TITLE: DISCOUNTING BY MEASURING THE ASSET LIABILITY MISMATCH

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ABSTRACT Traditionally, loss reserves have been discounted by calculating the net present value of a series of projected future payments at some "suitable" interest rate. What constitutes a suitable rate is the subject of much debate and has no simple answer. In this paper we demonstrate a simple technique for determining the present value of the loss and loss adjustment expense liabilities of a company without selecting a discount rate in the traditional sense. Instead the asset and liability cash flows are compared and used to determine the present value.

Traditionally, loss reserves have been discounted by determining the net present value of a series of future payments using some "suitable" interest rate. What constitutes a suitable interest rate is not a simple question and has several possible answers (the recent AICPA draft, Statement of Position, devotes some thirty pages to this question).

In this paper we introduce a technique for determining the present value of loss reserves without selecting an interest rate in the traditional sense. Instead, the difference between the asset and liability cash flows (the asset/liability mismatch) is combined with assumed reinvestment and "borrowing" rates and used to determine the present value of the company's loss and loss adjustment expense liabilities.

In the process, we will demonstrate the degree to which the asset cash flow matches the projected future liability cash flow[1]. This question is important to every insurer, even those who do not discount.

It is relatively easy to envision a situation in which a company is solvent on a statutory accounting basis but is completely unable to meet its obligations. We can construct a very simple example by considering a company whose assets consist of a single

[1] Pennsylvania requires a statement concerning to this effect for workers' compensation insurance.

6% zero coupon bond with par value of $100 million dollars maturing in 10 years. This bond has a book value of $55.8 million dollars (as carried on the NAIC blank). If the company's liabilities consist of one $50 million payment to be made during the next year, it is clear that the company is solvent on a statutory basis. Unfortunately, if interest rates are substantially above 6%, the market value of the bond will be less than the $55.8 million stated and the company, though technically solvent, will be unable to meet its liabilities (a 10% interest assumption implies a current market value of less than $40 million).

A mismatch in cash flows can also cause problems if interest rates fall as demonstrated by this example from [1]. During the early 1980's, transactions in which a premium was paid today in anticipation of a balloon payment at some future date were quite common. Even in this situation, a mismatch between asset and liability cash flows could have dire consequences. For example, consider a transaction performed in 1982 in which a $100 million payment in 2002 was to be funded by a single 12% bond (purchased in 1982). The present value of this payment (at 12%) was approximately $10.4 million in 1982. If this was a zero coupon bond, the asset and liability cash flows match each other and no problems arise. If, however, this is not a zero coupon bond, only some $24.9 million in interest will be provided by the coupons of this bond, which leaves a required $64.8 million in interest to be earned on reinvestment (in other words, when this transaction was made, over 70% of the anticipated investment

income was in the form of interest on the interest). A change in interest rates will have a major impact on "the" present value of this liability due to the mismatch in cash flows.

Consequently, not only should investment strategy recognize the necessity of matching asset and liability cash flows, but the reserving actuary must consider this match (or mismatch) when determining a company's required loss reserves.

The remainder of this note considers a simple technique for discounting loss reserves in terms of this match (mismatch).

We begin by considering the prominent insurer, Unbelievable Re. Although insolvent on a statutory basis management has been approached about possible sale of the company. Our goal is to determine the present value of this company's unpaid loss liabilities by comparing asset and liability cash flows and determining the degree of mismatch. This will produce a present value of unpaid losses which is supportable by the assets of the company (as opposed to being based on some selected rate of interest).

Exhibit A, Sheet 1 displays the Unbelievable bond portfolio. Since Unbelievable Re has cash holdings of $600,000, their total assets as at 12/31/86 are $6,841,361. If we turn to Sheet 2 of Exhibit A, we see that unpaid losses at 12/31/86 total $7,500,000 from which it is easy to conclude that Unbelievable Re is insolvent. [We will assume that the estimated unpaid

losses, together with the projected payments have been determined in a suitably conservative manner. Discounting optimistic reserve estimates is not recommended.]

If however, we examine column (5) of this same Exhibit it is easy to see that 1989 is the only year in which Unbelievable Re is unable to meet the required loss payments. Obviously, it is a simple matter for this company to sell some assets in 1989 in order to meet their liabilities. As an alternative to selling assets they could borrow money to meet those liabilities and repay the loan from future surpluses.

In addition, Unbelievable Re could, of course, invest any surplus cash and apply any interest earned towards meeting the liabilities. (This situation is shown in column (6) with a reinvestment rate of 5% and a borrowing rate of 10% -- for the sake of simplicity, we have assumed that all transactions take place in mid year.)

Consequently, the "value" of Unbelievable Re at 12/31/93 is $1,489,016 which is equivalent to $927,285 at 12/31/86 (at 7%). As a result, one could argue that the discounted present value of the losses is $5,914,076 (i.e., $6,841,361 - $927,285) as at 12/31/86. Traditional discounting techniques would require a discount rate of approximately 9.7% to obtain this answer.

The 7% used to calculate the present value of $927,285 above was selected as being easily obtainable over the seven year period

12/31/86-12/31/93. It would not be unreasonable to utilize a rate obtainable on a seven year bond -- however, it should be noted that the higher the rate used, the larger the present value of the liabilities as at 12/31/86.

We remark that, if Unbelievable Re is a going concern, they would probably choose to fund any shortfall in cash flows by means of current premium writings rather than borrowing money. Although this method eliminates any "borrowing cost", it only serves to replace it by an "opportunity cost" which is, in some sense, just as real. Such an opportunity cost, if not recognized, has the effect of transferring costs from one time period to another or from loss lines to profit lines, etc.

However, things are not always quite so straight forward. The choice of reinvestment and borrowing rates is not simply an academic exercise. Depending on the degree of mismatch between assets and liabilities (and which cash flow "matures" first) these rates can have a substantial effect on the outcome of our discounting exercise.

For example, our second company, Mythical Re has attempted to perform a similar exercise. Exhibit B shows the cash flow of the Mythical portfolio together with the corresponding liability cash flow. [Since construction of the asset cash flow is a largely mechanical exercise we have omitted that exhibit in this case.]

Mythical Re has assumed that money can be reinvested at 5% and borrowed at 9% and have arrived at a final position in column (6) which shows they are solvent. Or are they?

Sheet 2 shows the situation where money must be borrowed at 10% - in this case Mythical Re is insolvent. One response to this scenario is, of course, that future investment rates may well be higher than 5%. Sheet 3 shows that a reinvestment rate of 7% still does not suffice to support Mythical Re's liabilities. Sheet 4 summarizes a variety of interest rate scenarios.

It is important to note that the solvency of Mythical Re is more dependent on the borrowing rate than on the investment rate - not an unexpected result of the substantial early negative cash flows.

It is apparent that the choice of reinvestment rate and borrowing rate can be crucial to this exercise. If, as in the case of Unbelievable Re, the asset and liability cash flows are relatively well matched, it is clear that the selection of rates is not as critical as in a case, such as Mythical Re, where the asset and liability cash flows differ widely.

One distinct advantage of this technique lies in the fact that our assumed reinvestment and borrowing rates apply only to the cash surpluses and shortfalls and not to the actual loss payments themselves. Since, in general, these differences are smaller than the actual loss payments and the differences are

being "brought forward" only one year at a time, it follows that the resulting present value is less sensitive to the selected interest rates than in the traditional methodology.

In practice, there are additional complications. The first that comes to mind is reinsurance and the time lag associated with receiving payments from reinsurers. Exhibit C expands upon the Unbelievable Re scenario already Treated in Exhibit A. Here we have assumed that Unbelievable Re actually retains a 50% quota share and that recoveries from the reinsurer are received, on the average, six months after Unbelievable remakes the initial payment. In order to make Exhibit C and Exhibit A as directly comparable as possible we have assumed that all cash flows occur at the same time as in Exhibit A (i.e., all "primary" transactions take place on June 30 and all reinsurance transactions take place on December 31).

The remainder of the spreadsheet is similar to Exhibit A.

Another obvious complication arises when the assets of the company are not entirely in the form of bonds. If the assets are wholly or largely cash the above exercise with a conservative new money rate reduces to the traditional net present value exercise. If the assets are composed of stocks or other investments with uncertain dividend or maturity schedules the exercise is more complicated and may, in fact, be impractical.

One other observation has to be made. The present value of the losses as determined by this method is based on the present value of the assets, i.e., the present value of the liabilities equals the present value of the asset less the present value of the cash remaining when all payments have been made and all the assets redeemed. Consequently, if bonds were valued on a different basis (market value perhaps) the present value of the losses would change by an equal amount.

The principal advantages of the methodology we have discussed over the traditional present value technique are:

(1) the present value of the outstanding losses is determined in terms of the company's existing asset portfolio (and, in the process, we can whether the company's assets will meet its liabilities);

(2) it is not necessary to select a discount rate in the traditional sense;

(3) since reinvestment and borrowing rates apply only to the cash shortfall and surplus at a given point in time rather than the entire payment stream, the technique is less sensitive to the selected interest rates and is based on a more realistic model than the "traditional" approach to discounting;

(4) demonstrates the degree to which asset maturities match the projected future payments. (A statement to this effect is required in Pennsylvania under the new regulations for workers' compensation.)

BIBLIOGRAPHY

1. Ferguson, Ronald E., Duration, PCAS, Volume LXX, Page 265.

UNBELIEVABLE RE
Cash Flow of Assets

Maturity Year	Maturity Date	Bond	Par Value	Book Value @ 12/31/86	Monthly Interest Receivable	Interest and maturity payments 1987	1988	1989	1990	1991	1992	1993
1987	30-Jun-87	Bond Number 1	200,000	183,486	0	200,000	0	0	0	0	0	0
1988	01-Mar-88	Bond Number 2	532,000	577,886	5,542	66,500	543,083	0	0	0	0	0
1990	15-Feb-90	Bond Number 3	400,000	411,000	3,917	47,000	47,000	47,000	407,833	0	0	0
1989	16-Oct-89	Bond Number 4	200,000	188,000	1,208	14,500	14,500	212,083	0	0	0	0
1990	15-Nov-90	Bond Number 5	400,000	438,000	3,667	44,000	44,000	44,000	440,333	0	0	0
1990	15-Jul-90	Bond Number 6	200,000	209,500	2,271	27,250	27,250	27,250	213,625	0	0	0
1988	16-Feb-88	Bond Number 7	800,000	673,344	0	0	800,000	0	0	0	0	0
1989	16-Feb-89	Bond Number 8	320,000	247,099	0	0	0	320,000	0	0	0	0
1992	16-Feb-92	Bond Number 9	750,000	447,200	0	0	0	0	0	0	750,000	0
1991	16-Feb-91	Bond Number 10	300,000	194,979	0	0	0	0	0	300,000	0	0
1993	15-Sep-93	Bond Number 11	200,000	215,500	2,313	27,750	27,750	27,750	27,750	27,750	27,750	220,813
1991	19-May-91	Bond Number 12	400,000	451,000	3,625	43,500	43,500	43,500	43,500	418,125	0	0
1989	15-Apr-89	Bond Number 13	500,000	580,626	5,990	71,875	71,875	517,969	0	0	0	0
1987	15-Jul-87	Bond Number 14	200,000	206,000	1,833	213,750	0	0	0	0	0	0
1990	10-Feb-90	Bond Number 15	800,000	566,740	0	0	0	0	800,000	0	0	0
1987	31-Jul-87	Bond Number 16	200,000	211,000	2,250	218,000	0	0	0	0	0	0
1991	28-Nov-91	Bond Number 17	400,000	440,000	4,292	51,500	51,500	51,500	51,500	447,208	0	0
			6,802,000	6,241,361	36,906	1,025,625	1,670,458	1,291,052	1,984,542	1,193,083	777,750	220,813

Note: (a) The cash flow is determined as the monthly interest until maturity plus the maturity amount.

UNBELIEVABLE RE

Comparison of asset cash flow and liability cash flow.

Year	Asset Cash Flow	Liability Cash Flow	Net Cash Flow (2)-(3)	Accumulated Cash Position 0% interest	Cash Position Including Reinvestment & Borrowing	NPV of Final Position at 7.0%
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Cash	600,000		600,000	600,000		927,284
1987	1,025,625	1,536,678	(511,053)	88,947	103,764	
1988	1,670,458	1,662,724	7,734	96,681	116,686	
1989	1,291,052	1,443,958	(152,905)	(56,224)	(30,385)	
1990	1,984,542	967,424	1,017,118	960,893	983,694	
1991	1,193,083	798,581	394,502	1,355,396	1,427,382	
1992	777,750	629,739	148,011	1,503,407	1,646,761	
1993	220,813	460,896	(240,084)	1,263,323	1,489,016	
Totals	8,763,323	7,500,000	1,263,323			

Note: (a) We have assumed a reinvestment rate of 5% and a borrowing rate of 10%.

Mythical Reinsurance
Cash Flow of Assets maturing after December 31, 1986

Exhibit B
Sheet 1

Year	Asset Cash Flow	Required Loss Payments	Net Cash Flow (2)-(3)	Cummulative Cash Position	Cash Position with Interest	NPV of Final Position at 5.0%
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Cash	1,191,898		1,191,898	1,191,898	1,191,898	7,016
1987	2,201,144	2,412,712	(211,568)	980,330	1,009,764	
1988	2,151,367	2,614,306	(462,939)	517,391	597,313	
1989	1,349,573	2,264,418	(914,845)	(397,454)	(287,666)	
1990	685,125	1,502,266	(817,141)	(1,214,595)	(1,130,697)	
1991	626,667	1,232,224	(605,557)	(1,820,152)	(1,838,017)	
1992	1,300,417	962,184	338,233	(1,481,919)	(1,665,205)	
1993	146,125	692,142	(546,017)	(2,027,936)	(2,361,091)	
1994	2,008,355	422,102	1,586,253	(441,683)	(987,336)	
1995	740,667	225,042	515,625	73,942	(560,571)	
1996	750,750	152,060	598,690	672,632	(12,333)	
1997	25,443	0	25,443	698,075	12,000	
Totals	13,177,531	12,479,456	698,075			

Note: We have assumed a reinvestment rate of 5% and a borrowing rate of 9%.

Mythical Reinsurance
Cash Flow of Assets maturing after December 31, 1986

Exhibit B
Sheet 2

Year	Asset Cash Flow	Required Loss Payments	Net Cash Flow (2)-(3)	Cummulative Cash Position	Cash Position with Interest	NPV of Final Position at 5.0%
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Cash	1,191,898		1,191,898	1,191,898	1,191,898	(68,572)
1987	2,201,144	2,412,712	(211,568)	980,330	1,009,764	
1988	2,151,367	2,614,306	(462,939)	517,391	597,313	
1989	1,349,573	2,264,418	(914,845)	(397,454)	(287,666)	
1990	685,125	1,502,266	(817,141)	(1,214,595)	(1,133,574)	
1991	626,667	1,232,224	(605,557)	(1,820,152)	(1,852,488)	
1992	1,300,417	962,184	338,233	(1,481,919)	(1,699,504)	
1993	146,125	692,142	(546,017)	(2,027,936)	(2,415,471)	
1994	2,008,355	422,102	1,586,253	(441,683)	(1,070,765)	
1995	740,667	225,042	515,625	73,942	(662,217)	
1996	750,750	152,060	598,690	672,632	(129,749)	
1997	25,443	0	25,443	698,075	(117,281)	
Totals	13,177,531	12,479,456	698,075			

Note: We have assumed a reinvestment rate of 5% and a borrowing rate of 10%.

Mythical Reinsurance

Cash Flow of Assets maturing after December 31, 1986

Exhibit B
Sheet 3

Year	Asset Cash Flow	Required Loss Payments	Net Cash Flow (2)-(3)	Cumulative Cash Position	Cash Position with Interest	NPV of Final Position at 7.0%
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Cash	1,191,898		1,191,898	1,191,898	1,191,898	(8,048)
1987	2,201,144	2,412,712	(211,568)	980,330	1,021,341	
1988	2,151,367	2,614,306	(462,939)	517,391	629,896	
1989	1,349,573	2,264,418	(914,845)	(397,454)	(240,857)	
1990	685,125	1,502,266	(817,141)	(1,214,595)	(1,082,083)	
1991	626,667	1,232,224	(605,557)	(1,820,152)	(1,795,849)	
1992	1,300,417	962,184	338,233	(1,481,919)	(1,637,200)	
1993	146,125	692,142	(546,017)	(2,027,936)	(2,346,937)	
1994	2,008,355	422,102	1,586,253	(441,683)	(995,378)	
1995	740,667	225,042	515,625	73,942	(579,291)	
1996	750,750	152,060	598,690	672,632	(38,530)	
1997	25,443	0	25,443	698,075	(16,940)	
Totals	13,177,531	12,479,456	698,075			

Note: We have assumed a reinvestment rate of 7% and a borrowing rate of 10%.

Mythical Reinsurance
Comparison of results at a variety of interest rates

Reinvestment Rate	Borrowing Rate	Will Assets Meet Liabilities
5.0%	9.0%	Yes
5.0%	10.0%	No
7.0%	10.0%	No
7.0%	12.0%	No
8.0%	10.0%	Yes
10.0%	12.0%	No
4.0%	9.0%	No
4.0%	8.0%	No

UNBELIEVABLE RE

Exhibit C

Comparison of asset cash flow and liability cash flow.

Transaction Date	Asset Cash Flow	Primary Liability Cash Flow	Reinsurance Recovery	Net Cash Flow (2)-(3)	Accumulated Cash Position 0% interest	Cash Position Including Reinvestment & Borrowing	NPV of Final Position at 7.0%
	(1)	(2)	(3)	(3a)	(4)	(5)	(6)	(7)
Cash @12/86	600,000			600,000	600,000		726,875
6/87	1,025,625	3,073,356		(2,047,731)	(1,447,731)	(1,418,446)	
12/87			1,536,678	1,536,678	88,947	49,000	
6/88	1,670,458	3,325,448		(1,654,990)	(1,566,043)	(1,604,781)	
12/88			1,662,724	1,662,724	96,681	(20,384)	
6/89	1,291,052	2,887,915		(1,596,863)	(1,500,182)	(1,618,242)	
12/89			1,443,958	1,443,958	(56,225)	(253,269)	
6/90	1,984,542	1,934,848		49,694	(6,531)	(215,937)	
12/90			967,424	967,424	960,893	740,948	
6/91	1,193,083	1,597,162		(404,079)	556,814	355,166	
12/91			798,581	798,581	1,355,395	1,162,518	
6/92	777,750	1,259,478		(481,728)	873,667	709,498	
12/92			629,739	629,739	1,503,406	1,356,759	
6/93	220,813	921,792		(700,980)	802,426	689,284	
12/93			460,896	460,896	1,263,323	1,167,202	
Totals	8,763,323	15,000,000	7,500,000	802,426	2,261,518		

Note: (a) We have assumed a reinvestment rate of 5% and a borrowing rate of 10%.
(b) For simplicity of presentation we have assumed all interest and principal is received on 6/30.
(c) We have assumed that primary loss payments are made at 6/30 and recoveries are received at 12/31.