RECENT DEVELOPMENTS IN RESERVING FOR LOSSES IN THE LONDON REINSURANCE MARKET

BY

H.E. CLARKE

Summary

The paper describes in detail a new method which can be applied by any insurance company to its own data to set reserves for outstanding losses (including IBNR) and to calculate a confidence interval for these reserves. The method has also opened up a whole range of interesting ways of looking at data. Although the method can be applied to any sort of business it is particularly helpful in looking at long tail business, such as that written by reinsurers, for which other methods have proved less satisfactory. The methodology can also be applied by a supervisory authority to establish minimum reserving standards for companies where global general market data on run-offs for different classes of business is available. A new method of setting minimum reserves for individual syndicates based on the methodology in the paper is currently being tested by Lloyd's of London. This work is briefly described in the final section of the paper.

Author

Harold Clarke is a Partner in the UK firm of consulting actuaries Bacon & Woodrow whom he joined in 1973. He qualified as a Fellow of the Institute of Actuaries in 1975 and has just finished a six year stint as a member of the Committee of the Institute of Actuaries Students' Society. His previous papers are:

"Mathematical Density Functions Applied to a Liability Insurance Portfolio", H.E. Clarke & L.M. Eagles, Transactions of the 21st International Congress of Actuaries;
"Should Actuaries be Random?", R.D. Campbell & H.E. Clarke, Journal of the Institute of Actuaries Students' Society, Vol 25.

He has a MA in Mathematics from Cambridge University and a MSc in Statistics from Sussex University.

1. INTRODUCTION

This paper describes a system which our firm has developed and refined over the last 5 years to enable us to comment on reserves set up for outstanding and IBNR claims by companies writing marine, aviation, liability and reinsurance accounts or alternatively to advise on such reserves. The companies we have advised have been operating in the London Market in the UK of which Lloyd's is the centre. The London Market underwrites a significant part of the world's insurance and in particular its reinsurance and is a dominating influence on insurance world-wide. Although the system described is particularly suitable for reserving for reinsurance accounts it is also applicable to all other types of casualty business. The system is fully operational on our main frame computer. It has been used many times and it is stable.

In the London Market details of numbers of claims are generally not available or not relevant. Data is usually available for each "account year", i.e. for all risks written in a particular accounting year which is usually a calendar year. The items normally available are:

(i) Premiums paid to date

(ii) Claims paid to date

(iii) Claims outstanding, i.e. the case estimates as notified by the brokers to the companies for outstanding claims.

Further details of the constraints and problems posed by the data are given in Section 2.

The system had therefore to be able to generate estimates of the reserves from this limited amount of data. The method works by estimating the Ultimate Loss Ratio ("ULR") for each account year, from which the necessary reserve is easily derived. An important innovation of the method is that a confidence interval is produced for the ULR and hence for the reserves. An outline of the method is given in Section 3, a detailed worked example in Section 4 and some further problems and considerations are discussed in section 5. The method is very graphical and so easy to see and present to actuaries and non-actuaries.

In the final section of the paper, Section 6, we describe an application of the method to setting minimum reserves at Lloyd's which is currently being tested. The method can also be used in that way to set minimum reserves for companies operating in any insurance market where industry wide statistics are available.

The method starts from an idea put forward in a paper by D.H. Craighead (1) to the Institute of Actuaries. Inside our firm we have considerably refined and extended this idea. A detailed description of the potential use of the method by Lloyd's together with an outline of the general method is given in the paper by my colleagues S. Benjamin and L.M. Eagles (2) to the Institute of Actuaries. In this paper the emphasis is reversed with considerably greater detail being given about the general method. We also wish to thank A.B. English for the programming and application of the curve fitting algorithm and for much other programming.

2. DATA

As previously mentioned the data available for setting reserves in the London Market is sparser than that usually available from companies writing mainly domestic risks. The reasons for that are outlined below.

For risks written in the London Market cover is usually given for one year. The premiums are received over a period of typically three years. This delay can be due, for instance, to excess of loss treaties being rated on a burning cost basis or to delays in monies being forwarded by brokers. The incidents which take place during the year of cover give rise to claims which may not be reported for many years and then may take several years to settle. The main reason for this delay is that the London market tends to deal in reinsurance where the information is "second-hand" in the sense that it comes from a primary insurer which may itself be subject to delays of information. For instance suppose you are writing a catastrophe excess of loss treaty covering property damage exceeding $10 million in aggregate for any one incident for a Californian company. The reinsurer may not hear anything from the Californian company until its own claims reach the agreed limit. The final outcome for the reinsurer in the London Market may then take a long time to become fully known. Further as this example illustrates the concept of number of claims is not meaningful in this market.

Also the risk will often be placed on a coinsurance basis, often with 20 or 30 different underwriters. Detailed data may be available to the leading underwriter, but that detailed information may not be available to others on the risk and will not be recorded centrally. Statistics have in fact tended to be subordinate to accounting data, which is therefore the only data commonly available. This also has the problem that if an error is discovered in the statistics (e.g. an outstanding claim has been notified in Italian lire rather than US dollars) it will be corrected from discovery, but the history will be left unchanged so that the statistics still reconcile with the published accounts.

The data is usually available for each account year. Thus the method described in this paper will be presented for data collected on that basis. However as will become clear the method is equally applicable to data collected on an accident year basis. It is common for the data to be missing for early account years or early years of development, often due to computerisation of the accounting function taking place at that point.

In the case of Lloyd's further problems arise from the use of very broad risk categories which cannot be assumed to be homogeneous over time. The classic example of this is Non-marine All Other which can include marine business written by non-marine syndicates. Further the data collected centrally consist only of premiums received and claims paid, both net of reinsurance. After the end of the third year of development of an account year future premiums received are set off against future claim payments in the statistics.

More information on the operation of the London Market in general and Lloyd's in particular is given in the paper by D.H. Craighead (1).

The techniques described in the paper can be applied to gross data, net data, paid losses, paid plus outstanding losses. That is why we have not defined closely the basis of the data.

3. SYSTEM REQUIREMENTS AND OUTLINE OF METHOD

For the data described in the previous section most of the reserving methods commonly in use break down. We needed a method which:

(i) Was able to cope with long tail business.

(ii) Used only information on premiums, paid claims and claims outstanding as notified.

(iii) Could provide estimates where there were missing items of information from the run-off triangle.

(iv) Could handle multi currency portfolios. Most of the companies whose reserving we examine write substantial US dollar business even though they report in pounds sterling.

(v) Would enable us to set a range of values within which reserves would be acceptable. After all no single estimate can be correct unless we have business which has completely run off. We would expect in the early years of development of an account year that the range would be relatively wide and should reduce as development increases.

(vi) Where necessary would use market information or information from other similar businesses to establish reserves for a particular insurer.

It was vital that the system should be able to cope with all the preliminary data handling, and would be flexible enough to allow the data to be looked at in a variety of ways. Data can be accepted in a variety of formats. The data can be either cumulative or incremental. Claims data can show paid claims and claims outstanding either separately or summed, and can be expressed either as loss ratios or cash. Development time intervals can be either quarterly, half-yearly or annual. The system can accommodate several currencies, which can be combined or not at the user's discretion. When currencies are combined, uniform exchange rates are assumed to apply for all periods of origin and development. Data from up to 99 separate long and short tail categories can be accepted in any of the currencies, and again at user option any or all categories can be combined.

A major consideration underlying our whole approach is that for the classes of business we are considering, standard assumptions, e.g. homogeneous account from year to year, standard pay out pattern, no change in speed of claims advice, etc., would almost certainly all be violated. This suggested as a basic starting point that we examine the run-off of each account year separately. It also suggested that we look at the development of loss ratios rather than losses. Empirical considerations suggested that if we were seeking a smooth curve to fit the shape of the loss ratio at development time t, plotted against t, that curve would have a negative exponential shape.

In the remainder of this section we outline the reserving method we have developed to meet the above criteria. A worked example of the method is then given in section 4 to expand on the outline.

(a) Run-off triangles are drawn up for as many account years as possible showing the development year by year (or quarter by quarter) of premiums and claims.

(b) An estimate of the ultimate premiums receivable is made for each account year. If we have to calculate the estimate then we simply apply development factors calculated from the data without smoothing. Other methods could be used in appropriate circumstances. Often we use the underwriters' estimates since they have a better feel for the way, in practice, policies are being signed down.

(c) The estimates of ultimate premiums are divided into the relevant claims to give a run-off triangle of loss ratios.

(d) Separately for each account year for which there is sufficient development (this depends on the length of the tail of the business) a curve of negative exponential form is fitted to the loss ratio development for that account year. From this curve a preliminary estimate of the ULR for that account year can be made. In certain cases we can fix some of the parameters in the negative exponential curve from our knowledge of the values of the parameters for the same class of business in other companies, or on an industry wide basis.

(e) For each year of development, e.g. year r, we then combine the results obtained in (d) to give a table of the loss ratios at the end of year r and the corresponding estimated ULR's. A line is fitted to these points by standard linear regression techniques. Then given the loss ratio at the end of development year r a best estimate of the ULR for that account year can be obtained from the fitted line. Further a confidence limit for the ULR can also be obtained.

For an account year which is well developed the estimate of the ULR is obtained from (d) so no range is quoted, or usually needed. For a year with little development the ULR and accompanying confidence interval from (e) is quoted. For intermediate years the method depends on one's judgement.

4. WORKED EXAMPLE TO ILLUSTRATE METHOD

The approach outlined in the previous section is illustrated below by means of an example based on typical medium tail data. The data is available for account years by quarters of development up to 1st July 1985. This is the date as at which the reserves for outstanding claims are being calculated. For early years of development for the earlier account years the data is missing. It will be seen that this does not cause a problem to the system. Appendix 1 contains computer produced tables and graphs for the example. These are typical of the output produced by the computer system.

Estimating Ultimate Premiums

In this example we assume that no premiums are received after the end of development year 5. We thus need to estimate the ultimate premiums to be received for account years 1981 to 1984 (1985 is omitted from our consideration since half way through the year is too early to establish reserves). The estimates of ultimate premiums are given in Table 1.1 of Appendix 1. The numbers above the dotted line are the cumulative premiums to date. The numbers below the dotted line are the estimates of cumulative premiums for future development years estimated by development factors. Thus for each account year the last number in the column of data for that year is the estimate of total premiums receivable that we intend to use for that year.

Triangle of Loss Ratios

The estimates of total premiums are then divided into the cumulative development of incurred claims (i.e. claims paid plus notified claims outstanding) to generate the cumulative incurred loss ratios, based on ultimate premiums. Details of the loss ratios are given in Table 1.2 of Appendix 1.

Estimation of ULR by Curve Fitting

We now make a first estimate of the ULR's for each account year by fitting a suitable curve to the loss ratio development for that account year. Over the years we have tried a number of different families of curves for this purpose. The family of curves should satisfy the two criteria:

(i) For an account year where the ULR is already known with a fair degree of certainty the curve must level out at a value near that loss ratio.

(ii) For later account years the curve must fit the known data well and also allow for a reasonable amount of future development. In most cases this will mean a development period similar to the more fully developed years.

The curve we have found most suitable is:

$$L_t = A \times [1 - \exp(-[t/B]^C)]$$

where t is the development period and L_t the loss ratio for that development period. There are 3 parameters A, B and C. A determines the ULR while B and C determine the length of the tail and the way in which it approaches the ULR. The curve was originally suggested in a paper by D.H. Craighead (1). The method of fitting the curve given in that paper is not optimal and more powerful numerical methods than those described by Mr. Craighead should be used. In Appendix 2 we give examples of the effect on the shape of the curve of changing the parameters B and C. These illustrate the wide variety of run off shapes which can be fitted by this curve.

This family of curves is used to give estimates of ULR's for account years 1971 to 1981. For later years, not enough development has yet taken place for a satisfactory curve to be fitted. In Figures 1.3 to 1.13 of Appendix 1 we give the graphs of the curves fitted (the solid curves) in this example together with the developed loss ratios. Each loss ratio is represented by a vertical line, with the dotted line joining up the developed loss ratios. The quality of the goodness of fit can be tested by eye by comparing the closeness of the dotted and solid curves. The comparison should obviously concentrate on the later years of development. At the bottom of each curve we give the values of A, B and C fitted together with the mean squared error. In this particular example C was set equal to 1.5 and only A and B were fitted. We discuss the selection of the parameters to be fitted and the choice of the developed loss ratios to be included in the fitting in Section 5. The graphs need not be studied in detail but should just be looked through quickly to see how well, in general, the curves fit the data.

On occasions we have found that the graph produced by the computer does not suggest a smooth curve. Particularly when looking at incurred loss ratios we have found that the development can oscillate violently. An advantage of the system is that since it presents this in visual form it can be discussed with the underwriter. The most common explanations we have found for odd patterns are:

(a) Miscoding of data either by currency or category

(b) Data corrections that have not been carried back to the beginning of the account year

(c) Delays in reinsurance recoveries.

Thus the system is acting as a powerful check on the data.

In the particular example being used it appears likely that initially some claims for 1978 development year 7 and 1980 development year 5 have been coded as 1979 development year 6 with the error not being fully corrected retrospectively.

Estimation of ULR's by "line of best fit"

We have so far analysed the run-off of one account year at a time. We now analyse the run-off by examining one development year at a time for all account years together. Thus we use all the information in the run-off triangle.

For example at development year 3 we have the following data:

Account year	Loss ratio at development year 3	Estimated ULR from previous curve fitting
	%	%
1973	53.1	91.0
1974	65.8	92.1
1975	50.3	75.7
1976	43.6	70.2
1977	46.2	70.0
1979	73.5	103.8
1980	40.4	69.6
1981	39.1	72.2

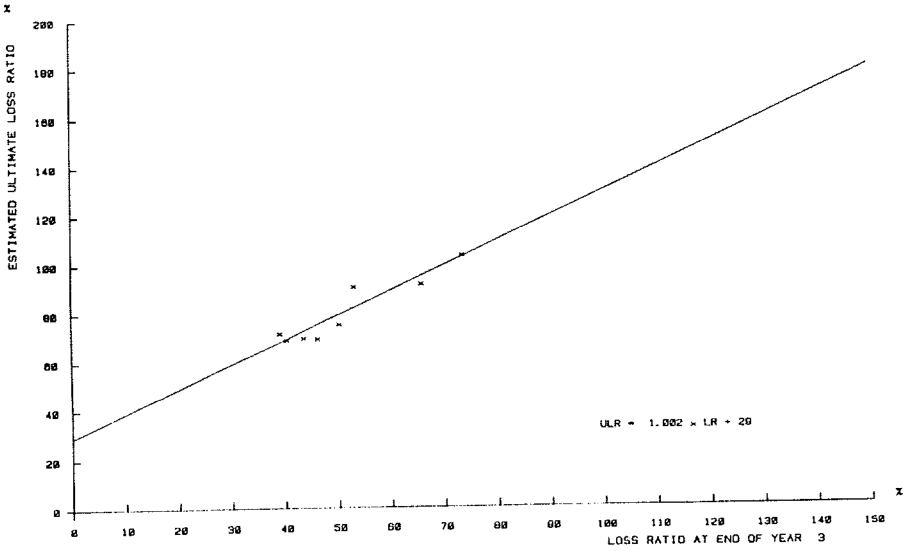
Account years 1971 and 1972 are omitted because the loss ratios for early development years are missing and 1978 is omitted because the run-off curve for that year seems to be a different shape from the other years.

The points are then plotted and the plot is examined to see if there is a statistically significant relationship between the loss ratio at development year 3 and the ULR. The method we use is to fit a regression line and test whether the gradient is significantly different from zero.

In this case the regression line is:

Estimated ULR = 1.002 x Year 3 Loss Ratio + 29.00%.

The fitted line is shown below, together with the 8 points to which it was fitted.



Account years fitted: 73, 74, 75, 76, 77, 79, 80, 81

To test if the gradient is significantly different from zero we use a t-test, with 2 degrees of freedom less than the number of points fitted. In this case we have $t_6 = 6.55$ which is significant at the 99% level. Thus the line is a good fit and the gradient is non zero, which supports the evidence available from inspection of the fitted line.

From the fitted line we can estimate the ULR for 1983 (where development year 3 is the latest known loss ratio) as:

$$1983\ \text{ULR} = 1.002 \times 39.57 + 29.00$$
$$= 68.65\%.$$

Since we have fitted a regression line we can also construct a confidence interval for this estimate of the ULR. There are two alternative methods, one empirical and the other mathematical.

The empirical method is to take the historical point furthest from the regression line and state that the true result for the year is unlikely to fall outside the historical maximum. This gives a likely variation of the result of $\pm$ 8.8% in this particular case.

The mathematical method is to derive the statistical confidence interval from the regression line fit. We have found that a 90% confidence interval does the right job for our analyses of individual portfolios. This gives a confidence interval in the example of $\pm$ 10.9%. Obviously the width of the confidence interval depends on where the point lies on the regression line.

The choice of method is a matter of personal preference. The advantage of the maximum deviation is the ease of presentation to the underwriter of the rationale for the range. The advantage of the second method is that it is statistically based and does allow properly for the number of points to which the line is fitted. It should be noted that underlying the second method as well as the t-test is the assumption that the underwriting results for different account years are independent identically distributed random variables. Such limited investigations as we have carried out suggest that this is a reasonable assumption.

If the gradient of the regression line is found to be not significantly different from zero then that implies that there is no correlation between the loss ratios at year 3 (say) and the ULR. In this case we would estimate the ULR as the average of the historic ULR's and obtain a confidence interval using the maximum deviation. However this also tells us something very useful about the data for that account year. It says that effectively there is no information in the data showing the development of the account year so far to indicate how the year will turn out ultimately in practice. Although this is a negative statement we feel that it is a fact that is often not fully appreciated by management, particularly with regard to long tail business. However it is clearly illustrated by the plots of loss ratios against ULR from which it is often easy to see that there is no relationship.

For our example the regression lines fitted for development years 2 to 10 together with the account years for which they are fitted are shown in Figures 1.14 to 1.22 of Appendix 1. Looking through the regression lines you will see how the fit gets better as the development year increases. When we reach the year of development where the "tail" of claims has effectively run off the loss ratio will equal the ULR. The regression line will pass through the origin of the graph and the slope of the line will be "1 in 1" i.e. 45%. You will see from Figure 1.22 that for the class of business being used for the example this position has almost been reached by the end of year 10. A summary of the lines fitted and the statistics is given in Table 1.23 of Appendix 1. From this you will see that for 1984 it is not appropriate to fit a regression line, since the t-test statistic is not significant at the 95% level.

Final estimates of ULR.

In this example we consider that the estimates of ULR obtained from the curve fits are the appropriate ones to use for account years 1971 to 1977. Clearly for the early account years one can not use the regression lines to estimate the ULR's since the lines would be based on too few data items to be credible. For account years 1979 to 1984 the results from the line of best fit seem most suitable. As previously stated for 1978 the position is difficult because the shape is different from the other account years and we have therefore used the curve fit. Although no confidence interval can be calculated for this year it is obvious from looking at the curve fit that in order to convey the correct information to management that one should be quoted. This has arbitrarily been taken to be the same as 1979. We have on this occasion used 90% confidence intervals rather than maximum deviation intervals.

The final results of the analysis are set out in Table 1.24 in Appendix 1.

Further considerations

We have already mentioned how this approach suggests how much information about the ULR is contained in the development to date of the relevant account year. The other useful thing that we find comes out of this approach is that it shows senior management that the estimate of the ULR is just that - an estimate. Thus the actual result will be better or worse than that estimate. The confidence intervals give an indication to senior management of the range in which the result will in fact lie. It thus enables them to assess the implications of establishing reserves based on particular estimates of the ULR. The closer to the upper limit of the ULR that the reserve is established the more likely it is that in practice the reserve will turn out to be more than adequate and the excess may be released as a profit in the future. The nearer to the lower limit of the range of the ULR that the reserve is established the more likely that the reserve will turn out to be inadequate. That would mean that additional cash would have to be found in the future either by restricting divided payments or raising new capital.

5. FURTHER DETAIL ON THE RESERVING METHOD

In this section we consider some of the practical problems that arise from using the approach to reserving discussed in the preceding two sections and describe some of the methods we have used to overcome these problems. Although a few of these problems and solutions were mentioned in the previous section we have covered all of these in this section for completeness.

Problems encountered with curve fitting

The curve we fit has 3 parameters A, B and C. Initially for each account year we fit the curve allowing all 3 parameters to vary. This is because a free fit allows the curve to reflect the data as accurately as possible given the constraints of the curve. However where there is an error in the data, or some other reason, one can find that the fit to the early years of development is satisfactory but it is rather less good to the later years of development. In such cases we fix either B or C in order to try and make the curve fit the later years of development better at the expense of a worse fit in the earlier years. We prefer to fit C as this allows more freedom in the shape of the curve than fitting B. If we have to fix a parameter for a particular account year then if most of the other account years are fitting well on a free fit we would take the values of the parameters of those other years into account when deciding on the values of the parameters to be fixed. Alternatively we would take into account the values of the parameters we have found suitable for similar classes of business either for other companies or on an industry wide basis.

As already mentioned we do not fit curves to recent account years since for such years there is insufficient development to permit a curve to be fitted. For longer tail categories we usually omit the first 8 to 12 quarters of data in fitting the curve to ensure that the fit is reasonable to the later development. This also solves the problem that for some of the earlier account years this early development can be missing from the data. Finally we sometimes find that the curve is approaching the value of A slowly so that A is probably too high an estimate of the ULR. In such cases we assume that the development is completed after a reasonable period, say 15 to 20 years for the longer tail classes, and take the value of L_t for that development period as the estimate of the ULR.

Problems encountered with "line of best fit"

One important problem that is often encountered is where a particular account year has a significantly different speed of development from all the other account years for that class. This may be due for example to writing a peculiar treaty or treaties in that year. That such a thing is happening is usually clear from the graphs of the curves and the reason can often be found from discussion with the underwriter. In these cases that account year is omited from the calculation of the line of best. A good example of this was the omission of account year 1978 from the calculation of the lines of best fit in the previous section.

Another problem is where the data is very variable particularly in the early years of development so that there are significant random fluctuations on top of the basic run off pattern. In this case we have found that it is better to use the developed loss ratios obtained from the fitted curves rather than the actual values. This smooths out the random fluctuations which one may consider are not being repeated in the account year for which one is using the line of best fit to calculate a ULR. Alternatively the data for early years of development for some account years may be missing and using the modelled data will permit the inclusion of those years in the calculation of the line of best fit. Because of the smoothing that takes place with modelled data it will be found that the confidence intervals are narrower than those brought out by using the unadjusted data. They should therefore either be quoted with a cautionary note that they underestimate the true amount of fluctuation or not quoted at all.

It is interesting comparing the line of best fit approach with the approach using development factors. The development factor approach is equivalent to fitting a line for ULR against developed loss ratio that passes through the origin. Our experience is that for early years of development the lines of best fit often miss the origin by a wide margin. However as one progresses to the lines of best fit for the later development years they become closer and closer to lines through the origin. If in looking at some lines of best fit we do not see this pattern then this suggests that something is awry. The most probable reason is an error in the data.

As will be apparent from the example and the above discussion the method is not an automatic method for setting loss reserves. It requires one to use one's judgement at all stages of the process. In particular we have found that a careful study of the graphs of the curve fits and the linear regressions is very important in deciding upon an appropriate best estimate of the ULR and the accompanying confidence intervals. Although the method described uses a curve fitting approach to obtain the initial estimates of ULR's there is no reason why alternative methods, as for example described in the paper by J.R. Berquist and R.E. Sherman (3), should not be used to obtain these initial estimates. However we would emphasise that in practice we have found the curve fitting approach to be very flexible and more than adequate for calculating values of ULR's to use in the line of best fit. The alternative methods are found to be more necessary to assist in estimating the ULR's for the early account years where the line of best fit is not going to be used as part of the estimating process.

6. APPLICATION OF METHOD TO LLOYD'S

One important use of the method we have developed, and in fact one of the reasons for developing it, was to provide a new method for calculating the minimum reserves to be established by Lloyd's syndicates. This is described in considerable detail in the paper by S. Benjamin and L.M. Eagles (2) and we shall therefore give only a brief outline of the method for setting minimum reserves here.

The syndicates in Lloyd's are the bodies in Lloyd's equivalent to companies that underwrite the risks. Collectively the syndicates comprise Lloyd's. The syndicates each maintain their own statistics and also certain statistics are collected centrally. Among other things the central statistics are used to help set minimum levels of the reserves for each account year to be established by the syndicates.

The current method of setting minimum reserves is by the use of the "Lloyd's audit percentages" which are set by Lloyd's centrally. Under this present method percentages are supplied for use as at the end of each calendar year separately for each class of business and each account year in which business was written. The minimum reserve for claims outstanding and IBNR at the end of that calendar year for the class of business and account year is the premium advised to date multiplied by the relevant percentage. Thus the minimum level for the total claims expected to be paid by the syndicate is the claims paid to date plus the minimum reserve. Suppose under the present method the paid loss ratio to date is, say 10% and the audit percentage for the minimum reserve is 78%. Then under the present method we have

Paid Loss Ratio	=	10%
Reserve (Audit Percentage)	=	78%
(Implied) ULR	=	88%

It will be clear that this method does not reflect the progress of the individual syndicates.

Under the proposed new method two figures are used instead of one. In this particular case instead of 78% the two figures are 3.4 and 33% and the calculation is as follows:

ULR	=	3.4 x Paid Loss Ratio + 33%	
	=	3.4 x 10% + 33%	= 67%
Paid Loss Ratio	=		10%
(Implied) Reserve	=		57%

Thus two figures are provided for each class of business and account year for which currently one audit percentage is provided. The proposed new method has been tried on a limited experimental basis for three years. The evidence so far is favourable and the experiment is currently being widened to cover the whole market.

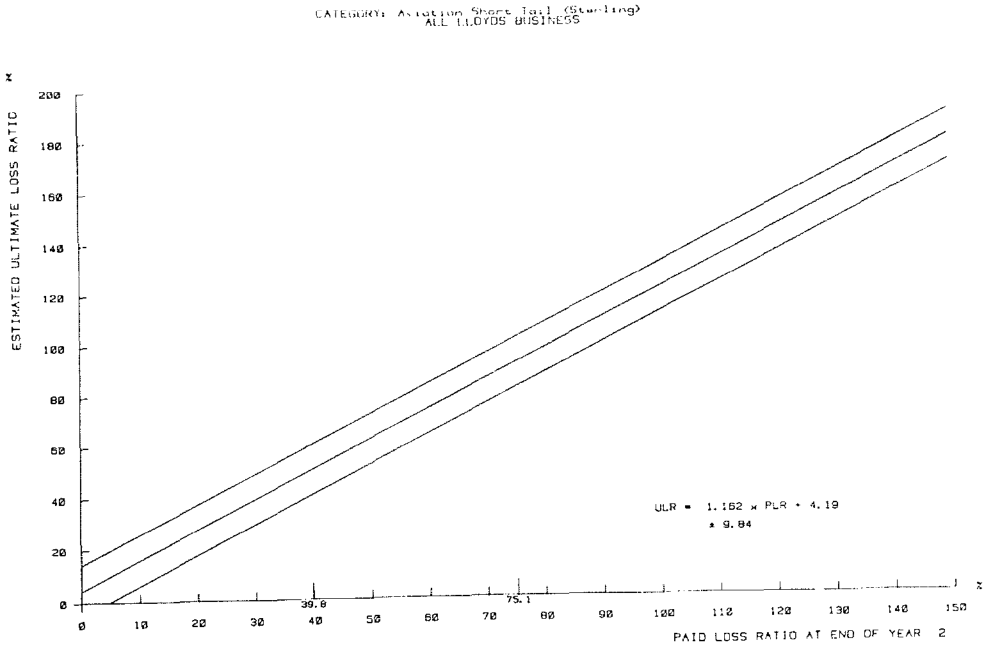
The two figures under the new method are calculated by applying the general method described in the preceding sections to the data collected centrally at Lloyd's for each class of business. For each class of business if one carries out that process one produces for each account year or year of development a line of best fit, together with an associated confidence interval, based on the point furthest from the line of best fit. The two numbers under the proposed method are the parameters that define the line of best fit. Thus in the example 3.4 gives the slope of the line and 33% its intercept on the vertical axis. There was considerable discussion inside the working party which reported to the Audit Committee as to whether the line of best fit or one of the other lines should be used to set minimum reserves. In the end the upper edge of the confidence interval seemed too high, the lower too low. The use of the line of best fit as a minimum allowed one to say that the total reserves set up in Lloyd's were at least as great as the average indicated by past experience, which seemed to be a useful statement to make. Underlying this approach to setting reserves is the assumption that for any class of business the business written by a syndicate will be similar to that "written" by all of Lloyd's combined. Incorporating the paid loss ratio in the calculation of the ULR in the way proposed then allows the quality of the business written by a particular syndicate to be reflected in the ULR in what appears intuitively to be a reasonable way. Also the new method would be easy to implement requiring very little change by individual syndicates in the work they carry out.

In addition to being provided with the new figures for calculating the minimum reserves the syndicates are also provided with graphs for each class of business and year of development showing:

(i) The lines of best fit together with the lines based on the point furthest from the line of best fit

(ii) The historic range of paid loss ratios.

Thus the syndicates are provided with graphs looking like this



The syndicates are being encouraged to plot their own data on the graphs to see how their experience compares with that of all of Lloyd's combined. It is hoped that as a result they will obtain useful information about their experience. For example if a syndicate's own path was narrow and different from the all-Lloyd's path then that would demonstrate in a very vivid way that it was writing a different class of business.

Clearly this approach can be adopted by any supervisory authority which wishes to set reserving standards for companies where global general market data of run-offs for the different classes of business is available.

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APPENDIX 1

DATA AND OUTPUT FOR WORKED EXAMPLE

Table 1.1

Estimation of Ultimate Premiums

Development Quarter 2

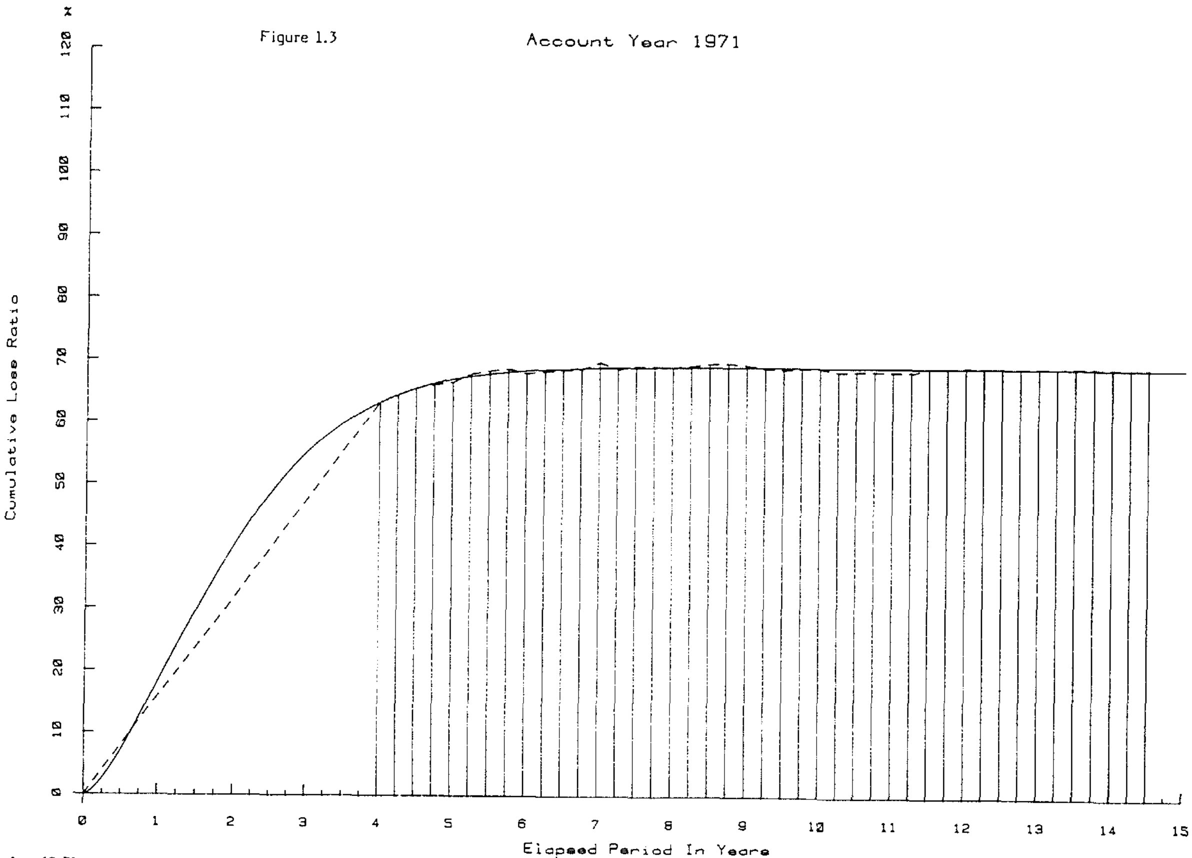
Development Year	Account Year: 1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
1				2,706	3,714	3,751	5,550	6,580	6,774	9,098	12,214	11,611	15,541	20,082
2			3,524	4,489	5,869	6,439	8,475	9,712	9,797	13,173	17,839	17,901	23,250	29,902
3		3,355	3,924	4,821	6,393	7,109	8,800	10,083	10,670	14,613	19,927	19,322	25,602	32,928
4	5,189	3,373	4,040	4,876	6,473	7,067	8,894	10,142	10,978	15,123	20,570	19,967	26,457	34,027
5	5,240	3,415	3,999	4,928	6,521	7,081	8,942	10,161	11,035	15,356	20,887	20,275	26,865	34,551
6	5,126	3,432	4,027	4,894	6,557	7,065	8,981	10,250	11,147					
7	5,279	3,449	4,024	4,911	6,570	7,091	9,006	10,329						
8	5,297	3,446	4,040	4,917	6,592	7,046	9,030							
9	5,300	3,452	4,035	4,896	6,580	7,070								
10	5,301	3,454	4,036	4,898	6,585									
11	5,288	3,455	4,037	4,894										
12	5,286	3,476	4,027											
13	5,284	3,474												
14	5,284													

Table 1.2

Loss Ratios

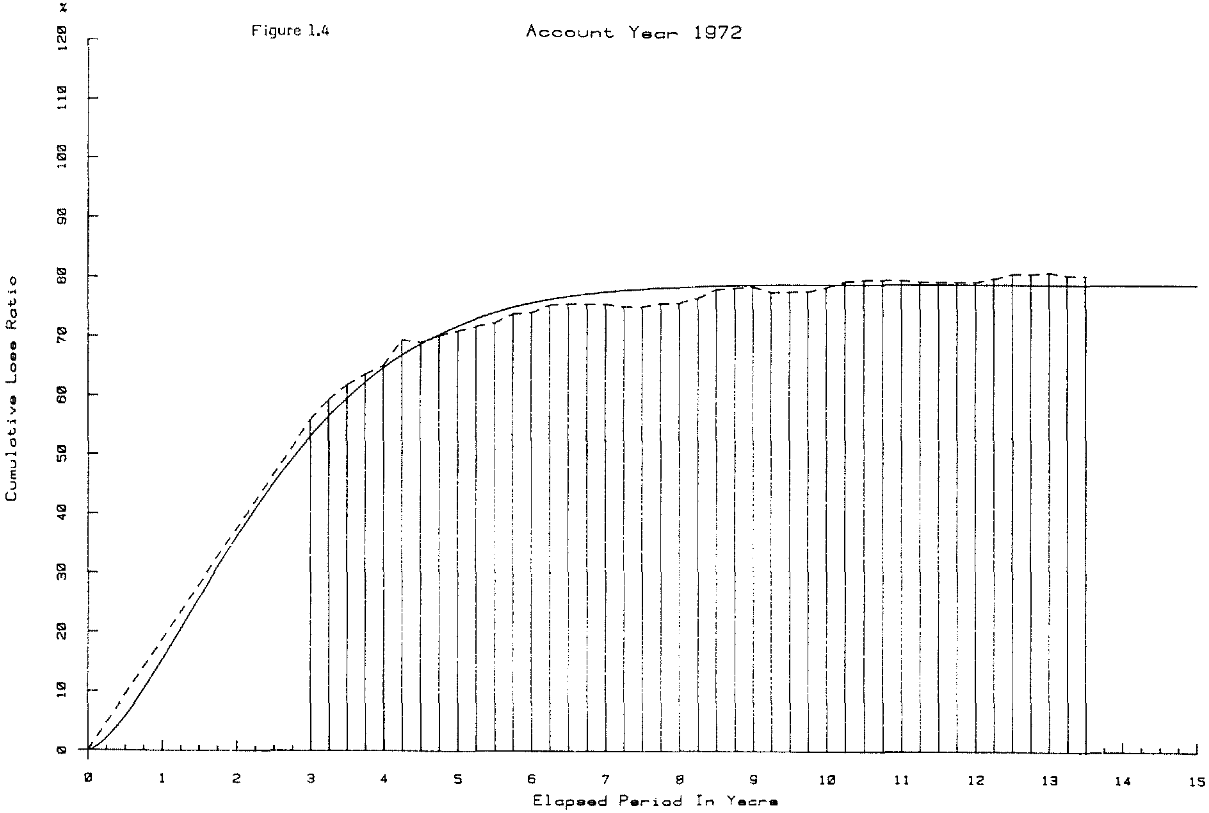
Quarter of Development	Accident Year: 1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
1					0.1	-0.6	1.2	0.1	1.0	0.0	0.1	0.0	0.1	0.9
2					0.8	-0.4	2.3	0.3	2.1	1.0	0.4	3.0	2.0	2.3
3					2.9	4.5	5.0	1.8	7.1	4.2	1.4	7.0	5.0	5.0
4				7.7	7.0	5.1	8.1	3.4	9.6	12.1	6.5	8.3	12.2	11.6
5				14.5	18.7	12.0	12.4	7.3	17.7	2.0	9.3	8.6	17.1	12.1
6				25.5	22.5	15.7	21.1	12.5	21.4	10.1	17.2	11.3	20.7	23.0
7				40.0	30.8	22.6	30.0	19.6	31.2	20.7	25.1	17.4	25.2	
8			41.6	48.2	39.1	37.1	35.6	26.0	63.1	29.9	30.7	25.4	29.6	
9			47.7	60.8	45.6	38.5	42.0	38.8	64.6	32.9	36.5	3.00	34.5	
10			53.1	65.8	50.3	43.6	46.2	40.7	73.5	40.4	39.1	33.5	39.6	
11			59.9	71.6	55.9	48.2	49.2	40.8	77.3	47.2	45.2	40.3		
12		55.9	64.9	76.0	61.5	52.3	51.0	46.8	75.5	44.2	47.8	43.8		
13		59.4	67.8	76.7	64.2	56.4	51.3	39.9	89.6	47.7	49.9	46.1		
14		61.9	73.5	79.1	67.4	58.3	52.9	41.9	87.2	50.5	53.2	47.5		
15		63.7	76.4	82.4	70.2	59.7	55.8	43.3	92.2	53.3	55.4			
16	63.2	65.2	78.3	83.4	71.4	60.9	60.7	43.6	92.6	55.7	56.6			
17	64.4	69.5	80.7	86.6	72.7	61.5	63.3	46.6	95.9	56.3	58.5			
18	65.5	69.1	81.4	85.0	72.6	64.3	65.4	48.8	96.2	58.2	60.6			
19	66.0	70.3	82.7	86.2	74.7	65.6	67.3	50.4	99.0	59.8				
20	66.5	71.1	84.1	86.6	75.5	67.3	68.7	51.7	107.3	60.5				
21	67.9	71.8	84.7	89.0	73.6	66.5	67.8	52.3	111.1	62.4				
22	68.4	72.4	87.2	89.2	72.9	67.2	69.5	58.1	117.7	63.8				
23	68.7	73.9	88.2	89.7	73.3	67.3	70.1	61.4	96.7					
24	68.0	74.1	88.2	90.5	73.0	67.9	71.3	57.9	96.0					
25	68.4	75.4	88.9	91.6	72.8	68.6	72.4	56.1	92.4					
26	68.6	75.6	89.8	92.5	73.8	69.1	72.1	55.8	94.0					
27	68.8	75.6	90.4	92.7	73.8	70.2	70.2	79.9						
28	69.9	75.6	91.0	93.1	74.0	71.4	69.7	82.0						
29	68.8	75.2	91.0	93.8	75.3	70.5	69.0	82.6						
30	69.3	75.2	91.6	93.8	75.7	72.0	68.5	86.3						
31	69.3	75.8	91.9	94.5	75.8	71.5	69.1							
32	69.2	75.9	91.7	94.1	76.2	70.6	68.6							
33	69.5	76.7	91.5	92.3	75.0	68.4	66.2							
34	69.8	78.2	91.7	92.8	75.3	68.6	65.8							
35	69.9	78.4	91.6	92.9	76.2	68.9								
36	69.6	78.6	91.0	92.3	75.5	68.8								
37	69.1	77.7	91.6	91.9	74.6	68.7								
38	68.9	77.8	91.6	91.9	74.4	69.8								
39	69.1	77.9	91.9	91.6	74.5									
40	69.1	78.6	91.6	91.6	75.1									
41	68.6	79.6	90.4	90.6	75.7									
42	68.7	79.9	90.4	90.6	75.7									
43	68.7	79.9	90.5	90.7										
44	68.6	79.9	90.6	90.7										
45	68.6	79.6	90.5	90.7										
46	69.3	79.6	90.5	90.7										
47	69.3	79.6	90.5											
48	69.5	79.6	90.8											
49	69.3	80.2	88.8											
50	69.3	81.1	88.8											
51	69.4	81.1												
52	69.4	81.3												
53	69.3	80.8												
54	69.5	80.8												
55	69.4													
56	69.4													
57	69.3													
58	69.4													

Figure 1.3



A = 69.3%, B = 2.21, C = 1.50, Mean squared error = 1.1

Figure 1.4 Account Year 1972



A = 79.2%, B = 2.80, C = 1.50, Mean squared error = 18.5

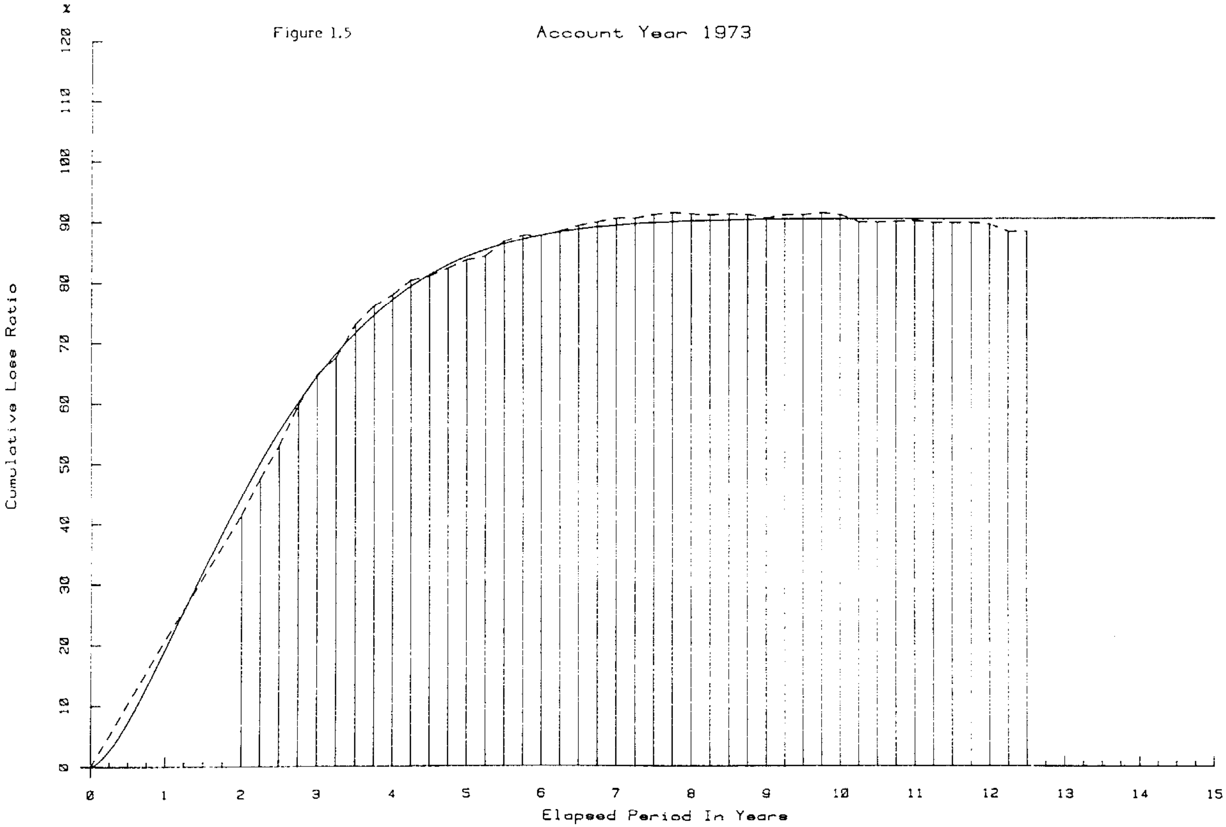
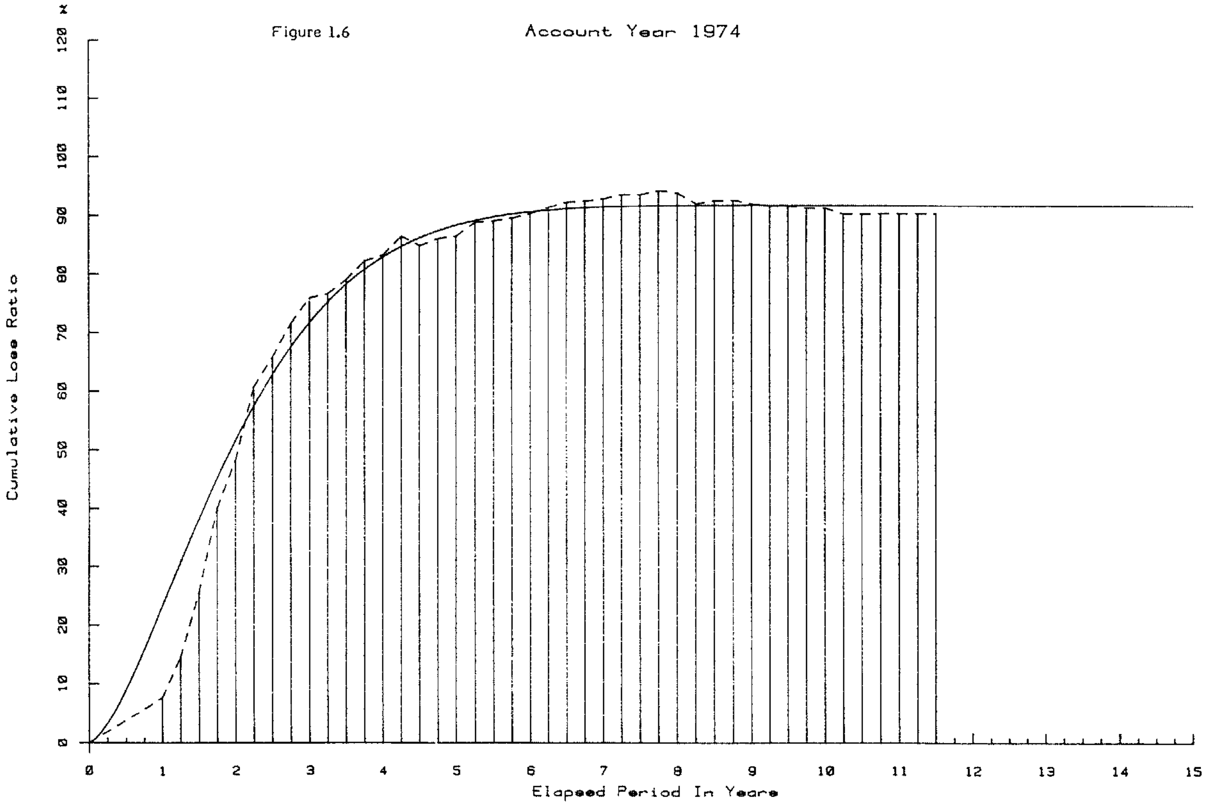


Figure 1.5 Account Year 1973

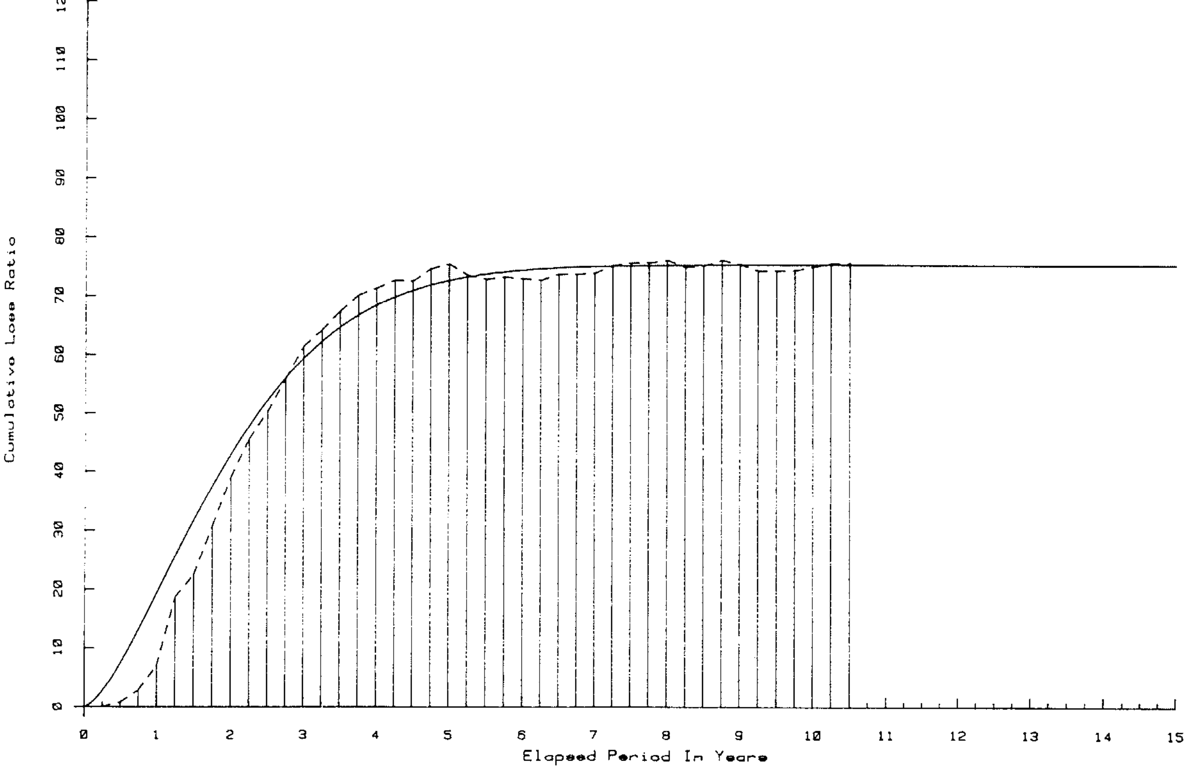
A = 91.0%, B = 2.60, C = 1.50, Mean squared error = 8.1

Figure 1.6 Account Year 1974



A = 92.1%, B = 2.28, C = 1.50, Mean squared error = 28.0

Figure 1.7 Account Year 1975



A = 75.4%, B = 2.24, C = 1.50, Mean squared error = 18.2

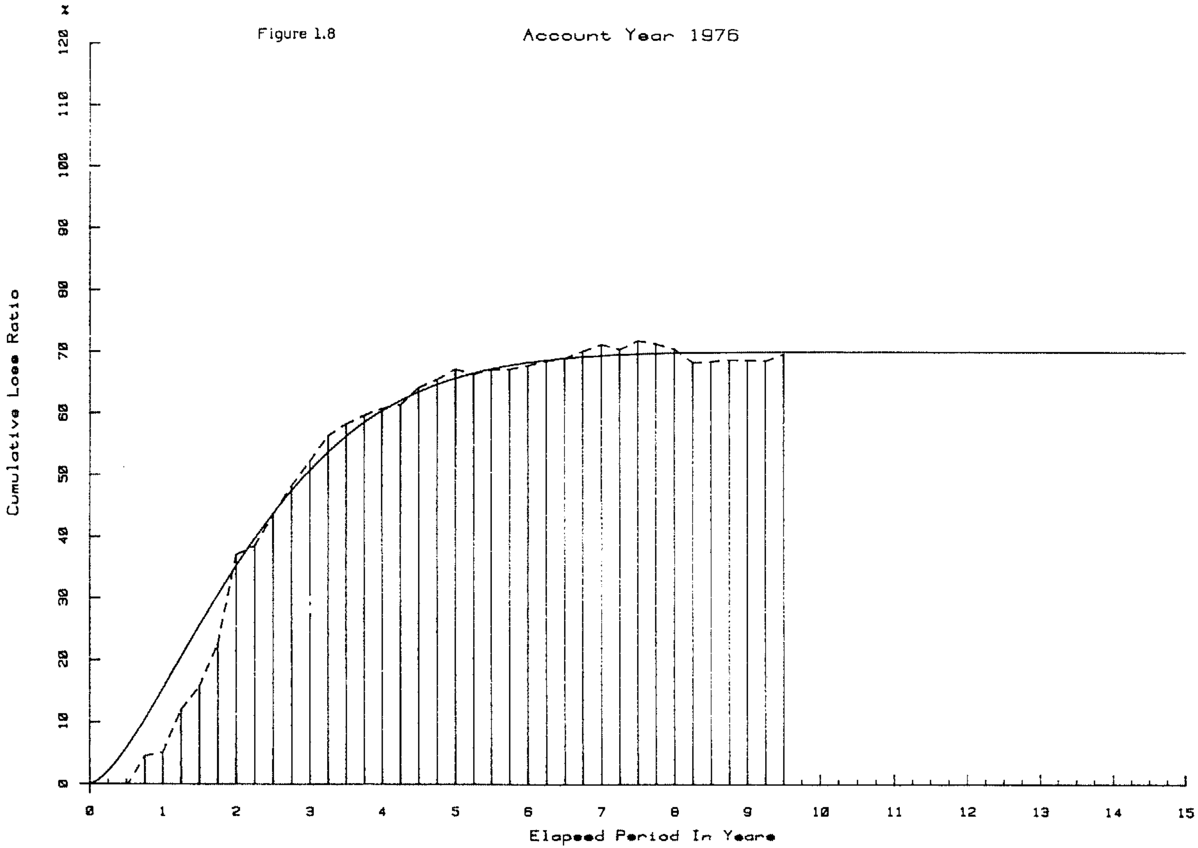
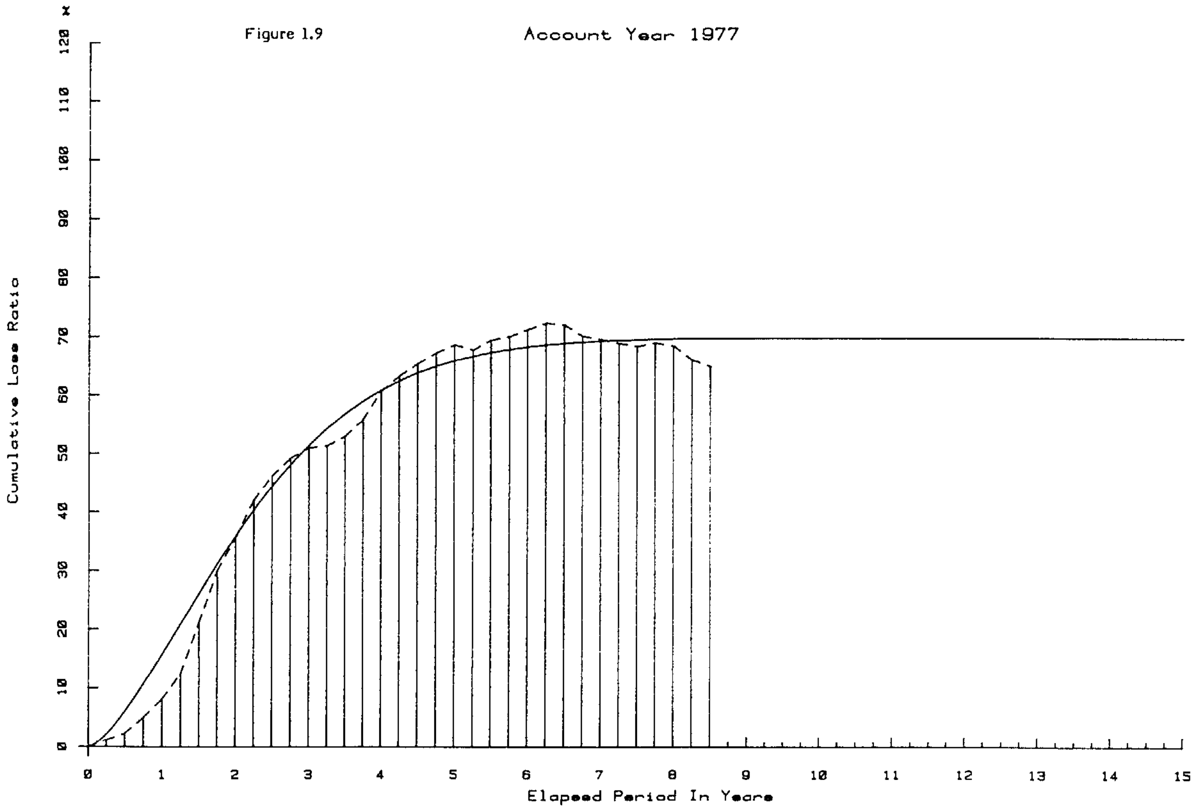


Figure 1.8

A = 70.2%, B = 2.54, C = 1.50, Mean squared error = 18.2

Figure 1.9 Account Year 1977



A = 70.0%, B = 2.49, C = 1.50, Mean squared error = 31.8

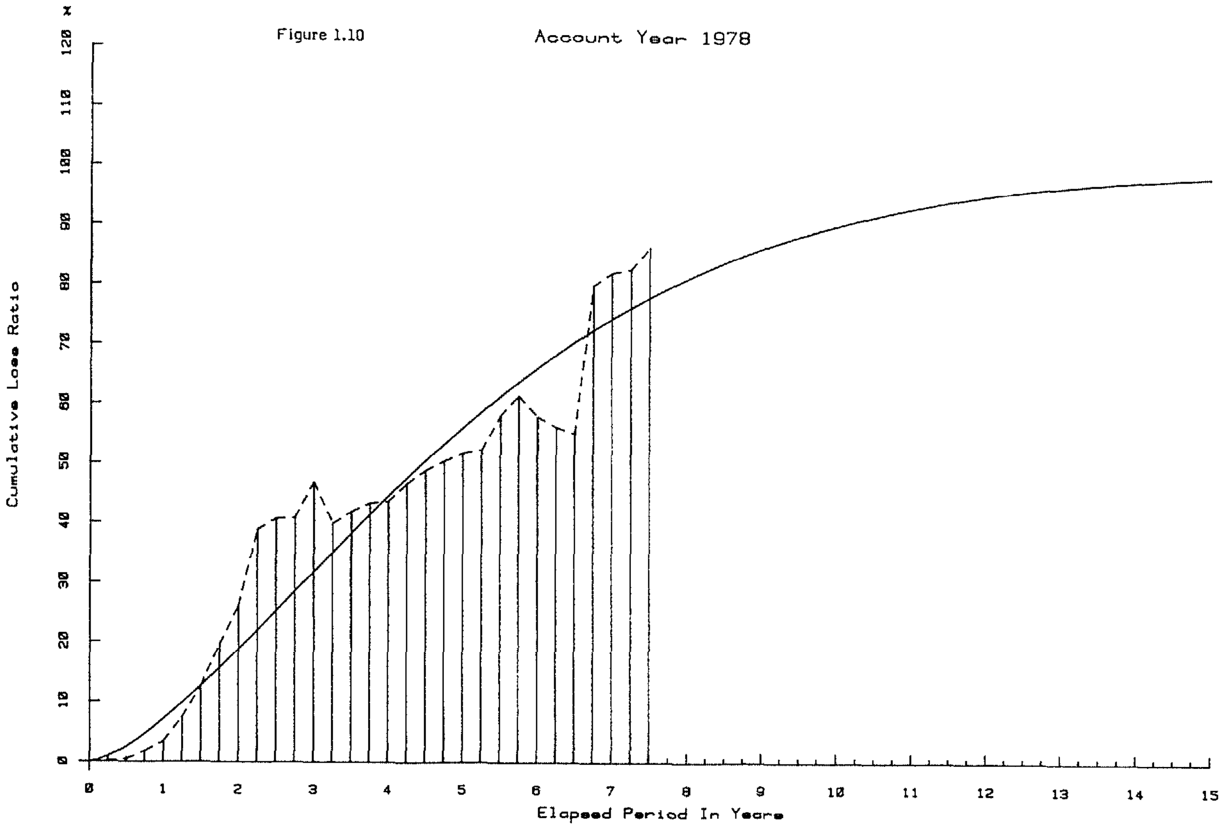
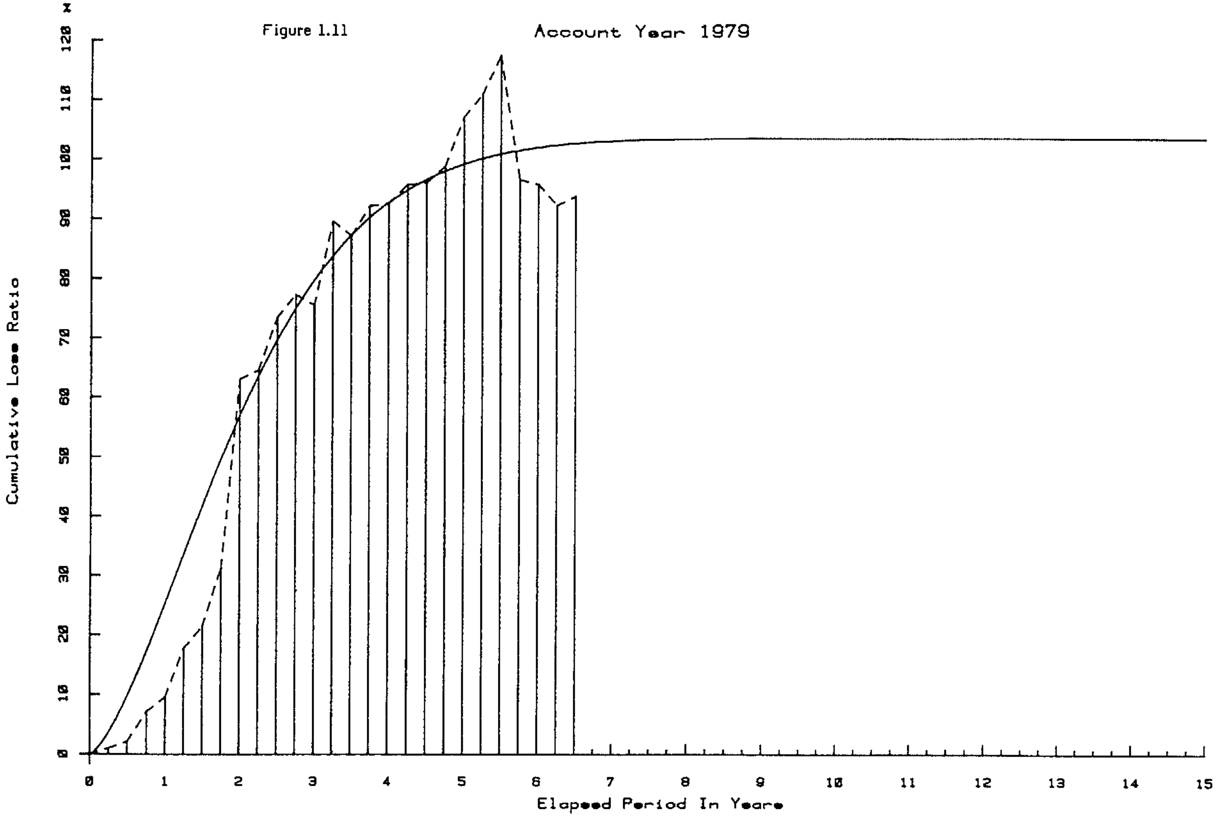


Figure 1.10 Account Year 1978

A = 99.9%, B = 5.69, C = 1.50, Mean squared error = 289.6

Figure 1.11



A = 103.8%, B = 2.33, C = 1.50, Mean squared error = 265.3

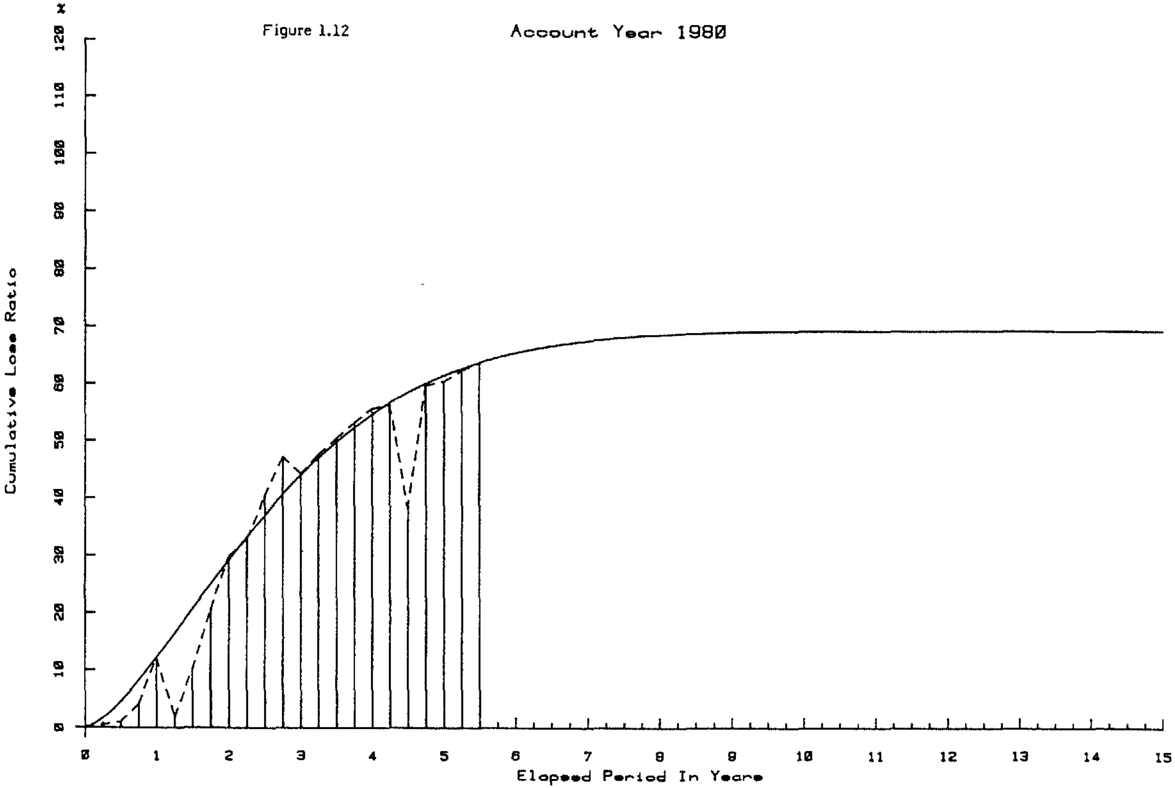
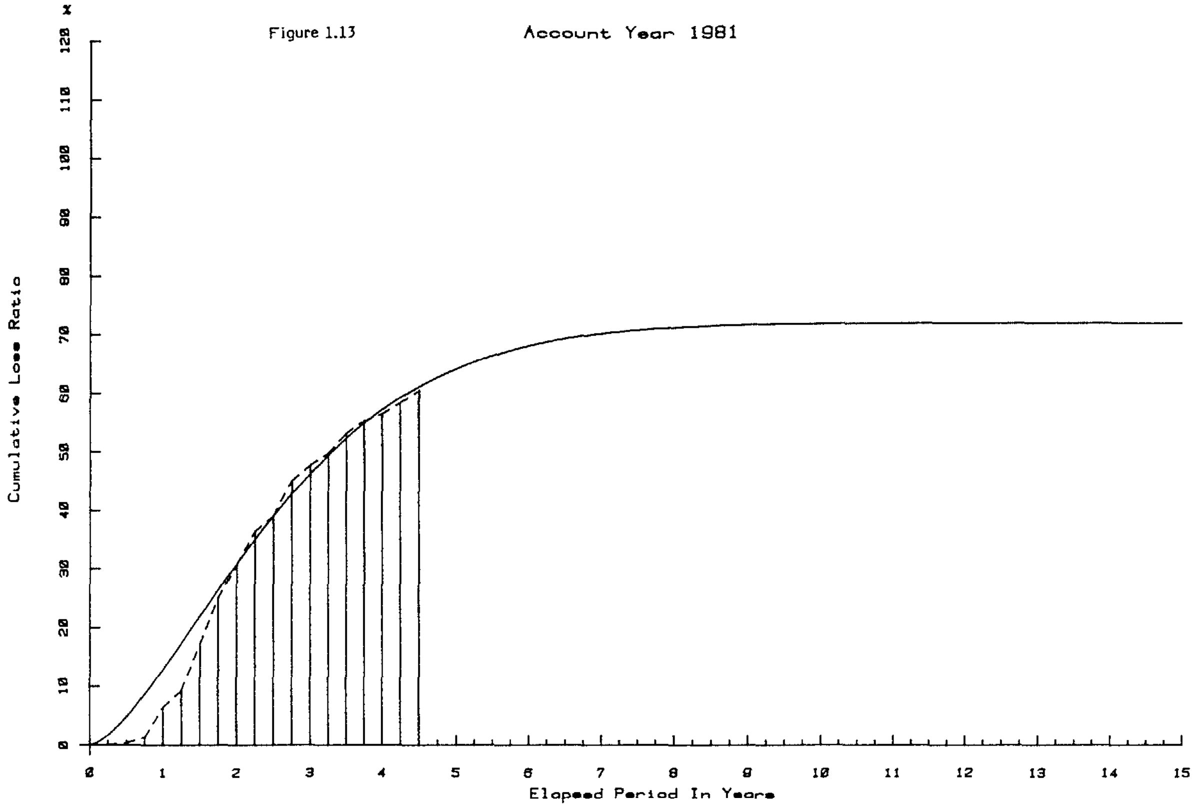


Figure 1.12

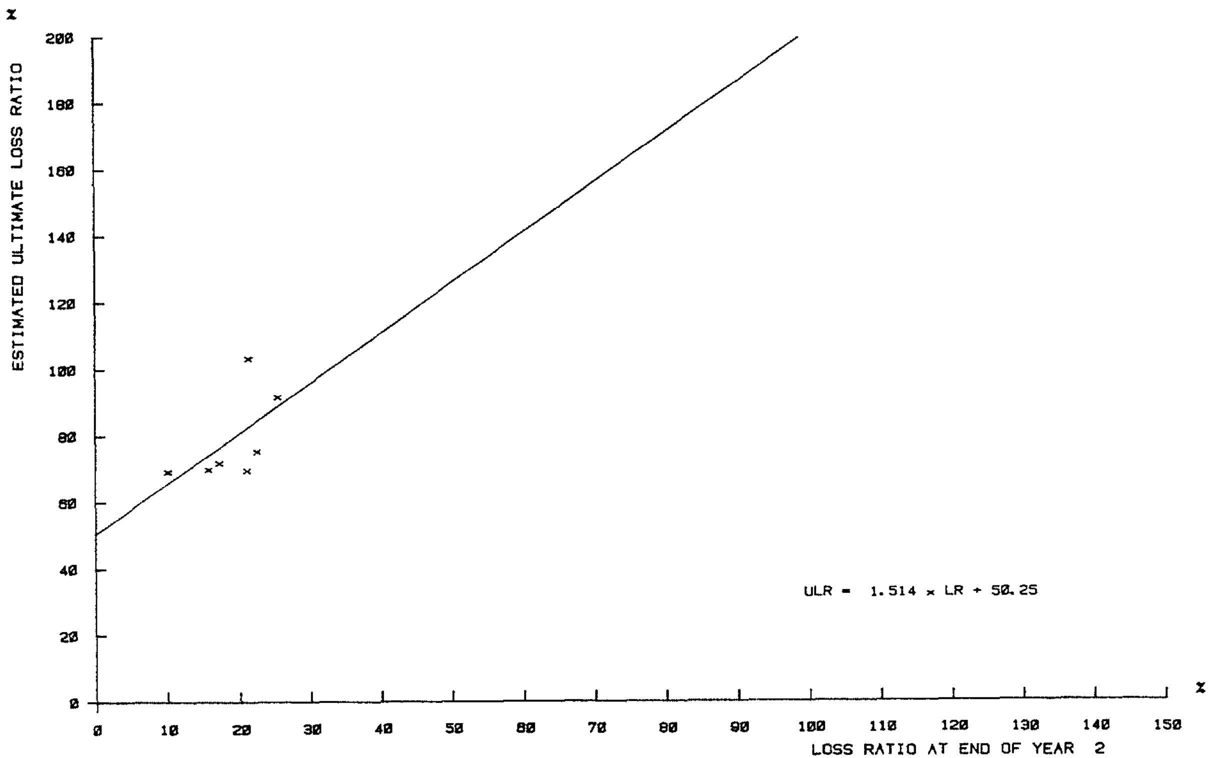
A = 69.6%, B = 3.00, C = 1.50, Mean squared error = 34.7

Figure 1.13 Account Year 1981



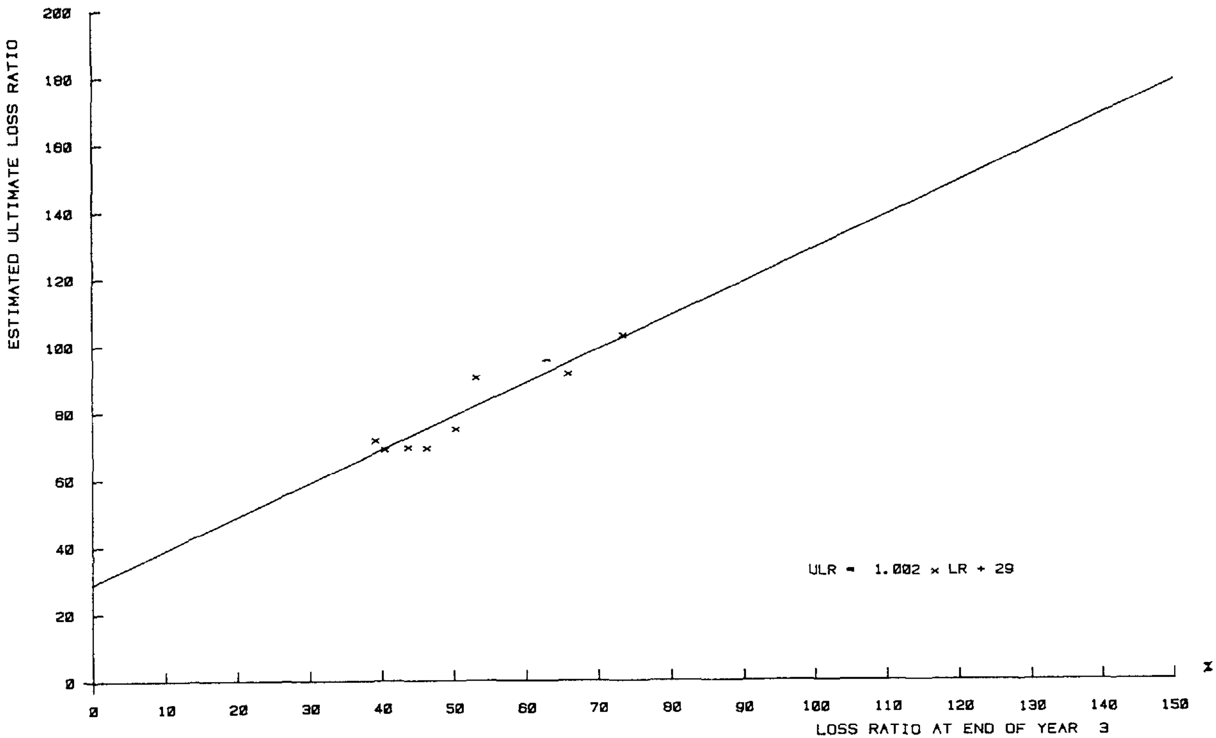
A = 72.2%, B = 2.95, C = 1.50, Mean squared error = 11.2

Figure 1.14



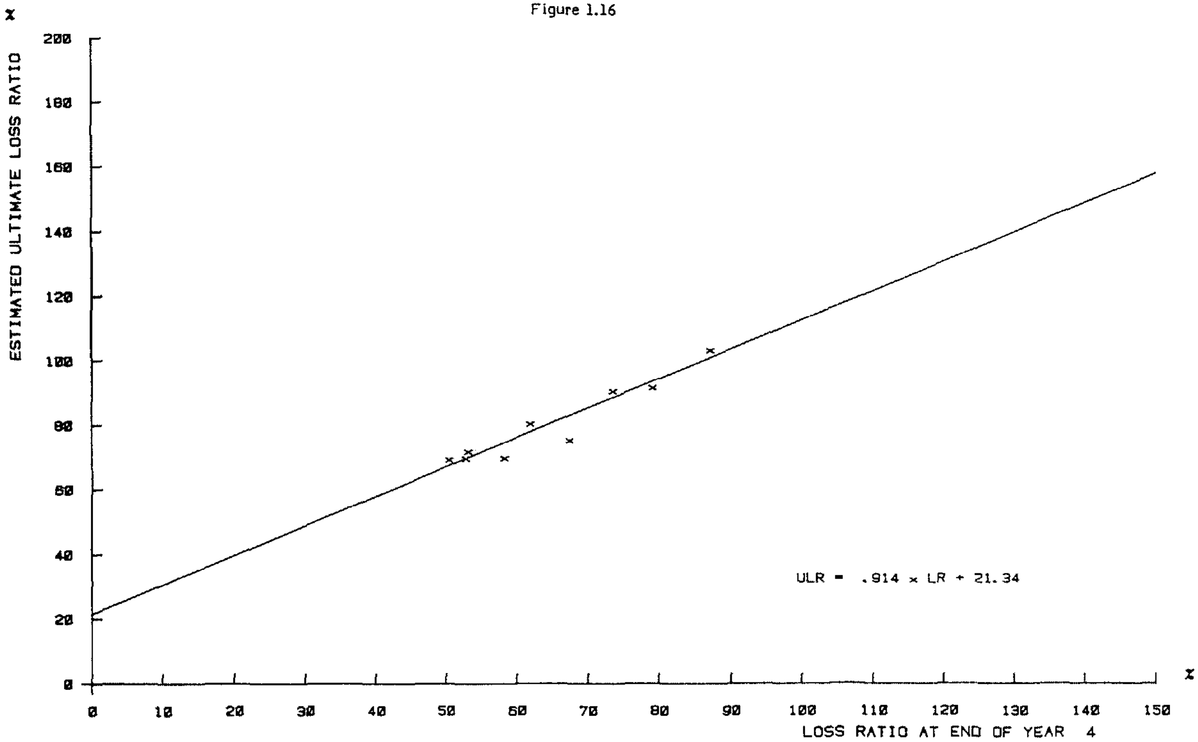
Account years fitted: 74, 75, 76, 77,79, 80, 81

Figure 1.15



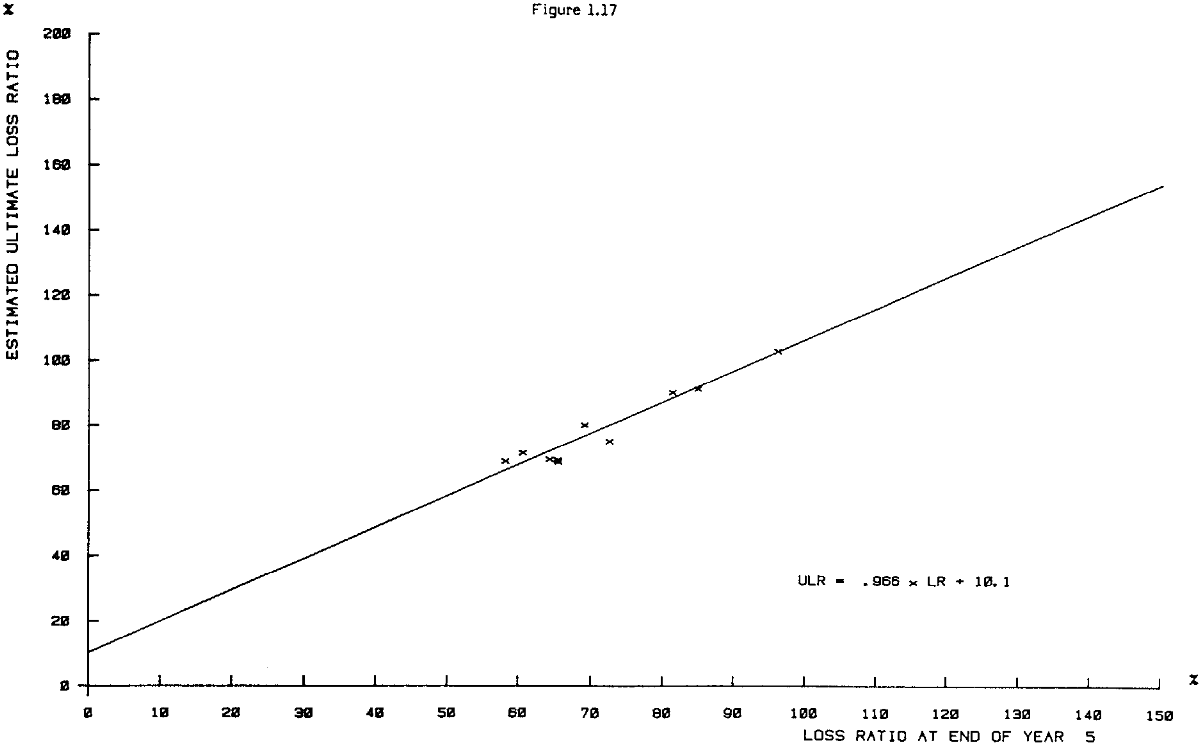
Account years fitted: 73, 74, 75, 76, 77, 79, 80, 81

Figure 1.16



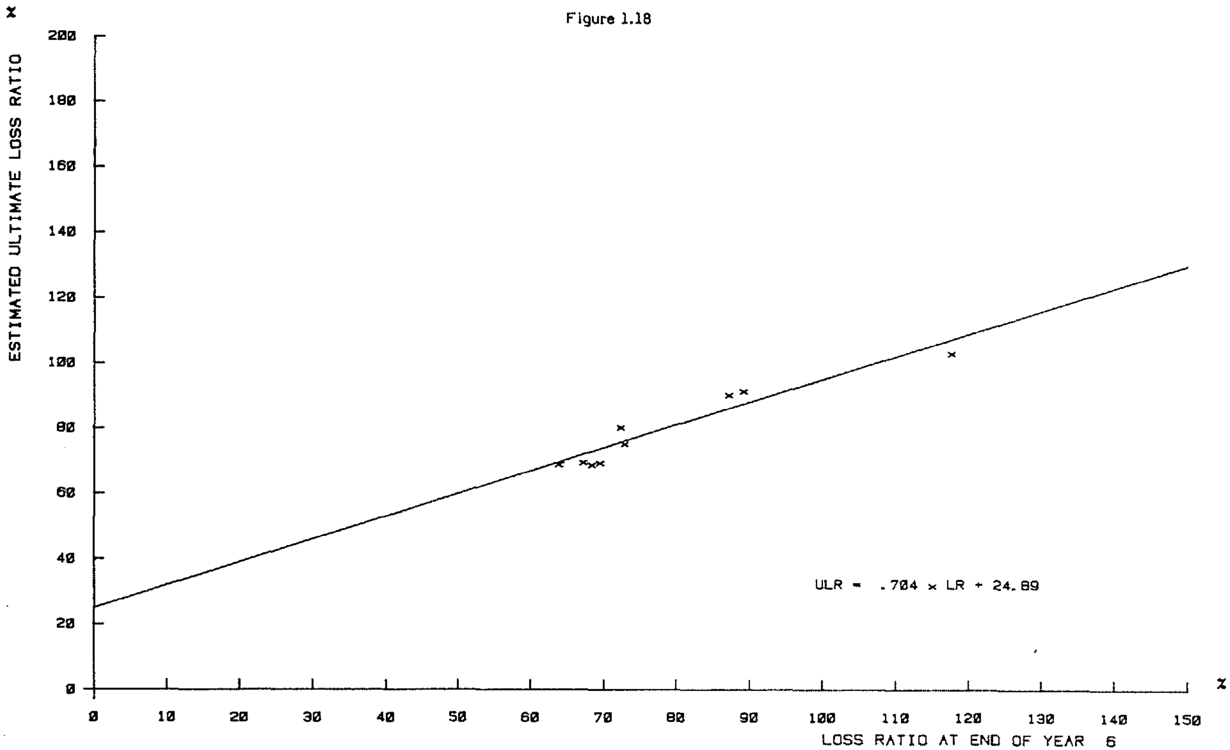
Account years fitted: 72, 73, 74, 75, 76, 77, 79, 80, 81

Figure 1.17



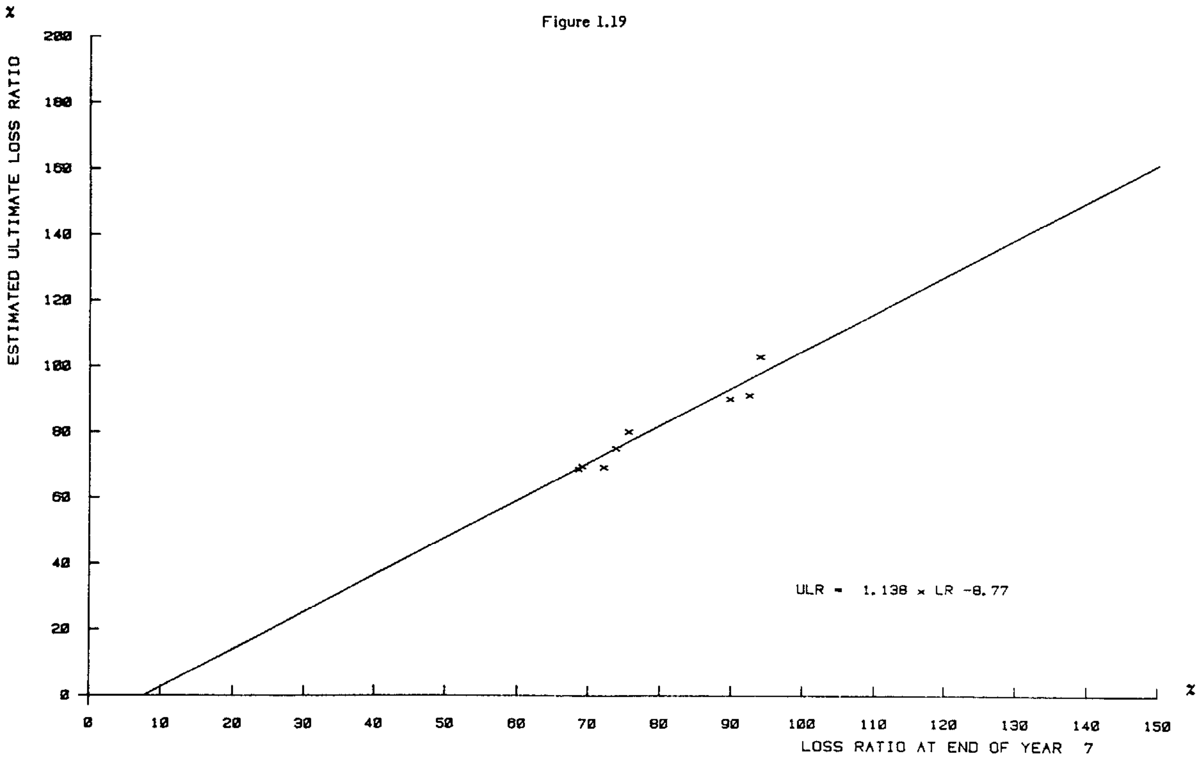
Account years fitted: 71, 72, 73, 74, 75, 76, 77, 79, 80, 81

Figure 1.18



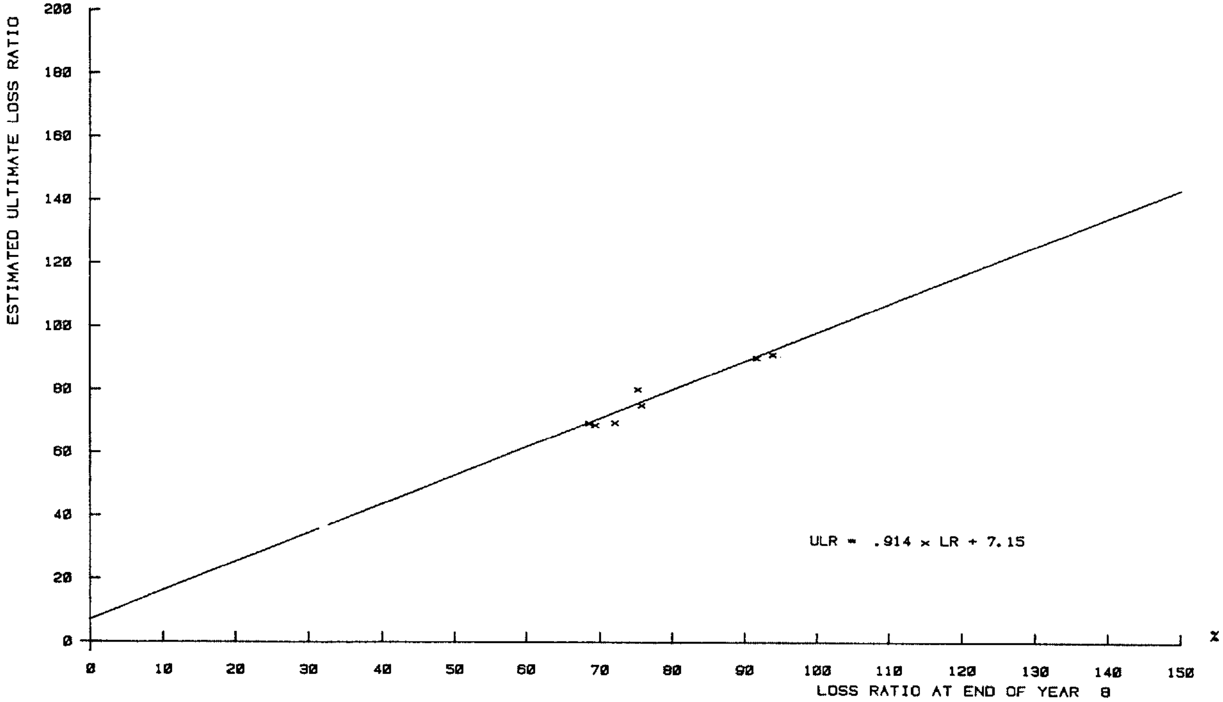
Account years fitted: 71, 72, 73, 74, 75, 76, 77, 79, 80

Figure 1.19



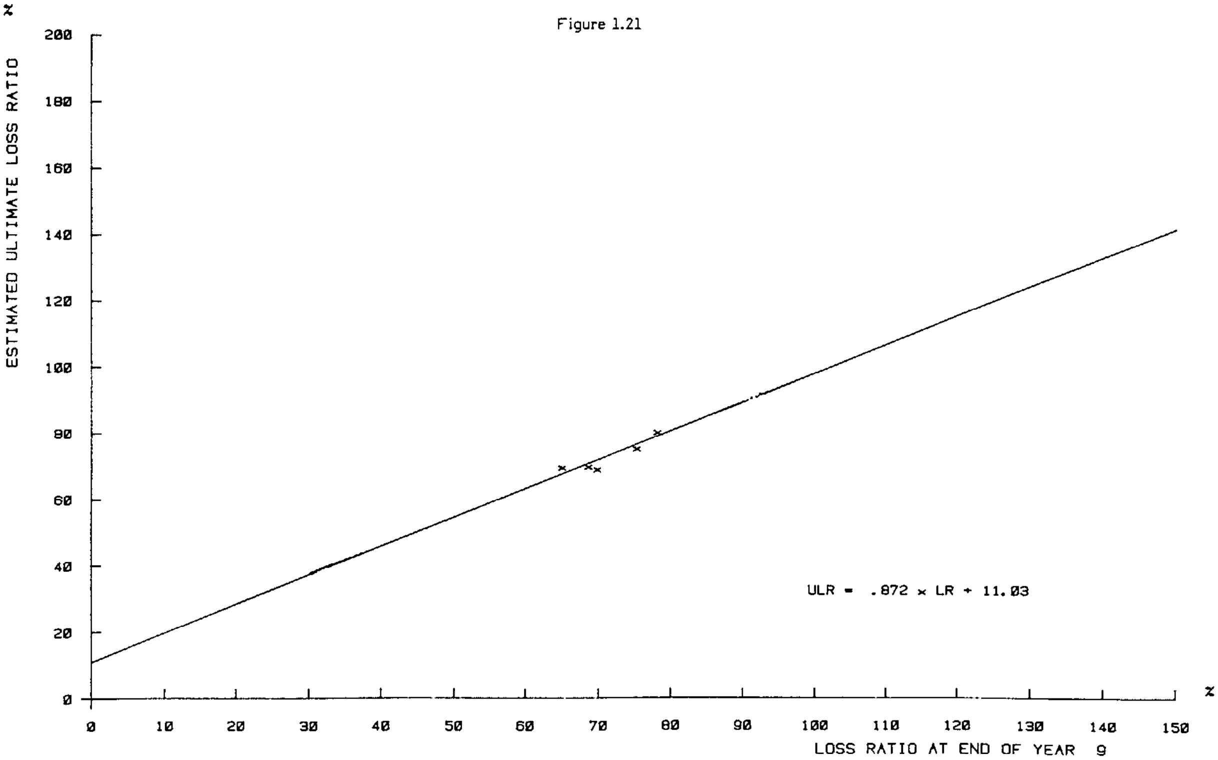
Account years fitted: 71, 72, 73, 74, 75, 76, 77, 79

Figure 1.20



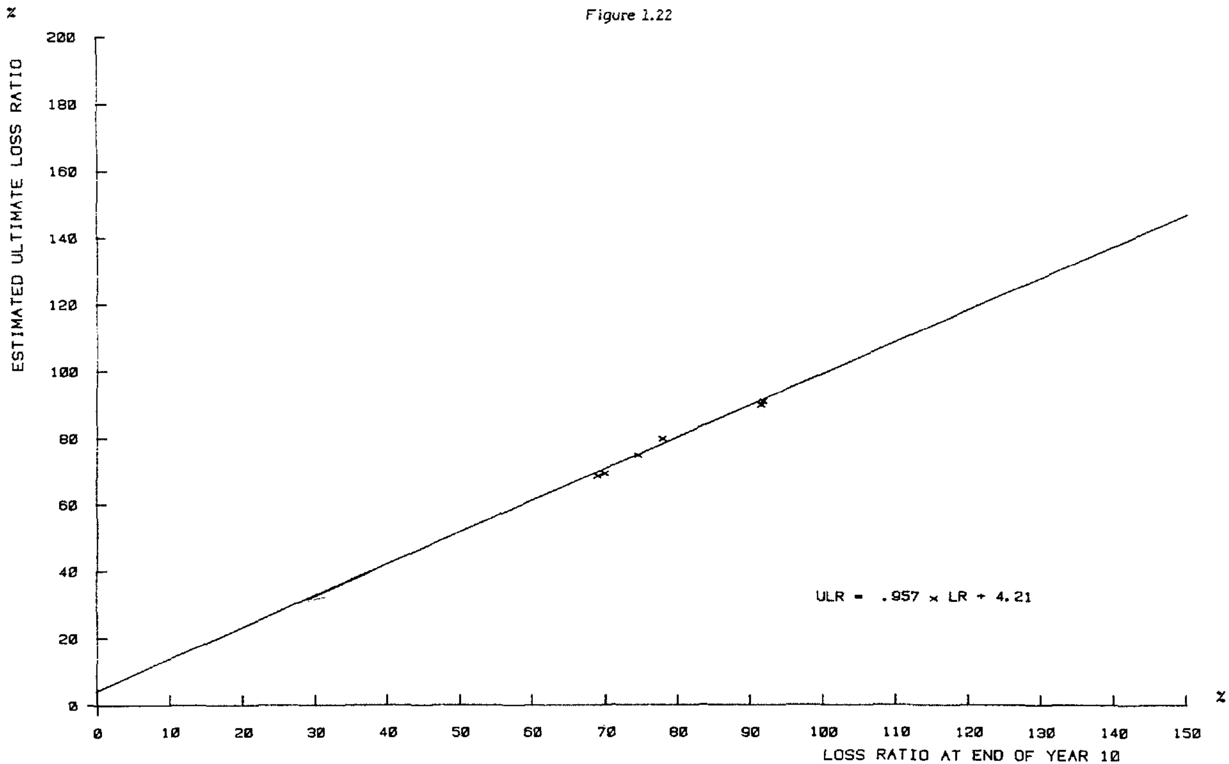
Account years fitted: 71, 72, 73, 74, 75, 76, 77

Figure 1.21



Account years fitted: 71, 72, 73, 74, 75, 76, 77

Figure 1.22



Account years fitted: 71, 72, 73, 74, 75, 76

Table 1.23

Summary of regression lines fitted

Account year	Corresponding development year	Regression line: Slope	Constant	t-test statistic: Value	Degrees of freedom
1984	2	1.514	50.25	1.58	5
1983	3	1.002	29.00	6.55	6
1982	4	.914	21.34	8.54	7
1981	5	.966	10.10	10.11	8
1980	6	.704	24.89	8.41	7
1979	7	1.138	-8.77	8.85	6
1978	8	.914	7.15	9.00	5
1977	9	.872	11.03	13.41	5
1976	10	.957	4.21	16.95	4

Account year	Latest loss ratio	Estimated ULR	Maximum deviation	90% confidence interval
	%	%	%	%
1984	23.05	85.15	21.15	27.07
1983	39.57	68.65	8.75	10.86
1982	47.48	64.74	7.23	8.54
1981	60.63	68.67	4.55	7.05
1980	63.75	69.77	4.98	8.41
1979	93.97	98.17	5.63	8.40
1978	86.30	86.03	4.89	5.83
1977	64.96	67.68	2.48	4.14
1976	69.84	71.05	2.17	3.19

Table 1.24

Recommended estimates of ULR

Account year	Loss ratio to date	Estimated ULR	Confidence interval (+ or -)
	%	%	%
1971	69.4	69.4	-
1972	80.8	80.8	-
1973	88.8	91.0	-
1974	90.7	92.1	-
1975	75.7	75.7	-
1976	69.8	70.2	-
1977	65.0	70.0	-
1978	86.3	103.8	8.4
1979	94.0	98.2	8.4
1980	63.8	69.8	8.4
1981	60.6	68.7	7.0
1982	47.5	64.7	8.5
1983	39.6	68.6	10.9
1984	23.0	85.1	27.1

APPENDIX 2

EFFECT ON SHAPE OF CURVE $L_t = A \times [1 - \exp(-[t/B]^c)]$

OF CHANGING VALUES OF PARAMETERS B AND C

Figure 2.1

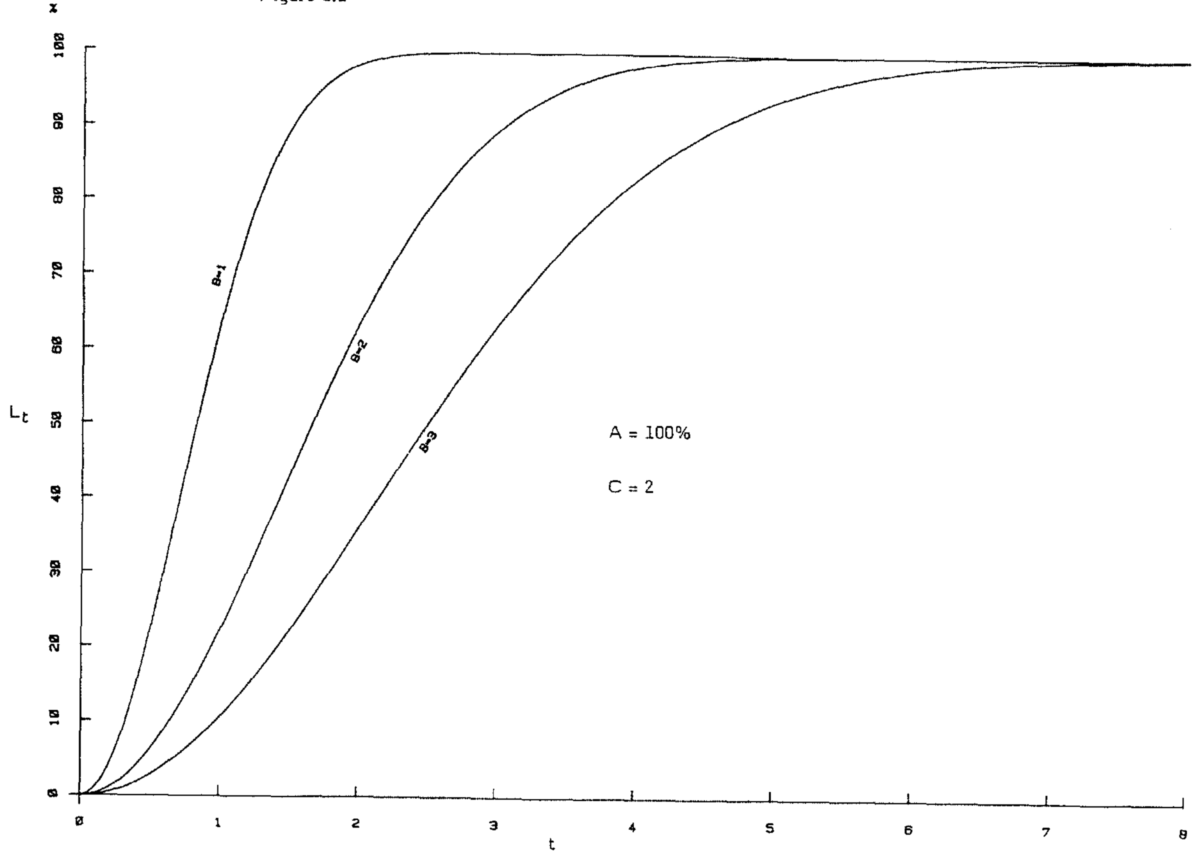


Figure 2.2