v*** crucial.

One generalization that can be made here is that slow developing lines tend to be more volatile than fast developing ones. This may be caused by the inordinate length of time required to formulate an accurate estimate of rate level need. In addition these lines tend to be characterized by high-severity and low frequency, which produces volatility.

This kind of reasoning could be used to create an ordinal ranking of segments by their expected loss ratio variances. Such a ranking has limited value in a multi-segment analysis since covariances play an important role.

A selection of v should make maximum use of available data. While this is true of any estimate, it should be stressed that intuition in this area is generally not too good. Estimates based on judgement will tend to be biased and difficult to support in this area. Actuaries do not have great deal of experience in performing these estimates.

(D) The loss ratio distribution must be chosen. This paper will not deal with the particulars of this process. To the extent that the distributions they discuss are relevant, Hogg and Klugman [8] present a clear discussion of how this can be done.

An alternative to the approach presented in the following section is the calculation of means, variances, and covariances directly as sample statistics. Such an approach is appropriate if the business mix within and across segments has remained stable over time. However, if premium volume has grown at different rates in the various segments or if it is desirable to include the effects of loss reserve volatility with pure loss ratio volatility, the procedure outline here will be beneficial.

The estimation process consists of regressing the ultimate, discounted loss ratio for each segment on the discounted loss ratio estimates at 12 months development for all lines. As a result the combined mean squared deviation of reserving misestimation and loss fluctuation (process error) is estimated.

This mean squared deviation can be decomposed into three components: (i) random noise, (ii) discounted, ultimate loss ratio variance combined with reserve deviations and (iii) correlation between (i) and (ii).

Estimation Process

The necessary data consists of ultimate, discounted loss ratios, by business segment and accident year. In addition, the corresponding estimates of ultimate loss ratios are made for reserving purposes at 12 months maturity for each accident year.

Say that T (t=1,...,T) accident years are included in the study. Then 2 T x N matrices can be constructed, namely ***LR*** and ***ELR***. Where the *ti*th element of ***LR***, LR_{ti} is the discounted, ultimate loss ratio in year *t* for segment *i*. ***ELR*** denotes the discounted loss ratio using estimated incurred losses at 12 months maturity in lieu of current ultimates. Let $\boldsymbol{LR}_{.i}$ and $\boldsymbol{ELR}_{.i}$ be the *i*th columns of ***LR*** and ***ELR***, respectively.

The first set of estimates made is through the use of a series of N regressions, each of the form:

$$LR_{ti}=\alpha_i+\boldsymbol{ELR}_{t.}\boldsymbol{\beta}_i+e_{ti}. \tag{10}$$

In words, the regression is estimating the ultimate loss ratio in year *t*, segment *i* as a linear function of the 12 month maturity estimates (for year *t*) of loss ratios for all segments. The random noise term e_{ti} has mean 0 and variance σ^2.

The use of ^ above a character will denote regression estimates. So that

$\hat{LR}_{ti}=\hat{\alpha}_i+\boldsymbol{ELR}_{t.}\hat{\boldsymbol{\beta}}_i$ and $\hat{e}_{ti}=LR_{ti}-\hat{LR}_{ti}$.

It is possible to compose the N x N matrix $\hat{\boldsymbol{\beta}}$ whose *i*th column, $\hat{\boldsymbol{\beta}}_i$, is the vector estimated in the segment *i*th regression. In addition, let $\hat{\boldsymbol{\alpha}}$ be the 1 x N matrix composed of the estimates $\hat{\alpha}_i$. Then the system of TN equations can be written as

(11) $$\boldsymbol{LR}=\mathbf{1}_T\hat{\boldsymbol{\alpha}}'+\boldsymbol{ELR}\hat{\boldsymbol{\beta}}+\hat{\boldsymbol{e}}.$$

With $\mathbf{1}_T$ being the T x 1 matrix with each element equal to 1.

Let the vector $\boldsymbol{m}$ be composed of N elements, the *i*th being $m_i=\sum_{t=1}^{T}\frac{\hat{LR}_{ti}}{T}$. In other words, the estimated mean for segment *i* is the mean of the regression estimates. These estimates are unbiased. To the extent that a segment is correlated to others, the use of *ELR* from all segments will reduce the variance of the estimated mean for the *i*th segment.

Now define the N x N matrix u as follows:

(12) $$u = \frac{(\boldsymbol{LR}-\boldsymbol{ELR})'(\boldsymbol{LR}-\boldsymbol{ELR})}{T-1}$$

where u can be calculated as the sum of the following component matrices

(13.i) $$\frac{\hat{\boldsymbol{e}}'\hat{\boldsymbol{e}}}{T-1}+$$

(variance-covariance matrix of residuals)

(13.ii) $$\frac{\hat{\mathbf{a}}\mathbf{1}_T'\mathbf{1}_T\hat{\mathbf{a}}'+(\boldsymbol{\beta}-\mathbf{1})\,\boldsymbol{ELR}'\boldsymbol{ELR}(\boldsymbol{\beta}-\mathbf{1})'}{T-1}+$$

(variance-covariance matrix of loss ratios loaded for reserve error)

(13.iii) $$\frac{2}{T-1}\left(\hat{\mathbf{a}}\mathbf{1}_T'+(\boldsymbol{\beta}-\mathbf{1}_{NN})'\boldsymbol{ELR}'\right)\hat{\boldsymbol{e}}$$

(covariance matrix of residuals and reserve errors).

The matrix $\mathbf{1}_{NN}$ is N x N with each element equal to unity.

The decomposition is useful for checking the reservist's track record. By comparing (13.ii) to the sample variance of the LR_i's the attenuation or exacerbation of volatility attributable to reserve error can be calculated. More to the point of the ROE analysis, the future effect of reserve errors may be expected to differ from past errors. If this is the case, (13.ii) can be adjusted in calculating u.

4. SAMPLE CALCULATION

Industry data taken from the 1990 edition of *Best's Averages & Aggregates* has been used to illustrate the procedure described above. Two lines, workers compensation (WC) and private passenger auto liability (AL), were selected to serve as the two segments for our sample insurance operation. Schedule P paid and incurred development was used to produce discounted, ultimate loss ratios. Incurred losses at 12 months maturity were divided by premiums to produce the *ELR*'s needed.

Since Schedule P contains 10 accident years of data we have T=10 with N=2. Previously it was mentioned that a long time series is of great benefit when data suffers from cyclicality, the 10 year period used here does not speak to this issue; however, for the sake of illustration it will suffice.

Average lag to payment was calculated to be 4.4 years for WC and 2.5 years for AL. The interest rate used for discounting is the weighted average of the nearest two Treasury note rates for the year in which the premium was earned.

Tables 4.1 and 4.2 contain statistics calculated from the data.

Table 4.1
Workers Compensation (i=1)

Year	Undisc., Ultimate Loss Ratio	Interest Rate	Disc. Ultimate Loss Ratio	ELR
1980	67.7%	11.4%	46.3%	51.6%
1981	68.9	14.2	44.0	45.5
1982	76.9	13.1	50.4	48.3
1983	87.1	10.7	60.8	54.8
1984	101.2	12.2	67.9	56.8
1985	79.2	10.2	56.0	59.6
1986	92.6	7.3	71.1	62.6
1987	90.3	7.8	68.5	61.9
1988	94.2	8.4	70.2	61.2
1989	89.5	8.6	66.3	61.7

Table 4.2

Private Passenger Auto Liab. (i=2)

Year	Undisc., Ultimate Loss Ratio	Interest Rate	Disc. Ultimate Loss Ratio	ELR
1980	70.4%	11.6%	54.5%	56.3%
1981	76.1	14.4	55.8	56.2
1982	77.8	13.0	58.7	58.9
1983	81.4	10.3	64.7	63.2
1984	87.6	11.8	67.6	63.5
1985	90.1	9.6	72.6	68.3
1986	86.2	7.0	73.3	71.5
1987	84.6	7.4	71.4	71.0
1988	85.9	8.1	71.4	70.9
1989	88.3	8.6	72.6	71.2

Based on the regression procedures we calculated m_1 = .601 and m_2 = .662. Evaluation of (13.i-iii) yields the following:

(13.i')

$$\hat{\varepsilon}'\hat{\varepsilon}=\begin{bmatrix} .019201 & .000897 \\ .000897 & .002566 \end{bmatrix}$$

(13.ii')

$$\hat{\alpha}1_T'1_T\hat{\alpha}'+(\beta-1)'ELR'ELR(\beta-1)'=\begin{bmatrix} 1.895773 & 1.771147 \\ 1.771147 & 2.38666 \end{bmatrix}$$

(13.iii')

$$2\left[\hat{\alpha}1_T'+(\beta-1_{NN})'ELR\right]\hat{\varepsilon}=\begin{bmatrix} 4.7x10^{-10} & -3.2x10^{-16} \\ 4.0x10^{-10} & -1.8x10^{-16} \end{bmatrix}$$

Summing the (13.i'-iii') yields the estimated u matrix

(14)

$$u=\begin{bmatrix} 1.914974 & 1.772044 \\ 1.772044 & 2.389226 \end{bmatrix}$$

Assume that both segment loss ratios are lognormally distributed and that $a_1 = a_2 = .50$. Ignoring covariance, the premium to surplus ratio could be calculated for each line using (3). The results are summarized as follows for the 99.99% confidence level: The cv's for segments 1 and 2 are .767512 and .778304. When combined as indicated by the a_i's the overall cv is found to be .739371. The indicated premium to surplus ratios are .31 and .27 for segments 1 and 2 on a stand alone basis. The benefit to diversification is found to be 7%, i.e. $Q = 1.07$. *On a combined basis the indicated premium to surplus ratio is .31.*

The indication is far from the industry standard of 2.0. A possible cause of such an unexpected outcome may be that the lognormal distribution is not appropriate for modelling industry-wide loss ratios. Another possible cause is that actual diversification benefits will be greater as more than two segments are mixed together. Greater diversification, of course, allows for more leveraged writings.

Finally, and probably most significantly, the data used clearly exhibited a trend in loss ratios. No correction was made for this trend prior to performing the regressions. Consequently, true loss ratio variance probably was overestimated. Connected to this are previous comments regarding the length of the time series used.

5. CONCLUSION

This study presents a method for statistical estimation of the appropriate premium to surplus ratio for insurance segments. The results could be used to perform ROE analyses. Insurers are gradually moving away from reliance upon "industry standards." One illustration is the recent movements towards a risk based capital model. The author believes the model presented here possesses a certain appeal because it provides for estimation of an industry standard concept using statistical methods.

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