AN INTEGRATED APPROACH TO RESERVE FOR ASSUMED REINSURANCE

FRANK D. PIERSON

Frank D. Pierson is Second Vice President and Actuary at John Hancock Reinsurance Company. He was graduated from Colgate University with a BA in mathematics. He attained his FCAS in 1984 and is a Member of the American Academy of Actuaries. He currently serves on the PCAS editorial staff.

ABSTRACT

Standard reserving techniques of squaring the triangle are difficult or impossible to apply to a portfolio of assumed reinsurance. A portfolio of assumed reinsurance is typically comprised of very dissimilar risks. This paper outlines a method to reserve at the individual contract level as a means to develop a reserve for the entire portfolio. The method also generates an aggregate loss distribution for each contract, and through a Monte Carlo simulation, an aggregate loss distribution for the portfolio.

INTRODUCTION

The procedure outlined in this paper to develop reserves does not set forth any new, state-of-the-art actuarial techniques. Instead, this paper outlines a framework in which fairly standard actuarial methods can be used to establish the loss(loss as used in this paper includes allocated loss expenses) reserve for a portfolio of assumed reinsurance by developing a separate reserve for each reinsurance contract within that portfolio. This paper also outlines a means to explicitly recognize the inherent variability in the reserve of an individual contract by building an aggregate loss distribution for each contract rather than developing a single point estimate. These distributions are then used to develop an aggregate loss distribution for the entire portfolio of assumed contracts.

BACKGROUND DISCUSSION

Reinsurance is inherently volatile. Volatility would usually argue for reserving and pricing using as wide an aggregation of data as possible. Militating against such an aggregation, however, is the extreme heterogeneity of reinsurance contracts. In fact, it is fair to say that each reinsurance contract is virtually unique--each having a different set of terms, underlying business, different management philosophies, etc. In addition, these factors change from year to year so that a reinsurance portfolio could be different every year even if composed of the identical set of reinsureds each year. This high degree of heterogeneity pulls the actuary away from using aggregate data towards reserving and pricing at the individual contract level. The reserving methodology described below attempts to recognize the uncertainty in the reserve indication for an individual contract by explicitly generating a probability of loss distribution instead of a single point estimate for each contract.

Because of the high degree of heterogeneity of a portfolio of reinsurance contracts, it is very difficult to apply standard actuarial techniques of squaring the data triangles. These standard techniques assume a high degree of homogeneity in the data or that the data can be so adjusted. This is not to say, however, that aggregate methods of reserving have no place in reinsurance. In fact, we use these methods at my company as a test of reasonableness of the total indication produced by reserving at the individual contract level. They are also used between reserve studies to check whether significant changes have occurred since the previous reserve study. Monitoring reserves using aggregate methods is needed because, unfortunately, reserving at the individual contract level is time-consuming and most companies cannot afford to do so more than once a year.

Reserving at the individual contract level satisfies the five guidelines for loss reserving enunciated by Mssrs. Berquist and Sherman[1]:

1. Reserving at the individual contract level forces the actuary to develop a thorough understanding of the book of business for which the reserve is being established.

2. By reserving at the individual contract level, the actuary automatically subdivides the data in a way that increases homogeneity and minimizes distortions due to underlying changes in the data.

3. Adjusting the data for underlying changes is easiest at the individual contract level.

4. Various reserving methods can be combined to develop the reserve estimates.

5. This procedure provides a means to retrospectively test the reserving methodologies.

THE MODEL

Reserving and pricing of individual reinsurance contracts are different sides of the same coin. Both attempt to determine the ultimate losses expected under a given contract. Reserving can be thought of as retrospective pricing, or "repricing", of a contract given current knowledge. With appropriate adjustments, any techniques used to prospectively price a contract can be used to reprice that contract for reserving purposes once that contract is on the books.

The goal of pricing or reserving any insurance or reinsurance contract is to estimate the expected ultimate losses under that contract. In reinsurance, it is common to estimate the ratio of the losses to the subject premium base for the contract. This ratio is referred to as the "burning cost" of the contract. The goal, then, of either reserving or pricing is to calculate the Burning Cost at Ultimate(BCU).

There are two traditional techniques for estimating the BCU of a treaty. The first uses Increased Limits Factors(ILFs) and is called the "exposure rating method." This method will be abbreviated "XRM." This method assumes that the reinsured's excess loss potential, as a function of total losses, is similar to that for the industry as a whole. See Miccolis[4] for a discussion of this method. The second method uses the reinsured's own actual losses to predict the BCU and is called the "experience rating method." This method will be abbreviated "ERM." This method assumes that past experience, appropriately adjusted, can be indicative of the future experience of the contract being priced.

The model described herein assumes that the BCU can be broken down into its component pieces, each of which is assumed to be easier to estimate than the BCU taken as a whole. For the XRM, the BCU can be expressed as:

$$BCU = \frac{XL}{SP} = \frac{XL}{TL} \cdot \frac{TL}{SP}$$

where XL = expected excess losses, TL = total subject losses, and SP = subject premium. The ratio of XL/TL is the ratio of excess losses to total subject losses and is calculated from ILFs. The ratio of TL/SP is the reinsured's loss ratio. This ratio is the same as that underlying the subject premium base for the contract. In many cases, this ratio is the reinsured's anticipated direct loss ratio.

For the ERM, the BCU can be expressed as:

$$BCU = \frac{XL}{SP} = \frac{XL}{N} \cdot \frac{N}{SP}$$

where XL and SP are defined as above, and N is the number of excess losses. The ratio XL/N is the average excess loss(or average excess severity) assuming that a loss, or losses occur. The ratio N/SP is referred to as the frequency of excess loss. The average severity and the frequency of excess loss are determined by trending and developing the reinsured's "large" losses and then applying the treaty's retention and limit to generate "as if" losses to the treaty. The ratio N/SP does not fit the standard definition of frequency since SP is not a true exposure base. However, SP is a proxy for exposure since it is used as the base against which the reinsurance rate is applied to generate the reinsurance premium.

Since the rest of the paper applies equally to both the experience and exposure rating methodologies, the remainder of the paper assumes that the ERM is used to reserve all treaties. This is a fairly realistic assumption. The XRM is used more heavily for initial pricing and when contracts are immature, or when there is little or no actual loss experience. As actual losses emerge, it is appropriate to rely more on the ERM.

Although actual data has started to emerge, there is still much uncertainty about the BCU since the ultimate average cost and the ultimate number of losses are still unknown. This uncertainty stems from the long delay in reporting of losses to reinsurers(and hence there is potential for a significant number of IBNR losses) and to the volatility of the size of large losses. Losses which were originally expected to settle within the reinsured's retention can quickly become a reinsurance claim. In addition, due to the small number of claims under a given contract, the average cost may be sensitive to the presence or absence of one large, or total, loss. Of course, the sensitivity is dependent on both the number of claims and on the size of the reinsurance protection offered by the contract.

The model attempts to recognize the uncertainty in the BCU by explicitly recognizing the uncertainty in its components. Instead of estimating a single value for the average severity (XL/N) and a single value for the frequency (N/SP), four values are selected representing a Very Optimistic(VO), Optimistic(O), Pessimistic(P) and Very Pessimistic(VP) value for each of the variables. Each of the values are assigned a probability of occurrence.

The values and probabilities for each variable are determined based on fairly standard actuarial techniques. Typically, individual losses are trended and developed to ultimate and then the treaty terms are imposed on them. Based on a given set of assumptions, a value(s) for the average severity and a value(s) for the frequency are selected. Given the information provided by the reinsured and the general conditions in the insurance marketplace and the economy as a whole, the assumptions of trend, development, rate level changes, subject premium estimates, etc. are then varied to produce a number of different indications of average severity and frequency. Based on the sensitivity of these variables to changes in the underlying assumptions, probabilities are subjectively assigned to the values chosen for each variable. These four values and their assigned probabilities will be used as a discrete representation of each variable's "true" continuous Probability Distribution Function(PDF).

A discrete, subjective PDF is used instead of a continuous PDF for a number of reasons. First, the construction of the discrete PDF is easier to explain to non-actuarial professionals, including management, underwriting and claims. Pricing and reserving are no longer mysterious. Once these disciplines understand the model, their knowledge and expertise can be systematically incorporated into the price and reserve. This ensures that the actuary receives valid and relevant feedback and that the underwriter can be comfortable using the results. This interaction among disciplines, moreover, forces the actuary to understand the practice of reinsurance pricing in addition to its theory.

Second, given the amount of data usually available to price a treaty, parameterizing a continuous PDF may not be possible. Pricing a reinsurance treaty is a very subjective exercise. It is not always possible to perform a detailed actuarial analysis. Indeed, many times it is a gut call on the part of the underwriter. The discrete PDF allows the underwriter to put his subjectivity down on paper. Once on paper, it is easier to test the assumptions for reasonableness and to ensure consistency between contracts and from year to year.

Third, each value of the variables can represent a different, fairly independent set of assumptions(including different approaches to estimating these values) regarding the underlying business and the economic conditions. As such, using a discrete PDF is a systematic way to explicitly recognize these different assumptions. In contrast, fitting a continuous PDF implicitly implies a single set of assumptions, which may not be sufficient to predict the full range of possible outcomes. Although multiple continuous PDFs could be used, any procedure using more than one PDF can quickly become unruly. A multiple PDF approach is still subjective; either in the choice of the form of the PDF to be used or in assigning probabilities to each one. The increased complexity involved with using multiple continuous PDFs may not be warranted by any increase in accuracy.

Fourth, a discrete, subjective PDF can be used for those contracts which are expected to be "loss free," e.g., high excess of loss contracts for which the probability of loss is very small. For a high excess contract, the underwriter may expect a loss only once every 100 years, i.e., corresponding to a probability of loss of 1%. As long as the underwriter can quantify his subjective evaluation of loss potential, a PDF can be created. For these high excess contracts, the frequency of loss is the difficult variable to estimate. The average severity is typically assumed to be distributed uniformly across the layer, i.e., all sizes of loss to the contract, if a loss occurs, are equally likely.

Although this paper uses a discrete, subjective PDF, the approach is equally applicable to use of a continuous PDF. For a discussion of pricing/reserving of excess of loss treaties using a continuous PDF, see Patrick and John[4] and the discussion of their paper by Miccolis[3].

Once the discrete PDFs for the average severity and the frequency are developed, it is possible to calculate the discrete PDF for the BCU. Multiplying each average severity by each frequency yields a matrix of sixteen estimates for the BCU. Multiplying the probability of each average severity by the probability of each frequency yields a sixteen point matrix of probabilities. These two matrices form the discrete PDF for the BCU.

Given a PDF of the BCU, we can analyze the impact of the treaty's terms on the expected losses in a systematic way which would be impossible if we had developed only a single point estimate for the BCU. For example, there are treaty terms that directly affect the expected losses rather than the rate. One example is an Annual Aggregate Deductible(AAD) where the reinsured pays the losses up to the amount of the AAD(these losses would, without the AAD, be subject to the treaty) and the reinsurer pays only the losses that exceed the AAD. Another example is an annual aggregate cap on the amount the reinsurer will pay. The reinsurer will pay losses only up to the value of the cap and the reinsured will pay all losses over the top of the cap. The reinsured is, in effect, providing an aggregate stop-loss treaty to its reinsurer.

Depending on the shape of the PDF, the AAD or the loss cap can have a significant or minimal effect on the expected losses. Exhibit I shows a simplified version of the pricing worksheet in use at my company. The matrix labelled "Unlimited Burning Cost at Ultimate" is the discrete PDF before the application of a 5% AAD. The matrix labelled "Limited Burning Cost at Ultimate" is the PDF after the application of the AAD. The expected BCU without an AAD is 5%. The expected BCU with the AAD is 0.8%. Thus, although the AAD equals the expected losses before application of the AAD, the expected losses after the AAD is greater than zero. This is not an uncommon outcome and one which would be difficult to assess without the PDF.

USE OF THE MODEL FOR LOSS RESERVING

Once a PDF for a contract is developed, it is possible to calculate the various moments of the distribution and, in particular, the expected value of the BCU. Once the expected BCU is calculated, the reserve can be calculated as the difference between the expected BCU and the inception-to-date paid losses. In addition, the reserve can be calculated at any, say the 95%, confidence level. By calculating reserves at various confidence levels, a meaningful range around the expected value can be developed. By calculating the reserve at the 5% and 95% confidence levels, for example, a range can be established for the reserves such that there is only a 10% probability that the actual reserve will fall outside this range.

For a portfolio of contracts, it is possible to calculate the reserve by summing the expected values of each of the individual contracts. This is not the strength of PDF approach, however, since this would be true even if only a single expected value for each contract were calculated. The strength of creating a PDF for each contract is that it provides a means to develop a PDF of the BCU of the portfolio. With a portfolio PDF, it is possible to calculate a meaningful range around the expected. Typically, when using aggregate data, an arbitrary range is established around the expected value by creating a low and high indication which reflect a less or more severe set of assumptions, respectively. Unfortunately, it is impossible to assign a probability to that range, or more importantly, a probability to exceeding the high end of the range.

The portfolio PDF is developed by running a Monte Carlo simulation across all treaties in the portfolio. Another advantage to using a discrete PDF is the ease and speed of running a simulation. It is not necessary to calculate the inverse of the integral of the PDF, which for many continuous distributions is impossible to do in closed form, nor is it necessary to construct the n-fold convolution of the severity and frequency distributions, nor to select individual claim amounts for the selected number of claims(in effect running two simulations: one for frequency and one for severity). In addition, it is a fairly straightforward calculation and can be programmed in Lotus 1-2-3.

To run the simulation, it is necessary to construct the Cumulative Distribution Function(CDF). The CDF is calculated by first sorting the sixteen estimates of the BCU in ascending order. The CDF is then:

$$F_i = prob(x<=BCU_i) = \sum_{j=1}^{i} prob(x=BCU_j)$$

where x is the actual BCU.

Once the CDF F is developed, the simulation proceeds as follows:

1. For each contract, generate a random number(r) between 0 and 1.

2. For the value in (1), select the BCU such that $F(BCU_i) = r$.

3. Multiply the BCU from (2) by the subject premium for the treaty to generate dollars of expected loss.

4. Add the result from (3) to a running total for this iteration.

5. Repeat (1) -(4) for all treaties in the portfolio.

Exhibit II shows the input for the simulation of a portfolio of seven contracts. These contracts represent a real, albeit small, portfolio for which our company establishes a separate reserve. The block of data for each treaty is automatically created when the pricing model is run. The second row is the cumulative distribution of the BCUs shown in row three. Row one is the CDF offset by one BCU. This row is needed so that the Lotus macro selects the correct BCU -- this compensates for how Lotus selects values when looking up values in a data table.

Exhibit III shows the results of a single iteration of the simulation. The incurred amount equals the subject premium times the reinsurer's share times the selected BCU. As a check on the simulation, the expected value for each contract is calculated as the product of the expected average severity times the expected number of claims times the reinsurer's share. The sum of these expecteds is then compared to the average incurred produced by the 1000 simulations. The difference is less than 0.5%. The actual Lotus macro is shown at the bottom of this exhibit.

Exhibit IV shows the distribution of 1000 iterations around the average incurred amount. Exhibit V is a graph of the PDF for the portfolio based on these 1000 simulations.

Exhibit VI shows the incurred amounts at various confidence levels. The expected losses for this portfolio are $14.5 million. If we assume that the inception-to-date paid losses are $4 million, then the reserve based on the expected losses is $10.5 million. From this exhibit, a confidence interval can be established around the expected incurred. For example, to establish a 90% confidence interval around the expected incurred, select the lower limit at $13 million, corresponding to the 5% level, and the upper limit at $16.5 million, corresponding to the 95% level. This will correspond to a reserve range of $9 million to $12.5 million.

APPLICATION OF MODEL TO OTHER RESERVES

There are reserves other than loss & LAE reserve that a reinsurer must establish. Most of these reserves are sensitive to the level of losses. As such, this method can be used for establishing these reserves.

Retrospectively Rated Contracts

Many reinsurance contracts are retrospectively rated instead of prospectively rated. Retrospectively rated contracts are commonly referred to as swing-rated contracts. The rate for a swing-rated contract is a function of the losses subject to a prospectively agreed upon minimum and maximum. The typical formula is

Rate = Loss * Loss Conversion Factor + Fixed Expense Load

Once a PDF for the losses has been created, the PDF for the swing rate is automatically created. Based on this PDF, an expected rate, and hence, an expected premium, for each contract can be determined. Without a PDF for the losses, it would be very difficult to estimate the expected rate for a given contract. The expected rate is not necessarily equal to the rate generated by applying the retrospective rating formula to the expected losses. For example, the expected rate for the contract in Exhibit I is 1.71%, or a premium of $2.9 million. The rate based on applying the swing-rate formula to the expected loss of .8% is 1.50%(=.8%*1.25+.5%), or a premium of $2.5 million, a $400,000 difference in the indicated premium.

Typically, the reinsured pays premium using a provisional rate for a specified period of time. At the end of that period, the rate is calculated using the swing-rate formula. Although the provisional rate is usually set near the initial expected losses for the treaty, as actual losses emerge the revised expected losses may be significantly different from the original expected. For those contracts for which the revised expected losses are less than the original estimate, the reinsurer should set up a reserve for premium which is anticipated to be returned to the reinsured.

Exhibit VII is a representative listing of some of the swing-rated treaties written by my company in the last three years. Column (1) lists the subject premium(my company's share) for each contract. Column (2) lists the contractual provisional rate. Column (3) lists the expected rate for each contract based on the latest review. The return rate in column (4) is equal to the expected rate less the provisional rate. It is set to zero if the difference is positive since that would represent additional premium. Column (5) converts the return rate into return premium. Column (6) is the additional rate and column (7) is the additional premium.

My company reserves for the return premium only. We do not take credit for potential additional premium on those swing-rated contracts that are expected to generate a rate greater than the provisional. The reserve for return premium is included in my company's unearned premium reserve. In other words, we do not recognize as earned all the premium reported to us as earned by our reinsureds. Of course, as the actual premium for these contracts exceed the provisional or the expected, we allow that premium to flow through as earned. We do not establish a return premium reserve for treaties in prior years, since the premiums for those treaties are no longer reported at the provisional rate. They are now reported at the developed rate, i.e., they are calculated by the swing-rate formula.

Contingent Profit/Sliding Scale Commission

Many reinsurance contracts include a provision for the reinsured to participate in the profits of the contract. The profit sharing can be through either a contingent profit commission or a sliding scale commission. Both of these mechanisms are a function of the losses of the contract. As such, they are simply a different form of swing rating with the provisional rate equal to the maximum rate. Since the final commission is a function of losses, the PDF of the losses automatically creates a profit sharing PDF. Based on this PDF, an expected profit sharing amount for each contract can be determined. In addition, it is possible to calculate a confidence interval around the expected reserve. Exhibit VIII shows the calculation of the contingent profit commission for the same reinsured as shown in

Exhibit I. The contract is prospectively rated with a contingent profit commission rather than being swing-rated. This set of terms does not include an AAD.

In a manner similar to that for determining the return premium for swing-rated contracts, it is possible to calculate the reserve for the profit-sharing contracts. The reserve is for only those treaties for which additional commission must be paid to the reinsureds. No recognition is given to any commission which is expected to be returned to the reinsurer.

Unallocated Loss Adjustment Reserves

Unallocated Loss Adjustment Expense(ULAE) is a function of the number of claim files that must be handled by the claims department. The ULAE reserve is a function of more than just those claims which ultimately become losses. For most excess of loss treaties, most claims are closed without payment by the reinsurer. Many claims are only precautionary in nature, i.e., the current incurred amount is less than the contract's attachment point, but the potential exists to become a loss to the reinsurer. In fact, many reinsurance contracts specify when precautionary claims must be reported to the reinsurer.

A natural by-product of reserving at the individual contract level is an estimate of the ultimate number of claims for each contract. By summing up these expected number of claims, the total expected number of claims for the entire portfolio can be determined. Using historical ratios of claims closed without payment to claims closed with payment, it is possible to estimate the expected number of claim files to be handled by the claims department. The ULAE is then established as a function of the expected number of claim files.

FUTURE DIRECTIONS

Our company has used a contract by contract review to set the bulk of its casualty reserves for two years. It has yet to use the portfolio PDF to set a confidence interval around the expected, except in a limited number of cases. There are a number of reasons for this. Among them are:

1) We only review contracts representing 75% of our casualty portfolio. We do not review the entire portfolio because it is not cost effective to do so. Although we do review the majority of the book from either a loss or premium viewpoint, we only review approximately 40% of the number of contracts. This non-reviewed portfolio causes some problems when attempting to calculate the portfolio PDF. One solution is to assume that the reserve for the non-reviewed portfolio is a constant. The PDF for the portfolio would then be a function of the PDFs of the reviewed contracts only. Another solution is to attempt to develop a PDF for the non-reviewed portfolio and treat it as a single contract in the simulation. Of course, this would tend to reduce the variation in the resulting

portfolio PDF. The size of the confidence interval around the expected could be affected.

2) The simulation procedure assumes that all of the individual contracts are independent; many of the contracts, however, are, in fact, related. For example, a reinsurer may write more than one layer of reinsurance coverage for a single reinsured. This is commonly referred to as a reinsurance program. Therefore, a single loss may affect more than one contract. Conversely, if the bottom layer is loss free, then a higher layer cannot have any losses. It is, therefore, necessary to run the simulation using program PDFs rather than PDFs for individual contracts. We have not yet developed a routine way to convert the contract PDFs into PDFs that would be appropriate for an entire program.

The ERM assumes that the average severity PDF is independent of the frequency PDF. However, this is not technically true. The variance of an average value is inversely proportional to the number of items used in calculating the average. This implies that in our model, as the frequency of loss increases, the variance of the average severity decreases; the PDF of the average severity is, therefore, not independent of the PDF of the frequency. The variance of the resulting PDF for the BCU is overstated. However, given that the probabilities given to the VO and VP values are usually small, say 5%, the distortion in the PDF should not be significant. In addition, given the uncertainty involved in pricing and reserving reinsurance, this built-in conservatism could be considered prudent.

ADVANTAGES

Reserving at the individual contract level has many advantages over traditional aggregate methods. Among them:

1) All available information is systematically combined into the reserve estimate. Information at the treaty level is an integral part of the reserve calculation. In addition, there is typically more data available at the individual contract level than at the portfolio level. The reinsured usually supplies several years of historical data with his request for reinsurance. This data typically includes a listing of individual losses including losses as yet unknown to the reinsurer, as well as a history of subject premiums and rate level changes. In many cases, enough information is available to estimate development. By reserving at the individual contract level, this information is explicitly reflected rather than implicitly included in the portfolio-wide assumptions made when using aggregate assumptions.

2) This method provides a systematic means for combining the input from the various disciplines: actuarial, underwriting and claims. The underwriter can bring his qualitative analysis to bear on the assumptions underlying the reserves. The actuary's assumptions are displayed in a manner which can be understood by the underwriter. The claimsman can provide input on the quality of the cedant's claims department and the validity of the numbers being used by the actuary.

3) Changes in the reinsurer's book are automatically included in the reserve estimate. These changes include shifts in the type of contracts written, e.g., writing excess of loss rather than pro-rata; changes in the companies being reinsured; changes in the contract terms, e.g., imposition of annual aggregate caps, writing at higher limits or retentions; changes in the underlying business written by the reinsureds; etc. It is not always possible to estimate the impact of these changes on the overall portfolio without estimating the impact at the contract level.

4) Since the same model is used to price new contracts as well as to reserve ones already on the books, this method provides a means to test our pricing models a posteriori. Exhibit IX shows the original and revised expected losses for all treaties that incepted in 1986 for which my company used this method to originally price and then reprice during our recent reserve review. As you can see, although there is some movement in the expected losses for individual contracts from one evaluation to the next; the expected losses for the portfolio as a whole, however, changed very little.

5) Since each contract is reviewed, the actuary gains a better understanding of the portfolio. He now longer has to guestimate about the portfolio. In addition, it is possible to determine if an observed change in the total reserve level is due to movement in the portfolio as a whole or can be traced back to an individual contract or group of contracts. This gives management a better feel for why

the reserves are changing. If only a limited number of contracts are causing the observed movement, then it may be possible to prevent these contracts from affecting the reserve further by monitoring them closely or by commuting them.

6) A reinsurer is also a buyer of reinsurance. That reinsurance may protect the entire portfolio, or it may protect an individual contract. It is difficult to estimate the extent of reinsurance recoverable at the portfolio level(except for proportional reinsurance covering the entire portfolio). Reserving at the individual contract level, and further by estimating the size of individual claims, reinsurance recoverables can be estimated more accurately.

7) Since these contracts are repriced using all contract terms, a wide range of additional information is produced: average contingent commission, ultimate expenses, ultimate number of claims, etc. In addition, the method provides a confidence interval around the expected value of each of these variables. This information can be used as input into the budget process, as well as providing guidance when creating alternative scenarios for strategic planning.

8) Since a reserve is established for each contract, a separate reserve can be established for any group of contracts. For example, the overall portfolio can be grouped by type of treaty, e.g., pro-rata vs excess of loss, or further, by type of excess of loss treaty. By analyzing the portfolio in this detail, it may be possible to identify which segments of the reinsurance market are the most profitable at

any given point in the cycle since not all segments of the market respond equally to changes in the market.

9) Since this analysis is used to initially price new contracts, this method can be used to establish the reserve for all the new business. Because no actual experience is available, the reserve for these contracts has a large confidence interval around it. It may be appropriate to set the reserve for these contracts as something greater than the expected value. The PDF is a guide to where the reserve should be set so that management is comfortable with the reserve.

10) As stated above, a probabilistic confidence interval is a natural by-product of this method. This helps management understand the true volatility of the reserve estimate.

11) The portfolio is split into the most homogenous groupings possible. In addition, assumptions are made at the individual contract level rather than at the portfolio level. Since we know a lot about the terms of an individual contract, and about the business subject to the treaty, it is easier to make assumptions at the individual contract level than at the portfolio level. For example, whether the cedant's Allocated Loss Adjustment Expense(ALAE) is considered as part of loss or is covered only in proportion to loss is difficult to determine at the portfolio level. However, the treatment of ALAE is known at the contract level. Another example is whether treaties are on a loss occurring during(i.e., accident year) basis or

on a risk attaching(i.e., policy year) basis. This is known at the treaty level, but the extent that the portfolio is one or the other can only be estimated.

To the extent that there is no built-in bias in the pricing methodology, the impact on the overall indication of errors in assumptions at the treaty level will be damped. On the other hand, an error in assumptions at the portfolio level would impact the indication for the entire portfolio. Given the apparent randomness of the changes in expected losses shown in Exhibit IX, there does not appear to be any significant bias in the methodology.

DISADVANTAGES

1) This method of reserving is extremely time consuming. To reserve each contract, it is necessary to review the experience; treaty terms and underwriting information; check the accounting of the treaty; meet with the underwriters and claimsmen; etc. The time required can be reduced by reviewing only the largest treaties or by sampling. The time required may prevent a company from performing this kind of review more than once a year. Some other method must be used to check reserve levels between reserve reviews based on this method.

2) The credibility of the indications of the individual contracts are difficult, if not impossible, to determine because of the subjectiveness of the PDFs. The PDFs are not rigorously, from a

mathematical sense, developed and, therefore, standard credibility measurements cannot be used.

CONCLUSIONS

Despite the time required to perform a contract by contract review, I believe that the advantages mentioned above make it a worthwhile exercise. For any insurer to survive, it must know its book of business inside and out. This is doubly true for a reinsurer.

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Insurance Comapny
aty # 999999 500000 excess 500000
E Share: 0.05

Expected Premium Rates

quency	VO	O	P	VP	Avg
	Severity				
VO	1.00%	1.00%	1.00%	1.00%	1.00%
O	1.00%	1.00%	1.00%	1.00%	1.00%
P	1.00%	1.28%	3.62%	5.00%	2.38%
VP	1.91%	4.09%	5.00%	5.00%	4.28%
Avg	1.05%	1.27%	2.25%	2.80%	1.71%

Expected Loss + LAE Ratio

Frequency	VO	O	P	VP	Avg
	Severity				
VO	0.00%	0.00%	0.00%	0.00%	0.00%
O	0.00%	0.00%	0.00%	25.00%	1.25%
P	0.00%	48.78%	68.97%	75.00%	61.80%
VP	59.02%	70.23%	110.00%	145.00%	92.67%
Avg	5.38%	31.07%	56.67%	70.54%	46.16%

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UNLIMITED BURNING COSTS AT ULTIMATE

	Frequenc /$mill	Severity Prob	VO 175 10.0%	O 225 45.0%	P 300 40.0%	VP 350 5.0%	Avg 256 100.0%
VO	0.100	10.0%	1.75%	2.25%	3.00%	3.50%	2.56%
O	0.150	45.0%	2.63%	3.38%	4.50%	5.25%	3.84%
P	0.250	40.0%	4.38%	5.63%	7.50%	8.75%	6.41%
VP	0.350	5.0%	6.13%	7.88%	10.50%	12.25%	8.97%
Avg	0.195	100.0%	3.41%	4.39%	5.85%	6.83%	5.00%

Additional Exposure Factor(eg, Clash) 100.00%

LIMITED BURNING COSTS AT ULTIMATE

	Severity Prob	VO 10.0%	O 45.0%	P 40.0%	VP 5.0%	Avg 100.0%
VO	10.0%	0.00%	0.00%	0.00%	0.00%	0.00%
O	45.0%	0.00%	0.00%	0.00%	0.25%	0.01%
P	40.0%	0.00%	0.62%	2.50%	3.75%	1.47%
VP	5.0%	1.12%	2.88%	5.50%	7.25%	3.97%
Avg	100.0%	0.06%	0.39%	1.27%	1.97%	0.79%

ing Parameters
1.00% = Min Rate
2.50% = Prov Rate
5.00% = Max Rate

125.00% = Loss Conversion Factor
0.50% = Fixed Expense Load

5.00% = Other U/W Expense
10.00% = Commission & Brokerage

Loss Corridor fr 0.00% to 5.00%
Annual Loss Cap ########

Expected Values ($000's)	100%	JHRe Share
Subject Premium	167,425	NA
R/I Premium	2,871	144
Other U/W Expenses	144	7
Incurred Losses	1,325	66
Comm & Broke	287	14
Underwriting Gain(Loss)	1,115	56
# of Claims to Treaty *	5.2	5.2
Total # of Claims(inc Loss Corr)*	32.6	32.6
Totals Funded @ Max(RDR(P)=0)	14.2	14.2

* Assumes avg loss 256

Exhibit II

Development of Portfolio Probability Distribution of Loss
Summary of Input Data for Simulation

	Outcome i															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Treaty # 1																
F[x(i-1)]	0.0%	1.0%	6.0%	11.0%	14.5%	39.5%	42.5%	43.0%	60.5%	75.5%	78.0%	88.5%	89.5%	91.0%	96.0%	99.5%
F[x(i)]	1.0%	6.0%	11.0%	14.5%	39.5%	42.5%	43.0%	60.5%	75.5%	78.0%	88.5%	89.5%	91.0%	96.0%	99.5%	100.0%
BCU(i)	1.5%	1.7%	1.8%	1.9%	2.0%	2.1%	2.1%	2.2%	2.3%	2.5%	2.5%	2.6%	2.9%	2.9%	3.1%	3.6%
#Clms(i)	5.2	5.8	6.1	6.4	6.7	6.9	7.2	7.4	7.7	8.4	8.5	8.7	9.6	9.6	10.6	12.0
Treaty # 2																
F[x(i-1)]	0.0%	0.5%	5.5%	8.0%	12.0%	37.0%	57.0%	58.5%	73.5%	74.0%	74.5%	86.5%	91.5%	94.0%	98.0%	99.5%
F[x(i)]	0.5%	5.5%	8.0%	12.0%	37.0%	57.0%	58.5%	73.5%	74.0%	74.5%	86.5%	91.5%	94.0%	98.0%	99.5%	100.0%
BCU(i)	27.0%	28.2%	29.3%	30.0%	30.6%	32.5%	33.8%	35.3%	36.0%	36.0%	37.5%	37.6%	39.0%	40.0%	45.0%	48.0%
#Clms(i)	55.4	57.8	60.0	61.5	62.7	66.7	69.2	72.3	73.8	73.8	76.9	77.1	80.0	82.1	92.3	98.5
Treaty # 3																
F[x(i-1)]	0.0%	1.0%	6.0%	9.5%	13.5%	33.5%	37.5%	51.5%	52.0%	72.0%	86.0%	88.0%	90.0%	91.0%	96.0%	99.5%
F[x(i)]	1.0%	6.0%	9.5%	13.5%	33.5%	37.5%	51.5%	52.0%	72.0%	86.0%	88.0%	90.0%	91.0%	96.0%	99.5%	100.0%
BCU(i)	36.3%	38.9%	40.7%	41.2%	44.1%	44.1%	46.2%	46.3%	47.3%	49.5%	52.5%	56.3%	56.8%	60.9%	63.8%	72.5%
#Clms(i)	33.9	36.3	38.0	38.4	41.2	41.2	43.2	43.2	44.1	46.2	49.0	52.5	53.1	56.9	59.6	67.7
Treaty # 4																
F[x(i-1)]	0.0%	1.0%	6.0%	10.5%	33.0%	36.5%	40.5%	41.0%	56.8%	76.8%	77.3%	79.8%	82.0%	96.0%	97.8%	99.8%
F[x(i)]	1.0%	6.0%	10.5%	33.0%	36.5%	40.5%	41.0%	56.8%	76.8%	77.3%	79.8%	82.0%	96.0%	97.8%	99.8%	100.0%
BCU(i)	4.3%	4.7%	4.7%	5.1%	5.3%	5.4%	5.8%	5.8%	5.8%	6.0%	6.2%	6.5%	6.7%	7.1%	7.5%	8.0%
#Clms(i)	11.3	12.1	12.2	13.1	13.9	14.1	15.0	15.0	15.2	15.6	16.2	16.9	17.3	18.5	19.5	20.8
Treaty # 5																
F[x(i-1)]	0.0%	1.0%	5.5%	9.0%	13.5%	14.5%	34.8%	50.5%	54.5%	59.0%	77.0%	77.5%	91.5%	93.8%	97.8%	99.5%
F[x(i)]	1.0%	5.5%	9.0%	13.5%	14.5%	34.8%	50.5%	54.5%	59.0%	77.0%	77.5%	91.5%	93.8%	97.8%	99.5%	100.0%
BCU(i)	0.9%	1.0%	1.2%	1.4%	1.4%	1.6%	1.9%	2.1%	2.3%	2.3%	2.8%	2.9%	3.1%	3.4%	3.8%	4.5%
#Clms(i)	4.1	4.6	5.7	6.6	6.7	7.4	9.1	9.9	10.7	11.1	13.2	13.6	14.8	16.1	18.1	21.4
Treaty # 6																
F[x(i-1)]	0.0%	1.0%	6.0%	9.0%	13.0%	33.0%	34.0%	38.0%	50.0%	70.0%	71.0%	75.0%	87.0%	92.0%	95.0%	99.0%
F[x(i)]	1.0%	6.0%	9.0%	13.0%	33.0%	34.0%	38.0%	50.0%	70.0%	71.0%	75.0%	87.0%	92.0%	95.0%	99.0%	100.0%
BCU(i)	1.2%	1.3%	1.6%	1.6%	1.8%	2.0%	2.1%	2.2%	2.3%	2.6%	2.7%	2.8%	2.8%	3.4%	3.5%	4.3%
#Clms(i)	6.5	7.3	8.7	9.0	10.0	10.9	11.5	12.0	12.7	13.9	15.0	15.3	15.5	18.5	19.1	23.2
Treaty # 7																
F[x(i-1)]	0.0%	0.5%	5.5%	7.5%	9.5%	29.5%	33.5%	34.0%	54.0%	54.5%	70.5%	75.5%	77.5%	93.5%	95.5%	99.5%
F[x(i)]	0.5%	5.5%	7.5%	9.5%	29.5%	33.5%	34.0%	54.0%	54.5%	70.5%	75.5%	77.5%	93.5%	95.5%	99.5%	100.0%
BCU(i)	9.3%	9.9%	10.0%	10.6%	10.6%	10.8%	11.3%	11.3%	11.4%	11.5%	12.0%	12.2%	12.3%	13.0%	13.0%	13.8%
#Clms(i)	26.2	28.0	28.0	29.8	29.9	30.3	31.6	31.8	32.0	32.4	33.7	34.2	34.5	36.4	36.5	38.7

Exhibit III

Development of Portfolio PDF

Result of One Iteration

Treaty #	(1) 100% Subject Premium	(2) Random Number (r)	(3) Burning Cost (BCU)	(4) Reinsurer's Share	(5) Reinsurer's Incurred =(1x3x4)		Check on Simulation: # Clms	Average Values from Pricing: 100% Avg Sev	Reinsurer's Share	Reinsurer's Incurred
1	700,000	0.13765	1.9%	10%	1,320		8	2,075	10.0%	1,556
2	50,000	0.20976	30.6%	25%	3,819		69	244	25.0%	4,209
3	50,000	0.90049	56.8%	25%	7,105		44	535	25.0%	5,885
4	18,000	0.23486	5.1%	25%	228		15	69	25.0%	255
5	23,000	0.61318	2.3%	35%	189		10	49	35.0%	170
6	30,000	0.42729	2.2%	95%	627		12	55	95.0%	627
7	70,000	0.66425	11.5%	23%	1,854		32	249	23.0%	1,833
					15,141		Expected Total Loss(Pricing)			14,535
							Expected Total Loss(Simulation)			14,509

Percent Difference -0.2%

Lotus Macro for Simulation	Comments
\s {windowsoff}{paneloff}{calc}	Disable Screen to speed up macro.
/rvtime~strttime~	Record running time.
{goto}sim1~	Goto location to record first iteration
/rvsumsim~~	Copy value of iteration to storage location.
{for counter,1,999,1,sim}	Loop through 999 more iterations
{calc}/dsddistval~pdistval~a~g	Once simulation is completed, sort the iterations by size.
/dddistval~distout~{calc}{beep 3}	Then calculate distribution as in Exhibit IV).
{panelon}{windowson}	Enable screen again.
Sim {recalccol outsim}	Recalculate only the area under
{down}	the heading "Result of One Iteration."
/rvsumsim~~	This speeds up the simulation--you don't need
	to recalculate the entire worksheet for each iteration.
	Then move down one row to store result of that iteration.
Counter 1	Keep track of number of iterations

Finish 05:39:33
Start 13:49:41
Elapsed Time 15:49:52

Exhibit IV

Aggregate Loss Distriution Around Mean
Based on 1000 Simulations

+/- Std Dev Around Mean	Incurred Loss Range Lower Limit	Upper Limit	Simulated Total Loss Number In Range	% of Total In Range	Cumulative Percent
-4	0	10,525	9	0.9%	0.9%
-3.5	10,526	11,056	2	0.2%	1.1%
-3	11,057	11,587	0	0.0%	1.1%
-2.5	11,588	12,118	0	0.0%	1.1%
-2	12,119	12,649	3	0.3%	1.4%
-1.5	12,650	13,181	38	3.8%	5.2%
-1	13,182	13,712	114	11.4%	16.6%
-0.5	13,713	14,243	229	22.9%	39.5%
0	14,244	14,774	250	25.0%	64.5%
0.5	14,775	15,305	187	18.7%	83.2%
1	15,306	15,836	75	7.5%	90.7%
1.5	15,837	16,368	42	4.2%	94.9%
2	16,369	16,899	26	2.6%	97.5%
2.5	16,900	17,430	20	2.0%	99.5%
3	17,431	17,961	4	0.4%	99.9%
3.5	17,962	18,492	1	0.1%	100.0%
4	18,493	19,023	0	0.0%	100.0%
Above 4			0	0.0%	100.0%
Total			1000		

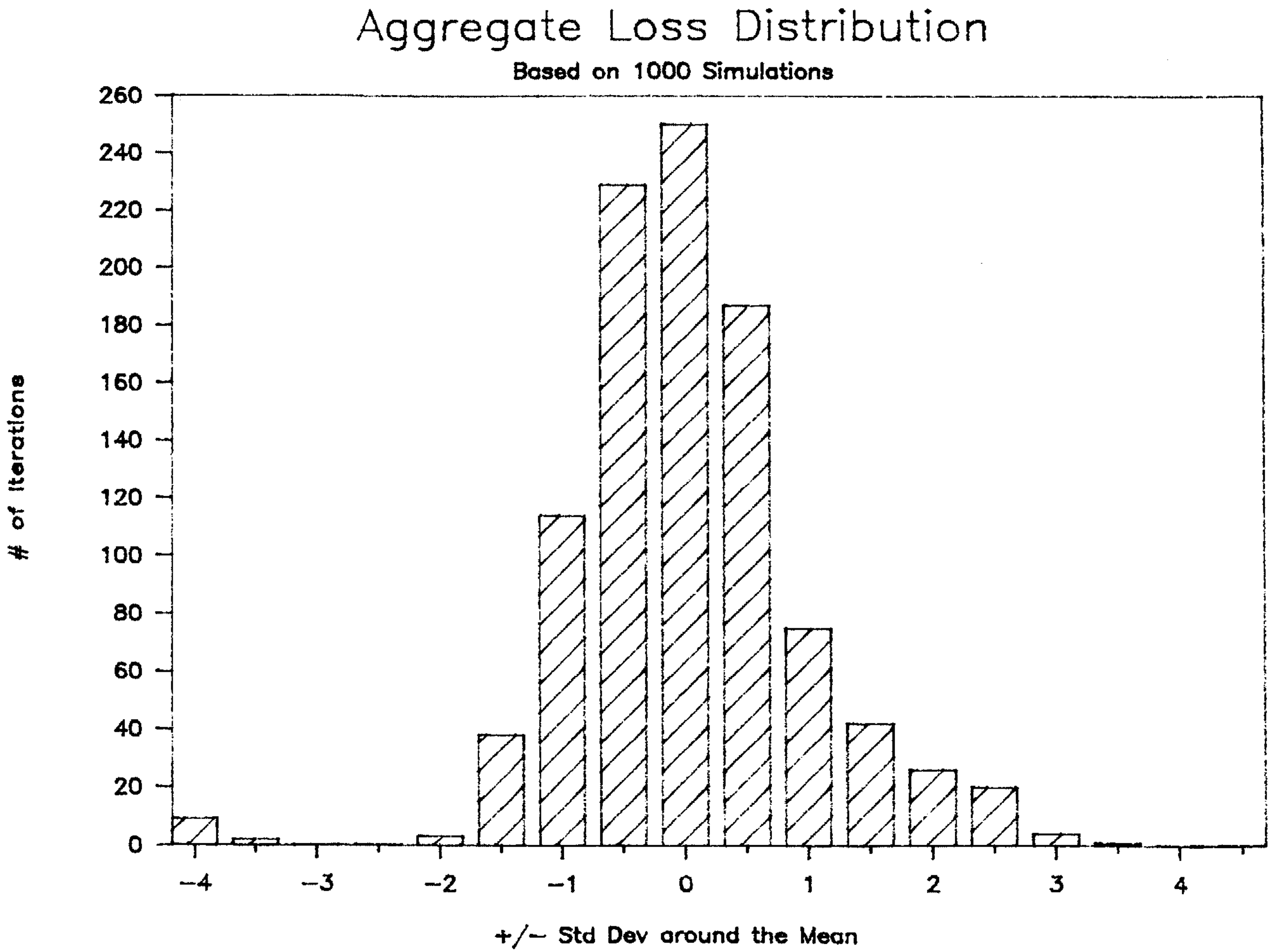
Aggregate Loss Distribution
Based on 1000 Simulations
# of Iterations
260
240
220
200
180
160
140
120
100
80
60
40
20
0
−4
−3
−2
−1
0
1
2
3
4
+/− Std Dev around the Mean

Exhibit V

Exhibit VI

Cumulative Distribution Function
Based on 1000 Simulations

Probability Total Loss <= $X	Incurred Loss Amount $X
5%	12,981
10%	13,350
15%	13,583
20%	13,761
25%	13,876
30%	13,983
35%	14,085
40%	14,212
45%	14,301
50%	14,406
55%	14,485
60%	14,630
65%	14,743
70%	14,851
75%	14,970
80%	15,113
85%	15,294
90%	15,578
95%	16,123
99.9%	17,058

Exhibit VII

Determination of Return Premium Reserve

Und. Year	Treaty Number	Subject Premium	Prov Rate	Inc'd Loss	Devel Rate	Expect Rate	Return Rate	Return Premium	Add'l Rate	Add'l Premium
86	11001	14,490	13.00%	87.3	7.50%	15.00%	0.00%	0	2.00%	290
87	11001	2,509	10.25%	0	6.00%	14.70%	0.00%	0	4.45%	112
86	11002	2,878	12.00%	0	6.50%	10.70%	-1.30%	(37)	0.00%	0
86	11003	1,888	7.00%	2.8	2.50%	8.80%	0.00%	0	1.80%	34
87	11003	192	10.00%	0	3.00%	14.70%	0.00%	0	4.70%	9
87	11004	217	7.50%	16.6	15.16%	15.07%	0.00%	0	7.57%	16
86	11005	6,239	4.00%	0	3.20%	4.03%	0.00%	0	0.03%	2
87	11006	1,342	4.00%	54.1	5.38%	4.54%	0.00%	0	0.54%	7
85	11007	4,034	17.50%	224.3	13.06%	12.50%	-5.00%	(202)	0.00%	0
86	11008	3,231	4.00%	0	2.00%	4.00%	0.00%	0	0.00%	0
86	11009	924	2.50%	0	1.00%	3.61%	0.00%	0	1.11%	10
87	11010	36,000	1.50%	76.9	0.80%	1.84%	0.00%	0	0.34%	122
87	11011	320	20.00%	0	9.00%	18.00%	-2.00%	(6)	0.00%	0
85	11012	505	2.50%	0	1.00%	4.50%	0.00%	0	2.00%	10
87	11013	20,833	9.00%	0	6.00%	15.00%	0.00%	0	6.00%	1,250
								(246)		1,863

Notes:

1. Prov Rate = Provisional Rate as specified in treaty.
2. Expect Rate = Average Rate as determined in pricing/reserving.
3. Return Rate = Expect Rate - Prov Rate, if negative.
4. Add'l Rate = Expect Rate - Prov Rate, if positive.
5. Return Premium = Return Rate * Subject Premium
6. Add'l Premium = Add'l Rate * Subject Premium
7. Subject Premium reflects reinsurer's share only.

ABC Insurance Company
Treaty # 999999 500000 excess 500000
JHRE Share: 0.05

Expected Contingent Profit

Frequency	Severity VD	D	P	VP	Avg
VD	0.91%	0.79%	0.60%	0.35%	0.70%
D	0.69%	0.51%	0.22%	0.00%	0.39%
P	0.26%	0.00%	0.00%	0.00%	0.03%
VP	0.00%	0.00%	0.00%	0.00%	0.00%
Avg	0.51%	0.31%	0.16%	0.03%	0.25%

Expected Loss + LAE Ratio

Frequency	Severity VD	D	P	VP	Avg
VD	29.17%	37.50%	50.00%	66.67%	43.13%
D	43.75%	56.25%	75.00%	100.00%	64.69%
P	72.92%	93.75%	125.00%	166.67%	107.81%
VP	102.08%	131.25%	175.00%	233.33%	150.94%
Avg	56.88%	73.13%	97.50%	130.00%	84.09%

****

UNLIMITED BURNING COSTS AT ULTIMATE

	Frequenc /$mill	Severity Prob	VD 175 10.0%	D 225 45.0%	P 300 40.0%	VP 400 5.0%	Avg 259 100.0%
VD	0.100	10.0%	1.75%	2.25%	3.00%	4.00%	2.59%
D	0.150	45.0%	2.63%	3.38%	4.50%	6.00%	3.88%
P	0.250	40.0%	4.38%	5.63%	7.50%	10.00%	6.47%
VP	0.350	5.0%	6.13%	7.88%	10.50%	14.00%	9.06%
Avg	0.195	100.0%	3.41%	4.39%	5.85%	7.80%	5.05%

Additional Exposure Factor(eg, Clash) 100.00%

LIMITED BURNING COSTS AT ULTIMAT

	Severity Prob	VD 10.0%	D 45.0%	P 40.0%	VP 5.0%	Av 100.
VD	10.0%	1.75%	2.25%	3.00%	4.00%	2.5
D	45.0%	2.63%	3.38%	4.50%	6.00%	3.8
P	40.0%	4.38%	5.63%	7.50%	10.00%	6.4
VP	5.0%	6.13%	7.88%	10.50%	14.00%	9.C
Avg	100.0%	3.41%	4.39%	5.85%	7.80%	5.C

Rating Parameters

6.00% = Offered Rate

25.00% = Contingent Profit Commission
10.00% = R/I Expense Allowance

5.00% = Other U/W Expense
10.00% = Commission & Brokerage

Loss Corridor from 0.00% to 0.00%
Annual Loss Cap ********

Expected Values ($000's)

	100%	JHR Shar
Subject Premium	167,425	
R/I Premium	10,046	5
Other U/W Expenses	502	
Incurred Losses	8,448	4
Comm & Broke	1,431	
Underwriting Gain(Loss)	(336)	(
# of Claims to Treaty *	32.6	32
Total # of Claims(inc Loss Corr) *	32.6	32
Totals Funded @ Max(RDR(P)=0)	17.1	17

* Assumes avg loss = 2

Determination of Incurred Losses for
Treaties Incepting in 1986
Comparison of Original Estimate to Current Estimate

Und. Year	Contract Number	Expected Losses Based on Review in 1987	1986	Percentage Change From '86 to '87
86	10001	30,206	66,741	-54.7%
86	10002	151,191	111,316	35.8%
86	10003	1,508,797	2,127,462	-29.1%
86	10004	1.112.940	864,000	28.8%
86	10005	170,573	78.803	116.5%
86	10006	287,539	158,642	81.2%
86	10007	891,000	880,000	1.3%
86	10008	464,983	503,732	-7.7%
86	10009	687,091	388,533	76.8%
86	10010	7,938,363	7,938,363	0.0%
86	10011	5,606,032	5,606,032	0.0%
86	10012	86,221	80,447	7.2%
86	10013	393,808	411,718	-4.4%
86	10014	1,712,813	1,771,875	-3.3%
86	10015	192,548	144,411	33.3%
86	10016	89,936	74,197	21.2%
86	10017	1,881,600	2,150,400	-12.5%
86	10018	2,662,200	3,519,000	-24.3%
86	10019	132,569	692,066	-80.8%
86	10020	36,032	44,565	-19.1%
86	10021	21,722	31,377	-30.8%
86	10022	71,718	75,028	-4.4%
86	10023	380,064	446,832	-14.9%
86	10024	542,500	840,000	-35.4%
86	10025	708,750	787,500	-10.0%
86	10026	1,488,163	1,337,463	11.3%
	Total	29,249,269	31,130,504	-6.0%
	Loss Ratio	82.1%	87.4%	

Notes:

1. The expected losses estimated in 1986 represent the original estimates underlying the pricing of each treaty at inception.

2. The expected losses estimated in 1987 represent the estimates generated using the latest information available at the time of the latest reserve review.