THE MEASUREMENT AND MANAGEMENT OF INTEREST RATE RISK

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ABSTRACT:

The intent of this paper is to provide some basic tools for the measurement and management of interest rate risk. Interest rate risk has been present in the P/C industry since inception of the first insurance policy. Recent (1980's) results of the P/C industry have heightened the awareness of the importance of investment income and its associated risk. Proper management of this risk is a key to the economic success of a P/C company. The actuary should play an important role in the evaluation of this risk and in further developing management techniques. While this paper goes beyond the work previously published in CAS materials, there is much need for additional work in this area.

Note to Reader: It is recommended that the reader be reasonably familiar with recent publications on similar topics.

THE MEASUREMENT AND MANAGEMENT OF INTEREST RATE RISK

HENRY FORD II....

> "NOBODY CAN REALLY GUARANTEE THE FUTURE. THE BEST WE CAN DO IS SIZE UP THE CHANCES, CALCULATE THE RISKS INVOLVED, ESTIMATE OUR ABILITY TO DEAL WITH THEM, AND THEN MAKE OUR PLANS WITH CONFIDENCE."

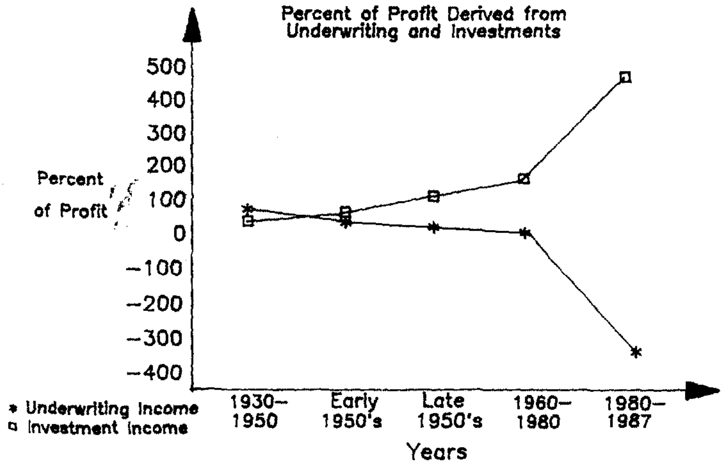
INTRODUCTION

Casualty actuaries have concentrated their efforts and expertise on the underwriting performance of the Property/Casualty (P/C) industry with little regard to investment performance, interest rate risk or asset portfolios. This concentration seemed appropriate in the past, however, the segregation of underwriting and investment results has become clouded in recent years. For some long tail lines it is unrealistic to segregate underwriting and investment results since investment income is directly reflected in the pricing of the insurance product. Many states require the pricing of insurance to reflect investment income. For all lines of business, the Tax Reform Act of 1986 has forced the P/C industry to discount reserves for future investment income when determining federal income taxes. Under these circumstances, if the actuary has any amount of responsibility for total results (i.e., contributions to surplus) from the business of writing insurance, and we believe this to be the case, then the actuary can no longer ignore investment performance levels, interest rate risk or the asset side of the balance sheet.

In this paper, we will discuss methods that can be used to help manage the investment risk of P/C companies through Asset/Liability Management (ALM). We will concentrate on the actuary's role in ALM, the overall purpose of ALM and how to measure the "matching" of assets and liabilities in order to measure interest rate risk.

WHY THE NEED FOR ASSET/LIABILITY MANAGEMENT

Prior to the late 1970's, interest rates were relatively low and stable with investment income that was predictable. P/C insurance companies concentrated their efforts on underwriting profits. Over the years, however, the relative contributions to profit from underwriting income and investment income have shifted (see chart below and Exhibit 1).



Pre-1980 investment strategy was yield oriented with a buy and hold mentality. The portfolios were predominantly long term (20-30 years) and heavily weighted in the Municipal sector. This strategy contributed, in some ways, to the poor underwriting results of the 1980's.

In the early 1980's, the high interest rate environment resulted in a strong desire for additional funds in order to take advantage of the higher yields. P/C companies quickly realized that they had to find ways to generate extra cash. In general, it was undesirable to sell current, long-term, low-yielding bonds at a loss, which would result in statutory surplus drain. The alternative chosen was to write more business at higher combined ratios. In other words, to avoid current accounting losses the industry took on future economic losses.

The above circumstances bring to the forefront the magnitude of interest rate risk. This risk had previously gone unmanaged in the P/C industry. The increased volatility of interest rates during the past 10 years, coupled with the substantial reliance on investment income for profitability, results in increased financial risk to a company's true economic net worth. This financial risk needs to be managed, thus the need for ALM.

There are additional factors, as well, that point to a need for ALM. The P/C industry's assets have grown faster than surplus in recent years (see Exhibit 2) such that the exposure level of surplus to changes in interest rates is quite large. By 1987 year end, the ratio of assets to surplus was 3.5 and a 10% change in invested asset market values would result in a 35% change in surplus. Also, ALM has become more feasible through the increased availability of financial products to obtain desired cash flows (i.e., growth in size and liquidity of derivative financial products such as options, futures, collateralized mortgage obligation vehicles, etc.).

Further, certain regulatory and accounting issues have surfaced. At least two states (Kentucky and Pennsylvania) have regulations in effect or proposed which require that the actuary address the reasonableness of the matching of invested assets and loss reserves when providing a statement of opinion on discounted loss reserves. Also, the Financial Accounting Standards Board has proposed disclosure requirements for financial instruments, including information as to the timing of the expected future receipts or payments as well as interest rates, market value and credit risk. At this time, the proposed requirements would apply fully to insurance company financial assets as well as liabilities. ALM is necessary in order to appropriately respond to these issues.

WHAT IS ASSET/LIABILITY MANAGEMENT

It is not our goal to in any way instruct on the management of invested assets. The management of invested assets is the responsibility of those professionals in the investment banking field. It is our intent, however, to instruct or inform the reader as to methods and the need for measuring and managing the level of the "matching" of assets and liabilities.

ALM involves techniques to measure the matching of assets and liabilities, thereby assisting in prudent management of the investment portfolio. The objective of ALM does not necessarily imply achieving a perfect match of assets and liabilities. It is not the exact match that is important but rather the prudent management of mismatch. The focus should be risk control since mismatch will almost always be present.

A fully matched position would be one where changes in the present value of assets equal changes in the present value of liabilities. (In this paper we have simplified the treatment of interest rate to ignore the effect of inflation, i.e., assuming nominal interest rate equals real interest rate. Since inflation can affect assets and liabilities differently, further consideration of inflation may be needed in actual applications of ALM.) The results of fully matching assets and liabilities will insulate economic net worth from interest rate changes since:

Economic Net Worth = Market value of Assets - Market value of Liabilities

The purpose of ALM, therefore, is not to project interest rates but rather, to the extent management chooses, to insulate its effects.

HOW TO MEASURE "MATCHING"

Methods of measuring matching, or mismatch, are still in the developmental stages. This paper will not be able to provide an ultimate answer to the measurement questions, but will take the method beyond that of simple duration analysis as presented in R.E. Ferguson's paper.[1]

Duration is one measure of price sensitivity to interest rates. It is defined as the weighted average maturity in which the weights are stated in present value terms. (See Appendix 1 for the calculation of duration for a single asset, a portfolio of assets and a liability stream.)

For coupon bearing bonds, duration must be modified as follows in order to be an indicator of price movement.

$$\text{Modified Duration (MD)} = D1/[1 + (\text{yield to maturity}/\text{number of coupons per year})]$$

Note that for zero coupon bonds and liabilities, MD = D1, thus the above is a general formula for duration.

To help comprehend duration, we must obtain an understanding of what influences it. It is clear that the components of duration for a bond are the maturity date, the coupon payments, and the discount rate. However, it may not be clear how their interrelationship affects duration. For example, in the case of a zero coupon bond, as maturity increases, so does duration. In the case of a deeply discounted bond (i.e., a 3% bond priced to yield 15%), duration at first increases with an increasing maturity, then eventually decreases (see Exhibit 3). This is important since in recent years we have seen stated coupons ranging from 0% to 17% resulting in bonds with the same maturities, but with wide variances in cash flows.

Duration is not without problems. It is best when it is used as a measure of relative price volatility. There are problems, however, with using duration as a complete measure of interest rate risk. Duration(D1) is an appropriate measurement technique only under the following conditions:

(1) Infinitesimal changes in interest rates

(2) Parallel shifts in yield curves

(3) Instantaneous shifts in yield curves

(4) Flat yield curves

Because of these restrictions, duration analysis works best in times of stable interest rates, but is inaccurate in volatile interest rate environments. Unfortunately, this is when ALM is needed the most. Given large shifts in interest rates, such as 100 basis points (i.e., interest rates changing from 8% to 9%), potential errors develop when using only the simple D1 measurement.

Empirical tests indicate that simple duration(D1) matching can remove about 70% of interest rate risk. The use of a multi-factor matching concept (i.e., matching both duration(D1) and convexity(D2)) will eliminate more risk. Each duration measure within a multi-factor model measures a type of interest rate risk. When used collectively, a portfolio can be insulated from almost any type of interest risk. The use of two duration measures, D1 and D2, removes close to 90% of interest rate risk.[14]

Convexity (D2) is defined as the change in duration relative to changes in yield. It is calculated as the weighted average of the square of the time to maturity of the cash flows. Convexity helps explain the difference between the actual price of a bond and the price estimated by using D1. An example of the calculation of convexity is shown in Appendix 3. Further discussions and examples of convexity can be found in a number of the referenced articles.[6,8,9]

The affect of adding the element of convexity to duration is to change duration's suggested linear relationship between price and yield to a more representative curved relationship. Appendix 2A displays the calculations of both duration and convexity for a zero coupon bond. Appendix 2B graphically displays the variance (gain/(loss) from convexity) when duration alone is used for the estimation of price from the above calculation. When convexity is added to the pricing formula, this variance is dramatically reduced as shown in Appendix 2C.

Using D1 and D2, the equation for the change in price of a single bond can now be written as:

$$chgP = -DD1(chgI) + 1/2\ DD2\ (chgI)^2 + R$$

where:

chgP	=	Change in price
DD1	=	Dollar duration (price x modified duration)
chgI	=	Change in interest rate
DD2	=	Dollar convexity (price x convexity)
R	=	Residual

For an entire portfolio, the change in present value or market value would be measured similarly. The values of D1 and D2 would merely be the weighted present values of each.

Like duration, to fully understand convexity we need to understand its characteristics:

(1) Positive convexity exists when decreases in yield rates result in a larger percentage movement in price than increases in yield rates. (This relationship would result in a convex curve.)

(2) Negative convexity, therefore, would employ the opposite relationship and produce a concave curve.

(3) Given Bonds of equal duration -- the higher the coupon, the higher the convexity. Therefore, zero coupons (as well as liability cash flow streams) will have the least convexity.

(4) Doubling the duration will more than double the convexity.

The mere matching of duration(D1) does not ensure the protection of surplus from interest rate volatility. P/C insurance company assets and liabilities are quite often of opposite convexity. It is reasonable to assume that a convex-shaped (positive convexity) market value profile of liabilities, as shown in Exhibit 5, exists of the P/C industry. D. F. Babble and R. Stricker stated that this would be the case because:

(1) When interest rates rise, the decrease in liability market values may be slowed down by the effect of inflation on ultimate claim costs (increasing severity).

(2) When interest rates fall, the increase in liability market values may be accelerated by the effect of an increase in claim frequency caused by an increase in moral hazards.

Conversely, a concave-shaped (negative convexity) market value profile of assets (as shown in Exhibit 5) is typical for a P/C company. One reason for this is that many of the industry's investments in securities display negative convexity characteristics, such as Mortgage Backed Securities (MBS), Callable Bonds, Derivative Products, etc. The enticements to attract investors are higher yields offered for these securities relative to similar quality instruments. For example, a typical MBS security with approximately the same credit strength as Treasuries may be purchased with a yield spread of 50-150 basis points above a comparable Treasury security.

In order to determine duration or convexity, present values must be determined. Assets should be discounted to present value using the implied market required rate of return for that asset. The discount rate for liabilities is not so straight forward. Since our purpose in ALM is to determine the effect interest rate changes have on the market values, the appropriate value of liabilities should be discounted to present value using current market rates of risk free securities.

Additional complications arise in the actual practice of calculating duration and convexity. Some of the more common ones are discussed further in Appendix 5.

APPLYING ALM TO THE P/C INDUSTRY

There are basically three methods of ALM applications used in other industries. The first is a maturity gap approach, which is currently used by most banks and thrifts. Second, simulation approaches, which are computer designed modeling techniques, each of which have varying levels of sophistication. The third, and most modern, is the duration gap approach, which is still in the development stage.[4] The three methods are described further in Appendix 6. In this paper we will apply the duration gap approach to the P/C industry.

The first step in the P/C industry application of ALM is to select the subset of assets that are required to be specifically matched with liabilities. This exercise is not trivial and requires at least temporary resolution of the following issues:

(1) How should the operation be reviewed (liquidation, runoff or ongoing concern)?

(2) Which assets and liabilities should be included?

(3) How should the uncertainty associated with the loss and loss expense amounts be considered (including ultimate value, expected payout and selected payout pattern)?

It does not seem desirable for most companies to approach their day-to-day management with a liquidation mentality. W. H. Penning recommends taking an approach that views the company as an ongoing concern. Theoretically, this is probably the best and most sophisticated approach; however, it is our position that this is an advanced extension of the runoff concern approach and should only be considered by an organization with sufficient experience and expertise with the less complex situation. This paper will limit the discussion to the runoff scenario.

Since the objective of ALM is to determine and manage the extent to which surplus is immunized from interest rate changes, the definition of included assets or liabilities should encompass any sub-category whose market value is affected by interest rate changes.

On the asset side, with the exception of owned real estate, the above definition naturally includes all other invested assets as well as the market value of other reported assets such as agents' balances or uncollected premiums, bills receivable, reinsurance recoverable on loss payments, federal income tax recoverable, receivables from parents or subsidiaries, etc. For all of these assets, the company has entered into an agreement in which some future steam of cash is expected to be received. In the case of owned real estate, management has made the decision to use a portion of their assets to purchase property in lieu of the periodic cash payment of rent. Management has, in essence, "matched" these assets with the applicable liability. Therefore, for practical purposes, the owned real estate asset (as well as any mortgage liability) can be removed from further ALM analysis. This line of reasoning can be extended to other "non-invested" assets (i.e., data processing equipment).

On the liability side, this definition would require the inclusion of numerous liabilities in addition to the stated loss and loss adjustment reserves. In the case of the unearned premium reserves, one should estimate and include the projected loss and expense payments that are associated with these reserves. Additionally, funds such as contingent commissions, other expenses, taxes, fees, borrowed money, drafts outstanding, etc. should be included within the liability category.

The remaining issues involve the loss and loss expense amounts (i.e., the uncertainty in ultimate values, actual payout and payout patterns). In addition to the "best estimates" of these values, the liabilities utilized for ALM should include "safety margins" to account for the risk of adverse development. The magnitude of these margins should be a reflection of management's adversity to risk (as respects its remaining surplus). Significant additional actuarial research needs to be done in this area of quantifying such "safety margins". Another way of reflecting "safety margin" is to reduce the discount rate for liabilities below the risk free market rate.

It is recommended that the assets, once defined, be segregated into two categories. The first category (Asset I) is that of assets at market value, supporting the market value of the liabilities. The second category (Asset II) is that of the remaining assets, which corresponds to the economic net worth of surplus. By segregating the assets, we are able to separately measure the levels of interest rate risk associated with the liability funds versus the surplus funds.

Exhibit 6 displays graphically our typical insurer. Here, the minimum value of matched assets has been determined to be equal to the present value of reported liabilities. Assuming that a change in interest rates is the only factor that can alter the value of the firm's assets or liabilities, the modified duration measurement will totally reflect the change in value (assuming equal convexities). If the modified durations of assets and liabilities, in present value terms, are equal, there will always be a sufficient market value of assets to cover claims. While cash flow matching is required for total interest rate immunity, it is not cost effective to implement due to the limitations it places on the selection of investment vehicles. Alternatively, the duration(s) matched approach can utilize the entire universe of available investments while still effectively controlling interest rate risk.

Once the assets and liabilities have been determined, the duration gap can be measured using appropriate duration measures and market values. In the following examples we have used the modified duration (MD, as previously discussed on page 6) measurement for simplicity. As discussed in the previous section, actual application in the P/C industry should normally include convexity.

In order to calculate the modified duration for surplus, we must account for the dollar value affected by interest rates, known as dollar duration (DD). dollar duration is calculated by multiplying market values by their respective modified durations. Appendix 4A displays an insurer whose ratio of surplus to assets, in market value terms, is 25%. Given the modified duration for assets and liabilities of seven and four, respectively, the duration mismatch is three years. Appendix 4A also shows how the above three year duration mismatch of assets and liabilities can result in a duration gap of 16 for surplus. In other words, a change in rates of 100 basis points will produce an approximate 16% change in the true economic value of surplus. The modified duration for surplus is thus compounded by a levered effect caused when assets are not equal to liabilities.

Given the above assumptions we can easily derive the proper modified duration (MD) for assets in order to totally insulate surplus from interest rate risk. Appendix 4B shows the calculation of the proper asset MD to be 3 under scenario (A) [using all assets]. Due to the levered impact (i.e., when assets are greater than surplus) the required duration of assets will not equal that of liabilities. Under scenario (A) we have now achieved the following:

Changes in the Market Value of Assets =
Changes in the Market Value of Liabilities

Scenario (B) segregates the assets into the two categories. In order to maintain the same insulated surplus position as in Scenario (A), Scenario (B) places any remaining assets into cash (Asset II), having a duration of zero (see Appendix 4C).

What remains is to derive the basis risk (i.e., the impact on surplus if yields change by 100 basis points) implied when altering the investment assumptions for the remaining assets (Asset II), which are not required to immunize the interest rate effects on liabilities. Appendix 4C shows how to quantify the interest rate risk under different duration scenarios. Under Scenarios B, C and D the following relationship has been maintained:

Changes in the Market Value of Liabilities =
Changes in the Market Value of Asset I

The impact on economic surplus from interest rate changes is isolated to the exposure of Asset II. At this point it becomes a management decision as to how much basis risk (or surplus volatility) is acceptable. In this way, the stockholders and policyholders are assured the fulfillment of the company's underwriting obligation by the matching of liabilities with the required amount of assets. Additionally, management has been given better control of its exposure to interest rate risk.

Maintenance of a matched asset/liability portfolio is by no means a buy-hold strategy. As time passes, the dollar durations (under D1) of the assets and liabilities will tend to drift apart, resulting in dollar duration mismatch. To avoid this, the portfolio must be rebalanced periodically to bring the dollar durations back in line. The problem of dollar duration drift is reduced if dollar convexity is utilized. In a totally immunized scenario, where the present value of assets exceeds the present value of liabilities, "the convexity of these assets should exceed the convexity of the liabilities"[8]. Some observations of duration drift of assets follow:

(1) As time passes, the duration of any asset shortens (given no change in interest rates).

(2) Zero coupon bond durations shorten linearly year to year.

(3) Coupon bond durations shorten more slowly than zero coupon bond durations.

For P/C industry liabilities, the age of the accident years that comprise a particular line's liability portfolio will affect the duration of that line. However, due to duration drift, the relationship between claim age and accident year duration cannot be generalized. At first glance it might appear that as an accident year ages, its duration should get shorter. Such is not always the case. If payments are heavily concentrated in the first year, a new accident year may have a shorter duration than an older accident year. In a study done by Goldman Sachs, the durations of both workers' compensation and general liability increase dramatically before ultimately declining. On the other hand, medical malpractice duration remains below its original level as the accident year ages. The duration of a P/C company's liabilities will depend on the following:

(1) Mix of business
(2) Relative age of loss and LAE reserves
(3) Pattern of growth in new business

CONCLUSION

The actuary is now challenged to utilize ALM methods to:

(1) Determine the P/C company's level of interest rate risk underlying its current liabilities by measuring the amount of mismatch of Asset I and liabilities; and

(2) Aid company management in its understanding of the total level of interest rate risk inherent in the company's current investment strategy.

Once the knowledge of investment rate risk for a company is understood, it can be managed through specific investment strategies that correspond to overall investment policies. In addition, actual results can be measured against the established policy objectives to determine if a portfolio strategy was truly effective.

Through ALM, management can assess and control the level of interest rate risk on the true economic net worth of the company. Only through informed decision making can a company understand the implied risk it has assimulated in order to attain a desired level of acceptable returns. This risk is a composite of certain assumed interest rate forecasts, the perceived strength of surplus and the adequacy of reserves. Positive changes in economic net worth will ensure that potential growth in premium sales is supported by growth in statutory surplus. Additionally, since the market places a premium on consistency, sustained growth should allow for higher acceptable P/E ratios for a firm's stock.

With certain exceptions, the P/C industry is inherently similar to other financial institutions and thus its true economic net worth is subject to interest rate risk. Because of the size of its invested asset base, surplus leverage, the dependency on investment income, the future growth (and possibly survival) of a P/C company will be a function of management's ability to understand and control its exposure to interest rate risk.

Through ALM, a link between assets and liabilities can be established to control this risk. It can also provide the mechanism for the creation of an objective portfolio strategy consistent with the company's stated goals and allow management to limit the extent to which external market forces affect is true underlying value. This paper has attempted to bring together many of the existing thoughts on ALM. However, considerable additional research needs to be done to make ALM an effective and meaningful tool that can be implemented on a practical, day-to-day basis within the P/C industry. The actuary can and should play a key role in developing and utilizing this very important management tool.

Exhibit 1

PROPERTY AND CASULATY

INSURANCE INDUSTRY PROFIT BREAKDOWN

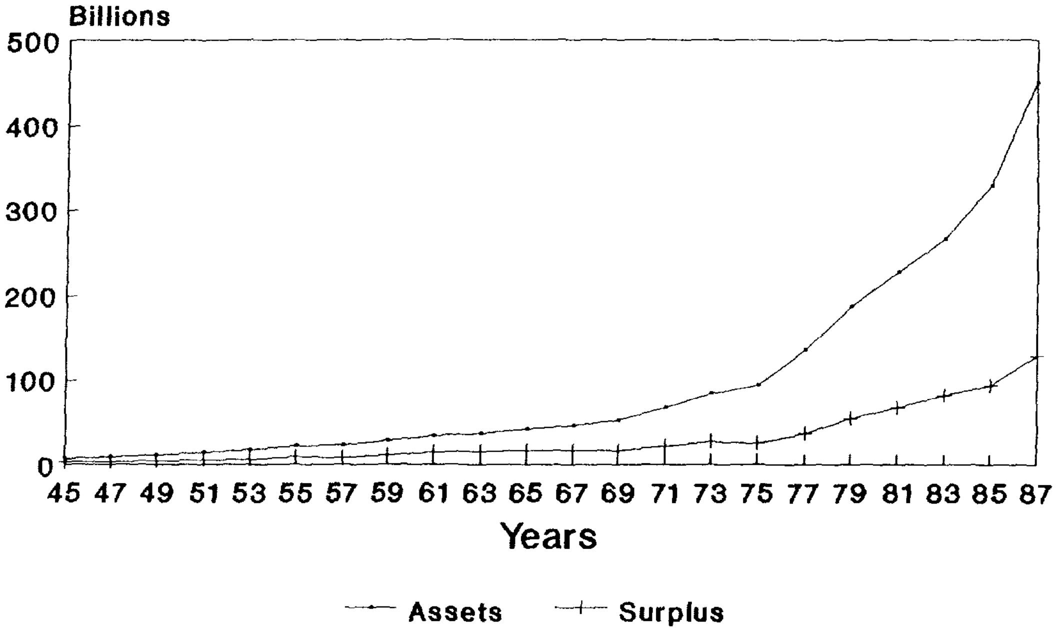
UNDERWRITING -vs- INVESTMENTS

	1987 (millions)	1987 (percentage)
Underwriting	($10,620)	-80%
Investments	$23,960	180%
Net Profit	$13,340	100%

	1930 - 1950	Early 1950's	Late 1950's	1960 - 1980	1980 -1987
Underwriting	63%	40%	10%	-15%	-343%
Investments	37%	60%	90%	115%	443%
Net Profit	100%	100%	100%	100%	100%

Source : Best's Aggregates & Averages

Property & Casualty Ins. Industry Growth
Assets vs. Surplus

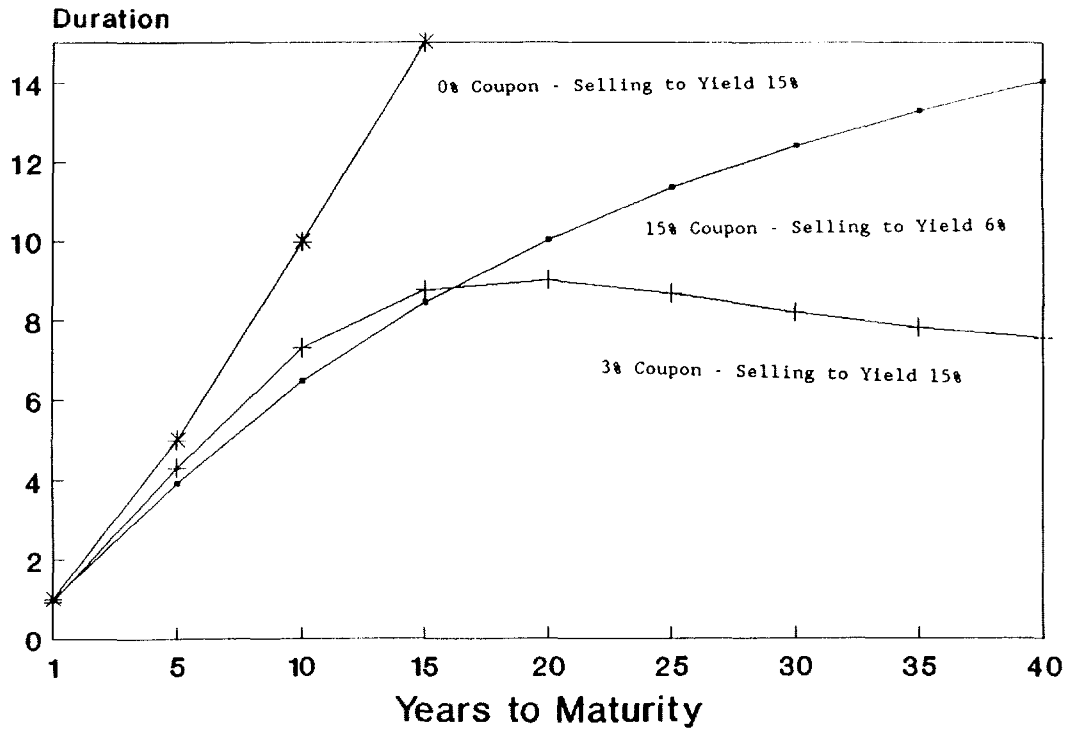


Source: Best's Aggregates & Averages

Exhibit 2

Exhibit 3

Duration vs. Maturity



Various Yield Curve Shifts

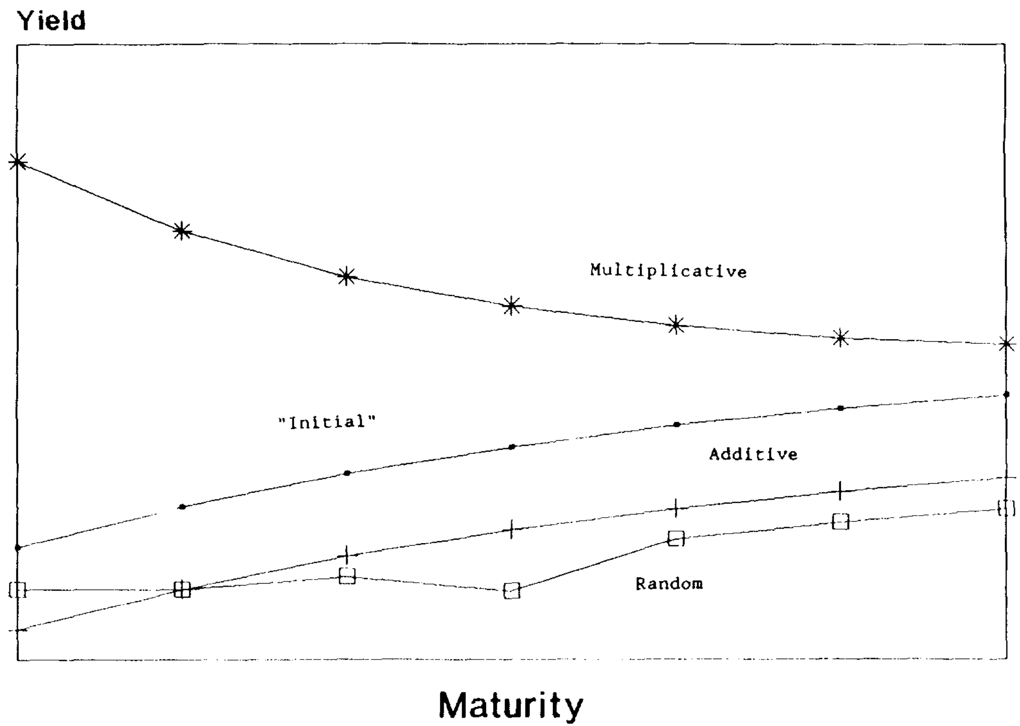


Exhibit 4

Assessment of Basis Risk in Surplus

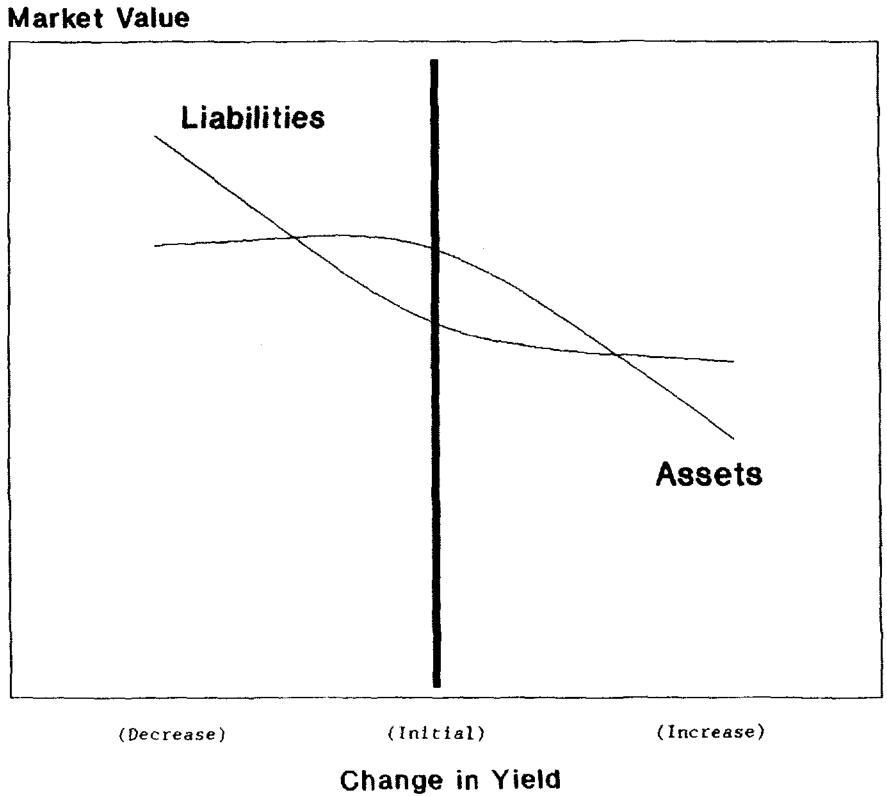
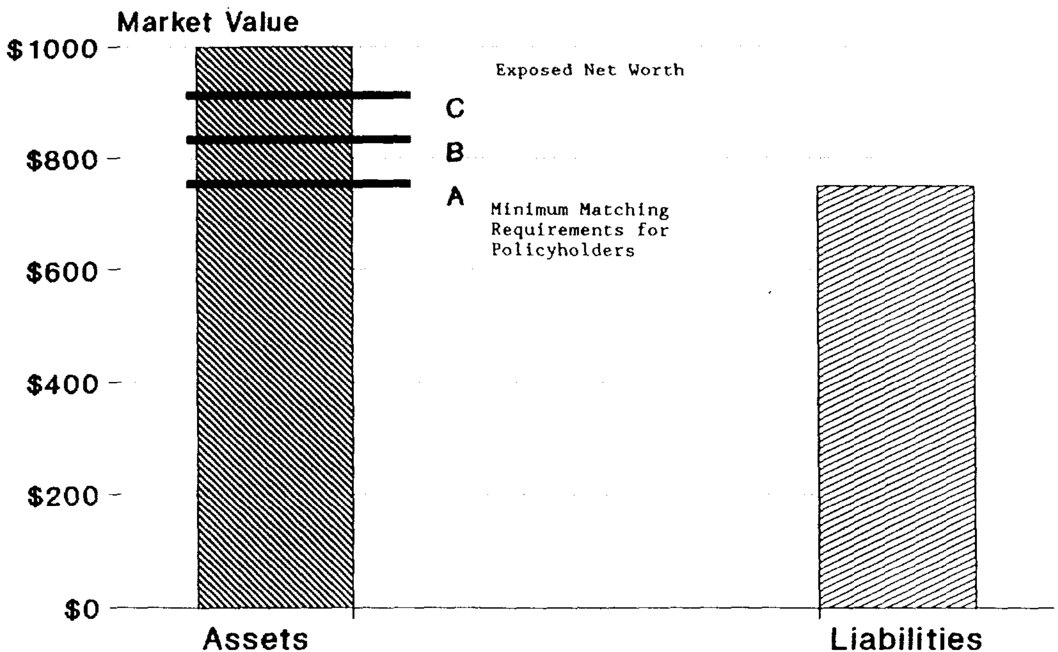


Exhibit 5

Determination of Assets
Dedicated to Liabilities



A - Initial Estimate of Liability Market Value
B - Adjustment for Variance Error in Ultimate Payout
C - Adjustment for Variance Error in Payout Pattern

WEIGHTED AVERAGE TERM TO MATURITY
(Assuming Annual Interest Payments)

Bond A

$1,000 Face Value with 4% coupon
Maturing in 10 years, discounted at 8%

1	2	3	4	5	6
Year	Present Value	Cash Flow	PV of Flow (2 * 3)	PV as % of Price	Duration Components (1 * 5)
1	0.9259	40	37.04	5.06%	0.0506
2	0.8573	40	34.29	4.69%	0.0937
3	0.7938	40	31.75	4.34%	0.1302
4	0.7350	40	29.40	4.02%	0.1608
5	0.6806	40	27.22	3.72%	0.1861
6	0.6302	40	25.21	3.45%	0.2067
7	0.5835	40	23.34	3.19%	0.2233
8	0.5403	40	21.61	2.95%	0.2363
9	0.5002	40	20.01	2.74%	0.2462
10	0.4632	1040	481.72	65.85%	6.5845
			731.60	100.00%	8.12

Weighted Average Term to Maturity : 8.12 years

Bond B

$1,000 Face Value with 10% coupon
Maturing in 12 years, discounted at 8%

1	2	3	4	5	6
Year	Present Value	Cash Flow	PV of Flow (2 * 3)	PV as % of Price	Duration Components (1 * 5)
1	0.9259	100	92.59	8.05%	0.0805
2	0.8573	100	85.73	7.45%	0.1490
3	0.7938	100	79.38	6.90%	0.2070
4	0.7350	100	73.50	6.39%	0.2555
5	0.6806	100	68.06	5.91%	0.2957
6	0.6302	100	63.02	5.48%	0.3286
7	0.5835	100	58.35	5.07%	0.3549
8	0.5403	100	54.03	4.70%	0.3756
9	0.5002	100	50.02	4.35%	0.3913
10	0.4632	100	46.32	4.03%	0.4025
11	0.4289	100	42.89	3.73%	0.4100
12	0.3971	1100	436.83	37.96%	4.5553
			1150.72	100.00%	7.81

Weighted Average Term to Maturity : 7.81 years

WEIGHTED AVERAGE TERM TO MATURITY
(Assuming Annual Interest Payments)

Given:

Bond A

$1,000 Face Value with 4% coupon
Maturing in 10 years, discounted at 8%
Priced at $731.60
Weighted Average Term to Maturity - 8.12 years

Bond B

$1,000 Face Value with 10% coupon
Maturing in 12 years, discounted at 8%
Priced at $1,150.72
Weighted Average Term to Maturity - 7.81 years

Calculation:

Portfolio Weighted Average Term to Maturity (D)
for Assets A and B

Formula:

$$\text{Portfolio (D)} = \frac{(\text{Price A} * \text{Duration A}) + (\text{Price B} * \text{Duration B})}{(\text{Price A} + \text{Price B})}$$

$$\text{Portfolio (D)} = \frac{(\$731.60 * 8.12) + (\$1,150.172 * 7.81)}{(\$731.60 * \$1,150.72)}$$

Portfolio (D) = 7.93 years

WEIGHTED AVERAGE TERM TO MATURITY
(Assuming Midyear Payments)

$1,000 Loss Reserve
Discounted at 9%

1	2	3	4	5	6	7
Year	Present Value	Payment Pattern	Cash Flow	PV of Flow (2 * 4)	PV as % of Price	Duration Components (1 * 6)
0.5	0.9578	9.2%	92.0	88.12	12.79%	0.0639
1.5	0.8787	16.2%	161.9	142.27	20.64%	0.3097
2.5	0.8062	14.7%	146.8	118.35	17.17%	0.4293
3.5	0.7396	15.1%	151.2	111.83	16.23%	0.5680
4.5	0.6785	11.0%	109.8	74.50	10.81%	0.4865
5.5	0.6225	8.9%	89.1	55.47	8.05%	0.4427
6.5	0.5711	5.1%	51.0	29.13	4.23%	0.2747
7.5	0.5240	4.3%	42.7	22.37	3.25%	0.2435
8.5	0.4807	2.2%	21.6	10.38	1.51%	0.1281
9.5	0.4410	1.0%	10.1	4.45	0.65%	0.0614
10.5	0.4046	1.0%	10.1	4.09	0.59%	0.0623
11.5	0.3712	1.0%	10.1	3.75	0.54%	0.0626
12.5	0.3405	1.0%	10.1	3.44	0.50%	0.0624
13.5	0.3124	1.0%	10.1	3.16	0.46%	0.0618
14.5	0.2866	1.0%	10.1	2.89	0.42%	0.0609
15.5	0.2630	1.0%	10.1	2.66	0.39%	0.0597
16.5	0.2412	1.0%	10.1	2.44	0.35%	0.0583
17.5	0.2213	1.0%	10.1	2.24	0.32%	0.0568
18.5	0.2031	1.0%	10.1	2.05	0.30%	0.0551
19.5	0.1863	1.0%	10.1	1.88	0.27%	0.0532
20.5	0.1709	1.0%	10.1	1.73	0.25%	0.0513
21.5	0.1568	1.0%	10.1	1.58	0.23%	0.0494
22.5	0.1438	0.3%	2.6	0.37	0.05%	0.0122
		100.00%	1000.0	689.1	100.00%	3.7

Weighted Average Term to Maturity : 3.7 years

DURATION and CONVEXITY CALCULATIONS
(Assuming Annual Interest Payments)

Bond C

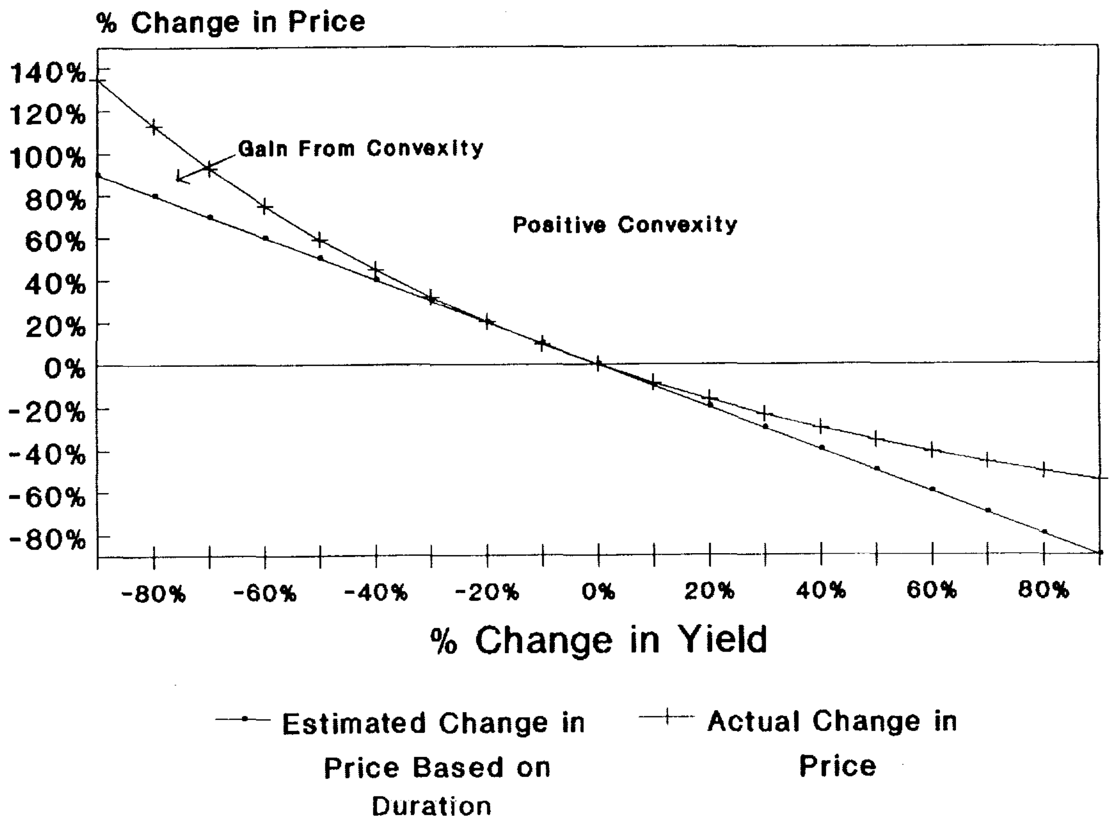
$1,000 Face Value with 0% coupon
Maturing in 10 years, discounted at 10%

1	2	3	4	5	6	7
Year	Present Value	Cash Flow	PV of Flow	PV as % of Price	Duration Components	Convexity Components
			(2 * 3)		(1 * 5)	$(1^2 * 5)$
1	0.9091	0	0.00	0.00	0.0000	0.00000
2	0.8264	0	0.00	0.00	0.0000	0.00000
3	0.7513	0	0.00	0.00	0.0000	0.00000
4	0.6830	0	0.00	0.00	0.0000	0.00000
5	0.6209	0	0.00	0.00	0.0000	0.00000
6	0.5645	0	0.00	0.00	0.0000	0.00000
7	0.5132	0	0.00	0.00	0.0000	0.00000
8	0.4665	0	0.00	0.00	0.0000	0.00000
9	0.4241	0	0.00	0.00	0.0000	0.00000
10	0.3855	1000	385.54	1.00	10.0000	100.00000
			385.54		10.00	100.00

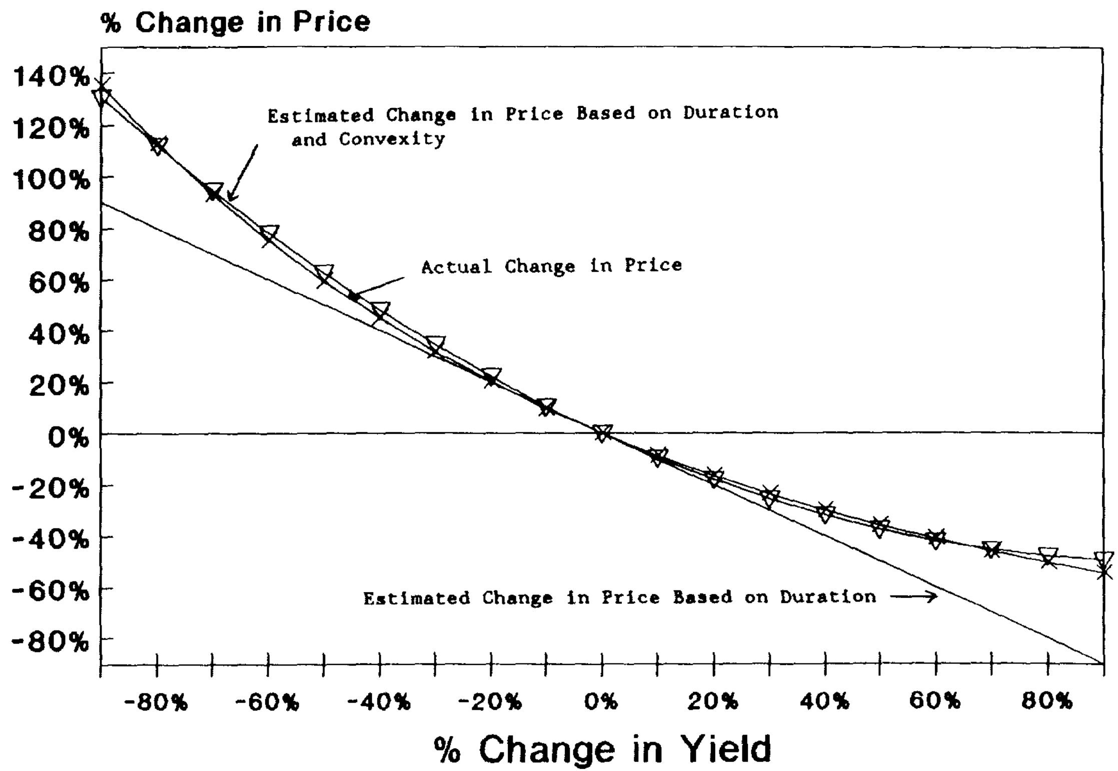
Weighted Average Term to Maturity : 10 years (Duration)

Weighted Average Term to Maturity Squared : 100 (Convexity)

Interest Rate Sensitivity



Interest Rate Sensitivity



DURATION and CONVEXITY CALCULATIONS
(Assuming Annual Interest Payments)

Bond D

$1,000 Face Value with 10.65% coupon
Maturing in 5 years, discounted at 10.65%

1	2	3	4	5	6	7	8
Year	Discount Rate	Present Value	Cash Flow	PV of Flow	PV as % of Price	Duration Components	Convexity Components
				(3 * 4)		(1 * 6)	$(1^2 * 6)$
1	10.65%	0.9038	106.5	96.25	0.10	0.0962	0.096
2	10.65%	0.8168	106.5	86.99	0.09	0.1740	0.348
3	10.65%	0.7382	106.5	78.61	0.08	0.2358	0.708
4	10.65%	0.6671	106.5	71.05	0.07	0.2842	1.137
5	10.65%	0.6029	1106.5	667.11	0.67	3.3355	16.678
				1000.00		4.13	18.97

Weighted Average Term to Maturity : 4.13 years (Duration)
Weighted Average Term to Maturity Squared : 18.97 (Convexity)

Bond D'

$1,000 Face Value with 10.65% coupon
Maturing in 5 years, discounted at implied Term Structure

1	2	3	4	5	6	7	8
Year	Discount Rate	Present Value	Cash Flow	PV of Flow	PV as % of Price	Duration Components	Convexity Components
				(3 * 4)		(1 * 6)	$(1^2 * 6)$
1	8.00%	0.9259	106.5	98.61	0.10	0.0986	0.099
2	9.05%	0.8409	106.5	89.56	0.09	0.1791	0.358
3	9.86%	0.7542	106.5	80.32	0.08	0.2410	0.723
4	10.42%	0.6727	106.5	71.64	0.07	0.2865	1.146
5	10.89%	0.5964	1106.5	659.92	0.66	3.2994	16.497
				1000.00		4.10	18.82

Weighted Average Term to Maturity : 4.10 years (Duration)
Weighted Average Term to Maturity Squared : 18.82 (Convexity)

CALCULATION OF MODIFIED DOLLAR DURATIONS

	(1) MARKET VALUE	(2) MODIFIED DURATION	(3) MODIFIED DOLLAR DURATION (1 * 2)
ASSETS	$1,000	7	$7,000
LIABILITIES	$750	4	$3,000
SURPLUS	$250	16	$4,000

EFFECT OF YIELDS INCREASING 100 basis points

	MARKET VALUE	CHANGE	CHANGE
ASSETS	$930	($70)	-7.00%
LIABILITIES	$720	($30)	-4.00%
SURPLUS	$210	($40)	-16.00%

DERIVATION OF MD FOR TOTAL ASSET PORTFOLIO TO IMMUNIZE SURPLUS

	Initial Portfolio			Scenario (A)		
	(1) Market Value	(2) Modified Duration	(3) Dollar Duration	(4) Market Value	(5) Modified Duration	(6) Dollar Duration
			(1)x(2)		(6)/(4)	
Assets	$1,000	7	$7,000	$1,000	3	$3,000
Liabilities	750	4	3,000	750 *	4 *	3,000 *
Surplus	$ 250	16	$4,000	$ 250	0	$ 0

	Effect of Yields Increasing 100 Basis Points			Effect of Yields Increasing 100 Basis Points		
	Market Value	Change	Change	Market Value	Change	Change
Assets	$ 930	($70)	-7.00%	$ 970	($30)	-3.00%
Liabilities	720	(30)	-4.00%	720	(30)	-4.00%
Surplus	$ 210	($40)	-16.00%	$ 250	$ 0	0.00%

* Constant

ASSESSMENT OF BASIS RISK ON SURPLUS

	ZERO BASIS RISK DERIVED FOR TOTAL PORTFOLIO			ALTERNATIVE ZERO BASIS RISK DERIVED FOR TOTAL PORTFOLIO		
	SCENERIO (A)			SCENERIO (B)		
	(1)	(2)	(3)	(4)	(5)	(6)
	MARKET VALUE	MODIFIED DURATION	DOLLAR DURATION	MARKET VALUE	MODIFIED DURATION	DOLLAR DURATION
ASSET (I)	$1,000	3	$3,000	$750	4	$3,000
ASSET (II)	$0	0	$0	$250	0	$0
TOTAL	$1,000	3	$3,000	$1,000	3	$3,000
LIABILITIES	$750	4	$3,000	$750	4	$3,000
SURPLUS	$250	0	$0	$250	0	$0
BASIS RISK :		0.00%			0.00%	

IMMUNIZE INTEREST RISK ON LIABILITIES ONLY

	(7)	(8)	(9)	(10)	(11)	12)
	MARKET VALUE	MODIFIED DURATION	DOLLAR DURATION	MARKET VALUE	MODIFIED DURATION	DOLLAR DURATION
	SCENERIO (C)			SCENERIO (D)		
ASSET (I)	$750	4	$3,000	$750	4	$3,000
ASSET (II)	$250	1	$250	$250	10	$2,500
TOTAL	$1,000	3.25	$3,250	$1,000	5.5	$5,500
LIABILITIES	$750	4	$3,000	$750	4	$3,000
SURPLUS	$250	1	$250	$250	10	$2,500
BASIS RISK :		1.00%			10.00%	

Basis Risk defined :
The impact on Surplus if yields change by 100 basis points.

CONSIDERATIONS WHEN APPLYING ALM

Exhibit 4 graphically displays three possible interest rate shifts [parallel (additive); multiplicative; and random]. The type of interest rate risks protected by D1 are small, parallel interest rate shifts. Matching D2 simultaneously can protect against large parallel shifts and small twists. While protecting against parallel shifts is comforting, most would agree that multiplicative shifts are closer to the real world. This type of risk is safeguarded by additionally matching D3, the weighted average time to maturity of the cash flows cubed. The matching of these three duration measures (D1, D2 and D3) removes 95% of the risk.

Managing multiple measures of duration allows management the ability to minimize and control interest rate risk. A perfect match of all duration measures would steer the portfolio to a true cash flow match. But, for most P/C companies, this would not be an acceptable portfolio management style. Like any other type of risk management, the degree and type of matching will be a matter of a company's policy and the type of investment portfolio. Policy will be a reflection of management's risk aversion and desired goals. The portfolio characteristics, on the other hand, will be less judgmental. For example, a portfolio composed of primarily Mortgage Backed securities will exhibit both positive and negative convexity under different interest rate environments. The rate of change of convexity (D3) is relatively more important in this situation when compared to a portfolio of pure fixed income, non-callable bonds.

The assumption of a flat yield curve is implied when all cash flows are discounted at the same rate. This had led some to believe that since each cash flow stream is discounted at the bond's yield to maturity, the weighted average duration of the portfolio is inaccurate. The way to eliminate this bias would be to work with the entire term structure of interest rates. While this may lead to a more accurate measure, the increase in precision is not material in most cases. Appendix 3 derives the above measures of D1 and D2 based on the following two methods:

(A) Derives and uses the term structure of interest rates implied by the relationship between the yield of a par bond and yields in zero coupon bonds (spot rates).

(B) Assumes a flat yield curve, discounting at the bond's yield to maturity.

It can be seen that the relative difference in (D1) and (D2) is insignificant. This can be extended to portfolio analysis. This conclusion is further supported in an article by D. R. Chambers appearing in the **Journal of Finance and Quantitative Analysis**, March, 1988.

Another consideration in practice is determining the duration of a mixed portfolio (i.e., taxable and tax-free bonds). An issue develops in using the overall portfolio duration if changes in the investment sectors are not identical. For example, a 1% change in the taxable rate may imply a 0.8% change in the municipal rate. In this situation, a present value average calculation of the portfolio duration is inaccurate. The duration of the municipal portfolio should first be multiplied by 0.8 before it is present value weighted into the overall portfolio duration.

Another example involves different asset qualities (i.e., AAA, BBB, etc.). In this case, assuming that the difference in spreads are constant, adding the present value weighted durations would be correct for the calculation of the overall portfolio duration. However, if the spreads are proportional to the level of interest rates, durations should be multiplied by the spread factor. For example, if BBB yields are 1.20 times greater than AAA yields, the durations of the BBBs should be first multiplied by 1.20 before being present value weighted into the overall portfolio duration.

A final example is the case where assets are of the same quality and type, but of different durations (i.e., a portfolio of 5 and 30 years U.S. Treasuries). If there exists a relationship between the 5 year and 30 year securities, the different duration assets would be multiplied by the appropriate factors.

All of the preceding examples assume some reference rate (i.e., 30 year AAA Treasuries). It is best to pick a reference rate that is most closely correlated with projected movements in portfolio value. Additionally, it is important to be consistent in reviewing how each asset value varies with respect to the selected reference rate. Naturally, these adjustments should be incorporated on the liability side, as well, to obtain the most accurate measure of total interest rate risk.[7]

The final issue to address is the calculation of the equity duration and its contribution to the total asset duration. Duration calculations for stocks have been addressed by several authors. There are generally two approaches presented.

One method is use of a dividend/earnings discount model. Dividend discount models will transform a stock investment into a stream of future cash flows. As shown before, given an estimated payment stream, the calculation of duration is straight forward. The problem with this method is determining credible estimates of the cash flows, specifically, growth rates of dividends.

A second method presented by M. L. Leibowitz draws upon parameters routinely used and accepted within most asset allocation studies. A duration measure can be derived for stocks once determinations are made of the variance of stock market returns, the variance of bond returns, and the correlation between the two asset classes. The resulting formulas from this approach would be as follows:[15]

Formula: $D_e = \frac{(sd_e)}{(sd_b)} \; @(E,B) \; D_b$

Where: D_e = estimated duration for the equity market
sd_e = standard deviation of stock market returns
sd_b = standard derivation of bond market index returns
@(E,B) = correlation of returns between the two markets
D_b = duration of a broad-based measure of the bond market

Formula: $D_{tp} = (W_{bp} \times D_{bp}) + (W_{ep} \times B_{ep} \times D_e)$

Where: D_{tp} = total asset portfolio duration
W_{bp} and W_{ep} are fractional allocations to bonds and stocks
D_{ep} = duration of the bond component
D_e = duration of the stock component
B_{ep} = beta value of equity portfolio

The calculation of the above stock duration is a statistically derived concept relating stock market returns to movements in long-term interest rates. As a final note, although it is true that the total return of stocks is a function of many variables (i.e., general economic conditions), the goal here is to insulate surplus from interest rate changes. It makes sense then to isolate and measure that influence via its correlation to interest rate movements as measured by a broad-based bond market measure.

THREE METHODS OF ALM APPLICATIONS

The maturity gap approach began in the 1970's. The intent of this approach is to match or intentionally mismatch the dollars of interest sensitive assets and liabilities which are scheduled to mature within a specified range of maturity dates (i.e., 0-1 year). Interest sensitive assets/liabilities are those that will experience a contractual change in interest rates during a selected accounting period. The focus of this approach is exclusively on net interest income. For example, if you want to hedge interest income for a period of one year from interest rate changes, you would set your one year gap equal to zero. Or, if you expect interest rates to rise over that period and want to take advantage of it, you would set your gap to some positive number.

One major problem is that the method does not account for the exact timing of the assets/liabilities repricing (e.g., assets mature on February 1st of the year; while liabilities are due on September 30th). This has led to a refinement called the periodic gap approach. Here, maturity buckets are used that would increase the precision of interest risk management. But, overall, problems still exist. These methods are basically static approaches and only focus on one target, a single period's interest income.

Simulation models provide results in a dynamic or forward looking context. They incorporate future assumptions (i.e., interest rate scenarios). Additionally, they measure results over time rather than at one point in time. Simulation models, however, are not without fault. As with the maturity gap method, the most serious of faults is that they are mainly focused on net interest income and ignore other goals. Also, they are often sold as "black boxes" having unknown internal structures. They rarely simulate the actual environment of the company being modeled.

Duration analysis is an index measure(s) of interest rate sensitivity for any series of cash flows. It takes into account both cash flow timing and magnitude. It does not ignore timing mismatch which is present in periodic gap models. Most importantly, with respect to the P/C industry, it permits measurement of Balance Sheet, as well as Income Statement, items.

BIBLIOGRAPHY

1) Ronald E. Ferguson. Duration. PCAS. Volume LXX. Page 265

2) G.O. Bierwag, George G. Kaufman, Alan Toevs. Duration: Its Development and Use in Bond Portfolio Management. Financial Analyst Journal. 1983

3) Peter D. Noris, CFA. Asset/Liability Management Strategies for Property & Casualty Companies. New York: Morgan Stanley. 1985

4) Alden L. Toevs, William C. Haney. Measuring and Managing Interest Rate Risk: A Guide to Asset/Liability Models Used in Banks and Thrifts. New York: Morgan Stanley. 1984

5) Alden L. Toevs. Uses of Duration Analysis for the Control of Interest Rate Risk. New York: Morgan Stanley. 1984

6) David P. Jacob, Graham Lord, James A. Tilley. Price, Duration, and Convexity of a Stream of Interest-Sensitive Cash Flows. New York: Morgan Stanley. 1986

7) David F. Babbel, David R. Klock, Paul V. Polachek. Assessing the Interest Rate Risk of Property and Casualty Companies. New York: Goldman Sachs. 1988

8) David F. Babbel, Robert Stricker. Asset/Liability Management for Insurers. New York: Goldman Sachs. 1987

9) Jess B. Yawitz. Convexity: An Introduction. New York: Goldman Sachs. 1986

10) W.H. Penning. Asset Liability Management: Beyond Interest Rate Risk. Page 322

11) J.A. Tilley. The Application of Modern Techniques to the Investment of Insurance and Pension Funds

12) Asset/Liability Management
Institute of Chartered Financial Analysts
Proceedings of Seminar - 1985

a) Brian F. Wruble, CFA. Asset/Liability Management in Perspective

b) James A. Tilley. A Framework for Measuring and Managing Interest Rate Risk

c) Alfred Weinberger. Strategies for Effective Asset/Liability Management

d) Terence I. Lennon. A Regulatory Perspective of Asset/Liability Management

e) William M. Bethke, CFA. Setting Corporate Goals and Objectives for Asset/Liability Management

f) Richard E. Horn. Investment Policy Considerations for Asset/Liability Management -- I

g) H. Dean Benner, CFA. Investment Policy Considerations for Asset/ Liability Management -- II

h) L. John Achenbach II, CFA. Problems and Pitfalls of Asset/Liability Management

13) S. Weiss. "Rocket Scientists are Revolutionizing Wall Street". Business Week. April 1986

14) Donald R. Chambers. Duration Vector Drops Risk. Pensions & Investment Age. March 1988

15) M.L. Leibowitz. A New Perspective in Asset Allocation. Institute of Chartered Financial Analysts. 1987

16) Frank J. Fabozzi, Irving M. Pollack. The Handbook of Fixed Income Securities. I.C.F.A. 1983

17) John L. Maginn, Donald L. Tuttle. Managing Investment Portfolios, 1985-1986 Update. I.C.F.A. 1985