SOURCES OF DISTORTION IN CLASSIFICATION RATEMAKING DATA AND THEIR TREATMENT

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ABSTRACT: Classification ratemaking represents an important role of most actuaries.

In order to increase stability when analyzing class relativities it is customary to combine the premium and loss experience of several years as well as from more than one state. This paper examines the possible distortions that may be introduced when such combinations are made. Two scaling factors are presented which address the distortion that has been detected.

The paper then shifts focus to the impact of the distortion on methods presented in two important papers published in the Proceedings of the Casualty Actuarial Society.

INTRODUCTION

At some point in the history of insurance, probably fairly soon after the first implementation of the concept of the pooling of risks, someone realized that it was possible to segment those risks into smaller, more homogeneous groups for more proper pricing. The quest for greater rate refinement has led to more and smaller classification cells, but this refinement has had a price. With smaller cells, less data is available for the calculation of rate relativities. The calculation of an overall rate level might be fully supported by the data from one year in one state, but the data for one small cell might be only a fraction of that necessary to support the rates for that cell. Thus the desire for more rate refinement leads directly to the need to combine data from more than one year and from more than one state. In fact, it is an accepted practice to incorporate premium and loss experience from several years and several states. However, when data is combined in this manner, it is necessary to ask if distortion can be created that is not present in the original data. Put another way, can ratemaking data carry information irrelevant to classification ratemaking that, in combination, creates false class relativity indications?

TRADITIONAL RATEMAKING METHODS

Actuarial literature outlines two general methods for classification[1] relativity analysis: the pure premium method and the loss ratio method. The pure premium method yields a class relativity directly. Each classification pure premium is divided by the base class[2] pure premium to produce the relativity. The loss ratio method yields an adjustment in the current classification relativity rather than the relativity itself. By dividing the classification loss ratio by the base class loss ratio, an indicated modification to the current relativity results.

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[1]The term classification includes all forms of stratification, including territory, zone, protection class, as well as conventional classifications (e.g. 1A, 1B, 1C in private passenger automobile).

[2]The base classification is generally that classification receiving a relativity of unity. This is usually one of the larger classifications and is often a classification receiving a relatively low rate, thus ensuring most relativities to be above 1.00.

The advantage of the pure premium method is that it requires no adjustment for past overall or classification relativity rate changes. The advantage of the loss ratio method is that, to the extent that the relativities for other strata are proper, any distortions due to the lack of independence between the stratum being examined and other strata are minimized. For example, if there is a preponderance of youthful operators in one territory, the pure premium method will yield an artificially high relativity for that territory while the loss ratio method will tend to adjust for the distortion. This occurs because, to the extent the youthful operator relativity is proper, the premium included in the loss ratio will reflect the higher youthful rate. Of course, to eliminate totally the distortion caused by distributional problems, an iterative process such as described by Bailey and Simon in their 1960 paper is required.[3]

A compromise method is attained if one modifies the premium to the base class rate level by dividing the premium by the class factor (or if extending exposures by utilizing only the base class relativity (unity) for each class) prior to calculating the loss ratio. This procedure (referred to in this paper as the modified loss ratio method) has an advantage of the loss ratio method since it adjusts, somewhat, for any of the distributional problems in the data described in the preceding paragraph (greater number of youthful operators in one territory), but it also has an advantage of the pure premium method since it directly yields the class factor.

SOURCES OF DISTORTION

Earlier the question was asked, "When data is combined, can distortion be created that is not present in the original data?" In order to answer this question a set of scenarios has been created illustrating the effect of two possible sources of distortion on traditional ratemaking methods. To simplify the situation, it is assumed that a company operates in two states and has two years of experience available with two classes in its rate structure. The input parameters are the current factors for each class, the required factor for each class, the base class loss ratio for each year and state, the distribution of exposures for each class in each year and state and the base rate for each state. The

[3]Bailey, Robert A. and Simon, Leroy J., "Two Studies in Automobile Insurance Ratemaking", PCAS XLVII, p1.

two assumed parameters (the sources of distortion) that are varied are the base class (class 01) loss ratio and the distribution of exposures by class. In each of the following six scenarios the premiums are created utilizing the assumed exposures, base rate and current factors, and the losses are created utilizing the assumed exposures, base rate, class 01 loss ratio and required factors. The derived loss experience illustrated in each scenario can be reproduced entirely from the displayed assumptions. In all of the scenarios, it is assumed that there has been no base rate change in either state, there has been no change in class factor during the experience period, that both classes are fully credible in total and that the required factor (that underlying the derived loss experience) for class 02 is 2.10. One would expect that a valid ratemaking methodology would produce the required factor as an indication.

The first three scenarios examine the single state, multiple year situation. Scenario 1 assumes an identical loss ratio for both years, but assumes that the class 02 exposures have increased from one third to one half of the total exposures. It can be verified that both of the traditional methods yield, for each year and for total, the required factor. That is, they yield a class 02 relativity of 2.10. For each of the scenarios in Exhibit I the modified loss ratio method would result in the same indicated relativity as does the loss ratio method and is therefore not displayed.

Scenario 2 illustrates a similar situation when the class 01 loss ratio increases in year two, but the distribution of exposures remains constant. In this situation both traditional methods, again, result in the required factor.

In Scenario 3, however, when both conditions are allowed to exist, that is an increasing loss ratio and different distribution of exposures, the situation changes. While both the traditional methods applied to each isolated year yield the required factor, when applied to the total data both methods yield the same incorrect answer (2.22). Class 02, because of its increase in exposure distribution, receives a proportionally higher contribution from year two's less adequate rate level, thus distorting its indicated factor upward. It is clear from this exhibit that caution must be used when combining more than one year's data to increase credibility.

At this point traditionalists may argue that since the total losses for the two years for class 02 are \$2,887,500 and the number of exposures is 20,000, why is the correct pure premium for class 02 not

$144.38? And similarly, why is the correct pure premium for class 01 not $65.00? The answer is that ratemaking in principle and, in fact, by statute is not to be designed to compensate insurers for past experience. Rather the object is to create the actuarially correct premium for the future considering past experience. In Scenario 3 there is no question that the proper relationship between the rates for class 01 and class 02 is 2.10. In fact, had this relativity been utilized, the loss ratio for each class in both years would have been equal.

Often, even when different states exhibit different rate levels, their classification experience is expected to be similar and the loss experience is combined for classification ratemaking. Scenario 4 displays a two state situation with identical class 01 loss ratios. State two has both a higher base rate and a preponderance of class 02 exposures. While the traditional methods result in the required factor when viewed separately by year, only the loss ratio method provides the correct answer when total data are utilized. The pure premium method is subject to distortion by the increased distribution of exposures of class 02 in the state with the higher rate level. The loss ratio method is immune to this distortion since both premiums and losses are identically affected.

When the combination of different loss ratios by state and different distribution of exposures by state are observed in Scenario 5, both conventional methods produce erroneous results. The pure premium method, however, is still subject to more distortion than is the loss ratio method.

Scenario 6 illustrates the full impact of each of the elements causing distortion. Admittedly, the parameters have been chosen so as to produce a significant and alarming result. In our dynamic industry, however, my assumptions are not beyond the realm of reality.

Appendix A displays the derivation of a symbolic representation of the distortion for both the pure premium method, the loss ratio method and the modified loss ratio method.

A summary of the results of each scenario is provided below:

					Type of Method			
	Class 01	Exposure			Pure Premium		Loss Ratio	
Scenario	Loss Ratio	Distribution	States	Years	Factor	Error	Factor	Error
1	Identical	Different	Single	Multiple	2.10	0.00%	2.10	0.00%
2	Different	Identical	Single	Multiple	2.10	0.00%	2.10	0.00%
3	Different	Different	Single	Multiple	2.22	5.71%	2.22	5.71%
4	Identical	Different	Multiple	Single	2.10	0.00%	2.30	9.52%
5	Different	Different	Multiple	Single	2.18	3.81%	2.39	13.81%
6	Different	Different	Multiple	Multiple	2.22	5.71%	2.59	23.33%

THE SOLUTION

In what manner do current ratemaking methods tend to mitigate the effects of the distortion that has been observed? The answer to this question lies in those adjustments that are normally performed on our ratemaking data. These adjustments include present level adjustments for overall rate level changes, application of trend factors, and in the case of accident year or policy year data, application of development factors. These adjustments serve to equalize the loss ratios from different years, thus reducing distortion. In fact, as the example in Scenario 1 suggests, even with a different exposure distribution in a multiple year situation, equalizing the loss ratio for the base class reduces the error in the relativity produced by both traditional methods.

As shown in Scenario 4, if the data from two states are combined to produce the same loss ratio for the base class, the loss ratio method yields the correct result. One would expect that, if the data could be adjusted to produce identical loss ratios for the base class for each state and year segmentation of data without altering the relationship between classifications in that data, the distortion would be eliminated. The adjustments traditionally applied to premiums and losses in ratemaking enumerated above are not sufficient to equalize the base class loss ratios for a number of reasons. First, loss experience is subject to a considerable "noise" which serves to allow indications derived from different years to vary somewhat. Also trend factors rarely will operate uniformly on the loss data from different years. The factors that impact loss costs seem to operate in spurts rather than at a constant rate. The loss ratio may differ by state because of differing regulatory climate, different laws and tort environment. In addition, those companies which maintain profit centers by state may, in fact, have different expected loss ratios in different states and will have different loss ratios in states even though they have equivalent rate adequacy. It is therefore quite unlikely that there will be identical base class loss ratios for each state and for each year even after the application of traditional loss experience adjustments.

Is it possible to scale the premiums or losses (or both) in such a manner that the distortion is removed when the data from one state or year are combined with that of another state or year? What characteristics should such a scaling factor possess? There are two criteria that must be met by any scaling factor candidate:

Criterion 1. The scaling factor should maintain the relationship between class loss ratios by year and state.

Criterion 2. The scaling factor should reduce the method error to zero.

Any scaling factor that is applied uniformly to each class within a specific state for a particular year or is applied to both premiums and losses for a specific class will fulfill the requirements of criterion 1. When either the exposure distribution or the base class loss ratio remain constant the distortion is not present, therefore any scaling factor that stabilizes either the base class loss ratio or the exposure distribution should fulfill the requirements of criterion 2.

Appendix B displays the derivation of two scaling factors that meet the needs of both of the criteria. The first scaling factor that is considered is the reciprocal of the base class loss ratio for each state and year. By applying this factor uniformly to the losses for each class the relationship between each of the class loss ratios is maintained (Criterion 1) while the method error is reduced to zero (Criterion 2).

Table II displays the effect of scaling losses with the reciprocal of base class loss ratio. Both Tables II and III utilize input parameters that are identical to those of Scenario 6. The modified loss ratio method is utilized on this exhibit. The premium is modified to the base class rate level by dividing by the class factor prior to calculating the loss ratio. For each class, the losses are scaled by the base class adjusted loss ratio for that year and state. For example, the incurred losses for state 01, year 1 (500,000) are multiplied by the reciprocal of class 01 loss ratio (1.00/0.50=2.00) to yield the scaled losses of $1,000,000. The class 02 incurred losses (525,000) are also multiplied by this factor to yield the scaled losses for that class of $1,050,000. These scaled losses maintain the relationship between the class loss ratios, but lose any information regarding the actual base class loss ratio. It is possible to apply a scaling factor (the base class loss ratio in this case rather than its reciprocal) to the premium rather than the losses. This method should be used only for larger, more stable lines of business. In cases where even the base class loss ratio can fluctuate wildly, it is more appropriate to scale the losses.

The second scaling factor derived in Appendix B addresses the different exposure distribution by year and state. The ratio of the total exposures for each class to the total exposures for the base class is multiplied by the ratio of the base class exposures in each state and year to the class exposures in each state and year to provide the scaling factor. As opposed to the first scaling factor the second scaling factor is different for each class, year and state; however, since the factor is applied to both premiums and losses, this scaling factor also satisfies the requirements of Criterion 1. When e'_{iys} replaces e_{iys} in the equation for the error developed in Appendix A, that error is reduced to zero, thus satisfying the requirements of Criterion 2. Table III displays the effect of utilizing the second scaling factor.

The advantages of the first scaling factor are:

1. Ease of use. The base class loss ratio is directly obtainable from the data already necessary for modified loss ratio method.
2. Since the scaling factor is applied uniformly for each class, the premium distribution by class for each year and state is left unaltered.
3. Elimination of some of the traditional adjustments to classification data (see next section).

The advantage of the second scaling factor is that if the exposure distribution is more stable than the base class loss ratio from year to year (as is probably expected) then the second scaling factor will result in less abrupt adjustments for most classes than will the first scaling factor.

OTHER CONSIDERATIONS

The acceptance of the hypothesis that the combination of more than one year of classification data may cause distortion in that data leads to the fear that a similar distortion may be caused in combining monthly or quarterly data into annual experience periods. This distortion might arise if there is a rapidly changing rate adequacy coupled with an unrelated, but also rapidly changing exposure distribution. The conscientious actuary should consider utilizing quarterly or semi annual data. It is reasonable to assume, however, that in most cases an acceptable level of precision is achieved utilizing annual data. Additionally as the data are segmented into smaller experience periods, seasonal exposure distribution changes and chance variations in base class loss ratio may artificially justify the need for such segmentation.

One of the advantages of the first scaling factor is that most of the traditional adjustments to premium and loss data are no longer necessary. Any adjustment that applies uniformly to the premiums or losses of all classes is nullified by the application of that scaling factor. These adjustments would include present level adjustments for overall rate changes, development factors and trend factors. If, however, these adjustments are not applied uniformly by class, they will still be necessary. For example, if trend factors are applied by cause of loss, these factors will need to be applied prior to the scaling process.

Earlier mention was made of the need to use an iterative approach to remove distortion caused when more than one classification structure is utilized and these segmentations are not independent. Two papers written in the early 1960's provide such approaches. "Two Studies in Automobile Insurance"[4] is a paper which describes a technique that minimizes the Chi square statistic to produce the class relativity estimate for each stratum. "Insurance Rates with Minimum Bias"[5] is a paper that derives a similar formula utilizing the average difference for each class. These papers are essential reading for the student of classification ratemaking. Of interest is whether or not these techniques

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[4]IBID

[5]Bailey, Robert A., "Insurance Rates with Minimum Bias", PCAS L, p4.

address the distortion found earlier in this paper and if not, whether the scaling factors relieve the distortion. Both papers discuss the applicability of multiplicative factors ("percents") versus additive factors ("cents"). Thus, it is necessary to test the suitability of the scaling methodology with both data that manifest multiplicative characteristics and data that manifest additive characteristics. Table IV displays the assumptions that were made in this analysis. Tables V and VI display observations made concerning several methods operating on data with underlying multiplicative and additive characteristics respectively. *To promote brevity these tables illustrate only the effects of the first scaling factor (the reciprocal of the base class loss ratio).* The current and required factors for both situations are displayed in Table IV. The class structure is that used in the first paper. The Classes 1, 2, 3, 4 and 5 are conventional automobile classes (age, sex and marital status) and classes A, X, Y and B are merit rating assignments. The x's are class relativities for the classes 1, 2, 3, 4 & 5 and the y's are classification relativities for merit rating assignments A, X, Y and B. The current factors assigned are similar to those in the original paper. In order to illustrate the distortion, it is assumed that the data from two years are being analyzed and that the class 1B (the base class) loss ratio is 50% in year 1 and 75% in year 2. Table IV also displays the assumed exposure distribution by class for both years.

I have selected four methods for observation from the referenced papers:

1. Method 1 - Bailey and Simon refer to this as the "customary method". It is an apt description. The relativities yielded by method 1 are the class loss ratios (at 1B rates) divided by the total loss ratio (also at 1B rates). It is essentially the modified loss ratio approach.
2. Method 2 - This is the minimized Chi square class relativity estimate that is utilized if it is assumed that multiplicative factors are most appropriate.
3. Method 3 - This is the minimized Chi square class relativity estimate that is utilized if it is assumed that additive factors are most appropriate. Both method 2 and 3 are shown on Tables V and VI to display the results when an inappropriate method is applied.

4. Minimum Bias - The minimum bias method uses the method detailed in Bailey's 1963 paper. The method used on each table is that appropriate for the underlying structure (multiplicative or additive).

Table V analyzes the factors yielded by each of four methods when the underlying data exhibit multiplicative characteristics. The left column (in each box) displays the raw relativities produced by each particular method. The second column displays these relativities adjusted to class 1B base for comparison with the required factors (third column). The adjusted values are straightforward for those methods which are multiplicative (Method 1, Method 2 and Minimum Bias on Table V) and are simply each factor divided by the base class (class 1 for the x's and class B for the y's). The adjustments for the additive methods are not as straightforward and are given by the following formulae: adjusted x_i = $(x_i + y_4) / (x_1 + y_4)$ and adjusted $y_i = (y_i - y_4) / (x_1 + y_4)$.

It is obvious from the factors displayed in the boxes on the left side of both tables V and VI that the traditional method of combining premium and loss data from different years fails to allow any of the methods to yield the required factors. Table V displays that both method 2 and the minimum bias methods, however, yield the required factors after the losses have been scaled. Method 1 fails to do so, since that method is inadequate for reducing bias in class relativity estimates. Method 3 is clearly inappropriate since it yields additive factors. Table VI displays that after the application of scaling factors, Method 3 and the minimum bias method yield the required factors, as expected. It is apparent from Tables V and VI that the scaling process helps these methods yield the required factors.

CONCLUSIONS

The combination of data from more than one year may cause distortion in traditional classification ratemaking techniques if each body of data represents a different base rate adequacy and different exposure distribution by class. The combination of data from more than one state may cause distortion in the traditional pure premium method if the base rate from each state is different and possesses a different exposure distribution by class. The combination of data from more than one state may cause distortion in both of the traditional methods if the base rate from each state is different, the base class loss ratio is different and the state/year data exhibit a different exposure distribution by class. It is more than likely that these conditions will exist within most bodies of ratemaking data.

These distortions may be remedied by the application of a scaling factor to the data from each year and each state. This scaling factor may address either the exposure distribution or the base rate adequacy.

The scaling factors appear to improve the output from two iterative classification methods outlined in two papers: "Two Studies in Automobile Insurance" and "Insurance Rates with Minimum Bias".

Single State – Multiple Year Situation

Identical Loss Ratios – Different Distribution

Table I
Scenario 1

Assumptions

Class Factors Underlying Experience

Class	Current Factor	Required Factor
01	1.00	1.00
02	2.00	2.10

Class 01 Loss Ratio

Year	Loss Ratio
1	75%
2	75%

Distribution of Exposures

Class	Year 1	Year 2	Total
01	10,000	15,000	25,000
02	5,000	15,000	20,000
Total	15,000	30,000	45,000

Base Rate = $100

Derived Loss Experience

Year	Class	Exposures	Earned Premium	Incurred Losses	Loss Ratio	Pure Premium
1	01	10,000	$1,000,000	$750,000	75.00%	$75.00
	02	5,000	$1,000,000	$787,500	78.75%	$157.50
	Total	15,000	$2,000,000	$1,537,500	76.88%	$102.50
2	01	15,000	$1,500,000	$1,125,000	75.00%	$75.00
	02	15,000	$3,000,000	$2,362,500	78.75%	$157.50
	Total	30,000	$4,500,000	$3,487,500	77.50%	$116.25
Total	01	25,000	$2,500,000	$1,875,000	75.00%	$75.00
	02	20,000	$4,000,000	$3,150,000	78.75%	$157.50
	Total	45,000	$6,500,000	$5,025,000	77.31%	$111.67

Indicated Class 2 Relativity

Loss Ratio Method: (78.75% / 75.00%) x 2.00 = 2.10
Pure Premium Method: 157.50 / 75.00 = 2.10

Single State – Multiple Year Situation

Different Loss Ratios – Identical Distribution

Table I

Scenario 2

Assumptions

Class Factors Underlying Experience

Class	Current Factor	Required Factor
01	1.00	1.00
02	2.00	2.10

Class 01 Loss Ratio

Year	Loss Ratio
1	50%
2	75%

Distribution of Exposures

Class	Year 1	Year 2	Total
01	10,000	15,000	25,000
02	5,000	7,500	12,500
Total	15,000	22,500	37,500

Base Rate = $100

Derived Loss Experience

Year	Class	Exposures	Earned Premium	Incurred Losses	Loss Ratio	Pure Premium
1	01	10,000	$1,000,000	$500,000	50.00%	$50.00
	02	5,000	$1,000,000	$525,000	52.50%	$105.00
	Total	15,000	$2,000,000	$1,025,000	51.25%	$68.33
2	01	15,000	$1,500,000	$1,125,000	75.00%	$75.00
	02	7,500	$1,500,000	$1,181,250	78.75%	$157.50
	Total	22,500	$3,000,000	$2,306,250	76.88%	$102.50
Total	01	25,000	$2,500,000	$1,625,000	65.00%	$65.00
	02	12,500	$2,500,000	$1,706,250	68.25%	$136.50
	Total	37,500	$5,000,000	$3,331,250	66.63%	$88.83

Indicated Class 2 Relativity

Loss Ratio Method: (68.25% / 65.00%) x 2.00 = 2.10

Pure Premium Method: 136.50 / 65.00 = 2.10

Single State – Multiple Year Situation

Different Loss Ratios – Different Distribution

Table I
Scenario 3

Assumptions

Class Factors Underlying Experience

Class	Current Factor	Required Factor
01	1.00	1.00
02	2.00	2.10

Class 01 Loss Ratio

Year	Loss Ratio
1	50%
2	75%

Distribution of Exposures

Class	Year 1	Year 2	Total
01	10,000	15,000	25,000
02	5,000	15,000	20,000
Total	15,000	30,000	45,000

Base Rate = $100

Derived Loss Experience

Year	Class	Exposures	Earned Premium	Incurred Losses	Loss Ratio	Pure Premium
1	01	10,000	$1,000,000	$500,000	50.00%	$50.00
	02	5,000	$1,000,000	$525,000	52.50%	$105.00
	Total	15,000	$2,000,000	$1,025,000	51.25%	$68.33
2	01	15,000	$1,500,000	$1,125,000	75.00%	$75.00
	02	15,000	$3,000,000	$2,362,500	78.75%	$157.50
	Total	30,000	$4,500,000	$3,487,500	77.50%	$116.25
Total	01	25,000	$2,500,000	$1,625,000	65.00%	$65.00
	02	20,000	$4,000,000	$2,887,500	72.19%	$144.38
	Total	45,000	$6,500,000	$4,512,500	69.42%	$100.28

Indicated Class 2 Relativity

Loss Ratio Method: (72.19% / 65.00%) x 2.00 = 2.22
Pure Premium Method: 144.38 / 65.00 = 2.22

Multiple State – Single Year Situation
Identical Loss Ratios – Different Distribution

Table I
Scenario 4

Assumptions

Class Factors Underlying Experience

Class	Current Factor	Required Factor
01	1.00	1.00
02	2.00	2.10

Class 01 Loss Ratio

State	Loss Ratio
1	75%
2	75%

Distribution of Exposures

Class	State 1	State 2	Total
01	10,000	15,000	25,000
02	5,000	15,000	20,000
Total	15,000	30,000	45,000

State 1 Base Rate = $100
State 2 Base Rate = $200

Derived Loss Experience

State	Class	Exposures	Earned Premium	Incurred Losses	Loss Ratio	Pure Premium
1	01	10,000	$1,000,000	$750,000	75.00%	$75.00
	02	5,000	$1,000,000	$787,500	78.75%	$157.50
	Total	15,000	$2,000,000	$1,537,500	76.88%	$102.50
2	01	15,000	$3,000,000	$2,250,000	75.00%	$150.00
	02	15,000	$6,000,000	$4,725,000	78.75%	$315.00
	Total	30,000	$9,000,000	$6,975,000	77.50%	$232.50
Total	01	25,000	$4,000,000	$3,000,000	75.00%	$120.00
	02	20,000	$7,000,000	$5,512,500	78.75%	$275.63
	Total	45,000	$11,000,000	$8,512,500	77.39%	$189.17

Indicated Class 2 Relativity

Loss Ratio Method: (78.75% / 75.00%) x 2.00 = 2.10
Pure Premium Method: 275.63 / 120.00 = 2.30

Multiple State - Single Year Situation

Different Loss Ratios - Different Distribution

Table I
Scenario 5

Assumptions

Class Factors Underlying Experience

Class	Current Factor	Required Factor
01	1.00	1.00
02	2.00	2.10

Class 01 Loss Ratio

State	Loss Ratio
1	50%
2	75%

Distribution of Exposures

Class	State 1	State 2	Total
01	10,000	15,000	25,000
02	5,000	15,000	20,000
Total	15,000	30,000	45,000

State 1 Base Rate = $100
State 2 Base Rate = $200

Derived Loss Experience

State	Class	Exposures	Earned Premium	Incurred Losses	Loss Ratio	Pure Premium
1	01	10,000	$1,000,000	$500,000	50.00%	$50.00
	02	5,000	$1,000,000	$525,000	52.50%	$105.00
	Total	15,000	$2,000,000	$1,025,000	51.25%	$68.33
2	01	15,000	$3,000,000	$2,250,000	75.00%	$150.00
	02	15,000	$6,000,000	$4,725,000	78.75%	$315.00
	Total	30,000	$9,000,000	$6,975,000	77.50%	$232.50
Total	01	25,000	$4,000,000	$2,750,000	68.75%	$110.00
	02	20,000	$7,000,000	$5,250,000	75.00%	$262.50
	Total	45,000	$11,000,000	$8,000,000	72.73%	$177.78

Indicated Class 2 Relativity

Loss Ratio Method: (75.00% / 68.75%) x 2.00 = 2.18
Pure Premium Method: 262.50 / 110.00 = 2.39

Multiple State – Multiple Year Situation
Different Loss Ratios – Different Distribution

Table I
Scenario 6

Assumptions

Class Factors Underlying Experience

Class	Current Factor	Required Factor
01	1.00	1.00
02	2.00	2.10

Class 01 Loss Ratio

State	Loss Ratios Year 1	Loss Ratios Year 2
1	50%	75%
2	60%	85%

Distribution of Exposures

	State 1		State 2		
Class	Year 1	Year 2	Year 1	Year 2	Total
01	10,000	15,000	10,000	15,000	50,000
02	5,000	15,000	15,000	45,000	80,000
Total	15,000	30,000	25,000	60,000	130,000

State 1 Base Rate = $100
State 2 Base Rate = $200

(The Derived Loss Experience is shown on the next page.)

Indicated Class 2 Relativity

Loss Ratio Method: (81.19% / 71.67%) x 2.00 = 2.27
Pure Premium Method: 284.16 / 107.50 = 2.64

Multiple State – Multiple Year Situation

Table I

Different Loss Ratios – Different Distribution

Scenario 6
(cont.)

Derived Loss Experience

	Class	Exposures	Earned Premium	Incurred Losses	Loss Ratio	Pure Premium
State 1						
Year 1	01	10,000	$1,000,000	$500,000	50.00%	$50.00
	02	5,000	$1,000,000	$525,000	52.50%	$105.00
	Total	15,000	$2,000,000	$1,025,000	51.25%	$68.33
Year 2	01	15,000	$1,500,000	$1,125,000	75.00%	$75.00
	02	15,000	$3,000,000	$2,362,500	78.75%	$157.50
	Total	30,000	$4,500,000	$3,487,500	77.50%	$116.25
All Years	01	25,000	$2,500,000	$1,625,000	65.00%	$65.00
	02	20,000	$4,000,000	$2,887,500	72.19%	$144.38
	Total	45,000	$6,500,000	$4,512,500	69.42%	$100.28
State 2						
Year 1	01	10,000	$2,000,000	$1,200,000	60.00%	$120.00
	02	15,000	$6,000,000	$3,780,000	63.00%	$252.00
	Total	25,000	$8,000,000	$4,980,000	62.25%	$199.20
Year 2	01	15,000	$3,000,000	$2,550,000	85.00%	$170.00
	02	45,000	$18,000,000	$16,065,000	89.25%	$357.00
	Total	60,000	$21,000,000	$18,615,000	88.64%	$310.25
All Years	01	25,000	$5,000,000	$3,750,000	75.00%	$150.00
	02	60,000	$24,000,000	$19,845,000	82.69%	$330.75
	Total	85,000	$29,000,000	$23,595,000	81.36%	$277.59
All States						
Year 1	01	20,000	$3,000,000	$1,700,000	56.67%	$85.00
	02	20,000	$7,000,000	$4,305,000	61.50%	$215.25
	Total	40,000	$10,000,000	$6,005,000	60.05%	$150.13
Year 2	01	30,000	$4,500,000	$3,675,000	81.67%	$122.50
	02	60,000	$21,000,000	$18,427,500	87.75%	$307.13
	Total	90,000	$25,500,000	$22,102,500	86.68%	$245.58
All Years	01	50,000	$7,500,000	$5,375,000	71.67%	$107.50
	02	80,000	$28,000,000	$22,732,500	81.19%	$284.16
	Total	130,000	$35,500,000	$28,107,500	79.18%	$216.21

Multiple State – Multiple Year Situation

Table II

Different Loss Ratios – Different Distribution

Derived Loss Experience

	Class	Exposures	Earned Premium	Class Factor	Adjusted Premium	Incurred Losses	Adjusted Loss Ratio	Scaled Losses	Modified Loss Ratio
State 1									
Year 1	01	10,000	$1,000,000	1.00	$1,000,000	$500,000	50.00%	$1,000,000	100.00%
	02	5,000	$1,000,000	2.00	$500,000	$525,000	105.00%	$1,050,000	210.00%
	Total	15,000	$2,000,000		$1,500,000	$1,025,000	68.33%	$2,050,000	136.67%
Year 2	01	15,000	$1,500,000	1.00	$1,500,000	$1,125,000	75.00%	$1,500,000	100.00%
	02	15,000	$3,000,000	2.00	$1,500,000	$2,362,500	157.50%	$3,150,000	210.00%
	Total	30,000	$4,500,000		$3,000,000	$3,487,500	116.25%	$4,650,000	155.00%
All Years	01	25,000	$2,500,000		$2,500,000	$1,625,000	65.00%	$2,500,000	100.00%
	02	20,000	$4,000,000		$2,000,000	$2,887,500	144.38%	$4,200,000	210.00%
	Total	45,000	$6,500,000		$4,500,000	$4,512,500	100.28%	$6,700,000	148.89%
State 2									
Year 1	01	10,000	$2,000,000	1.00	$2,000,000	$1,200,000	60.00%	$2,000,000	100.00%
	02	15,000	$6,000,000	2.00	$3,000,000	$3,780,000	126.00%	$6,300,000	210.00%
	Total	25,000	$8,000,000		$5,000,000	$4,980,000	99.60%	$8,300,000	166.00%
Year 2	01	15,000	$3,000,000	1.00	$3,000,000	$2,550,000	85.00%	$3,000,000	100.00%
	02	45,000	$18,000,000	2.00	$9,000,000	$16,065,000	178.50%	$18,900,000	210.00%
	Total	60,000	$21,000,000		$12,000,000	$18,615,000	155.13%	$21,900,000	182.50%
All Years	01	25,000	$5,000,000		$5,000,000	$3,750,000	75.00%	$5,000,000	100.00%
	02	60,000	$24,000,000		$12,000,000	$19,845,000	165.38%	$25,200,000	210.00%
	Total	85,000	$29,000,000		$17,000,000	$23,595,000	138.79%	$30,200,000	177.65%
All States									
Year 1	01	20,000	$3,000,000		$3,000,000	$1,700,000	56.67%	$3,000,000	100.00%
	02	20,000	$7,000,000		$3,500,000	$4,305,000	123.00%	$7,350,000	210.00%
	Total	40,000	$10,000,000		$6,500,000	$6,005,000	92.38%	$10,350,000	159.23%
Year 2	01	30,000	$4,500,000		$4,500,000	$3,675,000	81.67%	$4,500,000	100.00%
	02	60,000	$21,000,000		$10,500,000	$18,427,500	175.50%	$22,050,000	210.00%
	Total	90,000	$25,500,000		$15,000,000	$22,102,500	147.35%	$26,550,000	177.00%
All Years	01	50,000	$7,500,000		$7,500,000	$5,375,000	71.67%	$7,500,000	100.00%
	02	80,000	$28,000,000		$14,000,000	$22,732,500	162.38%	$29,400,000	210.00%
	Total	130,000	$35,500,000		$21,500,000	$28,107,500	130.73%	$36,900,000	171.63%

Indicated Class 2 Relativity

Modified Loss Ratio Method: (210.00% / 100.00%) x = 2.10

Multiple State – Multiple Year Situation

Table III

Different Loss Ratios – Different Distribution

Derived Loss Experience

	Class	Exposures	Earned Premium	Class Factor	Incurred Losses	Scaling Factor	Scaled Premium	Scaled Losses	Modified Loss Ratio
State 1 Year 1	01	10,000	$1,000,000	1.00	$500,000	1.000	$1,000,000	$500,000	50.00%
	02	5,000	$1,000,000	2.00	$525,000	3.200	$1,600,000	$1,680,000	105.00%
	Total	15,000	$2,000,000		$1,025,000		$2,600,000	$2,180,000	83.85%
Year 2	01	15,000	$1,500,000	1.00	$1,125,000	1.000	$1,500,000	$1,125,000	75.00%
	02	15,000	$3,000,000	2.00	$2,362,500	1.600	$2,400,000	$3,780,000	157.50%
	Total	30,000	$4,500,000		$3,487,500		$3,900,000	$4,905,000	125.77%
All Years	01	25,000	$2,500,000		$1,625,000		$2,500,000	$1,625,000	65.00%
	02	20,000	$4,000,000		$2,887,500		$4,000,000	$5,460,000	136.50%
	Total	45,000	$6,500,000		$4,512,500		$6,500,000	$7,085,000	109.00%
State 2 Year 1	01	10,000	$2,000,000	1.00	$1,200,000	1.000	$2,000,000	$1,200,000	60.00%
	02	15,000	$6,000,000	2.00	$3,780,000	1.067	$3,200,000	$4,032,000	126.00%
	Total	25,000	$8,000,000		$4,980,000		$5,200,000	$5,232,000	100.62%
Year 2	01	15,000	$3,000,000	1.00	$2,550,000	1.000	$3,000,000	$2,550,000	85.00%
	02	45,000	$18,000,000	2.00	$16,065,000	0.533	$4,800,000	$8,568,000	178.50%
	Total	60,000	$21,000,000		$18,615,000		$7,800,000	$11,118,000	142.54%
All Years	01	25,000	$5,000,000		$3,750,000		$5,000,000	$3,750,000	75.00%
	02	60,000	$24,000,000		$19,845,000		$8,000,000	$12,600,000	157.50%
	Total	85,000	$29,000,000		$23,595,000		$13,000,000	$16,350,000	125.77%
All States Year 1	01	20,000	$3,000,000		$1,700,000		$3,000,000	$1,700,000	56.67%
	02	20,000	$7,000,000		$4,305,000		$4,800,000	$5,712,000	119.00%
	Total	40,000	$10,000,000		$6,005,000		$7,800,000	$7,412,000	95.03%
Year 2	01	30,000	$4,500,000		$3,675,000		$4,500,000	$3,675,000	81.67%
	02	60,000	$21,000,000		$18,427,500		$7,200,000	$12,348,000	171.50%
	Total	90,000	$25,500,000		$22,102,500		$11,700,000	$16,023,000	136.95%
All Years	01	50,000	$7,500,000		$5,375,000		$7,500,000	$5,375,000	71.67%
	02	80,000	$28,000,000		$22,732,500		$12,000,000	$18,060,000	150.50%
	Total	130,000	$35,500,000		$28,107,500		$19,500,000	$23,435,000	120.18%

Indicated Class 2 Relativity

Modified Loss Ratio Method: (150.50% / 71.67%) x = 2.10

Assumptions

Table IV

	Multiplicative Case		Additive Case	
	Current Factors	Required Factors	Current Factors	Required Factors
x1	1.000	1.000	1.000	1.000
x2	1.650	1.650	1.100	1.100
x3	1.650	1.750	1.200	1.300
x4	2.400	2.500	1.500	1.600
x5	1.650	1.650	2.000	2.100
y1	0.650	0.650	−0.350	−0.350
y2	0.800	0.750	−0.200	−0.250
y3	0.900	0.850	−0.100	−0.150
y4	1.000	1.000	0.000	0.000

Class 1B	Year 1	Year 2
Loss Ratio	50.00%	75.00%

Year 1
Exposure Distribution

	A 1	X 2	Y 3	B 4	Total
1	30.00%	6.00%	9.00%	15.00%	60.00%
2	5.00%	1.00%	1.50%	2.50%	10.00%
3	7.50%	1.50%	2.25%	3.75%	15.00%
4	5.00%	1.00%	1.50%	2.50%	10.00%
5	2.50%	0.50%	0.75%	1.25%	5.00%
Total	50.00%	10.00%	15.00%	25.00%	100.00%

Year 2
Exposure Distribution

	A 1	X 2	Y 3	B 4	Total
1	20.00%	5.00%	10.00%	15.00%	50.00%
2	4.00%	1.00%	2.00%	3.00%	10.00%
3	6.00%	1.50%	3.00%	4.50%	15.00%
4	6.00%	1.50%	3.00%	4.50%	15.00%
5	4.00%	1.00%	2.00%	3.00%	10.00%
Total	40.00%	10.00%	20.00%	30.00%	100.00%

Multiplicative Case ("Percents")

Table V

	Traditional Method			Using Scaling Factors		
Class	Method 1	Adjusted	Required	Method 1	Adjusted	Required
x1	0.6870	1.0000	1.0000	0.7060	1.0000	1.0000
x2	1.1580	1.6856	1.6500	1.1670	1.6530	1.6500
x3	1.2280	1.7875	1.7500	1.2380	1.7535	1.7500
x4	1.8300	2.6638	2.5000	1.7740	2.5127	2.5000
x5	1.2420	1.8079	1.6500	1.1730	1.6615	1.6500
y1	0.7980	0.6205	0.6500	0.8180	0.6451	0.6500
y2	0.9450	0.7348	0.7500	0.9470	0.7468	0.7500
y3	1.1060	0.8600	0.8500	1.0800	0.8517	0.8500
y4	1.2860	1.0000	1.0000	1.2680	1.0000	1.0000

Class	Method 2	Adjusted	Required	Method 2	Adjusted	Required
x1	0.6721	1.0000	1.0000	0.7068	1.0000	1.0000
x2	1.1290	1.6798	1.6500	1.1666	1.6505	1.6500
x3	1.1976	1.7819	1.7500	1.2372	1.7505	1.7500
x4	1.7783	2.6459	2.5000	1.7670	2.5000	2.5000
x5	1.2027	1.7894	1.6500	1.1666	1.6505	1.6500
y1	0.8214	0.6248	0.6500	0.8219	0.6501	0.6500
y2	0.9687	0.7368	0.7500	0.9483	0.7501	0.7500
y3	1.1286	0.8584	0.8500	1.0747	0.8501	0.8500
y4	1.3148	1.0000	1.0000	1.2642	1.0000	1.0000

Class	Method 3	Adjusted	Required	Method 3	Adjusted	Required
x1	0.6847	1.0000	1.0000	0.7091	1.0000	1.0000
x2	1.1349	1.4715	1.6500	1.1542	1.4654	1.6500
x3	1.2043	1.5442	1.7500	1.2241	1.5385	1.7500
x4	1.8025	2.1707	2.5000	1.7558	2.0945	2.5000
x5	1.2120	1.5522	1.6500	1.1558	1.4671	1.6500
y1	−0.1606	−0.4511	0.6500	−0.1532	−0.4188	0.6500
y2	−0.0389	−0.3236	0.7500	−0.0439	−0.3045	0.7500
y3	0.1014	−0.1767	0.8500	0.0703	−0.1851	0.8500
y4	0.2701	0.0000	1.0000	0.2473	0.0000	1.0000

Class	Minimum Bias	Adjusted	Required	Minimum Bias	Adjusted	Required
x1	0.7639	1.0000	1.0000	0.7824	1.0000	1.0000
x2	1.2833	1.6798	1.6500	1.2914	1.6505	1.6500
x3	1.3613	1.7819	1.7500	1.3696	1.7505	1.7500
x4	2.0213	2.6459	2.5000	1.9561	2.5000	2.5000
x5	1.3670	1.7894	1.6500	1.2914	1.6505	1.6500
y1	0.7227	0.6248	0.6500	0.7425	0.6501	0.6500
y2	0.8523	0.7368	0.7500	0.8567	0.7501	0.7500
y3	0.9930	0.8584	0.8500	0.9709	0.8501	0.8500
y4	1.1567	1.0000	1.0000	1.1420	1.0000	1.0000

Additive Case ("Cents")

Table VI

	Traditional Method			Using Scaling Factors		
Class	Method 1	Adjusted	Required	Method 1	Adjusted	Required
x1	0.7670	1.0000	1.0000	0.7870	1.0000	1.0000
x2	0.8820	1.1499	1.1000	0.8870	1.1271	1.1000
x3	1.0790	1.4068	1.3000	1.0870	1.3812	1.3000
x4	1.4320	1.8670	1.6000	1.3880	1.7637	1.6000
x5	1.9950	2.6010	2.1000	1.8880	2.3990	2.1000
y1	0.8340	0.6786	-0.3500	0.8550	0.7060	-0.3500
y2	0.9570	0.7787	-0.2500	0.9590	0.7919	-0.2500
y3	1.0910	0.8877	-0.1500	1.0650	0.8794	-0.1500
y4	1.2290	1.0000	0.0000	1.2110	1.0000	0.0000

Class	Method 2	Adjusted	Required	Method 2	Adjusted	Required
x1	0.7463	1.0000	1.0000	0.7673	1.0000	1.0000
x2	0.8553	1.1459	1.1000	0.8640	1.1261	1.1000
x3	1.0467	1.4025	1.3000	1.0578	1.3787	1.3000
x4	1.3860	1.8571	1.6000	1.3486	1.7576	1.6000
x5	1.9315	2.5880	2.1000	1.8334	2.3896	2.1000
y1	0.8654	0.6858	-0.3500	0.8830	0.7112	-0.3500
y2	0.9855	0.7810	-0.2500	0.9852	0.7935	-0.2500
y3	1.1171	0.8853	-0.1500	1.0872	0.8757	-0.1500
y4	1.2618	1.0000	0.0000	1.2416	1.0000	0.0000

Class	Method 3	Adjusted	Required	Method 3	Adjusted	Required
x1	0.7322	1.0000	1.0000	0.7468	1.0000	1.0000
x2	0.8436	1.1124	1.1000	0.8464	1.1000	1.1000
x3	1.0406	1.3111	1.3000	1.0456	1.3000	1.3000
x4	1.3895	1.6629	1.6000	1.3444	1.6000	1.6000
x5	1.9516	2.2299	2.1000	1.8424	2.1000	2.1000
y1	-0.1206	-0.3831	-0.3500	-0.0994	-0.3500	-0.3500
y2	-0.0064	-0.2679	-0.2500	0.0002	-0.2500	-0.2500
y3	0.1197	-0.1408	-0.1500	0.0998	-0.1500	-0.1500
y4	0.2593	0.0000	0.0000	0.2492	0.0000	0.0000

Class	Minimum Bias	Adjusted	Required	Minimum Bias	Adjusted	Required
x1	0.7687	1.0000	1.0000	0.7886	1.0000	1.0000
x2	0.8811	1.1462	1.1000	0.8882	1.1000	1.1000
x3	1.0785	1.4030	1.3000	1.0874	1.3000	1.3000
x4	1.4284	1.8581	1.6000	1.3862	1.6000	1.6000
x5	1.9908	2.5897	2.1000	1.8842	2.1000	2.1000
y1	-0.1603	-0.3873	-0.3500	-0.1412	-0.3500	-0.3500
y2	-0.0436	-0.2698	-0.2500	-0.0416	-0.2500	-0.2500
y3	0.0847	-0.1406	-0.1500	0.0580	-0.1500	-0.1500
y4	0.2243	0.0000	0.0000	0.2074	0.0000	0.0000

APPENDIX A

SYMBOLIC REPRESENTATION OF DISTORTION

Let e_{iys} = Earned exposures for class i, year y, state s

r_{ys} = Base class loss ratio for year y, state s

B_{ys} = Base Rate for year y, state s

c_i = Current class factor for class i ($c_b = 1$)

f_i = Required factor for class i ($f_b = 1$)

g_i = Factor yielded by method for class i

E_i = Total earned exposures for class i = ${}_y\Sigma\ {}_s\Sigma\ e_{iys}$

P_i = Total earned premiums for class i on present rates = ${}_y\Sigma\ {}_s\Sigma\ e_{iys} B_{ys} c_i$

L_i = Total incurred losses for class i = ${}_y\Sigma\ {}_s\Sigma\ e_{iys} r_{ys} B_{ys} f_i$

b = base class subscript

Pure Premium Method

$$g_i = \frac{\dfrac{{}_y\Sigma\ {}_s\Sigma\ e_{iys} r_{ys} B_{ys} f_i}{{}_y\Sigma\ {}_s\Sigma\ e_{iys}}}{\dfrac{{}_y\Sigma\ {}_s\Sigma\ e_{bys} r_{ys} B_{ys} f_b}{{}_y\Sigma\ {}_s\Sigma\ e_{bys}}}$$

error in method = $(g_i / f_i) - 1$

$$= \frac{{}_y\Sigma\ {}_s\Sigma\ e_{iys} r_{ys} B_{ys}}{{}_y\Sigma\ {}_s\Sigma\ e_{iys}} \bullet \frac{{}_y\Sigma\ {}_s\Sigma\ e_{bys}}{{}_y\Sigma\ {}_s\Sigma\ e_{bys} r_{ys} B_{ys}} - 1$$

$$= \frac{{}_y\Sigma\ {}_s\Sigma\ e_{iys} r_{ys} B_{ys}}{E_i} \bullet \frac{E_b}{L_b} - 1$$

Loss Ratio Method

$$g_i \quad = \quad \frac{\dfrac{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ r_{ys}\ B_{ys}\ f_i}{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ B_{ys}\ c_i}}{\dfrac{{}_y\Sigma\ {}_s\Sigma\ e_{bys}\ r_{ys}\ B_{ys}\ f_b}{{}_y\Sigma\ {}_s\Sigma\ e_{bys}\ B_{ys}\ c_b}}$$

error in method = $(g_i / f_i) - 1$

$$= \quad \frac{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ r_{ys}\ B_{ys}}{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ B_{ys}\ c_i} \bullet \frac{{}_y\Sigma\ {}_s\Sigma\ e_{bys}\ B_{ys}}{{}_y\Sigma\ {}_s\Sigma\ e_{bys}\ r_{ys}\ B_{ys}} - 1$$

$$= \quad \frac{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ r_{ys}\ B_{ys}}{P_i} \bullet \frac{P_b}{L_b} - 1$$

Modified Loss Ratio Method

$$g_i \quad = \quad \frac{\dfrac{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ r_{ys}\ B_{ys}\ f_i}{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ B_{ys}\ c_b}}{\dfrac{{}_y\Sigma\ {}_s\Sigma\ e_{bys}\ r_{ys}\ B_{ys}\ f_b}{{}_y\Sigma\ {}_s\Sigma\ e_{bys}\ B_{ys}\ c_b}}$$

error in method = $(g_i / f_i) - 1$

$$= \quad \frac{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ r_{ys}\ B_{ys}}{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ B_{ys}\ c_b} \bullet \frac{{}_y\Sigma\ {}_s\Sigma\ e_{bys}\ B_{ys}}{{}_y\Sigma\ {}_s\Sigma\ e_{bys}\ r_{ys}\ B_{ys}} - 1 \qquad [1]$$

$$= \quad \frac{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ r_{ys}\ B_{ys}}{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ B_{ys}\ c_b} \bullet \frac{P_b}{L_b} - 1$$

APPENDIX B

DERIVATION OF SCALING FACTORS

Criterion 1. The scaling factor should maintain the relationship between class loss ratios by year and state.

Criterion 2. The scaling factor should reduce the method error to zero.

First Scaling Factor

Consider equation [1] from Appendix A:

error in method = $(g_i / f_i) - 1$

$$= \frac{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ r_{ys}\ B_{ys}}{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ B_{ys}\ c_i\ {}_y\Sigma\ {}_s\Sigma\ e_{bys}\ r_{ys}\ B_{ys}} \bullet \frac{{}_y\Sigma\ {}_s\Sigma\ e_{bys}\ B_{ys}}{} - 1$$

$$= \frac{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ r_{ys}\ B_{ys}}{{}_y\Sigma\ {}_s\Sigma\ e_{iys}\ B_{ys}\ c_b} \bullet \frac{P_b}{L_b} - 1$$

if each $r_{ys} = 1$ then error in method = 0

therefore $1/r_{ys}$ is a scaling factor

Second Scaling Factor

$$\text{Let } e'_{iys} = \frac{{}_y\Sigma\ {}_s\Sigma\ e_{iys}}{{}_y\Sigma\ {}_s\Sigma\ e_{bys}} \bullet e_{bys}$$

error in method = $(g_i / f_i) - 1$

$$= \frac{{}_y\Sigma\ {}_s\Sigma\ e'_{iys}\ r_{ys}\ B_{ys}}{{}_y\Sigma\ {}_s\Sigma\ e'_{iys}\ B_{ys}\ c_i} \bullet \frac{{}_y\Sigma\ {}_s\Sigma\ e'_{bys}\ B_{ys}}{{}_y\Sigma\ {}_s\Sigma\ e'_{bys}\ r_{ys}\ B_{ys}} - 1$$

$$= \frac{e'_i\ {}_y\Sigma\ {}_s\Sigma\ e_{bys}\ r_{ys}\ B_{ys}}{e'_i\ {}_y\Sigma\ {}_s\Sigma\ e_{bys}\ B_{ys}} \bullet \frac{e'_i\ {}_y\Sigma\ {}_s\Sigma\ e_{bys}\ B_{ys}}{e'_i\ {}_y\Sigma\ {}_s\Sigma\ e_{bys}\ r_{ys}\ B_{ys}} - 1$$

$$= 0 \qquad \text{where } e'_i = \frac{{}_y\Sigma\ {}_s\Sigma\ e_{iys}}{{}_y\Sigma\ {}_s\Sigma\ e_{bys}}$$

So this scaling factor satisfies Criterion 2.

Since this scaling factor is applied to premiums *and* losses by class each class loss ratio remains unchanged satisfying Criterion 1.

$$\text{Scaling factor} = \frac{{}_y\Sigma\ {}_s\Sigma\ e_{iys}}{{}_y\Sigma\ {}_s\Sigma\ e_{bys}} \bullet \frac{e_{bys}}{e_{iys}}$$