

EVALUATING CONTINGENT PREMIUM LIABILITIES

FOR EXCESS-OF-LOSS SWING PLANS

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BIOGRAPHY:

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ABSTRACT:

Swing Plans - or retrospectively rated contracts with maximum and minimum final premiums - are commonplace in excess-of-loss reinsurance treaties. Key provisions include the provisional premium, maximum and minimum premiums, an aggregate deductible, the loss conversion factor for claims in the reinsured layer, and the attachment point itself, which is usually indexed. It is not uncommon to find an attachment point for multiple defendant cases which is different than that for single defendant claims. The minimum premiums can operate in one of two ways - the "Subject to" minimum or the "Minimum Plus" variation.

Swing plans create a contingent premium liability for the reinsured carrier. Using medical professional liability as the illustrative line of business, this paper's objective is to (a) review the basic retro-rating fundamentals to develop a "probable final" swing premium concept and (b) derive an approximation to the underlying excess layer probability distribution, using a Monte Carlo model, to be employed in the determination of the probable final premium.

THE SWING-RATED EXCESS-OF-LOSS REINSURANCE CONTRACT

Of the many optional terms which can be included in an excess-of-loss reinsurance contract,¹ a "swing", or retrospective, rating feature is one which appears to be very prevalent in today's market, particularly for those treaties negotiated with the London segment. In basic terms, with a swing-rated contract the reassured tenders a deposit, or provisional, premium to the reinsurer of the excess loss layer in question, and then, at some future time designated in the contract, a final premium is established as a function of the actual losses incurred in the reinsured layer, subject to a maximum premium and also (in one way or another) to a minimum premium.

The existence of a retrospective rating provision in the excess-of-loss contract creates for the reassured company a contingent liability for additional future premium applicable to the current policy period, representing some part of the difference between the maximum swing premium and the provisional premium. The objective of this paper is to set forth a procedure to determine what "part" of that difference should logically be set up by the reassured company as a liability. More specifically, our objective is to estimate the probable final swing premium for a given set of reinsurance parameters. This procedure will be based on (a) generally recognized relationships which are common to all retro rating actuarial problems, and, more importantly, (b) the development of a Monte Carlo simulation model to approximate the underlying probability distribution of losses in a defined excess layer.

TYPICAL PROVISIONS IN SWING-RATED CONTRACTS

As an illustrative line of business, this paper will use Medical Professional Liability. This line was chosen because (a) for many of the doctor-owned companies formed during the past 10-12 years, the

¹ A full description of excess-of-loss reinsurance will not be attempted in this paper. For such a discussion, the author recommends LeRoy J. Simon, "The Excess of Loss Treaty in Casualty Insurance," in *Reinsurance* (New York: The College of Insurance, 1980, R. W. Strain, Ed.) p.213.

excess-of-loss reinsurance contract was and is the *sine qua non* for their maintaining the capacity to write liability coverage for their medical society members, and (b) there does not appear, from this author's perspective, to be a strong consensus among these companies in the methods used to account for the accompanying contingent premium liability.

To illustrate some of the typical provisions of an excess-of-loss contract with Lloyd's, we are reproducing in Appendix A a "Cover Note" summary of contract provisions, prepared by London brokers.² For some of the provisions, we have supplied optional language in brackets, which will be referred to later.

From the standpoint of our objectives in this paper, there are several key provisions in the the typical excess-of-loss contract which will have a large bearing on the construction of a simulation model to estimate contingent swing premium liability, as follows:

Indexing. The indexing of the excess layer attachment point is a device which was initiated by reinsurers in an attempt to neutralize the adverse effects of inflation on reinsured layers, especially in long tailed lines.³ With the inclusion of an index clause, the attachment point becomes a variable which is dependent not only on the size of the random claim but also on the **calendar year of payment**. When one considers the strong correlation between calendar year of payment and the size of the claim, as will be brought out later, one has to immediately start thinking about splitting the accident year and report year into calendar year pieces.

Per-occurrence vis-a-vis per-policy retentions. Excess-of-loss contracts typically set forth the attachment point on a per-occurrence basis

² We wish to thank the London brokerage firm of Ballantyne, McKean & Sullivan, Ltd. for permitting us to use some of their sample cover note language as illustrations. The actual contracts would contain language which would be extraneous for purposes of this paper.

³ For a full discussion on how indexing works, see R. E. Ferguson, "Non-Proportional Reinsurance and the Index Clause," PCAS LXI (1974), P. 141.

(all defendants in one incident) or specify the reassured's retention per occurrence separately from the retention per policy (defendant). Sometimes the retention per occurrence is the same as per defendant and other times it may be some multiple of the per-policy retention. In any event, the utilization of the two bases for retention -- per policy and per occurrence -- leads to some recognition of the distribution of the number of defendants per occurrence in the design of the excess loss simulation model.

The claims-made form. The predominance of the claims-made form in today's medical professional market brings the focus of attention to the year that claims are **reported**, in addition to the year that they are incurred. Since, as will be brought out later, there is a correlation between the incurred-to-report lag time and the size of the claim, it would follow that our simulation model should incorporate the report year pattern of any accident year sampled.

Allocated loss adjustment expense (ALAE) provisions. Most excess-of-loss contracts provide that ALAE on a claim (occurrence) is recoverable "pro rata," i.e., the percentage of the ALAE which is recoverable in a claim is the same as the percentage of the gross indemnity amount which is recoverable. Some contracts (relatively infrequent) set forth a retention level based on the **sum** of the indemnity and ALAE for one claim. In any case, the interaction between ALAE and indemnity would be an important consideration in our simulator. Treating ALAE as a constant percentage of indemnity (like tax and gratuity) would clearly not reflect the real world.

Aggregate deductibles. As the sample cover notes illustrate, many excess of loss contracts include a provision whereby the reinsured company retains the first X dollars of any losses in the policy term in the defined reinsured layer. Although this provision would have no direct bearing in the design of the excess loss simulation model (which deals with losses in a defined layer, gross of any internal aggregate), the aggregate deductible does play a role, obviously, in the final estimation of the contingent swing premium liability.

Maximums, minimums, and loss conversion factors. None of these parameters would have any bearing on the simulated distribution of losses in a defined excess loss layer, but they undoubtedly have a significant impact on the estimation of the contingent premium liability. As noted in the sample cover notes in Appendix A, there have evolved two distinct variations in the way that minimum premiums are applied in excess-of-loss contracts, which can be described as follows:

(a) **"Subject to" minimums.** In this variation, the final premium is determined by the losses in the layer, loaded by the loss conversion factor, **subject to** the minimum and maximum premium. In other words, if the loaded losses were less than the minimum premium, the minimum would apply.

(b) **"Minimum Plus."** In this second variation (which seems to be replacing the "subject to" provisions on most current contracts) the final premium is determined as the loaded losses **plus** the minimum premium, the sum of which is subject to the maximum premium. Normally, the loss conversion factor is lower for "minimum plus" contracts than for "subject to" contracts.

EXPECTED FINAL SWING PREMIUMS: BASIC MATHEMATICS

The "subject to" minimum option

The mathematical expressions representing the probable final premium in conjunction with swing plan excess-of-loss reinsurance treaties are fairly straightforward. A quick tour through these relationships will establish the foundation upon which we will later lay the necessary building blocks, one step at a time. First, starting with the "subject to" minimum premium option,

let $f(x)$ = probability density function of excess losses
in the layer in question

and

$$\int_0^{\infty} f(x)dx = 1 .$$

Then

let M_1 = Minimum premium
 M_2 = Maximum premium
 C = Loss conversion factor
 A = Aggregate deductible .

Then

let P = final swing premium
 $= C*(X-A)$
where X = excess loss amount
and $M_1 < P < M_2$.

Further

let L_1 = excess loss level corresponding to M_1
 L_2 = excess loss level corresponding to M_2 .

Since, by definition

$$C*(L_1-A) = M_1 \quad \text{and} \\ C*(L_2-A) = M_2$$

then

$$L_1 = M_1/C + A \quad \text{and} \\ L_2 = M_2/C + A \quad .$$

It follows, then, that the final probable swing premium can be expressed as the sum of three pieces:

(a) when the minimum premium applies,

$$M_1 \int_0^{L_1} f(x)dx ,$$

(b) when losses fall between the minimum and maximum loss levels,

$$\int_{L_1}^{L_2} C(X-A)f(x)dx ,$$

(c) when the maximum premium applies,

$$M_2 \int_{L_2}^{\infty} f(x)dx .$$

Thus,

$$E[P] = M_1 \int_0^{L_1} f(x)dx + \int_{L_1}^{L_2} C(X-A)f(x)dx + M_2 \int_{L_2}^{\infty} f(x)dx .$$

The "Minimum Plus" Variation

For the "minimum plus" option, the mathematics is a little different. The final premium is defined as the converted losses (after the aggregate deductible) plus the minimum premium, and the sum of these two quantities is subject to the maximum premium. Thus,

P = final swing premium

$$= M_1 + C*(X-A)$$

where X = excess loss amount

and $P < M_2$.

Further, in terms of our original definitions for the "subject to" option, the L_1 factor effectively becomes zero and

$$L_2 = (M_2 - M_1) / C + A .$$

Then

$$E[P] = \int_0^{L_2} [M_1 + C(X-A)]f(x)dx + M_2 \int_{L_2}^{\infty} f(x)dx .$$

Clearly, no matter which variation of the minimum premium is employed, the key ingredient in the expression for E[P] is the excess loss p.d.f., f(x), on which we will now focus our attention.

THE INDEMNITY SIZE OF LOSS DISTRIBUTION

The NAIC Closed Claim Studies

The nucleus of our procedure to determine contingent excess-of-loss swing premiums is the distribution of indemnity amounts (from ground up, with no limit) for one accident year. Using medical professional liability as the line of business in question, we referred to the NAIC closed claim study.⁴ For this study, some 75,000 claims closed during the period 1975-78 were recorded. Among many other items of information, the accident dates, report dates, closed dates, and indemnity and ALAE amounts were included.

It has been shown by many researchers⁵ that, in order for any calendar year closed claim distribution to accurately represent the claim-size distribution applicable to an accident year, some trending adjustments are necessary for both claim frequency and claim severity. For this author's model claim-size distribution, we first devised annual indices of claim severity and frequency (both accident year) from available national data covering a period of about 20 years up to calendar year 1978 (the final

⁴ National Association of Insurance Commissioners, **NAIC Malpractice Claims**, 1980.

⁵ See, for example, Archer McWhorter, Jr., "Drawing Inferences from Medical Malpractice Closed Claim Studies", **The Journal of Risk and Insurance**, XLV, no. 1 (March, 1978) and Michael R. Lamb, "Uses of Closed Claim Data for Pricing," **Pricing Property and Casualty Insurance Products**, 1980 C.A.S. Discussion Paper Program, p. 219.

closing year of the study). The frequency and severity indices for each year were then expressed in terms of the 1978 index equal to 1.0. Then to each detail claim record,⁶ based on the accident date, we applied the reciprocal of the frequency index to the claim count (1 per record, initially) and the reciprocals of both the frequency and severity indices to the indemnity and ALAE amounts. As a result of this exercise, we produced a claim size distribution adjusted to represent the **accident year 1978**.

A printout of the trend-adjusted claim size distribution (indemnity) is shown in Appendix B, page 1. The brackets of indemnity size are set up on logarithmic (geometric) scale, with the end point of each bracket a constant factor (about 1.3335) times the end point of the previous bracket. A plot of the histogram for the non-zero members of this adjusted distribution is displayed on page 2 of Appendix B. The cumulative distribution ogive is then plotted on page 3. But the most revealing and useful plot of this accident-year adjusted distribution is shown on pages 4-5, on which we have plotted the cumulative distribution on lognormal probability graph paper, the grids of which are constructed so that the cumulative distribution ogive of a lognormal probability distribution is a straight line.

The lognormal model has been used extensively to represent claim size distributions in property and casualty lines.⁷ Finger, in particular, used the lognormal model to determine implied increased limit factors for medical professional liability. It would follow, then, that the lognormal would be a good candidate to investigate for modelling losses ceded in excess-of-loss reinsurance treaties.

On the first page of our cumulative distribution graph (claims up to \$100,000), the lognormal fit -- a straight line drawn through the points

⁶ In addition to referring to the hard-copy NAIC report, we also purchased the detail data tape from the association.

⁷ See, for example, Charles C. Hewitt, Jr., "Credibility for Severity," *PCAS*, LVII (1970), p. 148; David R. Bickerstaff, "Automobile Collision Deductibles and Repair Cost Groups: the Lognormal Model," *PCAS*, LIX (1972), p. 68; and Robert J. Finger, "Estimating Pure Premiums by Layer -- an Approach," *PCAS*, LXIII (1976), p. 34.

strictly by sight -- clearly is good enough to represent the actual data. On the continuation of the distribution on page 5, it can be noted that for values above about \$500,000 the actual data points veer out above the hand-selected lognormal line. There is a very plausible explanation for this. If the lognormal model does in fact provide a good representation of the claim size distribution with no limit, then the imposition of policy limits on the bigger claims in the data base itself would have had a dampening effect on the relative frequency of these claims in the higher, potentially excess, layers. It can be approximated from the graph, for example, that the extension of the lognormal line would indicate a frequency of claims in the \$2 million plus range about 4 to 5 times greater than the actual data points would indicate. For this reason, more than any other, this author disdained any idea of walking through a rigorous, analytical curve-fitting choreography, which would have generated a "best fitting" line that understates the potential for big claims.

The selected lognormal parameters for indemnity

We estimated a mean and variance from our fitted lognormal claim size distribution by marking off the median and standard deviation directly from the graph, using the 50 percentile and +1 standard deviation marks on the vertical scale, as follows:

$$\text{Observed median} = e^{\mu} = 10650 .$$

$$\text{Observed } \sigma = \log_e(68000) - \log_e(10650) = 1.853$$

Our final selected value for the mean is, then

$$\exp(\log_e(10650) + (1.853)^2/2) = 59300 .$$

The coefficient of variation (standard deviation divided by the mean) of the fitted distribution is calculated as follows:

$$\begin{aligned} (CV)^2 &= e^{\sigma^2} - 1 \\ &= 29.988 . \end{aligned}$$

Thus, for future modelling purposes, we set the CV value = $\sqrt{30}$.

Working Size of Loss Model for Indemnity

The absolute values of the 1978 NAIC closed claim distribution, even after adjusting for frequency and severity trends, are not particularly important to us - especially in 1987-88. The **shape** of the adjusted, fitted distribution is the key parameter, measured by the CV. We believe that it is reasonable to assume that as the average unlimited indemnity increases over time or from one territory to another, the $(CV)^2$ should remain relatively constant. This also implies that as the **average** unlimited mean increases k percent from one point in time to another, it is reasonable to expect that the entire distribution of claims moves up about k per cent. Put another way, an \$800,000 claim has about the same relative niche in a distribution whose unlimited mean is \$100,000 as a \$400,000 claim in a distribution with half the unlimited mean.

Our working indemnity distribution can, then, be represented by a lognormal distribution whose unlimited mean is 1.0 and whose $(CV)^2$ is 30, as shown in page 6 of Appendix B. The top line represents the basic distribution of claims by size and the bottom line depicts the **first moment** distribution.⁸ To illustrate how this graph is read, from the top line one can note that about 82.5% of all claims are less than or equal to the mean and about 96.5% of the claims are less than or equal to five times the mean. From the bottom line, one can further note that about 18% of the total **dollars** in the distribution come from claims which are less than or equal to the mean and about 47% of the dollars from claims below five times the mean.

Generation of random claim amounts from lognormal model

To tabulate sample claims from the lognormal distribution, our Monte Carlo model employs a random number generator which generates **normal** random

⁸ For a discussion of moment distributions and other attributes of the lognormal distribution, see J. Aitchison and J. A. C. Brown, **The Lognormal Distribution**, (Cambridge University Press, 1969).

numbers.⁹ The sample random claim size (indemnity) is determined from the following formula:

$$X = \exp(\mu + N\sigma)$$

where μ = mean of the logs of the distribution
 σ = S.D. of " " " "
 N = normal random number (mean 0, var. 1) .

From the basic relationships of the lognormal distribution,

$$M = \exp(\mu + \sigma^2/2)$$

where M = mean of the distribution .

Then we have

$$\mu = \log_e(M) - \sigma^2/2$$

and then the sample claim would be generated with

$$X = \exp(\log_e(M) - \sigma^2/2 + N\sigma) .$$

REPORT YEAR / CALENDAR YEAR STRATIFICATION OF ACCIDENT YEAR

The use of indexed attachment points, the claims-made form, the well-recognized correlation between payment lag and payment size, and other considerations related to the typical excess-of-loss treaty have led us to introduce a form of stratification in the sampling of medical professional claim amounts. To accomplish this, we first set forth some basic relationships between report year and calendar year severities, within the accident year:

⁹ A full discussion of random number generation is beyond the scope of this paper. For further reference, we recommend G. S. Fishman, **Principles of Discrete Event Simulation** (New York: John Wiley & Sons, 1978), chap. 8-9.

Let $R(i)$ = Frequency of claims reported in report year i of acc. year,
relative to total accident year

$C(j)$ = Freq. of claims of one rep. year paid in cal. year j ,
relative to total report year

S_1 = Severity of claims of report year i , relative to total
accident year severity

T_j = Severity of claims of calendar year j , relative to
total severity of report year

n = total report years in accident year

m = total calendar year's payout for each report year

Then you have

$$\sum_{i=1}^n R(i) = 1$$

$$\sum_{j=1}^m C(j) = 1$$

and, by definition,

$$\sum_{i=1}^n S_1 R(i) = 1$$

$$\sum_{j=1}^m T_j C(j) = 1 \quad .$$

The total accident year can then be stratified into $n \cdot m$ report year/calendar year cells. The cell identified by the i th report year and the j th relative calendar year in that report year would have a claim frequency of $R(i) \cdot C(j)$

times the total accident year frequency and a severity of $S_i \cdot T_j$ relative to the total accident year severity. It also holds that the mean severity over all $n \cdot m$ cells is

$$\sum_{i=1}^n \sum_{j=1}^m S_i T_j C(j) R(i) = 1$$

Since the above mean = 1, the coefficient of variation squared over all $n \cdot m$ cells is:

$$C^2 = \sum_{i=1}^n \sum_{j=1}^m [S_i T_j]^2 C(j) R(i) - 1 .$$

Modified CV's for stratified sampling

We earlier developed a model indemnity size-of-loss distribution for an entire accident year, with a CV^2 of 30. But instead of simply sampling indemnity amounts from the entire accident year distribution, our Monte Carlo model will first select (randomly) a report year and then a calendar year paid for each random claim and then, based on the relative severity levels discussed above, sample from an indemnity distribution the mean of which has been adjusted to the levels corresponding to that report year and relative calendar year. Consequently, it becomes necessary to modify the CV applicable to each RY/CY stratum so that when you combine the sampled claims from the various RY/CY cells, you achieve the desired composite accident year $CV^2 = 30$.

To accomplish the desired approximation of the modified CV applicable to each RY/CY cell, we used a method first advanced by Hewitt.¹⁰ He demonstrated that, if (a) a random variable Y were stratified into groups and (b) the means of the groups were lognormally distributed and (c) the

¹⁰ Hewitt, *op. cit.*, Appendix A, p. 167.

variance of the logs of the means were S^2 , and (d) if the variance of the logs of each group were $(\sigma_Y)^2$, (a constant), then the variance of the logs of the combined distribution of all groups would be $S^2 + (\sigma_Y)^2$. The "spread parameter" S^2 over the $n \cdot m$ report year/calendar cells can be determined directly from the C^2 , calculated above:

$$C^2 = e^{S^2} - 1$$

$$S^2 = \log(C^2 + 1) .$$

Thus,

$$\log(C^2 + 1) + (\sigma_Y)^2 = \text{Log}(31)$$

and

$$(\sigma_Y)^2 = \text{Log}(31) - \log(C^2 + 1) .$$

It should be emphasized that the above expression is an "approximation" of the modified variance (of the logs) to be used in the stratified sampling, since some of Hewitt's prerequisites are not necessarily met. Therefore, it is appropriate to perform a test of the stratified sampling, using sample values of $R(i)$, $C(j)$, S_i , and T_j , to determine if the overall accident year CV is achieved within an acceptable tolerance.

Testing the stratified sampling parameters

To determine appropriate values for the distributions of $R(i)$, $C(j)$, S_i , and T_j , we referred again the NAIC closed claim studies. Using the detail NAIC data base, after the frequency and severity trend adjustments, we constructed a report year/calendar year matrix as shown in Appendix C. The entire claim data base, now adjusted to represent an accident year, was stratified into cells defined by ten report years and 16 calendar years (relative to the accident year). Each cell contains the (adjusted) claim counts, amounts, and averages. From the totals by report year, we derived the percentages of total claims by report year and the relative severity for each report year. On pages 5-6 of that same Appendix we determined relative

severity values for calendar years, relative to report years. The values from this matrix will, then, be a starting point to determine the $R(i), C(j), S_i,$ and T_j values for a specific case (it should be pointed out that the actual historical report year and calendar year patterns for a given jurisdiction and company, to the extent that they are credible, should be given more weight than the NAIC numbers).

For this paper's case study, we have selected the report year and calendar year distributions shown in Exhibits 1 and 2 of Appendix C. We have used a total of seven report years ($n = 7$) and seven relative calendar years ($m = 7$). The relative severity factors have been selected (roughly from the NAIC matrix) and then adjusted so that the sums of the products of the frequency times the relative severities are 1.0. The $(CV)^2$ of the cell means,

$$\begin{aligned}
 C^2 &= \sum_{i=1}^n \sum_{j=1}^m [S_i T_j]^2 C(j) R(i) - 1 \\
 &= .2607 .
 \end{aligned}$$

Thus,

$$\begin{aligned}
 (\sigma_Y)^2 &= \text{Log}(31) - \log(1.2607) \\
 &= 3.20232
 \end{aligned}$$

$$\text{and } \sigma_Y = 1.7895 .$$

Thus, while the standard deviation (of the logs) of the entire accident year is $\sqrt{\log(31)} = 1.8531$, the standard deviation applicable to each cell will be reduced to 1.7895.

The results of our test of the stratified sampling versus unstratified is summarized in Appendix D. Rather than sampling from the lognormal distribution with no limit, we sampled successively from distributions with limits of \$50,000, \$100,000, \$500,000, \$1,000,000, \$10,000,000, and \$25,000,000. In each case, the unlimited mean was \$100,000. For each limit,

we (a) calculated the mean and CV directly,¹¹ (b) generated a sample mean and CV from the unstratified distribution, and (c) generated a sample mean and CV using the RY/CY strata with the adjusted means and appropriately reduced variance. To make sure we covered a full spectrum of possibilities, we used three values for (CV)²: 10, 20, and 30. The report year and calendar year distributions were similar, but not identical, to those in Exhibits 1-2 of Appendix C. For each combination, 100,000 claims were sampled.

The test samples demonstrated that the composite means and CV's derived from the stratified process were a good approximation to the direct calculation, within an acceptable tolerance.

THE ALAE COMPONENT IN THE MODEL

The ALAE-Indemnity relationship

It should be emphasized that in our excess claim model the ALAE for the sampled claim is not treated as a constant factor related to the indemnity size, but rather the expected ALAE (mean value of a separate ALAE distribution) is established, given the sample observed value of the indemnity. To treat ALAE otherwise would result in an understatement in the overall variability of the aggregate excess loss distribution.

To determine the functional relationship (if indeed a measurable relationship exists) between ALAE size and indemnity size for medical professional liability claims, we turned again to the NAIC Closed Claim Study.¹² As shown in Appendix E, Page 1, the average ALAE was calculated for each of several brackets of indemnity size. After plotting the average ALAE in each bracket against the corresponding average indemnity for the bracket,

¹¹ The calculation of the moments of a lognormal distribution limited (censored) by some limit L is fairly straightforward but is not covered here.

¹² NAIC, *op. cit.*.

using logarithmic X and Y axes (see Appendix E, page 2), it was observed that a reasonably good straight line fit was obtainable, implying that the ALAE-indemnity relationship was representable by a member of the "power" curve family, $Y=AX^B$.

The equation used to regress the ALAE means with the indemnity values (grouped into brackets) is:

$$\text{Log}_e(Y) = A + B*\text{Log}_e(X).$$

The weighted least squares best fit coefficients, using the number of claims in each indemnity bracket as weights, were

$$A = 3.66331$$

$$B = .482945$$

From the same data base which was used to develop this relationship between average ALAE and indemnity, it was also determined that the average indemnity was \$53,363. Thus,

$$\text{Let } I = \text{average indemnity} = 53363.$$

Then restate the regression formula above by expressing both ALAE and indemnity as a ratio to the average indemnity over the entire distribution, as follows:

$$Y' = Y/I$$

$$X' = X/I$$

Then the restated expression becomes:

$$\text{Log}_e(I*Y') = B*\text{Log}_e(I*X') + A .$$

Simplifying, you get

$$\begin{aligned}\text{Log}_e(Y') &= B \cdot \text{Log}_e(X') + B \cdot \text{Log}_e(I) + A - \text{Log}_e(I) \\ &= B \cdot \text{Log}_e(X') + (B-1) \cdot \text{Log}_e(I) + A.\end{aligned}$$

Then let

$$C = (B-1) \cdot \text{Log}_e(I) + A = -1.964768.$$

You then have

$$\text{Log}_e(Y') = B \cdot \text{Log}_e(X') + C$$

and

$$Y' = e^C X'^B = .1401884 * X'^{.482945}$$

For future reference, we call

$$D = e^C.$$

From the above expression, it can be noted that, in approximate terms, the expected ALAE varies in proportion to the square root of the sample indemnity.

Distribution of ALAE per claim, independent of indemnity

The next step of our treatment of ALAE in the model is to examine the distribution of ALAE per claim (defendant), irrespective of indemnity amounts. To do this, we again investigated the NAIC closed claim study.¹³ The distribution is graphed in Appendix E, page 3. Using lognormal

¹³ For this distribution, we chose, for the sake of conservatism, the earlier 1975 version of the NAIC study, since the plotted CV was higher than that of the 1978 release.

probability graph paper, the near straight line plot of the cumulative distribution function suggests that, just as was the case for the distribution of indemnity values by size, the ALAE amounts also can be represented quite adequately by the lognormal model.

We determined a mean and variance for the ALAE distribution two ways: first, we calculated the mean and variance directly from the data and then we followed the same procedure used for the indemnity graph. After drawing a straight line fit for the cumulative distribution function on the lognormal probability graph paper (the plotted points from the actual data were close enough to a straight line to allow us to simply draw the fitted line free-hand), we "picked off" the median and standard deviation directly from the graph, using the 50 percentile and +1 standard deviation marks on the vertical scale, as follows:

$$\text{Observed median} = e^{\mu} = 1355 .$$

$$\text{Observed } \sigma = \log_e(5200) - \log_e(1355) = 1.345$$

Our final selected value for the mean is, then

$$\exp(\log_e(1355) + (1.345)^2/2) = 3348 .$$

Of more importance, as will become clear later, our selected value for the variance was $(1.345)^2$, or 1.809.

Parameters for conditional ALAE distribution

We established earlier that, for purposes of sampling ALAE for any Monte Carlo simulation model, the expected ALAE in the distribution sampled from will be dependent on the sample indemnity value, or

$$E[Y|X] = DX^B ,$$

where

Y = random variable ALAE, conditional on value of indemnity, X

D = .1401884

B = .482945

and both Y and X are expressed relative to the unlimited mean indemnity.

Aitchison and Brown¹⁴ have shown that if the random variable X is lognormally distributed with parameters μ and σ^2 , then DX^B is also lognormally distributed with parameters $\log(D) + B\mu$ and $B^2\sigma^2$. The parameters are the mean and variance, respectively, of the logs of the random variables.

We now let

$$\begin{aligned} S^2 &= \text{variance of the logs of ALAE means } E[Y|X], \text{ conditional on} \\ &\quad \text{sample indemnity values} \\ &= B^2 \sigma^2 \\ &= (.482956)^2(1.8531)^2 \\ &= .8009 \end{aligned}$$

Again employing Hewitt's method of isolating the "spread parameter",¹⁵ we can solve for the variance applicable to each ALAE "group", $(\sigma_Y)^2$, defined as the sample ALAE given the sample indemnity mean:

We earlier derived an approximation for the combined variance

$$S^2 + (\sigma_Y)^2 = 1.809$$

then

$$\begin{aligned} (\sigma_Y)^2 &= 1.809 - .8009 \\ &= 1 \text{ (approx.)} \end{aligned}$$

In a word summary, then, we have established that the sample ALAE (relative to the unlimited mean indemnity) would be drawn from a lognormal distribution whose mean is $.1401884X^{.482945}$ and the variance of whose logs is 1.0, where X represents the sample indemnity, relative to the unlimited mean indemnity.

¹⁴ *op. cit.*, p. 11

¹⁵ Hewitt, *loc. cit.*

Testing the sampled ALAE values, conditional on sample indemnity

Using the parameters estimated above, a test was set up to randomly sample 100,000 claims to make sure that the resulting overall ALAE sample moments were sufficiently close to those from direct calculations. For all ALAE combined, the coefficient of variation $(CV)_a$ is determined:

$$\begin{aligned}(CV_a)^2 &= e^{S^2 + \sigma^2} - 1 \\ &= 5.104 \\ CV_a &= 2.259 \ .\end{aligned}$$

From our sample of 100,000 claims, the sample CV for ALAE was 2.24363.

THE DEFENDANT-PER-INCIDENT DISTRIBUTION

As pointed out earlier, the distinction between retention per defendant and retention per incident in the typical excess-of-loss contract points to a need to incorporate in any excess loss simulation model a distribution of incidents by number of defendants. (It should be pointed out, also, that the average indemnity used in the model will be on a per-defendant basis.) To produce such a distribution for medical professional liability, we again turned to the NAIC closed claim studies.

In Appendix F we have outlined the results of a special study of the defendant distribution. Starting first with a "universe" of all insurers, we then devised a means of approximating the defendant-per-incident distribution for a given insurer with a "penetration factor" p ($0 < p < 1$), which leads to a ratio of defendants to incidents of $1 + p$. For a given case study, the value of p is selected which is appropriate for the company

in question.¹⁶

THE MONTE CARLO MODEL

Having highlighted the key actuarial considerations in approximating the excess loss probability density function, we are now ready to describe the Monte Carlo model in some detail. The use of Monte Carlo models shows up with increasing regularity in the actuarial literature.¹⁷ But despite the general agreement, in risk theory circles, that Monte Carlo models are an acceptable technique for approximating these distributions, this author perceives that any number of the direct approximation methods¹⁸ are considered superior, assuming that the mean and variance of the distribution can be calculated directly and precisely.

Given all of the interactions between the many variables discussed above -- e.g., the calendar year severity factors and indexed retention, the ALAE-indemnity relationship, the defendant distribution and the retention per occurrence -- this author is hard pressed to identify any direct approximation formula from any risk theory text which will yield adequate results for the defined problem. The use of a Monte Carlo model, in which all of the interactions can be adequately defined and programmed into one composite risk process, would appear to be the only answer.

A full description of our excess-of-loss Monte Carlo model is included in Appendix G. In the first section, we have listed the miscellaneous

¹⁶ The selection of p does not necessarily represent a precise estimate of a company's actual market "penetration," but rather it is selected to incorporate most of the "contagion" phenomena which affect the proportions of multi-defendant claims. In actual practice, we have used a p factor as high as .7 for some of the larger doctor-owned carriers and as low as .3 for the smaller ones.

¹⁷ See, for example, P. E. Heckman and G. G. Meyers, "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions," *PCAS*, LXX (1983), p. 22.

¹⁸ No attempt will be made to provide a list of these methods here.

assumptions, the input parameters, and the various distributions from which samples are made. Since, for this case study, the excess layer pertains to the claims-made year 1987, the exposure, frequency, severity, and policy limit values would be required for the seven accident years 1981-87 (but since 1983 was the company's first year, we use only 1983-87). In the second section the actual simulation process for one trial (normally, at least 1,000 trials are run for a given case study) is outlined in pseudo code. Tracking the program flow through this pseudo code will reveal how the many variables interact with each other.

DERIVATION OF THE PROBABLE FINAL SWING PREMIUM FROM THE LOSS DISTRIBUTION

The Simulated Loss Distribution

With the Monte Carlo model loaded up with the appropriate input parameters and distributions, we can now make the run for the case study at hand. The resulting printout of the distribution, generated from 1,000 trials of the model, is shown in Appendix H, page 1. The results of the 1,000 trials have been tabulated and summarized into 31 intervals of total loss in the defined layer, including the number of "hits" in each bracket and also the total losses in each bracket. For the loss levels corresponding to the minimum and maximum premiums (L_1 and L_2), which are calculated on spreadsheets according to our original formulas, we have interpolated the appropriate values in the distribution.

The histogram of the sample distribution and cumulative distribution ogive are shown on pages 2 and 3. These plots display a fairly smooth and regular contour -- so much so that I am sure that, with enough effort and with an appropriate set of parameters, someone could uncover some exotic probability density function which would supply an acceptable "fit" to this curve. But what purpose would this serve? It would be unlikely that such a curve, or even a member of its immediate family, would adequately fit another case defined by an entirely different set of initial variables (retentions, index factors, unlimited means, defendant-per-incident distribution, etc.). Thus, the final estimated excess loss distribution in Appendix H, generated solely for this one particular case, is simply what it is. It needs no name.

The Probable Final Swing Premium

We have actually defined two sets of reinsurance parameters for the calculation of the final swing premium -- one employing the "subject to" minimum concept and the other using the "minimum plus" formula. The actual excess loss layer, defined by the retentions and the indemnity and ALAE means -- is the same for each option, so the loss distribution $f(x)$ is the same for both options. The parameters are as follows:

	"Subject to"	"Minimum +"
	-----	-----
Gross Net Earned Premium Income	\$15,000,000	\$15,000,000
Maximum Premium (30% of GNEPI)	4,500,000	4,500,000
Minimum Premium	1,500,000	1,125,000
Loss Conversion factor	1.25	1.10
Aggregate Deductible	1,125,000	1,125,000
Provisional Premium (20% of GNEPI)	3,000,000	3,000,000

The final swing premiums for the two options are developed separately in the spreadsheets in pages 4 and 5 of Appendix H. The relationships in the spreadsheets follow the basic mathematical relationships developed earlier. For the "subject to" option, the expected final premium is \$3,958,784. For the "minimum plus" version, the final premium is \$4,196,040. The contingent premium liability, then, would be booked as the difference between these probable final premiums and the provisional premiums.

SUMMARY AND CONCLUSIONS

In this paper we have developed a procedure to estimate the contingent premium liability encountered by reinsured companies in conjunction with swing plan excess-of-loss reinsurance treaties. To determine this liability, the terms of the contract themselves (provisional/minimum/maximum premiums, aggregate deductibles, and loss conversion factors) are combined into formulas which include the probability distribution of the excess losses in the reinsured layer. This distribution is approximated with a

Monte Carlo simulation model, incorporating the interaction of many variables. The resulting swing premium liability estimate is, like other reserve entries on the balance sheet, a "best estimate" or "most likely" entry, and is, therefore, consistent with the rest of the balance sheet.

[Sample Cover Note for Excess of Loss Contract]

REINSURED : XYZ INSURANCE COMPANY
Somewhere, USA

PERIOD : 12 months at 1st January, 1987 to expire 31st December, 1987 covering claims-made and losses occurring on original policies including extended reporting endorsements issued by Reinsured. However, this contract may be renewed for a further period of twelve months until 31st December, 1988 by mutual consent between the parties hereto.

TYPE : EXCESS OF LOSS REINSURANCE.

CLASS : Professional Liability Insurance Policies including Premises Liability issued to Physicians and Surgeons.

COVERAGE : A. The reinsurance coverage will be the difference between \$1,000,000 each loss, each insured and \$250,000 each loss, each insured, indexed as outlined below, plus pro rata loss adjustment expenses.

: B. The reinsurance coverage will be the difference between \$1,000,000 each occurrence and \$375,000 each occurrence, indexed as outlined below, plus pro rata loss adjustment expenses. Section A will inure to the benefit of Section B (e.g., Section A recoveries will be made first, and the Company's accumulated Section A retentions will then be submitted to Section B).

INDEX CLAUSE : A. The initial retention hereunder at the beginning of each calendar year shall be \$250,000 as respects Section A and \$375,000 as respects Section B and shall be increased at each January 1 thereafter by \$25,000 for Section A and \$37,500 for Section B, notwithstanding the date of loss occurrence.

: B. The date of payment of subject losses shall be used in determining the Company's indexed retention.

PREMIUM : The Company shall pay the reinsurer a deposit premium of \$3,000,000 in four equal installments of \$750,000 on January 1, March 31, June 30 and September 30 of the contract year. The rate shall be the following percentages of subject gross net earned premium income cumulative for the adjusted period:

Provisional	20.0%
Developed	125% of Reinsurers' incurred losses for the period

Subject to:

Minimum	10% of GNEPI
Maximum	30% of GNEPI

[Optional Language]

[Minimum	7.5%]
[Provisional	20.0%]
[Maximum	30.0%]
[Developed	7.5% plus 110% of Reinsurers' losses for the period.]

GENERAL CONDITIONS : Service of Suit Clause.
 Insolvency Clause.
 Insolvency Funds Exclusion Clause.
 Ultimate Net Loss Clause (including 80% E.C.O. Loss).
 Excess of Original Policy Limits Clause.
 Commutation Clause - Reassured to have right at any time after 3 years from inception of policy year to commute losses at established reserves and relieve Underwriters of all further liability hereunder provided rate at time below maximum hereon.
 Claims Review Clause. (Costs to be borne by current Reinsurers hereon).
 L.O.C. for difference between actual paid and Maximum Rate.
 If required, Reinsurers to appoint independant actuary to assess rating procedurs of Reassured (Cost to be borne by Reinsurers on current year hereon).
 Pro-rata costs in addition.
 Extended Reporting Endorsements - Any claim made under an Extended Reporting Endorsement shall be deemed to have been made during the term of the original policy to which the Endorsement attached. The premium for such Extended Reporting Endorsements shall be considerd fully earned on the date the Endorsement is issued.
 Date of Loss Clause.

WORDING : To be agreed.

HEREETO : 100%

EFFECTED WITH : Lloyd's Underwriters
XX.XX% London, England
 (Participation hereon percentage part of 100%)

NAIC CLOSED CLAIM DATA BASE - ADJUSTED FOR FREQUENCY/SEVERITY INDICES

Distribution by Size of Loss

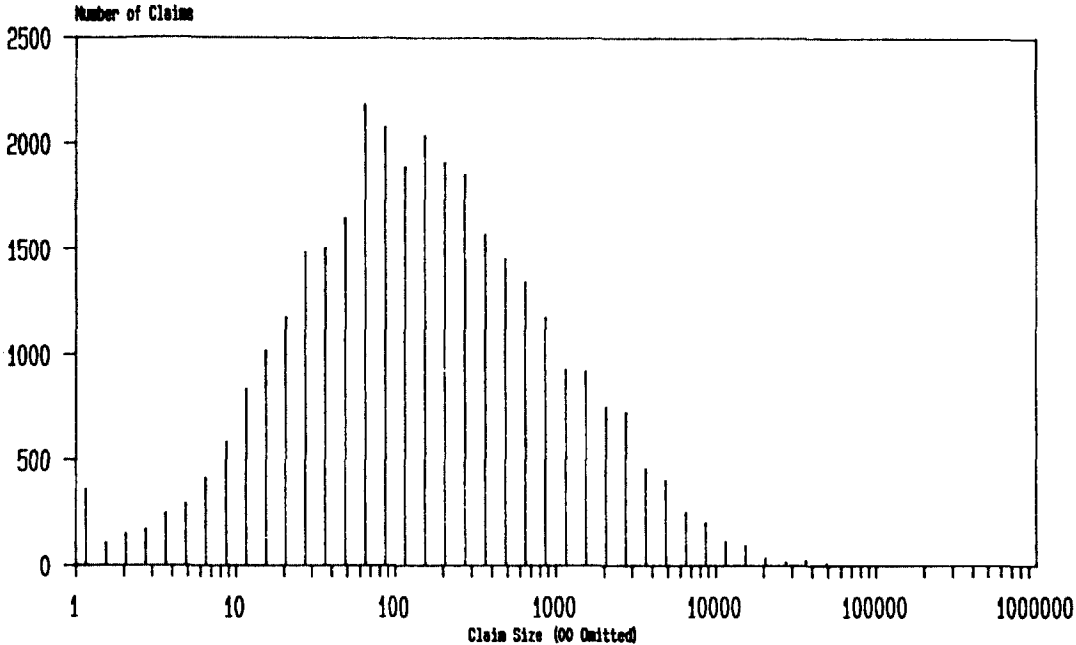
All Claims Combined

Bracket*	# Claims	Cum. # Claims	Indem.Amount	Avg. Indem.	Exp.Amount	Avg.Expense
0	51607.8	51607.8	0	0	133432000	2586
100	358.3	51966.1	18105	51	82011	229
133	103.2	52069.3	11821	115	28022	272
178	145.3	52214.6	22401	154	24138	166
237	167.7	52382.3	34386	205	65789	392
316	242.8	52625.1	67813	279	127067	526
422	292.9	52918.0	108852	372	120612	412
562	411.8	53329.8	201463	489	306647	745
750	581.2	53911.0	379945	654	409760	705
1000	828.3	54739.3	720464	870	767031	926
1334	1015.0	55754.3	1167310	1150	1408230	1387
1778	1170.2	56924.5	1831020	1565	1483850	1268
2371	1477.1	58401.6	3059210	2071	2794090	1892
3162	1499.5	59901.1	4177710	2786	2815350	1878
4217	1640.8	61541.9	6069360	3699	3594630	2191
5623	2180.2	63722.1	10755100	4933	5663140	2598
7499	2071.1	65793.2	13590200	6562	6580210	3177
10000	1884.5	67677.7	16401600	8703	5619610	2982
13335	2029.0	69706.7	23358300	11512	7190910	3544
17783	1906.4	71613.1	29460500	15453	9797740	5139
23714	1848.9	73462.0	37950200	20526	8096010	4379
31623	1564.3	75026.3	42906200	27428	8307880	5311
42170	1448.2	76474.5	53156900	36705	8734200	6031
56234	1340.3	77814.8	65590800	48937	9357350	6982
74989	1171.7	78986.5	76561700	65342	9231510	7879
100000	926.5	79913.0	79771100	86099	7090310	7653
133352	917.8	80830.8	105277000	114706	8637350	9411
177828	746.2	81577.0	114798000	153843	10081600	13511
237137	722.3	82299.3	148033000	204947	10681500	14788
316228	456.1	82755.4	124647000	273289	6077140	13324
421697	402.6	83158.0	145920000	362444	7202570	17890
562341	247.9	83405.9	120768000	487164	4983840	20104
749894	199.7	83605.6	129525000	648598	7204110	36075
1000000	112.6	83718.2	97909200	869531	2094480	18601
1333520	93.3	83811.5	106538000	1141890	2284480	24485
1778280	34.0	83845.5	50086600	1473140	1177000	34618
2371370	15.1	83860.6	30357800	2010450	434327	28763
3162280	22.4	83883.0	62135900	2773920	978374	43677
4216970	4.9	83887.9	19205700	3919530	206093	42060
5623410	0.0	83887.9	0	0	0	0
7498940	0.0	83887.9	0	0	0	0
10000000	0.0	83887.9	0	0	0	0
TOTALS		83887.9	1722570000	20534	295171000	3519
TOTAL, EXCL. CNP'S		32280.1	1722570000	53363	161739000	5011

*End point of interval of indemnity amount

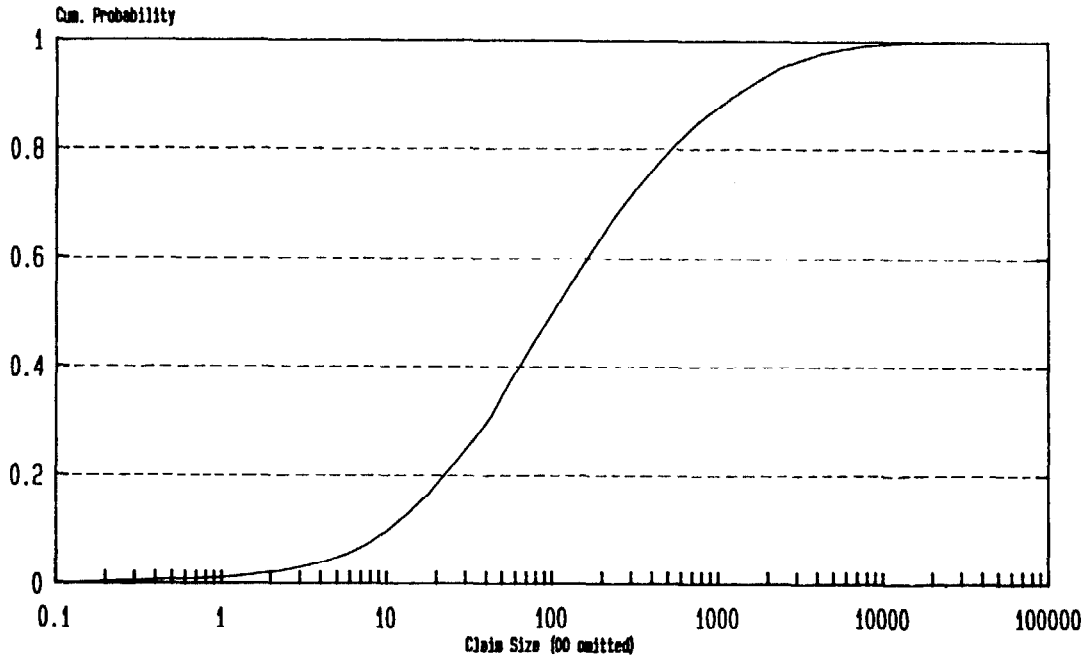
NAIC Closed Claim Data Base

Distribution by Size of Loss



NAIC Closed Claim Data

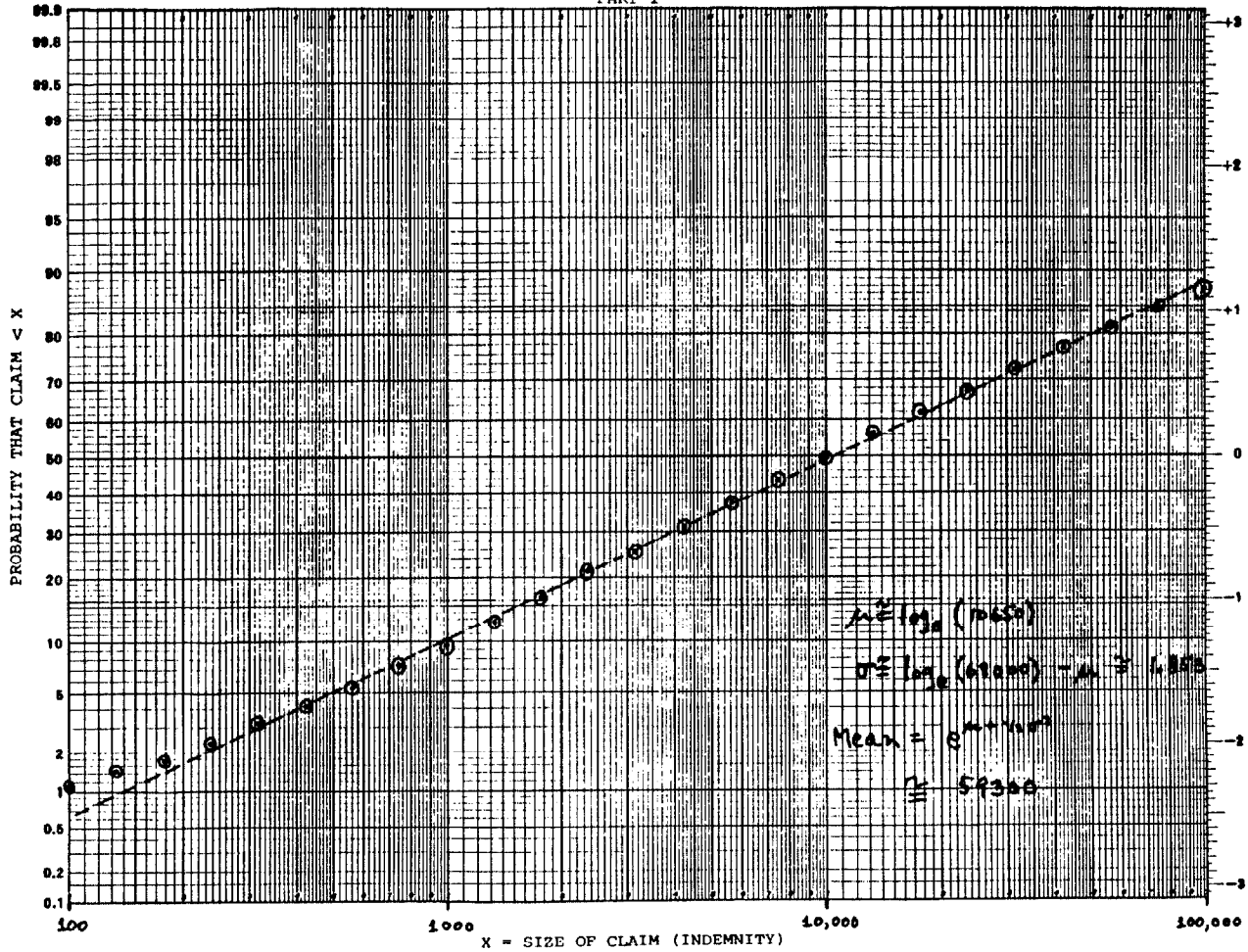
Distribution by Size of Loss



Indemnity Excluding CMP

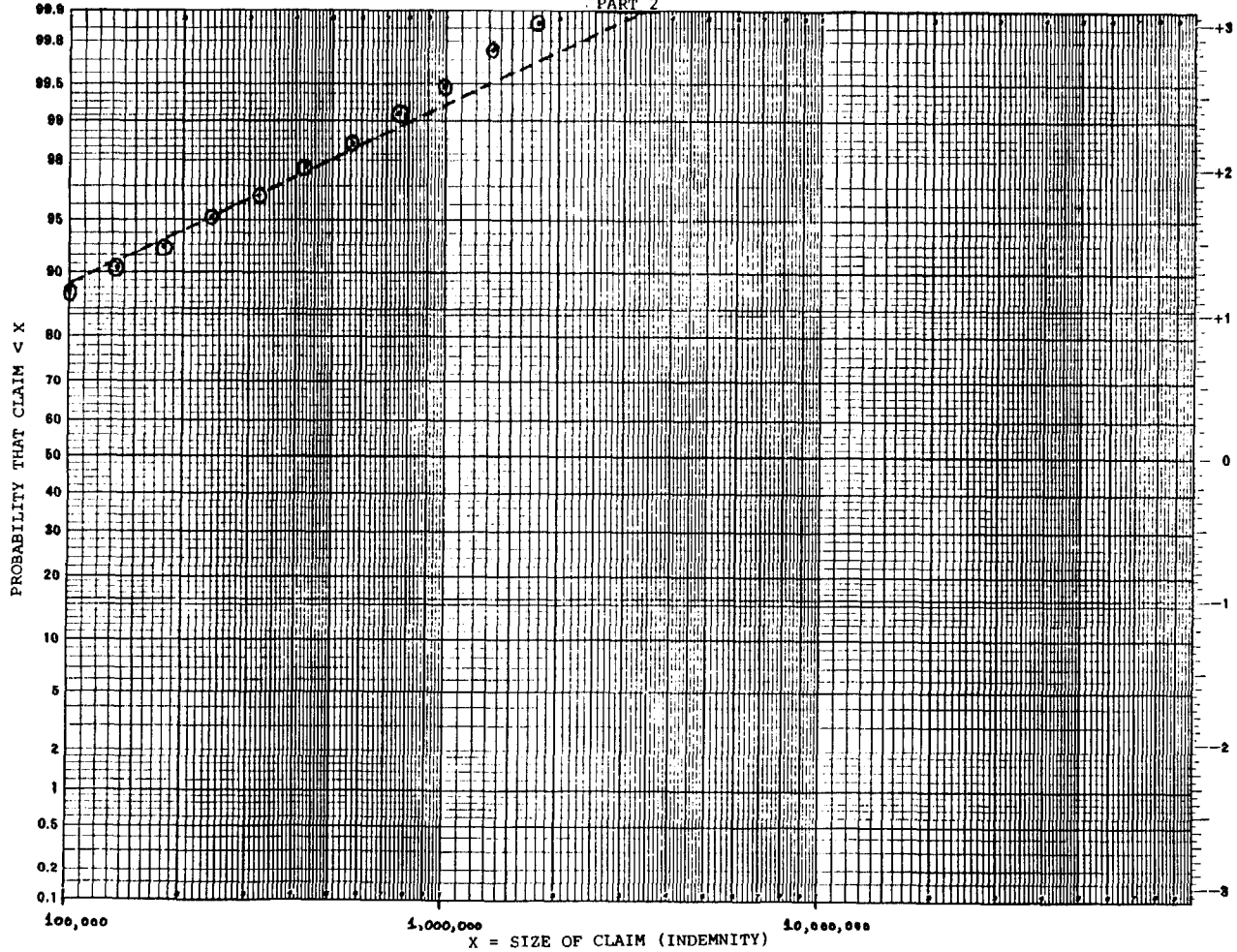
NAIC CLOSED CLAIM STUDY
CUMULATIVE DISTRIBUTION BY SIZE OF LOSS

PART 1



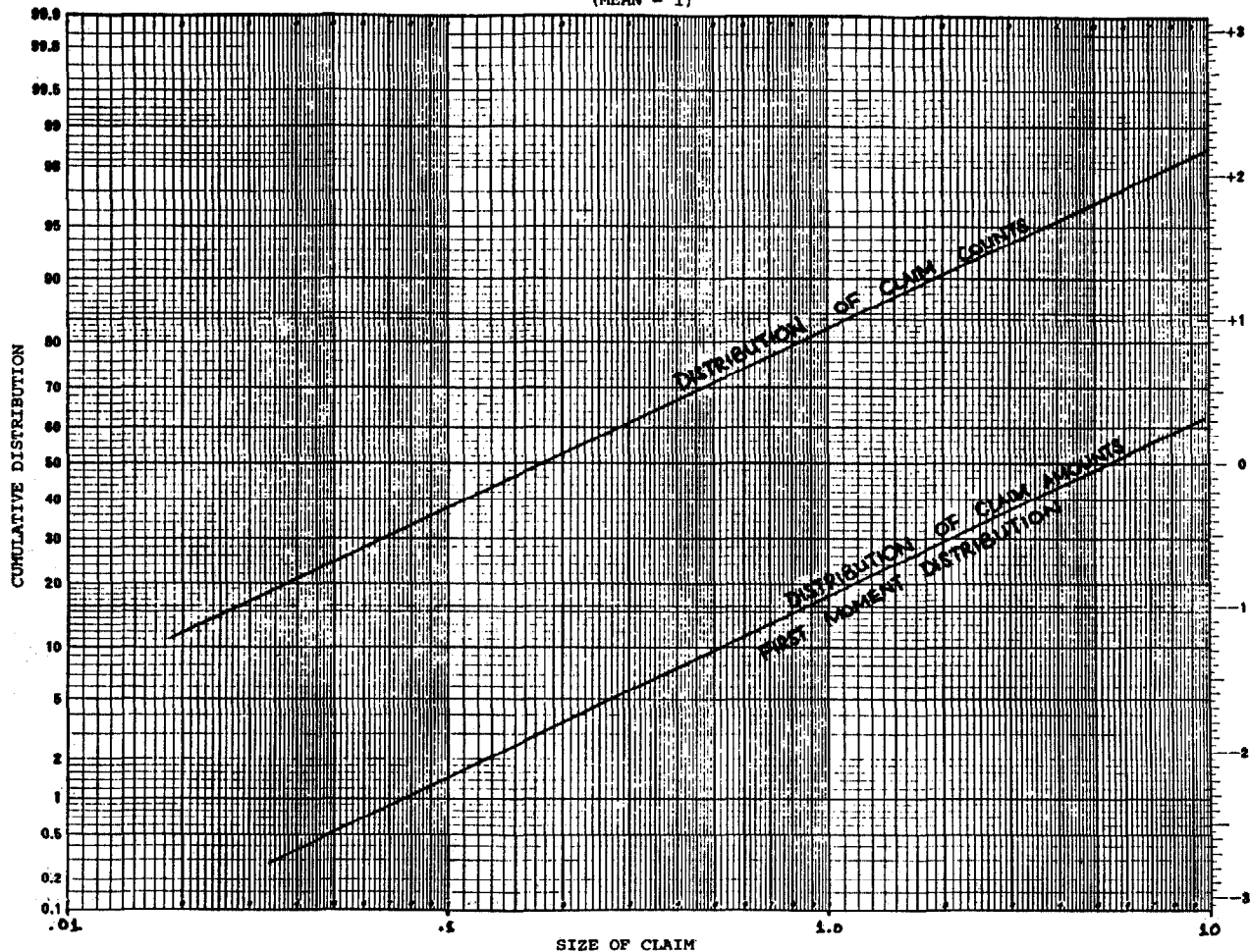
NAIC CLOSED CLAIM STUDY
 CUMULATIVE DISTRIBUTION BY SIZE OF LOSS

PART 2



100

MEDICAL PROFESSIONAL LIABILITY - MODEL CLAIM SIZE DISTRIBUTION
(MEAN = 1)



101

NAIC CLOSED MEDICAL LIABILITY CLAIMS - ADJUSTED FOR FREQUENCY/SEVERITY TRENDS

REPORT YEAR/CALENDAR YEAR MATRIX FOR LOSSES OF ONE ACCIDENT YEAR

page 1 of 4

Cal. Year	Report Year										Total CY
	1	2	3	4	5	6	7	8	9	10+	
1:											
#CMI	4218.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4218.5
INDEM	32999300	0	0	0	0	0	0	0	0	0	32999300
#CMI/CWE	2822.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2822.5
ALAE	1985220	0	0	0	0	0	0	0	0	0	1985220
#CNP	11648.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11648.7
AVG. INDEM	7823	0	0	0	0	0	0	0	0	0	7823
AVG. ALAE	703	0	0	0	0	0	0	0	0	0	703
2:											
#CMI	3400.8	998.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4399.5
INDEM	97259700	15218500	0	0	0	0	0	0	0	0	112478000
#CMI/CWE	4685.5	1305.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5990.8
ALAE	11119700	2209980	0	0	0	0	0	0	0	0	13329700
#CNP	6591.1	2076.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8667.3
AVG. INDEM	28599	15238	0	0	0	0	0	0	0	0	23566
AVG. ALAE	2373	1693	0	0	0	0	0	0	0	0	2225
3:											
#CMI	2771.4	2013.0	659.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5445.8
INDEM	114001000	63847100	11433500	0	0	0	0	0	0	0	189282000
#CMI/CWE	4473.3	4075.9	1418.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	9967.7
ALAE	18193800	10811900	2336750	0	0	0	0	0	0	0	31342400
#CNP	1754.2	2141.0	766.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4862.1
AVG. INDEM	41135	31717	17339	0	0	0	0	0	0	0	34770
AVG. ALAE	4067	2653	1647	0	0	0	0	0	0	0	3144
4:											
#CMI	2059.9	2065.8	1183.4	196.7	0.0	0.0	0.0	0.0	0.0	0.0	5505.8
INDEM	119169000	97840500	37162100	5590990	0	0	0	0	0	0	259762000
#CMI/CWE	3439.6	3882.1	2687.6	425.0	0.0	0.0	0.0	0.0	0.0	0.0	10434.3
ALAE	22287000	22019200	9143340	750406	0	0	0	0	0	0	34200000
#CNP	512.5	700.7	945.5	367.6	0.0	0.0	0.0	0.0	0.0	0.0	2526.3
AVG. INDEM	57852	47362	31403	28424	0	0	0	0	0	0	47180
AVG. ALAE	6480	5672	3402	1766	0	0	0	0	0	0	5194
5:											
#CMI	1287.5	1494.5	1353.5	365.7	100.6	0.0	0.0	0.0	0.0	0.0	4601.8
INDEM	90294500	99669400	60844300	19196000	4988230	0	0	0	0	0	274992000
#CMI/CWE	2012.5	2716.1	2502.3	852.4	165.1	0.0	0.0	0.0	0.0	0.0	8248.4
ALAE	15079500	18122900	12210800	2898220	360409	0	0	0	0	0	48671900
#CNP	221.8	359.9	411.7	357.0	175.5	0.0	0.0	0.0	0.0	0.0	1525.9
AVG. INDEM	70132	66691	44953	52491	49585	0	0	0	0	0	59758
AVG. ALAE	7493	6672	4880	3400	2183	0	0	0	0	0	5901

NAIC CLOSED MEDICAL LIABILITY CLAIMS - ADJUSTED FOR FREQUENCY/SEVERITY TRENDS

REPORT YEAR/CALENDAR YEAR MATRIX FOR LOSSES OF ONE ACCIDENT YEAR

page 2 of 4

Cal. Year	Report Year										Total CY
	1	2	3	4	5	6	7	8	9	10+	
6:											
WCWI	637.0	954.0	936.5	448.5	175.6	40.5	0.0	0.0	0.0	0.0	3192.1
INDEM	62810500	68229600	51776400	29412800	7451200	2176270	0	0	0	0	221857000
WCWI/CWE	1047.0	1611.6	1645.2	822.7	354.8	72.2	0.0	0.0	0.0	0.0	5553.5
ALAE	8918380	15749100	11532300	5062490	1394870	124000	0	0	0	0	42781100
WCNP	114.7	234.5	257.7	144.7	186.3	89.3	0.0	0.0	0.0	0.0	1027.2
AVG. INDEM	98604	71520	55287	65580	42433	53735	0	0	0	0	69502
AVG. ALAE	8518	9772	7010	6154	3931	1717	0	0	0	0	7703
7:											
WCWI	312.0	457.1	501.4	288.3	159.3	91.9	35.5	0.0	0.0	0.0	1845.5
INDEM	25695300	43347500	36378300	28472600	7354830	7186140	1840770	0	0	0	150276000
WCWI/CWE	519.9	842.0	850.2	588.1	311.2	186.9	64.2	0.0	0.0	0.0	3362.5
ALAE	5202720	7814970	7501700	5747590	1468480	847159	119039	0	0	0	28701700
WCNP	63.4	118.7	161.5	64.2	72.1	118.8	76.6	0.0	0.0	0.0	675.3
AVG. INDEM	82357	94832	72553	98760	46170	78195	51853	0	0	0	81428
AVG. ALAE	10007	9281	8823	9773	4719	4533	1854	0	0	0	8536
8:											
WCWI	166.6	169.9	246.1	154.4	107.2	86.1	69.5	48.6	0.0	0.0	1048.4
INDEM	22553100	20196800	24585500	23101900	14556600	8527560	15314600	3321850	0	0	132158000
WCWI/CWE	261.4	343.5	508.7	319.7	174.7	138.9	138.0	52.6	0.0	0.0	1937.5
ALAE	2390770	3691680	5584570	2777160	1436690	1067770	561138	106096	0	0	17615900
WCNP	53.6	57.0	60.5	24.4	33.7	32.2	78.4	56.9	0.0	0.0	396.7
AVG. INDEM	135373	118875	99900	149624	135789	99043	220354	68351	0	0	126057
AVG. ALAE	9146	10747	10978	8687	8224	7687	4066	2017	0	0	9092
9:											
WCWI	91.9	125.3	138.1	87.3	55.5	76.6	56.4	45.1	12.6	0.0	688.8
INDEM	16032600	16306000	28059500	18619100	4322220	16250000	3333660	6686580	553794	0	110163000
WCWI/CWE	178.4	208.7	264.1	197.0	108.4	104.9	95.6	99.4	17.8	0.0	1274.3
ALAE	2408700	2365090	4652560	2499220	1299700	798795	524145	559968	17809	0	15126000
WCNP	17.6	27.6	46.9	22.8	16.0	21.9	50.8	61.2	31.2	0.0	296.0
AVG. INDEM	174457	130136	203182	215277	77878	212141	59107	148261	43945	0	159935
AVG. ALAE	13502	11332	17617	12686	11990	7615	5483	5633	1001	0	11870
10:											
WCWI	38.2	59.6	102.1	64.4	30.8	33.5	33.0	46.3	40.9	9.4	458.2
INDEM	6153010	10470000	25091200	11632900	7919250	9263940	6371450	7106910	6234870	550632	90794100
WCWI/CWE	77.4	99.1	153.7	100.6	70.3	59.8	61.3	70.1	72.5	21.1	785.9
ALAE	767007	1593940	2390640	1606360	1013300	495524	477845	302084	354016	791526	9792240
WCNP	7.0	12.2	30.3	19.0	2.2	0.0	9.8	7.4	30.5	7.1	125.5
AVG. INDEM	161074	175671	245751	180635	257119	276536	193074	153497	152442	58578	198154
AVG. ALAE	9910	16084	15554	15968	14414	8286	7795	4309	4883	37513	12460

NAIC CLOSED MEDICAL LIABILITY CLAIMS - ADJUSTED FOR FREQUENCY/SEVERITY TRENDS

REPORT YEAR/CALENDAR YEAR MATRIX FOR LOSSES OF ONE ACCIDENT YEAR

page 3 of 4

Cal Year	Report Year										Total CY
	1	2	3	4	5	6	7	8	9	10+	
11:											
OCMI	32.4	28.4	50.5	27.5	10.6	12.8	40.6	20.3	30.8	49.0	302.9
INDEM	4386450	4233490	5834620	5565890	1339760	765109	3187830	3816980	1481460	4948140	35559700
OCMI/CWE	47.1	55.4	80.9	50.7	23.5	14.9	67.9	56.2	59.2	54.5	510.3
ALAE	747815	842896	1031990	859194	329762	109646	626445	510024	462201	185338	5705310
OCNP	10.5	12.1	10.4	4.8	0.0	7.6	5.3	2.6	17.6	46.0	116.9
AVG. INDEM	135384	149067	115537	202396	126393	59774	78518	188029	48099	100982	117398
AVG. ALAE	15877	15215	12756	16947	14032	7359	9226	9075	7807	3401	11180
12:											
OCMI	8.3	24.1	24.2	16.0	12.8	16.5	13.4	26.0	28.1	57.6	227.0
INDEM	1117480	6275840	4763150	2770500	2870580	4038150	1583540	2216640	5530560	6412130	37578600
OCMI/CWE	32.7	51.7	37.0	19.0	18.1	19.3	29.8	28.3	44.1	109.9	389.9
ALAE	545520	764081	426788	214047	146696	109696	443067	434754	1042200	13770900	17897800
OCNP	8.0	0.0	10.8	10.7	5.2	0.0	0.0	2.7	0.0	47.1	84.5
AVG. INDEM	134636	260408	196824	173156	224264	244736	118175	85255	196817	111322	163544
AVG. ALAE	16683	14779	11535	11266	8105	5684	14868	15362	23633	125304	45904
13:											
OCMI	5.4	5.3	22.4	2.8	5.3	2.8	5.2	8.9	24.8	57.5	140.4
INDEM	1103330	1078500	6978730	137336	560984	610383	1687220	10049400	5933220	6842250	34981300
OCMI/CWE	8.2	16.5	30.6	5.4	13.9	2.7	13.6	8.9	24.8	98.2	222.8
ALAE	117986	235890	551016	97325	50023	29515	212639	367939	349327	571351	2583010
OCNP	2.8	2.6	2.7	0.0	0.0	2.8	2.6	8.5	13.6	24.8	60.4
AVG. INDEM	204320	203491	311550	49049	105846	217994	324464	1129140	239243	118996	249155
AVG. ALAE	14389	14296	18007	18023	3599	10931	15635	41341	14086	5818	11593
14:											
OCMI	8.8	5.8	14.4	0.0	0.0	5.5	0.0	2.8	5.7	66.7	109.7
INDEM	162381	519054	1590490	0	0	615932	0	2136340	385388	14013500	19423100
OCMI/CWE	9.0	8.8	23.0	5.7	0.0	5.5	0.0	2.8	14.3	89.6	158.7
ALAE	27520	178291	246741	26013	0	60947	0	1896	525605	805173	1872190
OCNP	5.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	17.8	23.6
AVG. INDEM	18452	89492	110451	0	0	111988	0	762979	67612	210098	177056
AVG. ALAE	3058	20260	10728	4564	0	11081	0	677	36756	8986	11797
15:											
OCMI	5.7	3.1	0.0	0.0	0.0	0.0	0.0	3.1	0.0	14.8	26.7
INDEM	1076180	274813	0	0	0	0	0	1544170	0	1588750	4483920
OCMI/CWE	14.5	3.1	0.0	3.0	0.0	0.0	0.0	3.1	3.0	43.9	70.6
ALAE	221683	99595	0	80071	0	0	0	609400	29850	473084	1513680
OCNP	0.0	0.0	0.0	0.0	0.0	0.0	3.0	0.0	0.0	5.8	8.8
AVG. INDEM	188804	88650	0	0	0	0	0	498121	0	107348	167937
AVG. ALAE	15288	32127	0	26690	0	0	0	196581	9950	10776	21440

NAIC CLOSED MEDICAL LIABILITY CLAIMS - ADJUSTED FOR FREQUENCY/SEVERITY TRENDS

REPORT YEAR/CALENDAR YEAR MATRIX FOR LOSSES OF ONE ACCIDENT YEAR

page 4 of 4

Cal. Year	Report Year										Total CY	
	1	2	3	4	5	6	7	8	9	10+		
16:												
#CWI	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	70.4	70.4
INDEM	0	0	0	0	0	0	0	0	0	0	15784300	15784300
#CWI/CWE	0.0	0.0	0.0	3.1	0.0	3.1	0.0	0.0	0.0	0.0	80.3	86.5
ALAE	0	0	0	16284	0	65949	0	0	0	0	1951080	2033320
#CNP	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.4	21.4
AVG. INDEM	0	0	0	0	0	0	0	0	0	0	224209	224209
AVG. ALAE	0	0	0	5253	0	21274	0	0	0	0	24297	23507
Total												
Rep. Year												
#CWI	15044.4	8404.6	5232.0	1651.6	657.7	366.2	253.6	201.1	142.9	325.4	32279.5	
INDEM	594814000	447507000	294498000	144500000	51363700	49433500	33319100	36878800	20119200	501597001	722570000	
#CWI/CWE	19629.0	15219.8	10201.8	3392.4	1240.0	608.2	470.4	321.4	235.7	497.5	51816.2	
ALAE	90013400	86499500	57609200	22634400	7499930	3709000	2964320	2892160	2781010	18548500	295151000	
#CNP	21011.7	5742.5	2904.9	1015.2	491.0	272.6	226.5	139.3	92.9	170.0	32066.6	
AVG. INDEM	39537	53246	56288	87491	78096	134990	131384	183386	140792	154086	53364	
AVG. ALAE	4586	5683	5647	6672	6048	6098	6302	8999	11799	37283	5696	
Ratio, avg. indemnity to total acc.vr.	.74	1.00	1.05	1.64	1.46	2.53	2.46	3.44	2.64	2.89		
Smoothed avg. indem. ratio	.74	1.00	1.20	1.40	1.60	2.00	2.35	2.70	2.845	3.00		
Ratio, total # claims to total acc.vr.	.484	.250	.156	.053	.021	.011	.008	.005	.004	.008		

Source: NAIC Malpractice Claims: Medical Malpractice Closed Claims, 1975-78, National Association of Insurance Commissioners, 1980. Adjustments for frequency/severity trends performed by the author on the detail data base purchased from NAIC. Accordingly, the conclusions drawn from the adjusted data are those of the author and not necessarily those of the NAIC.

NAIC CLOSED MEDICAL LIABILITY CLAIMS - ADJUSTED FOR FREQUENCY/SEVERITY TRENDS

REPORT YEAR/CALENDAR YEAR MATRIX FOR LOSSES OF ONE ACCIDENT YEAR

Average Indemnity by Calendar Year Components of Report Year
Page 1 of 2

Cal. Year	Report Year										
	1	2	3	4	5	6	7	8	9	10+	
1:	7823 0.198	Average indemnity									
		Ratio, avg. indemnity to avg.ind.,total report year									
2:	28599 0.723	15238 0.286									
3:	41135 1.040	31717 0.596	17339 0.308								
4:	57852 1.463	47362 0.890	31403 0.558	28424 0.325							
5:	70132 1.774	66691 1.253	44953 0.799	52491 0.600	49585 0.635						
6:	98604 2.494	71520 1.343	55287 0.982	65580 0.750	42433 0.543	53735 0.398					
7:	82357 2.083	94832 1.781	72553 1.289	98760 1.129	46170 0.591	78195 0.579	51853 0.395				
8:	135373 3.424	118875 2.233	99900 1.775	149624 1.710	135789 1.739	99043 0.734	220354 1.677	68351 0.373			
9:	174457 4.412	130136 2.444	203182 3.610	213277 2.438	77878 0.997	212141 1.572	59107 0.450	148261 0.808	43945 0.312		
10:	161074 4.074	175671 3.299	245751 4.366	180635 2.065	257119 3.292	276536 2.049	193074 1.470	153497 0.837	152442 1.083	58578 0.380	Rel. CY 1
11:	135384 3.424	149067 2.800	115537 2.053	202396 2.313	126392 1.618	59774 0.443	78518 0.598	188029 1.025	48099 0.342	100982 0.655	Rel. CY 2
12:	134636 3.405	260408 4.891	196824 3.497	173156 1.979	224264 2.872	244736 1.813	118175 0.899	85255 0.465	196817 1.398	111322 0.722	Rel. CY 3
13:	204320 5.168	203491 3.822	311550 5.535	49049 0.561	105846 1.355	217994 1.615	324465 2.470	1129150 6.157	239243 1.699	118996 0.772	etc.
14:	18452 0.467	89492 1.681	110451 1.962	0 0.000	0 0.000	111988 0.830	0 0.000	762979 4.161	67612 0.480	210097 1.364	
15:	188804 4.775	88650 1.665	0 0.000	0 0.000	0 0.000	0 0.000	0 0.000	498119 2.716	0 0.000	107348 0.697	
16:	0 0.000	0 0.000	0 0.000	0 0.000	0 0.000	0 0.000	0 0.000	0 0.000	0 0.000	224209 1.455	
Total	39537 rep.vr 1.000	53246 1.000	56288 1.000	87491 1.000	78096 1.000	134990 1.000	131384 1.000	183386 1.000	140792 1.000	154086 1.000	

NAIC CLOSED MEDICAL LIABILITY CLAIMS - ADJUSTED FOR FREQUENCY/SEVERITY TRENDS

REPORT YEAR/CALENDAR YEAR MATRIX FOR LOSSES OF ONE ACCIDENT YEAR

Average Indemnity by Calendar Year Components of Report Year
Page 2 of 2

Composite Average Indemnity by Relative Calendar Year Cells

	Smoothed

relative cal. year 1 avg. = 0.233	.25
relative cal. year 2 avg. = 0.669	.67
relative cal. year 3 avg. = 0.891	.89
relative cal. year 4 avg. = 1.295	1.30
relative cal. year 5 avg. = 1.531	1.53
relative cal. year 6 avg. = 2.125	2.13
relative cal. year 7 avg. = 2.623	2.60
relative cal. year 8 avg. = 3.173	2.80
relative cal. year 9 avg. = 2.972	3.00

XYZ INSURANCE COMPANY

Assumed Distribution of Claims By Report Year

For Claims Incurred in One Accident Year

Report Year*	(1) Ratio, Number of Claims Reported to Total Accident Year Claims	(2) Ratio, Average Indemnity to Average for Entire Accident Year	(3) Ratio, Amount of Indemnity to Total Accident Year Amount = (1) x (2)
	-----	-----	-----
1	.3783	.7137	.270
2	.2522	.9516	.240
3	.1839	1.1420	.210
4	.1126	1.3323	.150
5	.0394	1.5226	.060
6	.0210	1.9033	.040
7	.0126	2.3791	.030
Total	1.0000		1.000

* Relative to accident year.

XYZ INSURANCE COMPANY

Assumed Distribution of Claims By Calendar Year of Payment

For Claims Incurred in One Report Year

Calendar Year**	(1) Ratio, Number of Claims Paid to Total Report Year	(2) Ratio, Average Indemnity to Average for Entire Accident Year	(3)* Ratio, Amount of Indemnity to Total Report Year Amount
1	.0591	.2538	.015
2	.2058	.6803	.140
3	.3873	.9037	.350
4	.2803	1.3200	.370
5	.0418	1.5535	.065
6	.0162	2.1627	.035
7	.0095	2.6399	.025
Total	1.0000		1.000

* Column (1) x Column (2).

** Relative to report year.

MEDICAL PROFESSIONAL LIABILITY CLAIM SIZE DISTRIBUTION

TEST OF SAMPLED MEANS AND CV'S, STRATIFIED AND UNSTRATIFIED
COMPARED TO DIRECT CALCULATIONS, WITH VARIOUS POLICY LIMITS

Lognormal distribution with Unlimited mean = 100,000

Each sample = 100,000 random trials

	Unlim. CV ² =10		Unlim. CV ² =20		Unlim. CV ² =30	
	Limited Mean	Limited CV	Limited Mean	Limited CV	Limited Mean	Limited CV
Limit=50,000						
Direct Calc.	29686	0.6361	26076	0.7511	24185	0.8173
Sample, unstrat.	29716	0.6352	26119	0.7502	24231	0.8164
Sample, strat.	29242	0.6525	25717	0.7655	23861	0.8309
Limit=100,000						
Direct Calc.	43878	0.8464	38297	0.9696	35416	1.0413
Sample, unstrat.	43960	0.8453	38370	0.9681	35476	1.0395
Sample, strat.	43245	0.8614	37723	0.9831	34868	1.0544
Limit=500,000						
Direct Calc.	77888	1.4981	70163	1.6635	65847	1.7595
Sample, unstrat.	77742	1.4948	69996	1.6605	65667	1.7566
Sample, strat.	77020	1.5166	69251	1.6829	64935	1.7796
Limit=1,000,000						
Direct Calc.	88071	1.8412	81451	2.0531	77437	2.1725
Sample, unstrat.	87797	1.8374	81158	2.0508	77136	2.1711
Sample, strat.	87386	1.8657	80648	2.0786	76594	2.1988
Limit=10,000,000						
Direct Calc.	99499	2.8548	98364	3.5134	97273	3.8728
Sample, unstrat.	98367	2.7628	96964	3.4215	95784	3.7946
Sample, strat.	99335	2.9231	98250	3.5966	97164	3.9585
Limit=25,000,000						
Direct Calc.	99916	3.0473	99582	3.9620	99169	4.4987
Sample, unstrat.	98575	2.8535	97794	3.7435	97141	4.2810
Sample, strat.	99619	3.0336	99436	3.9946	99192	4.5895

Notes:

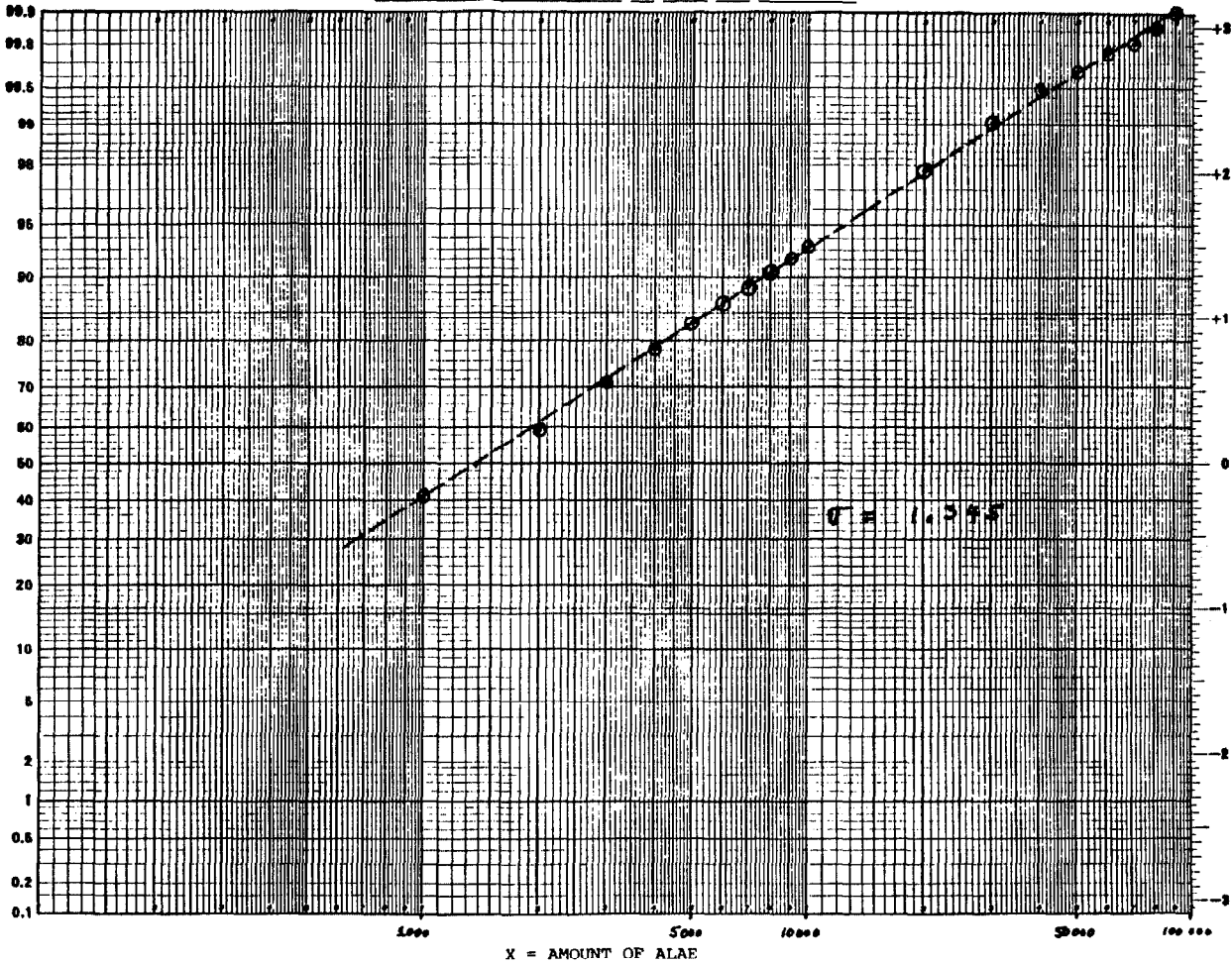
The objective of this test is to establish the reliability of the Monte Carlo simulation process in sampling indemnity amounts, both stratified and unstratified. The stratified process samples from distributions for assigned report year/calendar year subsets of an accident year. Prior to each RY/CY sampling, the report year and calendar year are selected randomly from RY/CY distributions. For the selected subset, the mean has been adjusted by report year and calendar year severity relativity factors and the variance has been adjusted downward from the variance for the entire accident year, so that the total sample variance for all subsets combined will approximate that of the overall accident year. The unstratified sampling bypasses the partitioning of the accident year into report year/calendar cells and simply samples from the total accident year distribution, using the accident year mean and overall variance.

NAIC CLOSED CLAIM STUDY

CUMULATIVE DISTRIBUTION OF ALAE PER DEFENDANT

111

PROB. ALAE AMT. < X



MAIC CLOSED CLAIM STUDY

REGRESSION OF AVG. EXPENSE VERSUS AVG. INDEMNITY

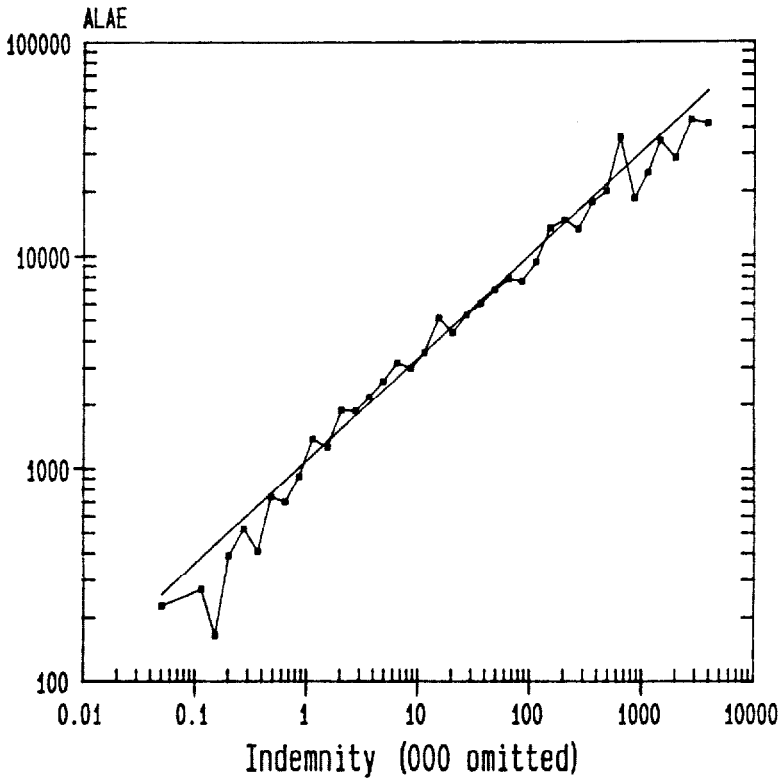
X = AVERAGE INDEMNITY BRACKET -----	Y = AVERAGE ALAE IN BRACKET -----	WEIGHT (NUMBER OF CLAIMS) -----	COMPUTED Y -----
51	229	358.3	259.2
115	272	103.2	384.9
154	166	145.3	444.3
205	392	167.7	509.9
279	526	242.8	591.9
372	412	292.9	679.5
489	745	411.8	776.0
654	705	581.2	892.6
870	926	828.3	1024.6
1150	1387	1015.0	1172.5
1565	1268	1170.2	1360.5
2071	1892	1477.1	1557.7
2786	1878	1499.5	1797.6
3699	2191	1640.8	2061.3
4933	2598	2180.2	2368.8
6562	3177	2071.1	2718.7
8703	2982	1884.5	3116.1
11512	3544	2029.0	3566.7
15453	5139	1906.4	4111.7
20526	4379	1848.9	4715.8
27428	5311	1564.3	5424.5
36706	6031	1448.2	6244.1
48937	6982	1340.3	7174.5
65342	7879	1171.7	8249.5
86099	7653	926.5	9425.2
114706	9411	917.8	10825.7
153844	13511	746.2	12474.7
204947	14788	722.3	14328.0
273289	13324	456.1	16464.3
362444	17890	402.6	18869.6
487164	20104	247.9	21766.5
648598	36075	199.7	24993.1
869532	18601	112.6	28794.1
1141890	24485	93.3	32843.9
1473140	34618	34.0	37143.0
2010450	28763	15.1	43161.8
2773930	43677	22.4	50421.5
3919530	42060	4.9	59583.4

B = 0.48294500 A = 3.66331000

EQUATION: LOG (Y)=A+B*LOG(X)

NAIC Closed Claim Study

Regression of Avg. ALAE vs. Avg. Ind.



—●— Observed ALAE — Computed ALAE

A = 3.66331 B = 0.482945

EQUATION: $\text{LOG}(Y) = A + B \cdot \text{LOG}(X)$

DISTRIBUTION OF NUMBER OF DEFENDANTS PER INCIDENT

(Incidents Closed With Payment)

<u>Number of Defendants(X)</u>	<u>Number of Incidents with X Defendants</u>	<u>Ratio To Total</u>	<u>Indemnity Average</u>
1	12,187	.478	\$ 14,795
2	6,111	.240	33,038
3	3,307	.130	51,850
4	1,699	.067	67,411
5	910	.036	87,841
6	450	.018	91,726
7	285	.011	77,805
8	173	.007	100,157
9 & Over	387	.015	111,322
Total	25,509		

Source: **NAIC Malpractice Claims: Medical Malpractice Closed Claims, 1975-78**, National Association of Insurance Commissioners, 1980, Table 2.10, p. 68.

**DERIVATION OF GENERAL MODEL DISTRIBUTION OF
NUMBER OF DEFENDANTS PER INCIDENT**

I. Assume, for the entire population, the distribution is as follows:

<u>x = No. of defendants</u>	<u>f(x)</u>
1	.50
2	.25
3	.125
.	.
.	.
.	.
x	$(.5)^x$

Total defendants = $\sum_{x=1}^{\infty} x (1/2)^x = 2$ (per incident)

II. Let p = penetration factor for a given insurer
= probability that a given defendant in the population is insured by that insurer

III. Determine the distribution by number of defendants for the given insurer.

Number of Defendants (K)	Population Prob. Incident with K defs.	Number of defs. Insured by Insurer (N)	Prob. of N Insurer's defs.
1	$(1/2)$	0	$(1/2)(1-p)$
		1	$(1/2)(p)$
2	$(1/2)^2$	0	$(1/2)^2 \binom{2}{0} p^0 (1-p)^2$
		1	$(1/2)^2 \binom{2}{1} p (1-p)$
		2	$(1/2)^2 \binom{2}{2} p^2$
3	$(1/2)^3$	0	$(1/2)^3 \binom{3}{0} (1-p)^3$
		1	$(1/2)^3 \binom{3}{1} p (1-p)^2$
		2	$(1/2)^3 \binom{3}{2} p^2 (1-p)$
		3	$(1/2)^3 \binom{3}{3} p^3$

and so on

IV. Generalizing.

let

$I_p(K)$ = probability that an incident of a given insurer with penetration factor p will have exactly K defendants ($K \geq 0$)

$$= \sum_{N=K}^{\infty} (1/2)^N \binom{N}{K} p^K (1-p)^{N-K}$$

$$\text{and } \sum_{K=0}^{\infty} I_p(K) = 1$$

Note that the distribution of incidents by number of defendants for this one insurer now includes incidents with zero defendants for that insurer. To back out these zero defendant situations and thereby create a modified distribution for that insurer, you make the adjustment

$$I'_p(K) = \frac{I_p(K)}{1 - I_p(0)}$$

for all $K > 0$

V. A useful property of the defendant distribution is that the expected number of defendants per incident is $1+p$, that is

$$\sum_{K=1}^{\infty} \frac{K \cdot I'_p(K)}{p} = 1+p$$

With this property, the reasonableness of the assumed penetration factor for the insured in question can be tested quickly and conveniently by simply determining the ratio of defendants to incidents over an appropriate claim history period.

VI. The following table displays the defendant distribution $I'_p(K)$ for

values of p from .1 to 1.

MEDICAL PROFESSIONAL LIABILITY
PROBABILITY OF NUMBER OF DEFENDANTS PER INCIDENT

Num. of defs.	Penetration factor									
	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
1	.9091	.8333	.7692	.7143	.6667	.6250	.5882	.5556	.5263	.5000
2	.0826	.1389	.1775	.2041	.2222	.2344	.2422	.2469	.2493	.2500
3	.0075	.0231	.0410	.0583	.0741	.0879	.0997	.1097	.1181	.1250
4	.0007	.0039	.0095	.0167	.0247	.0330	.0411	.0488	.0559	.0625
5	.0001	.0006	.0022	.0048	.0082	.0124	.0169	.0217	.0265	.0313
6	.0000	.0001	.0005	.0014	.0027	.0046	.0070	.0096	.0126	.0156
7	.0000	.0000	.0001	.0004	.0009	.0017	.0029	.0043	.0059	.0078
8	.0000	.0000	.0000	.0001	.0003	.0007	.0012	.0019	.0028	.0039
9	.0000	.0000	.0000	.0000	.0001	.0002	.0005	.0008	.0013	.0020
> 9	.0000	.0000	.0000	.0000	.0001	.0001	.0003	.0007	.0012	.0019
Expected Number:	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0

**DESCRIPTION OF MONTE CARLO MODEL TO GENERATE PROBABILITY DISTRIBUTION
OF LOSSES IN FIRST EXCESS LAYER**

I. Miscellaneous Assumptions, Input Parameters, and Distributions

(a) Report year distribution of accident year losses, with relative severity factors by report year - see Appendix C, Exhibit 1.

(b) Calendar year distribution of report year losses, with relative severity factors by calendar year - see Appendix C, Exhibit 2.

(c) Distribution of claims (indemnity, defendant basis) by size - see Appendix B.

Note: The basic distribution applies to all claims of one accident year, using the overall mean value for the entire year. The model stratifies the claims first into 49 report year/calendar year cells, each with a modified mean value from (a) and (b) above. Accordingly, the variance applicable to each cell has been reduced from the overall variance for random selection purposes, such that the combined sample variance over all 49 cells will approximate the entire accident year distribution.

(d) Average unlimited indemnity by year (per defendant) - used as the parameter in the size of loss distribution for each accident year:

1983	\$180,000
1984	200,000
1985	230,000
1986	260,000
1987	290,000

(e) Policy limit distribution by year:

Year	500/1M	1M/3M
----	-----	-----
1983	12%	88%
1984	10	90
1985	9	91
1986	8	92
1987	7	93

(f) Average claim expense by year. Based on the functional relationship derived between the expected average ALAE and the sample indemnity value (see Appendix E), the sample ALAE is SELECTED from a distribution the mean of which is determined as a function of the sample indemnity. The starting values for the average ALAE for the entire accident year, over all indemnity values are:

1983	\$7,000
1984	7,500
1985	7,900
1986	9,000
1987	9,500

(g) Claim frequency by year (class 1 equivalent), including claims closed with indemnity (CWI), with expense only (CWE), and with no payment (CNP):

1983	.088
1984	.093
1985	.089
1986	.102
1987	.107

(h) Exposures by year (class 1 equivalent):

1983	2,400
1984	2,700
1985	2,900
1986	3,100
1987	3,200

It is assumed in this case study that all exposures are claims-made and the company commenced claims-made coverage in 1983, writing the 2,400 exposures at maturity year 1 (retroactive date 1/1/83). It is further assumed that there were no terminations and that all new entrants were started at maturity year 1 (no prior acts coverage).

(i) Percentages for claims disposed, all years:

CWI	35%
CWE	45%
CNP	20%

(j) Distribution of number of defendants per incident - see Appendix F, for penetration factor = .4 (expected incident frequency = defendant frequency from (g) divided by (1.4)).

(k) Layer of excess in question:

\$750,000 excess of \$250,000 per defendant, with retention indexed at \$25,000 per year (based on calendar year paid). Total retention per occurrence (all defendants combined) = 150% of applicable (indexed) per-defendant retention. ALAE retained in one occurrence is proportional to indemnity retained, relative to total indemnity for occurrence.

(l) Parameter variance (uncertainty factor). These values are expressed in relation to the expected population frequency and severity. In this case study we are assuming a "standard error" of 20% for both claim frequency and average indemnity.

II. The Monte Carlo Simulation Process (In Pseudo Code):

Accumulators Set Up:

Excess losses paid by actual calendar year (including pro rata ALAE) for one trial.

Aggregate excess loss brackets (31) for all trials combined (probability distribution). One accumulator for counts (number of trials falling into bracket) and another for total excess loss dollars.

Input:

Uncertainty factor for population mean frequency and severity (parameter variance).

Retention per defendant and index amount.

For each trial

[1,000 trials run]

For each accident year 1983 to 1987 (the year in question)

SELECT expected population frequency and severity for year
(using normal distribution and parameter variance, or
uncertainty factors, which were input).

SELECT number of incidents

For each incident:

- (a) SELECT report year, relative to accident year and adjust mean indemnity by relative severity factor corresponding to that report year.
- (b) If report year not equal to year in question (1987), skip to next incident.
- (c) SELECT number of defendants from distribution of number of defendants per incident (see Appendix F).
- (d) SELECT calendar year paid relative to report year and further modify mean indemnity by calendar year severity factor.
- (e) Establish retention per defendant applicable to calendar year, including index.
- (f) For each defendant (from (c)):
 - (1) SELECT applicable policy limit
 - (2) SELECT mode of closure (CWI, CWE, CNP). If CWE, SELECT ALAE amount only and then skip to next defendant. If CNP, go to next defendant.
 - (3) SELECT gross (unlimited) indemnity from size of loss distribution, the mean of which was adjusted by report year and calendar year severity factors from (a) and (d).
 - (4) Limit indemnity amount to policy limit, as necessary
 - (5) Bump direct loss accumulator for the single incident.
 - (6) Based on indemnity amount, adjust expected ALAE, and SELECT sample ALAE from distribution. Then bump direct ALAE accumulator for the incident.

(7) Limit indemnity to retention per defendant for calendar year and add to retained amount for this incident.

Next defendant

(g) For all defendants, limit total net indemnity to applicable retention per occurrence (incident). If total direct loss (indemnity) for incident exceeds total net, then add the excess to the excess loss accumulator corresponding to calendar year paid.

(h) If there is excess in (g) above, add pro rata ALAE to same accumulator.

(i) Reinitialize accumulators for incident.

Next incident

Next year

Tally excess losses (optional: present value of losses) in accumulators for this trial. Then determine which one of the 31 brackets of aggregate losses this trial falls in and bump the corresponding accumulators for counts (1) and total excess dollars.

Re-initialize all accumulators, except aggregate excess loss brackets.

Next trial

Print out probability distribution

Note: Each time the word "SELECT" is used in the above process the program randomly samples from the appropriate distribution described in Part I, using a random number generator.

XYZ INSURANCE COMPANY

MONTE CARLO SIMULATION OF THE
DISTRIBUTION OF CEDED LOSSES - 1987

\$750,000 excess of \$250,000 per defendant, indexed \$25,000/year
Maximum retention per occurrence = 1.5 X retention per defendant

(Gross of any aggregate deductible)

Interval end point	Number of trials in interval	Cumulative # trials	Total losses	Cumulative Tot. Losses
0	0	0	0	0
500000	0	0	0	0
586051	1	1	556139	556139
686912	1	2	598031	1154171
805131	1	3	791586	1945756
943696	0	3	0	1945756
1106108	0	3	0	1945756
1296472	2	5	2463088	4408845
1519598	5	10	7045101	11453946
1781124	7	17	11618442	23072388
2087659	8	25	15525836	38598223

*2325000		41.51		76775029

2446950	25	50	57792841	96391064
2868076	34	84	91482954	187874018
3361679	68	152	213118730	400992748
3940231	85	237	312092391	713085139

#4193182		281.76		905151344

4618354	120	357	514901379	1227986519

*4725000		377.93		1333005518

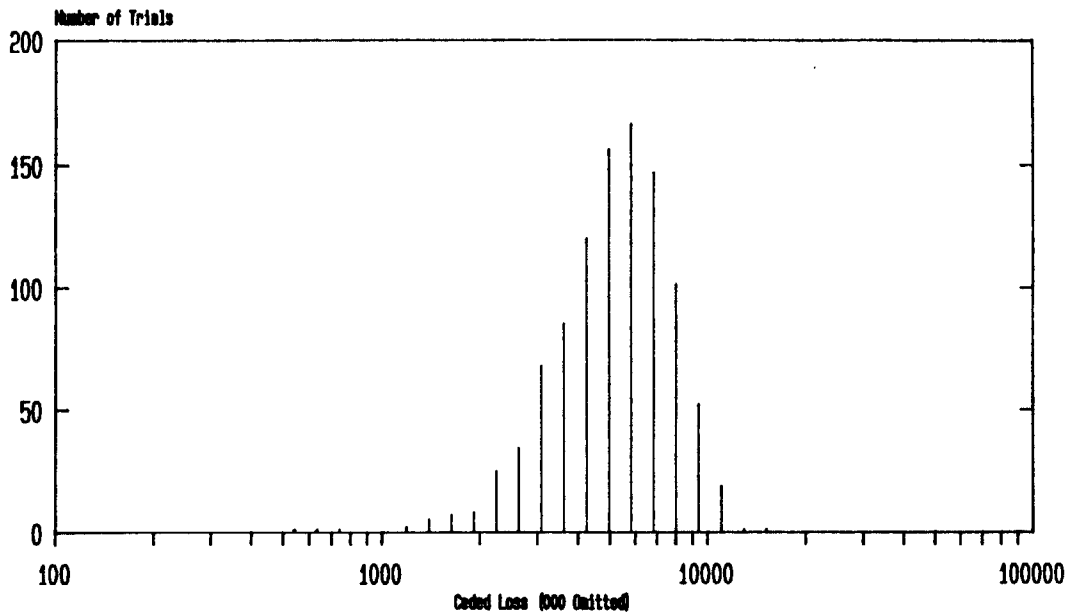
5413184	156	513	782705965	2010692484
6344805	166	679	971158691	2981851175
7436761	147	826	1001225587	3983076762
8716644	101	927	806017263	4789094025
10216799	52	979	486329911	5275423935
11975133	19	998	206340176	5481764112
14036081	1	999	13325644	5495089755
16451723	1	1000	14296444	5509386200
19283103	0	1000	0	5509386200
22601769	0	1000	0	5509386200
26491585	0	1000	0	5509386200
31050848	0	1000	0	5509386200
36394770	0	1000	0	5509386200
42658394	0	1000	0	5509386200
50000000	0	1000	0	5509386200

Notes:

- * Interpolated values for Minimum and Maximum loss levels for "subject to minimum" option.
- # Interpolated value for Maximum loss level for "minimum plus" option.

XYZ Insurance Company

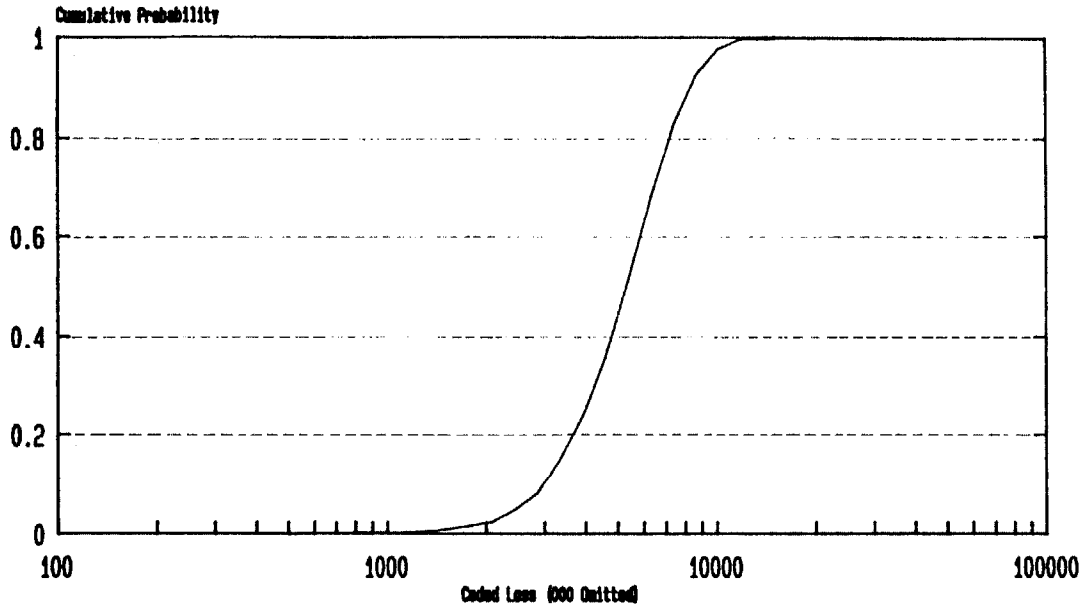
Distribution of Ceded Losses



75000 x 250000 per defendant, indexed
25000/year.
Max Retention=1.5 x defendant retention

XYZ Insurance Company

Distribution of Coded Losses



750000 x 250000 per defendant, indexed
25000/year.
Max Retention-1.5 x defendant retention

XYZ INSURANCE COMPANY
Derivation of Probable Final 1987 Retrospective Reinsurance Premium
Using "Subject to Minimum" Formula
GNEPI = \$15,000,000

(1) Minimum Premium --10% of GNEPI	1,500,000
(2) Maximum Premium --30% of GNEPI	4,500,000
(3) Loss Conversion Factor	1.25
(4) Aggregate Deductible --7.5% of GNEPI	1,125,000
(5) Loss Level Corresponding to Minimum $[(1)/(3)]+(4)$	2,325,000
(6) Probability of Losses Less than Minimum (per 1000 trials)	41.51
(7) Aggregate Losses for trials less than or equal to minimum	76,775,029
(8) Loss Level Corresponding to Maximum $[(2)/(3)]+(4)$	4,725,000
(9) Probability of Losses Less than Maximum (per 1000 trials)	377.93
(10) Aggregate Losses for trials less than or equal to maximum	1,333,005,518
(11) Minimum Premium Paid per 1000 trials $(1) \times (6)$	62,271,757
(12) Maximum Premium Paid per 1000 trials $(2) \times [1000 - (9)]$	2,799,309,664
(13) Premium Paid for trials between Min. and Max. $(3) \times [(10) - (7)] - (4) \times [(9) - (6)]$	1,097,202,154
(14) Expected Final Reinsurance Premium $[(11) + (12) + (13)] / 1000$	3,958,784

Note : Lines (6) through (10) from excess loss distribution.

XYZ INSURANCE COMPANY
Derivation of Probable Final 1987 Retrospective Reinsurance Premium
Using "Minimum Plus" Formula
GNEPI = \$15,000,000

(1) Minimum Premium --7.5% of GNEPI	1,125,000
(2) Maximum Premium --30% of GNEPI	4,500,000
(3) Loss Conversion Factor	1.10
(4) Aggregate Deductible --7.5% of GNEPI	1,125,000
(5) Loss Level Corresponding to Minimum	0
(6) Probability of Losses Less than Minimum	0.00
(7) Aggregate Losses for trials less than or equal to minimum	0
(8) Loss Level Corresponding to Maximum $[(2)-(1)]/(3)+(4)$	4,193,182
(9) Probability of Losses Less than Maximum (per 1000 trials)	281.76
(10) Aggregate Losses for trials less than or equal to maximum	905,151,344
(11) Minimum Premium Paid per 1000 trials (1)x(6)	0
(12) Maximum Premium Paid per 1000 trials (2)x[1000-(9)]	3,232,071,620
(13) Premium Paid for trials between Min. and Max. $(1)x[(9)-(6)]+(3)x\{[(10)-(7)]-(4)x[(9)-(6)]\}$	963,968,269
(14) Expected Final Reinsurance Premium $[(11)+(12)+(13)]/1000$	4,196,040

Note : Lines (6) through (10) taken from excess loss distribution.

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