# OVERLAP REVISITED:

# THE "INSURANCE CHARGE REFLECTING LOSS LIMITATION" PROCEDURE

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## ABSTRACT:

Several years ago, the retrospective rating plan premium formula was modified with the intention of eliminating "overlap" between its insurance charge and excess loss premium components. Under the modified formula, the excess loss premium is reduced via subtraction of an Excess Loss Adjustment Amount (ELAA). Now, the ELAA approach is likely to be supplanted by an alternative methodology, called ICRLL (Insurance Charge Reflecting Loss Limitation), under which the insurance charge is calculated net of overlap and no adjustment is made to the excess loss premium. ICRLL uses the standard system of equations, modified to correctly reflect the loss limit, along with a column shifting procedure to arrive at a better set of insurance charge values to use for solving the equations.

The paper starts with an introductory discussion of overlap and a brief review of the literature. After that, the ELAA methodology is described and its vulnerability to inconsistency is explained and demonstrated. The final section of the paper is devoted to an exposition of the ICRLL procedure. As a topic for future research, the author also presents an alternative "variance matching" approach to the selection of insurance charge values.



## BIOGRAPHY:

The author is an Assistant Vice-President at CIGNA where he is the Director of Actuarial Research. He has a Phd in Mathematics (1980) from Rutgers University and received his undergraduate degree from Michigan State in 1973. The author has attended NCCI Individual Risk Rating Plan Committee meeting for many years and currently chairs the committee. He has also taught retrospective rating seminars for the CAS Part 9 exam.

# OVERLAP REVISITED -

## The "Insurance Charge Reflecting Loss Limitation" Procedure

## I. INTRODUCTION

A retrospective rating plan is a plan in which the premium is adjusted based on actual loss experience under the policy contract. The "retro" premium is recalculated at each of various evaluation dates using per accident limited losses and is subject to "Max" and "Min" premium constraints. Under the National Council on Compensation Insurance (NCCI) Retrospective Rating Plan Option V (formerly Retro D), the per accident "Loss Limit" is elective, and, within allowable bounds, the choice of the Max and Min is up to the policyholder. Between the Max and the Min, the retro premium can be expressed as the premium tax loaded sum of a fixed component and a component proportional to limited losses.

The fixed component includes an "insurance charge" to cover losses over the Max, and an offset "insurance savings" provision for losses below the Min. The fixed component also includes an "Excess Loss Premium" for losses in excess of the Loss Limit. Thus the total charge for premium limitation and per accident limitation is the sum of the Excess Loss Premium and net insurance charge, where the net insurance charge is the difference between the insurance charge and the insurance savings.

The net insurance charge is included in what is called the "Basic" portion of the retrospective premium. The Basic also contains an expense component. Actually, expenses may be recouped in whole or part through application of a factor (the Loss Conversion Factor or LCF) to the limited losses, net insurance charge, and excess loss premium, or via the expense component of the Basic. The selection of the LCF is up to the insured, though it is frequently chosen to cover loss adjustment expenses. To summarize, retro premium is given as the sum of the Basic plus Converted Excess Loss Premium plus Converted Limited Losses, all times a Tax Multiplier and subject to Max and Min constraints.

The overlap problem arises if the net insurance charge is calculated as if there were no loss limit. If one assumes that the Excess Loss Premium exactly covers losses in excess of the loss limit, then the resulting fixed charges will be in error and, most likely, be redundant. A bias for redundancy arises because large accidents increase the odds of "blowing the max". This can lead to some double-counting since the expected losses from large accidents will contribute to the insurance charge as well as to the Excess Loss Premium.

There is also a more subtle source of error which stems from the neglect of the excess loss premium in the insurance charge calculation. Consider that a savings has been credited for loss scenarios in which the formula retro premium, sans the excess loss premium, falls below the min. However, once the excess loss premium is included, the formula retro premium could well turn out to exceed the min, even if there are no losses. From a more general perspective, it follows that an excess insurance savings is credited when the net insurance charge is calculated while neglecting the excess loss premium. In rare cases, the impact could be so large that it offsets the bias towards redundancy in the insurance charge. In those cases, the correct fixed component is higher, rather than lower, when a loss limit is elected.

This "underlap" phenomenon is relatively rare and is so counterintuitive as to be hard to believe. Skeptics should consider the case in which the Max is huge and the Min is big enough to force the net insurance charge to be substantially negative. If a Loss Limit of $0 is now elected, the Excess Loss Premium is theoretically equal to total expected Losses. The sum of the Excess Loss Premium and net insurance charge is thus less than total expected losses; hence there is underlap.

In keeping with the usual terminology, "overlap" shall be used to refer to any inaccuracy in the total charge for premium limitation (between the max and min) and per accident loss limitation. After discussing the literature briefly and reviewing the current method for handling overlap, a new method will be presented.

## II. OVERLAP IN THE LITERATURE - A BRIEF REVIEW

Before turning to the new method for solving the overlap problem, it is useful to review the literature and the current overlap adjustment procedure.

In Meyers [4], overlap was calculated by finding the correct net insurance charge using a retrospective premium adequacy equation. Under his approach, it was the net insurance charge and not the ELPF that was modified to account for overlap. Because his calculation of the correct insurance charge was fairly difficult, he did not advocate using such an approach in practice. He also observed that, when ELPF were adequate, the correct insurance charges (net of overlap) were roughly equal for risks of different severity.

Overlap under California's tabular retrospective rating plan has been handled via the "Table L" approach as discussed by Skurnick [3]. This entails defining a Table L charge to include both the Excess Loss Premium and Net Insurance Charge together in one Table L Charge which is net of overlap. The general opinion of this solution has been that it is workable only when there is a rather restricted choice of loss limits and only a small number of differing ELPF tables. Since the NCCI plan allows wide latitude in loss limits and since ELPF vary by Hazard Group and by state, the feeling was that the Table L approach would result in a "ruinous tide of paper".

In his review of Meyers, Feibrink [5] also advocated adjustment of the insurance charge in order to remove overlap. However he partly disputed Meyers observation about equality of (correct) insurances across severity class. Feibrink also noted that the calculation of the correct insurance charge could be done in more traditional fashion, without using the Retrospective Premium Adequacy equation.

The traditional calculation is described in both Snader[2] and Stafford [1]. It is an iterative procedure in which one seeks to find maximum and minimum "entry ratios". With entry ratios, one expresses a dollar loss amount as a fraction relative to expected (unlimited) loss. The "max" and "min" entry ratios correspond to the loss amounts at which the Max and Min premium constraints will be reached. The "ratio difference" between the max and min entry ratios may be computed by differencing the Max and Min premium factors and then dividing by the converted expected loss ratio grossed up for premium tax. Using a mathematical relation between the insurance charge and saving and writing out the equation for the loss necessary to achieve the Min, one can arrive at an equation for the "value difference" between the insurance charge at the min less the insurance charge at the max. This is all described in more detail in Appendix A. What Feibrink may have been alluding to was the possibility that such a "ratio difference - charge difference" system could be used with appropriate modification when there was a loss limit.

## III. <u>THE ELAA PROCEDURE TO ELIMINATE OVERLAP</u>

The National Council on Compensation Insurance (NCCI) 1984 Retrospective Rating Plan Manual prescribed a retrospective premium calculation for Rating Option V (formerly Retro D) slightly different from that of it predecessors. In particular, that manual featured an adjustment to the excess loss premium. This adjustment was designed to eliminate "overlap" between the net insurance charge and excess loss premium.

It should be stressed that the net insurance charge calculation procedure was not altered. Target "charge difference" and "ratio difference" numbers are computed based on plan parameters which do <u>not</u> depend on the loss limit. Neither the Table of Insurance Charges nor the net insurance charge calculation routine was altered in any way to reflect the impact of loss limitation.

The excess loss premium before adjustment is computed as the product of Standard Premium times the appropriate Excess Loss Factor (ELF). The terminology here was newly introduced in order to clearly distinguish between excess loss premium before and after adjustment. Theoretically, the ELF is the ratio of expected losses in excess of the per accident loss limit over Standard Premium. ELF are displayed in state tables by hazard group and loss limit.

The excess loss premium after adjustment is the product of a computed Excess Loss Premium Factor (ELPF) times Standard Premium. The ELPF is obtained by subtracting an Excess Loss Adjustment Amount (ELAA) from the ELF. In addition, a constraint is imposed so that the ELPF is at least 10% of the original ELF.

## III.1. ELPF Formula

$$ELPF = \text{Max}\,(ELF - ELAA,\ 10\%\ ELF)$$

where

ELPF = Excess Loss Premium Factor
ELF = Excess Loss Factor
ELAA = Excess Loss Adjustment Amount

The ELAA for Retro Rating Option V are displayed in various expected loss tables according to loss limit and maximum entry ratio. Interpolation is used to derive the ELAA for intermediate values.

The ELAA approach has some obvious practical strongpoints. It handles the overlap problem with a few relatively small grids of numbers and no unduly cumbersome calculations.

However there are problems of a practical nature with the ELAA methodology; in particular concerning state exceptions, and trend updates. First, since the ELAA are listed as fractions relative to Standard Premium, though they ultimately depend on the expected loss, there should be different ELAA tables depending on the ELR (Expected Loss Ratio ) parameter used in retro rating. Because ELRs are updated periodically and are not necessarily the same for all states, ELAA tables have been changed over time, and there are now several sets of current ELAA tables.

Another practical difficulty is that there seems to be no relatively easy way to update the ELAA for trend. Yet, in principle, the ELAA should be updated along with the updates of the Expected Loss ranges used for entering Table M.

In addition to the practical updating problems, there are several problems arising from the nature of the ELAA methodology. First, observe that the ELAA not depend on the state or Hazard Group or the ELF of a risk. Thus, the procedure presupposes that the adjustment for overlap at a particular loss limit is the same for risks with different severity distributions. Initially, this may not seem troubling if one relies on the observation made by Meyers [4] to conclude that overlap should be relatively constant for risks of different severity. While the relative independence of (overlap-adjusted) insurance charges on risk severity was disputed by Feibrink [5], perhaps the loss of accuracy is slight, and not too large a price to pay.

However, consideration of severity raises not only questions of accuracy, but also the spectre of anomalies and inconsistencies. Indeed, in the procedure that was implemented, a "10% floor" was adopted to prevent a possible anomaly. In certain cases, if the ELF is small, the subtraction of the ELAA from the ELF could even incorrectly result in a negative number. The "10% floor" rule prevents such an overcorrection for overlap.

Unfortunately, there is a problem of inconsistency which a simple "floor" rule cannot resolve. The problem arises in the comparison of plans with different loss limits. If the ELAA at two different loss limits differ by more than the difference between the ELFs, then an inconsistency will result. This may be seen by comparing the total charges for premium limitation and per accident loss limitation. The total will be lower for the plan with the lower loss limit. While this inconsistency is rare, it can happen as is shown in Exhibit 1.

Such inconsistencies between ELF gradation by limit for particular states and hazard groups versus the gradation of countrywide ELAA does not necessarily mean that either are incorrect. Rather, it means that when it comes to overlap, one set of ELAA does not necessarily work for all states and hazard groups.

The other major objection against the ELAA procedure is that it does not directly consider the Min. Recall from the initial discussion that part of the error in total limitation charges is due to the "underlap" overstatement of savings below the Min. However, to reflect this "Min dependence" explicitly would require indexing ELAA with both the "min" as well as the "max" entry ratio. When the ELAA procedure was first adopted this idea was briefly considered, but was rejected as being impractical. Thus, we have ELAA generated to be theoretically accurate at most for a particular Min.

In addition to the inaccuracy, neglect of the Min can also lead to inconsistency when comparing plans with different Mins. The problem is not that the plan with the higher Min has a lower total limitation charge; that is to be expected. What is problematic is that the retro premium for the "High Min" plan could turn out to always be less than the retro premium for the "Low Min" plan. This can happen if the "Low Min" plan has a fixed component exceeding the min of the "High Min" plan. An example of this is shown in Exhibit 2.

## IV. THE ICRLL PROCEDURE

A procedure called the ICRLL (Insurance Charge Reflecting Loss Limitation) procedure has recently been approved by the NCCI Individual Risk Rating Subcommittee (IRRPS) as the latest solution to the overlap problem. Under ICRLL, the insurance charge is calculated net of overlap, and no adjustment is made to the excess loss premium. Pending approval by the appropriate NCCI committees and subsequent approval by state regulators, ICRLL will supplant the ELAA methodology.

The strategy behind the ICRLL procedure is to follow the theoretically exact algorithm to the degree possible allowing some compromise of accuracy, but not of consistency, in order to attain a practical method. The theoretically correct solution is derived in Appendix B. There it is shown that the correct net insurance charge can be found using a "ratio difference - value difference" system similar to the traditional one, except that all entry ratios and insurance charge values should be considered in relation to expected limited, rather than unlimited, losses. The author has long advocated use of this "limited loss" system of equations as an essential facet of any solution to the overlap problem.

To obtain a theoretically correct solution would also require that the table of insurance charges be based on limited losses. Note that the required limited loss insurance charge is not the same as Skurnick's Table L charge. The Table L charge includes the excess losses and is expressed as a ratio against expected unlimited losses. The limited loss insurance charge value is a ratio relative to expected limited losses and it does not include the excess losses.

Thus, in principle, a separate table of insurance charges would be needed for each loss limit and state and hazard group. Such a multitude of tables would scarcely be practical. However, at a small cost in accuracy, one can make do with but a single table of insurance charges. The trick is to select the column of insurance charges within that table in such a way as to reasonably approximate the column of charges from a theoretically correct table. Instead of having multiple tables, one shifts columns within a single table. Under ICRLL, insurance charge column selection depends not only on expected unlimited losses, but also on risk severity and the loss limit.

Based on a proposal made by Robin Gillam of the NCCI and refined by the IRRPS, the ICRLL procedure features a column selection algorithm only slightly more complicated than the current assignment via expected (unlimited) loss range. Before discussing the ICRLL adjustment, it should be noted that a separate State/Hazard Group adjustment has been approved by the IRRPS. Under this "S/H" severity adjustment, expected unlimited losses are multiplied by a S/H severity factor in order to determine the loss amount used in entering the table of Expected Loss ranges. For example, if a particular State and Hazard Group has average severity 25% above the national average, then the S/H multiplier would be .8 (1/1.25). If expected unlimited losses were $100,000, then a figure of $80,000 would be used to enter the Loss Group ranges. In theory, if a uniform severity transformation (akin to uniform trend) explained all differences in severity, this S/H adjustment procedure would be exactly correct.

Under ICRLL, a further multiplier is applied in calculating the "losses" to be used in selecting the Loss Group. To summarize

IV.1. **"Losses" Used for Loss Group Selection**

$$LUGS = E[L] \bullet m(S/H)\, m(k)$$

where

LUGS = "Losses" Used for Loss Group Selection

E[L] = Expected (Unlimited) Losses

m(S/H) = State/Hazard Group Severity Multiplier

m(k) = ICRLL Multiplier for reflecting loss limit k.

It should be noted that the "m(k)" adjustment can move the loss group to a more appropriate selection, but there is no general simple assumption under which it is theoretically exact. This is because loss limitation should change the table of charges and not just the column out of which the charges are taken.

Given that there is no "exact" formula, a reasonably behaved formula is used in the ICRLL procedure under which the multiplier is expressed as a function of the loss elimination ratio (LER).

## IV.2. Multiplier for Reflecting Loss Limitation

$$m(k) = (1 + \alpha \bullet LER)/(1-LER)$$

where

$\alpha$ = constant

LER = Loss Elimination Ratio = ELF/ELR

The constant, $\alpha$, can be chosen so that the ICRLL plan gives answers fairly close to theoretically correct ones. An examples of the ICRLL procedure is shown in Exhibit 3. This should be compared to the examples of the ELAA procedure shown in Exhibit 2. Note that the ICRLL procedure forestalls occurrence of the "min" problem.

The biggest advantage of ICRLL is its near invulnerability to inconsistencies. This follows directly from its solid theoretical foundation. As well, it promises to be more accurate over a wider range of scenarios. Finally, it has no updating problems, because there is nothing to update.

Probably its biggest disadvantage is that it requires an iterative recalculation of the insurance charge every time one wants to price another loss limit. This is not as telling a disadvantage as it might seem to be. In the computer age, such calculations can be done rather quickly. With this in mind, some have even suggested using an aggregate distribution simulation or approximation model to generate limited loss insurance charges for each pricing quote. While this could be done, it would turn the insurance charge calculation into a "black box" methodology. Under ICRLL, the underwriter or actuary can still use the manual to verify any particular quote. The computer is thus a labor-saving device under ICRLL and not a mystic oracle.

From a theoretical perspective, the most vulnerable step of the ICRLL calculation is in the selection of the Table M column. The selection process is reasonable and should lead to no great error, but it can likely be improved with future research.

To understand the issues here, consider the multiplier formula. Note that when there is no loss limit, the LER becomes zero, and the multiplier is unity. This is as it should be. However, as the loss limit approaches zero, the multiplier asymptotically approaches infinity. The LUGS also approaches infinity and thus, the loss group of an infinitely large risk. While this is certainly in the right direction, the author believes that it may go too far. In principle, as the loss limit declines, one should end up with a distribution in which all variation is due to the variability of frequency and all variability due to severity is eliminated. Thus the insurance charges asymptotically approach the ones generated solely from the claim count distribution. These are not the same as the insurance charge values for an "infinitely large" risk. However, the multiplier formula, in the extreme case, does move all risks to the insurance charge curve of an infinitely large risk. Thus, it is likely that the multiplier approach underestimates the insurance charge values for low loss limits.

Under ICRLL, this inaccuracy in insurance charge values does not result in any great inaccuracy in the dollar amount of insurance charge. When the loss limit is zero, the correct ICRLL equations force the insurance charge to zero (dollars) and the ELPF to be equal to the ELR. Since the multiplier formula parameter "$\alpha$" is chosen to give fairly accurate answers for intermediate loss limits, and the ICRLL equations guarantee correct answers as the loss limit approaches zero, it follows that the only potential error of any significance exists at low (but non-zero) loss limits.

While recognizing the likelihood that the multiplier methodology suffers from, at worst, fairly minor inaccuracies, the author would nonetheless propose research on an alternative way of selecting insurance charge columns. Discussion of the alternative can be expedited by referring to each Table M column as an MCOL.

Currently, each MCOL is numbered by its charge at the unity entry ratio and the selection of an MCOL is governed by Expected Loss Group ranges. The alternative approach to MCOL selection would entail characterizing Table M columns, not only by their charge at the unity entry ratio, but also by their associated entry ratio variance. A formula could then be used to approximate the impact of loss limitation on the variance of the entry ratio. The idea of "variance matching" to select MCOLs is described in Appendix D. Note that it is not part of ICRLL as currently conceived.

In conclusion, despite some room for marginal improvement in the selection of insurance charge values, the ICRLL procedure provides a consistent and fairly accurate way to solve the "overlap" problem. The ICRLL system of "ratio difference - value difference" equations relative to limited losses is an intuitively appealing as well as mathematically valid solution. The use of a single Table M is a sensible compromise in the interests of practicality.

## References

[1] John R. Stafford, Retrospective Rating, Fifth Edition, 1981, J & M Publications, Palatine, Illinois.

[2] Richard H. Snader, Fundamentals of Individual Risk Rating and Related Topics, CAS Study Note, CAS, 1980.

[3] David Skurnick, "The California Table L", PCAS LXI, 1974.

[4] Glenn G. Meyers, "An Analysis of Retrospective Rating", PCAS LXVII, 1980.

[5] Mark E. Fiebrink, "An Analysis of Retrospective Rating, Glenn G. Meyers, Volume LXVII, Discussion by Mark E. Fiebrink", PCAS LXVIII, 1981.

[6] National Council on Compensation Insurance, Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance, 1984 Edition, (Updates through 7/89).

EXHIBIT 1

# Inconsistency of Certain State ELFs with ELAA Table

Alabama

Hazard Group IV

| Limit | Original ELPF (ELF) | Expected Losses | $r_{Max}$ | ELAA | Final ELPF |
|---|---|---|---|---|---|
| 35,000 | .289 | 65,000 | 1.00 | .201 | .088 |
| 40,000 | .273 | 65,000 | 1.00 | .172 | .101 |

South Dakota

Hazard Group I

| Limit | Original ELPF (ELF) | Expected Losses | $r_{Max}$ | ELAA | Final ELPF |
|---|---|---|---|---|---|
| 30,000 | .257 | 100,000 | .75 | .222 | .035 |
| 40,000 | .221 | 200,000 | .75 | .176 | .045 |

EXHIBIT 2
Sheet 1

# ELAA INCONSISTENCY
# RETRO PREMIUM REDUCTION
# BY INCREASING THE MIN
# - AN - EXAMPLE -

| | | RP | RP* |
|---|---|---|---|
| **Retro Plan Parameters** | | | |
| 1. | Standard Premium (SP) | 100,000 | 100,000 |
| 2. | Maximum Premium Factor (Mx) | 1.350 | 1.350 |
| 3. | Minimum Premium Factor (Mn) | 0.550 | .590 |
| 4. | Tax Multiplier (TM) | 1.050 | 1.050 |
| 5. | Loss Conversion Factor (LCF) | 1.125 | 1.125 |
| 6. | Expected Loss Ratio (ELR) | 0.650 | .650 |
| 7. | Loss Limit | 25,000 | 25,000 |
| 8. | Excess Loss Factor (ELF) | 0.310 | .310 |
| 9. | Expense Factor (inc profit, exc tax) (XPR) | 0.214 | .214 |
| **Expense Component of Basic** | | | |
| 10. | Expense Factor (XPR) | 0.214 | .214 |
| 11. | Expected Recovery Due to LCF ((LCF-1)*ELR) | 0.081 | .081 |
| 12. | Expense Component of Basic (BX)=(XPR-((LCF-1)*ELR) | 0.133 | .133 |
| **Insurance Charge** | | | |
| 13. | Expected Losses (ELR*SP) | 65,000 | 65,000 |
| 14. | Expected Loss Group (MCOL) | 48 | 48 |
| 15. | Ratio Difference ((Mx-Mn)/(TM*LCF*ELR)) | 1.04 | .99 |
| 16. | Value Difference (ELR+XPR-(Mn/TM))/(LCF-ELR) | .465 | .413 |
| 17. | Maximum Entry Ratio (rmax) | 1.15 | 1.17 |
| 18. | Minimum Entry Ratio (rmin) | 0.11 | .18 |
| 19. | Insurance Charge at Max (X(rmax)) | .441 | .436 |
| 20. | Insurance Savings at Min (S(rmin)) | .014 | .031 |
| 21. | Net Insurance Charge (I)=(ELR*(X(rmax)-S(rmin))) | .278 | .263 |
| 22. | Converted Net Insurance Charge (LCF*(I)) | .312 | .296 |
| **Excess Loss Premium** | | | |
| 23. | Excess Loss Factor (ELF) | 0.310 | .310 |
| 24. | Excess Loss Adjustment Amount (ELAA) | .189 | .185 |
| 25. | Excess Loss Premium Factor (ELPF) | .121 | .125 |
| 26. | Converted ELPF(LCF*ELPF) | .136 | .141 |
| **Basic, Total Limitation, and Fixed Charges** | | | |
| 27. | Basic (B)=((BX)+(LCF*I)) | .445 | .429 |
| 28. | Total Limitation Charges ((I)+(ELPF)) | .399 | .388 |
| 29. | Fixed Charges (TM)*((B)+(LCF*ELPF)) | .610 | .598 |

EXHIBIT 2
Sheet 2

## RETRO PREMIUM AT SAMPLE LIMITED LOSS RATIOS

| Limited Loss Ratio | RP | RP* |
|---|---|---|
| 0% | 61,017 | 59,801 |
| 10% | 72,830 | 71,613 |
| 20% | 84,642 | 83,426 |
| 30% | 96,455 | 95,238 |
| 40% | 108,267 | 107,051 |
| 50% | 120,080 | 118,863 |
| 60% | 131,892 | 130,676 |
| 70% and over | 135,000 | 135,000 |

## ICRLL CALCULATION

| | | RP* |
|---|---|---|
| **Retro Plan Parameters** | | |
| 1. | Standard Premium (SP) | 100,000 |
| 2. | Maximum Premium Factor (MxPF) | 1.350 |
| 3. | Minimum Premium Factor (MnPF) | .590 |
| 4. | Tax Multiplier (TM) | 1.050 |
| 5. | Loss Conversion Factor (LCF) | 1.125 |
| 6. | Expected Loss Ratio (ELR) | .650 |
| 7. | Loss Limit | 25,000 |
| 8. | Excess Loss Factor (ELF) | .310 |
| 8a | Limited Loss Ratio (ELLR)=(ELR-ELF) | .340 |
| 9. | Expense Factor (inc profit, exc tax) (XPR) | .214 |
| **Expense Component of Baisc** | | |
| 10. | Expense Factor (XPR) | .214 |
| 11. | Expected Recovery Due to LCF ((LCF-1)*ELR) | .081 |
| 12. | Expense Component of Basic (BX)=(XPR-((LCF-1)*ELR) | .133 |
| **Insurance Charge** | | |
| 13. | Expected Losses (ELR*SP) | 65,000 |
| 13a | State/Harzard Group Severity Multiplier (M(S/H)) | 1.000 |
| 13b | Loss Elimination Ratio (LER)=(ELF/ELR) | .477 |
| 13c | Loss Limitation Multiplier M(k)=(1+.8*LER)/(1-LER) | 2.641 |
| 13d | Losses Used for Group Selection (LUGS) | 171,676 |
| 14. | Expected Loss Group (MCOL) | 36 |
| 15. | Ratio Difference ((Mx-Mn)/(TM*LCF*ELLR)) | 1.89 |
| 16. | Value Difference (ELR+XPR-(Mn/TM)/LCF•ELLR) | .790 |
| 17. | Maximum Entry Ratio (rmax) | 1.92 |
| 18. | Minimum Entry Ratio (rmin) | .03 |
| 19. | Insurance Charge at Max(X(rmax)) | .180 |
| 20. | Insurance Savings at Min (S(rmin)) | .000 |
| 21. | Net Insurance Charge (I)=(ELLR*(X(rmax)-S(rmin))) | .061 |
| 22. | Converted Net Insurance Charge (LCF*(I)) | .069 |
| **Excess Loss Premium** | | |
| 23. | Excess Loss Factor (ELF) | .310 |
| 24. | Excess Loss Adjustment Amount (ELAA) | .000 |
| 25. | Excess Loss Premium Factor (ELPF) | .310 |
| 26. | Converted ELPF (LCF*ELPF) | .349 |
| **Basic, Total Limitation, and Fixed Charges** | | |
| 27. | Basic (B)=(BX)+(LCF*I)) | .202 |
| 28. | Total Limitation Charges ((I)+(ELPF)) | .371 |
| 29. | Fixed Charges (TM)*((B)+(LCF*ELPF)) | .578 |

Note: If Mn is below ≈ .58, the system fails to have a solution.

Exhibit 3
Sheet 2

RETRO PREMIUM
AT SAMPLE LIMITED
LOSS RATIO

| Limited Loss Ratio | RP* |
|---|---|
| 0% | 59,000 |
| 10 | 69,599 |
| 20 | 81,412 |
| 30 | 93,224 |
| 40 | 105,037 |
| 50 | 116,849 |
| 60 | 128,662 |
| 70% and over | 135,000 |

Exhibit 4

# Insurance Charge Integrals For Some Table M Columns

| MCOL or ELG# | 100 x Integral |
|---|---|
| 37 | 131.80 |
| 36 | 125.65 |
| 35 | 119.72 |
| 34 | 114.86 |
| 33 | 110.03 |
| 32 | 105.38 |
| 31 | 99.88 |
| 30 | 95.05 |
| 29 | 91.73 |
| 28 | 86.90 |
| 27 | 82.17 |
| 26 | 78.01 |
| 25 | 74.59 |
| 24 | 73.21 |
| 23 | 71.79 |
| 22 | 70.43 |
| 21 | 69.02 |
| 20 | 67.20 |
| 19 | 65.41 |
| 18 | 63.48 |
| 17 | 61.68 |

Exhibit 5
Sheet 1

Comparison Example
of
Variance Matching
vs
Loss Group Multiplier Factor
for
Selecting Table M Columns

| | |
|---|---|
| Expected Unlimited Losses | 280,000 |
| MCOL for Unlimited Losses | 33 |
| Insurance Charge Integral (x100) | 110.03 |

Multiplier Method

| LER | Multiplier | LUGS | MCOL |
|---|---|---|---|
| .0 | 1.0000 | 280,000 | 33 |
| .1 | 1.200 | 336,000 | 31 |
| .2 | 1.450 | 406,000 | 30 |
| .3 | 1.771 | 496,000 | 29 |
| .4 | 2.000 | 616,000 | 28 |
| .5 | 2.800 | 784,000 | 26 |
| .6 | 3.700 | 1,036,000 | 24 |
| .7 | 5.200 | 1,456,000 | 23 |
| .8 | 8.200 | 2,296,000 | 20 |

$$\text{Multiplier} = (1 + \alpha\ \text{LER}) / (1 - \text{LER})$$

$$\alpha = .8$$

# Comparison Example of Variance Matching vs Loss Group Multiplier Factor for Selecting Table M Columns

| LER | "b" Factor | (ICI) Insurance Charge Integral (x 100) | MCOL |
|---|---|---|---|
| 0 | 1.00 | 110.03 | 33 |
| .1 | .957 | 105.34 | 32 |
| .2 | .914 | 100.64 | 31 |
| .3 | .871 | 95.95 | 30 |
| .4 | .829 | 91.25 | 29 |
| .5 | .786 | 86.56 | 28 |
| .6 | .743 | 81.87 | 27 |
| .7 | .700 | 77.17 | 26 |
| .8 | .657 | 72.48 | 23 |

$$\beta = 2$$

$$\theta = 1.5$$

$$b = (\beta + \theta(1-LER))/(\beta + \theta)$$

$$ICI(LER) = .5 + (ICI(0)-.5) \bullet b(LER)$$

# APPENDIX A - RETROSPECTIVE PREMIUM CALCULATION

The retrospective premium formula can be written as:

## A.1. Retrospective Premium Formula

$$RP = SP \bullet TM\,((LR_k + ELPF\,) \bullet LCF + B)$$

subject to a minimum of $SP \bullet Mn$

and a maximum of $SP \bullet Mx$

where

| | | |
|---|---|---|
| RP | = | Retrospective Premium |
| SP | = | Standard Premium |
| TM | = | Tax Multiplier |
| $LR_k$ | = | Loss Ratio (Relative to Standard) computed with accidents capped by the loss limit k. |
| ELPF | = | Excess Loss Premium Factor for loss limit k. |
| LCF | = | Loss Conversion Factor. |
| B | = | Basic Premium Factor. |
| Mn | = | Minimum Premium Factor. |
| Mx | = | Maximum Premium Factor. |

Initially Retro premium is set equal to Standard. At 18 months and at annual intervals thereafter, the formula is used to determine Retro premium. A loss development factor is allowed, but is seldom used in practice and has been omitted for ease of presentation.

The ELPF term is a fixed charge for capping accidents by the loss limit selected by the insured. If no loss limit is elected, the ELPF term disappears and one writes LR instead of $LR_k$. ELPF are published by state, hazard group, and loss limit.

The Basic Premium Factor can be expressed as the sum of the expense component of the Basic and the net insurance charge. The expense component of the Basic varies depending on the LCF in such a way that together all underwriting expense (except premium tax), profit, and loss adjustment expense is covered.

A.2. <u>Basic</u>

$$B = BX + LCF \bullet ELR\ I$$

where

$BX$ = Expense Component of Basic

$ELR$ = Expected Loss Ratio (Relative to Standard)

$I$ = Net Insurance Charge Factor (Relative to Expected Loss)

The expense component of the basic satisfies

A.3. Expense Component of the Basic

$$BX = XPR - (LCF-1)ELR$$

where

$XPR$ = Expense Ratio (Including Profit and LAE, Excluding Premium Tax)

The net insurance charge factor is included to reflect the maximum and minimum constraints on Retro premium. Theoretically, if losses are unlimited, it is the solution to a balance equation.

## A.4. Net Insurance Charge - Balance Equation

$$ELR = E[\min(R_{Max}, \max(R_{Min}, LR + ELR \bullet I))]$$

where

$$R_{Max} = (Mx/TM - BX) \div LCF$$

$$R_{Min} = (Mn/TM - BX) \div LCF$$

In practice, the net insurance charge is found by expressing it as the difference between insurance charge and insurance savings factors [1]. First define the insurance charge and insurance savings factors mathematically as a function of entry ratio r.

## A.5. Insurance Charge and Insurance Savings

$$X(r) = \int_r^{\infty} (t - r)\, dF(t)$$

$$S(r) = \int_0^{r} (r - t)\, dF(t)$$

where

F(t) = Cumulative distribution at loss ratio t relative to expected loss ratio

= Prob (Aggregate Loss $\leq$ t•SP•ELR)

The net insurance charge is found by solving the system:

## A.6. Net Insurance Charge-System of Equations

( i) $R_{Max} = ELR \bullet ( r_{Max} + X(r_{Max}) - S(r_{Min}) )$

(ii) $R_{Min} = ELR \bullet ( r_{Min} + X(r_{Max}) - S(r_{Min}) )$

where

$$ELR = \text{Expected Loss Ratio (to Standard)}$$

$$r_{Max} = \text{Entry Ratio at Max}$$

$$r_{Min} = \text{Entry Ratio at Min}$$

$$R_{Max} = (Mx/TM - BX) \div LCF$$

$$R_{Min} = (Mn/TM - BX) \div LCF$$

Subtracting (ii) from (i), one finds the "ratio difference":

A.7. Ratio Difference

$$r_{Max} - r_{Min} = \frac{Mx - Mn}{TM \bullet ELR \bullet LCF} = \frac{R_{Max} - R_{Min}}{ELR}$$

Using the easily proved formula:

A.8. Charge and Savings Relation

$$X(r) - S(r) = 1 - r$$

and A.6 (ii), one calculates the "charge difference",

A.9. Charge Difference

$$X(r_{Min}) - X(r_{Max}) = 1 - \frac{R_{Min}}{ELR} = \frac{ELR + XPR - Mn/TM}{ELR \bullet LCF}$$

In practice, one enters Table M, and by an iterative process finds the $r_{Max}$ and $r_{Min}$ that satisfy the ratio difference and charge difference equations. Note that A.8 will fail to have a solution if the charge difference exceeds unity.

Table M is a display of insurance charge ratios at .01 entry ratio intervals. Columns of insurance charge ratios are given corresponding to Expected Loss Groups. The Expected Loss Groups are numbered according to the insurance charge at the 1.0 entry ratio. For instance, Expected Loss Group 15 shows an insurance charge factor of .15 at the unity entry ratio. To determine the appropriate column of Table M for a given insured, one uses the table of Expected Loss Group Ranges.

This brief description of traditional procedure is not intended to be complete or exhaustive, and the interested reader is referred to [1], [2], and the body of NCCI material on this subject for further detail.

## The Theoretically Correct Insurance Charge Calculation

The overlap issue can be analyzed by comparing the theoretically correct system of net insurance charge equations against the traditional one.

As in Appendix A, let k denote the loss limit and use k as a subscript on other variables to indicate their dependence on the loss limit. Also, use superscript "primes" as necessary to distinguish the entry ratios and net insurance charge from those of the previous section. Using this notation, define the insurance charge factor reflecting the loss limit as follows:

B.1. **Insurance Charge Reflecting Loss Limit**

$$X_k(r) = \int_r^{\infty} (t - r)dF_k(t)$$

where $F_k(t) = \text{Prob (Aggregate Limited Loss} \leq t \bullet ELR_k \bullet SP)$

The insurance savings factor reflecting a loss limit is defined in similar fashion, by explicitly subscripting the loss limit dependence of all variables in A.4.

The theoretically correct net insurance charge system of equations reflecting loss limitation is:

B.2. <u>Net Insurance Charge Reflecting Loss Limit - System of Equations</u>

$$\text{( i)} \quad \frac{Mx}{TM} - BX - ELPF_k \bullet LCF = ELR_k \bullet LCF \bullet (r'_{max} + X_k(r'_{max}) - S_k(r'_{min}))$$

$$\text{(ii)} \quad \frac{Mn}{TM} - BX - ELPF_k \bullet LCF = ELR_k \bullet LCF \bullet (r'_{min} + X_k(r'_{max}) - S_k(r'_{min}))$$

Using this system, one can derive revised ratio difference and charge difference equations:

B.3. <u>Ratio Difference</u>

$$r'_{Max} - r'_{Min} = \frac{Mx - Mn}{TM \bullet ELR_k \bullet LCF}$$

B.4. <u>Charge Difference</u>

$$X_k(r'_{Min}) - X_k(r'_{Max}) = \frac{(ELR + XPR - Mn/TM)}{LCF \bullet ELR_k}$$

Note that the system fails to have a solution if the charge difference exceeds unity.

In comparing this with Appendix A, one should note that not only do the insurance charge and savings factors reflect the loss limit, but also that the expected loss ratio, ELR, is replaced with the expected limited loss ratio, $ELR_k$. Further, the ELPF occurs explicitly in the system of net insurance charge equations.

This system ultimately stems from a balance condition appropriately modified in comparison with A.3 to reflect loss limitation.

## B.5. Theoretical Net Insurance Charge - Balance Equation

$$ELR = E[\min(R_{Max}, \max(R_{Min}, LR_k + ELPF + ELR_k \bullet I'))]$$

where $I'$ = Net Insurance Charge Factor Reflecting Loss Limitation (Relative to Expected Limited Loss)

$$= X_k(r'_{max}) - S_k(r'_{min})$$

Overlap may be defined as the difference between the traditional insurance charge as calculated in Appendix A and the theoretically correct one.

## B.6. Overlap - Definition

$$ER = ELR \bullet I - ELR_k \bullet I'$$

where

$ER$ = Overlap

## APPENDIX C - FIXED AND VARIABLE COMPONENTS

In order to facilitate comparisons, it is useful to group all excess loss premium and insurance charges together into a "total limitation charge". Recall that in [3], Skurnick defined a Table L charge which incorporates both per accident and aggregate limitation. Thus, in that context, the Table L charge is the same as the total limitation charge. In the context of the 1984 Retrospective Rating Manual, the total limitation charge is obtained as the sum of the insurance charge and the excess loss premium after adjustment.

It is also useful to split retro premium into fixed and variable components.

### C.1. Retro Premium - Fixed and Variable Components

$$RP = SP \bullet \min(Mx, \max(Mn, FX + VC \bullet LR_k))$$

where

| | | |
|---|---|---|
| RP | = | Retro Premium |
| Mx | = | Maximum Premium Factor |
| Mn | = | Minimum Premium Factor |
| FX | = | Fixed Component Factor |
| VC | = | Variable Component Coeffecient |
| $LR_k$ | = | Loss Ratio relative to Standard Premium subject to loss limit k. |

The total limitation charge factor plus a factor for expenses (to be assessed as non-loss sensitive charges) together comprise the fixed component factor.

In the context of Retro Rating Option V as per the 1984 manual, the various formulas are:

**C.2. Retro Option V - Total Limitation Charge, Fixed Component, & Variable Component Coeffecients**

A. $TLC = ELR \cdot I + ELPF$

B.(i) $FX = TM\ (B + LCF \cdot ELPF)$

(ii) $FX = TM\ (BX + LCF \cdot TLC)$

C. $VC = TM \cdot LCF$

where

TM = Tax Multiplier

B = Basic Premium Factor

LCF = Loss Conversion Factor

BX = Expense Component of the Basic

TLC = Total Limitation Charge Factor

I = Net Insurance Charge Factor (Relative to Expected Losses)

ELR = Expected Loss Ratio

Using these concepts, certain consistency requirements for a retrospective rating plan can be stated.

C.3. **Consistency Requirements**

A. FX and TLC are decreasing functions of the maximum premium factor and the minimum premium factor assuming all else is fixed.

B. FX and TLC are decreasing functions of the loss limit assuming all else is fixed.

There is another sort of requirement that a retro calculation procedure ought to satisfy. To state this requirement, suppose one has a retro plan with parameters yielding the retro formula denoted RP. Now suppose that by changing the maximum premium factor, the minimum premium factor, or the loss limit, one arrives at a formula with different rating factors. Denote the other formula as RP*. A limited uniform adequacy requirement is:

C.4. **Uniform Adequacy Requirement**

$$E[RP] = E[RP^*]$$

where $E[\ ]$ = Expectation Value

One way to prove that the uniform adequacy condition is not met is to show, for instance, that $RP \geq RP^*$ for any possible loss outcomes, with strict inequality in at least one non-zero probability situation.

C.4 can also be viewed as a well-definition or anti-selection safety condition. If insureds are able to alter various options and obtain lower expected retro premiums, it follows that rational insureds may search for the minimizing combination. The uniform adeqacy condition says that such a search will be fruitless.

## APPENDIX D - MATCHING ENTRY RATIO VARIANCE

Insurance charges relate to the variance, skewness, and higher moments of the entry ratio random variable. Assigning a column of insurance charges to a risk on the basis of its expected losses is thus necessarily indirect and potentially inaccurate. The idea in this Appendix is to explore the possibility of using a variance matching approach to determine an appropriate Table M column (MCOL) of insurance charges for a risk reflecting its severity and elected loss limit.

In the following, integrals involving the insurance charge will be expressing in terms involving moments of the aggregate loss distribution. The moments of the aggregate distribution can be written in terms of moments of the accident count and accident severity distributions. The idea is to associate the aggregate distribution with an MCOL by \`matching´ second moments as close as possible.

Before explaining the details of this procedure, one should note that no modification of the MCOL can ever exactly capture the effect of loss limitation. This is true because all the Table M distributions are theoretically based on unlimited loses.

Turning now to the mathematical development, a proposition will be stated showing the relation between various intergrals of the insurance charge (viewed as a function of entry ratio) and moments of the aggregate loss distribution.

D.1. <u>Propostion</u>

Let L denote the aggregate loss random variable. Then for n = 0, 1, 2, ...,

$$(n+1)\,(n+2)\int_0 r^n\, X(r)dr = \frac{E[L^{n+2}]}{E[L]^{(n+2)}}$$

<u>Proof</u>

Let $F_R$ be the cdf of the entry ratio random variable , R, where R = L/E[L].

Compute

$$\int_0^\infty r^n\, X(r)dr = \int_0^\infty r^n \left( \int_r^\infty (s-r)\, dF_R(s)\right)dr$$

$$= \int_0^\infty \left(\int_0^s r^n\, (s-r)\, dr\right) dF_R(s)$$

$$= \int_0^\infty \frac{s^{n+2}}{n+1} - \frac{s^{n+2}}{n+2}\, dF_R(s) = \int_0^\infty \frac{s^{n+2}}{(n+1)(n+2)}\, dF_R(s)$$

Substitute $x = s \cdot E[L]$ to obtain

$$(n+1)(n+2) \int_0^\infty r^n X(r)dr = \int_0^\infty x^{n+2}\, dF(x) \div E[L]^{n+2}$$

where F is the cumulative distribution of aggregate loss.

The result is now obvious.

□

Using the Proposition in the case $n = 0$, a formula for the variance of aggregate loss can be derived:

D.2. <u>Variance Formula</u>

$$(Var(L)/(E[L]^2)) = (2 \int_0^\infty X(r)dr - 1)$$

<u>Proof</u>

The formula follows using Proposition D.1, the equation $Var(L) = E[L^2] - E[L]^2$, and a little algebra.

□

The integral of the insurance charge can be approximated without too much difficulty. Values for several MCOLs are shown in Exhibit 4.

Now let C be the severity random variable and write $C_k$ to denote the associated limited loss severity when a loss limit, k, is elected. Further let N denote the number of accidents. Then the following holds true.

## D.3. Impact of Loss Limitation on Insurance Charge Integral

If $\quad Var(N) = \beta \cdot E[N]$, then

$$\int_0^\infty X_k(r)dr = \frac{1}{2} + \left(\int_0^\infty X(r)dr - \frac{1}{2}\right) \cdot \frac{h_k}{h}$$

where

$$h = \beta + Var(C)/E[C]^2$$

$$h_k = \beta + Var(C_k)/E[C_k]^2$$

**Proof**

This follows from D.2. using the well known formula:

$$Var(L) = E[N]Var(C) + E[C]^2\ Var(N) \qquad \square$$

Note that the assumption that Var(N) is proportional to E[N] is not unduly restrictive. The negative binomial, in particular, has this property. For Poisson, the proportionality constant is unity.

D.3. suggests that a formula of the following sort might be reasonable.

D.4. Approximation to the Limited Loss Insurance Charge Integral

$$\int_0^\infty X_k(r)\,dr \simeq \tfrac{1}{2} + \left(\int_0^\infty X(r)dr - \tfrac{1}{2}\right) b(LER)$$

where

$$b(LER) = (\beta + \theta(1-LER))/(\beta + \theta)$$

A comparison example of this the "variance matching" approximation method is shown in comparison against the "loss multiplier" method in Exhibit 5. The parameters used, $\beta = 2$ and $\theta = 1.5$ were selected out of thin air. Thus the comparison should merely be regarded as demonstrative of the practicality of variance matching. Nonetheless, the results do seem fairly reasonable.