REGULATORY STANDARDS FOR RESERVES

By

OAKLEY E. VAN SLYKE

The Author

O.E. (Lee) Van Slyke is Director, Casualty Actuarial Services, for Coopers & Lybrand in Los Angeles. He is a Fellow of the Casualty Actuarial Society, a Member of the American Academy of Actuaries, and an Associate of the Society of Actuaries. Mr. Van Slyke holds a 1969 B.S. degree from Massachusetts Institute of Technology. He has been a consulting actuary for 13 years.

Mr. Van Slyke has published two previous reviews on the subject of risk measurement and reserve adequacy. The first was a review of "Reinsuring the Captive/Specialty Company" by Lee R. Steeneck, PCAS, 1985, and a review of "REROSHE: The Concept of a Risk-Free Equivalent Return on Shareholders' Equity" by Alastair G. Longley-Cook, Transactions of the Society of Actuaries, 1983.

Abstract

The loss and loss expense reserves of a property and casualty company are the most important items on the balance sheet and the most difficult to value. Regulators' concerns include but go beyond the question of reserve adequacy, and include a knowledge that the company will be reasonably secure even if the claims environment unfolds in unexpected ways. Some regulators are also concerned that assets appropriately match liabilities.

The paper suggests several concepts about the calculation of risk margins that have been used for insurance pricing. It shows how these concepts can be applied to determine a reserve value that reflects the risk associated with the possible eventual claim payments as well as with the expected value of those payments. In addition to concepts, the paper presents specific techniques for distinguishing between individual possible payment outcomes and entire scenarios of associated payments, as well as a basic set of mathematical formulas. The methods suggested by the paper are intended to be a part of an extensive review of reserves, not a modest test that can be applied in the absence of an understanding of a company's operations. As a part of an extensive reserve review the methods presented in the paper contribute a valuable amount of information for a modest additional investment of time.

REGULATORY STANDARDS FOR RESERVES

I. INTRODUCTION

The loss and loss expense reserves of a property and casualty company are the most important items on the balance sheet and the most difficult to value. Regulators have expressed increasing concern over the adequacy of reserves carried in statutory financial reports in recent years, in large part because of the inadequacy of reserves reported by many companies at year-end 1983 and year-end 1984.

Although a general sense of reserve adequacy is important, regulators' concerns go beyond the elementary questions of reserve adequacy. Regulators rightly want to know that the insurer and its certifying actuary have given reasonable attention to the problems the company potentially faces in the course of paying outstanding claim liabilities. Regulators want to know that the company has reasonable reserves for claim payments in spite of shortcomings in data about exposures and losses, recent changes in the nature of the insurer's book of business, changes in the adequacy of premium rates, potentially uncollectible reinsurance, and other causes of unexpected shortfalls in reserves.

In addition, the claim liabilities are secured by assets that may or may not be invested in ways that expose the company to financial hardship if interest rates depart from projected levels. Regulators are concerned with the possible adverse impact of changes in interest rates. Although anticipated interest income may have been a reasonable margin for contingencies in the past, the impact of long-tailed lines on the balance sheets of most property-casualty companies in the past 20 years has increased the pressure to find a margin for contingencies that recognizes the nature of the investment portfolio as well as the underwriting portfolio.

Finally, regulatory standards for the valuation of reserves (and assets) have always reflected a desire to stabilize the statutory surplus of companies against fluctuations in outside forces such as interest rates. These standards have included the valuation of Schedule P liabilities at minimum statutory values and the valuation of assets at book value. If reasonable, however, regulatory standards should be much closer to the economic values in the marketplace.

This paper introduces regulators to several concepts of risk margins developed for insurance pricing. With this set of tools, the paper argues that reserves should be valued for statutory purposes at estimated market value, and that if reserves are valued at market value, the statutory bottom line can be made more stable and asset matching can be encouraged by valuing assets at market value as well. Valuing liabilities at market value requires consideration of all of the factors discussed above, and so responds to all of the regulator's concerns.

The techniques to determine risk margins that are presented here are an integral part of the proposed standards. A regulatory standard for reserves will be implemented only if regulators, accountants and actuaries can agree on how to estimate the values required by the regulatory standard. The regulatory standards discussed here are practical because they are accompanied by a method of calculating reserve values that can be implemented by a wide range of professionals concerned with insurance company reserves.

The material that follows is based on well established principles for evaluating risk by a decision-maker who is averse to risk. Some of the notation is new, in hopes that the new notation will make the fundamental concepts clear. The development of this material can be found in Raiffa [3], Cozzolino [4], Van Slyke [5] and Van Slyke [6], among others. The approach requires that a utility be associated with each cash flow, but plays down the role of any particular utility function. The result is a straightforward (although tedious) analysis of the financial strength of a business as reflected in its anticipated future cash flows.

In my experience, a good review of the reserves of a multi-line company can take as much as 1000 hours. The only existing statutory standards for reserves (those in Schedule P) can be computed in a few minutes. The proposed standards are designed to go hand-in-hand with a thorough review, not to replace the current standards in Schedule P. They add little work to a thorough review - only the time required to document the review in a certain way and enter the results into a spreadsheet.

This paper, then, is both theoretical and practical. It may be controversial. If it is, I hope it will not be because of the particular method of valuing risk charges it employs, but because it advocates that risk charges should be explicitly calculated and included in property-casualty insurance company reserves. That debate - the proper level of strength to be included in reserves - deserves a great deal more attention than it has received.

II. BRIEF OVERVIEW OF INTEREST AND DISCOUNTING

The reader is trained and experienced in the theory and practice of investment calculations. These few comments are intended to highlight the concepts that will be used later rather than to present any new material.

Interest rates vary from time to time. The same investment, such as the purchase of a one-year U.S. Treasury bill, may yield 5% one year and 10% another.

Interest rates also vary from one investment to another. Bonds issued by companies with marginal financial histories have higher yields than bonds with the same term issued by the U.S. government. Writers have discussed many reasons for these variations, but for our purposes we can consider this variation simply as a reflection of the risk that the principal or anticipated interest will not be fully realized.

Whatever the reason for the interest level, an anticipated cash flow of $1.00 in the future is worth less than $1.00 in hand. We can say that an income of $1.00 deferred some length of time is less useful than $1.00 in hand, or that it has lower utility.

The extent to which a cash flow of $1.00 at time t is worth less than $1.00 is usually expressed as $\$1v^t$, where t is in years, and

$v = (1+i)^{-1}$. More generally, we may write \$1v(t), which allows the interest rate i to vary over time and avoids questions about the meaning of i over a fraction of a year. For any cash flow of x at time t, then, the value today is xv(t).

In today's financial marketplace we can generally purchase nearly risk-free investments maturing at nearly any future date. For practical purposes this allows us to distinguish that part of v(t) that is due to the time the payment is deferred without risk from that part that is due to risk. For example, a return of 10.24% on a one-year investment when risk-free investments are earning 6% suggests that the additional return for risk is 4%. We shall return to this with some notation after introducing utilities in Section IV.

III. BRIEF REVIEW OF PROBABILITY

The reader is also familiar with probability and we shall keep this section brief. The concept of the <u>event</u> plays such a crucial role, however, that we must be careful to define events and probabilities at this point.

We often say that an event i has a probability p_i. (The subscript i should be easily distinguished from the interest rate i. Both bits of nomenclature are firmly established in actuarial literature, and it seems unnecessary to change notation here.) We define the events to be mutually exclusive events, so that

$$\sum_{\text{all } i} p_i = 1. \tag{III-1}$$

The concept of an event must be contrasted with the concept of a scenario. Both are used in valuing insurance products and insurance companies, but they are different concepts. Specifically, a scenario usually refers to a set of closely related possible outcomes. An "inflationary scenario," for example, refers to a description of the world in which price inflation is a dominant concern. Many possible events might ensue.

Within a particular scenario, however, the probabilities of particular events are linked to one another. For example, Scenario 1 might be High Interest Rates. In a High Interest Rate scenario, investment income might be high in early years but loss costs might also be increased, particularly in later years, if the high interest rates are associated with a long-term decline in the purchasing power of the dollar. These offsetting profits and losses are linked together by the scenario.

The concept of the scenario is extremely important if one is to adequately measure risk. A particular economic situation may cause a company to be making an unexpected investment in a line of business, or reap an unexpected short-term reward, but the long-term effect of the investment or reward might be much different than the short-term effect. A valid way of measuring the value of loss reserves will reflect later costs that are directly associated with early benefits, and later benefits that are directly associated with early costs.

Using the definitions set forth above, any particular scenario can be analyzed into many scenarios, each with a description within the more general framework. As a scenario is analyzed into mutually exclusive sets of outcomes (such as "slightly inflationary, with tight credit") it becomes, in the extreme, a list of possible events, each with a specific probability. Any framework for evaluating insurance products or insurance companies should give results that are consistent for various ways of identifying the scenarios and the events that may result under those scenarios (Van Slyke [6]).

In general for a set of scenarios denoted by the subscript j, with j = 1,...,m, each with n_j possible events, the total probability associated with all possible outcomes with all possible events is unity. This can be written algebraically as:

$$\sum_{\text{all } j} \sum_{\text{all } i_j} p_j p(i,j|j) = 1 \qquad \text{(III-2)}$$

In this expression $p_j p(i,j|j)$ is the probability of occurrence of event i under scenario j.

IV. BRIEF REVIEW OF DECISION THEORY

This section introduces some material that may be new to the reader, and notation that is almost certainly new. There are only three concepts, however, and we have included several examples of each to help the reader gain an intuitive feel for the concepts.

Of course, the expected value of a set of cash flows x_i with probability p_i at time 0 is:

$$EV = \sum_{\text{all } i} p_i x_i \tag{IV-1}$$

If cash flows may occur in the future we have familiar formulas for the expected value and the present value:

$$EV = \sum_{\text{all } i} \sum_{t} p_{i,t}\, x_{i,t} \tag{IV-2}$$

$$PV = \sum_{\text{all } i} \sum_{\text{all } t} p_{i,t}\, v_{i,t}\, x_{i,t} \tag{IV-3}$$

where

$p_{i,t}$ = probability of event i leading to cash flow $x_{i,t}$ at time t

$v_{i,t}$ = value of \$1 cash flow at time t given occurrence of event i

We can extend this to a problem in which the decision-maker has defined many scenarios. The formulas are:

$$EV = \sum_{\text{all } j} \sum_{\text{all } i} \sum_{\text{all } t} p_j \, p(i,j|j) \, x_{i,j,t} \qquad \text{(IV-4)}$$

$$PV = \sum_{\text{all } j} \sum_{\text{all } i} \sum_{\text{all } t} p_j \, p(i,j|j) \, v_{j,t} \, x_{i,j,t} \qquad \text{(IV-5)}$$

where

p_j = probability of scenario j

$p(i,j|j)$ = probability of cash flow $x_{j,i,t}$ at time t in scenario j, given that scenario j has been realized.

$v_{j,t}$ = value of \$1 cash flow at time t in scenario j

Formula IV-4 says that the expected value of a set of cash flows over a range of scenarios is the sum over all possible events over all possible times within each possible scenario of the cash outflow at a particular time arising out of a particular event for a particular scenario times the probability of that event arising during that scenario. Formula IV-5 says that the present value can be similarly constructed, except that each cash flow should be discounted by a factor that depends on the scenario and the time of the cash flow.

The third concept (after expected value and present value) is utility. The utility of a particular cash flow is the value that a particular decision-maker attaches to that cash flow. We have mentioned that \$1v(t) is a measure of the worth (today) of a cash flow of \$1 at time t. If the decision-maker uses present-value calculations,

$$U(x) = \sum_{\text{all } t} x_t v(t) \qquad \text{(IV-6)}$$

In general, we can simply write the utility of a cash flow of x as U(x), the utility of a cash flow of x at time t as U(x,t), and the utility of a cash flow x at time t under scenario j as U(x,t,j).

The thrust of this paper is that the utility of a cash flow should be considered because any reasonable decision or evaluation should weigh bad outcomes more severely than favorable outcomes. Expected value calculations are merely a special case of utility calculations in which the evaluator's aversion to risk is negligible. Expected value calculations fall short of the needs of regulators, investors, product developers and others because it is appropriate for these decision-makers to be averse to risk.

As the pioneers of decision theory (e.g., Raiffa [3]) showed, consistent decisions can be developed from a given set of estimates of probabilities only when the probabilities and utilities are combined in the following way:

$$U. = \sum_{\text{all } i} p_i \; u(x_i) \tag{IV-7}$$

Note that the utility of an outcome must be evaluated independently of its probability. The utilities of the possible outcomes must be determined, then averaged using their probabilities as weights. The decision-maker will not be able to reach a consistent set of decisions if the utility of the possible events is measured by the sum of the $U(p_i x_i)$, unless the decision-maker has no aversion to risk.

The concepts of present value and utility can be applied to the cash flows associated with loss and loss expense payments alone. However, investment risk is an integral part of any present value factor whenever the investment anticipates a return greater than the risk-free rate of return. The use of the utility frees the evaluator to give greater weight to significant adverse cash flows than to modest cash flows or to favorable cash flows. A correct treatment of risk for an insurance enterprise as a whole will divide each discount factor into a risk-free present value factor and a utility adjustment for risk. That is, all of the cash flows, investment income as well as loss and loss expense payments arising out of the assets and liabilities, should be analyzed together at each point in time under each scenario. The possible cash flows at each point in time under each scenario should be expressed as a set of probabilities that possible cash flows will equal various amounts. Then the utility of each possible cash flow can be weighted by a probability and the utility of all possible cash flows can be discounted at a risk-free present value factor.

This treatment of interest is important for two reasons:

1. It supports the important concept of immunization. Specifically, to the extent the company has cash outflows equal to cash inflows, the net cash flow is zero and the company has removed the element of risk from both the investment portfolio and the underwriting portfolio. This support comes directly from the careful distinction between risk-free return and uncertainty in the dollar amount of return.

2. It provides to the final result the important associative property of algebra. Specifically, a risk-adjusted value can be associated with any set of risky outcomes. The risk-adjusted value for the entire set of outcomes does not depend on how finely the evaluator enumerates the possible outcomes.

Exponential Utility

The particular utility functions to be used will not play an important role in this article. Often we will write simply U(x). But simple examples will employ an exponential utility function because the exponential function is reasonably easy to understand and is probably a reasonable approximation to most functions that will be used in practice.

The exponential utility function may be defined as follows: the utility of a cash flow (in) of x_i with probability p_i is

$$-c \ln \left(p_i e^{\frac{x_i}{-c}} + (1-p_i)\right) \tag{IV-8}$$

In this utility calculation a gain of x_i is related to some scale c, and then decreased by the exponentiation, then weighted by its probability, p_i. The value of a gain of zero (which is unity) is given a weight of $1-p_i$. Then the exponentiation and scale shift in c are undone in order to develop a result in the same scale as x_i.

Example 1.

$$p_1 = 0.1$$
$$x_1 = \$10 \text{ million}$$
$$c = \$150 \text{ million}$$
$$EV_1 = 0.1 \bullet \$10 \text{ million} = \$1.0 \text{ million}$$

$$\text{Utility} = -\$150 \text{ million} \ln \left(0.1e^{\frac{10}{-150}} + (1-0.1)\right)$$

$$= \$0.97 \text{ million}$$

Example 2.

$$p_2 = 0.01$$
$$x_2 = -\$100 \text{ million (loss)}$$
$$c = \$150 \text{ million}$$

$$EV_2 = .01 \bullet (-\$100 \text{ million}) = -\$1 \text{ million}$$

$$\text{Utility} = -\$150 \text{ million} \ln \left(.01e^{\frac{100}{150}} + (1-0.01)\right)$$

$$= -\$1.41 \text{ million}$$

In Example 1, the incoming cash flow had an expected value of \$1.0 million and a utility of \$0.97 million. In Example 2, the outgoing cash flow had an expected value of \$1 million and a utility of loss of \$1.41 million. The utility function accomplishes its objective of making cash inflows and modest cash outflows of less concern than large cash outflows.

This definition allows the outcomes to be defined in any degree of detail. In particular, if we associate with every possible outcome a value x_i and a probability p_i, then the utility of the set of outcomes is

$$U. = -c \ln \sum_{\text{all } i} p_i \, e^{\frac{x_i}{-c}} \tag{IV-9}$$

This definition can be expanded easily to include sums over several scenarios:

$$U. = -c \ln \sum_{\text{all } j} \sum_{\text{all } i|j} p_j \, p(i,j|j) \, e^{\frac{x_{i,j}}{-c}} \tag{IV-10}$$

where p_j is the probability of the jth scenario.

<u>Risk-Adjusted Value (RAV)</u>

Whenever a utility measure is in the same units as the cash flow it makes intuitive sense to call it the <u>Risk-Adjusted Value</u>, or <u>RAV</u>, of the set of outcomes.

A feel for utility calculations in general, and exponential utility in particular, can be gained from considering the value of RAV at extreme values of p_i and c. The reader can verify the following results for the exponential utility calculations:

Result	Implications
A. $\lim_{p_1 \to 1} RAV = x_1$	As the probability of event 1 increases toward certainty, the Risk-Adjusted Value of all outcomes approaches the value x_1.
B. The addition of a certain cash flow x (one with probability one) to a set of cash flows increases their RAV by x.	Amounts that are absolutely certain can be handled outside the RAV calculation.
C. $\lim_{c \to \infty} RAV = EV$	As one's scale factor increases toward infinity, the decision or evaluation approaches a simple expected-value calculation.
D. $\lim_{c \to 0} RAV = \min (x_i)$	As the scale factor decreases toward zero, the Risk-Adjusted Value approaches the cost of the worst possible cash outflow (the minimum cash inflow).

We call the scale factor c the evaluator's <u>risk capacity</u>.

The last two observations can be restated in common sense ways. If an evaluator uses exponential utility calculations, then:

- An evaluator with very large risk capacity (in relation to the outcomes) is an expected-value decision-maker.

- An evaluator with a very limited risk capacity (in relation to the outcomes) will behave as if the worst possible outcome were certain to occur. This is the premise of Game Theory developed by Von Neumann and others following World War II.

Of course in most practical business situations those who take risks have a risk capacity that is significant in relation to the assumed risks, yet assume risks that are significant in relation to this risk capacity.

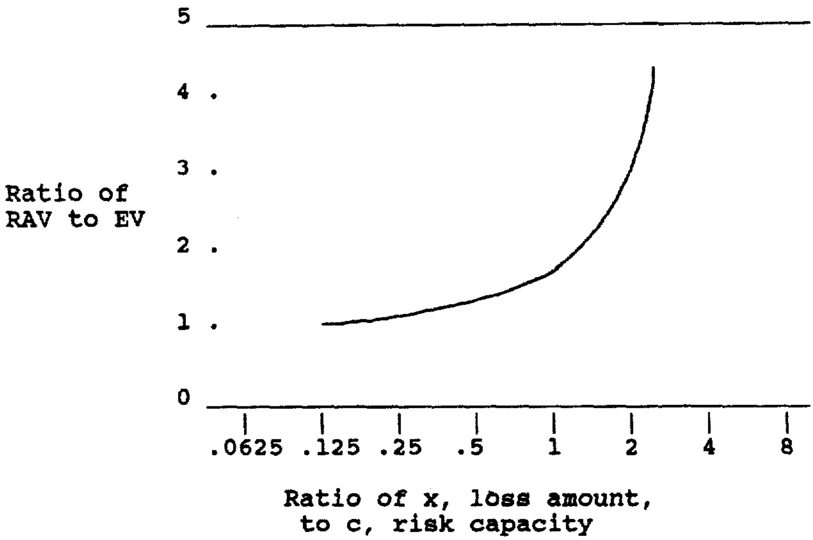
<u>Determining Risk Capacity</u>

From the point of view of one who must carry out the RAV calculations, the risk capacity is no more than the scale factor, c, that must be chosen to calculate a numerical value for RAV. In an important sense, however, a firm's risk capacity is a measure loss. For example, in the presence of a competitive but not overzealous reinsurance market, a company might be expected to buy reinsurance to protect against a financial loss greater than its risk capacity.

Note that the value, x, the risk capacity, c, and the risk-adjusted value, RAV, are all in the same units. If the units of x are cash in U.S. dollars, c and RAV are cash in U.S. dollars. If the units of x are statutory earnings, c and RAV are statutory earnings.

One corollary to the concept of risk capacity that has an intuitive feel is that a "flinch point" exists at which even a small possibility of a loss causes us to move to an attitude of risk aversion. Figure 1 illustrates the "flinch point" by considering the utility of a small chance of a cash outflow of x for various values of risk capacity c. One's flinch point is at roughly twice one's risk capacity. (The curve in Figure 1 was computed by using values of $p(x) = .001$; $EV = .001\ x$; and $RAV = -c \ln(.001 \exp(x/c) + .999)$.)

Figure 1
The Flinch Point



Under exponential utility, the risk-adjusted value of a small chance of losing x increases dramatically when the loss x exceeds about twice the evaluator's risk capacity, c. This is the evaluator's flinch point.

Practical experience suggests that a flinch point or a value of c can be identified for any particular valuation problem. For example, in valuing a company with assets of $10 million it might be appropriate to use a flinch point of $1 million, corresponding to a risk capacity of about $500,000. In valuing a company with assets of $1 billion, it might be appropriate to use a flinch point of $50 million, corresponding to a risk capacity of about

$25 million. These examples reflect an assumption that the large company is more conservative in percentage terms than the small company, yet has greater risk capacity in absolute terms.

Discussion of utility and risk capacity should not give the impression that risk capacity is objective. Neither should we leave the impression that risk capacity is arbitrary. In practice, the selection of an appropriate value for c requires judgment, but c is confined within a certain range by the uncertainties being considered. (E.g., "I'd like to think the ABC company is risk averse, but when I review their underwriting commitments, their risk capacity is clearly between $10 million and $50 million.") The value of risk capacity, c, can be selected with the same degree of confidence that the probabilities can be estimated. Most important for insurance company valuation, a consistent set of valuations for a number of companies can be calculated by consistently setting c to be a simple fraction of assets such as 3%.

Other Utility Functions

The specific results above depend on the exponential utility function. Other utility functions might be considered as well. Our impression is that other utility functions that provide substantially all of the desirable properties offered by the exponential will generally produce results similar to those of the exponential.

V. THE IMPORTANCE OF CASH FLOW

Insurance companies make commitments to pay claims many years in the future. Elaborate accounting conventions have evolved to promote early recognition of financial situations that may lead to later inability to pay claims. These accounting conventions, including both GAAP and Statutory, are continuing to evolve to promote early recognition of situations that may lead to financial difficulty. This paper suggests revisions to GAAP and Statutory accounting to improve their ability to promote early recognition of potential difficulties.

The Committee on Valuation and Related Problems of the Society of Actuaries began studying measures of valuing life insurance companies in 1977. Mateja and Geyer [1] describe the history of work by this committee and its members. As they explain:

> This committee published its landmark paper in 1979[1] which established a conceptual framework for the balance sheet of an insurance enterprise. Assets and liabilities were viewed as cash flow streams. The value assigned to the assets was the present value of all cash

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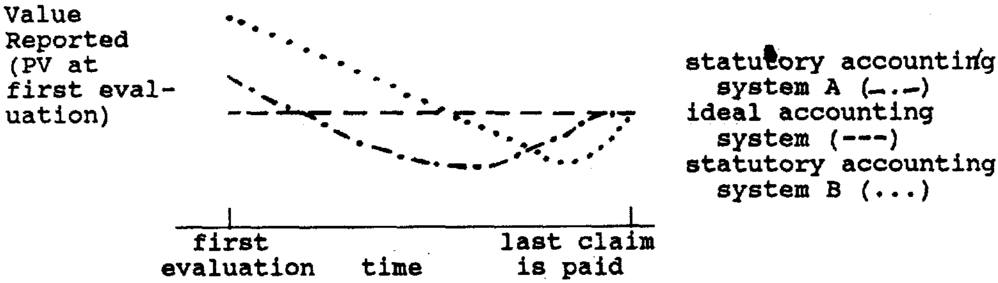
1/ "Valuation, Surplus and Related Problems," Record, Society of Actuaries; Volume 5, No. 1, 1979; Pages 256-284.

flows generated by the assets, and the value assigned to the liabilities was the present value of all cash flows generated by the liabilities. Surplus then was defined in accordance with traditional accounting conventions as the difference between the asset value and the liability value.

Advocates of cash flow measures of financial condition have sometimes been criticized because forecasts of future cash flows are subject to error. Accounts based on historical measures of sales commitment are in some sense more subject to objective audit. This is true, but when the last claim is paid, both cash flow-based measures and "objective" measures must converge to the same result. The track records of three valuation systems are illustrated in Figure 2.

In Figure 2, statutory accounting system A withholds recognition of future investment income but fails to detect a deterioration in reserve adequacy a few years after the first evaluation. Reserves are redundant at first, but insufficient in many years. Statutory system B is almost the same, but the premiums were high enough to lead to statutory reserve penalties in Schedule P for several years.

Figure 2
Accounting Systems Over Time



Accounting systems have several goals. One is to reflect at any point in time the value of the company in light of its future obligations. If the business takes on no new obligations, all accounting systems will eventually produce the same answer.

In short, estimates of future cash flow are subjective but subject to rational analysis. When the last claim is paid, cash flow is objective, and all other accounting systems converge to accumulated cash flow.

Mateja and Geyer [2] demonstrate two important points that we will take as given. These are:

1. Cash flows must be evaluated after-tax and discounted at after-tax rates of return if they are to provide consistent results.

2. The present value of a company's cash flow is the sum of its current cash, the present value of future dividends, and the present value of the increase in its cash.

In these points, present value includes expected value (as it does in the section on expected value above), and all calculations run until the last claim is closed.

Regulatory standards for reserves, and Statutory standards for company valuation, must of course evolve over a period of time. We do not advocate a rapid and radical departure from current statutory accounting conventions. Nontheless, to the extent that statutory accounting can be augmented by regulatory standards for reserves that reflect the risk-adjusted values for future cash flows, the regulatory objectives of Statutory accounting will be better served.

VI. SYNTHESIS OF CASH FLOW AND RAV

The final step in company valuation measuring cash flows in light of uncertain future payments is to synthesize the focus on cash flows into the Risk-Adjusted Value calculations outlined above.

Figure 3 summarizes the risk-adjusted value of the loss payments and investment return cash flows of a hypothetical insurance company. Appendix A gives a brief description of the company and the assumptions about future losses and investment returns. In this simple example, we have considered only four scenarios. The exhibit shows the risk-adjusted value of each cash flow under each scenario and the total risk-adjusted value of the business under all scenarios.

Figure 3 provides a reasonable reflection of the risk associated with the company's reserves and investment strategy. The actuary's best estimate or expected value of loss reserves is $400 million. The present value of the reserves (at the targeted rates of return) is about $320 million. The book value of the assets is $500 million. Ignoring investment opportunity and risk, then, the company would have a value of $180 million. After adjusting for investment opportunity and risk, however, the company has a value of $138 million.

Figure 3

Total RAV of Four Scenarios

	Scenario	Probability of Scenario	Present Value of Accumulated RAV (millions of dollars) 12 Years	22 Years
1.	Low Litigation Low Inflation Low Interest	25%	$190.9	$207.5
2.	Modest Litigation Modest Inflation Modest Interest	25%	162.5	203.3
3.	High Litigation High Inflation High Interest	25%	132.5	197.4
4.	High Litigation High Inflation Low Interest	25%	61.3	98.7
	Total RAV (Eqn. A-6)		$ 98.9	$137.6

Source: Appendix A

The results tabulated in Figure 3 highlight two important points about the risks the company faces. First, the total risk-adjusted value of the future cash flows is much greater if the cash flows in years 13 to 22 are included. This is because all of the scenarios included an assumption that after all claims had been paid the company would be positioned to earn an after-tax return greater than the risk-free return.

Second, the total risk-adjusted value is much less than the simple average of the values for the four scenarios. The computed total risk-adjusted value is strongly influenced by the worst-case scenario because the difference among the scenarios is greater than the company's risk capacity.

Inspection of the supporting details in Appendix A yields another observation relevant to risk and to asset-liability matching. In each scenario there is one point at which the loss payment stream poses a real risk to the company (embodied in year 3), and one point at which the loss payments offset the investment income (embodied in year 6).

The corporate model in Appendix A is quite crude. Corporate models have been widely discussed elsewhere. Our purpose here is merely to illustrate the risk-adjustment calculations.

Large negative cash flows will play a key role in the actual value of RAV(c). Recall that an evaluation with exponential utility with a limited risk capacity in relation to outcomes will give results that correspond to the outcome of the worst possible result. The calculations can be inspected for such values to see the sensitivity of the value of RAV(c) to specific assumptions about the cash flows.

Large negative values of possible cash flow may be due to poor assumptions or to real risks the insurer faces. In the latter case, the company may want to adjust its underwriting or investment portfolio to reduce the magnitude of the possible loss.

To a regulator, one advantage of requesting the risk-adjusted value calculations is that it puts the responsibility for identifying possible adverse situations and possible large negative values of cash flow squarely on the shoulders of the individuals responsible for preparing the regulatory information. Whether the responsibility for this information continues to rest with the officers of the insurance company or is shifted to a consulting actuary, the regulator will benefit greatly from this type of disclosure. In a sense, this is merely an exhaustive way to ask for information which has in the past been asked in a few simple questions in the General Interrogatories of Statutory Blanks.

VII. ARE RAV CALCULATIONS WORTH THE EFFORT?

RAV calculations will be worth the effort only if the advantages of the RAV results are important. These advantages are:

1) The results of the Risk-Adjusted Value calculations are objective in an important sense: different regulators and actuaries working with the same set of assumptions will reach substantially the same results. As a corollary, although different RAV's may be developed by several regulators and actuaries, the differences can be traced to specific differences in assumptions.

   This advantage is held by existing Statutory and GAAP calculations, but not by the judgemental comparisons of surplus levels to underwriting and investment risks which regulators must make in order to use existing Statutory and GAAP values.

2) The RAV calculations in all their detail lead to better understanding of a company's financial strength and the sources of that strength. Those details also help pinpoint events of high possible loss which can be reduced by changes in underwriting and investment strategies. The RAV effort may lead to better management.

3) The RAV approach can be generalized. For example, a regulator reviewing a merger of two insurance companies can consider the RAV of the two companies and of the new consolidated company. A wise merger would have a consolidated RAV greater than the sum of the two separate RAV's.

4) The RAV calculations not only reflect the extent of immunization but show the sensitivity of the valuation to changes in assumptions about underwriting losses and investment returns.

Considering the enormous effort currently spent on solvency regulation, the potentially harmful effects of poor solvency regulation, and the better management that might come from the RAV calculations themselves, the RAV calculations do seem worth the effort.

BIBLIOGRAPHY

[1] James A. Geyer and Michael E. Mateja, "Cash Flow Based Surplus, Draft 7/1," unpublished, dated July 2, 1985.

[2] James A. Geyer and Michael E. Mateja, "Cash Flow Based Surplus," unpublished, dated November 11, 1985.

[3] Raiffa, H., "Decision Analysis - Introductory Lectures on Choices Under Uncertainty," Addison-Wesley Publishing Company, Inc., Reading, Massachusetts.

[4] Cozzolino, John M., "A New Method for Risk Analysis," Sloan Management Review, Massachusetts Institute of Technology, Spring, 1979.

[5] Van Slyke, Oakley E., discussion of "REROSHE: The Concept of a Risk-Free Equivalent Return on Shareholders' Equity," by Alastair G. Longley-Cook, Transactions of the Society of Actuaries, 1983.

[6] Van Slyke, Oakley E., discussion of "Reinsuring the Captive/Specialty Company," by Lee R. Steeneck, Proceedings of the Casualty Actuarial Society, 1985.

APPENDIX A

Risk-Adjusted Value Calculations

Appendix A

Risk-Adjusted Value Calculations

This appendix gives an example of risk-adjusted value calculations for a hypothetical insurance company. The calculations employ the exponential utility function. Only four scenarios are considered in order to simplify the presentation.

1. Description of the Company

The company has sold workers' compensation insurance in California for two years. It has accumulated assets of $500 million. Claim and expense liabilities are reported to be $410 million. The company's reinsurance is with a strong company and limits each loss to $250,000. The company has invested primarily in short-term investments.

2. General Assumptions

The regulator should give exceptional weight to a loss of more than $60 million. This is the "flinch point." The company's risk capacity is about $30 million.'

Loss payments will be more affected by litigation rates than by factors associated with investment income.

Calculations

Each $RAV_{j,t}(c)$ is the risk-adjusted value of all events that might occur at time t under scenario j. This value is:

$$RAV_{j,t}(c) = -c \ln \sum_{\text{all } i} p(i,j|j) \quad e^{-c \frac{x_{i,j,t}}{}} \qquad \text{(A-1)}$$

Consider, for example, the Risk-Adjusted Value of the possible cash flows at time 3 under Scenario 1. This is $RAV_{1,3}$ (c). $RAV_{1,3}$ (c) comprises

$$\begin{array}{l} x_{1,1,3};\ p_{1,1,3} \\ x_{2,1,3};\ p_{2,1,3} \\ \quad\vdots \\ x_{n,1,3};\ p_{n,1,3} \end{array}$$

In this appendix these probability distributions are presumed to be continuous functions, not discrete. For example, the probability distribution in scenario 1 at time 3 is assumed to be a gamma distribution with mean \$75 million and a precision parameter of 4. This continuous probability distribution was used because it is a simple way to reflect both the expected cash flow and the risk of other, adverse cash flows. The RAV of a gamma distribution is (see Van Slyke [5]);

$$RAV = c\alpha \ \ln(1 + mean/c\alpha) \quad \text{(A-2)}$$

Several examples of gamma distributions are shown in Appendix B.

In the general case, $RAV_{j,t}(c)$ comprises the following elements:

$$\begin{array}{l} x_{1,j,t};\ p_{1,j,t} \\ x_{2,j,t};\ p_{2,j,t} \\ \quad\vdots \\ x_{n,j,t};\ p_{n,j,t} \end{array}$$

The amounts and probabilities set forth must reflect all risk if risk is to be treated consistently regardless of its source. A statement such as, "Under Scenario 1 at time 10, there is a 1% chance of a cash inflow of $100," should reflect all sources of uncertainty about the amount of the cash flow. Some of the cash flow may be investment income, some may be late reported premium, some may be administrative expense, some may be loss and loss adjustment expense, and so on. Each source presumably has an expected value and an uncertainty about it. The pairs of x_i, p_i for a particular set of j and t should reflect all sources of cash flow and the uncertainty about that total cash flow.

These calculations reflect a particular way to handle investment risk. As we discussed in Section II, investment risk is an integral part of any present value factor whenever the investment anticipates a return greater than the risk-free rate of return. The discussion of utility in Section IV pointed out that risk aversion implies the evaluator should give greater weight to significant adverse cash flows than to modest adverse cash flows or to favorable cash flows. Each discount factor (the v(t)'s of Section II) is divided into a risk-free present value factor and a utility adjustment for risk.

Of course, the principle of applying a utility function to reflect risk aversion could be applied to loss reserves, and to cash flows associated with losses alone. It is not arithmetically necessary to consider investment income from the asset side as well as loss and loss expense payments from the liability side. But because it is possible to protect an insurance enterprise against interest rate fluctuations (at least to some extent) by a suitable choice of investment maturities and because insurance companies that do get into financial trouble often have their troubles compounded by having to sell assets at values less than book, we recommend that all cash flows be considered.

Within a given scenario, early investments in underwriting may be offset by later gains, or early high premium revenues may be offset by later high loss costs. The present-value calculation should bring together these investment-return situations.

The risk-adjusted values at each point in time for each scenario have been adjusted to present value using a risk-free rate of return. A risk-free rate of return is appropriate because all investment risk has been reflected in the Risk-Adjusted Value.

In the calculations here, reinvested returns were included in the cash flow. The present-value formula was:

$$RAV_j\ (c) = v_n \sum_{t=0}^{n} RAV_{j,t}\ (c) \tag{A-3}$$

If returns had not been reinvested, the present value formula would be:

$$RAV_j(c) = \sum_t v_t\ RAV_{j,t}(c) \tag{A-4}$$

$$= -c \sum_t v_t \ln \sum_{\substack{\text{all } i \\ \text{within } j}} p(i,j|j)\ e^{-\frac{x_{i,j,t}}{c}} \tag{A-5}$$

The overall risk-adjusted value is determined from these RAV_j's. The risk-adjusted value for each scenario must be given proper adjustment for risk and probability. The formula is

$$RAV(c) = -c \ln \sum_j p_j \, e^{\frac{-RAV_j(c)}{c}} \qquad \text{(A-6)}$$

These calculations allow early investments to be offset by later rewards, while preserving the flexibility to envision scenarios in which the anticipated rewards do not materialize.

General Form

The intermediate steps shown above are intended to make the rationale more clear. They are not essential. The formulas for $RAV_j(c)$ and $RAV_{j,t}(c)$ can be replaced with their equivalent expressions in terms of the p_i's and $x_{i,j,t}$'s. The results are:

$$RAV(c) = -c \ln \sum_j p_j \, e^{\sum_t v_t \ln \sum p(i,j|j)\, e^{\frac{x_{i,j,t}}{-c}}} \qquad \text{(A-7)}$$

Every atom of cash flow under any scenario is weighted by the utility function, which in this case is $e^{-x/c}$, and by its probability, which is $p_j p(i,j|j)$.

EXAMPLE OF RAV CALCULATIONS

SCENARIO 1

Low litigation; Low inflation: Low interest

Risk Capacity:	$30 million
Total Losses:	360 million after tax
Initial Assets:	500 million

All cash flows are after tax
Precision denotes the ratio of the square of the mean payment to the variance of the loss payments.
Probability 2x Mean denotes the probability that the cash flow will be within $1 million of twice the mean.
Risk-free rate of return: 4.5 %

	Loss Payments		Investment Income			After-tax cash flow						
Year	%	Amount	% ret.	Amount	End. assets	Mean	Precision	Probability 2x Mean	RAV	Accumulated RAV	Risk-Free Return	Present Value Of Accumulated RAV
0					$500.0					500.0		
1	17	$61	8.5	$39.9	478.7	($21.3)	4	0.011	(23.5)	476.5	0.96	456.0
2	21	75.6	7.0	32.8	435.9	($42.8)	4	0.005	(53.0)	423.6	0.92	387.9
3	28	100.8	6.0	25.3	360.4	($75.5)	4	0.003	(119.0)	304.6	0.88	266.9
4	14	50.4	5.0	17.7	327.7	($32.7)	4	0.007	(38.2)	266.4	0.84	223.4
5	8	28.8	5.0	16.2	315.0	($12.6)	4	0.018	(13.3)	253.0	0.80	203.0
6	5	18	5.0	15.6	312.7	($2.4)	4	0.097	(2.4)	250.6	0.77	192.5
7	2	7.2	5.0	15.6	321.0	$8.4	4	0.027	8.1	258.7	0.73	190.1
8	1	3.6	5.0	16.0	333.5	$12.4	4	0.018	11.8	270.6	0.70	190.2
9	1	3.6	5.0	16.6	346.5	$13.0	4	0.018	12.4	282.9	0.67	190.4
10	1	3.6	5.0	17.3	360.2	$13.7	4	0.017	13.0	295.9	0.64	190.5
11	1	3.6	5.0	18.0	374.6	$14.4	4	0.016	13.6	309.5	0.62	190.7
12	1	3.6	5.0	18.7	389.7	$15.1	4	0.015	14.2	323.7	0.59	190.9
13		0	5.0	19.5	409.2	$19.5	4	0.012	18.1	341.8	0.56	192.9
14		0	5.0	20.5	429.7	$20.5	4	0.011	18.9	360.7	0.54	194.8
15		0	5.0	21.5	451.1	$21.5	4	0.011	19.8	380.4	0.52	196.6
16		0	5.0	22.6	473.7	$22.6	4	0.010	20.7	401.1	0.49	198.3
17		0	5.0	23.7	497.4	$23.7	4	0.010	21.6	422.7	0.47	200.0
18		0	5.0	24.9	522.3	$24.9	4	0.009	22.6	445.3	0.45	201.6
19		0	5.0	26.1	548.4	$26.1	4	0.009	23.6	469.0	0.43	203.2
20		0	5.0	27.4	575.8	$27.4	4	0.008	24.7	493.6	0.41	204.7
21		0	5.0	28.8	604.6	$28.8	4	0.008	25.8	519.5	0.40	206.1
22		0	5.0	30.2	634.8	$30.2	4	0.008	27.0	546.4	0.38	207.5

EXAMPLE OF RAV CALCULATIONS

SCENARIO 2

Modest litigation; Modest inflation: Modest interest

Risk Capacity:	$30 million
Total Losses:	400 million after tax
Initial Assets:	500 million

All cash flows are after tax
Precision denotes the ratio of the square of the mean payment to the variance of the loss payments.
Probability 2x Mean denotes the probability that the cash flow will be within $1 million of twice the mean.
Risk-free rate of return: 4.5 %

	Loss Payments		Investment Income			After-tax cash flow						Present Value
Year	%	Amount	% ret.	Amount	End. assets	Mean	Precision	Probability 2x Mean	RAV	Accumulated RAV	Risk-Free Return	Of Accumulated RAV
0					$500.0					500.0		
1	17	$68	8.5	$39.6	471.6	($28.4)	4	0.008	(32.4)	467.6	0.96	447.5
2	21	84	7.0	32.3	419.9	($51.7)	4	0.004	(67.7)	399.9	0.92	366.2
3	28	112	7.0	28.4	336.3	($83.6)	4	0.003	(143.1)	256.8	0.88	225.1
4	14	56	6.0	19.8	300.1	($36.2)	4	0.006	(43.1)	213.7	0.84	179.2
5	8	32	6.0	17.8	285.8	($14.2)	4	0.016	(15.2)	198.5	0.80	159.3
6	5	20	6.0	17.0	282.8	($3.0)	4	0.076	(3.0)	195.5	0.77	150.1
7	2	8	6.0	16.9	291.7	$8.9	4	0.026	8.6	204.1	0.73	150.0
8	1	4	6.0	17.5	305.2	$13.5	4	0.017	12.8	216.8	0.70	152.5
9	1	4	6.0	18.3	319.5	$14.3	4	0.016	13.5	230.3	0.67	155.0
10	1	4	6.0	19.1	334.6	$15.1	4	0.015	14.3	244.6	0.64	157.5
11	1	4	6.0	20.0	350.7	$16.0	4	0.014	15.1	259.7	0.62	160.0
12	1	4	6.0	21.0	367.7	$17.0	4	0.013	15.9	275.6	0.59	162.5
13		0	6.0	22.1	389.7	$22.1	4	0.010	20.3	295.8	0.56	166.9
14		0	6.0	23.4	413.1	$23.4	4	0.010	21.4	317.2	0.54	171.3
15		0	6.0	24.8	437.9	$24.8	4	0.009	22.5	339.7	0.52	175.5
16		0	6.0	26.3	464.2	$26.3	4	0.009	23.8	363.5	0.49	179.7
17		0	6.0	27.9	492.0	$27.9	4	0.008	25.0	388.5	0.47	183.8
18		0	6.0	29.5	521.6	$29.5	4	0.008	26.4	414.9	0.45	187.9
19		0	6.0	31.3	552.9	$31.3	4	0.007	27.8	442.7	0.43	191.8
20		0	6.0	33.2	586.0	$33.2	4	0.007	29.3	472.0	0.41	195.7
21		0	6.0	35.2	621.2	$35.2	4	0.007	30.8	502.9	0.40	199.5
22		0	6.0	37.3	658.5	$37.3	4	0.006	32.5	535.3	0.38	203.3

EXAMPLE OF RAV CALCULATIONS

SCENARIO 3

High litigation; High inflation: High interest

Risk Capacity: $30 million
Total Losses: 440 million after tax
Initial Assets: 500 million

All cash flows are after tax
Precision denotes the ratio of the square of the mean payment to the variance of the loss payments.
Probability 2x Mean denotes the probability that the cash flow will be within $1 million of twice the mean.
Risk-free rate of return: 4.5 %

	Loss Payments		Investment Income			After-tax cash flow						
Year	%	Amount	% ret.	Amount	End. assets	Mean	Precision	Probability 2x Mean	RAV	Accumulated RAV	Risk-Free Return	Present Value Of Accumulated RAV
0					$500.0					500.0		
1	17	$75	8.5	$39.3	464.5	($35.5)	4	0.006	(42.1)	457.9	0.96	438.2
2	21	92.4	8.0	36.3	408.4	($56.1)	4	0.004	(75.6)	382.4	0.92	350.1
3	28	123.2	8.0	31.6	316.8	($91.6)	4	0.002	(173.1)	209.2	0.88	183.4
4	14	61.6	7.0	21.7	276.9	($39.9)	4	0.006	(48.5)	160.7	0.84	134.8
5	8	35.2	7.0	19.1	260.8	($16.1)	4	0.014	(17.3)	143.4	0.80	115.1
6	5	22	7.0	18.1	256.9	($3.9)	4	0.058	(4.0)	139.4	0.77	107.1
7	2	8.8	7.0	17.9	266.0	$9.1	4	0.025	8.8	148.2	0.73	108.9
8	1	4.4	7.0	18.6	280.2	$14.2	4	0.016	13.4	161.6	0.70	113.7
9	1	4.4	7.0	19.6	295.3	$15.2	4	0.015	14.3	175.9	0.67	118.4
10	1	4.4	7.0	20.6	311.6	$16.2	4	0.014	15.2	191.1	0.64	123.1
11	1	4.4	7.0	21.8	328.9	$17.4	4	0.013	16.2	207.4	0.62	127.8
12	1	4.4	7.0	23.0	347.5	$18.6	4	0.012	17.3	224.7	0.59	132.5
13		0	7.0	24.3	371.9	$24.3	4	0.009	22.2	246.8	0.56	139.3
14		0	7.0	26.0	397.9	$26.0	4	0.009	23.6	270.4	0.54	146.0
15		0	7.0	27.9	425.8	$27.9	4	0.008	25.0	295.4	0.52	152.6
16		0	7.0	29.8	455.6	$29.8	4	0.008	26.6	322.0	0.49	159.2
17		0	7.0	31.9	487.4	$31.9	4	0.007	28.3	350.3	0.47	165.8
18		0	7.0	34.1	521.6	$34.1	4	0.007	30.0	380.3	0.45	172.2
19		0	7.0	36.5	558.1	$36.5	4	0.006	31.9	412.2	0.43	178.6
20		0	7.0	39.1	597.1	$39.1	4	0.006	33.8	446.0	0.41	184.9
21		0	7.0	41.8	638.9	$41.8	4	0.005	35.9	481.9	0.40	191.2
22		0	7.0	44.7	683.7	$44.7	4	0.005	38.0	519.9	0.38	197.4

EXAMPLE OF RAV CALCULATIONS

SCENARIO 4

High litigation; High inflation: Low interest

Risk Capacity: $30 million
Total Losses: 440 million after tax
Initial Assets: 500 million

All cash flows are after tax
Precision denotes the ratio of the square of the mean payment to the variance of the loss payments.
Probability 2x Mean denotes the probability that the cash flow will be within $1 million of twice the mean.
Risk-free rate of return: 4.5 %

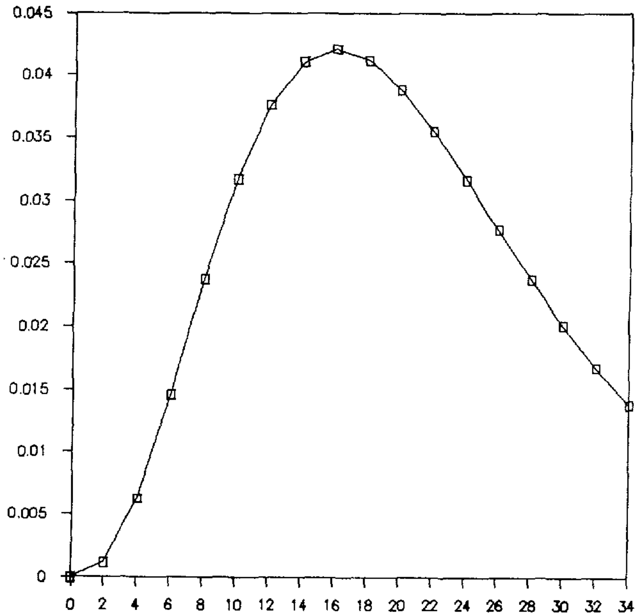
	Loss Payments		Investment Income			After-tax cash flow						
Year	%	Amount	% ret.	Amount	End. assets	Mean	Precision	Probability 2x Mean	RAV	Accumulated RAV	Risk-Free Return	Present Value Of Accumulated RAV
0					$500.0					500.0		
1	17	$75	8.5	$39.3	464.5	($35.5)	4	0.006	(42.1)	457.9	0.96	438.2
2	21	92.4	7.0	31.8	403.9	($60.6)	4	0.004	(84.4)	373.5	0.92	342.0
3	28	123.2	6.0	23.4	304.1	($99.8)	4	0.002	(213.9)	159.7	0.88	139.9
4	14	61.6	5.0	14.9	257.4	($46.7)	4	0.005	(59.2)	100.4	0.84	84.2
5	8	35.2	5.0	12.7	234.8	($22.5)	4	0.010	(25.0)	75.5	0.80	60.6
6	5	22	5.0	11.6	224.4	($10.4)	4	0.022	(10.9)	64.6	0.77	49.6
7	2	8.8	5.0	11.2	226.8	$2.4	4	0.097	2.3	67.0	0.73	49.2
8	1	4.4	5.0	11.3	233.7	$6.9	4	0.033	6.7	73.7	0.70	51.8
9	1	4.4	5.0	11.7	241.0	$7.3	4	0.032	7.0	80.7	0.67	54.3
10	1	4.4	5.0	12.0	248.6	$7.6	4	0.030	7.4	88.1	0.64	56.8
11	1	4.4	5.0	12.4	256.6	$8.0	4	0.029	7.7	95.9	0.62	59.1
12	1	4.4	5.0	12.8	265.0	$8.4	4	0.027	8.1	104.0	0.59	61.3
13		0	5.0	13.3	278.3	$13.3	4	0.017	12.6	116.6	0.56	65.8
14		0	5.0	13.9	292.2	$13.9	4	0.016	13.2	129.7	0.54	70.1
15		0	5.0	14.6	306.8	$14.6	4	0.016	13.8	143.5	0.52	74.2
16		0	5.0	15.3	322.1	$15.3	4	0.015	14.4	158.0	0.49	78.1
17		0	5.0	16.1	338.2	$16.1	4	0.014	15.1	173.1	0.47	81.9
18		0	5.0	16.9	355.1	$16.9	4	0.014	15.8	188.9	0.45	85.5
19		0	5.0	17.8	372.9	$17.8	4	0.013	16.6	205.5	0.43	89.0
20		0	5.0	18.6	391.6	$18.6	4	0.012	17.3	222.8	0.41	92.4
21		0	5.0	19.6	411.1	$19.6	4	0.012	18.1	240.9	0.40	95.6
22		0	5.0	20.6	431.7	$20.6	4	0.011	19.0	259.9	0.38	98.7

APPENDIX B

Examples of Gamma Distributions

Distribution of Possible Negative Cash Flows

Scenario 1
Year 1



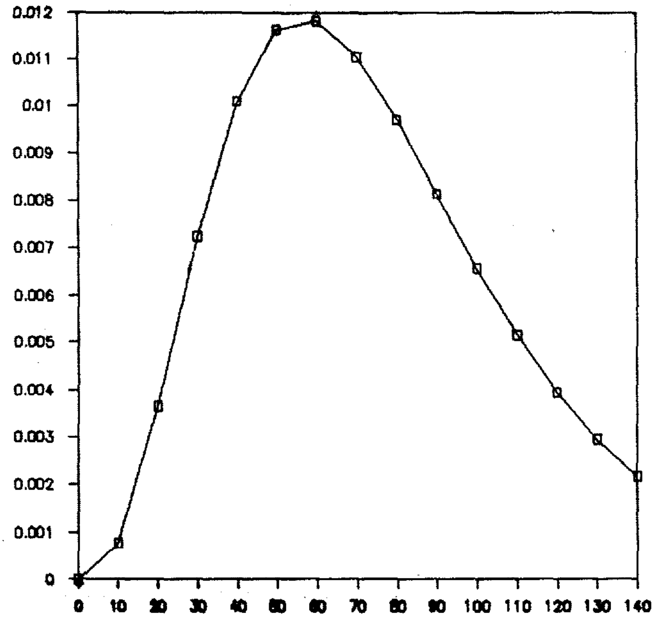
(millions of dollars)

Mean = $21.3 million

Precision (α) = 4

Distribution of Possible Negative Cash Flows

Scenario 1
Year 3



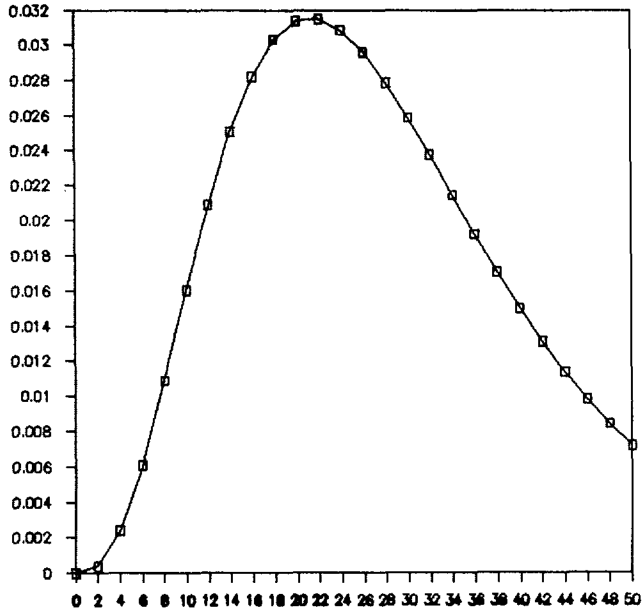
(millions of dollars)

Mean = $75.5 million

Precision (α) = 4

Distribution of Possible Negative Cash Flows

Scenario 2
Year 1



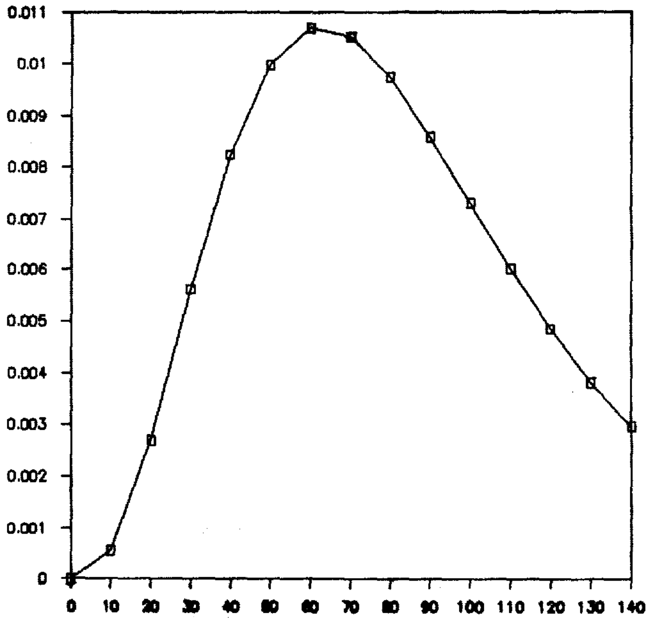
(millions of dollars)

Mean = \$28.4 million

Precision (α) = 4

Distribution of Possible Negative Cash Flows

Scenario 2
Year 3

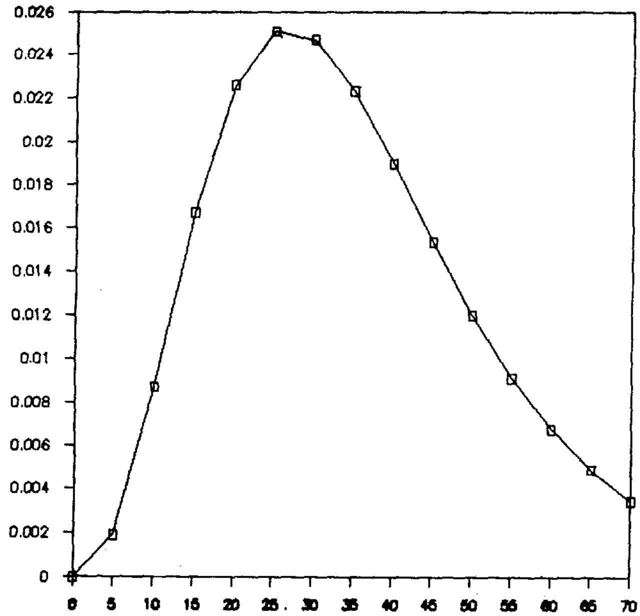


Mean = $83.6 million

Precision (α) = 4

Distribution of Possible Negative Cash Flows

Scenario 3
Year 1



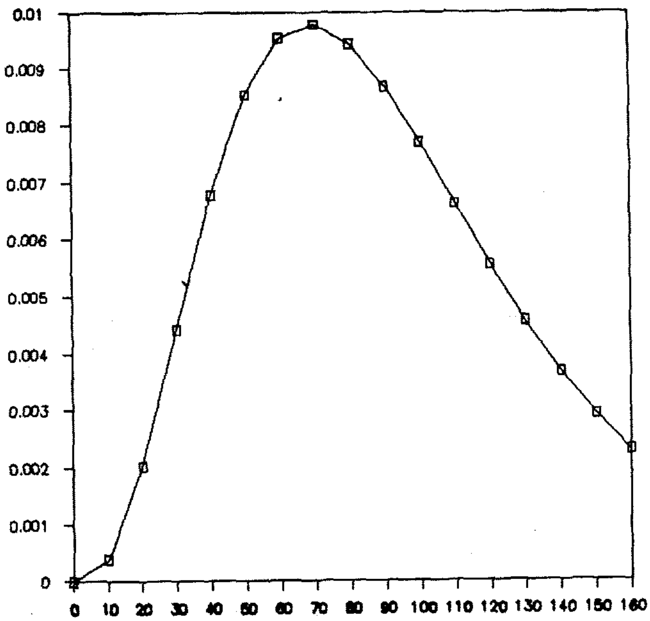
(millions of dollars)

Mean = $35.5 million

Precision (α) = 4

Distribution of Possible Negative Cash Flows

Scenario 3
Year 3



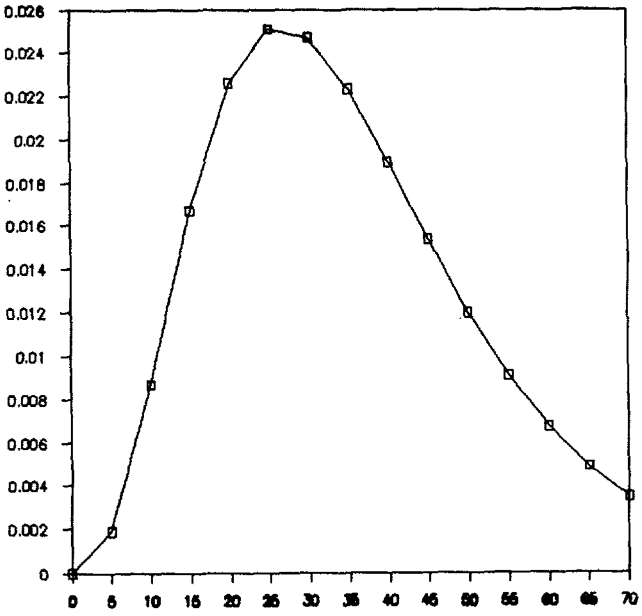
(millions of dollars)

Mean = $91.6 million

Precision (α) = 4

<u>Distribution of Possible Negative Cash Flows</u>

Scenario 4
Year 1



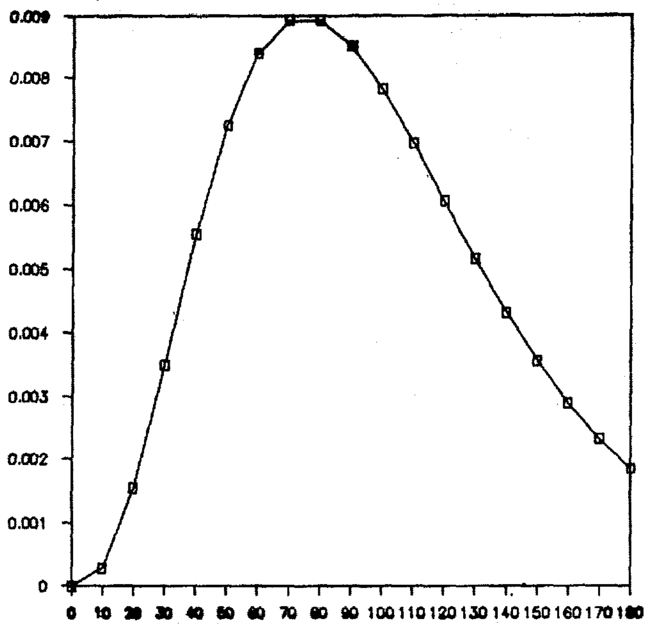
(millions of dollars)

Mean = $35.5 million

Precision (α) = 4

Distribution of Possible Negative Cash Flows

Scenario 4
Year 3



(millions of dollars)

Mean = $99.8 million

Precision (α) = 4