BASIC AND INCREASED LIMITS RATEMAKING:

AN INTEGRATED APPROACH

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BIOGRAPHY:

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ABSTRACT:

Casualty actuaries commonly treat basic and increased limits ratemaking for liability insurance as two completely separate projects. Though this separation arises quite naturally, several inequities may arise from such an approach. This paper proposes a model that partially resolves some of these problems by deriving basic and increased limits rate indications simultaneously using a pure premium approach. The model takes into account the mean and standard deviation of the projected severity distribution, investment income, differences in loss and adjustment expense payment patterns by policy limit, fixed and variable expenses, and risk and profit loadings. The paper provides an example of the model's use, in which the risk-and-profit component of the indicated rates is clearly shown. The paper also tests the model's sensitivity to changes in assumptions and suggests areas for further study.

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PERSPECTIVE

Casualty actuaries commonly treat basic and increased limits ratemaking for liability insurance as two completely independent projects. The Insurance Services Office, for example, usually computes, files and distributes basic limits advisory rates--from now on, loss costs--and increased limits factors separately. Insurers that make rates independently often introduce new basic limits rates and increased limits factors simultaneously, but the rate indications still may be the result of two completely separate calculations.

There is one obvious reason for separating the process into two tasks. Loss data loses credibility as the limit of liability increases. As a result, it may be necessary to use a much broader data base for calculating increased limits factors than for developing basic limits rates. In many cases, individual state data are used for basic limits rates, but increased limits factors are determined from countrywide data. Thus the separation of the basic and increased limits ratemaking tasks occurs quite naturally.

Yet several inequities may arise from such an approach.

Adding Risk Loads to the Profit Margin

Increased limits factors often contain risk loads to reward the insurer with a greater return when it assumes a greater risk (i.e., writes a policy at a higher limit of liability). Appropriate risk loads are sometimes assumed to be proportional to the standard deviation of the size-of-loss distribution for each policy limit. That assumption does not seem unreasonable, but it yields only the relative size of the risk loads. How should the proportionality constant be determined?

Our actuarial literature includes several different models that might be used to determine an appropriate average return on an insurer's book of business. These models may help the insurer to determine the magnitude of the risk loads. However, in many cases, the insurer has already used one of these models to determine the profit loading in the basic limits rates. And the models are sometimes applied without considering the below-average risk of basic limits exposures. Thus the average return the insurer desires on all policy limits may be built into the basic limits ratemaking process. Then when risk loads are included in the increased limits factors, there may be an overlap.

One recent increased limits rate filing received by a state regulator stated: "We have selected a proportionality constant . . . such that the total additional dollars available . . . due to risk load averages to 6.5% of premiums for all commercial liability lines." As the corresponding basic limits rates included the traditional 5% loading for underwriting profit and contingencies, approximately 11.5% of the total limits premium was budgeted for profit and contingencies.

Allocating Fixed Expenses

Increased limits factors are generally based solely on loss and loss adjustment expense data; expenses other than adjustment expense are usually ignored. This procedure does not cause a problem if all expenses vary with premium. But if, in basic limits ratemaking, a significant portion of the expenses are considered to be fixed--that is, a constant amount per policy or per unit of exposure--insureds with high limits of liability will pay more "fixed" expenses than those with low limits, because the fixed expenses in the basic limits rate are multiplied by the increased limits factor.

Accounting for Investment Income

When insurers use investment income data in ratemaking, they often incorporate it as an adjustment to the loading for profit and contingencies in the basic limits rates. Another common approach is to use discounted pure premiums in building the basic limits rates. But explicit consideration of the different loss payment patterns associated with basic and increased limits is unusual. In general, the higher the policy limit, the slower the loss pay-out, and hence the greater the benefit from investment income.

In some cases, financial data, which are on a total limits basis, are used to make the investment income adjustment for basic limits rates. Often these data are net of reinsurance. But because of the differing loss payment patterns by policy limit and the effects of reinsurance on the data, it is questionable whether the resulting adjustment is valid for basic limits ratemaking.

A PROPOSED MODEL

This paper proposes a model that--at least, partially--resolves the problems mentioned above. The model provides a method of establishing basic and increased limits rates simultaneously, using a pure premium approach.

Assumptions

The following assumptions are made in the use of this model:

1. The ratemaker has determined an appropriate total limits return on the book of business to be rated. This return includes both underwriting profit (or loss) and investment income on the cash flow generated by the book of business. Regardless of how the appropriate return was determined, it is now expressed as a percentage of total limits premium.

2. The distribution of exposures by policy limit has been estimated. (This distribution may affect the selection of an appropriate return.)

3. Credibility and data base differences between basic and increased limits have been resolved. For example, basic limits losses may be projected from individual state data. On the other hand, the ratemaker may use countrywide data to estimate a distribution of losses by size. These two projections might be combined by adjusting the countrywide loss distribution so that its mean matches the projected severity for the state, before the model is applied.

4. All allocated loss adjustment expense (ALAE) is included in the basic limits rate. (The proposed model could be modified to accommodate different assumptions regarding ALAE.)

5. Losses, loss adjustment expenses, and a portion of the insurer's other expenses can be expressed as a fixed (dollar or other currency) amount for each unit of exposure.

With respect to Assumption 5, note that a pure premium approach requires the ratemaker to express losses and (usually) loss adjustment expenses as a certain amount per exposure. But often a different measure of exposure is more appropriate with respect to fixed expenses. For example, pure premium ratemaking for physicians' and surgeons' professional liability sometimes involves measuring exposures in terms of base class equivalents. In the calculation of pure premiums, one neurosurgeon's exposure may be regarded as equivalent to that of six physicians in family practice, so one neurosurgeon insured for one year would be counted as six units of exposure. On the other hand, the fixed expense of writing a policy for a neurosurgeon may be no greater than that of writing one for a family practitioner. Fixed expenses may be the same amount for every specialty, so that one neurosurgeon insured for one year should be counted as one unit of exposure from the expense viewpoint.

Assumption 5 is satisfied only when the same measure of exposure is appropriate for expressing both pure premiums and fixed expenses. If this assumption is not satisfied, the ratemaker may be able to divide the book of business into segments (groups of classes, for example) so that the assumption holds (or is nearly true) for each segment. The model would then produce separate rate schedules or increased limits tables for each segment. If such a division is impractical, fixed expense considerations can be deleted from this model. There may be other equitable methods of treating fixed expenses--such as by using an expense constant or policy fee.

Some additional assumptions concerning the availability of data are implicit in the notation defined below.

Notation

Let P_x denote the indicated rate per unit of exposure at policy limit x.

The loss and adjustment expense components of P_x, which comprise the projected pure premium for policy limit x, will be denoted as follows:

l_x: projected loss per exposure
a_x: projected allocated loss adjustment expense (ALAE) per exposure
u_x: projected unallocated loss adjustment expense (ULAE) per exposure

Because of our assumption that all ALAE is to be included in the basic limits rate, a_x is the same for all values of x. This common value will be denoted a.

Let v denote the expected variable expense ratio, excluding any provision for profit.

Let f represent the expected amount of fixed expense per exposure.

Let p be the desired return, expressed as a proportion of anticipated total limits premium, as discussed under Assumption 1.

The letter d will be used to denote discount factors, which express the ratio of the present value to the nominal value of loss, ALAE, or ULAE. Although these factors depend on the selected interest rate or discount rate, the notation will not be encumbered by this dependence. The following notation will be used, with x again representing the policy limit:

$d_l(x)$: discount factor for loss

$d_a(x)$: discount factor for ALAE

$d_u(x)$: discount factor for ULAE

Assumption 4 implies that d_a is constant as a function of x, so we will write d_a in place of $d_a(x)$.

We will use the letter e to represent exposures. The proportion of the exposures expected to be written at policy limit x will be denoted e_x.

Finally, s_x will denote the standard deviation of the size-of-loss distribution truncated at the limit x. The proportionality constant, denoted k, will be defined so that the risk load--or, more accurately, risk-and-profit load--in the indicated rate for each limit can be expressed as $r_x = ks_x$.

Basic Formulas

Given the above notation, it is not difficult to write down a formula for the indicated rate at policy limit x:

$$P_x = \frac{l_x d_l(x) + a d_a + u_x d_u(x) + k s_x + f}{1 - v} \qquad (1)$$

If b denotes the basic limit, the increased limit factor for policy limit x can be expressed as

$$\frac{P_x}{P_b} = \frac{l_x d_l(x) + a d_a + u_x d_u(x) + k s_x + f}{l_b d_l(b) + a d_a + u_b d_u(b) + k s_b + f} \qquad (2)$$

To solve for the proportionality constant k, we equate two expressions for the average profit over all policy limits:

$$\sum_x e_x r_x = \sum_x p e_x P_x$$

Making substitutions in both sides of this equation, we obtain:

$$k \sum_x e_x s_x = \frac{p}{1 - v} \sum_x e_x (l_x d_l(x) + a d_a + u_x d_u(x) + k s_x + f)$$

After we combine terms and solve for k, we find that

$$k = \frac{p}{1 - p - v} \cdot \frac{a d_a + f + \sum_x e_x (l_x d_l(x) + u_x d_u(x))}{\sum_x e_x s_x} \qquad (3)$$

A NUMERICAL EXAMPLE

The Data Base

This example is based on an insurance company's data for commercial auto liability. The primary components of the data base were paid and incurred loss development triangles at various loss limits: $10,000, $25,000, $50,000, $100,000, and $300,000. Development triangles for reported claims, closed claims, and paid ALAE were also available. We made the reasonable assumption that most commercial policies are written at limits of at least $300,000, so that the data-censoring effect of policy limits under $300,000 is negligible in this case.

Preliminary Estimates

The proposed model requires several estimates that are commonly made in other ratemaking models. Pure premium techniques require the ratemaker to project pure premium, which is often expressed as the product of projected frequency and projected severity. The trending procedures, etc., by which these projections are made are not the subject of this paper. Methods for obtaining increased limits factors generally require the estimation of the distribution of losses by size. The selection of a loss distribution type and the estimation of its parameters are, again, outside the scope of this paper. A lognormal distribution is used in this example.

Estimation of loss payment patterns using paid loss development data is also a common actuarial procedure. The unusual aspect of this model is that a separate pattern must be estimated for each loss limit. For this example, the payment streams were discounted to present value using an interest rate of 8% per annum. Discount factors for limits above $300,000 were obtained by extrapolation. A payment pattern and discount factor for ALAE were also estimated.

Other Assumptions

At each policy limit, we assumed ULAE to be proportional to the sum of loss and ALAE. Because ULAE is a relatively small component of the pure premium, we also made the simplifying assumption that an appropriate discount factor for ULAE is the average of 1.000 and the discount factor for loss.

For this example, we arbitrarily selected an anticipated distribution of exposures by policy limit. However, this distribution is generally not difficult to estimate using data on policies in force.

The basic limit was assumed to be $25,000.

Results

Table 1 presents the results of our example. We first used the estimates and assumptions to solve (3) for the proportionality constant k. Then, using (1), we obtained the indicated rate P_X for each policy limit. We used (2) to produce the indicated increased limit factors. (Note that the projected loss per exposure and ALAE per exposure needed to apply the formulas are calculated by multiplying frequency [Table 1, line (3)] by the mean loss [column (10)] and mean ALAE [line (7)], respectively.) The formulas also enabled us easily to express the risk-and-profit component of the indicated rate for each policy limit.

Table 1
Example Results

(1)	Target Risk/Profit Load	.075
(2)	Interest Rate	8%
(3)	Projected Frequency	.083
(4)	Fixed Expense per Exposure	$50
(5)	Variable Expense Factor	.280
(6)	ULAE/(Loss + ALAE)	.080
(7)	ALAE per Claim	$968
(8)	ALAE Discount Factor	.760

Policy Limit ($000)	Exposure Distribution	Size-of-Loss Distribution		Loss Discount Factor
		Mean	Standard Deviation	
	(9)	(10)	(11)	(12)
10	.00	2,338	3,323	.889
25	.01	3,430	6,364	.869
50	.02	4,312	9,893	.853
100	.05	5,161	14,545	.840
300	.23	6,289	24,450	.833
500	.36	6,673	29,899	.830
750	.04	6,922	34,557	.829
1,000	.29	7,111	38,550	.828

(13) Proportionality Constant .0023681

Policy Limit ($000)	Indicated Rate	Risk and Profit		Increased Limits Factor
		Amount	Loading (15)/(14)	
	(14)	(15)	(16)	(17)
10	434	8	.018	0.78
25	557	15	.027	1.00
50	656	23	.036	1.18
100	754	34	.046	1.35
300	900	58	.064	1.62
500	956	71	.074	1.72
750	996	82	.082	1.79
1,000	1,028	91	.089	1.85
Average	945	71	.075	1.70

We will consider the sensitivity of the model in two respects. First, we will ask whether the considerations introduced in the model make any real difference, when the model results are compared with a more traditional approach. Second, we will review the effects of varying several of the model's parameters, using the example above.

Comparison with Traditional Approach

The model itself can be used to approximate the results of applying a more traditional approach to the same data. First, the fixed expense is assumed to be zero, and the variable expense ratio is increased accordingly. The variable expense ratio is also adjusted to include the desired provision for underwriting profit in the basic limits rates. (Note that the increased limits factors, as produced by (2), are independent of these changes to the variable expense ratio.) Finally, a common discount factor is used for all policy limits. If investment income is not to be reflected at all, this common factor is equal to one. The risk-and-profit load in the model becomes simply the risk load in the increased limits factors.

These new assumptions were applied to the example discussed in the previous section, assuming a zero interest rate, no fixed expenses, a 33.3% variable expense loading plus a 5% basic limits underwriting profit margin, and an average risk load that would produce an additional 6.5% of total limits premium in the increased limits factors. The resulting rates were much higher, of course, because of these assumptions. And the average increased limits factor was 1.84, rather than 1.70.

The indicated rates and increased limits factors based on these new assumptions may be regarded as comparable to bureau advisory rates. Companies commonly use bureau increased limits factors, but often deviate from the basic limits advisory rates. It might be instructive, then, to deviate from these indications by reducing the rates across the board so that they average $945, as in the earlier example.

The comparison is shown in Table 2. As we would expect, the deviated rates at the higher limits—where most of the policies are written—are about the same as the indicated rates from the example. The most significant differences are at lower limits, where policyholders would pay substantially lower rates under the more traditional approach. In fact, under the deviated rates, profit (including investment income) is negative for policy limits less than $100,000.

Table 2
Comparison with Traditional Approach

Policy Limit ($000)	Indicated Rate (1)	Increased Limits Factor (2)	Deviated Rate (3)	Rate from Example (4)	Percentage Difference (5)
10	495	0.74	382	434	-12.0
25	666	1.00	515	557	-7.5
50	810	1.22	626	656	-4.6
100	953	1.44	736	754	-2.4
300	1,160	1.74	896	900	-0.4
500	1,239	1.86	957	956	0.1
750	1,295	1.94	1,001	996	0.5
1,000	1,340	2.01	1,035	1,028	0.7
Average	1,223	1.84	945	945	0.0

In practice, the difference between the two approaches could be greater. Here we have chosen a deviation so that the average rates under the two approaches are the same. A company using only more traditional methods would not likely choose exactly this deviation.

Varying Assumptions within the Model

It is clear that different distributions of exposures by policy limit will produce different indicated rates and factors under the model. However, distribution changes ought to be offset by changes in the target risk-and-profit load. If, for example, a company wrote all policies at $1 million limits, it might be appropriate to assume an 8.9% return on premium instead of the 7.5% we used in the calculations above (see Column 16 of Table 1).

To explore the sensitivity of the rate indications to various changes in assumptions, we will compare the average increased limits factors resulting from different sets of assumptions. We will use the same data as in the example of the preceding section, but will allow the following variations:

Target Risk-and-Profit Load:	.025, .050, .075, .100
Interest Rate:	0%, 6%, 8%, 10%
Fixed Expense per Exposure:	$0, $50, $100

Table 3 shows the average increased limits factor for each combination of assumptions.

The trends that appear in Table 3 are not unexpected. If the target risk-and-profit load increases, the risk loads--which are higher at higher policy limits--contribute more to the indicated rates, so the average increased limits factor increases. If fixed expenses are increased, a greater proportion of the premium does not vary by policy limit, so the average increased limits factor is reduced. If interest rates increase, the payment pattern differences among policy limits become more significant, as the discount factors vary more widely. This variation then has a greater effect in partially offsetting the differences in expected losses, so the average increased limits factor decreases.

Table 3
Varying Assumptions - Average Increased Limits Factors

Risk and Profit	Fixed Expense	Interest Rate 0%	6%	8%	10%
.025	$ 0	1.75	1.72	1.71	1.70
.025	50	1.67	1.63	1.62	1.61
.025	100	1.61	1.56	1.55	1.54
.050	0	1.79	1.76	1.75	1.74
.050	50	1.71	1.67	1.66	1.65
.050	100	1.64	1.60	1.59	1.58
.075	0	1.83	1.80	1.79	1.78
.075	50	1.75	1.71	1.70	1.69
.075	100	1.69	1.64	1.63	1.62
.100	0	1.88	1.84	1.83	1.82
.100	50	1.80	1.75	1.74	1.73
.100	100	1.73	1.68	1.67	1.66

These results suggest that the model is not very sensitive to changes in the interest rate, though there is a clear difference between discounting at, say, 8%, and not discounting at all. For a longer-tailed line of business, such as general liability or medical malpractice, the sensitivity to changes in the interest rate would be somewhat greater.

The selected values for fixed expense per exposure represent approximately 0%, 5%, and 10%, respectively, of the indicated average rate. If values such as

5% and 10% are not atypical, the model suggests that ignoring fixed expense can lead to significant distortions in the relationships among rates at various policy limits.

AREAS FOR FURTHER STUDY

Modifications and Simplifications

The model could be altered or simplified in several ways:

If the ratemaking situation does not permit the reflection of fixed expense in the increased limits factors, the model can be applied on the more traditional per claim basis (rather than per exposure) to develop increased limits factors.

One of the time-consuming aspects of the model is the necessity of estimating loss payment patterns for many different loss limits. This problem might be solved by using a regression technique to interpolate or extrapolate the loss discount factors for most of the desired policy limits.

As we noted earlier, we made a simplifying assumption to deal with ULAE in the example. There may be better ways to incorporate ULAE into the model.

Defining Risk-and-Profit Load

The model assumes that the risk-and-profit load should be proportional to the standard deviation of the limited size-of-loss distribution. The load is thus related to the risk inherent in the projected claim severity process. The model can easily be generalized by replacing the standard deviation with an unspecified function of the parameters of the severity distribution. But for pure premium ratemaking, it is likely more appropriate to make the risk load proportional to the standard deviation (or another function) of the pure premium distribution, which would include the process risk in the frequency component of the pure premium. How should frequency variation be considered by the ratemaker in developing appropriate risk loads?

The risk loads, as they are defined both traditionally and in this model, do not account for any parameter risk. The parameters of the size-of-loss distribution, as well as many other estimated quantities, are presumed to be known with certainty. The parameter risk, of course, depends on the data and the methods by which parameters are estimated, and many of those methods are outside of the model proposed here. But it still seems appropriate to ask what bearing, if any, parameter risk should have on the relationships between rates at various policy limits.

CONCLUSION

In response to regulators' concerns, the Insurance Services Office has recently decided to distribute basic limits loss cost statistics instead of advisory rates. Much of the property-casualty insurance industry is thus at the threshold of a new era in ratemaking. This paper offers a model that may assist some insurers with tasks they have not previously performed on their own--tasks such as determining an appropriate profit loading for basic limits rates, or taking investment income into account.

The model proposed here may also help to clarify the relationship between profit margins and risk loads. It suggests that the magnitude of the risk loads that are often a component of increased limits factors is based on overall profit margin considerations, and not wholly determined by the loss data. This point may be important in discussions of rating organizations' role in developing increased limits factors for liability insurers.

In some situations, the proposed model may yield rate indications that are not much different from traditional approaches; in other instances, there could be substantial differences. Some of the adjustments made in the model appear to offset each other. But this phenomenon should not lead the ratemaker to assume that the adjustments exactly offset each other and can therefore be disregarded. Similarly, it is sometimes assumed that loss development and discounting of loss reserves probably offset each other, yet actuaries have learned that it is important to perform both of these calculations.

The model should be seen as an initial attempt to integrate two ratemaking tasks--basic and increased limits--that should never have been completely separated. Further work on this problem may prove fruitful.

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