RESERVE REVIEW OF A REINSURANCE COMPANY

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ABSTRACT

The estimation of reserves for a reinsurance company is conceptually similar to the estimation of reserves for a primary insurance company. However, due to a number of differences between primary and reinsurance companies, the actual practice of reserving for reinsurance companies involves a process which is quite distinct from methods commonly used for primary companies. In this discussion paper we hope to accomplish two goals:

- discuss a reserving approach which is appropriate for reinsurance companies, and

- provide a methodology for analyzing development patterns which is especially applicable to reinsurance companies, but also has applications for primary companies.

In this paper we will assume the reader is generally familiar with reserving practices for primary companies. We will focus on those situations which are unique or different for reinsurance companies.

I. RESERVING CONSIDERATIONS

The first step in any review of reserves for a reinsurance company is to define the scope of the project.[1] At one end of a spectrum, we may perform a cursory review of results, intended primarily to highlight potential problem areas, but not intended to produce reserve recommendations. Another possibility would be a methodology review, in which the actuary might review the procedures used by the company and the data used in these procedures for reasonableness, but with little emphasis on the bottom-line reserve. At the other end of the spectrum, a company may wish to consider a combination of portfolio transfers and commutations.[2] In this situation, the company needs more than a single, bottom-line reserve estimate; it also needs anticipated payout patterns associated with the liabilities, and possibly reserve estimates and payout patterns for individual treaties, or small groups of treaties.

In this discussion paper, we will assume that the company requires an analysis which will produce a reserve estimate for financial statement purposes, but does not require the detailed information necessary for commutation or portfolio transfer analysis.

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[1] This paper is written from the point of view of an external consultant; the discussion would also apply generally to an in-house actuary.

[2] Portfolio transfers involve transfer of liabilities forward to an unrelated party, commutations involve transfer of liabilities back to the original ceding company.

Once the project scope has been defined, it is necessary to obtain details of the company's operation and the types of business that it has written. This information will generally be gathered from several types of sources:

- Public sources - such as Best's reports, and annual statements;
- Internal company documents;
- Interviews with company personnel;
- External company documents and personnel, e.g., ceding companies or intermediaries.

There are a large number of questions which should be asked in any reserve analysis. Rather than repeat them here, the reader is referred to Berquist and Sherman [1]. In this paper, we will concentrate on those issues which are most relevant to a reinsurance company.

DATA GROUPING

In any reserve review, one of the goals is to subdivide the company into a reasonable number of pieces that can each be analyzed separately. Each piece should be as homogeneous as is practical, yet large enough that random fluctuations will not materially distort the results.

In primary companies, this division is usually performed along lines of business. The major decisions involve what level of grouping of lines is appropriate. In a reinsurance company, the situation is very different. There are a large variety of ways in which reinsurance business can be categorized:

- Location of insured exposures (country or group of countries)
- Currency[3]
- Line of Business
- Form - Quota Share, Excess of Loss, Stop Loss
- Layer - Primary, Working Excess, High layer excess, Catastrophic covers
- Facultative versus Treaty
- Accounting Basis - Earned/Incurred versus Written/Paid
- Other Special Forms - e.g., Portfolio Transfers
- Active versus Cancelled Treaties

This list is far from exhaustive. Any particular company will have its own way of categorizing business which may add to the above list. For example, the company may have formed the business into pools, for the purpose of securing retrocessional coverage. In many cases, the formation of these pools does not conform to any one of the categorizations listed above. It may

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[3] Not necessarily the same as country. For example, many reinsurance contracts are denominated in $US or £ even though the exposures may be in other countries.

be necessary to analyze reserves separately by pool to produce results which can be reported to retrocessionnaires.

It is clearly impractical (using current techniques) to maintain all categories simultaneously. This would result in a categorization encompassing at least nine dimensions. The total number of categories could easily reach several hundred, most of which are likely to contain only one or two pieces of business. (The reader may recall Longley-Cook's, [4] delightful analogy - "We may liken our statistics to a large crumbly loaf cake, which we may cut in slices to obtain easily edible helpings. The method of slicing may be chosen in different ways--across the cake, lengthwise down the cake, or even in horizontal slices--but only one method of slicing may be used at a time. If we try to slice the cake more than one way at a time, we shall be left with a useless collection of crumbs.")

Before we discuss ways to reduce the number of categories, it will be helpful to review reasons for using categories at all. Some of the reasons include:

- **Improved Estimates** - Categories which are homogeneous will exhibit more stable development patterns and more reliable estimates of ultimate losses.

- **Statutory Reporting** - Statutory rules require reporting of results along statutory lines of business.

- **Contractual Reporting** - Retrocessional agreements require reporting of results for business covered by the reinsurance contract.

- **External Relevant Data** - There may be relevant development factors from outside sources which could be used if the data is organized in a similar fashion.

Any attempt to combine or eliminate categories should be done with the above goals or requirements in mind.

Each of these possible categorizations will be discussed in turn:

Location

Reporting patterns typically vary for exposures from foreign countries due to accounting lags. It should be emphasized that the location of the broker is often as important as the location of the insured. For example, U. S. insureds may be written by a U. S. company who purchases reinsurance through a London broker. The premium and loss reporting may be subject to longer lags than if the business had been placed by a U. S. broker.

Another reason for segregating business by country is that many retrocessionnaires may restrict coverage to U. S. business (sometimes U. S. and Canadian).[4]

Development patterns also differ by country. At present, reporting and payment patterns for liability coverages in the U. S. are significantly slower than most other countries. The counterpart of workers' compensation in other countries may differ considerably in terms of coverage characteristics and reporting and payment patterns.

Currency

International reinsurance transactions involving foreign currencies will need to be converted to U. S. dollars for financial reporting purposes. This conversion process can introduce material distortions into the reserve analysis. Typically, premiums will be converted to $US at the conversion rate in effect at the time they are written, paid losses will use the conversion rate in effect at the time the loss is paid, and outstanding losses will be converted based upon the financial statement date. Note that this means that incurred losses in a financial statement will not all be converted at the same rate. In addition, there may be departures from this practice. Some companies use the conversion rate in effect on the day a case

[4] More generally, it is common for some retrocessionnaires to impose limitations on the type of business they will accept. Typically these include certain lines of business, maximum (or minimum) attachment points, as well as location.

reserve is established, and then a more current conversion rate only on the change in reserve, not on the entire outstanding. Thus, a reserve at any point in time might be the result of several different conversion rates. Some companies use conversion rates which are spelled out in the contract.

The analysis of premium and loss development should be done with a data base that eliminates the distortions due to changing exchange rates. One method is to analyze the entire treaty in the original currency, and then apply conversion rates as a final step. A more common approach is to maintain data such that the entire incurred can be converted at a common exchange rate. This will ensure that development of premiums and losses will be true development and not due to fluctuating exchange rates. Paid losses will also have to be calculated at the conversion rate in effect at the time of payment so that financial statements will be correct. Because loss amounts may have to be available at more than one conversion rate (an historical rate for financial reporting purposes, and a current rate for reserve analysis), the analyst will have to be especially careful to use the correct values.

The advantage of converting to a common currency before analysis is that it allows combination of treaties. This may help remove one level of categorization.

Line of Business

It is rare to have line of business detail even approaching the level of detail available for most primary companies. A typical line of business breakdown for a small to medium reinsurer might include the following:

- Property
- Casualty
- Property/Casualty Combined
- Marine
- Other

Many of the larger reinsurers will have more detail available, but there usually are some significant treaties which cover broad categories of business. This makes allocation of results to statutory lines questionable at best. (This subject will be discussed in more detail later.)

Form

Reinsurance can generally be classed into two forms - either proportional or excess. Proportional forms include quota share and surplus share. Excess forms include excess of loss, aggregate excess (also called stop loss or aggregate stop loss), and catastrophe.

From a reserving point of view, policies written on a proportional form will have loss reporting and loss payment patterns similar to the patterns applicable to the underlying primary companies, although the patterns may be slower due to the accounting lag associated with the reporting of results from the primary company to the reinsurer. Excess forms will have a slower pattern because smaller losses are reported and paid faster (on average) than larger losses. Thus, the reinsured portion contains a disproportionate share of the slower reporting losses. This factor is in addition to the accounting lag which also affects excess policies.

Layer

A layer refers to the range of losses covered by the reinsurance contract. For example, if the contract covers the amount between $1,000,000 per occurrence and $6,000,000 per occurrence, this is referred to as a $5 million layer. The $1,000,000 is referred to as an attachment point. The coverage is usually written $5MM x $1MM, or $5M xs $1M. (Unfortunately, the letter M is used by some to mean thousands, by others to mean millions.)

Many combinations of layer size and attachment point are seen in practice. These can loosely be categorized into primary, working excess, and high excess layers, depending on the attachment point. Primary layers have an attachment point of zero, or some fairly small value. Working layers have attachment points over which the ceding company expects to have a number of claims each year. High excess layers are layers in which losses are not expected in every year.

Development patterns will generally be slower as the attachment point is raised. In theory, a formula might be derived which would specify the adjustment needed for any value of an attachment point but this author is not aware of any such study. Instead, business is categorized into the above rough categories and development factors are estimated for each group.

Facultative versus Treaty

Treaty business refers to a contract which provides automatic coverage of a defined portion of the business written by a company, typically all business written in specified classes such as casualty. Facultative business is written on specific individual risks.

While facultative business can be categorized into the same groups as treaty business, it is often necessary to keep facultative business separate from treaty because the pricing may be very different. In addition a block of facultative business will have a wide variety of attachment points and layers, while a particular treaty or group of treaties may have more homogeneous characteristics.

Accounting Basis

Development patterns can vary materially depending on the accounting basis of the contract. Some contracts are on a written/paid basis, meaning that premiums are remitted from the insurer to the reinsurer as they are written, and losses are received on a paid basis. Some contracts are on an earned/incurred basis, which technically means that premiums are remitted only as they are earned while losses are received on an incurred basis. In practice, losses are often still handled on a paid basis, with the outstanding being secured by a letter of credit.

Reinsurance reporting forms sometimes simply use the terms "premiums" and "losses" without further definition. The analyst must be careful to ascertain the correct nature of the data.

Other Special Forms

Portfolio transfers and commutations have become increasingly common in recent years. A portfolio transfer on a treaty or group of treaties rarely transfers the ultimate liability without limit. Thus, it will be necessary to determine if the current estimate of the ultimate liability exceeds the amounts transferred. Loss development data should be restated to eliminate any distortions that may have occurred due to the transaction. If a commutation is effected, the historical data may also have to be restated. A complicating factor is that there are differences of opinion as to the proper accounting treatment of these contracts.

The analyst should also keep in mind that portfolio transfers can be made from one policy year to the next, and some international contracts may be cancellable without runoff liability.

Active versus Cancelled Treaties

Reinsurers periodically undertake significant "reunderwriting" of their books of business. This typically includes cancellation of a large number of treaties. It may be natural for a company to track the active and cancelled treaties separately. While this may make sense in some situations, there are potential problems. In subsequent updates to data, a decision will have to be made whether the block of cancelled contracts will remain fixed. For example, if the split between active and cancelled treaties is first made on December 31, 1984, what will be done with the treaties which are cancelled during 1985? If they are included in the definition of cancelled treaties, then the runoff statistics and loss ratios may be meaningless for comparison purposes. If these treaties are not included with cancelled treaties, then we have the awkward situation that the category "active" treaties includes some cancelled treaties.

Another problem which may arise is the temptation to use the overall historical development on active treaties alone to project future development on these treaties.

It may still be appropriate to use total historical development on active treaties, depending on the reason for cancellation.

If cancellation was based on loss ratio, then it may be reasonable to assume that the non-cancelled treaties will have similar development. If cancellation was based on adverse development, then one must determine a development pattern for the active contracts. It is unlikely that sufficient historical experience exists to determine a development pattern solely based on active contracts. In addition, there is the possibility that the business underlying active contracts has changed over time.

OTHER KEY DIFFERENCES

The organization of data into groupings can be very different for reinsurance companies compared to primary companies as was shown in the previous section. There are a number of other items which are especially relevant to the reinsurance reserving analyst.

Premium Development

Premium development[5] is a critical item in reinsurance reserving for two main reasons. First, reserving techniques which use premiums (loss ratio, Bornhuetter-Ferguson) are more important in reinsurance reserving than for primary companies. Second, premium development is generally insignificant for primary companies[6] beyond 24 months. Premium development factors for reinsurance companies are often needed beyond 60 months, and often exceed 3.0 at 12 months, on an underwriting year basis.

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[5] For a more in-depth discussion of premium development, see Collins [3]. A discussion of alternative accounting treatments of premium development is included in Miccolis [6].

[6] An important exception is retro premiums, but that occurs only because they are a function of losses.

Premium development occurs primarily due to accounting lags in the reporting of premium. An original piece of business may be ceded as part of a treaty to a reinsurer, which in turn may be partially ceded to a retrocessionnaire. There often may be several levels of reinsurers. Each company will receive premium from its insureds and will pass on a portion to its reinsurer. There is typically a one quarter (3 months) lag between receiving premium and forwarding on the reinsurance premium. When several levels are involved, years can elapse.

Another reason for the long lag is due to the nature of an underwriting year. This will be discussed in the next section.

Losses are also subject to the same accounting lags in most cases. However, many treaties have provisions (typically referred to as "cash calls") which allow more rapid reporting of losses over a specified size, such as $50,000. It is not impossible for a property treaty to have premium development slower than loss development, if cash calls have occurred.

Underwriting Year

The concept of underwriting year is an important one in reinsurance reserving. An underwriting year consists of all treaties issued by a reinsurance company during the year. A typical treaty will cover the risks of a primary company. If the treaty is on a "risks attaching" basis, it will cover all policies issued by the primary company during the term of the treaty. The exposure arising from a one-year policy issued by a primary company on the last day of the treaty, for a treaty which incepts on the last day of the underwriting year, will extend three years from the beginning of the underwriting year.

More extreme examples can occur involving longer term policies, or more "layers" of reinsurers between the ultimate reinsurer and the primary company.

The actual pattern of exposures tends to vary more from the "ideal" underwriting year than is true for policy or accident years. There are several reasons for this:

- While treaties can incept on any day of the year, the most common date is January 1, followed by July 1. Typically, 50% to 75% of all treaties incept on one of these two dates.

- Three-year policies may be issued under some treaties, although this is becoming less common.

- Some treaties (particularly excess covers) are on a "losses occurring" basis, which means they cover all losses with an occurrence date during the treaty, rather than all policies with an inception date during the treaty.

- Some treaties have a provision which allows the reinsured to extend a policy period (typically up to a total of 18 months) in order to coordinate anniversary dates.

- In some cases, a treaty issued late in a year may be assigned to the subsequent underwriting year.

For the above reasons it may be necessary to determine the distribution of exposures within an underwriting year on a case-by-case basis. This distribution is helpful in determining reasonable development factors. For example, if exposures are weighted heavily toward the end of the underwriting year, loss development (and premium development) factors are likely to be higher. (In Section II, we discuss a more formal approach to the calculation of development factors.)

Lack of Data

Perhaps the most severe problem in reinsurance reserving is lack of useful data.

The shortage of data arises for several reasons.

High excess layers, catastrophe covers and clash covers are not expected to have many losses. Many treaties might expect only one or two incidents in a particular year. Traditional development factor methods depend on a relatively large number of claims to produce reasonable estimates.

Reinsurance statistical gathering is generally not as good as for primary companies. Many commercially available data processing systems for collecting statistical information are designed with primary companies in mind. These systems are often inadequate for reinsurance companies.

Information is often reported on a bordereau basis. This is typically a quarterly report which contains only summary premium and loss information. IBNR and bulk reserves as set by the reinsured are sometimes reported, but not always. Individual claim detail is seldom available. This means that many important actuarial techniques are not available - specifically, any techniques based upon counts or average claim sizes.

Another problem arises from the fact that the bulk of the claim reserving is performed at the primary company level. Reinsurers have claim departments, but they cannot afford to investigate each individual claim to the extent that a primary company can. This means that a reinsurance company's data is a mixture of results from reinsureds with a variety of case reserving methodologies and approaches.

Finally, the delayed reporting of information due to the accounting lags inherent in the system means that the data that is available tends to be much less current than for primary companies.

RESERVING METHODS

Many of the same reserving methods used by primary companies can be used by reinsurance companies. Common reserving techniques include:

- Paid Loss Development
- Incurred Loss Development
- Bornhuetter-Ferguson (can be applied on a paid or incurred basis)
- Loss Ratio

It is far easier to identify weaknesses associated with each method than to identify strengths. Paid loss development factors can become extraordinarily large due to the extremely slow payment patterns on many reinsurance books of business. Incurred loss development techniques suffer from the inconsistency of case reserving techniques and the inadequate collection of data in many cases. It is not unusual to have evaluation dates for which no change in case reserves has been received. Only cash items, such as losses paid and premiums written, may have been reported.

The loss ratio method's primary weakness is its ease of abuse. In theory, the loss ratio method can be a very reasonable and useful method. In practice, it is often used to reach a predetermined, and often optimistic, result. In many cases, the loss ratio is chosen simply by subtracting an expense loading from 100%. A more proper use of the loss ratio method is based upon actuarial analysis of the underlying pricing. In simple terms, the starting point is a base with which there is some degree of comfort regarding the overall appropriate loss ratio. This might include several older years of experience, where the losses are reaching a level of maturity, or it might be a primary layer of coverage. If adjustments are made for intervening inflation, changes in coverage and difference in limits, the results should be a reasonable current price for the coverage. This value, divided by the actual premium charged for the coverage, produces an estimated loss ratio. At early evaluation dates, this loss ratio may produce better reserves than a development technique.

The Bornhuetter-Ferguson technique represents a blend of the loss ratio and development techniques. While this combination is arguably the best combination of techniques, it is not a panacea. Erratic loss development and inappropriate loss ratios will generate unrealistic reserves no matter what technique is used. (For those unfamiliar with this technique, a brief review is provided in Appendix C.)

It is important to compare actual and expected losses for each underwriting year and to determine whether any differences indicate an inaccurate reporting pattern or initial expected loss ratio assumption. Of course, the same level of expected losses can be produced by an infinite number of loss ratio and reporting pattern assumptions. One must decide realistically whether a given level of reported losses is the result of a low loss ratio and a fast reporting pattern, or a high loss ratio and a slower reporting pattern, or something in between.

The most important statement we can make about the various methods is that one should never rely on any one of them alone. As many of these techniques should be used as possible, as well as other techniques which might be applicable depending upon data availability.

LINE OF BUSINESS

The decomposition of reinsurance experience into statutory lines of business is a valuable aid in reserve analysis. Loss development patterns are available from a variety of sources (Best's, RAA) which are segregated by major line of business. These patterns can be quite dissimilar. In some cases, loss detail will be available directly by line, but in other cases, some detective work may be needed. The reserve analyst should be cautioned not to accept loss information segregated by line blindly. In many cases, the breakdown may have been calculated by an allocation system. Nevertheless, even an approximate allocation of losses may be better than no breakdown at all. There are a number of ways to estimate the appropriate line of business breakdown.

In most cases, a ceding company or broker will supply an EPI, or estimated premium income. This is intended to provide an estimate of the amount of premium to be anticipated under the treaty. In some cases, the EPI values will be broken down by line.

In some cases, the actual premium reported can be segregated by line, even though losses cannot be similarly segregated.

The policy wording of the treaty should be helpful, as it typically states the classes of business accepted under the treaty. Unfortunately, a common description is "all classes of business written by John Doe," where John Doe is the name of an MGA or ceding company.

Underwriting files may be helpful, as they may contain a history of this treaty including some line of business detail. Direct discussions with the broker or producer of the business can sometimes produce line of business profiles. The value of this approach should not be underestimated. It is common for many reinsurers to accept a block of business based upon little more than the (presumed) reputation of the source. In these cases, the reinsurer's files may contain only limited information. Direct contact with the underwriters actually involved in the day-to-day selection of business may be the only reasonable way to understand the nature of the business.

Individual large losses may also be helpful. Although it would be dangerous to extrapolate an entire line of business profile from a few large losses, general liability losses are occasionally observed on a treaty described as property only. On the basis of that evidence, we reevaluated the line of business breakdown.

EXAMPLE

The following example illustrates a "typical" reserve review. This example is a composite of several actual reviews.

The company writes fifty treaties of varying size, but no facultative business. After reviewing each of the treaties, they are grouped into several categories:

- First, one very large treaty accounting for 30% of the company's entire business. We make arrangements to spend time at the ceding company's offices to review the business in detail.

- Next, the ten largest treaties will be reviewed in individual detail, although on-site visits will be limited to the largest treaty. These ten treaties comprise an additional 45% of the company's total earned premiums and carried reserves.

- The company participates in two large pools which are singled out for attention. Each of these pools has been subject to review by an independent actuarial consulting firm. We make arrangements to obtain copies of their most recent reports.

- Twelve other treaties are also singled out. In each case, our company is a quota-share participant in the treaty along with other companies. In each of these treaties, we have reviewed the experience on behalf of one of the other participants.

- The remaining treaties now account for less than 15% of the total business.

We now review each treaty, attempting to create a line of business profile for each one. Various methods are used as outlined in an earlier section. While this exercise is performed for every treaty, proportionately more time is spent on the larger treaties.

On the ten largest treaties we obtain historical paid and incurred development data and assemble standard triangles of data. Development factors are calculated for each one. We separately assemble "industry" loss development statistics from a variety of sources. From Best's Loss Development Reports, we put together paid and incurred patterns for auto, general liability, workers' compensation, multiple peril and medical malpractice. This is done separately for primary versus reinsurance companies. An adjustment is made for estimated inadequacy of industry results.

RAA data is also assembled by line. This data is on an accident year basis for excess of loss business. We would typically use this data to develop treaties which cover excess business. If the treaties being analyzed are on a policy year or underwriting year basis, we could make formal adjustments to the factors (as outlined in Section II). We could also make judgmental adjustments to reflect perceived differences in the layers or attachment points involved. Other sources include some special studies which produce factors for marine, property, and various components of London business. For each of the smaller treaties we assign weights to each of the possible industry patterns, based upon our line of business analysis.

We now can perform premium development, paid loss development, incurred loss development, Bornhuetter-Ferguson (both paid and incurred) and loss ratio analysis. We review the results of each method, look for consistency of loss ratios over time, and, with a heavy dose of judgment, select the final reserve estimates.

This description is far from complete. The diversity of reinsurance business produces many unique situations requiring unique approaches. However, the above approach represents a reasonable approach in the specific situations involved.

SECTION II

Loss development factors are one of the more important tools in any reserve analysis. Typically, historical data is segregated by some exposure basis. Evaluations of each group are obtained at regular intervals. Ratios of successive valuations are calculated (age-to-age development factors) and the results are averaged with various types of averages used and judgment applied when the ratios exhibit unusual patterns. The resulting factors are cumulatively multiplied together to produce age-to-ultimate development factors which can be used directly in a development factor method, or as an intermediate step in a Bornhuetter-Ferguson analysis [2].

There are a large number of variations of this method.

Historical data may be:

- Incurred Losses
- Paid Losses
- Claim Counts
- Allocated Expenses

Exposure period may be:

- Accident Year
- Policy Year
- Underwriting Year
- Report Year

Evaluation intervals may be:

- Yearly
- Quarterly
- Monthly

Of course, the above list is just illustrative; it is far from complete.

In most applications, the actuary may be working with several forms of historical data, but only a single exposure period and a single evaluation interval. In this situation, it is quite common to work with the empirical ratios as calculated. However, there are some situations where adjustments to this procedure are required. Examples of such situations include:

- Mismatched Evaluation Dates
- Tail Factors
- Exposure Period Conversion
- Evaluation Interval Conversion
- Rapid Exposure Growth

Mismatched Evaluation Dates

The evaluation dates available may not match the evaluation dates required or the data may consist of two or more subgroups with differing evaluation dates. This might occur if the anniversary date for a policy has been changed.

One way of handling this situation is interpolation. Alternatively, it may be preferable to fit a function to the development data. Several articles have discussed various functional forms, including Sherman [8] and McClenahan [5].

Tail Factors

Another reason for fitting a function to development data is to smooth erratic data points, or to project tail factors beyond the oldest available evaluation date.

Exposure Period Conversion

If the only available development factors are on an accident year basis, and the data you wish to apply it to is on a policy year basis, some conversion must be made.

Typically, the actuary adjusts the evaluation ages by the difference in average accident dates. For example, the average accident date for an accident year is six months after the beginning of the period. The average accident date for a policy year is twelve months after the beginning of the period. Thus, an accident year evaluated at 24 months is 18 months after the average accident date, which converts to 30 months after the beginning of a policy year. This conversion technique works reasonably well for evaluation dates beyond 24 months, but is less accurate at earlier periods, and is grossly inaccurate at ages less than 12 months.

Evaluation Interval Conversion

Data may be available on an accident year basis but needed on an accident quarter basis. The typical method of adjustment is similar to the previous paragraph, with similar problems at early evaluate dates.

Rapid Exposure Growth

If the exposures for a block of business have been growing rapidly, then development factors from another block of business with more even growth will not be applicable. One method of dealing with this has been described by Simon [9].

APPLICATION TO REINSURANCE

Each of the above reasons for adjustments to development factors occurs to some extent in primary insurance, but these reasons are much more important in reinsurance. In a typical reinsurance reserve review, the individual body of data being evaluated is not large enough to deserve 100% credibility. External development factors may be needed. In many cases, available external data may have different evaluation dates, or be of a different form. Typically, factors are needed on an underwriting year basis, while available factors are on an accident year or policy year basis.

The extremely long tail of reinsurance makes it more likely that tail factors will have to be estimated. The high volatility of reinsurance development, particularly excess covers, makes it more likely that a smoothing technique, such as fitting development factors to a functional form, will be needed.

Finally, it is common to have books of business that have grown very rapidly. More recently, we may have to deal with books of business which have decreased very rapidly.

For these reasons it is extremely necessary to have procedures to make adjustments to development factors. In this section, I will describe a procedure which can be extended to cover all of the above adjustments.

Underlying Adjustments

Many of the adjustments implicitly assume that a period such as an accident year can be approximated by a single point in time, typically the midpoint. Most of the functional forms used to fit development factors do not explicitly recognize that they are being fit to a period of time, rather than a point in time. An accident year of twelve months does not mean that every point in the accident year is twelve months old. The first point is twelve months old, the most recent point is zero months old, and the intermediate points are at various ages between zero and twelve.

Suppose that the development of a single point follows a particular functional form. It actually makes more sense that an exposure point would have a simple functional form than an exposure period. For example, growth within an exposure period includes the addition of new accident months as well as reevaluations of prior months. Even after the end of a period, the development contains diverse elements. For example, the 13th month of development of an accident year includes the 13th actual month of development on the first month, as well as the second month of development of the last month of the accident year.

Accident Year

Assume that development of a single exposure point can be described by a function F(t). For convenience, we will express all development factors as proportions of ultimate losses (or counts). If we represent typical age-to-ultimate factors as d_i, we will work with $p_i=1/d_i$. The development pattern of any exposure period can be expressed as an integral of F(t). In some cases, an exposure weight function will be required. For example, an accident year evaluated at age t (t>1) includes individual exposure points evaluated at ages ranging from t-1 to t. If we assume even exposures over the period, we can write the formula for accident year development as:

$$G(t) = \int_{t-1}^{t} F(z)dz \qquad t>1 \qquad (1)$$

If we require development factors for evaluation dates before the end of the period, the calculation is straightforward:

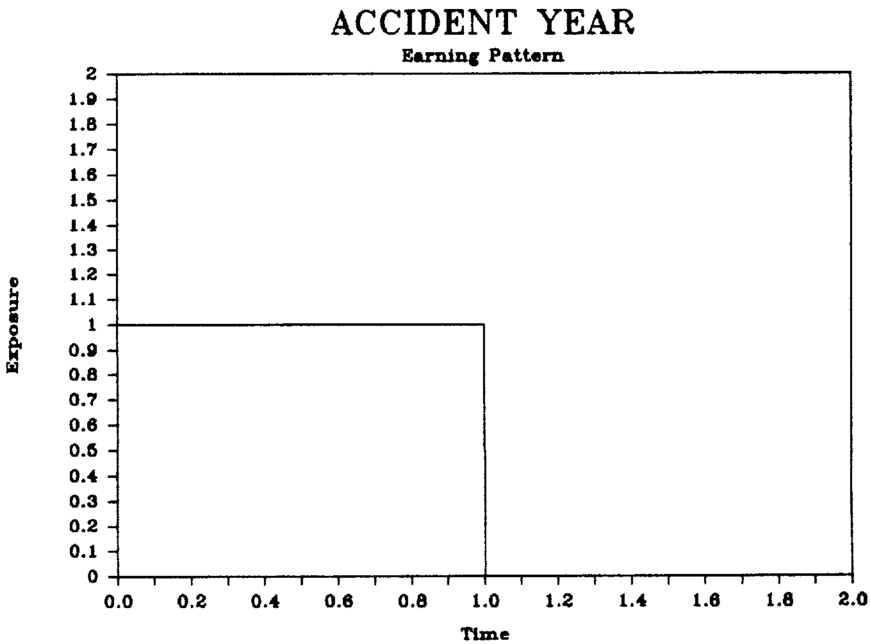
$$G(t) = \frac{\int_{0}^{t} F(z)dz}{t} \qquad t<1 \qquad (2)$$

Note that most other procedures do not work well at all in this situation. Also note carefully that the development factor does not represent the proportion of the entire period which has been reported (paid, etc.) at that point. For example, if we are interested in an accident year evaluated at six months, G(.5) represents only the proportion of the half-accident year. The percentage should be divided by two (or the corresponding age-to-ultimate factor should be doubled) to represent the correct proportion of the full year.

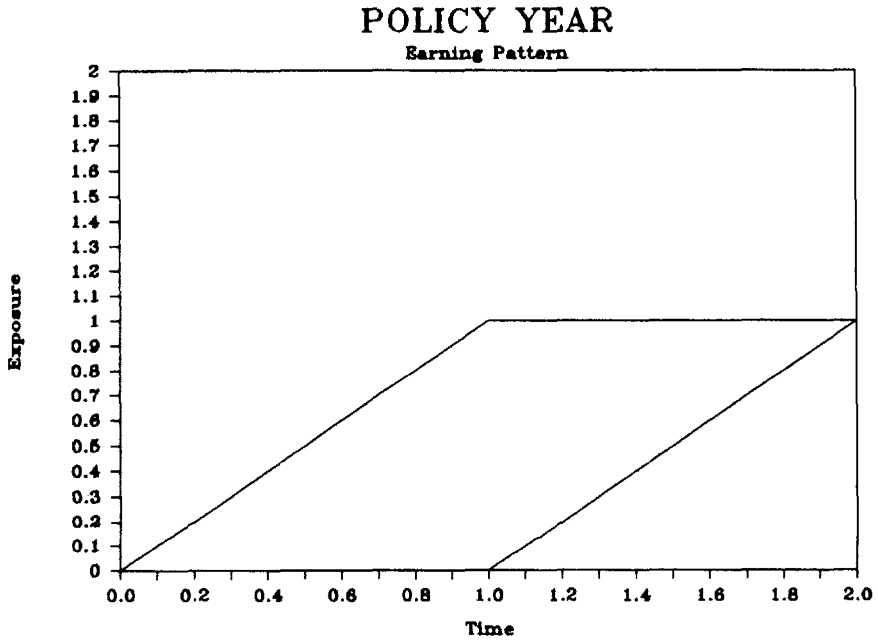
(We could rewrite the lower limit as Max (t-1, 0), the denominator as Min (t,1) and thereby use only a single equation. The reader may choose whichever notation they prefer.)

Policy Year

When we are interested in policy years, we have to introduce additional notation. Recall that an accident year can be represented as a rectangle of exposures, and a policy year can be represented as a parallelogram. Assume that we have one-year policies written evenly throughout the year. We can represent accident year exposures as:



We can represent policy year exposures as:



In the case of a single policy year, the exposure (represented by the earnings in each calendar unit of time) is not uniform over time. We can represent the exposure in each calendar point in time with the following:

$$w(t) = t \qquad 0 \leq t \leq 1 \tag{3a}$$

$$w(t) = 2 - t \qquad 1 < t \leq 2 \tag{3b}$$

We can then write the formula for development factors for a policy year as:

$$G(t) = \int_{t-2}^{t} w(t-z)\ F(z)dz \qquad t \geq 2 \tag{4}$$

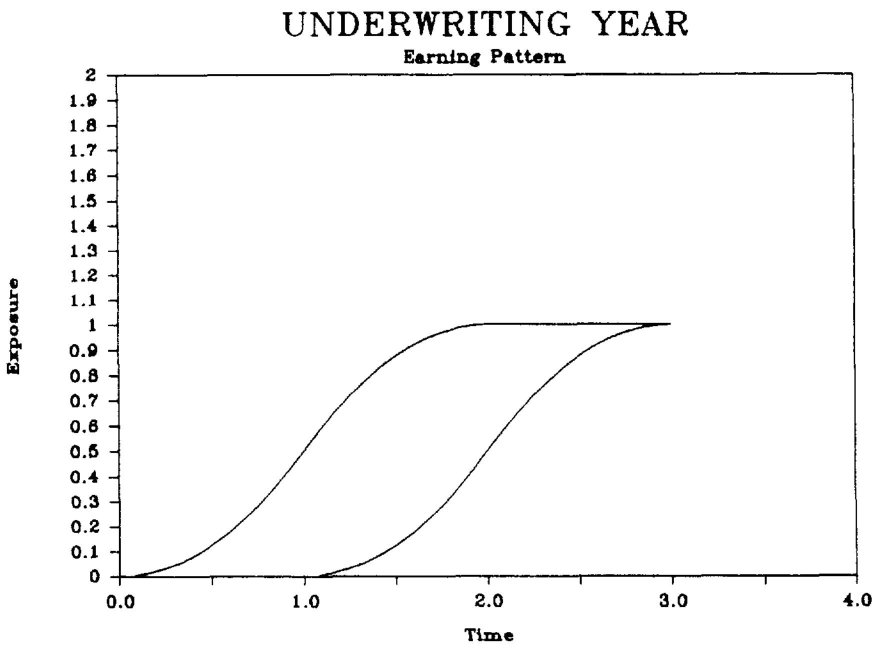
When we wish to evaluate a policy year before the end of the "year" (that is, prior to 24 months), we can use the following:

$$G(t) = \frac{\int_{0}^{t} w(t-z)\ F(z)dz}{\int_{0}^{t} w(t-z)dz} \qquad t<2 \tag{5}$$

Note the expression in the denominator. This expression technically belongs in the denominators of the other formulations, but it simplifies in most cases.

Underwriting Years

As discussed earlier, an underwriting year consists of the exposures covered by treaties issued during a year. If we assume that treaties are written evenly throughout the year, and the treaties are written on a risks attaching basis (with underlying policies written evenly throughout the year), the pattern of earned exposures will look as follows:



Note that an underwriting year extends through three calendar years. We fully recognize that this pattern does not represent a "typical" underwriting year for several reasons:

- Treaties are not written evenly throughout the year. A large portion of all treaties incept January 1. Another large group incepts July 1. These two groups typically account for more than half of all treaties.

- Many treaties, especially excess covers, are on a "losses occurring" rather than a "risks attaching" basis.

- Many treaties cover business written by a reinsurance company, rather than a primary company. In this case, the earnings might extend more than three years after the beginning of the period.

In actual practice, the earnings pattern will have to be generated as a discrete distribution. The calculation of the development patterns will still be an integral, however, it may have to be evaluated using numerical methods. For purposes of illustration, we will use the above form for our underwriting year.

The development factors for an underwriting year can be calculated as follows:

$$G(t) = \int_{t-3}^{t} w(t-z)\ F(z)dz \qquad t\geq 3 \qquad (6)$$

$$G(t) = \frac{\int_{0}^{t} w(t-z)\ F(z)dz}{\int_{0}^{t} w(t-z)dz} \qquad t<3 \qquad (7)$$

In the case of this specific underwriting year, the exposure function can be written as follows:

$$w(t) = t^2/2 \qquad 0\leq t\leq 1 \qquad (8a)$$

$$w(t) = -t^2 + 3t - 1.5 \qquad 1\leq t\leq 2 \qquad (8b)$$

$$w(t) = t^2/2 - 3t + 4.5 \qquad 2\leq t\leq 3 \qquad (8c)$$

Exposure Growth

Rapid growth in exposures (or any uneven exposure pattern) can be handled by this technique. The general form for accident year development factors is:

$$G(t) = \frac{\int_{t-1}^{t} w(t-z)\, F(z)dz}{\int_{t-1}^{t} w(t-z)\, dz} \qquad t \geq 1 \qquad (9)$$

$$G(t) = \frac{\int_{0}^{t} w(t-z)\, F(z)dz}{\int_{0}^{t} w(t-z)dz} \qquad t<1 \qquad (10)$$

If exposures are written evenly throughout the period, the function w(t) disappears. However, for uneven exposures, we simply specify the exposure growth as w(t) and integrate the more general form. If the exposure growth is a simple form, e.g. linear growth, we may be able to calculate G(t) analytically. Otherwise, we can express w(t) as a vector of exposure weights and use numerical integration.

Application

The real advantage of this procedure is not that we have a consistent notation for expressing development factors. The advantage is that with this approach, we can make adjustments to development factors that are appropriate and consistent. For example, if we are given a particular set of development factors, say, accident year factors, we can solve for the underlying F(x) and then create development factors for policy year, underwriting year, accident quarter, etc. that are consistent with the original factors. This procedure will allow us to calculate factors for evaluation dates within the period, correcting interpolate and extrapolate, and adjust for situations where exposure growth is uneven.

In actual practice, it will be desirable to create computer programs which can quickly solve the necessary equations and create the required factors, but these programs should not be particularly difficult.

Example

Suppose we are given the accident year development factors in Appendix B. Assume that the form of the underlying development is:

$$F(x) = 1 - e^{ax} \qquad (11)$$

The form of the development factors will be:

$$G(t) = 1 + e^{at}/a[e^{-a} - 1] \qquad (12)$$

The derivation of this result is shown in Appendix A. We can estimate the value of "a" using the Newton-Raphson (also discussed in Appendix A). The resulting estimate will be a = .5. In practice, each development factor will produce a different estimate of "a". A leasts-squares technique may be used to select a single value, or some other judgmental weighting method may be used.

If we now desire policy year development factors, we perform the integration in Equation (4). We then substitute a = .5. The resulting policy year factors are shown in Appendix B. Similarly, if we desire underwriting year factors, we perform the integration in Equation (6). After substituting a = .5, the result will be the underwriting year development factors shown in Appendix B.

This methodology is particularly useful for reinsurance problems. "Industry" development factors such as those calculated from Best's data can be converted to an underwriting year basis.

One other factor should be considered in any actual application. There is typically an accounting lag between the time a primary company receives data, and the time it is reported to a reinsurer. The methodology outlined above does not specifically recognize this lag. If the lag can be quantified, it should be a reasonably straightforward exercise to adjust for accounting lag.

Acknowledgement

I'd like to thank Doug Collins and Jerry Miccolis for numerous helpful suggestions, and John Yonkunas for his help with the mathematical derivations in Appendix A. Any errors remain the responsibility of the author.

CALCULATION OF ACCIDENT YEAR DEVELOPMENT FACTORS

Assume

$$F(x) = 1 - e^{ax}$$

We wish to evaluate G(t) where

$$G(t) = \int_{t-1}^{t} F(x)dx \qquad (1)$$

Substituting, we have

$$G(t) = \int_{t-1}^{t} [1 - e^{ax}]dx \qquad (2)$$

$$G(t) = \left[x - \frac{e^{ax}}{a} \right] \Big|_{t-1}^{t}$$

$$G(t) = \left[t - \frac{e^{at}}{a} \right] - \left[(t-1 - \frac{e^{a(t-1)}}{a} \right]$$

$$G(t) = 1 + \frac{e^{a(t-1)}}{a} - \frac{e^{at}}{a}$$

$$G(t) = 1 + \frac{e^{at}}{a}[e^{-9} - 1] \qquad (3)$$

We suggest the use of Newton-Raphson iteration to solve for the parameter "a". Restating (3) we have

$$H(t) = 1 + \frac{e^{a(t-1)}}{a} [1-e^{a}] - G(t) = 0 \qquad (4)$$

Differentiating (4) with respect to "a" yields

$$H'(t) = \frac{(t-1)e^{a(t-1)}}{a}[1 - e^{a}] - \frac{e^{a(t-1)}}{a^{2}}[1 - e^{a}] - \frac{e^{a(t-1)}e^{a}}{a} = 0$$

Simplifying,

$$H'(t) = \frac{e^{a(t-1)}}{a} \times \left[1 - e^{a}\right] \times \left[(t-1) - \frac{1}{a} - \frac{e^{a}}{1 - e^{a}}\right]$$

$$= [G(t) - 1] \left[(t-1) - \frac{1}{a} - \frac{e^{a}}{1 - e^{a}}\right] \qquad (5)$$

The nth iteration of Newton-Raphson can be stated as follows:

$$a_{n+1} = a_n - \frac{H(t)}{H^{1}(t)} \qquad (6)$$

Where H(t) and H'(t) are evaluated at a_n.

Continuing and substituting into (6)

$$a_{n+1} = a_n \frac{1 + \frac{e^{a(t-1)}}{a} \times [1 - e^a] - G(t)}{\frac{e^{a(t-1)}}{a} \times [1 - e^a] \times \left((t-1) - \frac{1}{a} - \frac{e^a}{1 - e^a} \right)} \qquad (7)$$

Formula (7) can be used to converge to a solution for "a" for a given "t" and G(t).

HYPOTHETICAL LOSS DATA

	ACCIDENT YEAR		POLICY YEAR		UNDERWRITING YEAR	
Months	% of Ultimate	Development Factor	% of Ultimate	Development Factor	% of Ultimate	Development Factor
12	22.93%	4.360	9.25%	10.814	2.95%	33.864
24	53.26%	1.878	40.61%	2.463	25.84%	3.869
36	71.65%	1.396	63.98%	1.563	54.23%	1.844
48	82.80%	1.208	78.15%	1.280	72.24%	1.384
60	89.57%	1.116	86.75%	1.153	83.16%	1.202
72	93.67%	1.068	91.96%	1.087	89.79%	1.114
84	96.16%	1.040	95.12%	1.051	93.81%	1.066
96	97.67%	1.024	97.04%	1.030	96.24%	1.039
108	98.59%	1.014	98.21%	1.018	97.72%	1.023
120	99.14%	1.009	98.91%	1.011	98.62%	1.014
132	99.48%	1.005	99.34%	1.007	99.16%	1.008
144	99.69%	1.003	99.60%	1.004	99.49%	1.005
156	99.81%	1.002	99.76%	1.002	99.69%	1.003
168	99.88%	1.001	99.85%	1.001	99.81%	1.002
180	99.93%	1.001	99.91%	1.001	99.89%	1.001
192	99.96%	1.000	99.95%	1.001	99.93%	1.001
204	99.97%	1.000	99.97%	1.000	99.96%	1.000
216	99.98%	1.000	99.98%	1.000	99.97%	1.000
228	99.99%	1.000	99.99%	1.000	99.98%	1.000
240	99.99%	1.000	99.99%	1.000	99.99%	1.000

Bornhuetter-Ferguson
Technique for Reserving

In 1972, an article was published with the title "The Actuary and IBNR" authored by Ron Bornhuetter and Ron Ferguson. The article discussed a number of ideas, but is best known for a reserving technique which now bears their names.

This technique is most easily explained if we reexamine some of the basics underlying reserving theory.

BACKGROUND

If we take a particular accident year, for example 1981, and review the loss data at some later point such as 1984, a portion of the losses will be paid and a portion will be outstanding. The sum of the paid and outstanding case basis reserves we call the case incurred. In many lines of business, we don't assume that the case basis reserves represent the ultimate incurred losses. Additional reserves are set up to account for two reasons:

1. True IBNR - additional claims reported after the evaluation date.

2. Case basis development - case basis reserves are only an estimate of the final settlement values, and the actual value can vary up or down from the estimate. Evidence shows that it is typical to expect some upward development.

If we use older years which have matured enough to provide reasonable estimates of ultimate, we can calculate the ratios of the case basis incurred to the ultimate losses at various stages of development. These ratios can be used to estimate development patterns of development factors.

EXAMPLE

Suppose, at the time the 1981 accident year is priced, the estimated losses are $10,000,000. Furthermore, assume that development patterns are examined and we expect to have 60% of the total known by 48 months of development. In other words, at 12/31/84, we expect that case incurred will be $6,000,000. If the case incurred at 12/31/84 turns out to be $6,000,000, of course we will be even more convinced that our original estimate of $10,000,000 for ultimate losses is reasonable.

Suppose, however, that case incurred at 12/31/84 is only $3,000,000. What should we conclude about the ultimate incurred? There are three types of conclusions that we can reach:

1. We can conclude that the fact that actual results differ from expected results is not sufficient evidence to change our original estimates of ultimate losses. Our reasons for rejecting the indications of actual experience might include:

   a. The volatile nature of the particular line of business.

   b. Possible inaccuracies in the presumed development pattern.

   In this case we would select $10,000,000 as the ultimate incurred, and set reserves equal to this amount less paid-to-date.

2. We could conclude that the development factors determined by the development curve are the best indicator of ultimate losses. The estimate that 60% of ultimate would be case incurred at 48 months means that a development factor of 1.67 (1 : .60) is appropriate. Accordingly, we would multiply the case incurred of $3,000,000 by 1.67 yielding an estimated ultimate of $5,000,000.

Another way of looking at this approach is that our assumption that 60% of ultimate incurred will be known at 48 months means that we expect that 40%, or $4,000,000 will be unknown (IBNR) at 48 months. To summarize:

Estimated Ultimate	$10,000,000
Expected known at 48 months	6,000,000
Expected IBNR at 48 months	$ 4,000,000

The actual known losses at 48 months were $3,000,000 or 1/2 of the expected. The development factor approach is equivalent to assuming that the better than expected actual experience will continue on a proportional basis. That is, the IBNR will only be 1/2 of the original estimate. One-half of 4,000,000 is 2,000,000. So our new estimate of ultimate incurred is:

Actual case incurred to date	$3,000,000
Revised IBNR estimate	2,000,000
Ultimate Incurred	$5,000,000

This agrees with the number calculated using the development factor.

3. The third alternative is that we assume the development patterns are reasonable. The fact that actual losses are less than the expected at this point in time is due to the volatile nature of this line. Here, we do not assume that the better than expected experience will continue into the future. In other words, we still believe that our original estimate of IBNR which would be remaining at 48 months is still reasonable. However, we are willing to recognize the difference between the actual and expected experience for the case basis portion. The current estimate of ultimate incurred would be calculated as:

Actual case incurred to date	$3,000,000
Original IBNR estimate	4,000,000
Ultimate Incurred	$7,000,000

The method described in #3 above is an application of the Bornhuetter-Ferguson approach. It allows recognition of favorable (or unfavorable) experience without creating the wide swings which would occur with the rote application of development factors.

SUMMARY

In the situations described above, we have made the following assumptions:

- Expected losses based on pricing assumptions are $10,000,000.

- Expected known losses (case basis) at 48 months of development are $6,000,000, hence, expected IBNR at 48 months is $4,000,000.

- Actual known losses at 48 months are $3,000,000.

Under these assumptions, we can summarize the reserving techniques as follows:

	Pricing Assumption	Loss Ratio Method	Bornhuetter-Ferguson Method	Development Factor Method
Known Losses	6,000,000	3,000,000	3,000,000	3,000,000
IBNR	4,000,000	7,000,000	4,000,000	2,000,000
Ultimate Incurred	10,000,000	10,000,000	7,000,000	5,000,000

BIBLIOGRAPHY

[1] Berquist, James R., and Sherman, Richard E., "Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach", PCAS LXIV, 1977, p. 123.

[2] Bornhuetter, Ronald L., and Ferguson, Ronald E., "The Actuary and IBNR", PCAS LIX, 1972, p. 181.

[3] Collins, Douglas J., "Measuring the Impact of Unreported Premiums on a Reinsurer's Financial Results", 1985 CAS Discussion Paper Program, p. 104.

[4] Longley-Cook, L. H., "An Introduction to Credibility Theory", PCAS XLIX, 1962, pp. 16-17.

[5] McClenahan, Charles L., "A Mathematical Model for Loss Reserve Analysis", PCAS LXII, 1975, p. 134.

[6] Miccolis, Jerry A., "Interaction of Claims, Underwriting & Actuarial FUnctions in Reinsurance Reserving", Transcripts of 1985 Casualty Loss Reserving Seminar (to be published).

[7] Miller, David L., and Davis, George E., "A Refined Model for Premium Adjustment", PCAS LXIII, 1976, p. 117.

[8] Sherman, Richard E., "Extrapolating, Smoothing, and Interpolating Development Factors", PCAS LXXI, 1984, p. 122.

[9] Simon, Leroy J., "Distortion in IBNR Factors", PCAS LVII, 1970, p. 64.