INDIVIDUAL RISK LOSS DEVELOPMENT

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Abstract:

Commercial lines prospective individual risk rating plans utilize the individual risk's own loss experience as one input into the determination of the premium charge for that risk. For long tail coverages such as General Liability, an accurate estimate of the risk's ultimate losses is an important and necessary pricing element.

Current plans utilize IBNR estimates which are based on either the risk's known losses or the risk's expected losses. This paper proposes an alternative hybrid approach which incorporates both methods. The impact that this approach has on the pricing of individual risks is shown in the context of the ISO General Liability Experience Rating Plan.

A sensitivity analysis is done based on the parameters of expected losses, expected loss development in the aggregate, expected loss development on reported claims, and the amount of reported losses.

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1. Introduction

In the pricing of commercial lines risks, prospective individual risk rating plans are available which utilize the individual risk's own loss experience as one input into the determination of the final premium charge for that risk. For long tail coverages, the accurate estimate of the risk's ultimate losses is an important and necessary pricing element. Prospective individual risk rating plans currently filed by ISO for Commercial General Liability utilize one of two methods to estimate the ultimate losses which are used in the plans.

To calculate IBNR in the ISO Commercial General Liability Experience and Schedule Rating Plan (17-400 7-88 Ed.) a factor is multiplied by the risk's expected losses. In the Loss Rating Procedure of the ISO Composite Rating Plan (90-1250 10-88 Ed.) a loss development factor is multiplied by the risk's known losses.

This paper will propose an alternative hybrid approach which incorporates both methods. This method is applied in the context of the ISO General Liability Experience and Schedule Rating Plan . The impact that this approach has on the pricing of individual risks is shown by comparing the experience mods obtained using this hybrid approach with the mods obtained using either of the alternative methods described above. A sensitivity analysis is done based on the parameters of expected losses(alias basic limits annual premium), expected loss development - in the aggregate (alias $LDF_k(t)$ which varies by subline), expected loss development - on reported claims (alias $P_k(t)$ which also varies by subline), and the amount of reported losses (alias KL(r,t)).

2. Definitions

The following abbreviations are defined.

KL(r,t) = the known losses for risk r evaluated t months from the beginning of its coverage period.

KL(r,*) = the ultimate losses for risk r,i.e.,the known losses at time t=*, where * is the first time when the known losses equal the ultimate losses.

EL(r,t) = an estimate of the ultimate losses for risk r (KL(r,*)) given knowledge of KL(r,t).

EL(r,0) = an estimate of the ultimate losses for risk r (KL(r,*)) prior to any knowledge of actual losses for that risk, i.e. at t=0.

EKL(r,t) = an estimate of KL(r,t).

IBNR(r,t) = an estimate of the quantity [KL(r,*)-KL(r,t)]. $IBNR_k(r,t)$ will denote an estimate which is a function of KL(r,t). $IBNR_e(r,t)$ will denote an estimate which is a function of EL(r,0).

Additional definitions will be stated as needed in the development of the discussion.

3. IBNR

One method of calculating IBNR(r,t) assumes that the ultimate losses can be estimated from the known losses. A loss development factor ($LDF_k(r,t)$) is multiplied by the risk's known losses as is done in the loss rating procedure of the ISO Composite Rating Plan.

$$1.\ EL(r,t) = KL(r,t) + IBNR_k(r,t) = KL(r,t) \times LDF_k(t)$$

By removing the known losses from eqn. 1, an estimate of the IBNR is given.

$$2.\ IBNR_k(r,t) = (LDF_k(t) - 1) \times KL(r,t)$$

The other method of calculating IBNR(r,t), which is used in the ISO Commercial General Liability Experience and Schedule Rating Plan utilizes a loss development factor ($LDF_e(t)$) which is multiplied by the expected losses for the risk (EL(r,0)).

$$3.\ IBNR_e(r,t) = LDF_e(t) \times EL(r,0)$$

A method of calculating EL(r,0) is shown in EQN 13. It is assumed that risk r's ultimate losses (KL(r,*)) are equal to its expected losses (EL(r,0)).

$$4.\ KL(r,*) = EL(r,0)$$

In practice, $LDF_e(t)$ is calculated from $LDF_k(t)$.

$$5.\ LDF_e(t) = (LDF_k(t) - 1)/LDF_k(t)$$

The two methods are equivalent if:

$$6.\ EL(r,0) = KL(r,t) \times LDF_k(t)$$

The first method assumes that IBNR(r,t) is best estimated from expected losses while the second method assumes that it is better to use known losses in the calculation of IBNR(r,t). Both methods assume stable loss development patterns. There are places where common sense would lead you to prefer one method over the other but a more "actuarial" approach would be to determine some type of weighting scheme so as to incorporate the strengths of both of the estimators.

An approach which uses weights which vary by age of development is called for. (An example of such an approach applied to a different problem can be seen in Berry's method of setting retro reserves[1].) This paper takes this approach by recognizing the distinction between loss development which arises from periodic evaluation of known losses as opposed to that which arises from claims not yet reported. (Recent articles by Robbin[2] and Venter[3] utilize a Bayesian approach to calculating weights.)

The article, "The Actuary and IBNR"[4] mentions the distinction between these two sources of loss development. For purposes of this paper, IBNER (incurred but not enough reported) will denote that portion of the loss development which is due to the future development on claims which have already been reported as of the evaluation date. IBNYR (incurred but not yet reported) will denote the loss development which is due to those claims which have not yet been reported as of the evaluation date which corresponds to t months of development.

$$7.\quad IBNR(r,t) = IBNER(r,t) + IBNYR(r,t)$$

In the discussion above, both $IBNR_e(r,t)$ and $IBNR_k(r,t)$ are methods of calculating $IBNR(r,t)$. This paper will develop a formula for $IBNER(r,t)$ similar to that shown for $IBNR_k(r,t)$ and a formula for $IBNYR(r,t)$ similar to that shown for $IBNR_e(r,t)$.

4. Weighting of Methods

One method of estimating the ultimate level of losses for aggregates of risks, is to tabulate $LDF_k(t)$'s from case incurred loss development triangles using historical data. Ideally, those coverages which have similar loss development patterns are grouped together. This is reflected in the ISO experience rating plan where there are different LDF_e's for the two sublines of Premises/Operations and Products/Completed Operations. These factors are shown on Table 1:e. They are appropriate for losses evaluated at a date 3 months prior to the renewal effective date.

Table 1:e

Percent of Ultimate Losses Not Reported

$LDF_e(t) \times 100$

t Months of Development	Premises/ Operations	Products/ Completed Operations
21	34.8%	72.0%
33	19.4	58.9
45	10.7	48.6

For reference purposes, the $LDF_k(t)$ which are implied by the $LDF_e(t)$ of Table 1:e are shown on Table 1:k.

Table 1:k

Loss Development Factor For KL(r,t)

$LDF_k(t)$

t Months of Development	Premises/ Operations	Products/ Completed Operations
21	1.534	3.571
33	1.241	2.433
45	1.120	1.946

At any evaluation date for a recent experience period, some claims will have already been reported while others are still not yet reported. By following the loss development of these two categories of claims seperately, another estimate of the ultimate losses for an individual risk can be made.

Consider the 34.8% for Premises/Operations as of 21 months of development from Table 1:e. This factor means that for every $1.00 of ultimate losses, $.652 is already reported and $.348 has yet to emerge. Part of the emergence will be due to the periodic reevaluation of known losses and part will emerge due to new claims being reported. Assume that $.212 of the $.348 is from the former and \$.136 is from the latter. This would mean that 61% of the total loss development will come from known claims while the remaining 39% will come from new claims.

Let $P_k(t)$ represent the portion of the total loss development which is expected to come from known claims.(A method of computing $P_k(t)$ is discussed in section 8.)

$$8.\ P_k(t) = IBNER(r,t)/IBNR(r,t)$$

By multiplying EQN 2 by $P_k(t)$, a formula for IBNER(r,t) is created as follows.

$$9.\ IBNER(r,t) = (LDF_k(t) - 1) \times P_k(t) \times KL(r,t)$$

Also, by multiplying EQN 3 by $(1 - P_k(t))$, a formula for IBNYR(r,t) is created as follows.

$$10.\ IBNYR(r,t) = LDF_e(t) \times (1 - P_k(t)) \times EL(r,0)$$

As stated above, the total IBNR(r,t) will be the sum of IBNER(r,t) and IBNYR(r,t). $P_k(t)$ is the weight that is used. By definition, $P_k(t)$ will take on the value of 0 at 0 months of development and 1 at that point in time after which no more new claims will be reported.

For perspective, Table 2 shows values of $P_k(t)$ which were estimated using an actual company's data for basic limits losses. Values of $P_k(t)$ will be influenced by claim reserving practices as well as the types of risks insured and the coverages provided and so would vary from company to company.

Table 2

Portion of Total Loss Development
Expected From Known Claims

$P_k(t)$

t Months of Development	Premises/ Operations	Products/ Completed Operations
21	.610	.330
33	.709	.480
45	.787	.500

For a given coverage at t of months of development there will be a specific $LDF_k(t)$ and $P_k(t)$ which apply. The formula for the estimate of IBNR(r,t) which was given in EQN 7 can be elaborated by incorporating EQNs 9 and 10.

$$11.\ \ IBNR(r,t) =$$

$$[(LDF_k(t) - 1) \times P_k(t) \times KL(r,t)] + [LDF_e(t) \times (1 - P_k(t)) \times EL(r,0)]$$

5. Expected Loss Development - Individual Risks

EQNs 2,3, and 11 provide three methods for calculating IBNR(r,t). Except for the situation where EQN 6 is satisfied, they will give different estimates. In order to compare and contrast these estimates, the following simple example may be helpful.

Assume that there is a single Premises/Operations risk which has EL(r,0) = $10,000. That is, prior to knowing anything else about the risk's known losses, it is expected to develop $10,000 in ultimate losses. By restating EQN 6 as EQN 12 below, an estimate of KL(r,t), EKL(r,t), can be stated in terms of EL(r,0) and $LDF_k(t)$.

$$12.\ \ EKL(r,t) = EL(r,0)\ /\ LDF_k(t)$$

Based on Table 1:k, and assuming EQN 12 is true, EKL(r,21) is $6,519 and all three EQNs give an IBNR of $3,481. A graphic display of what happens to these methods of calculating IBNR(r,t) when KL(r,21) varies from $6,519 can be seen on Exhibit I. Exhibit II displays this information for a Products/Completed Operations risk.

Now, assume that there are 40 individual Premises/Operations risks with the same size as the risk in the previous example, each having EL(r,0) = $10,000. At time t months, KL(r,t) will represent the known losses for the rth risk where the risks are sorted by the size of their known losses at that time.

Suppose that after the losses are evaluated at t=21 months, columns 1 through 3 on Table 3 are tabulated. This shows that 25 risks had no losses ,10 risks had $6,519 in losses each, and 5 risks had $39,113 in losses each.

This information along with the factors in Tables 1:e, 1:k, and 2, could be used to calculate columns 4 through 6 of Table 3 to show the various IBNR estimates which can be gotten using EQNs 2,3, and 11.

Table 3

40 Premises/Operations Risks
IBNR(r,21) Per Individual Risk
EL(r,0) = $10,000

(1)	(2)	(3)	(4)	(5)	(6)
			Estimate of IBNR(r,21)		
		Number	EQN 3	EQN 2	EQN 11
		of	.348 x	.534 x	.61x(5) +
r	KL(r,21)	Risks	EL(r,0)	KL(r,21)	.39x(4)
1 to 25	$ 0	25	$3,480	$ 0	$1,357
26 to 35	6,519	10	3,480	3,481	3,481
36 to 40	39,113	5	3,480	20,886	14,098

6. Expected Loss Development - In the Aggregate

Suppose some type of individual risk rating plan was being used to price each one of the risks represented on Table 3. Since the final price will be affected by the estimate of the IBNR(r,t), someone interested in pricing to the competition would be tempted to pick and choose among the estimates, namely pick the lowest estimate for each risk. That would amount to picking 0 for the first 25 risks and $3,480 for each of the next 15. This would come to $52,200 in the aggregate.

On Table 4, the aggregate values for the risks of Table 3 are shown. Except for slight differences due to rounding, the sum total of the IBNR(r,t) in the aggregate comes to slightly over $139,000, which is more than double what the total comes to when the lowest estimate was chosen. Thus it would seem to be important to have a consistent method of calculating IBNR(r,t) so that at least the sum total of IBNR(r,t) over all risks adds up to the aggregate amount expected. But is it enough just to have a consistent method of calculating IBNR(r,t), even if that method is accurate in the aggregate?

What would happen if in the process of the market place one third of the market adopted the method of EQN 3, another third of the market adopted the method of EQN 2, while the remaining third adopted the method of EQN 11?

With everything else being equal and price being the determining factor, the first 25 risks would end up with coverage purchased from EQN 2 companies, the next 10 risks would be randomly distributed among all companies, while the last 5 risks would end up with coverage in EQN 3 companies.

What kind of results would these companies get? Since the EQN 2 companies ignored the part of loss development which comes from IBNYR they would find themselves short by that amount for their risks,i.e., $33,925. The EQN 3 companies that wrote the last 5 risks completely ignored the information that they had concerning KL(r,21). Because of this, part of the loss development which comes from IBNER was ignored and they were short by $53,090. The $53,090 being the difference between columns 6 and 4 for the last 5 risks. The EQN 11 companies who found themselves with the smallest share of the market, at least that year, found that they had charged just enough.

Table 4

40 Premises/Operations Risks
IBNR(r,21) In the Aggregate
EL(r,0) = $400,000

(1)	(2)	(3)	(4)	(5)	(6)
				Estimate of IBNR	
	Known	Number	EQN 3	EQN 2	EQN 11
r	Losses	of Risks	.348xEL	.534xKL	.61x(5)+.39x(4)
1 to 25	$ 0	25	$87,000	$ 0	$33,925
26 to 35	65,190	10	34,800	34,810	34,810
36 to 40	195,565	5	17,400	104,430	70,490
Total	230,755	40	139,200	139,240	139,225

The point of the example is that in order to estimate ultimate losses certain assumptions need to be made. If it is assumed that:

1) the incurred loss development patterns are stable in the aggregate; and

2) the known losses do ultimately develop to equal the expected losses in the aggregate, then

EQNs 2,3, and 11 will produce the same estimate for IBNR(r,t) in the aggregate. However, even if these assumptions are valid in the aggregate the raison d'etre of insurance suggests that they will not be valid for an individual risk.

By taking into account the differences in development for individual risks by appropriately using all the information which is available, a better job of rating individual risks may be done. If the assumptions stated above were valid for individual risks, there would not be any point in using an individual risk rating plan since every risk would develop the expected losses.

7. Comparison of Experience Mods

The purpose of experience rating in general and ISO's General Liability Experience Rating Plan in particular, is to use a risk's own loss experience in an attempt to discern how suitable class rates are for that risk. Since class rates are established utilizing the experience of many risks, the suitability of that rate for any particular risk depends partly on how close the expected loss pure premium for the individual risk is to that of the whole class of risks. The problem is, that in most cases, an individual risk's own loss experience by itself is not suitable for answering this question with any degree of certainty. If it were, class rates would not be necessary since the proper rate for an individual risk could be determined from its own experience.

Because of this dilemna, the underwriter of the risk uses a weighted average of two indicated rates. This could be seen as insuring a portion of the risk at one rate and the rest of the risk at another rate. One of the rates is the class rate and the other rate is that which is indicated by the risk's own experience from the experience period.

For practical reasons, the weight that is given to the risk's own experience is a tabular value, known as the credibility, which is based on his renewal premium calculated using the current class rate. Adjustments are made to this renewal premium to take into account the length of time in the experience period ,the changes in exposures and coverage from the experience period, and the trend in loss severity from the experience period to the renewal period. The resulting measure of risk size is termed the subject premium. The expected losses for risk r are equal to the subject premium (SP(r)) times the expected loss ratio (ELR(r)).

$$13.\quad EL(r,0) = SP(r) \times ELR(r)$$

In actuality not all of the known losses get into experience rating. Only that part of the known basic limits loss which is less than the tabular maximium single loss is used in the experience rating calculation. Therefore, in the actual plan, the expected loss ratio, ELR(r), is replaced by the adjusted expected loss ratio, AELR(r), as shown in EQN 14. The ratio of AELR(r) to ELR(r) represents that portion of the basic limits losses and allocated loss adjustment expenses which remain after the individual losses are limited to the maximum single loss.

$$14.\quad EL(r,0) = SP(r) \times AELR(r)$$

For purposes of understanding how the IBNR affects the experience mod, example calculations are shown on Exhibits III - XI. In order to incorporate the potential for IBNER into the plan columns 11 and 12 were added, other than that all other calculations are done according to the ISO plan. The top half of the exhibits shows the calculation of the Subject Premium and Total Losses while the actual calculation of the experience mod is located in the lower left hand corner.

In order to test the impact on the mod under various scenarios, some of the assumptions which appear in the lower right hand corner of the exhibits were varied. These assumptions include the value for $P_k(t)$ and the ratio of the actual case losses (KL(r,t)) to the expected case losses (EKL(r,t)). These items show up in columns 15 and 17 respectively. The $LDF_k(t)$'s in column 14 are the same as those which appear on Table 1:k. While not shown explicitly on the exhibits, the expected losses (EL(r,0)) would be calculated according to EQN 14 for each of the three years of the experience period. The expected case losses in column 16 are then calculated according to EQN 12. These two calculations are combined in EQN 15.

$$15.\ EKL(r,t) = (SP(r) \times AELR(r))/LDF_k(t)$$

The example on these exhibits is for a Products/Completed Operations risk with $20,000 basic limits annual premium. A summary of the assumptions and results of these exhibits is shown on Table 5. The nine scenarios shown on the nine exhibits are as a result of varying the values in columns 15 and 17. That is, the aggregate loss development is fixed but the portion which comes from IBNER and the amount of actual case losses reported are varied. The resulting mods and the exhibit number from which it was obtained are shown in Table 5.

Table 5

Products/Completed Operations Risk
Basic Limits Annual Premium = $20,000
Example Experience Mods

Experience Mod / Exhibit Number

$P_k(t)$	Actual Case Losses / Expected Case Losses: 0	1	2
0	.87/III	1.00/VI	1.13/IX
Table 2	.79/IV	1.00/VII	1.21/X
1	.68/V	1.00/VIII	1.32/XI

When $P_k(t)$ is fixed at 0, EQN 12 simplifies to EQN 3 which is the method that is specified in the ISO plan. This assumption regarding IBNR is the least responsive to the level of actual case losses as the mods for $P_k(t) = 0$ are closer to 1.00 than the other assumptions about $P_k(t)$. When the actual case losses are equal to the expected case losses, the assumption regarding $P_k(t)$ does not matter since all three methods will give the same amount of IBNR in total. This is what happened at the point of intersection on Exhibits I and II. Finally, when $P_k(t)$ is fixed at 1, Eqn 12 simplifies to EQN 2 which assumes that all IBNR is IBNER.

If the assumptions regarding $P_k(t)$ from Table 2 are correct, then it is appropriate to consider the mods resulting from their use to be more predictive than the other two assumptions. In order to more clearly see the impact that the assumption regarding $P_k(t)$ has on the mod, the differences in the mods for $P_k(t)$ of 0 and 1 from those for $P_k(t)$ from Table 2 are shown below in Table 6.

Table 6

Products/Completed Operations Risk
Basic Limits Annual Premium = $20,000

Difference From Table 2 Mod

$P_k(t)$	Actual Case Losses / Expected Case Losses	
	0	2
0	+.08	-.08
1	-.09	+.11

While this Table is the result of one example risk representing a single subline and risk size, these same differences were calculated for various risk sizes for both sublines and the results are displayed on Exhibits XII and XIII. In general, the differences in the mods are the direct result of the impact that $P_k(t)$ has on the the calculation of the IBNR(r,t). The differences are greatest for the Products/Completed Operations subline since it has greater loss development in the aggregate.

Exhibit XII shows the difference in the mods for loss free risks. The mods calculated using the ISO factors, which assume that $P_k(t)$ is 0 for all t, are greater than the the mods calculated using the $P_k(t)$ from Table 2.

For risks with two times the expected losses as shown on Exhibit XIII, the mods calculated utilizing the ISO factors, are less than those using the $P_k(t)$ from Table 2, therefore they have negative differnces.

The mods calculated under the assumption that $P_k(t) = 1$ are in the opposite direction of those assuming $P_k(t) = 0$, therefore their differences are negative on Exhibit XII and positive on Exhibit XIII.

The assymptotic influence for larger risks is due to the credibility weighting procedure. That is, the difference in the mods continues to grow as more credibility (weight) is given to the individual risks own experience. Since the credibility is proportional to the square root of the subject premium, it grows most quickly at the smaller premium sizes and more gradually approaches unity until it is judgementally set to unity at the self rating point. The actual values of the mods are displayed on Exhibits XIV and XV.

8. Calculation of $P_k(t)$

In order to calculate values for $P_k(t)$ for the appropriate values of t, a loss development history data base must be available which allows the tracking of loss development for claims reported in a given interval of time. A simple example would be as follows.

Consider a Premises/Operations risk whose losses are evaluated at t=21,33, and 45 and then again at ultimate(t=*). The report date of the claim is used to group losses into four report date intervals. The first one being those claims reported in the first 21 months, the second one being claims reported between 21 and 33 months, the third one being claims reported between 33 and 45 months, and the fourth one being claims reported after 45 months. Table 7 shows the total amount of these claims at the indicated points of development.

Table 7

Premises/Operations Risk
Loss Development
By Report Date Interval and Months of Development

Report Date Interval	Months of Development 21	33	45	*
0 to 21 Months	$6,520	$7,460	$7,990	$8,640
21 to 33 ' '	-	600	690	800
33 to 45 ' '	-	-	250	330
45 to * ' '	-	-	-	230
0 to * ' '	6,520	8,060	8,930	10,000

The loss development factors, $LDF_k(t)$, are representative of the total development for an experience period without differentiating by report date. $LDF_k(21)$ is an estimate of the ratio KL(r,*)/KL(r,21), in this case, \$10,000/\$6,520 = 1.534. This is the same as the $LDF_k(21)$ on Table 1:k. Also $LDF_k(33)$ is the same as (\$10,000/\$8,060 = 1.241) and $LDF_k(45)$ is the same as (\$10,000/\$8,930 = 1.120).

According to Table 7, IBNR(r,21) = \$10,000 - \$6,520 = \$3,480. \$2,120 of this,(\$8,640 - \$6,520), is due to the development on the claim reported in the first 21 months, thus IBNER(r,21) = \$2,120. According to EQN 8, $P_k(21)$ = \$2,120/\$3,480 = .609, which is the same as on Table 2 except for rounding.

While the above example is primarily for illustrating the concepts involved, in actuality, the loss development factors would be calculated for claims falling into the report date intervals of interest using aggregate data. For this purpose the following abbreviations are defined.

KL(y,b.e,t) = the known losses for experience period y claims, which were first reported between time beginning at t=b and ending at t=e, evaluated t months from the beginning of the experience period.

$LDF_k(y,b.e,t+n/t)$ = the age-to-age development factor calculated by taking the ratio of KL(y,b.e,t+n) to KL(y,b.e,t).

$LDF_k(b.e,t+n/t)$ = the age-to-age development factor selected after reviewing the $LDF_k(y,b.e,t+n/t)$ for recent values of y.

$LDF_k(b.e,*/t)$ = the loss development factor to be applied to KL(y,b.e,t) to develop them to ultimate.

With these definitions and the previous example in mind, new equations can be stated to give IBNR(r,t) in terms of KL(r,t) and $LDF_k(0.*,*/t)$ and IBNER(r,t) in terms of KL(r,t) and $LDF_k(0.t,*/t)$, and thus using EQN 8, $P_k(t)$ can also be stated in terms of these amounts.

16. $IBNR(r,t) = KL(r,t) \times (LDF_k(0.*,*/t) - 1)$

17. $IBNER(r,t) = KL(r,t) \times (LDF_k(0.t,*/t) - 1)$

18. $P_k(t) = (LDF_k(0.t,*/t) - 1)/(LDF_k(0.*,*/t) - 1)$

The first step in developing the needed factors is to set up incurred loss development triangles by report date interval as shown in Table 8 for the interval 0.21. In this example the experience periods are in terms of accident years.

Table 8

Incurred Loss Development Triangle
Report Date Interval 0.21

KL(y,0.21,t)

t

y	21	33	45	
.	.	.	.	. . . . .
.	.	.	.	. . . .
81	KL(81,0.21,21)	KL(81,0.21,33)	KL(81,0.21,45)	. . .
82	KL(82,0.21,21)	KL(82,0.21,33)	KL(82,0.21,45)	. .
83	KL(83,0.21,21)	KL(83,0.21,33)	KL(83,0.21,45)	.
84	KL(84,0.21,21)	KL(84,0.21,33)	KL(84,0.21,45)	
85	KL(85,0.21,21)	KL(85,0.21,33)		
86	KL(86,0.21,21)			

With Table 8, the following age-to-age factors can be calculated and an appropriate factor can be selected for each t.

Table 9

Age-to-Age Loss Developmet Factors

LDF_k(y,0.21,t+12/t)

t

y	21	33	
	.	.	. . . . . .
	.	.	. . . . .
81	LDF_k(81,0.21,33/21)	LDF_k(81,0.21,45/33)	. . . .
82	LDF_k(82,0.21,33/21)	LDF_k(82,0.21,45/33)	. . .
83	LDF_k(83,0.21,33/21)	LDF_k(83,0.21,45/33)	. .
84	LDF_k(84,0.21,33/21)	LDF_k(84,0.21,45/33)	.
85	LDF_k(85,0.21,33/21)		
Selected	LDF_k(0.21,33/21)	LDF_k(0.21,45/33)	

The loss development factors which were used in the calculation of the $P_k(t)$ were based on actual company data which produced loss development patterns which are different than ISO's. For the presentation in this paper, the $P_k(t)$ were based on the company's patterns while the $LDF_k(t)$ are strictly ISO's. Thus the Premises/Operations age-to-age loss development factors which are shown in Table 10 are the result of this merging of patterns.

Table 10

Premises/Operations
Age-to-Age Loss Development Factors
Scaled to ISO's Patterns

$LDF_k(0,e,t+n/t)$

t+n/t

e	33/21	45/33	*/45
21	1.144	1.071	1.081
33		1.076	1.088
45			1.094
*	1.236	1.108	1.120

The to-ultimate loss development factors ($LDF_k(0,e,*/t)$ needed in order to calculate $P_k(t)$ in EQN 18 are gotten by accumulating the age-to-age factors on Table 10. They are shown on Table 11.

Table 11

Premises/Operations
To-Ultimate Loss Development Factors
Scaled to ISO's Patterns

$LDF_k(0,e,*/t)$

t

e	21	33	45
21	1.325	1.158	1.081
33		1.171	1.088
45			1.094
*	1.534	1.241	1.120

The graphs on Exhibits XVI and XVII display the development patterns by report date interval underlying the Premises/Operations and Products/Completed Operations respectively.

9. Claims Made Policies

The discussion and the examples provided up to this point have been centered on the occurrence form of the General Liability policy. The claims made form of the policy has certain idiosyncrasies regarding coverage which depend on the interaction of several different variables. The experience rating plan excludes those claims which the policy covers under it's "midi-tail" provision or those which are covered under a supplementary extended reporting period endorsement thus there is no IBNYR. The ISO factors also assume that there is no IBNER.

In order to test the assumption that there is no IBNER for claims made policies the appropriate loss development patterns must be generated. The losses would be grouped by report period instead of by experience period. Report period losses can be gotten through the appropriate summation of KL(y,b.e,t) elements as described below.

Consideration can be given to the maturity of the claims made policies involved. For example, a first year claims made policy would cover those claims which both occurred and were made during the policy period. The appropriate loss development triangles would consist of elements of the form KL(y,0.12,t). Loss development triangles for a second year claims made policy would consist of elements of the form (KL(y,0.12,t)+KL(y-1,12.24,t+12)). This pattern could be developed further for more mature claims made policies.

10. Conclusion

In order to price risks various assumptions must inevitably be made. Some are more believable than others. The market place will reward those insurers who end up with prices which accurately reflected the ultimate losses for the risks they insured. While there are practical limitations on the amount of information which can be used in establishing the price for a risk, those who can use the information which is available most efficiently will benefit from the effort in the long run. How much effort an insurer puts forth will depend on how close he believes it is possible to accurately predict the losses of an individual risk.

While it is admitted that any analysis which includes the phrase "With everything else being equal..." (as the example in section 6 did) is in some ways reminiscent of those stories which commence with the phrase "Once upon a time...", the ultimate question for the insurer who is about to underwrite a risk is whether or not the price is worth the risk. That is, the level of risk determines the price.

In the dizziness of the underwriting cycles the question can be rephrased to ask if the risk is worth the price. That is, given a certain level of surplus, an insurer may attempt to generate an appropriate level of premium. Given the marketplace, that premium level will be transformed into the price level which will attract the risks needed to generate the premium desired. It is probably true that one reason why the ultimate question is misapplied has to do with the overall uncertainty in the price needed for a given level of risk. There is room for judgement and the more room there seems to be the easier it is to let the market set the price instead of setting the price based on the risk.

Because of this, any practical improvement in the pricing of risks is worth the effort. Hopefully this paper has added to the tools which are available to those who are interested in pricing risks instead of risking prices.

I would also like to acknowledge my appreciation to fellow employees at Wausau Insurance Companies for the efforts required to put these ideas into practice.

REFERENCES

[1] Charles H. Berry,"A Method For Setting Retro Reserves",PCAS LXVII,1980,p. 226.

[2] Ira Robbin,"A Bayesian Credibility Formula For IBNR Counts",PCAS LXXIII, 1986,p.129.

[3] Gary Venter,"Incorporating Exposure Estimates in Excess of Loss IBNR",1988 Fall Casualty Loss Reserve Seminar, Exhibit 6.

[4] Ronald L. Bornhuetter and Ronald E. Ferguson,"The Actuary and IBNR",PCAS LIX,1972,p.181.

Premises/Operations Risk

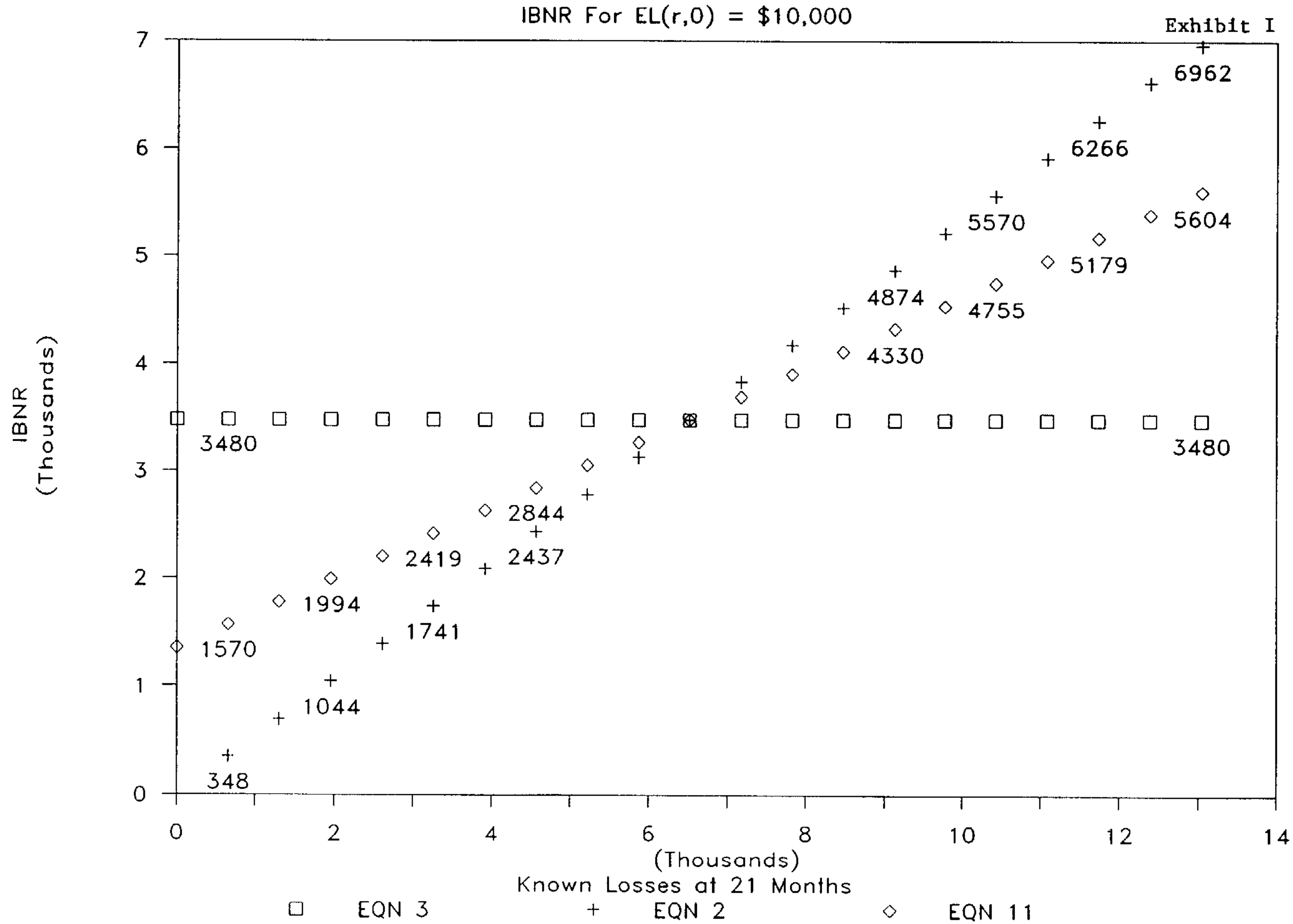
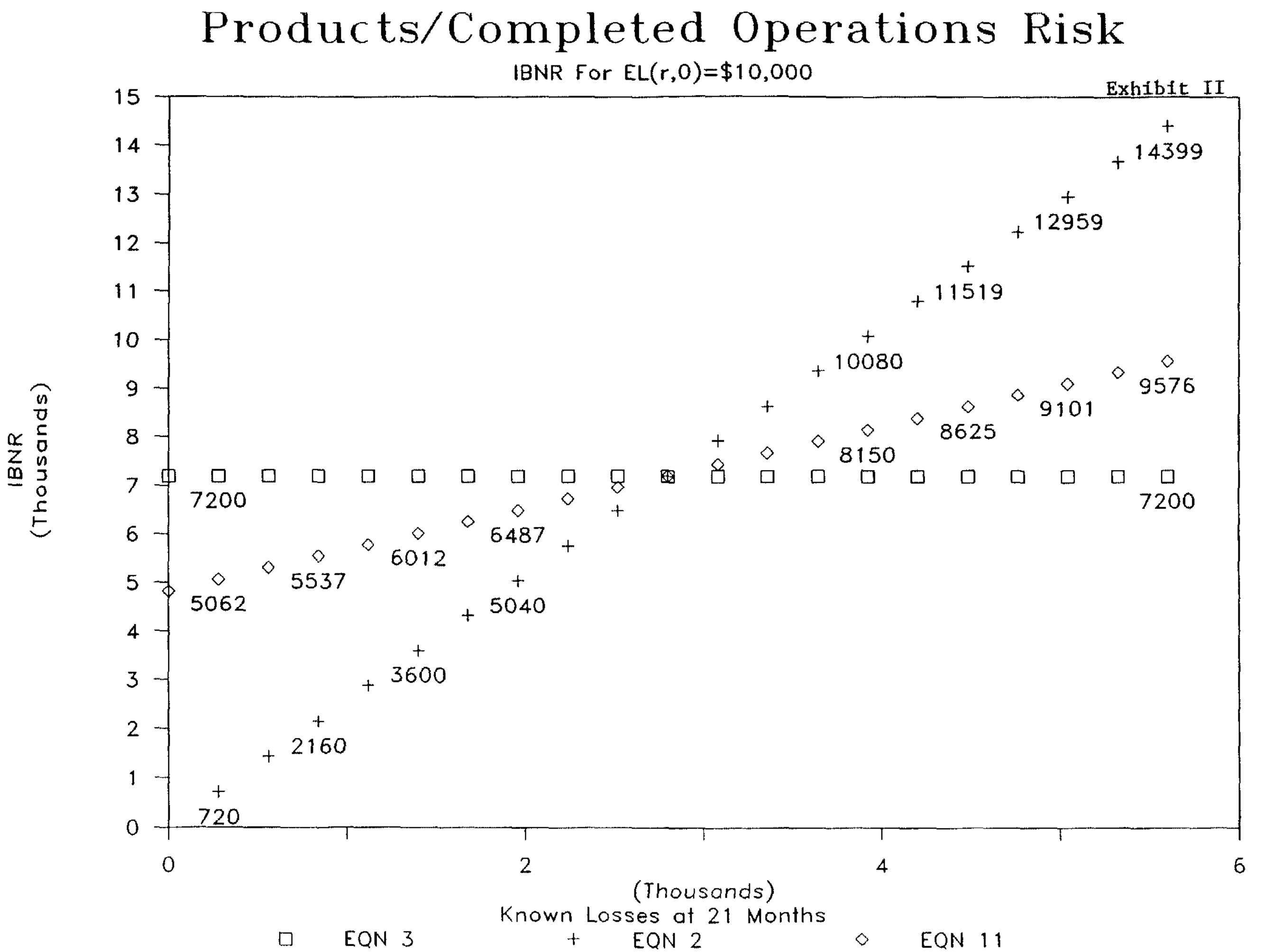


Exhibit II



Products/Completed Operations Risk
IBNR For EL(r,0)=$10,000
IBNR (Thousands)
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
(Thousands)
Known Losses at 21 Months
0
2
4
6
7200
7200
720
2160
3600
5040
10080
11519
12959
14399
5062
5537
6012
6487
8150
8625
9101
9576
EQN 3
EQN 2
EQN 11

Calculation of Experience Mod

Exhibit III

(1) t MONTHS DEVELOPED	(2) COVERAGE	(3) ANNUAL BASIC LIMITS PREMIUM	(4) POLICY ADJUSTMENT FACTOR	(5) DETREND FACTOR	(6) 3x4x5 SUBJECT PREMIUM	(7) AELR	(8) IBNR FACTOR	(9) 6x7x8 IBNR LOSSES	(10) ACTUAL CASE LOSSES	(11) CASE LDF	(12) 10x11 DEVELOPED CASE LOSSES	(13) 9+12 TOTAL LOSSES
21	PREM/OPS	$0	1.00	0.851	$0	0.381	0.348	$0	$0	1.000	$0	$0
21	PROD/C.OPS	$20,000	1.00	0.858	$17,160	0.381	0.720	$4,702	$0	1.000	$0	$4,702
									$0			
33	PREM/OPS	$0	1.00	0.785	$0	0.381	0.194	$0	$0	1.000	$0	$0
33	PROD/C.OPS	$20,000	1.00	0.794	$15,880	0.381	0.589	$3,560	$0	1.000	$0	$3,560
45	PREM/OPS	$0	1.00	0.724	$0	0.381	0.107	$0	$0	1.000	$0	$0
45	PROD/C.OPS	$20,000	1.00	0.735	$14,700	0.381	0.486	$2,720	$0	1.000	$0	$2,720
	TOTAL				$47,740			$10,982	$0		$0	$10,982

A. TOTAL LOSSES:	$10,982
B. SUBJECT PREMIUM:	$47,740
C. ALR:	0.230
D. AELR:	0.381
E. CREDIBILITY(Z):	0.32
F. EXPERIENCE MOD:	0.87
ANNUAL BL PREM:	$0 (PREM/OPS)
	$20,000 (PROD/C.OPS)

(1) t MONTHS DEVELOPED	(2) COVERAGE	(14) LDFk(t)	(15) Pk(t)	(16) EXPECTED CASE LOSSES	(17) ACTUAL/ EXPECTED CASE LOSSES
21	PREM/OPS	1.534	0.000	0	0.00
21	PROD/C.OPS	3.571	0.000	1829	0.00
33	PREM/OPS	1.241	0.000	0	0.00
33	PROD/C.OPS	2.433	0.000	2484	0.00
45	PREM/OPS	1.120	0.000	0	0.00
45	PROD/C.OPS	1.946	0.000	2875	0.00

NOTE: ALR = TOTAL LOSSES / SUBJECT PREMIUM

EXP MOD = 1 - Z + Z x (ALR/AELR)

IBNR FACTOR = ((LDFk(t)-1) x (1-Pk(t))

CASE LDF = 1 + ((LDFk(t)-1) x Pk(t))

Calculation of Experience Mod

Exhibit IV

(1) t MONTHS DEVELOPED	(2) COVERAGE	(3) ANNUAL BASIC LIMITS PREMIUM	(4) POLICY ADJUSTMENT FACTOR	(5) DETREND FACTOR	(6) 3x4x5 SUBJECT PREMIUM	(7) AELR	(8) IBNR FACTOR	(9) 6x7x8 IBNR LOSSES	(10) ACTUAL CASE LOSSES	(11) CASE LDF	(12) 10x11 DEVELOPED CASE LOSSES	(13) 9+12 TOTAL LOSSES
21	PREM/OPS	$0	1.00	0.851	$0	0.381	0.136	$0	$0	1.326	$0	$0
21	PROD/C.OPS	$20,000	1.00	0.858	$17,160	0.381	0.482	$3,151	$0	1.848	$0	$3,151
									$0			
33	PREM/OPS	$0	1.00	0.785	$0	0.381	0.057	$0	$0	1.171	$0	$0
33	PROD/C.OPS	$20,000	1.00	0.794	$15,880	0.381	0.306	$1,851	$0	1.688	$0	$1,851
45	PREM/OPS	$0	1.00	0.724	$0	0.381	0.023	$0	$0	1.094	$0	$0
45	PROD/C.OPS	$20,000	1.00	0.735	$14,700	0.381	0.243	$1,360	$0	1.473	$0	$1,360
	TOTAL				$47,740			$6,362	$0		$0	$6,362

A. TOTAL LOSSES: $6,362

B. SUBJECT PREMIUM: $47,740

C. ALR: 0.133

D. AELR: 0.381

E. CREDIBILITY(Z): 0.32

F. EXPERIENCE MOD: 0.79

ANNUAL BL PREM: $0 (PREM/OPS)
$20,000 (PROD/C.OPS)

(1) t MONTHS DEVELOPED	(2) COVERAGE	(14) LDFk(t)	(15) Pk(t)	(16) EXPECTED CASE LOSSES	(17) ACTUAL/ EXPECTED CASE LOSSES
21	PREM/OPS	1.534	0.610	0	0.00
21	PROD/C.OPS	3.571	0.330	1829	0.00
33	PREM/OPS	1.241	0.709	0	0.00
33	PROD/C.OPS	2.433	0.480	2484	0.00
45	PREM/OPS	1.120	0.787	0	0.00
45	PROD/C.OPS	1.946	0.500	2875	0.00

NOTE:

ALR = TOTAL LOSSES / SUBJECT PREMIUM

EXP MOD = 1 - Z + Z x (ALR/AELR)

IBNR FACTOR = ((LDFk(t)-1) x (1-Pk(t))

CASE LDF = 1 + ((LDFk(t)-1) x Pk(t))

Calculation of Experience Mod

Exhibit V

(1) t MONTHS DEVELOPED	(2) COVERAGE	(3) ANNUAL BASIC LIMITS PREMIUM	(4) POLICY ADJUSTMENT FACTOR	(5) DETREND FACTOR	(6) 3x4x5 SUBJECT PREMIUM	(7) AELR	(8) IBNR FACTOR	(9) 6x7x8 IBNR LOSSES	(10) ACTUAL CASE LOSSES	(11) CASE LDF	(12) 10x11 DEVELOPED CASE LOSSES	(13) 9+12 TOTAL LOSSES
21	PREM/OPS	$0	1.00	0.851	$0	0.381	0.000	$0	$0	1.534	$0	$0
21	PROD/C.OPS	$20,000	1.00	0.858	$17,160	0.381	0.000	$0	$0	3.571	$0	$0
									$0			
33	PREM/OPS	$0	1.00	0.785	$0	0.381	0.000	$0	$0	1.241	$0	$0
33	PROD/C.OPS	$20,000	1.00	0.794	$15,880	0.381	0.000	$0	$0	2.433	$0	$0
45	PREM/OPS	$0	1.00	0.724	$0	0.381	0.000	$0	$0	1.120	$0	$0
45	PROD/C.OPS	$20,000	1.00	0.735	$14,700	0.381	0.000	$0	$0	1.946	$0	$0
	TOTAL				$47,740			$0	$0		$0	$0

A.	TOTAL LOSSES:	$0
B.	SUBJECT PREMIUM:	$47,740
C.	ALR:	0.000
D.	AELR:	0.381
E.	CREDIBILITY(Z):	0.32
F.	EXPERIENCE MOD:	0.68
	ANNUAL BL PREM:	$0 (PREM/OPS)
		$20,000 (PROD/C.OPS)

(1) t MONTHS DEVELOPED	(2) COVERAGE	(14) LDFk(t)	(15) Pk(t)	(16) EXPECTED CASE LOSSES	(17) ACTUAL/ EXPECTED CASE LOSSES
21	PREM/OPS	1.534	1.000	0	0.00
21	PROD/C.OPS	3.571	1.000	1829	0.00
33	PREM/OPS	1.241	1.000	0	0.00
33	PROD/C.OPS	2.433	1.000	2484	0.00
45	PREM/OPS	1.120	1.000	0	0.00
45	PROD/C.OPS	1.946	1.000	2875	0.00

NOTE: ALR = TOTAL LOSSES / SUBJECT PREMIUM

EXP MOD = 1 - Z + Z x (ALR/AELR)

IBNR FACTOR = ((LDFk(t)-1) x (1-Pk(t))

CASE LDF = 1 + ((LDFk(t)-1) x Pk(t))

Calculation of Experience Mod

Exhibit VI

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
					3x4x5			6x7x8			10x11	9+12
t MONTHS DEVELOPED	COVERAGE	ANNUAL BASIC LIMITS PREMIUM	POLICY ADJUSTMENT FACTOR	DETREND FACTOR	SUBJECT PREMIUM	AELR	IBNR FACTOR	IBNR LOSSES	ACTUAL CASE LOSSES	CASE LDF	DEVELOPED CASE LOSSES	TOTAL LOSSES
21	PREM/OPS	$0	1.00	0.851	$0	0.381	0.348	$0	$0	1.000	$0	$0
21	PROD/C.OPS	$20,000	1.00	0.858	$17,160	0.381	0.720	$4,702	$1,829	1.000	$1,829	$6,531
									$0			
33	PREM/OPS	$0	1.00	0.785	$0	0.381	0.194	$0	$0	1.000	$0	$0
33	PROD/C.OPS	$20,000	1.00	0.794	$15,880	0.381	0.589	$3,560	$2,484	1.000	$2,484	$6,044
45	PREM/OPS	$0	1.00	0.724	$0	0.381	0.107	$0	$0	1.000	$0	$0
45	PROD/C.OPS	$20,000	1.00	0.735	$14,700	0.381	0.486	$2,720	$2,875	1.000	$2,875	$5,595
	TOTAL				$47,740			$10,982	$7,188		$7,188	$18,171

A.	TOTAL LOSSES:	$18,171
B.	SUBJECT PREMIUM:	$47,740
C.	ALR:	0.381
D.	AELR:	0.381
E.	CREDIBILITY(Z):	0.32
F.	EXPERIENCE MOD:	1.00
	ANNUAL BL PREM:	$0 (PREM/OPS)
		$20,000 (PROD/C.OPS)

(1)	(2)	(14)	(15)	(16)	(17)
t MONTHS DEVELOPED	COVERAGE	LDFk(t)	Pk(t)	EXPECTED CASE LOSSES	ACTUAL/ EXPECTED CASE LOSSES
21	PREM/OPS	1.534	0.000	0	1.00
21	PROD/C.OPS	3.571	0.000	1829	1.00
33	PREM/OPS	1.241	0.000	0	1.00
33	PROD/C.OPS	2.433	0.000	2484	1.00
45	PREM/OPS	1.120	0.000	0	1.00
45	PROD/C.OPS	1.946	0.000	2875	1.00

NOTE:

ALR = TOTAL LOSSES / SUBJECT PREMIUM

EXP MOD = 1 - Z + Z x (ALR/AELR)

IBNR FACTOR = ((LDFk(t)-1) x (1-Pk(t))

CASE LDF = 1 + ((LDFk(t)-1) x Pk(t))

Calculation of Experience Mod — Exhibit VII

(1) t MONTHS DEVELOPED	(2) COVERAGE	(3) ANNUAL BASIC LIMITS PREMIUM	(4) POLICY ADJUSTMENT FACTOR	(5) DETREND FACTOR	(6) 3x4x5 SUBJECT PREMIUM	(7) AELR	(8) IBNR FACTOR	(9) 6x7x8 IBNR LOSSES	(10) ACTUAL CASE LOSSES	(11) CASE LDF	(12) 10x11 DEVELOPED CASE LOSSES	(13) 9+12 TOTAL LOSSES
21	PREM/OPS	$0	1.00	0.851	$0	0.381	0.136	$0	$0	1.326	$0	$0
21	PROD/C.OPS	$20,000	1.00	0.858	$17,160	0.381	0.482	$3,151	$1,829	1.848	$3,381	$6,531
									$0			
33	PREM/OPS	$0	1.00	0.785	$0	0.381	0.057	$0	$0	1.171	$0	$0
33	PROD/C.OPS	$20,000	1.00	0.794	$15,880	0.381	0.306	$1,851	$2,484	1.688	$4,193	$6,044
45	PREM/OPS	$0	1.00	0.724	$0	0.381	0.023	$0	$0	1.094	$0	$0
45	PROD/C.OPS	$20,000	1.00	0.735	$14,700	0.381	0.243	$1,360	$2,875	1.473	$4,235	$5,595
	TOTAL				$47,740			$6,362	$7,188		$11,809	$18,171

A. TOTAL LOSSES:	$18,171
B. SUBJECT PREMIUM:	$47,740
C. ALR:	0.381
D. AELR:	0.381
E. CREDIBILITY(Z):	0.32
F. EXPERIENCE MOD:	1.00
ANNUAL BL PREM:	$0 (PREM/OPS)
	$20,000 (PROD/C.OPS)

(1) t MONTHS DEVELOPED	(2) COVERAGE	(14) LDFk(t)	(15) Pk(t)	(16) EXPECTED CASE LOSSES	(17) ACTUAL/ EXPECTED CASE LOSSES
21	PREM/OPS	1.534	0.610	0	1.00
21	PROD/C.OPS	3.571	0.330	1829	1.00
33	PREM/OPS	1.241	0.709	0	1.00
33	PROD/C.OPS	2.433	0.480	2484	1.00
45	PREM/OPS	1.120	0.787	0	1.00
45	PROD/C.OPS	1.946	0.500	2875	1.00

NOTE:

ALR = TOTAL LOSSES / SUBJECT PREMIUM

EXP MOD = 1 - Z + Z x (ALR/AELR)

IBNR FACTOR = ((LDFk(t)-1) x (1-Pk(t))

CASE LDF = 1 + ((LDFk(t)-1) x Pk(t))

Calculation of Experience Mod — Exhibit VIII

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
					3x4x5			6x7x8			10x11	9+12
t MONTHS DEVELOPED	COVERAGE	ANNUAL BASIC LIMITS PREMIUM	POLICY ADJUSTMENT FACTOR	DETREND FACTOR	SUBJECT PREMIUM	AELR	IBNR FACTOR	IBNR LOSSES	ACTUAL CASE LOSSES	CASE LDF	DEVELOPED CASE LOSSES	TOTAL LOSSES
21	PREM/OPS	$0	1.00	0.851	$0	0.381	0.000	$0	$0	1.534	$0	$0
21	PROD/C.OPS	$20,000	1.00	0.858	$17,160	0.381	0.000	$0	$1,829	3.571	$6,531	$6,531
									$0			
33	PREM/OPS	$0	1.00	0.785	$0	0.381	0.000	$0	$0	1.241	$0	$0
33	PROD/C.OPS	$20,000	1.00	0.794	$15,880	0.381	0.000	$0	$2,484	2.433	$6,044	$6,044
45	PREM/OPS	$0	1.00	0.724	$0	0.381	0.000	$0	$0	1.120	$0	$0
45	PROD/C.OPS	$20,000	1.00	0.735	$14,700	0.381	0.000	$0	$2,875	1.946	$5,595	$5,595
	TOTAL				$47,740			$0	$7,188		$18,171	$18,171

A. TOTAL LOSSES: $18,171

B. SUBJECT PREMIUM: $47,740

C. ALR: 0.381

D. AELR: 0.381

E. CREDIBILITY(Z): 0.32

F. EXPERIENCE MOD: 1.00

ANNUAL BL PREM: $0 (PREM/OPS)
$20,000 (PROD/C.OPS)

(1)	(2)	(14)	(15)	(16)	(17)
t MONTHS DEVELOPED	COVERAGE	LDFk(t)	Pk(t)	EXPECTED CASE LOSSES	ACTUAL/ EXPECTED CASE LOSSES
21	PREM/OPS	1.534	1.000	0	1.00
21	PROD/C.OPS	3.571	1.000	1829	1.00
33	PREM/OPS	1.241	1.000	0	1.00
33	PROD/C.OPS	2.433	1.000	2484	1.00
45	PREM/OPS	1.120	1.000	0	1.00
45	PROD/C.OPS	1.946	1.000	2875	1.00

NOTE:

ALR = TOTAL LOSSES / SUBJECT PREMIUM

EXP MOD = 1 - Z + Z x (ALR/AELR)

IBNR FACTOR = ((LDFk(t)-1) x (1-Pk(t))

CASE LDF = 1 + ((LDFk(t)-1) x Pk(t))

Calculation of Experience Mod — Exhibit IX

(1) t MONTHS DEVELOPED	(2) COVERAGE	(3) ANNUAL BASIC LIMITS PREMIUM	(4) POLICY ADJUSTMENT FACTOR	(5) DETREND FACTOR	(6) 3x4x5 SUBJECT PREMIUM	(7) AELR	(8) IBNR FACTOR	(9) 6x7x8 IBNR LOSSES	(10) ACTUAL CASE LOSSES	(11) CASE LDF	(12) 10x11 DEVELOPED CASE LOSSES	(13) 9+12 TOTAL LOSSES
21	PREM/OPS	$0	1.00	0.851	$0	0.381	0.348	$0	$0	1.000	$0	$0
21	PROD/C.OPS	$20,000	1.00	0.858	$17,160	0.381	0.720	$4,702	$3,658	1.000	$3,658	$8,360
									$0			
33	PREM/OPS	$0	1.00	0.785	$0	0.381	0.194	$0	$0	1.000	$0	$0
33	PROD/C.OPS	$20,000	1.00	0.794	$15,880	0.381	0.589	$3,560	$4,969	1.000	$4,969	$8,528
45	PREM/OPS	$0	1.00	0.724	$0	0.381	0.107	$0	$0	1.000	$0	$0
45	PROD/C.OPS	$20,000	1.00	0.735	$14,700	0.381	0.486	$2,720	$5,750	1.000	$5,750	$8,470
	TOTAL				$47,740			$10,982	$14,377		$14,377	$25,359

A.	TOTAL LOSSES:	$25,359
B.	SUBJECT PREMIUM:	$47,740
C.	ALR:	0.531
D.	AELR:	0.381
E.	CREDIBILITY(Z):	0.32
F.	EXPERIENCE MOD:	1.13
	ANNUAL BL PREM:	$0 (PREM/OPS)
		$20,000 (PROD/C.OPS)

(1) t MONTHS DEVELOPED	(2) COVERAGE	(14) LDFk(t)	(15) Pk(t)	(16) EXPECTED CASE LOSSES	(17) ACTUAL/ EXPECTED CASE LOSSES
21	PREM/OPS	1.534	0.000	0	2.00
21	PROD/C.OPS	3.571	0.000	1829	2.00
33	PREM/OPS	1.241	0.000	0	2.00
33	PROD/C.OPS	2.433	0.000	2484	2.00
45	PREM/OPS	1.120	0.000	0	2.00
45	PROD/C.OPS	1.946	0.000	2875	2.00

NOTE: ALR = TOTAL LOSSES / SUBJECT PREMIUM

EXP MOD = 1 - Z + Z x (ALR/AELR)

IBNR FACTOR = ((LDFk(t)-1) x (1-Pk(t))

CASE LDF = 1 + ((LDFk(t)-1) x Pk(t))

Calculation of Experience Mod

Exhibit X

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
					3x4x5			6x7x8			10x11	9+12
t MONTHS DEVELOPED	COVERAGE	ANNUAL BASIC LIMITS PREMIUM	POLICY ADJUSTMENT FACTOR	DETREND FACTOR	SUBJECT PREMIUM	AELR	IBNR FACTOR	IBNR LOSSES	ACTUAL CASE LOSSES	CASE LDF	DEVELOPED CASE LOSSES	TOTAL LOSSES
21	PREM/OPS	$0	1.00	0.851	$0	0.381	0.136	$0	$0	1.326	$0	$0
21	PROD/C.OPS	$20,000	1.00	0.858	$17,160	0.381	0.482	$3,151	$3,658	1.848	$6,762	$9,912
									$0			
33	PREM/OPS	$0	1.00	0.785	$0	0.381	0.057	$0	$0	1.171	$0	$0
33	PROD/C.OPS	$20,000	1.00	0.794	$15,880	0.381	0.306	$1,851	$4,969	1.688	$8,386	$10,237
45	PREM/OPS	$0	1.00	0.724	$0	0.381	0.023	$0	$0	1.094	$0	$0
45	PROD/C.OPS	$20,000	1.00	0.735	$14,700	0.381	0.243	$1,360	$5,750	1.473	$8,470	$9,830
					========			======	=======		=======	=======
	TOTAL				$47,740			$6,362	$14,377		$23,618	$29,980

A.	TOTAL LOSSES:	$29,980
B.	SUBJECT PREMIUM:	$47,740
C.	ALR:	0.628
D.	AELR:	0.381
E.	CREDIBILITY(Z):	0.32
F.	EXPERIENCE MOD:	1.21
	ANNUAL BL PREM:	$0 (PREM/OPS)
		$20,000 (PROD/C.OPS)

(1)	(2)	(14)	(15)	(16)	(17)
t MONTHS DEVELOPED	COVERAGE	LDFk(t)	Pk(t)	EXPECTED CASE LOSSES	ACTUAL/ EXPECTED CASE LOSSES
21	PREM/OPS	1.534	0.610	0	2.00
21	PROD/C.OPS	3.571	0.330	1829	2.00
33	PREM/OPS	1.241	0.709	0	2.00
33	PROD/C.OPS	2.433	0.480	2484	2.00
45	PREM/OPS	1.120	0.787	0	2.00
45	PROD/C.OPS	1.946	0.500	2875	2.00

NOTE:

ALR = TOTAL LOSSES / SUBJECT PREMIUM

EXP MOD = 1 - Z + Z x (ALR/AELR)

IBNR FACTOR = ((LDFk(t)-1) x (1-Pk(t))

CASE LDF = 1 + ((LDFk(t)-1) x Pk(t))

Calculation of Experience Mod

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
					3x4x5			6x7x8			10x11	9+12
t MONTHS DEVELOPED	COVERAGE	ANNUAL BASIC LIMITS PREMIUM	POLICY ADJUSTMENT FACTOR	DETREND FACTOR	SUBJECT PREMIUM	AELR	IBNR FACTOR	IBNR LOSSES	ACTUAL CASE LOSSES	CASE LDF	DEVELOPED CASE LOSSES	TOTAL LOSSES
21	PREM/OPS	$0	1.00	0.851	$0	0.381	0.000	$0	$0	1.534	$0	$0
21	PROD/C.OPS	$20,000	1.00	0.858	$17,160	0.381	0.000	$0	$3,658	3.571	$13,063	$13,063
									$0			
33	PREM/OPS	$0	1.00	0.785	$0	0.381	0.000	$0	$0	1.241	$0	$0
33	PROD/C.OPS	$20,000	1.00	0.794	$15,880	0.381	0.000	$0	$4,969	2.433	$12,088	$12,088
45	PREM/OPS	$0	1.00	0.724	$0	0.381	0.000	$0	$0	1.120	$0	$0
45	PROD/C.OPS	$20,000	1.00	0.735	$14,700	0.381	0.000	$0	$5,750	1.946	$11,190	$11,190
	TOTAL				$47,740			$0	$14,377		$36,341	$36,341

A.	TOTAL LOSSES:	$36,341
B.	SUBJECT PREMIUM:	$47,740
C.	ALR:	0.761
D.	AELR:	0.381
E.	CREDIBILITY(Z):	0.32
F.	EXPERIENCE MOD:	1.32
	ANNUAL BL PREM:	$0 (PREM/OPS)
		$20,000 (PROD/C.OPS)

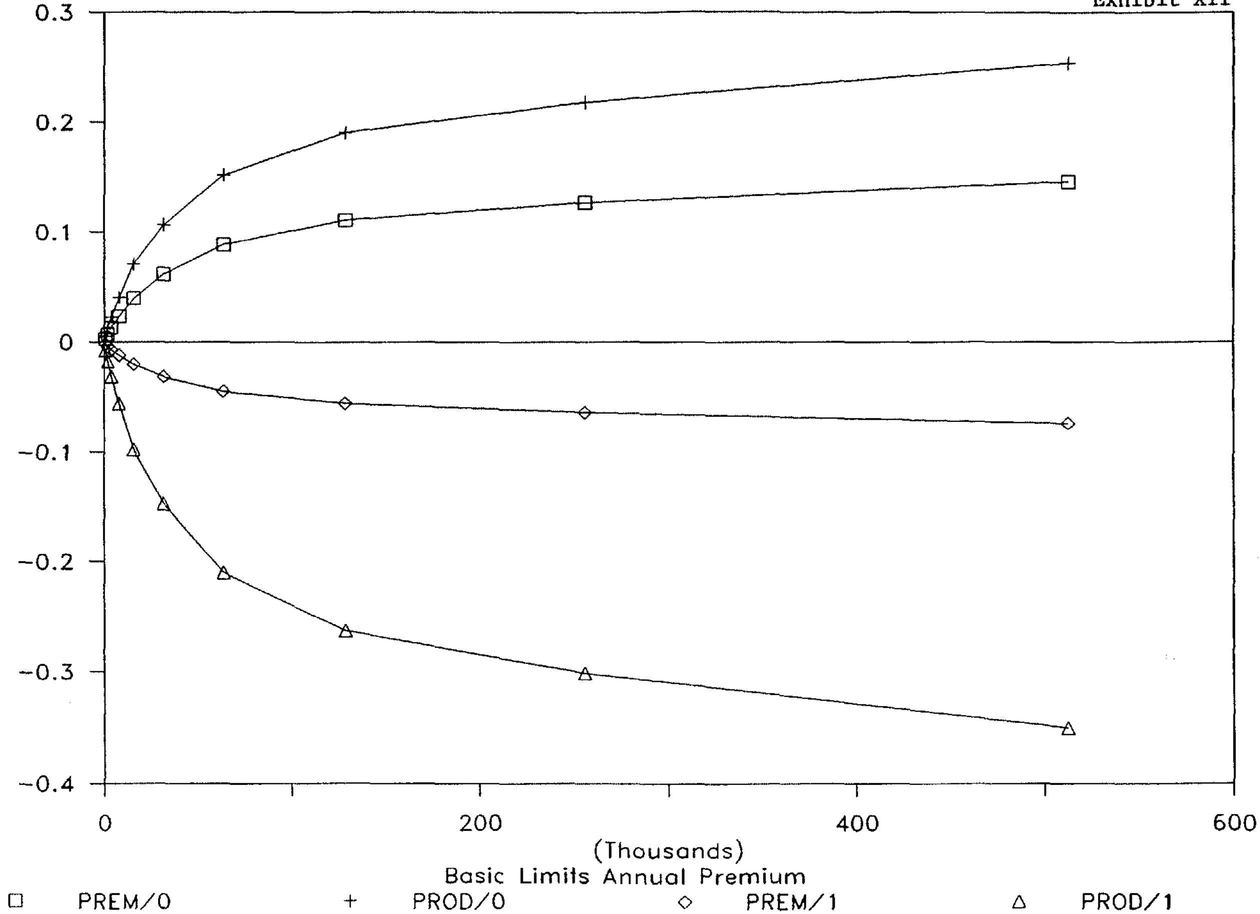
(1)	(2)	(14)	(15)	(16)	(17)
t MONTHS DEVELOPED	COVERAGE	LDFk(t)	Pk(t)	EXPECTED CASE LOSSES	ACTUAL/ EXPECTED CASE LOSSES
21	PREM/OPS	1.534	1.000	0	2.00
21	PROD/C.OPS	3.571	1.000	1829	2.00
33	PREM/OPS	1.241	1.000	0	2.00
33	PROD/C.OPS	2.433	1.000	2484	2.00
45	PREM/OPS	1.120	1.000	0	2.00
45	PROD/C.OPS	1.946	1.000	2875	2.00

NOTE: ALR = TOTAL LOSSES / SUBJECT PREMIUM
EXP MOD = 1 - Z + Z x (ALR/AELR)
IBNR FACTOR = ((LDFk(t)-1) x (1-Pk(t))
CASE LDF = 1 + ((LDFk(t)-1) x Pk(t))

Difference From Table 2 Mod

With No Losses

Exhibit XII



Difference From Table 2 Mod

With Two Times Expected Losses

Exhibit XIII

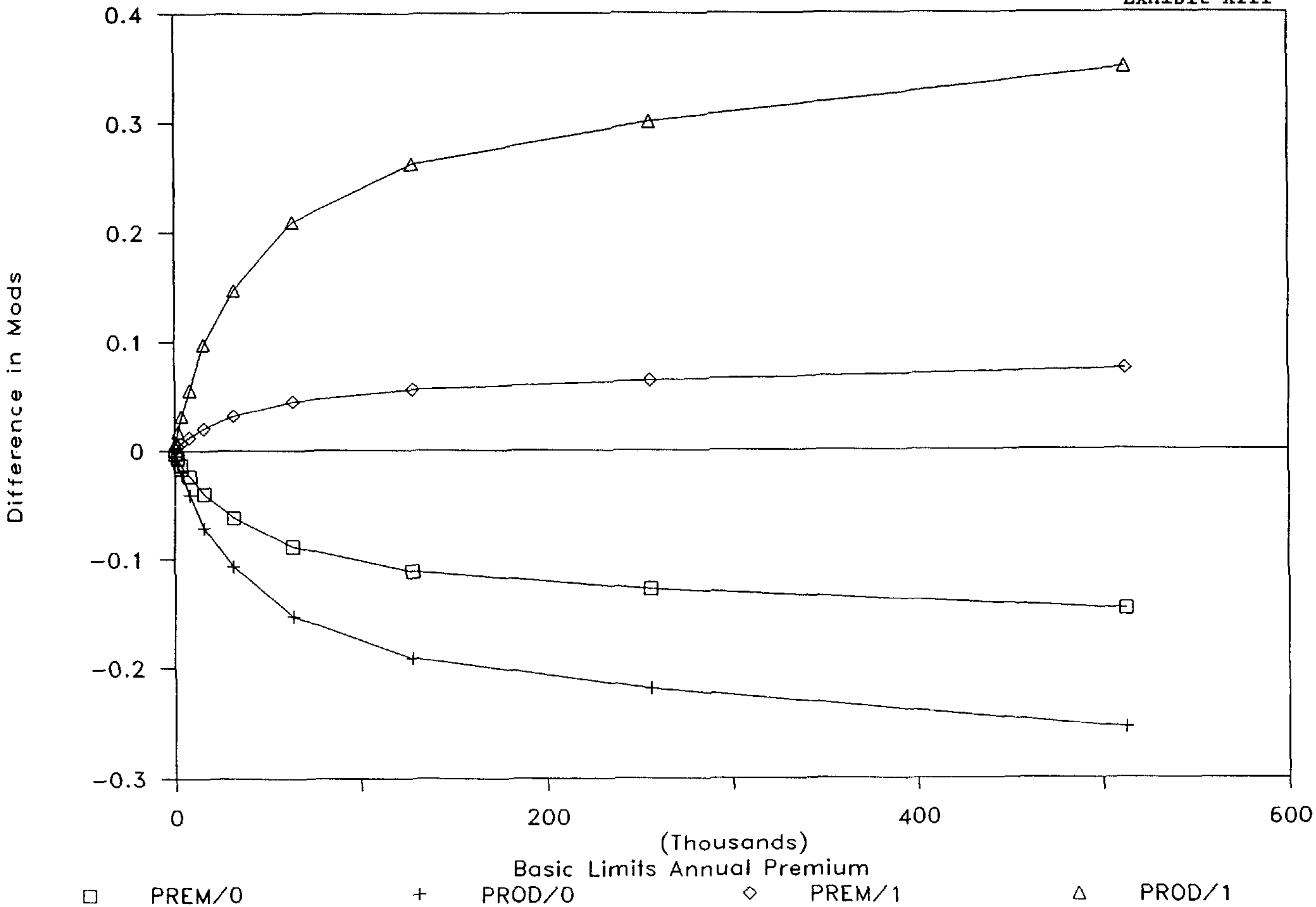


Exhibit XIV

Experience Mods
With No Losses

Basic Limits Annual Premium	Premises/Operations Pk(t) 0	Premises/Operations Pk(t) Table 2	Premises/Operations Pk(t) 1	Products/Completed Operations Pk(t) 0	Products/Completed Operations Pk(t) Table 2	Products/Completed Operations Pk(t) 1
1000	0.984	0.981	0.980	0.992	0.987	0.980
2000	0.961	0.954	0.950	0.980	0.968	0.950
4000	0.930	0.917	0.910	0.964	0.942	0.910
8000	0.876	0.852	0.840	0.937	0.896	0.840
16000	0.790	0.750	0.730	0.889	0.818	0.720
32000	0.674	0.611	0.580	0.834	0.727	0.580
64000	0.534	0.445	0.400	0.763	0.610	0.400
128000	0.417	0.306	0.250	0.703	0.513	0.250
256000	0.332	0.204	0.140	0.660	0.441	0.140
512000	0.231	0.084	0.010	0.604	0.350	0.000

Exhibit XV

Experience Mods

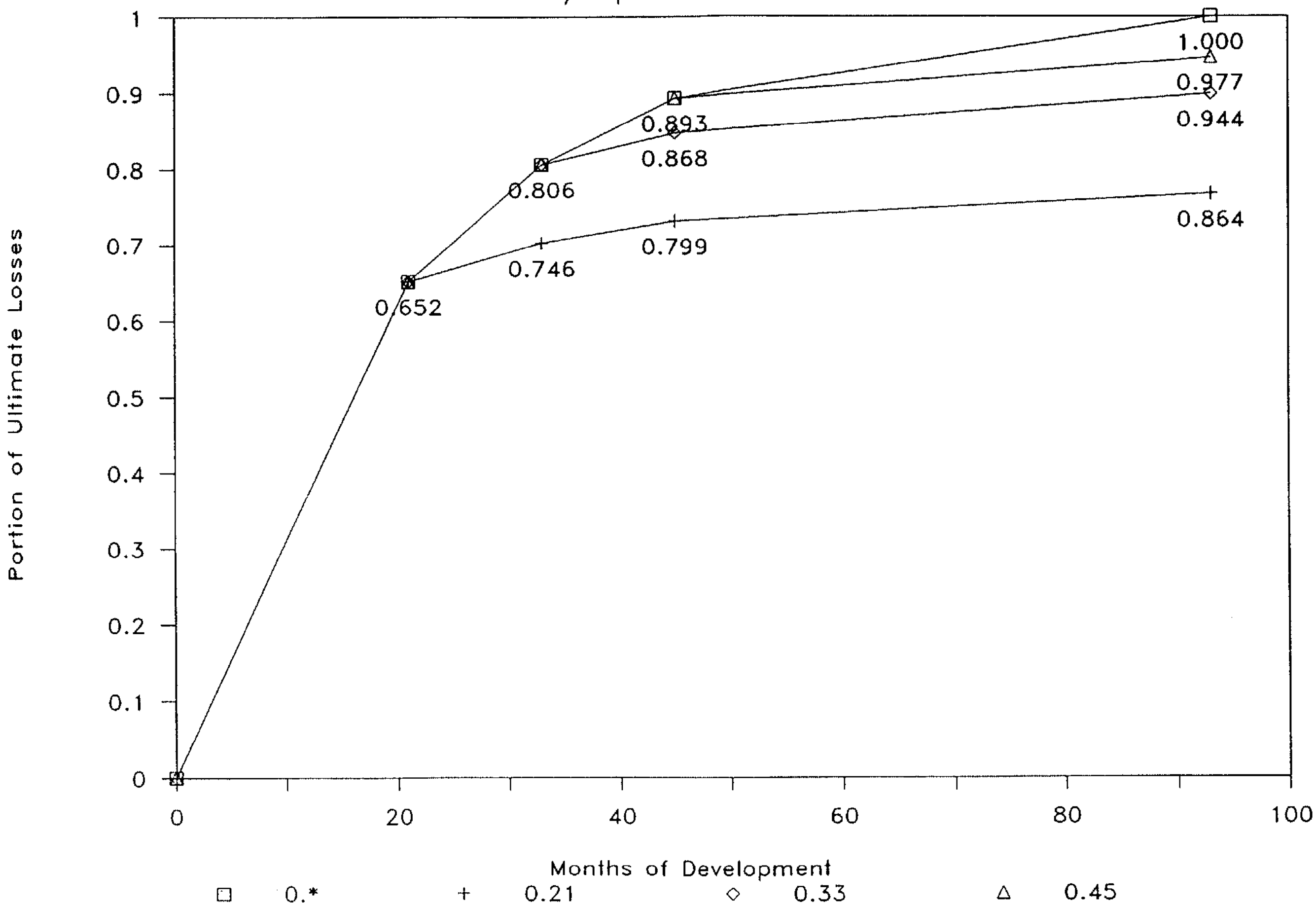
With Two Times Expected Losses

Basic Limits Annual Premium	Premises/Operations Pk(t)			Products/Completed Operations Pk(t)		
	0	Table 2	1	0	Table 2	1
1000	1.016	1.019	1.020	1.008	1.013	1.020
2000	1.039	1.046	1.050	1.020	1.032	1.050
4000	1.070	1.083	1.090	1.036	1.058	1.090
8000	1.124	1.148	1.160	1.063	1.104	1.160
16000	1.210	1.250	1.270	1.111	1.182	1.280
32000	1.326	1.389	1.420	1.166	1.273	1.420
64000	1.466	1.555	1.600	1.237	1.390	1.600
128000	1.583	1.694	1.750	1.297	1.487	1.750
256000	1.668	1.796	1.860	1.340	1.559	1.860
512000	1.769	1.916	1.990	1.396	1.650	2.000

Premises/Operations Development Pattern

By Report Date Interval

Exhibit XVI



Products/C.Ops Development Pattern

By Report Date Interval

Exhibit XVII