REINSURANCE

1986
DISCUSSION PAPER PROGRAM

Presented May 11-14, 1986
San Diego, California

CASUALTY ACTUARIAL SOCIETY
ORGANIZED 1914

AUTHOR CAS

TITLE Discussion Paper - 1986

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REINSURANCE

These papers have been prepared in response to a call for papers by the Casualty Actuarial Society to provide discussion material for its Spring meeting, May 11-14, in San Diego, California.

Actuaries are involved in the overall financial analysis of insurance products and programs. The effects of reinsurance are an essential component of that process. It is hoped that these papers will provide futher insights into the subject of reinsurance as they serve as a discussion vehicle for workshops at the Spring meeting.

Committee on Continuing Education

NOTICE

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REINSURANCE PRICING FOR THE NEW TRANSITIONAL CLAIMS-MADE GL PRODUCT

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Biographical Sketch

Mr. Asch is currently Vice President and Actuary for SCOR Reinsurance Company with responsibility for both Loss Reserving and pricing. Mr. Asch holds a 1971 BS Degree from Columbia University and a 1973 MBA from Tulane.

From 1973-1979 he held a variety of actuarial positions with CG/Aetna (CIGNA). From 1979-1982 he was Center Pricing Officer for Sentry Insurance Group. From 1982-1984 he was Vice President - Worldwide Casualty Underwriting for AFIA Worldwide Insurance.

Abstract

The losses that impact Casualty Excess of Loss Reinsurers are far different in nature and size than the losses that confront primary companies.

The new claims-made ISO GL policy forces new data requirements for accurate pricing of these new covers. Data is sparse and difficult to obtain. This is particularly true for reinsurers who cannot directly control their ceding company's data base.

An analysis was performed on the 329 individual reinsurance claims in this sample whose average individual loss size was $1.1 million.

These results are compared to the results ISO obtained from average primary claims experience.

The Marker-Mohl Backward Recursive method was applied to develop ultimate estimated losses by report lag subset.

Surprising results are obtained that may allow reinsurers to avoid seriously under pricing reinsurance for these new products in their first years of existence.

Some observations are also made about accurately assessing levels of exposure in pricing risks under a claims-made system, as well as discussion of what exposures would be covered in varying situations.

Effective January 1, 1986, ISO plans to introduce an updated edition of its standard CGL policy. It will introduce a claims-made coverage option for all insureds as well as the traditional coverage form based on the date of loss occurrence.

The purpose of this paper is to discuss several important transitional pricing implications for those firms that reinsure the new claims-made coverage option on an excess-of-loss basis. It is assumed throughout this paper that the reader is familiar with standard actuarial pricing techniques covered thoroughly elsewhere.

THE PROPOSED CLAIMS MADE FORM

The new coverage option covers the insured against the ultimate losses arising out of written claims for damages first made during the policy period, no matter when losses occur.

This paper concerns pricing procedures and quirks during the transition from an "occurrence-year" coverage to a "claims-made" coverage system.

A retroactive date limits coverage under a claims-made policy to occurrences only after that retroactive date. When the insured has had occurrence-year coverage previously, the retroactive date can be set to the policy inception date. All "prior acts" have been covered by previous policies.

There also can be an option for purchasing an Extended Reporting Period Endorsement or "Tail Coverage" with any new claims-made form.

The extended reporting option deems losses reported after the policy expiration date to still be covered by the earlier claims-made policy. The following chart may help illustrate the differences between the two forms in the initial transition years since it is valuable to easily examine exactly which exposures are actually covered under various possible permutations of claims-made coverage, retroactive dates and extended reporting periods. For ease of discussion, we assume here that all losses are reported after only two years. In practice, there are a potentially infinite number of "rows" corresponding to various loss reporting lags. Let us define this type of array as a Loss Reporting Matrix. Each row of this matrix corresponds to losses associated with a particular claim reporting time lag. Henceforth, we will refer to them as "lag n" losses. The first year of transitional pricing will only involve "lag 0" losses. The second year involves "lag 0" and "lag 1" only. This is a new dimension that comes with a claims-made policy.

	DATE OF LOSS OCCURRENCE				
CLAIM REPORTING TIME LAG	1984	1985	1986	1987	1988
0	A	B	C	D	E
1	F	G	H	I	J
2	K	L	M	N	O

POLICY COVERAGE	EXPOSURES COVERED
A. A claims-made overage for 1986 with a 1/1/86 retroactive date and no extended reporting period	C
A. With 1/1/85 Retroactive date	C + G
A. With 1/1/84 retroactive date	C + G + K
A. With extended reporting until 12/31/87	C + H
A. With extended reporting until 12/31/88	C + H + M
A. With extended reporting until 12/31/88 and A 1/1/84 retroactive date	C + H + M + G + K + L
A traditional 1986 occ. yr. policy	C + H + M

1988 claims-made policy	E + I + M
1988 occurrence policy	E + J + O
1988 claims made policy with unlimited extended reporting option and 1/1/86 retroactive date	E + I + M + J + O + N

The mature 1988 claims-made policy contains pieces that would have been covered by 1986 (M) and 1987 (I) and 1988 (E) occurrence basis policies at varying limits of liability and levels of exposure. All this is incorporated into the 1988 mature claims-made coverage. Note that each individual cell will vary with respect to average severity and its unique Loss Emergence Pattern. Generally, the later reported claims are larger and more complex. With the extended reporting option, a claims-made policy could cover far more exposures than an occurrence policy for the same year.

THE TRANSITIONAL PRICING PROBLEM

What percentage of the prior occurrence year premium should be charged for the claims-made option during each year of the initial transition period? That is perhaps the central question this study addresses. Each column of our Loss Reporting Matrix represents 100% of the former occurrence basis pure premium. We assume that no extended reporting period is being priced. That should be handled elsewhere. We also assume that if a risk begins the transition from an occurrence coverage to a claims-made coverage on January 1, 1986, then the retroactive date for the claims-made policy series will always be January 1, 1986. These are reasonable assumptions.

For the first year of claims-made coverage, the pure premium should only cover losses which occurred in 1986 and were reported in 1986. We will name these losses "lag 0" losses since the claim reporting lag is zero years. Also we define the former 100% occurrence basis pure premium as follows:

$$\text{OccPP} = \sum_{\forall i} \text{lag } i \text{ ultimate pure premium}$$

Then, in all cases, the first year transitional fraction (or pure premium incremental multiplier) to be charged is clearly: T(1) = (lag 0 ultimate pure prem)/OccPP.

For any lag n, therefore, the transitional claims-made multiplier (Tn) is defined as ($\sum_{i=1}^{n}$ lag i ult pure prem)/OccPP, or $\sum_{i=1}^{n} T(i)$.

OccPP clearly can always be alternatively expressed as 100%.

As more years transpire under a claims-made system, the fraction that is appropriate will increase. We can visualize this as more and more loss reporting lag "rows" within our Loss Reporting Matrix being exposed. The longer the loss reporting lag for the covered portfolio, the lower the initial transitional pure premium multipliers. Reinsurance portfolios evidence different characteristics than primary portfolios. A special type of loss data base is necessary to evaluate these transitional claims-made multipliers.

THE DATA PROBLEM

Industry and company statistics have historically been kept on an accident year basis. This is perfectly appropriate for occurrence based policies. Statutory Reporting also has stressed accident year loss development. Nowhere has there been an organized statistical data base congenial to the claims-made form or the transaction period between an occurrence based policy and a claims-made policy.

What we are saying is totally invisible in this data is the distribution between pure IBNR losses on late reported claims (IBNYR) and changes in the claims values related to known claims already reported (IBNER).

This additional dimension is absolutely crucial for accurate pricing of the claims made coverage in its first few years of existence.

This is precisely the dimension that is missing from the standard ISO, BESTS, NAII, RAA or Annual Statement data bases. These data bases track incurred losses by accident year as they are reported and later settled. There is currently no way to extract any data by reporting lag subset from any of these sources.

A concrete example should illustrate the point. Let us postulate a simple liability product with a uniform three year ultimate life span (or tail) for loss development. Data for this line in the traditional format is readily available.

TYPICAL YEAR OF OCCURRENCE	CUM. INCURRED LOSSES AS OF: 1980	1981	1982	Ult.
1980	1/3	2/3	1	1

Let us heroically assume that we know in advance how the loss development for this line ultimately distributes itself. We are starting the transition to claims-made system from an occurrence system.

The following chart shows a view of the coverage from an accident year perspective. One third of the ultimate incurred losses for this occurrence policy emerges in each of its three years of loss development. In reality, we never know in advance precisely what this pattern will be.

The columns to the right of the solid line analyze these losses from one year's occurrences by report lag. The first calendar year's one-third emergence relates completely to losses with no loss reporting lag by definition. The second calendar year's one third of the occurrence years ultimate losses relates 50% to late case loss reserve developments on claims immediately reported, and 50% to losses first reported with a one year loss reporting lag. We call the second subset lag 1 losses.

We see clearly here that any and all traditional accident year industry standard loss development data is of very little value in establishing the proper fraction of "occurrence coverage" rates to charge for transitional claims-made coverage.

The proper incremental allocation, in retrospect, would be 1/2 for the first year claims made policy, 1/3 for the second year and 1/6 for the third year. The traditional data bases may have indicated to the casual analyst that the three years should each have been charged one-third.

TRADITIONAL ACC. YEAR I.L. AFTER N YEARS DEVELOPMENT	LOSS COST % OF ULT. ACC. YEAR LOSSES	ULTIMATE AMOUNT RELATED TO REPORTING LAG YEARS			
		LAG YR 0		LAG YR 1	LAG YR 2
0	1/3	1/3			
1	1/3	1/6	+	1/6	
2	1/3			1/6	1/6
TOTALS	1	1/2		1/3	1/6

This short exercise tells us that for the first year transitional claims-made pricing, naive use of traditional data and analysis might lead you to feel a rate of 1/3 is adequate when actually 1/2 should be used. It also could lead people to naively analyze results one year after implementation and feel 1/3 was an adequate rate, when actually 1/2 is the needed rate. If only the exercise were that simple.

Let us create another Loss Reporting Matrix:

CLAIM REPORTING TIME LAG	LOSS OCCURRENCE DATES			
	1980	1981	1982	1983
0	A	B	C	D
1	E	F	G	H
2	I	J	K	L

We have assumed throughout that initial case claim reserves turn out to be exactly correct. Debate rages, but the general consensus is that the industry is 10% or more under-reserved. If you are under-reserved, it will be easy to feel you are profitable in this environment when you easily may not be.

Each entry on the above Loss Reporting Matrix represents initial losses. These amounts are not ultimate or static. Each cell of the matrix will have a loss development pattern of its own, which I will label a Loss Emergence Pattern.

(ie) Incurred Losses for Cell A as of:	1980	1981	1982	1983	1984	"Ultimate"
	70	70	70	70	100	100

There appeared no more intuitively appealing way to describe this three-dimensional phenomenon to me.

Let us assume that initial incurred loss estimates are 3/7 inadequate.

At the end of calendar year 0 our intrepid insurer does not see the ultimate 1/2 in losses or even the ultimate 1/3 related to matrix element A. One half would be precisely the appropriate percentage multiplier for the first year transitional claims-made coverage. He sees only (.333) (.7) = .2331 in losses. He has charged a rate of .33330 times the prior occurrence rate for the cover. He has "claims-made coverage". He believes he has no IBNR at all. Instead of feeling that his first year transitional rates were only (.50/.33) = 2/3 of being adequate, he feels they were redundant by .333/.2331 = 43%. There will be firms who will behave in this manner. Depending upon the level of inadequacy of their case reserves, and the time they take to develop, they may live in a fool's paradise for some time. This may be particularly true for certain excess-of-loss reinsurers in the coming transitional environment.

Next we will be dealing with hard actual data as opposed to the theoretical data we have used thusfar. We hope to introduce a dose of caution here to those who are naive enough to believe that the claims-made form will eliminate all concerns about adverse and unanticipated late loss development or IBNR for them after January 1, 1986.

A "REAL WORLD" EXAMPLE

The statistics shown in Exhibit 7 are drawn from a large, homogenous, mature claims-made program. Although we do not have statistics in occurrence date by report date detail, the nature of the exposures (building collapses, water damage, tiles falling, etc...) intrinsically keeps the lag time between date of loss occurrence and date of first claims notice minimal. The program was handled continuously by the same association - broker - underwriting company combination for many years prior to our observation of its loss experience. It has had a large and stable individual risk population. It is included here for emphasis. Let all primary and reinsurance firms note the serious incurred loss deterioration over 18 calendar months on a a series of old claims-made underwriting years. All those who feel "claims-made" has solved all their "IBNR" problems for GL business need to study this Exhibit more closely. The claims-made program will do nothing to mitigate late development of losses on old policy years arising from individual, case-basis, claim under-reserving.

This data shows how a mature claims-made program can still develop major increases in incurred losses many years after a claims made policy expires.

THE ISO APPROACH TO TRANSITIONAL CLAIMS MADE MULTIPLIERS

As stated before, no appropriate industry data base exists for claims-made pricing. ISO has done an excellent job of using limited data to estimate these percentages. ISO made one standard industrywide assumption regarding case claims reserve adequacy. They assumed that current case reserves turn out ultimately to be exactly correct. Later, I will have much to say regarding the sensitivity of rate adequacy to case claim reserve adequacy in this new environment. Their approach is described fully in ISO Circular GL-85-64. That is the ISO data continually referred to in the next section.(1)

A firm should not attempt to independently price these transitional covers unless it has access to a large data base of homogenous risks and is willing to commit a great deal of time and effort to this analysis.

After having said this, there probably will be instances where firms feel the industry average factors are not appropriate for their use.

A reinsurer in particular often faces a far different loss exposure profile than a primary company. The reinsurer also does not have direct control of his ceding company's claims handling, coding and reinsurance loss reporting systems. For these reasons, I felt it would be valuable to assemble the appropriate data base from a sample of GL excess-of-loss treaty submissions. After reviewing over 100 casualty treaty submissions, only four treaties contained the detailed individual claim data to perform the analysis I desired to make. This requires a data base of large GL claims where occurrence year and report year are available in individual claims detail. Also available is the gradual case reserve loss development in individual claim detail. This type of data was unavailable to ISO on any industrywide basis also.

Please remember this sample is heavily biased toward the very large claims a reinsurer will face. This should be interesting since experience tells us that these claims should take longer to report and to settle, on average, than normal primary claims.

A SAMPLE OF REINSURANCE DATA

The data base contains 329 individual GL claims from four large primary company's treaty submissions. In light of the extreme difficulty in assembling this sort of large loss data, credibility issues have been ignored. When trended to a common date of 1986, these claims have an average severity of $3.016 million each. Exhibit 1 gives some pertinent statistics relating to the sample.

My first objective was to duplicate, as closely as possible, the ISO Methodology to compare immature claims-made transitional "year-in-program" factors. My decision was to use the Products subline for comparison purposes since most of these reinsurance claims were products claims and products liability most closely resembles the "long-tail" large-loss reinsurance data base we are working with.

Therefore, the same annual trend factors ISO used for products were used to trend the losses in Exhibit 1 to a 1986 Calendar level. At this point, the same assumption that ISO made about subsequent case claims development is made for this sample: all case reserve estimates are exactly correct. More will be said and exhibited later regarding this point.

Exhibit 2 presents the pure premium multipliers from the reinsurance data base and the ISO products liability pure premium multipliers. No adjustments to gross rates for expenses are made since expense provisions can vary widely between reinsurers and primary firms. Neither was there any adjustment for any prior acts coverage. This allows for a simple and direct comparison.

Both Exhibits 1 and 2 contained a large number of surprises to me versus my intuitive a priori expectations. It was shocking to find that consistently derived pure premium multipliers for massive reinsurance claims were so very close to those for primary products-liability business. My expectations included both lots of reinsurance claims with very large reporting lags and 10% or 15% indicated first year pure premium multipliers. The data shattered both these intuitive assumptions. The reinsurer who believes that he writes only "long-tail" business, and therefore can heavily discount the initial ISO immature claims-made multipliers profitably, is probably going to receive a very expensive surprise. The intuitive belief that larger claims tend to be reported later is somewhat borne out by this data sample. It would seemingly break severity into subsets of lag 0 to lag 2 and "all others".

My initial expectation was that large reinsurance losses would show serious case reserve inadequacies over time. Although the new claims-made program does eliminate true late-reported IBNR, the lingering uncertainty surrounding errors in case claim reserve estimates remains.

The case reserve margins might also show different patterns between the various reporting lag subsets. Exhibit 4 displays the data over the longer loss development time horizon reinsurers must concern themselves with. To give a feeling for loss development, all final reserve estimates per claim are carried forward to the final evaluation point in the study. We show lag 0 subsequent case loss development out to a maximum of 15 years. It is amazing to see that, assuming outstanding case reserves are accurate, the case development to date on these massive claims has approached its "ultimate" 15 year value after only 5 years for lag 0. There are no obvious and massive case reserve deteriorations as we have seen in Exhibit 7. Exhibit 5 shows the percentage of assumed "ultimate" incurred losses that have emerged after each year of development. It is surprising to see the consistency of these patterns between the various lag subsets for lag 0 to lag 4. The patterns in higher lags are erratic due to the very small sample sizes involved.

The foregoing would lead one to believe that a firm could quickly predict its ultimate liabilities in a claims-made environment. The crucial assumption still being that the last set of case claim reserve estimates is perfectly accurate.

DEALING WITH POTENTIAL CASE RESERVE INACCURACIES

Up to this point, both ISO and this study have assumed that all case claim reserve estimates are final and perfectly adequate. This assumption needs to be emphasized and tested. It was for this reason that we insisted that every claim in this data base show annual updates of all case reserves as long as they remained outstanding.

At this point it was decided to test that assumption using techniques published by Marker & Mohl.(2) Factors for future case reserve development are applied only to the outstanding reserve portion of losses at each step of development. Using this sample, one large loss or some strange late development is always possible. My approach was to develop Marker-Mohl factors separately for each lag using a subset of already closed claims within that lag. This avoids any assumption being made about future loss development. Then these factors would be applied to all the loss data to predict ultimate losses. The method would yield perfectly stable ultimate loss projections from year 1 to the final year for the closed claims subset. To use them on data partially containing that data subset is to bias your result toward accurate ultimate loss projections from the first year.

The author realizes the many valid objections actuaries have raised to the use of closed claims data for performing IBNR studies. It was felt that, if ultimate losses could be early and accurately predicted, even with a flawed methodology, then the argument that claims-made reinsurance IBNR in a new claims-made environment is virtually zero would be very strong indeed.

The author could not think of any other satisfying test that did not somehow involve assuming your final ultimate incurred loss level. That would not be a test of any practical value to me.

Exhibit 6 shows the various Marker-Mohl ultimate estimates by lag subset as they develop. Another surprise was delivered. After all the above transpired my expectation was a very narrow range of ultimate forecasts forced by the data subset the factors were based on. What we find is the potential for ultimate loss estimates that are both too high and too low by wide margins. That is with the benefit of factors derived from the actual final development of a large subset of each data component. There are three possible explanations:

1. Reinsurance, casualty, excess-of-loss, loss development is simply too unstable to accurately predict.

2. The concept of using closed-claims data here is flawed, so the experiment fails.

3. Credibility issues.

Any and all of the above explanations may well apply.

However, we have, for this sample of data, tended to disprove the allegation that ultimate loss costs can be early and easily predicted in a claims-made environment.

SOME COMMENTS ON EVALUATING EXPOSURE LEVELS

1988
400

In the past, this was sufficient information to establish the level of exposure units to price a 1988 occurrence year coverage for most subclasses of GL. We realize that products liability exposures levels also bear a relationship to all past year exposures, where the product is still being used. In the coming mature claims-made environment, you will need much more information. You will need to know how many years of reporting lags are associated with this risk and the distribution of ultimate incurred losses by all elements in a loss emergence matrix.

Let us assume a simple 3 year loss pattern with ultimate losses distributed 50% to the first lag year, 30% to the second, and 20% to the third. Let us now compare the true exposure level for a 1988 mature claims-made policy in a growth mode versus a shrinkage mode.

	1986	1987	1988	1988 MATURE CLAIMS-MADE EXPOSURES
GROWTH MODE	200	300	400	200 + 90 + 40 = 330
SHRINKAGE MODE	600	500	400	200 + 150 + 120 = 470

	1988 CLAIMS-MADE POLICY GROWTH MODE	1988 OCCURRENCE POLICY	1988 CLAIMS-MADE POLICY SHRINKAGE MODE
1988 EXPOSURE LEVEL	330	400	470

Note that true exposure levels would always be higher in the shrinkage mode than the growth mode under a claims-made system. This is a pricing variable that will be important and can easily be underestimated. This variable clearly is sensitive to the historic rate of exposure growth or shrinkage as it relates to the loss reporting emergence pattern of the risk.

We can now see why the issue of extended reporting periods and E + O Coverage extensions for retired professionals have always been difficult problems for a claims-made insurance coverage system.

SUMMARY

The following are the key points that I have tried to emphasize:

1. All common historical industry data bases do not prepare any firm to price properly claims-made coverage neither do they guide you to assess properly the calendar year results arising from the early years of the transition to claims-made coverage from occurrence coverage.

2. The data-base that is crucial is a combination of the Loss Reporting Matrix and Loss Emergence Pattern described in this paper.

3. The IBNR exposure under a claims-made system totally excludes the true IBNR associated with late reported claims. You still are exposed to loss reserve developments. The practices of your own claims department here are paramount. An example shows how very major late loss development is possible in some situations.

4. A sample of very large historic GL claims does not show the expected greater loss-reporting lag than small claims exhibit.

Surprisingly, there is little evidence in this sample to support the belief that larger claims exhibit greater "late" loss development. Also, the comparison of ISO and Reinsurance sample immature pure premium multipliers showed small differences only.

5. When pricing mature or traditional claims-made covers, the calculation of the risks level of exposure will not be trivial. It theoretically is the historic number of calendar exposure units weighted by ultimate losses distributed by report lag periods or $\sum_{\forall i} \left[P(i) \text{ times } E(x-i) \right]$ where P(i) is the proportion of all ultimate losses with report lag i and E(x-i) is defined as the exposure level in Year x-i where we are pricing the risk for Calendar Year x. This is the solution to the problem as raised in the Marker and Mohl article.(2)

6. With differing retroactive dates and extended reporting periods available, the scope of coverage for a particular risks claims-made coverage can vary widely.

7. There are many non-trivial issues raised on retroactive date choices in later renewals when an insured may change carriers.

I realize more questions have been raised than answered. This topic needs a great deal of study and discussion in light of its easily growing importance to the entire industry. Hopefully, those facing new issues like these will not rely totally on intuition and assumption. There is merit to assembling a data base appropriate for your use, when possible, and testing any assumptions you may have.

Please remember that this study is restricted to only one sample of data. If anyone has another similar data base, the author would be pleased to compare evidence. To the author's knowledge, there is no data base of this kind in existence at the present time. If claims-made becomes a major CGL policy form, perhaps such a data base will become necessary on an industrywide basis.

EXHIBIT 1

Summary of Reinsurance Claims Data

Category	Number of Claims	($000) Total Loss* Values Trended To 1986	($000) Average Trended Size of Loss	Percentage Lag Factors
Lag 0	133	$310,453	$2334	31.3%
Lag 1	82	$192,444	$2347	19.4%
Lag 2	40	$101,085	$2527	10.2%
Lag 3	29	$118,836	$4098	12.0%
Lag 4	13	$ 67,525	$5194	6.8%
Lag 5	12	$ 87,747	$7312	8.5%
Lag 6	4	$ 18,217	$4554	1.8%
Lag 7	8	$ 41,296	$5162	4.1%
Lag 8	4	$ 34,247	$8562	3.4%
Lag 9	3	$ 18,831	$6277	1.9%
Lag 11	1	$ 4,650	$4650	0.5%
TOTAL	329	$995,331	$3025	100%

*The same 11% annual trend that ISO used for pricing its Products - Claims-Made Pure Premium multipliers is used here.

EXHIBIT 2

	Sample Reinsurance Data Lag Factor	11% Annual Trend Adjustment	Adjusted Lag Factor
Lag 0	.313	1.00	.313
Lag 1	.194	.901	.175
Lag 2	.102	.812	.083
Lag 3	.120	.732	.088
Lag 4	.068	.660	.045
Lag 5	.085	.595	.051
Lag 6	.018	.537	.010
Lag 7 & Rmdr	.100	.484	.048

YEAR-IN-PROGRAM

Adjusted	1st	2nd	3rd	4th	5th	6th	7th	Mature
Lag 0	.313	.313	.313	.313	.313	.313	.313	.313
1		.175	.175	.175	.175	.175	.175	.175
2			.083	.083	.083	.083	.083	.083
3				.088	.088	.088	.088	.088
4					.045	.045	.045	.045
5						.051	.051	.051
6							.010	.010
7								.048
SUM	.313	.488	.571	.659	.704	.755	.765	.813

EXHIBIT 3

Comparison of Pure Premium Multipliers For Transitional Claims-Made Pricing

CLAIMS-MADE Year-In-Program	REINSURANCE Data Sample	Multipliers For ISO Primary Products Liability
1	.313	.442
2	.488	.574
3	.571	.671
4	.659	.723
5	.704	.756
6	.755	.780
7	.765	.794
Mature	.813	.844

EXHIBIT 4

Untrended Incurred Loss Development by Report Lag Subset

Incurred Losses as of	($000) Lag 0	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7
1 YR	$71,890	44,407	20,096	38,849	10,898	16,691	2921	9347
2 YR	99,623	53,202	28,702	41,023	11,785	17,102	4253	7893
3 YR	117,523	53,874	30,061	42,122	14,712	17,611	4137	5806
4 YR	127,274	59,936	32,109	44,763	14,951	18,212	2822	6824
5 YR	133,066	61,110	33,999	48,134	14,994	16,286	2883	8324
6 YR	133,761	61,876	34,841	48,089	14,997	17,049	3097	8475
7 YR	138,368	61,318	36,157	48,227	14,997	19,514	3097	8475
8 YR	138,440	61,703	36,118	48,944	16,435	15,260	3209	8493
9 YR	139,380	62,135	36,186	50,121	17,649	-	1950	-
10 YR	139,129	71,529	36,221	49,759	17,963	-	-	-
11 YR	139,366	-	-	-	-	-	-	-
12 YR	139,366	-	-	-	-	-	-	-
13 YR	138,705	-	-	-	-	-	-	-
14 YR	136,860	-	-	-	-	-	-	-
15 YR	136,860	-	-	-	-	-	-	-

There is no need to apply trend to analyze loss development within each lag subset since each subset has one constant and unique average accident date. (The Last Case Reserve estimate is assumed to be perfectly accurate and is always carried forward)

EXHIBIT 5

Untrended Insured Loss as a Percentage of Mature Incurred Loss Levels

Incurred Losses as of	% Lag 0	% Lag 1	% Lag 2	% Lag 3	% Lag 4	% Lag 5	% Lag 6	% Lag 7
1 year	53	62	55	78	61	109	150	110
2 years	73	74	79	82	66	112	218	93
3 years	86	75	83	85	82	115	212	68
4 years	93	84	89	90	83	119	145	80
5 years	97	85	94	97	83	107	148	98
6 years	98	87	96	97	83	112	159	100
7 years	101	86	100	97	83	128	159	100
8 years	101	86	100	98	91	100	165	100
9 years	102	87	100	101	98	-	100	-
10 years	102	100	100	100	100	-	-	-
11 years	102	-	-	-	-	-	-	-
12 years	102	-	-	-	-	-	-	-
13 years	101	-	-	-	-	-	-	-
14 years	100	-	-	-	-	-	-	-
15 years	100	-	-	-	-	-	-	-

EXHIBIT 6

Evolving Ultimate Loss Estimates -
Marker-Mohl Method Applied Only
To Closed Claim Subset by Lag Subset

Ultimate Incurred Loss Estimate as of	Lag 0	Lag 1	Lag 2	Lag 3	Lag 4
1 YR	$206,896	66,004	40,142	63,339	12,131
2 YR	193,932	62,885	33,578	56,693	12,283
3 YR	188,322	65,182	35,904	58,031	13,923
4 YR	164,220	58,811	37,671	48,624	16,390
5 YR	147,150	62,404	40,425	48,203	17,697
6 YR	154,290	61,321	41,070	48,700	17,697
7 YR	139,803	62,499	37,788	48,571	17,697
Volume of Closed Claims Used	65,006	36,325	25,579	19,892	11,868
Latest Evaluation of Total Insured Losses	136,860	71,529	36,221	49,759	17,963
Closed Claims as a Percentage of Total Claims	47.5%	50.8%	70.6%	40.0%	66.1%

EXHIBIT 7

U/W YEAR	AS OF JUNE 1980			AS OF DEC 1981			% LOSS DEVELOPMENT OVER 18 MONTHS		
	Paid Losses	% Losses $000	Inc. Losses $000	Paid Losses $000	O/S Losses $000	Inc. Losses $000	Paid Losses $000	Q/S Losses $000	Inc. Losses $000
1973/74	868	316	1184	1271	626	1897	+ 46 %	+ 98 %	+ 60%
1974/75	419	671	1090	800	479	1279	+ 91 %	- 29 %	+ 17%
1975/76	336	762	1098	766	1164	1930	+ 128 %	+ 53 %	+ 76%
1976/77	417	1485	1902	1993	2257	4250	+ 378 %	+ 52 %	+ 123%
1977/78	128	985	1113	967	2955	3922	+ 655 %	+ 200 %	+ 252%
1978/79 *	65	599	664	868	2858	3726	+1235 %	+ 377 %	+ 461%

* Theorically, this u/w year has just expired. By one argument, there is absolutely no more potential for any true IBNR after this date (on that u/w year and all the prior years listed above) since no more new claims can be reported after this date. Theorically true - but wait.

(The local currency units actually are more valuable than dollars).

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RECENT DEVELOPMENTS IN RESERVING FOR LOSSES IN THE LONDON REINSURANCE MARKET

BY

H.E. CLARKE

Summary

The paper describes in detail a new method which can be applied by any insurance company to its own data to set reserves for outstanding losses (including IBNR) and to calculate a confidence interval for these reserves. The method has also opened up a whole range of interesting ways of looking at data. Although the method can be applied to any sort of business it is particularly helpful in looking at long tail business, such as that written by reinsurers, for which other methods have proved less satisfactory. The methodology can also be applied by a supervisory authority to establish minimum reserving standards for companies where global general market data on run-offs for different classes of business is available. A new method of setting minimum reserves for individual syndicates based on the methodology in the paper is currently being tested by Lloyd's of London. This work is briefly described in the final section of the paper.

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"Mathematical Density Functions Applied to a Liability Insurance Portfolio", H.E. Clarke & L.M. Eagles, Transactions of the 21st International Congress of Actuaries;
"Should Actuaries be Random?", R.D. Campbell & H.E. Clarke, Journal of the Institute of Actuaries Students' Society, Vol 25.

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1. INTRODUCTION

This paper describes a system which our firm has developed and refined over the last 5 years to enable us to comment on reserves set up for outstanding and IBNR claims by companies writing marine, aviation, liability and reinsurance accounts or alternatively to advise on such reserves. The companies we have advised have been operating in the London Market in the UK of which Lloyd's is the centre. The London Market underwrites a significant part of the world's insurance and in particular its reinsurance and is a dominating influence on insurance world-wide. Although the system described is particularly suitable for reserving for reinsurance accounts it is also applicable to all other types of casualty business. The system is fully operational on our main frame computer. It has been used many times and it is stable.

In the London Market details of numbers of claims are generally not available or not relevant. Data is usually available for each "account year", i.e. for all risks written in a particular accounting year which is usually a calendar year. The items normally available are:

(i) Premiums paid to date

(ii) Claims paid to date

(iii) Claims outstanding, i.e. the case estimates as notified by the brokers to the companies for outstanding claims.

Further details of the constraints and problems posed by the data are given in Section 2.

The system had therefore to be able to generate estimates of the reserves from this limited amount of data. The method works by estimating the Ultimate Loss Ratio ("ULR") for each account year, from which the necessary reserve is easily derived. An important innovation of the method is that a confidence interval is produced for the ULR and hence for the reserves. An outline of the method is given in Section 3, a detailed worked example in Section 4 and some further problems and considerations are discussed in section 5. The method is very graphical and so easy to see and present to actuaries and non-actuaries.

In the final section of the paper, Section 6, we describe an application of the method to setting minimum reserves at Lloyd's which is currently being tested. The method can also be used in that way to set minimum reserves for companies operating in any insurance market where industry wide statistics are available.

The method starts from an idea put forward in a paper by D.H. Craighead (1) to the Institute of Actuaries. Inside our firm we have considerably refined and extended this idea. A detailed description of the potential use of the method by Lloyd's together with an outline of the general method is given in the paper by my colleagues S. Benjamin and L.M. Eagles (2) to the Institute of Actuaries. In this paper the emphasis is reversed with considerably greater detail being given about the general method. We also wish to thank A.B. English for the programming and application of the curve fitting algorithm and for much other programming.

2. DATA

As previously mentioned the data available for setting reserves in the London Market is sparser than that usually available from companies writing mainly domestic risks. The reasons for that are outlined below.

For risks written in the London Market cover is usually given for one year. The premiums are received over a period of typically three years. This delay can be due, for instance, to excess of loss treaties being rated on a burning cost basis or to delays in monies being forwarded by brokers. The incidents which take place during the year of cover give rise to claims which may not be reported for many years and then may take several years to settle. The main reason for this delay is that the London market tends to deal in reinsurance where the information is "second-hand" in the sense that it comes from a primary insurer which may itself be subject to delays of information. For instance suppose you are writing a catastrophe excess of loss treaty covering property damage exceeding $10 million in aggregate for any one incident for a Californian company. The reinsurer may not hear anything from the Californian company until its own claims reach the agreed limit. The final outcome for the reinsurer in the London Market may then take a long time to become fully known. Further as this example illustrates the concept of number of claims is not meaningful in this market.

Also the risk will often be placed on a coinsurance basis, often with 20 or 30 different underwriters. Detailed data may be available to the leading underwriter, but that detailed information may not be available to others on the risk and will not be recorded centrally. Statistics have in fact tended to be subordinate to accounting data, which is therefore the only data commonly available. This also has the problem that if an error is discovered in the statistics (e.g. an outstanding claim has been notified in Italian lire rather than US dollars) it will be corrected from discovery, but the history will be left unchanged so that the statistics still reconcile with the published accounts.

The data is usually available for each account year. Thus the method described in this paper will be presented for data collected on that basis. However as will become clear the method is equally applicable to data collected on an accident year basis. It is common for the data to be missing for early account years or early years of development, often due to computerisation of the accounting function taking place at that point.

In the case of Lloyd's further problems arise from the use of very broad risk categories which cannot be assumed to be homogeneous over time. The classic example of this is Non-marine All Other which can include marine business written by non-marine syndicates. Further the data collected centrally consist only of premiums received and claims paid, both net of reinsurance. After the end of the third year of development of an account year future premiums received are set off against future claim payments in the statistics.

More information on the operation of the London Market in general and Lloyd's in particular is given in the paper by D.H. Craighead (1).

The techniques described in the paper can be applied to gross data, net data, paid losses, paid plus outstanding losses. That is why we have not defined closely the basis of the data.

3. SYSTEM REQUIREMENTS AND OUTLINE OF METHOD

For the data described in the previous section most of the reserving methods commonly in use break down. We needed a method which:

(i) Was able to cope with long tail business.

(ii) Used only information on premiums, paid claims and claims outstanding as notified.

(iii) Could provide estimates where there were missing items of information from the run-off triangle.

(iv) Could handle multi currency portfolios. Most of the companies whose reserving we examine write substantial US dollar business even though they report in pounds sterling.

(v) Would enable us to set a range of values within which reserves would be acceptable. After all no single estimate can be correct unless we have business which has completely run off. We would expect in the early years of development of an account year that the range would be relatively wide and should reduce as development increases.

(vi) Where necessary would use market information or information from other similar businesses to establish reserves for a particular insurer.

It was vital that the system should be able to cope with all the preliminary data handling, and would be flexible enough to allow the data to be looked at in a variety of ways. Data can be accepted in a variety of formats. The data can be either cumulative or incremental. Claims data can show paid claims and claims outstanding either separately or summed, and can be expressed either as loss ratios or cash. Development time intervals can be either quarterly, half-yearly or annual. The system can accommodate several currencies, which can be combined or not at the user's discretion. When currencies are combined, uniform exchange rates are assumed to apply for all periods of origin and development. Data from up to 99 separate long and short tail categories can be accepted in any of the currencies, and again at user option any or all categories can be combined.

A major consideration underlying our whole approach is that for the classes of business we are considering, standard assumptions, e.g. homogeneous account from year to year, standard pay out pattern, no change in speed of claims advice, etc., would almost certainly all be violated. This suggested as a basic starting point that we examine the run-off of each account year separately. It also suggested that we look at the development of loss ratios rather than losses. Empirical considerations suggested that if we were seeking a smooth curve to fit the shape of the loss ratio at development time t, plotted against t, that curve would have a negative exponential shape.

In the remainder of this section we outline the reserving method we have developed to meet the above criteria. A worked example of the method is then given in section 4 to expand on the outline.

(a) Run-off triangles are drawn up for as many account years as possible showing the development year by year (or quarter by quarter) of premiums and claims.

(b) An estimate of the ultimate premiums receivable is made for each account year. If we have to calculate the estimate then we simply apply development factors calculated from the data without smoothing. Other methods could be used in appropriate circumstances. Often we use the underwriters' estimates since they have a better feel for the way, in practice, policies are being signed down.

(c) The estimates of ultimate premiums are divided into the relevant claims to give a run-off triangle of loss ratios.

(d) Separately for each account year for which there is sufficient development (this depends on the length of the tail of the business) a curve of negative exponential form is fitted to the loss ratio development for that account year. From this curve a preliminary estimate of the ULR for that account year can be made. In certain cases we can fix some of the parameters in the negative exponential curve from our knowledge of the values of the parameters for the same class of business in other companies, or on an industry wide basis.

(e) For each year of development, e.g. year r, we then combine the results obtained in (d) to give a table of the loss ratios at the end of year r and the corresponding estimated ULR's. A line is fitted to these points by standard linear regression techniques. Then given the loss ratio at the end of development year r a best estimate of the ULR for that account year can be obtained from the fitted line. Further a confidence limit for the ULR can also be obtained.

For an account year which is well developed the estimate of the ULR is obtained from (d) so no range is quoted, or usually needed. For a year with little development the ULR and accompanying confidence interval from (e) is quoted. For intermediate years the method depends on one's judgement.

4. WORKED EXAMPLE TO ILLUSTRATE METHOD

The approach outlined in the previous section is illustrated below by means of an example based on typical medium tail data. The data is available for account years by quarters of development up to 1st July 1985. This is the date as at which the reserves for outstanding claims are being calculated. For early years of development for the earlier account years the data is missing. It will be seen that this does not cause a problem to the system. Appendix 1 contains computer produced tables and graphs for the example. These are typical of the output produced by the computer system.

Estimating Ultimate Premiums

In this example we assume that no premiums are received after the end of development year 5. We thus need to estimate the ultimate premiums to be received for account years 1981 to 1984 (1985 is omitted from our consideration since half way through the year is too early to establish reserves). The estimates of ultimate premiums are given in Table 1.1 of Appendix 1. The numbers above the dotted line are the cumulative premiums to date. The numbers below the dotted line are the estimates of cumulative premiums for future development years estimated by development factors. Thus for each account year the last number in the column of data for that year is the estimate of total premiums receivable that we intend to use for that year.

Triangle of Loss Ratios

The estimates of total premiums are then divided into the cumulative development of incurred claims (i.e. claims paid plus notified claims outstanding) to generate the cumulative incurred loss ratios, based on ultimate premiums. Details of the loss ratios are given in Table 1.2 of Appendix 1.

Estimation of ULR by Curve Fitting

We now make a first estimate of the ULR's for each account year by fitting a suitable curve to the loss ratio development for that account year. Over the years we have tried a number of different families of curves for this purpose. The family of curves should satisfy the two criteria:

(i) For an account year where the ULR is already known with a fair degree of certainty the curve must level out at a value near that loss ratio.

(ii) For later account years the curve must fit the known data well and also allow for a reasonable amount of future development. In most cases this will mean a development period similar to the more fully developed years.

The curve we have found most suitable is:

$$L_t = A \times [1 - \exp(-[t/B]^C)]$$

where t is the development period and L_t the loss ratio for that development period. There are 3 parameters A, B and C. A determines the ULR while B and C determine the length of the tail and the way in which it approaches the ULR. The curve was originally suggested in a paper by D.H. Craighead (1). The method of fitting the curve given in that paper is not optimal and more powerful numerical methods than those described by Mr. Craighead should be used. In Appendix 2 we give examples of the effect on the shape of the curve of changing the parameters B and C. These illustrate the wide variety of run off shapes which can be fitted by this curve.

This family of curves is used to give estimates of ULR's for account years 1971 to 1981. For later years, not enough development has yet taken place for a satisfactory curve to be fitted. In Figures 1.3 to 1.13 of Appendix 1 we give the graphs of the curves fitted (the solid curves) in this example together with the developed loss ratios. Each loss ratio is represented by a vertical line, with the dotted line joining up the developed loss ratios. The quality of the goodness of fit can be tested by eye by comparing the closeness of the dotted and solid curves. The comparison should obviously concentrate on the later years of development. At the bottom of each curve we give the values of A, B and C fitted together with the mean squared error. In this particular example C was set equal to 1.5 and only A and B were fitted. We discuss the selection of the parameters to be fitted and the choice of the developed loss ratios to be included in the fitting in Section 5. The graphs need not be studied in detail but should just be looked through quickly to see how well, in general, the curves fit the data.

On occasions we have found that the graph produced by the computer does not suggest a smooth curve. Particularly when looking at incurred loss ratios we have found that the development can oscillate violently. An advantage of the system is that since it presents this in visual form it can be discussed with the underwriter. The most common explanations we have found for odd patterns are:

(a) Miscoding of data either by currency or category

(b) Data corrections that have not been carried back to the beginning of the account year

(c) Delays in reinsurance recoveries.

Thus the system is acting as a powerful check on the data.

In the particular example being used it appears likely that initially some claims for 1978 development year 7 and 1980 development year 5 have been coded as 1979 development year 6 with the error not being fully corrected retrospectively.

Estimation of ULR's by "line of best fit"

We have so far analysed the run-off of one account year at a time. We now analyse the run-off by examining one development year at a time for all account years together. Thus we use all the information in the run-off triangle.

For example at development year 3 we have the following data:

Account year	Loss ratio at development year 3	Estimated ULR from previous curve fitting
	%	%
1973	53.1	91.0
1974	65.8	92.1
1975	50.3	75.7
1976	43.6	70.2
1977	46.2	70.0
1979	73.5	103.8
1980	40.4	69.6
1981	39.1	72.2

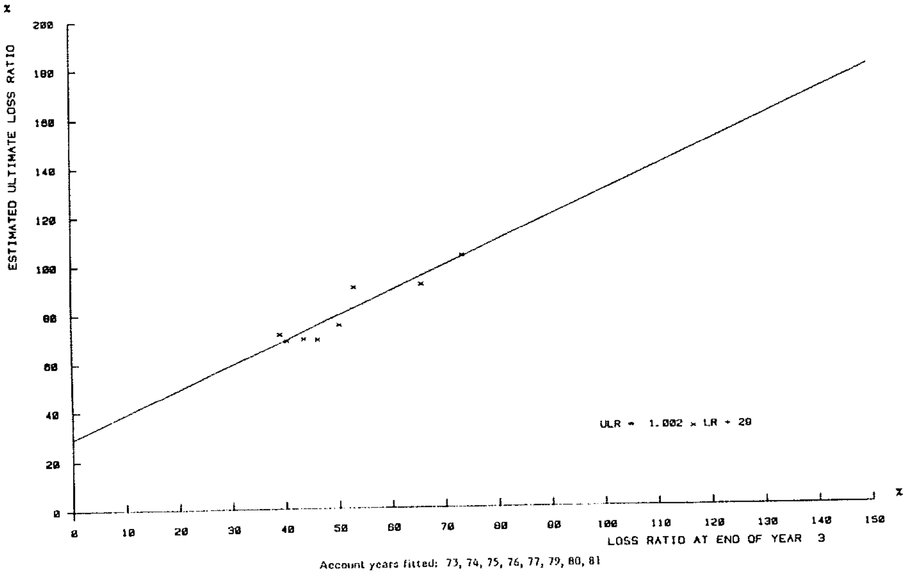
Account years 1971 and 1972 are omitted because the loss ratios for early development years are missing and 1978 is omitted because the run-off curve for that year seems to be a different shape from the other years.

The points are then plotted and the plot is examined to see if there is a statistically significant relationship between the loss ratio at development year 3 and the ULR. The method we use is to fit a regression line and test whether the gradient is significantly different from zero.

In this case the regression line is:

Estimated ULR = 1.002 x Year 3 Loss Ratio + 29.00%.

The fitted line is shown below, together with the 8 points to which it was fitted.



To test if the gradient is significantly different from zero we use a t-test, with 2 degrees of freedom less than the number of points fitted. In this case we have $t_6 = 6.55$ which is significant at the 99% level. Thus the line is a good fit and the gradient is non zero, which supports the evidence available from inspection of the fitted line.

From the fitted line we can estimate the ULR for 1983 (where development year 3 is the latest known loss ratio) as:

$$\begin{aligned} \text{1983 ULR} &= 1.002 \times 39.57 + 29.00 \\ &= 68.65\%. \end{aligned}$$

Since we have fitted a regression line we can also construct a confidence interval for this estimate of the ULR. There are two alternative methods, one empirical and the other mathematical.

The empirical method is to take the historical point furthest from the regression line and state that the true result for the year is unlikely to fall outside the historical maximum. This gives a likely variation of the result of $\pm$ 8.8% in this particular case.

The mathematical method is to derive the statistical confidence interval from the regression line fit. We have found that a 90% confidence interval does the right job for our analyses of individual portfolios. This gives a confidence interval in the example of $\pm$ 10.9%. Obviously the width of the confidence interval depends on where the point lies on the regression line.

The choice of method is a matter of personal preference. The advantage of the maximum deviation is the ease of presentation to the underwriter of the rationale for the range. The advantage of the second method is that it is statistically based and does allow properly for the number of points to which the line is fitted. It should be noted that underlying the second method as well as the t-test is the assumption that the underwriting results for different account years are independent identically distributed random variables. Such limited investigations as we have carried out suggest that this is a reasonable assumption.

If the gradient of the regression line is found to be not significantly different from zero then that implies that there is no correlation between the loss ratios at year 3 (say) and the ULR. In this case we would estimate the ULR as the average of the historic ULR's and obtain a confidence interval using the maximum deviation. However this also tells us something very useful about the data for that account year. It says that effectively there is <u>no</u> information in the data showing the development of the account year so far to indicate how the year will turn out ultimately in practice. Although this is a negative statement we feel that it is a fact that is often not fully appreciated by management, particularly with regard to long tail business. However it is clearly illustrated by the plots of loss ratios against ULR from which it is often easy to see that there is no relationship.

For our example the regression lines fitted for development years 2 to 10 together with the account years for which they are fitted are shown in Figures 1.14 to 1.22 of Appendix 1. Looking through the regression lines you will see how the fit gets better as the development year increases. When we reach the year of development where the "tail" of claims has effectively run off the loss ratio will equal the ULR. The regression line will pass through the origin of the graph and the slope of the line will be "1 in 1" i.e. 45%. You will see from Figure 1.22 that for the class of business being used for the example this position has almost been reached by the end of year 10. A summary of the lines fitted and the statistics is given in Table 1.23 of Appendix 1. From this you will see that for 1984 it is not appropriate to fit a regression line, since the t-test statistic is not significant at the 95% level.

Final estimates of ULR.

In this example we consider that the estimates of ULR obtained from the curve fits are the appropriate ones to use for account years 1971 to 1977. Clearly for the early account years one can not use the regression lines to estimate the ULR's since the lines would be based on too few data items to be credible. For account years 1979 to 1984 the results from the line of best fit seem most suitable. As previously stated for 1978 the position is difficult because the shape is different from the other account years and we have therefore used the curve fit. Although no confidence interval can be calculated for this year it is obvious from looking at the curve fit that in order to convey the correct information to management that one should be quoted. This has arbitrarily been taken to be the same as 1979. We have on this occasion used 90% confidence intervals rather than maximum deviation intervals.

The final results of the analysis are set out in Table 1.24 in Appendix 1.

Further considerations

We have already mentioned how this approach suggests how much information about the ULR is contained in the development to date of the relevant account year. The other useful thing that we find comes out of this approach is that it shows senior management that the estimate of the ULR is just that - an estimate. Thus the actual result will be better or worse than that estimate. The confidence intervals give an indication to senior management of the range in which the result will in fact lie. It thus enables them to assess the implications of establishing reserves based on particular estimates of the ULR. The closer to the upper limit of the ULR that the reserve is established the more likely it is that in practice the reserve will turn out to be more than adequate and the excess may be released as a profit in the future. The nearer to the lower limit of the range of the ULR that the reserve is established the more likely that the reserve will turn out to be inadequate. That would mean that additional cash would have to be found in the future either by restricting divided payments or raising new capital.

5. FURTHER DETAIL ON THE RESERVING METHOD

In this section we consider some of the practical problems that arise from using the approach to reserving discussed in the preceding two sections and describe some of the methods we have used to overcome these problems. Although a few of these problems and solutions were mentioned in the previous section we have covered all of these in this section for completeness.

Problems encountered with curve fitting

The curve we fit has 3 parameters A, B and C. Initially for each account year we fit the curve allowing all 3 parameters to vary. This is because a free fit allows the curve to reflect the data as accurately as possible given the constraints of the curve. However where there is an error in the data, or some other reason, one can find that the fit to the early years of development is satisfactory but it is rather less good to the later years of development. In such cases we fix either B or C in order to try and make the curve fit the later years of development better at the expense of a worse fit in the earlier years. We prefer to fit C as this allows more freedom in the shape of the curve than fitting B. If we have to fix a parameter for a particular account year then if most of the other account years are fitting well on a free fit we would take the values of the parameters of those other years into account when deciding on the values of the parameters to be fixed. Alternatively we would take into account the values of the parameters we have found suitable for similar classes of business either for other companies or on an industry wide basis.

As already mentioned we do not fit curves to recent account years since for such years there is insufficient development to permit a curve to be fitted. For longer tail categories we usually omit the first 8 to 12 quarters of data in fitting the curve to ensure that the fit is reasonable to the later development. This also solves the problem that for some of the earlier account years this early development can be missing from the data. Finally we sometimes find that the curve is approaching the value of A slowly so that A is probably too high an estimate of the ULR. In such cases we assume that the development is completed after a reasonable period, say 15 to 20 years for the longer tail classes, and take the value of L_t for that development period as the estimate of the ULR.

Problems encountered with "line of best fit"

One important problem that is often encountered is where a particular account year has a significantly different speed of development from all the other account years for that class. This may be due for example to writing a peculiar treaty or treaties in that year. That such a thing is happening is usually clear from the graphs of the curves and the reason can often be found from discussion with the underwriter. In these cases that account year is omited from the calculation of the line of best. A good example of this was the omission of account year 1978 from the calculation of the lines of best fit in the previous section.

Another problem is where the data is very variable particularly in the early years of development so that there are significant random fluctuations on top of the basic run off pattern. In this case we have found that it is better to use the developed loss ratios obtained from the fitted curves rather than the actual values. This smooths out the random fluctuations which one may consider are not being repeated in the account year for which one is using the line of best fit to calculate a ULR. Alternatively the data for early years of development for some account years may be missing and using the modelled data will permit the inclusion of those years in the calculation of the line of best fit. Because of the smoothing that takes place with modelled data it will be found that the confidence intervals are narrower than those brought out by using the unadjusted data. They should therefore either be quoted with a cautionary note that they underestimate the true amount of fluctuation or not quoted at all.

It is interesting comparing the line of best fit approach with the approach using development factors. The development factor approach is equivalent to fitting a line for ULR against developed loss ratio that passes through the origin. Our experience is that for early years of development the lines of best fit often miss the origin by a wide margin. However as one progresses to the lines of best fit for the later development years they become closer and closer to lines through the origin. If in looking at some lines of best fit we do not see this pattern then this suggests that something is awry. The most probable reason is an error in the data.

As will be apparent from the example and the above discussion the method is not an automatic method for setting loss reserves. It requires one to use one's judgement at all stages of the process. In particular we have found that a careful study of the graphs of the curve fits and the linear regressions is very important in deciding upon an appropriate best estimate of the ULR and the accompanying confidence intervals. Although the method described uses a curve fitting approach to obtain the initial estimates of ULR's there is no reason why alternative methods, as for example described in the paper by J.R. Berquist and R.E. Sherman (3), should not be used to obtain these initial estimates. However we would emphasise that in practice we have found the curve fitting approach to be very flexible and more than adequate for calculating values of ULR's to use in the line of best fit. The alternative methods are found to be more necessary to assist in estimating the ULR's for the early account years where the line of best fit is not going to be used as part of the estimating process.

6. APPLICATION OF METHOD TO LLOYD'S

One important use of the method we have developed, and in fact one of the reasons for developing it, was to provide a new method for calculating the minimum reserves to be established by Lloyd's syndicates. This is described in considerable detail in the paper by S. Benjamin and L.M. Eagles (2) and we shall therefore give only a brief outline of the method for setting minimum reserves here.

The syndicates in Lloyd's are the bodies in Lloyd's equivalent to companies that underwrite the risks. Collectively the syndicates comprise Lloyd's. The syndicates each maintain their own statistics and also certain statistics are collected centrally. Among other things the central statistics are used to help set minimum levels of the reserves for each account year to be established by the syndicates.

The current method of setting minimum reserves is by the use of the "Lloyd's audit percentages" which are set by Lloyd's centrally. Under this present method percentages are supplied for use as at the end of each calendar year separately for each class of business and each account year in which business was written. The minimum reserve for claims outstanding and IBNR at the end of that calendar year for the class of business and account year is the premium advised to date multiplied by the relevant percentage. Thus the minimum level for the total claims expected to be paid by the syndicate is the claims paid to date plus the minimum reserve. Suppose under the present method the paid loss ratio to date is, say 10% and the audit percentage for the minimum reserve is 78%. Then under the present method we have

Paid Loss Ratio	=	10%
Reserve (Audit Percentage)	=	78%
(Implied) ULR	=	88%

It will be clear that this method does not reflect the progress of the individual syndicates.

Under the proposed new method two figures are used instead of one. In this particular case instead of 78% the two figures are 3.4 and 33% and the calculation is as follows:

ULR	=	3.4 x Paid Loss Ratio + 33%	
	=	3.4 x 10% + 33%	= 67%
Paid Loss Ratio	=		10%
(Implied) Reserve	=		57%

Thus two figures are provided for each class of business and account year for which currently one audit percentage is provided. The proposed new method has been tried on a limited experimental basis for three years. The evidence so far is favourable and the experiment is currently being widened to cover the whole market.

The two figures under the new method are calculated by applying the general method described in the preceding sections to the data collected centrally at Lloyd's for each class of business. For each class of business if one carries out that process one produces for each account year or year of development a line of best fit, together with an associated confidence interval, based on the point furthest from the line of best fit. The two numbers under the proposed method are the parameters that define the line of best fit. Thus in the example 3.4 gives the slope of the line and 33% its intercept on the vertical axis. There was considerable discussion inside the working party which reported to the Audit Committee as to whether the line of best fit or one of the other lines should be used to set minimum reserves. In the end the upper edge of the confidence interval seemed too high, the lower too low. The use of the line of best fit as a minimum allowed one to say that the total reserves set up in Lloyd's were at least as great as the average indicated by past experience, which seemed to be a useful statement to make. Underlying this approach to setting reserves is the assumption that for any class of business the business written by a syndicate will be similar to that "written" by all of Lloyd's combined. Incorporating the paid loss ratio in the calculation of the ULR in the way proposed then allows the quality of the business written by a particular syndicate to be reflected in the ULR in what appears intuitively to be a reasonable way. Also the new method would be easy to implement requiring very little change by individual syndicates in the work they carry out.

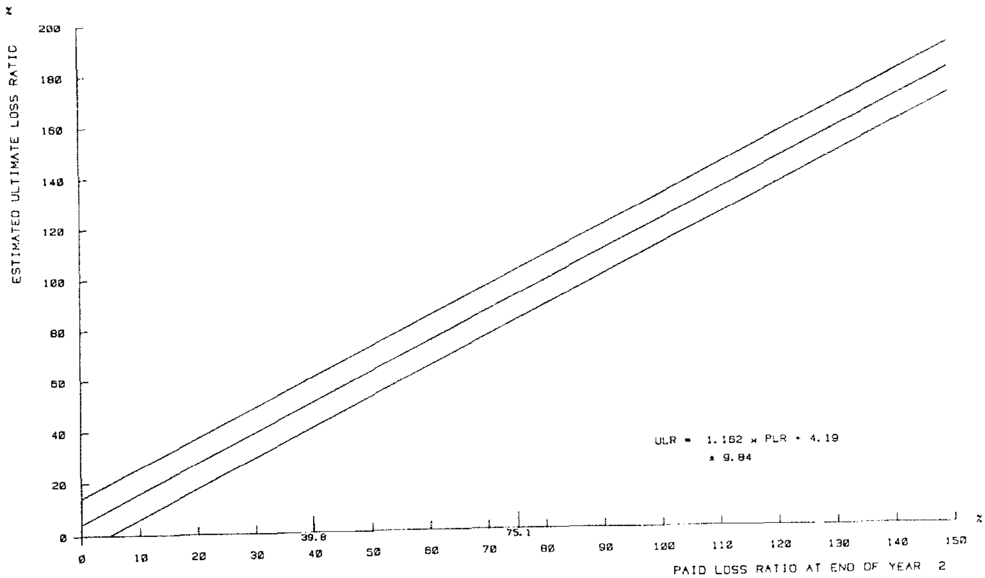
In addition to being provided with the new figures for calculating the minimum reserves the syndicates are also provided with graphs for each class of business and year of development showing:

(i) The lines of best fit together with the lines based on the point furthest from the line of best fit

(ii) The historic range of paid loss ratios.

Thus the syndicates are provided with graphs looking like this

CATEGORY: Aviation Short Tail (Sterling)
ALL LLOYDS BUSINESS



The syndicates are being encouraged to plot their own data on the graphs to see how their experience compares with that of all of Lloyd's combined. It is hoped that as a result they will obtain useful information about their experience. For example if a syndicate's own path was narrow and different from the all-Lloyd's path then that would demonstrate in a very vivid way that it was writing a different class of business.

Clearly this approach can be adopted by any supervisory authority which wishes to set reserving standards for companies where global general market data of run-offs for the different classes of business is available.

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APPENDIX 1

DATA AND OUTPUT FOR WORKED EXAMPLE

Table 1.1

Estimation of Ultimate Premiums

Development Quarter 2

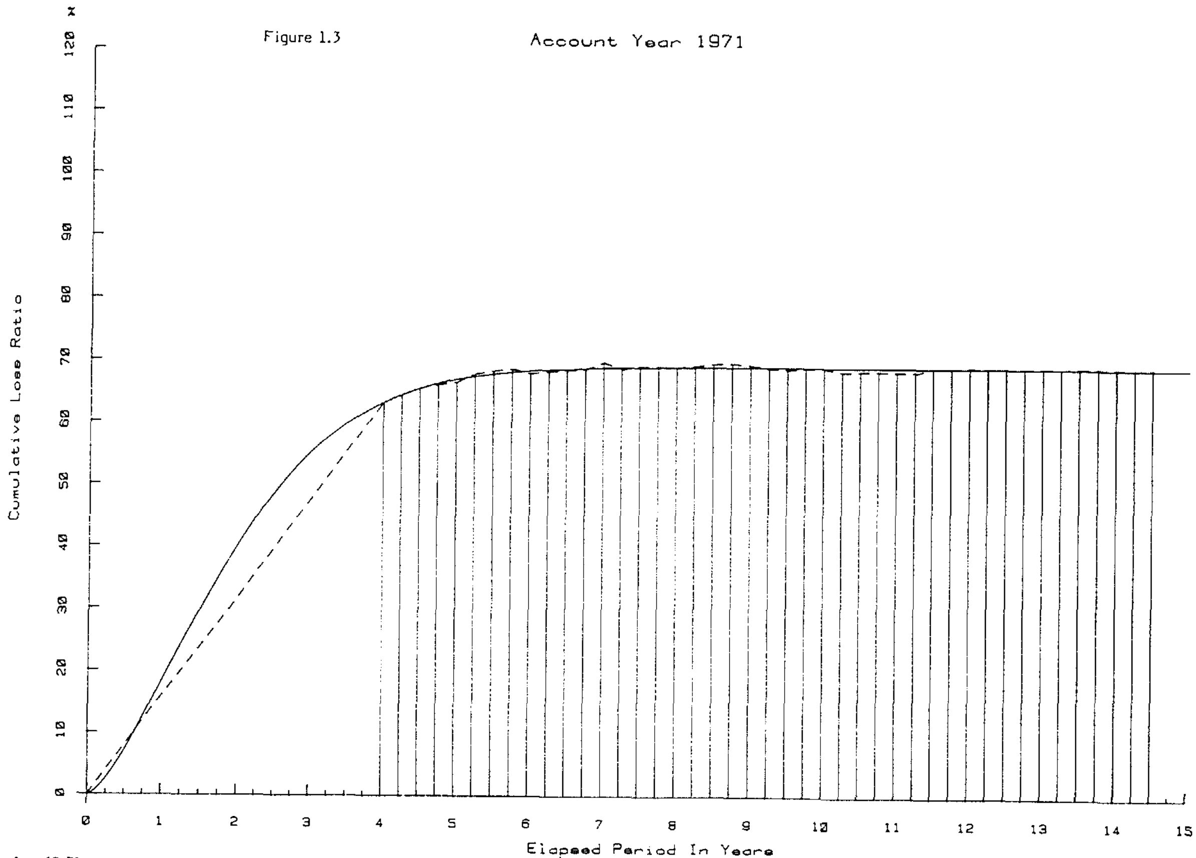
Account Year:

Development Year	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
1				2,706	3,714	3,751	5,550	6,580	6,774	9,098	12,214	11,611	15,541	20,082
2			3,524	4,489	5,869	6,439	8,475	9,712	9,797	13,173	17,839	17,901	23,250	29,902
3		3,355	3,924	4,821	6,393	7,109	8,800	10,083	10,670	14,613	19,927	19,322	25,602	32,928
4	5,189	3,373	4,040	4,876	6,473	7,067	8,894	10,142	10,978	15,123	20,570	19,967	26,457	34,027
5	5,240	3,415	3,999	4,928	6,521	7,081	8,942	10,161	11,035	15,356	20,887	20,275	26,865	34,551
6	5,126	3,432	4,027	4,894	6,557	7,065	8,981	10,250	11,147					
7	5,279	3,449	4,024	4,911	6,570	7,091	9,006	10,329						
8	5,297	3,446	4,040	4,917	6,592	7,046	9,030							
9	5,300	3,452	4,035	4,896	6,580	7,070								
10	5,301	3,454	4,036	4,898	6,585									
11	5,288	3,455	4,037	4,894										
12	5,286	3,476	4,027											
13	5,284	3,474												
14	5,284													

Table 1.2

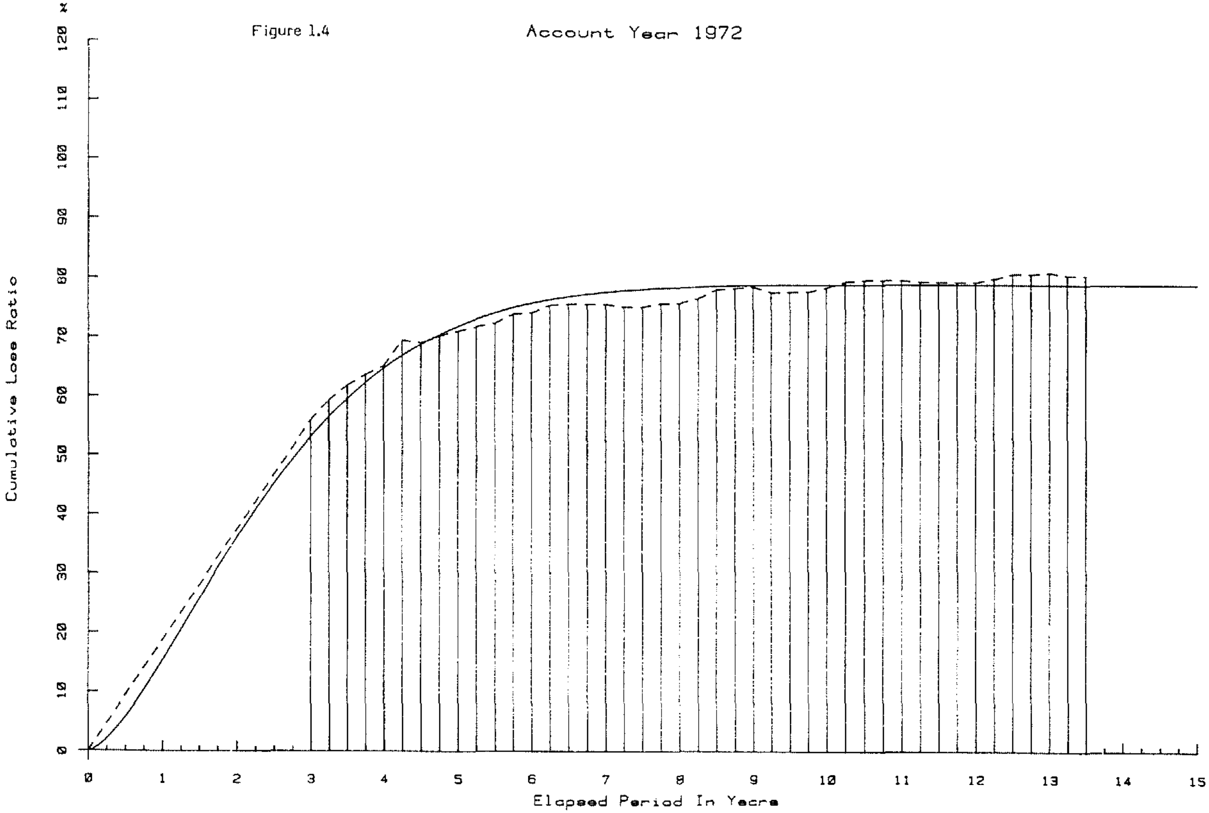
Loss Ratios

Quarter of Development	Accident Year: 1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
1					0.1	-0.6	1.2	0.1	1.0	0.0	0.1	0.0	0.1	0.9
2					0.8	-0.4	2.3	0.3	2.1	1.0	0.4	3.0	2.0	2.3
3					2.9	4.5	5.0	1.8	7.1	4.2	1.4	7.0	5.0	5.0
4				7.7	7.0	5.1	8.1	3.4	9.6	12.1	6.5	8.3	12.2	11.6
5				14.5	18.7	12.0	12.4	7.3	17.7	2.0	9.3	8.6	17.1	12.1
6				25.5	22.5	15.7	21.1	12.5	21.4	10.1	17.2	11.3	20.7	23.0
7				40.0	30.8	22.6	30.0	19.6	31.2	20.7	25.1	17.4	25.2	
8			41.6	48.2	39.1	37.1	35.6	26.0	63.1	29.9	30.7	25.4	29.6	
9			47.7	60.8	45.6	38.5	42.0	38.8	64.6	32.9	36.5	3.08	34.5	
10			53.1	65.8	50.3	43.6	46.2	40.7	73.5	40.4	39.1	33.5	39.6	
11			59.9	71.6	55.9	48.2	49.2	40.8	77.3	47.2	45.2	40.3		
12		55.9	64.9	76.0	61.5	52.3	51.0	46.8	75.5	44.2	47.8	43.8		
13		59.4	67.8	76.7	64.2	56.4	51.3	39.9	89.6	47.7	49.9	46.1		
14		61.9	73.5	79.1	67.4	58.3	52.9	41.9	87.2	50.5	53.2	47.5		
15		63.7	76.4	82.4	70.2	59.7	55.8	43.3	92.2	53.3	55.4			
16	63.2	65.2	78.3	83.4	71.4	60.9	60.7	43.6	92.6	55.7	56.6			
17	64.4	69.5	80.7	86.6	72.7	61.5	63.3	46.6	95.9	56.3	58.5			
18	65.5	69.1	81.4	85.0	72.6	64.3	65.4	48.8	96.2	58.2	60.6			
19	66.0	70.3	82.7	86.2	74.7	65.6	67.3	50.4	99.0	59.8				
20	66.5	71.1	84.1	86.6	75.5	67.3	68.7	51.7	107.3	60.5				
21	67.9	71.8	84.7	89.0	73.6	66.5	67.8	52.3	111.1	62.4				
22	68.4	72.4	87.2	89.2	72.9	67.2	69.5	58.1	117.7	63.8				
23	68.7	73.9	88.2	89.7	73.3	67.3	70.1	61.4	96.7					
24	68.0	74.1	88.2	90.5	73.0	67.9	71.3	57.9	96.0					
25	68.4	75.4	88.9	91.6	72.8	68.6	72.4	56.1	92.4					
26	68.6	75.6	89.8	92.5	73.8	69.1	72.1	55.0	94.0					
27	68.8	75.6	90.4	92.7	73.8	70.2	70.2	79.9						
28	69.9	75.6	91.0	93.1	74.0	71.4	69.7	82.0						
29	68.8	75.2	91.0	93.8	75.3	70.5	69.0	82.6						
30	69.3	75.2	91.6	93.8	75.7	72.0	68.5	86.3						
31	69.3	75.8	91.9	94.5	75.0	71.5	69.1							
32	69.2	75.9	91.7	94.1	76.2	70.6	68.6							
33	69.5	76.7	91.5	92.3	75.0	68.4	66.2							
34	69.8	78.2	91.7	92.8	75.3	68.6	65.0							
35	69.9	78.4	91.6	92.9	76.2	68.9								
36	69.6	78.6	91.0	92.3	75.5	68.8								
37	69.1	77.7	91.6	91.9	74.4	68.7								
38	68.9	77.8	91.6	91.9	74.4	69.8								
39	69.1	77.9	91.9	91.6	74.5									
40	69.1	78.6	91.6	91.6	75.1									
41	68.6	79.6	90.4	90.6	75.7									
42	68.7	79.9	90.4	90.6	75.7									
43	68.7	79.9	90.5	90.7										
44	68.6	79.9	90.6	90.7										
45	68.6	79.6	90.3	90.7										
46	69.3	79.6	90.3	90.7										
47	69.3	79.6	90.3											
48	69.5	79.6	90.8											
49	69.3	80.2	90.8											
50	69.3	81.1	90.8											
51	69.4	81.1												
52	69.4	81.3												
53	69.3	80.8												
54	69.5	80.8												
55	69.4													
56	69.4													
57	69.3													
58	69.4													



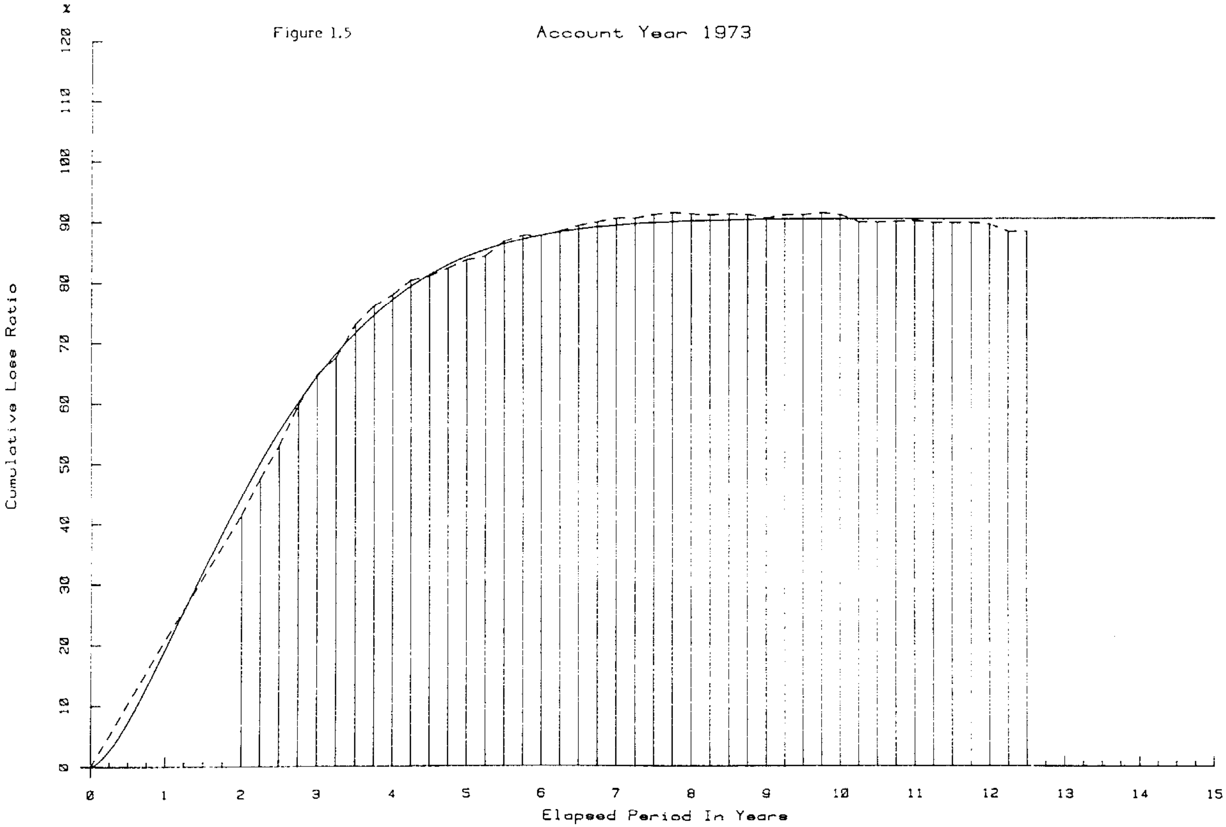
A = 69.3%, B = 2.21, C = 1.50, Mean squared error = 1.1

Figure 1.4



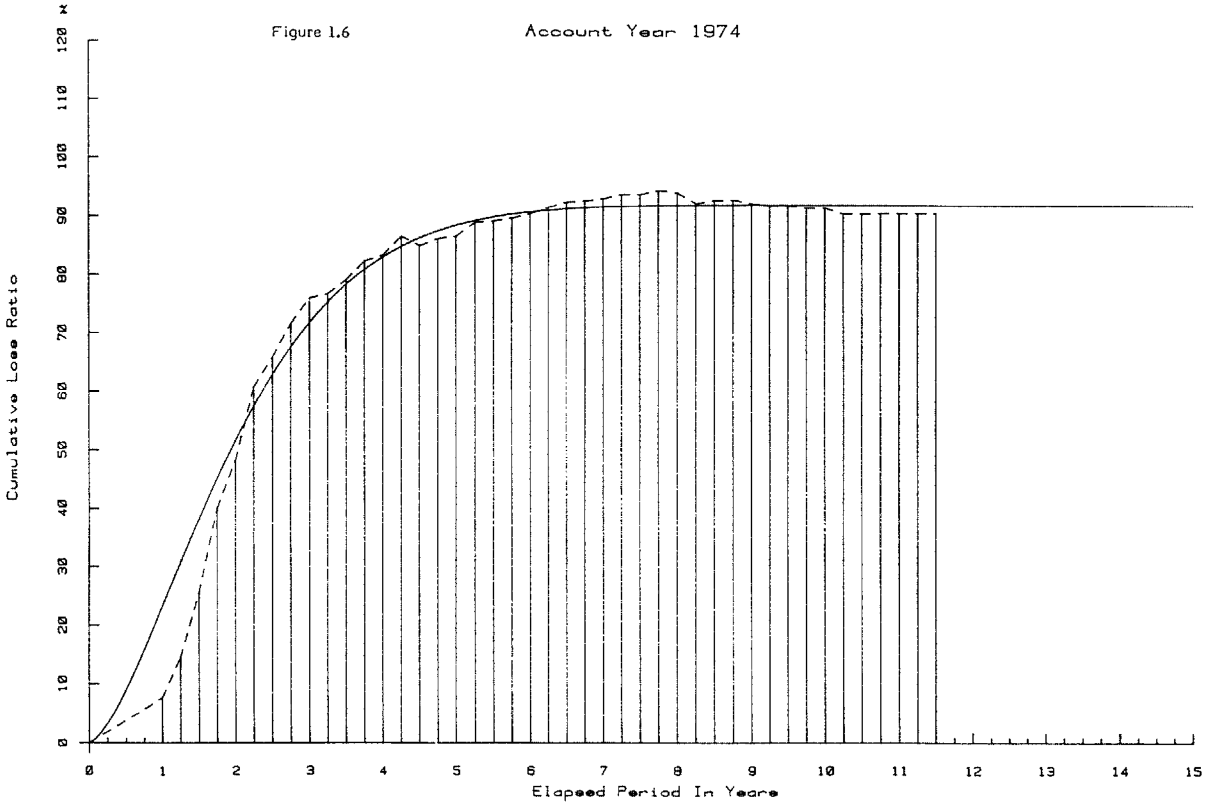
A = 79.2%, B = 2.80, C = 1.50, Mean squared error = 18.5

Figure 1.5 Account Year 1973



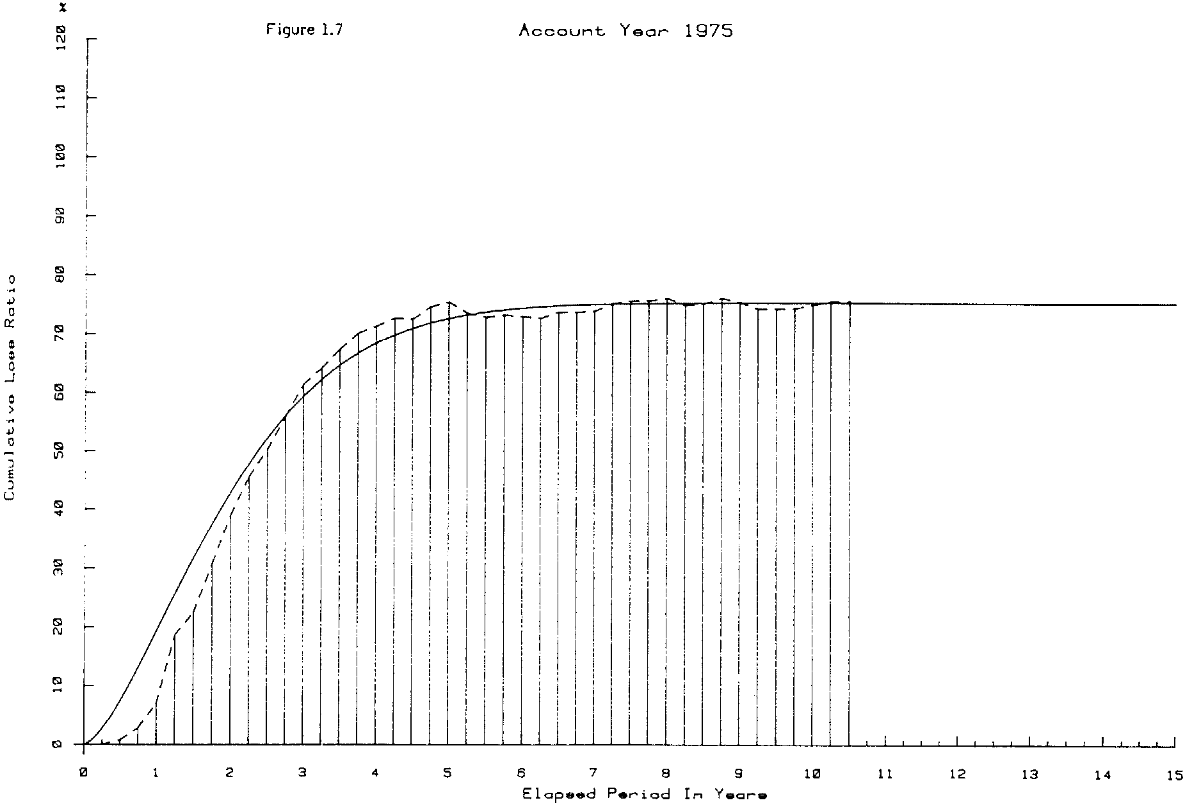
A = 91.0%, B = 2.60, C = 1.50, Mean squared error = 8.1

Figure 1.6 Account Year 1974



A = 92.1%, B = 2.28, C = 1.50, Mean squared error = 28.0

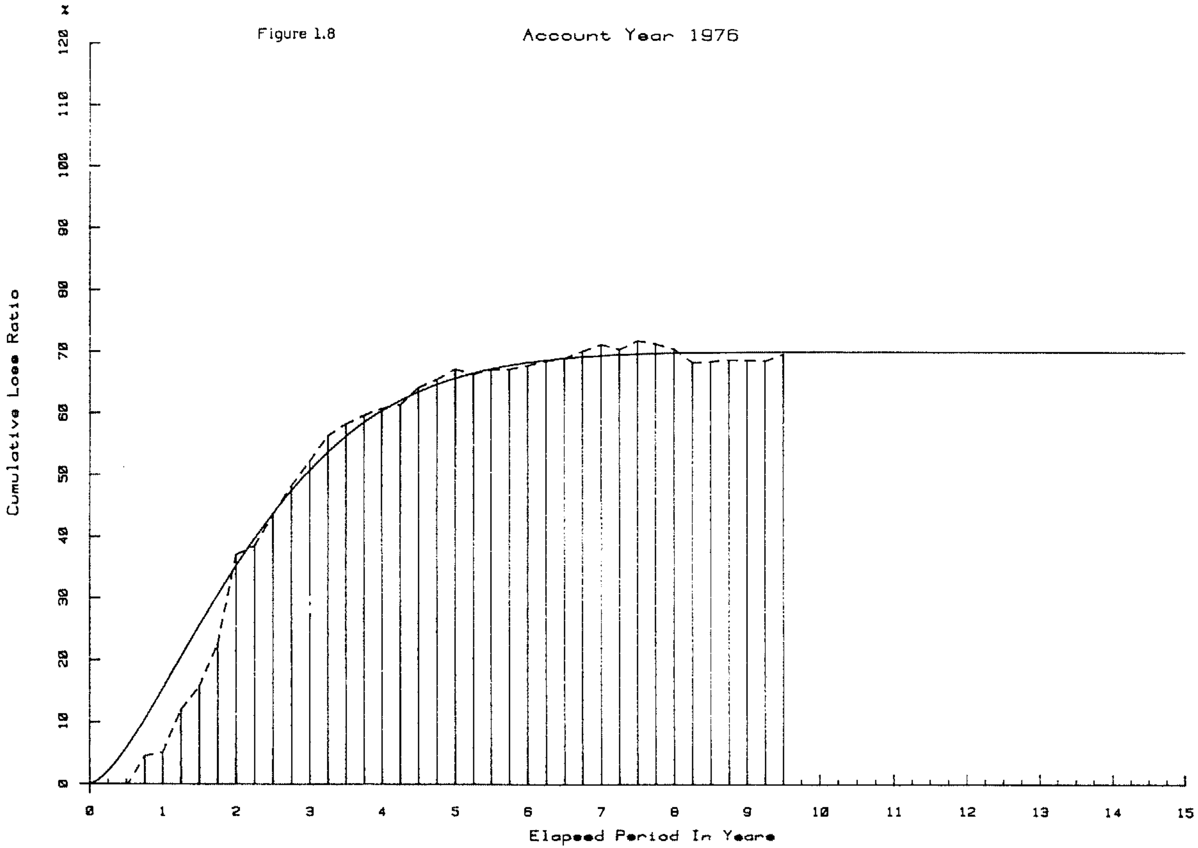
Figure 1.7 Account Year 1975



A = 75.4%, B = 2.24, C = 1.50, Mean squared error = 18.2

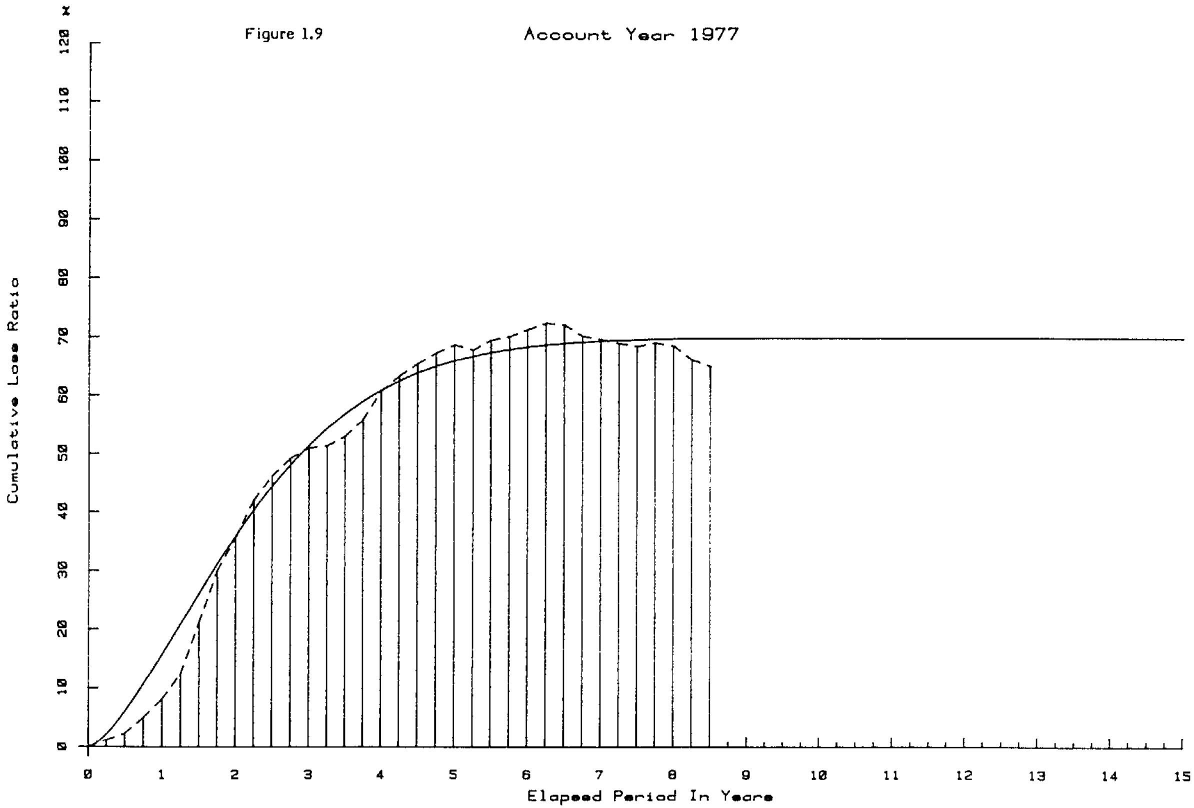
Figure 1.8

Account Year 1976



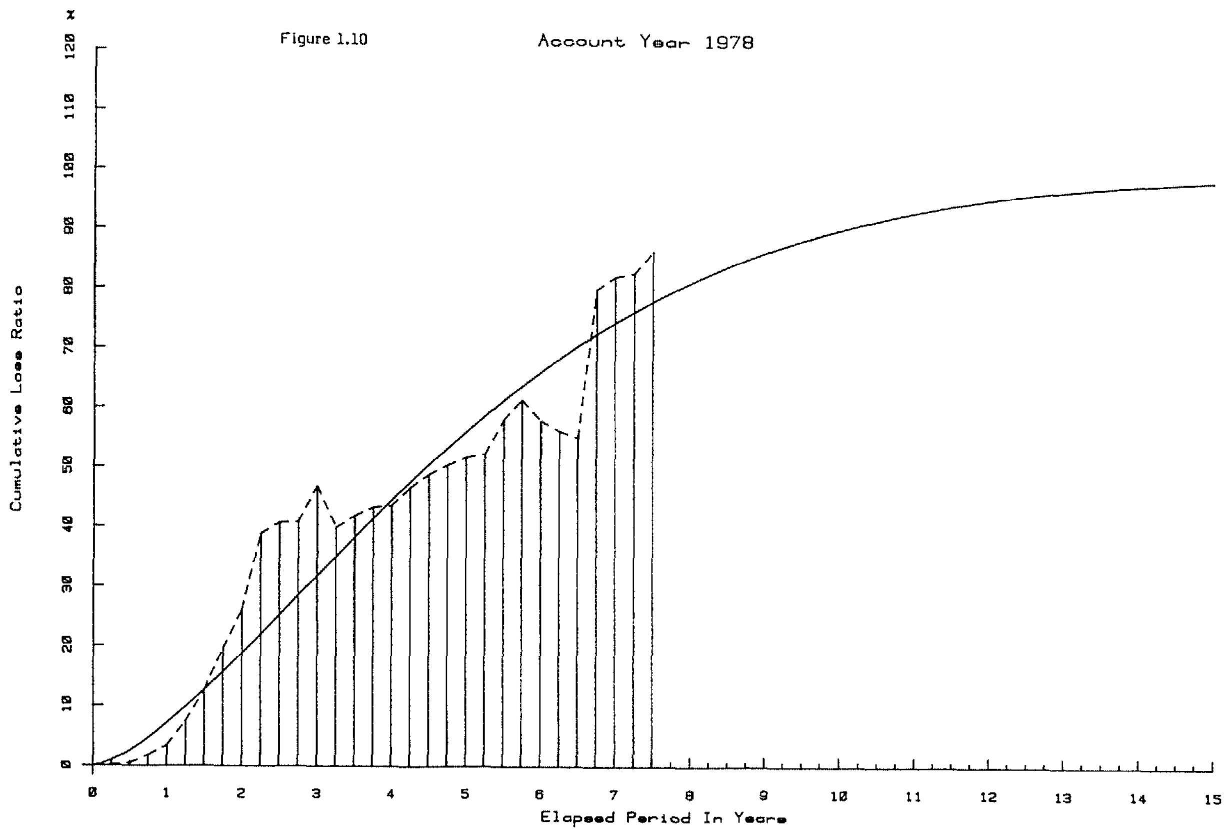
A = 70.2%, B = 2.54, C = 1.50, Mean squared error = 18.2

Figure 1.9



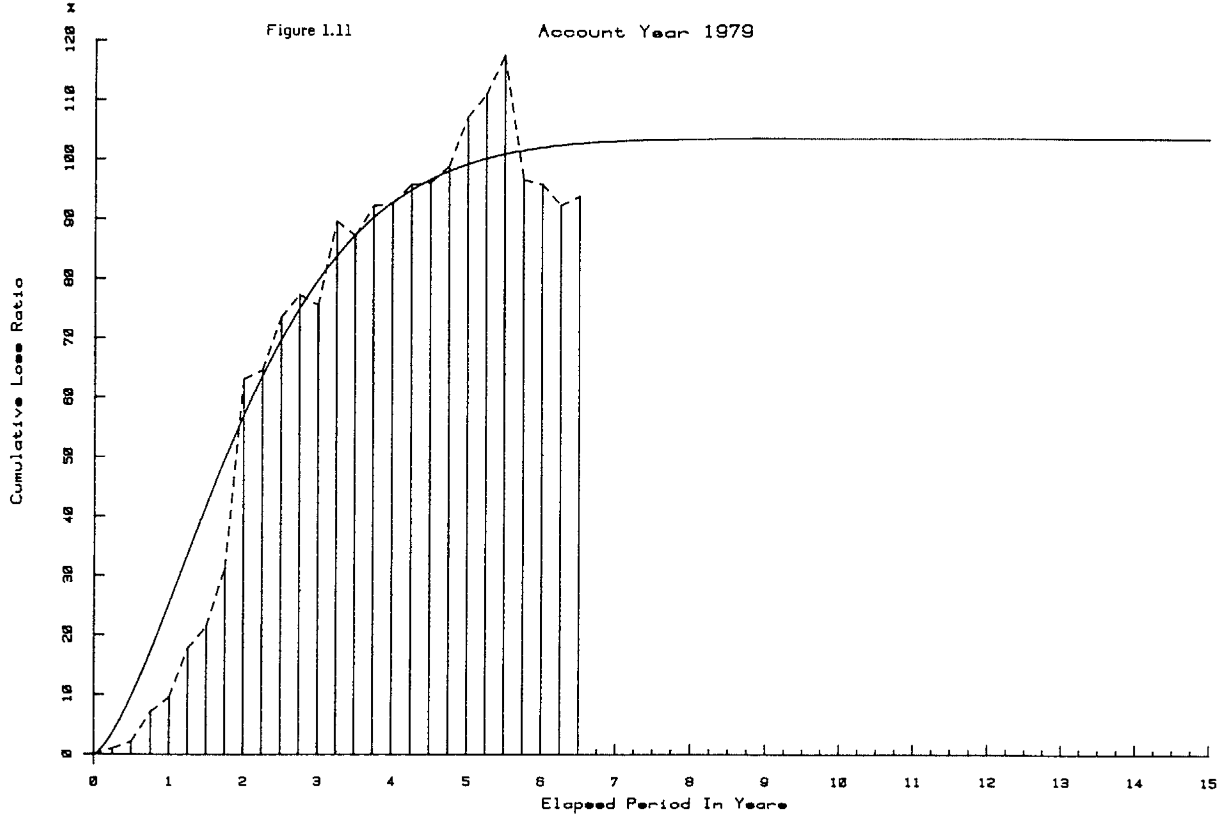
A = 70.0%, B = 2.49, C = 1.50, Mean squared error = 31.8

Figure 1.10



A = 99.9%, B = 5.69, C = 1.50, Mean squared error = 289.6

Figure 1.11



A = 103.8%, B = 2.33, C = 1.50, Mean squared error = 265.3

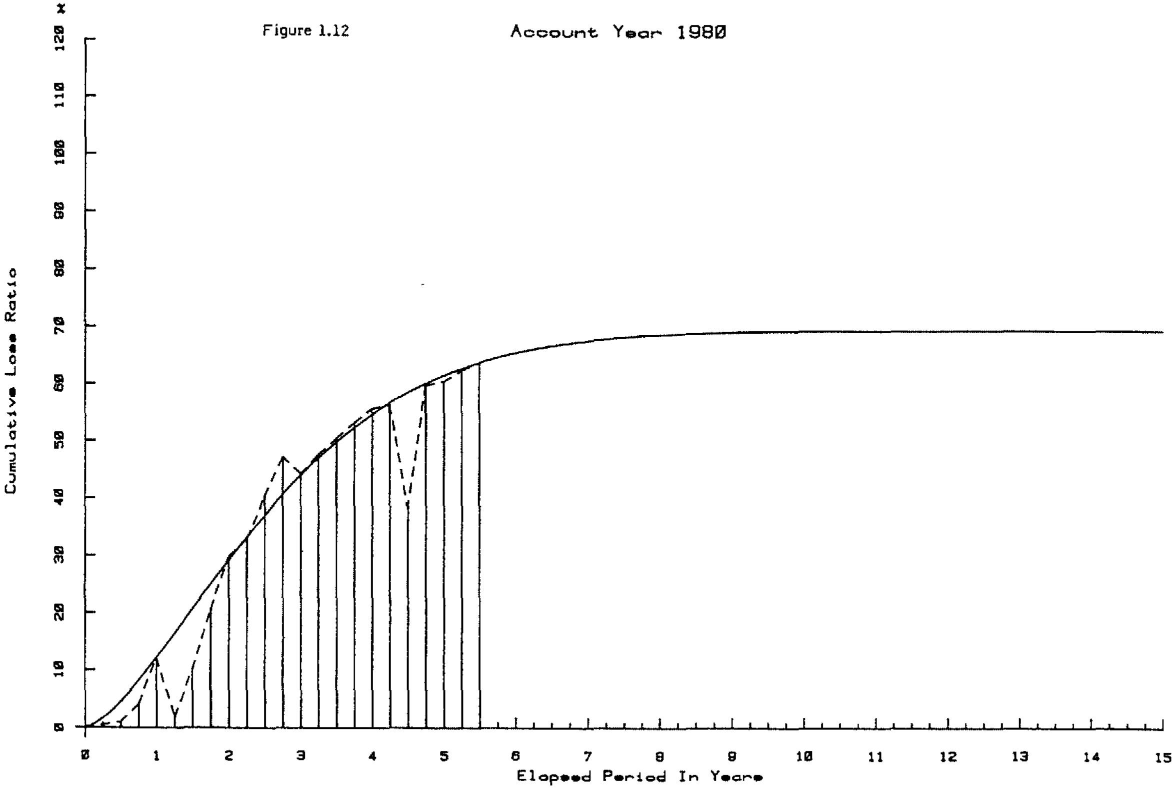
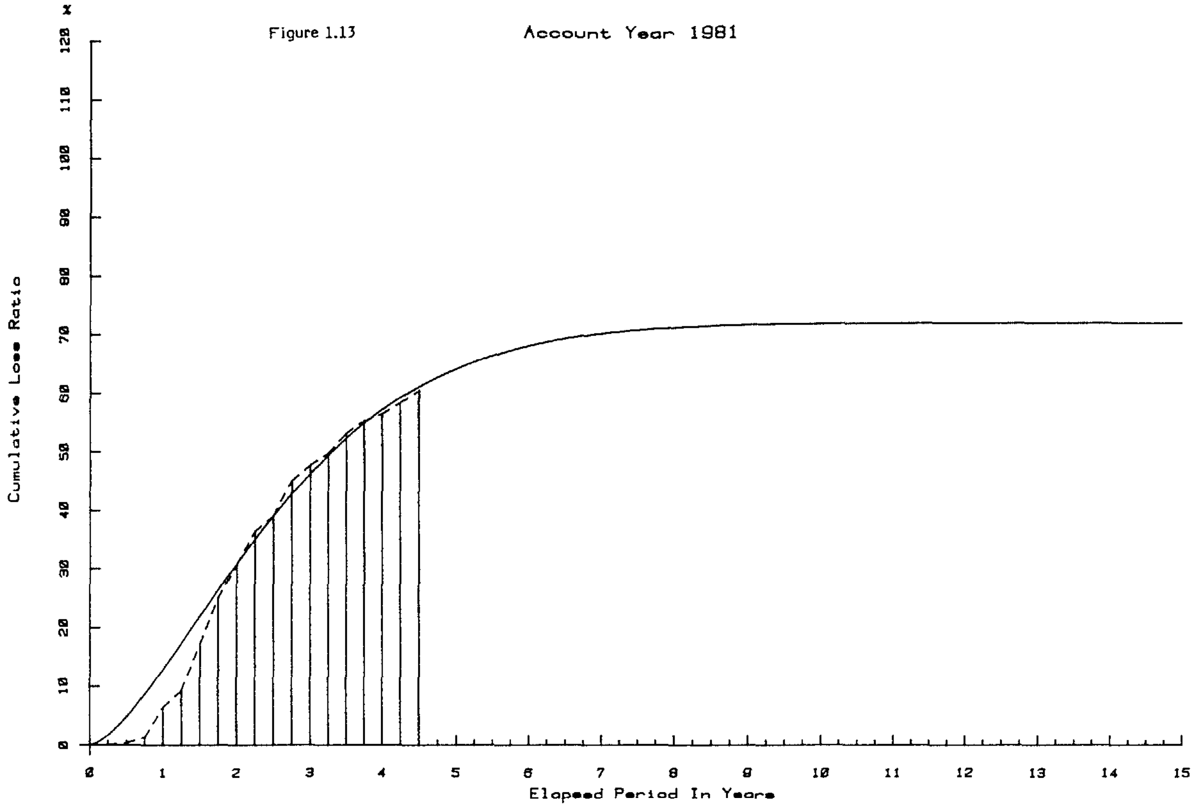


Figure 1.12

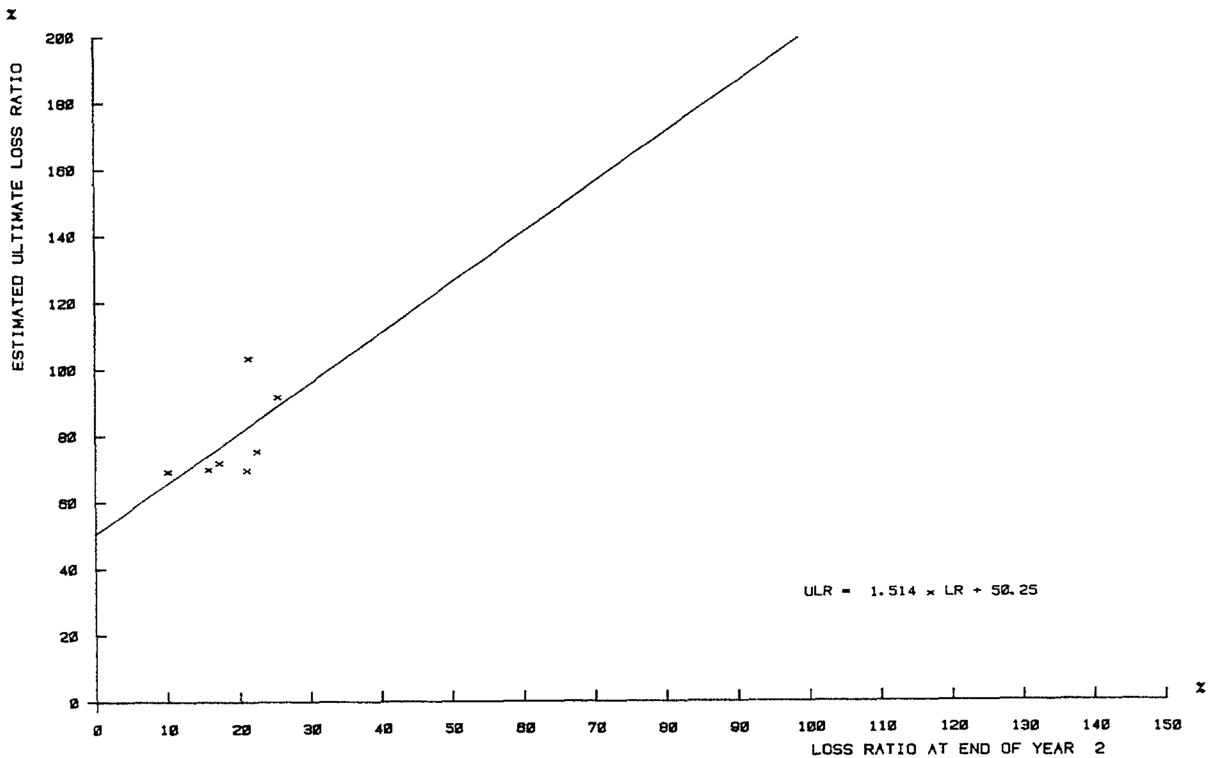
A = 69.6%, B = 3.00, C = 1.50, Mean squared error = 34.7

Figure 1.13



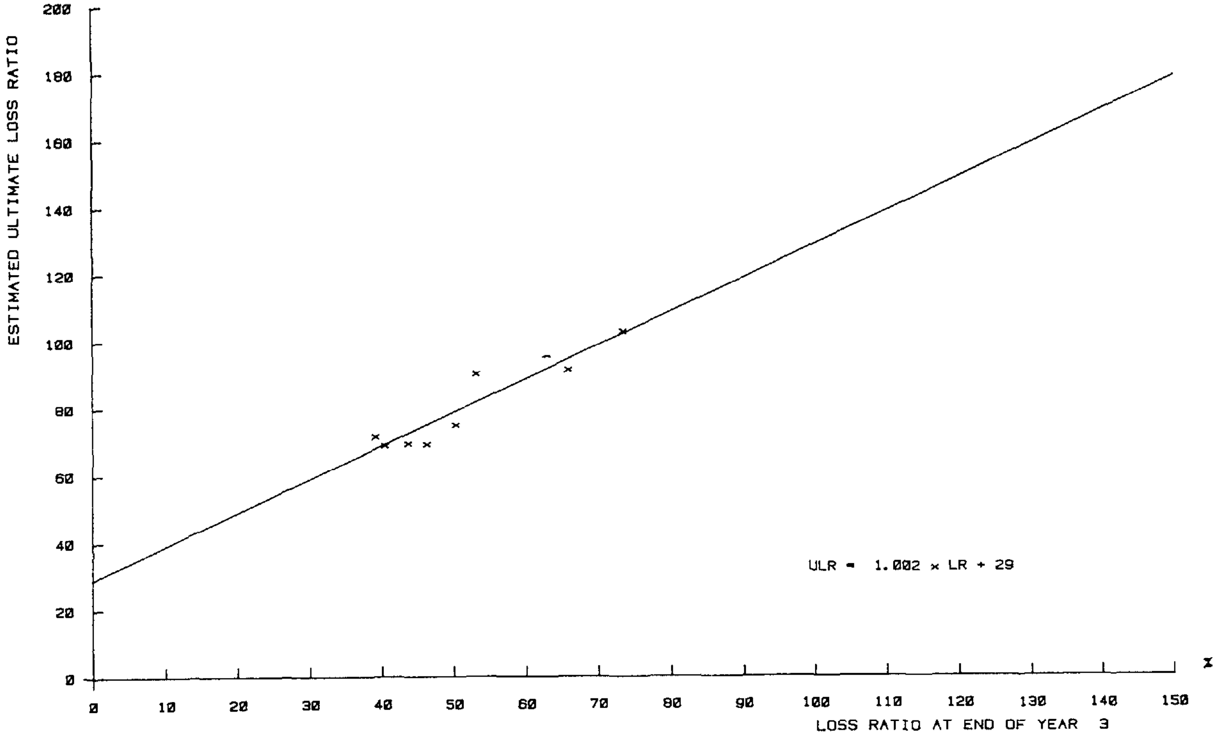
A = 72.2%, B = 2.95, C = 1.50, Mean squared error = 11.2

Figure 1.14



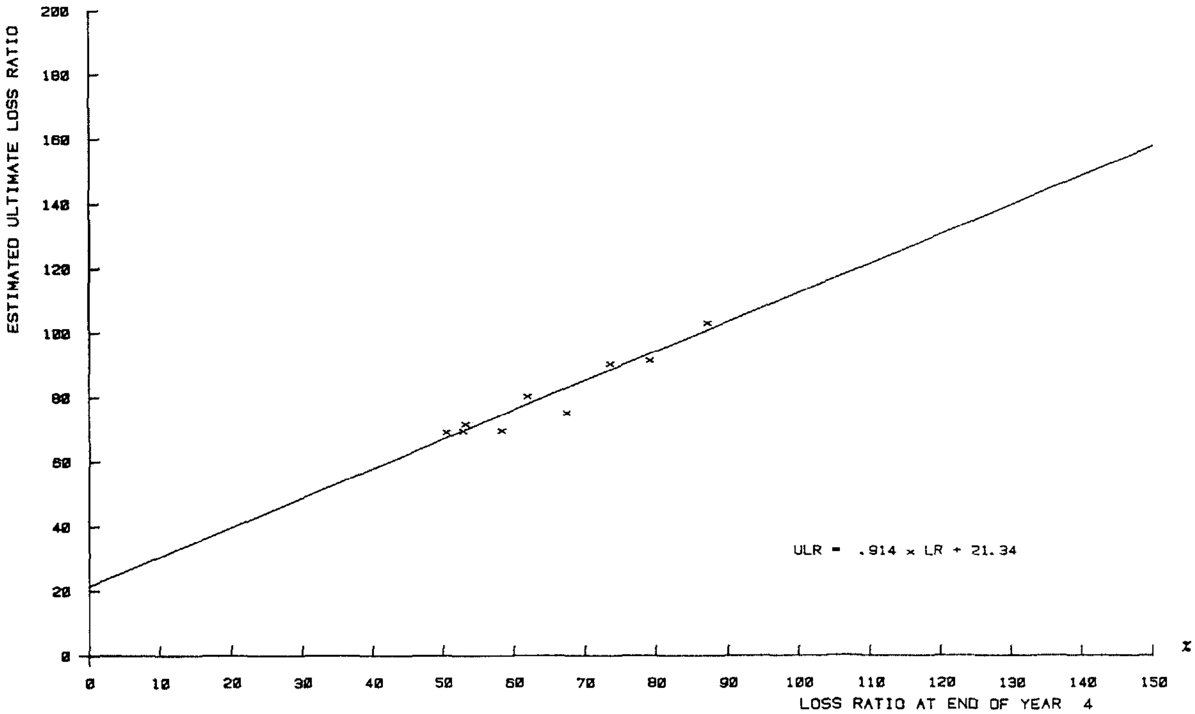
Account years fitted: 74, 75, 76, 77,79, 80, 81

Figure 1.15



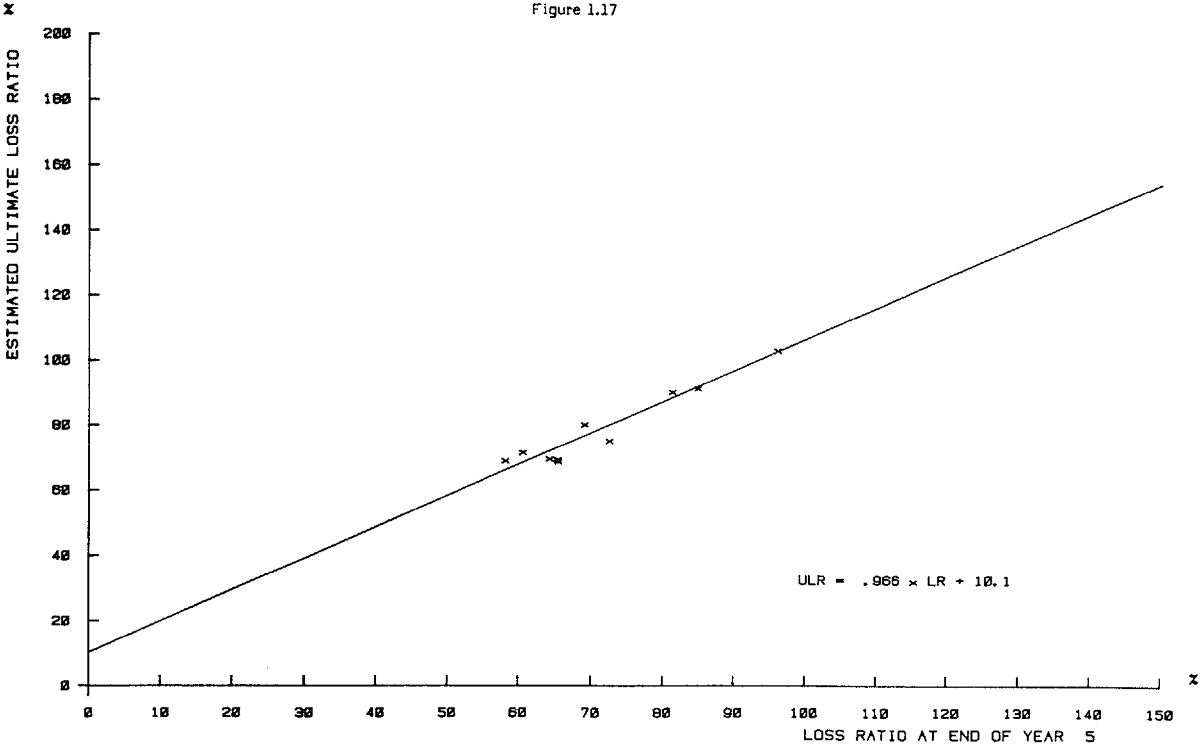
Account years fitted: 73, 74, 75, 76, 77, 79, 80, 81

Figure 1.16



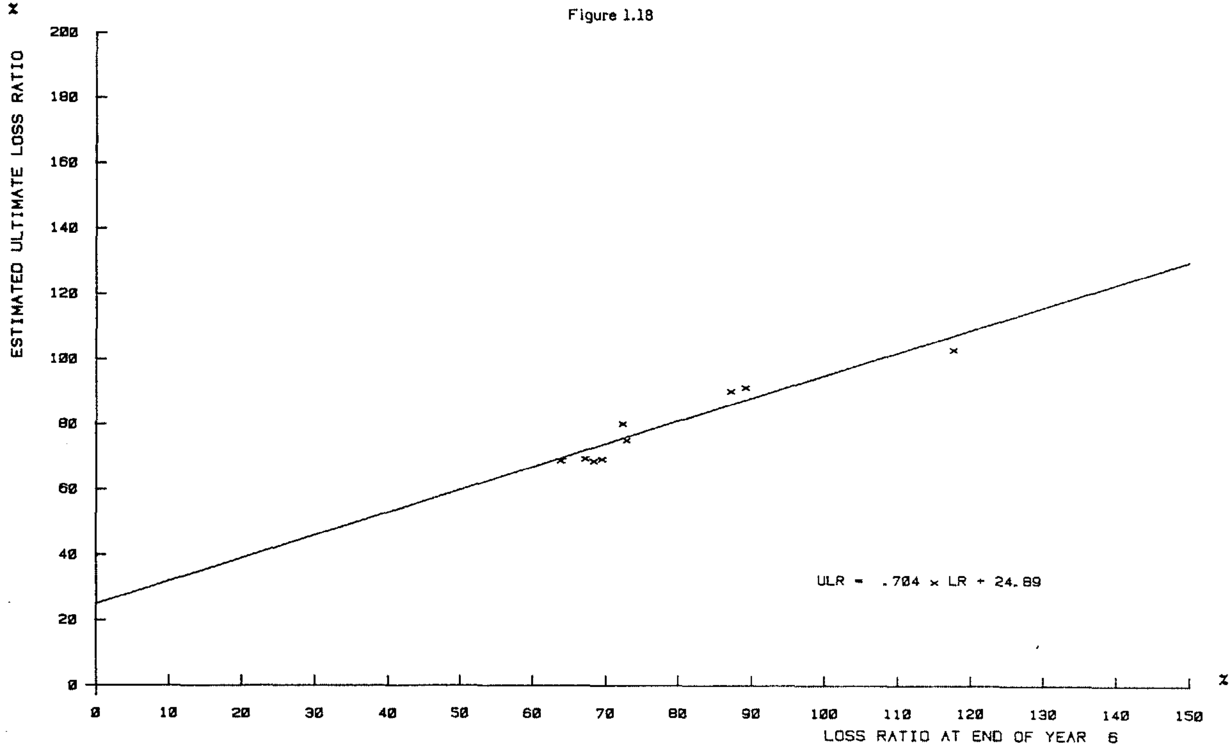
Account years fitted: 72, 73, 74, 75, 76, 77, 79, 80, 81

Figure 1.17



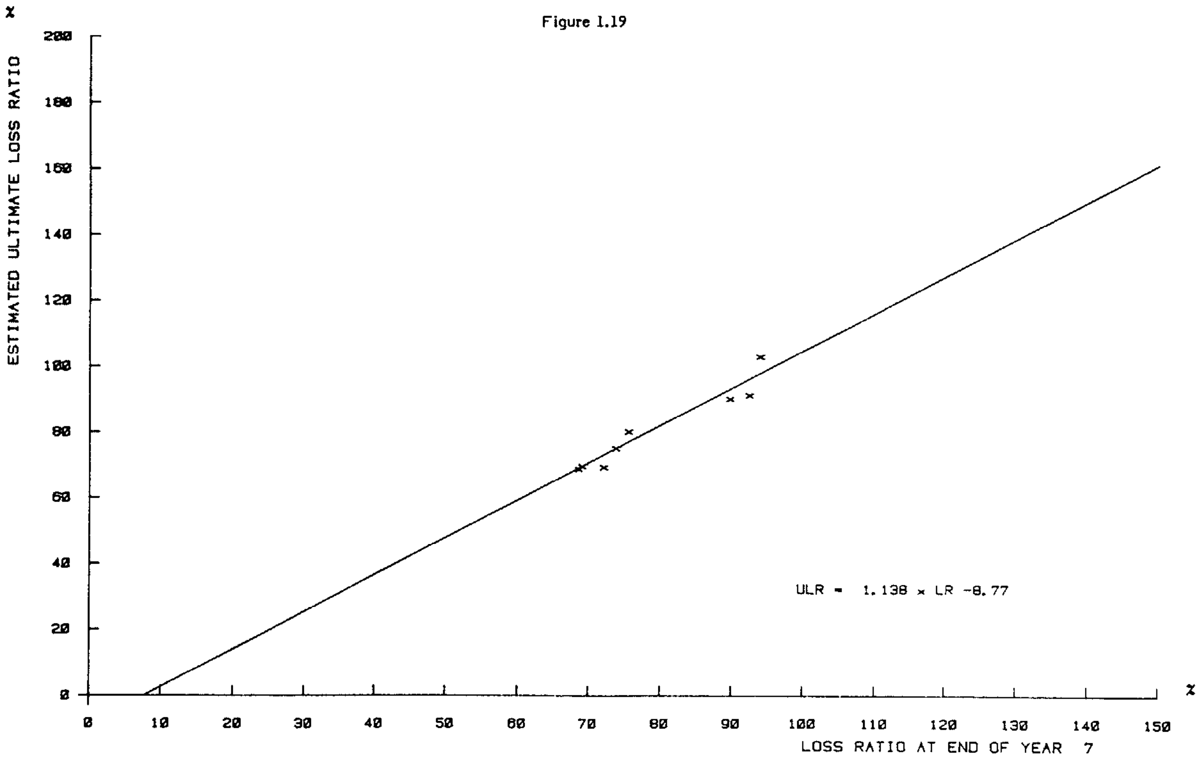
Account years fitted: 71, 72, 73, 74, 75, 76, 77, 79, 80, 81

Figure 1.18



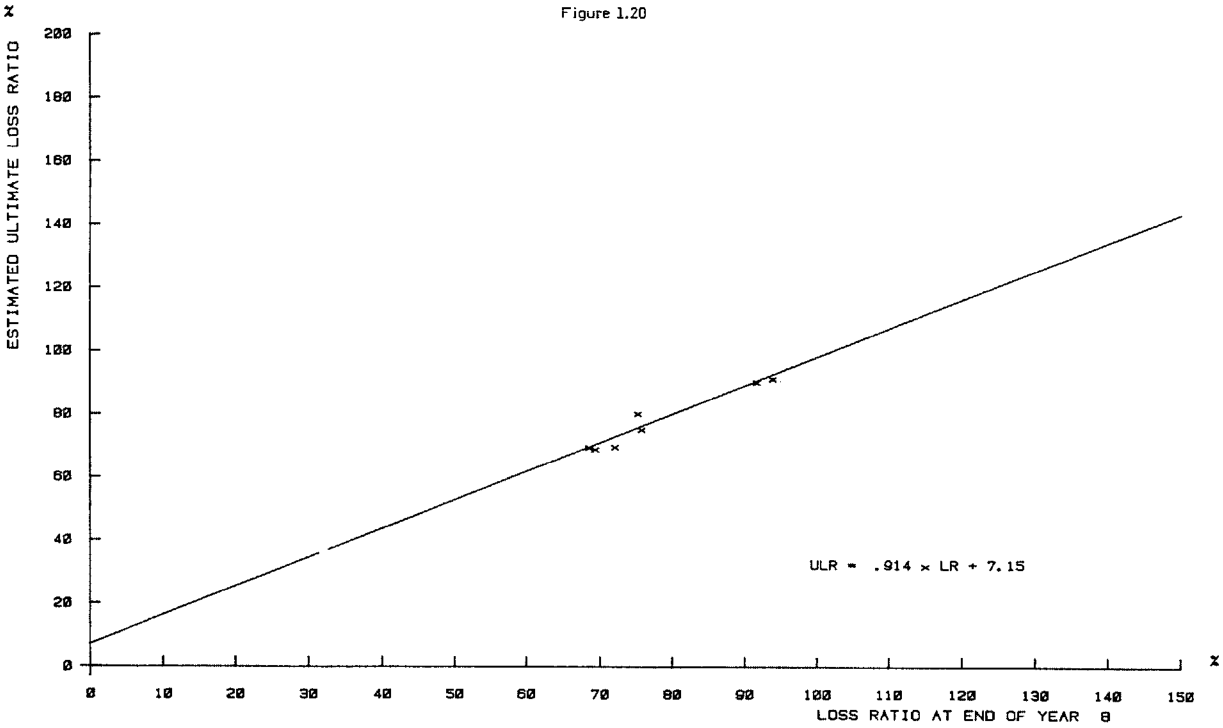
Account years fitted: 71, 72, 73, 74, 75, 76, 77, 79, 80

Figure 1.19



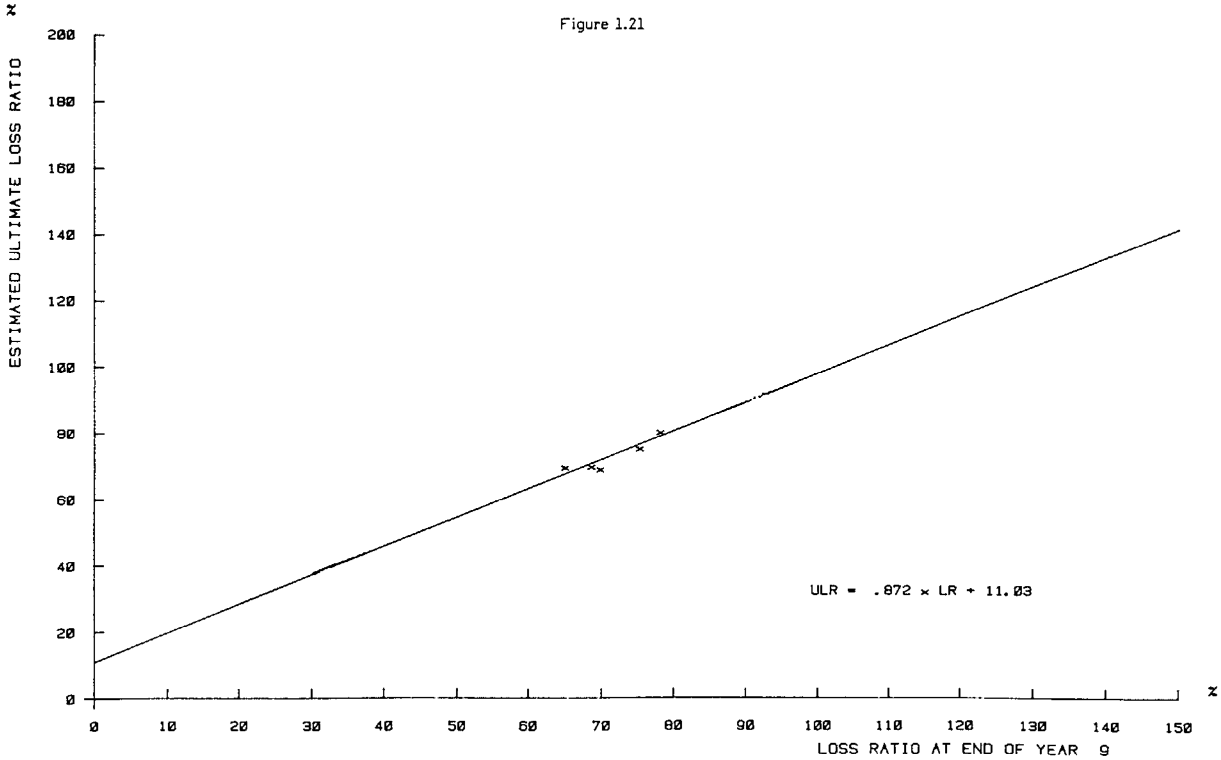
Account years fitted: 71, 72, 73, 74, 75, 76, 77, 79

Figure 1.20



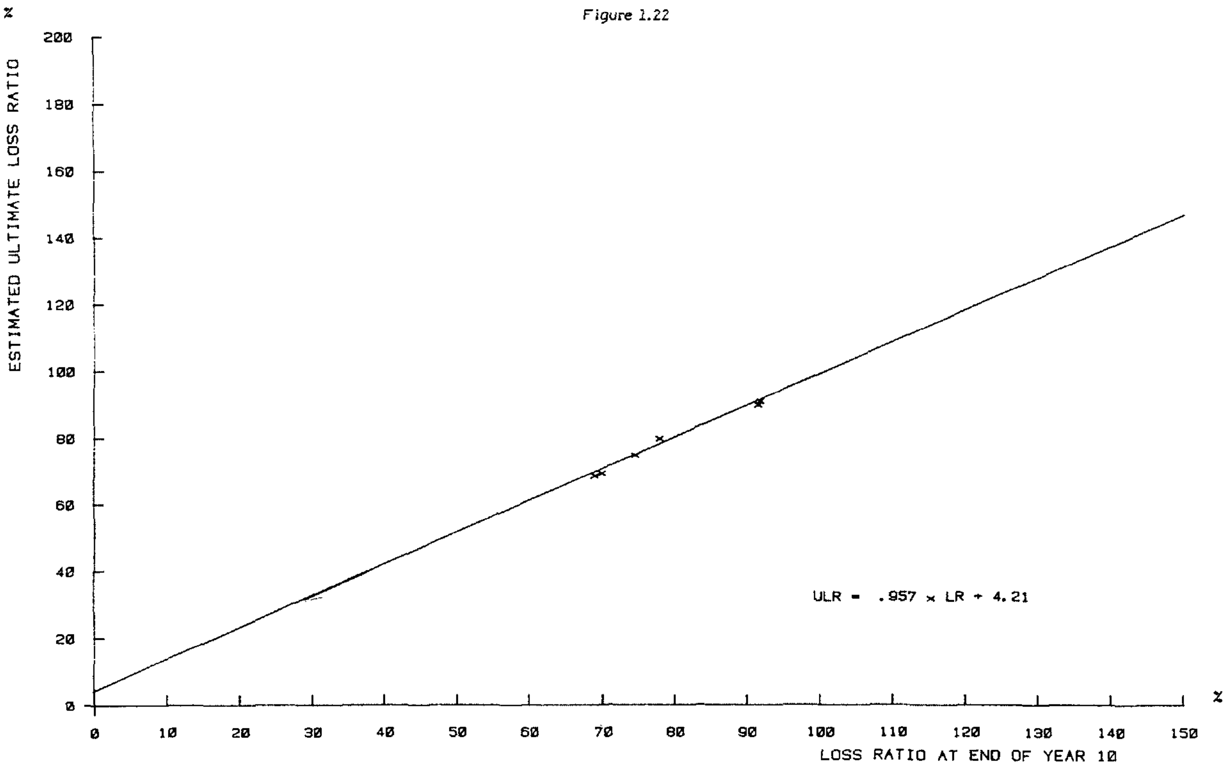
Account years fitted: 71, 72, 73, 74, 75, 76, 77

Figure 1.21



Account years fitted: 71, 72, 73, 74, 75, 76, 77

Figure 1.22



Account years fitted: 71, 72, 73, 74, 75, 76

Table 1.23

Summary of regression lines fitted

Account year	Corresponding development year	Regression line: Slope	Constant	t-test statistic: Value	Degrees of freedom
1984	2	1.514	50.25	1.58	5
1983	3	1.002	29.00	6.55	6
1982	4	.914	21.34	8.54	7
1981	5	.966	10.10	10.11	8
1980	6	.704	24.89	8.41	7
1979	7	1.138	-8.77	8.85	6
1978	8	.914	7.15	9.00	5
1977	9	.872	11.03	13.41	5
1976	10	.957	4.21	16.95	4

Account year	Latest loss ratio	Estimated ULR	Maximum deviation	90% confidence interval
	%	%	%	%
1984	23.05	85.15	21.15	27.07
1983	39.57	68.65	8.75	10.86
1982	47.48	64.74	7.23	8.54
1981	60.63	68.67	4.55	7.05
1980	63.75	69.77	4.98	8.41
1979	93.97	98.17	5.63	8.40
1978	86.30	86.03	4.89	5.83
1977	64.96	67.68	2.48	4.14
1976	69.84	71.05	2.17	3.19

Table 1.24

Recommended estimates of ULR

Account year	Loss ratio to date	Estimated ULR	Confidence interval (+ or -)
	%	%	%
1971	69.4	69.4	-
1972	80.8	80.8	-
1973	88.8	91.0	-
1974	90.7	92.1	-
1975	75.7	75.7	-
1976	69.8	70.2	-
1977	65.0	70.0	-
1978	86.3	103.8	8.4
1979	94.0	98.2	8.4
1980	63.8	69.8	8.4
1981	60.6	68.7	7.0
1982	47.5	64.7	8.5
1983	39.6	68.6	10.9
1984	23.0	85.1	27.1

APPENDIX 2

EFFECT ON SHAPE OF CURVE $L_t = A \times [1 - \exp(-[t/B]^c)]$

OF CHANGING VALUES OF PARAMETERS B AND C

Figure 2.1

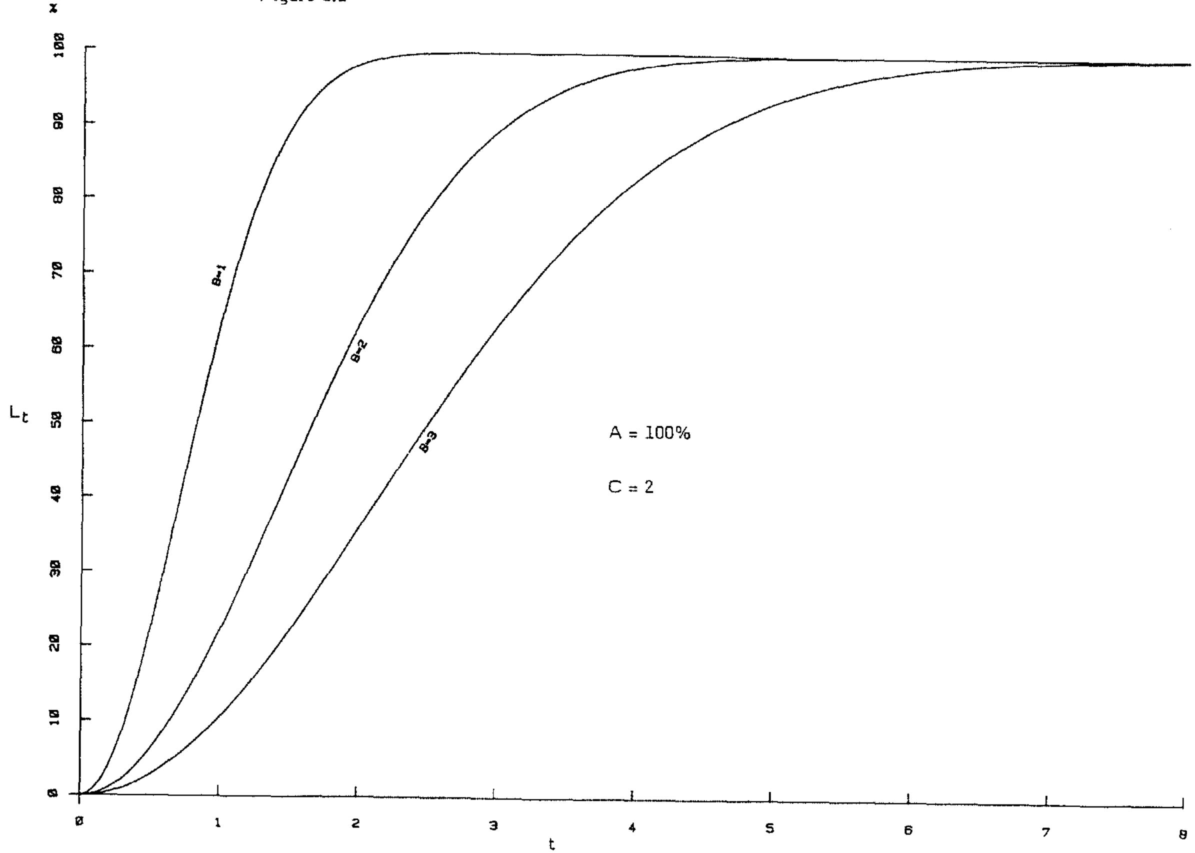
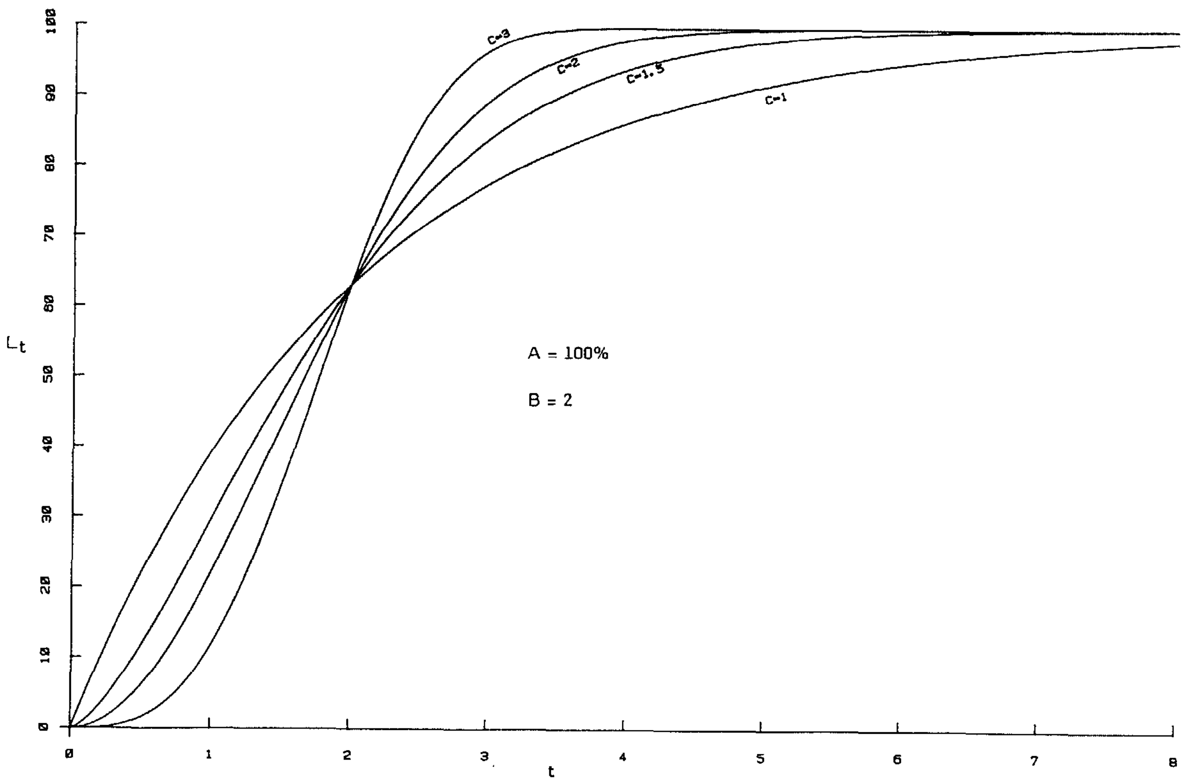


Figure 2.2



TITLE: FOREIGN EXCHANGE FLUCTUATIONS IN THE ANNUAL STATEMENT

AUTHOR: Mr. Kirk G. Fleming

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ABSTRACT: This paper is intended for those actuaries who want to find out how annual statements of foreign subsidiaries are translated into U.S. dollars. The paper will also present some of the problems that can result from these translation methods. The paper presents some samples of different translation methods used and then works through some simple examples to see the effect of these methods on several exhibits of the annual statement. Although written from the viewpoint of a reinsurer, the principles would apply equally to a primary insurer.

INTRODUCTION

Foreign reinsurance subsidiaries have a special importance in the international marketplace. Because of protectionist laws in many foreign countries, U.S. insurers are prohibited from writing primary business in many overseas countries. If a U.S. company is determined to write business in these countries, reinsurance may be the only way to do it.

This paper is for all those people who wonder how foreign exchange fluctuations affect the published results of those companies. The paper gives an overview of what FASB and the NAIC have to say about translating foreign subsidiaries annual statements into U.S. dollars so that they can be consolidated into the U.S. parent's annual statement. Then by starting with a simple manufacturing foreign subsidiary and working up to a more complicated reinsurance foreign subsidiary, we'll see how these rules are suppose to be applied and how they are applied in practice. We'll also see how the different ways of translating the annual statements of foreign subsidiaries affect the U.S. annual statement.

Before we start talking about methods, we should set up a criteria to judge any foreign currency translation method. The Financial Accounting Standards Board uses the following criteria:

1) a translation method should provide information that meets the goals of the report we are working with. As an example, certain exhibits of the annual statement are intended to show underwriting results. A translation method for those exhibits should not mix underwriting and investment information;

2) a translation method should provide information that is compatible with intuition. This statement seems obvious but is is a big reason why some translation methods have become obsolete in the past.

The Financial Accounting Standards Board has been grappling with the questions of foreign currency translation for many years and they have the most to say about it. Let's start with them.

FASB 52

FASB 52 spells out the rules of the game for translating the financial statements of foreign subsidiaries into U.S. dollars so that they can be consolidated with the parent's statements. Translating a foreign statement into U.S. dollars is not that difficult when you follow the methods of FASB 52. Basically, you get the exchange rate at the annual statement date between the foreign currency you're dealing with and the U.S. dollar. Then you use that rate to translate all the assets and liabilities. We all realize that the value of a foreign asset purchased over ten years ago will go up or down in terms of U.S. dollars depending upon whether the dollar has weakened or strengthened. The amount of a foreign liability will also change over time. Two questions arise when you try to account for foreign exchange gains or losses that accumulate over the years. The two questions that come up are 1) how do you measure that gain or loss and 2) when should that gain or loss be included in net income.

The answer to the second question first. FASB 52 gives different rules for including foreign exchange gains or losses in net income depending upon how the foreign entity operates and also depending upon the type of transaction that we are talking about.

If a foreign entity is relatively self-contained and integrated within a foreign country, then the foreign exchange gain or loss on that investment should not be included in current net income. An example of this type of operation would be a French subsidiary that collects premium, maintains its own reserves and funds, and pays losses all in it's local currency. If this subsidiary is ever sold or liquidated, then the accumulated foreign exchange gain or loss should be included in the net income of the parent at that time.

If a foreign entity is really an extension of the parent's domestic operation and it's operation directly affects the parent's cash flow, then any foreign exchange gains or losses from that operation should be included in the parent's current net income. An example of this type of operation is a parent that collects premium in Canadian dollars, converts them into U.S. dollars, maintains this money with its other domestic funds and buys Canadian dollars at the spot rate to pay losses and expenses. The spot rate is the exchange rate for immediate delivery of the currencies exchanged.

That covers the difference in rules that depend upon type of operation. Now for the distinctions that depend upon type of transaction.

If a foreign subsidiary or the parent conducts a transaction in a currency other than its functional currency (FASB 52 defines the primary local currency as the functional currency) and there is an actual foreign exchange gain or loss on this transaction, then that gain or loss should be included in the current net income of the parent. As an example, a French subsidiary writes a policy in Germany for a certain price in marks. These marks have a certain value in francs at the time of sale. When the French subsidiary collects these marks their value in francs may have gone up or down. This gain or loss should be included in the net income of the U.S. parent.

There is one exception to this rule. If the transaction is designated as and is effective as a hedge of a foreign currency commitment, then any foreign exchange gain or loss on that hedge is deferred and not included in current income. The gain or loss on the hedge is deferred and not included in current income. The gain or loss on the hedge should be deferred and used to offset the loss or gain from the designated foreign currency commitment.

That answers question 2. The best way to answer questions 1 -- how to measure foreign exchange gains or losses -- is to start with an example for a simple manufacturing subsidiary and then move on to a simple reinsurance subsidiary.

FASB 52 FOR A SIMPLE MANUFACTURING SUBSIDIARY

The foreign exchange gains or losses that result from exchange rates changing over time are called "translation adjustments" by FASB 52. Translation adjustments emerge from two sources. The first source is the assets and liabilities that were on the balance sheet at the beginning of the year. The second source is from revenues, expenses, gains and losses that occur during the year.

Translation adjustments have to be calculated when translating a subsidiary's financial statements from their functional currency into U.S. dollars. In order to see exactly what a transaction adjustment is, why don't we set up a foreign subsidiary at December 31, 1980, give it something to do during 1981 and then translate its 1981 year-end financial statement into dollars. In the process we will calculate the translation adjustment.

The company's year-end 1980 annual statement is shown on Exhibit I. This is a manufacturing firm so it has some inventory and some cash. It also has a large amount of liabilities. On July 1, 1981, the item will sell in item for FC20 (20 functional currency monetary units) which had been carried in inventory for FC10, so it made a profit of FC10. The exchange rate on December 31, 1980 was FC1 = $1. The exchange rate on December 31, 1981 was FC1 = $0.50. In this example the dollar strengthened.

On Exhibit I we see the effect of the above transaction on the foreign entity's 1981 year-end balance sheet. We see the shift of assets from inventory to cash and we see the increase in retained earnings.

On Exhibit 2 we have the income statement in both the functional currency and translated into U.S. dollars. Let's talk about how we translate the functional currency statement into U.S. dollars.

According to FASB 52, any revenue, expense, gain or loss should be translated into U.S. dollars using the exchange rate that existed at the date those elements were recognized. In our example where there is only one transaction, it is not that difficult to isolate an exchange rate. However, for a normal company, trying to keep track of exchange rates along with transactions could be an impractical task. Because of this FASB 52 allows weighted averages of exchange rates for the period to be used. In this example we will use the average 1981 exchange rate of FC1 = $0.75 to translate the income statement items.

This brings us to the first place where translation adjustments come from--Net Income. The FC10 that the company received on July 1, 1981 was equal to $7.50 on that date because the exchange rate was FC1 = $0.75. On December 31, 1981 that FC10 was worth only $5.00. So looking at it from the viewpoint of the parent, we lost $2.50 because of the change in exchange rates.

This is shown on Exhibit 3 where we calculate the total translation adjustment. In general, the difference between the year end exchange rate and the average exchange rate for the year should be multiplied times the net income to get the translation adjustment attributable to net income.

The next step in the translation process is expressing the current assets and liabilities in U.S. dollars. The 1981 Balance Sheet is shown in functional currency on Exhibit 1. By using an exchange rate of FC1 = $0.50 the assets and liabilities have been translated and are shown on Exhibit 4. We have the original capital carried over at the "historical" exchange rate. The "historical" exchange rate is the original rate used to translate the capital. We also have the net income of $7.50 which we calculated on Exhibit 3 added to Retain Earnings.

This translation process for assets and liabilities also created some translation adjustments and these are calculated on Exhibit 4. The total investment subject to exchange risk is sometime called the net assets and is equal to assets minus liabilities. This is FC20 in our example. These assets were worth $20 on December 31, 1980 and dropped to $10 on December 31, 1981. We had an unrealized foreign exchange loss or another way of looking at it is that the value of the parent's investment in the subsidiary declined over the year. This translation adjustment will be added to the translation adjustment due to Net Income for the total translation adjustment.

The total translation adjustment is made a separate component of equity. And now that the foreign subsidiary's balance sheet is translated into U.S. Dollars and the translation adjustments have been isolated and accounted for, the statement can be incorporated into the parent's annual statement following the rules of consolidation.

Using this general approach to translating annual statements, we get results that agree with our intuition. As the dollar strengthens, the value of an existing foreign subsidiary goes down. The amount of change is directly proportional to the length of time that we held the foreign asset i.e., if the dollar is strengthening and you have had one asset longer than another, then the first asset will have a bigger percentage decline in value than the second. All the exhibits show what we expect to see.

The method also produces an annual statement where all the important relationships that hold true in the functional currency also hold true in the translated currency. So the method meets both of our goals for a translation method.

One of the reasons why all these exhibits make sense is that all the transactions are simple and instantaneous. At the point of sale, cash is exchanged for an item and the transaction is over. There is no dispute about the amount of money involved. The amount that changes hands is very clear cut. This also holds true for assets and liabilities. Although their value may change over time, at any particular point in time their value can be determined in a straight-forward manner.

These are the main reasons why reinsurance company financial statements are not as easy to translate. A transaction has a definite beginning but it takes serveral years to call it complete. When an annual treaty is written it takes a year to earn the premium. It may take several years to pay all the losses from a treaty and during that time the ultimate amount of those losses will not be known.

Another complication with reinsurance companies' statutory annual statements is that they try to give an "historical" perspective to a company in addition to the "snapshot" perspective that balance sheets and income statements give. Schedule O and P, and the SEC disclosure are supposed to show the historical development pattern of losses. Trying to fit the rules of FASB 52 to these exhibits and the special statutory accounting rules provide some complications.

Let's take a simple reinsurance company and look at what happens when we apply FASB 52.

FASB 52 FOR A SIMPLE REINSURANCE COMPANY

Let's suppose that our simple reinsurance company has the December 31, 1980 balance sheet shown on Exhibit 5. The company has some cash and other assets. It also has outstanding losses of FC 80. Over the next few years, suppose we have the following sequence of events:

Date		Event
1.	January 1, 1981	The foreign subsidiary writes a one year liability policy for FC 100. The exchange rate is FC1 = $1.
2.	July 1, 1981	There is a loss under the policy but it is not reported until 1982.
3.	December 31, 1981	An IBNR reserve is established for FC40. The exchange rate is FC1 = $0.50.
4.	July 1, 1982	An initial case reserve of FC 50 is established and the IBNR reserve is taken down.
5.	December 31, 1982	The exchange rate is FC1 = $0.25.
6.	July 31, 1983	The loss is paid for FC 60.
7.	December 31, 1983	The exchange rate is FC1 = $0.50.

On Exhibit 5 we see the effect of the above transactions on the 1981, 1982 and 1983 functional currency balance sheets.

On Exhibit 6, we have the income statements for 1981, 1982 and 1983 in both the functional currency and translated into U.S. dollars. Just like the manufacturing firm, any revenue, expense, gain or loss has to be translated into U.S. dollars using the exchange rates that existed at the date these elements were recognized. In our example, the premium was earned uniformly throughout the year so a weighted average exchange rate should be calculated using earned premiums as weights. This weighted average exchange rate should be used to translate the premiums.

When it come to losses, we follow a similar procedure for calculating an exchange rate for translation. A weighted average exchange rate should be calculated using incurred losses as weights. In our example, all loss transactions take place midway through the year so the exchange rate to be used is the average rate.

On Exhibit 7, 8 and 9 we see the calculation of translation adjustments due to net income for 1981, 1982 and 1983.

These calculations follow the same procedure that we saw for the manufacturing firm. Also shown on these exhibits are the translation adjustments attributable to assets and liabilities. Once again the procedure matches that of the manufacturing firm.

All of these results come together on Exhibit 10 in the translated balance sheet. By using this general procedure we get results that we expect in the balance sheets and income statement. As the dollar strengthens, the value of our foreign subsidiary goes down. If the dollar had weakened, the value of the foreign subsidiary would go up.

One problem area that we run into is the historical exhibits. These exhibits are intended to show how our estimates of incurred losses change over time and they will be distorted if we let changes in exchange rates flow through them. We can see some of these problems on Exhibit 11 which shows the Schedule P and SEC disclosure for our example.

In our example, we revised our initial estimate of the loss upwards. This is what is shown in the functional currency exhibits. However, the translated exhibits show a completely different story. Here we see a pattern of wildly fluctuating results.

The problem comes up because of the changes in exchange rates. The dollar is strengthening in our example and as it does, the value of our foreign liabilities goes down. As we translate the development exhibits we get a mixture of underwriting and foreign investment results.

Please notice that the entire Schedule P is not restated each year using the latest exchange rate. It would be possible to do this but it would involve a large bookkeeping task. Companies that I have seen that follow FASB 52 leave the historical loss numbers at the historical exchange rate and add on the latest numbers using the latest exchange rates. I'll talk about possible reasons why a company would do this later on.

So far, We've been talking about the GAAP rules on foreign exchange. Let's talk about what the NAIC has to say and then we'll talk about other methods that companies use.

THE NAIC ON FOREIGN EXCHANGE

When compared to FASB the NAIC has very little to say on how to account for foreign exchange. They seems to allow a great deal of leeway. From conversations with people at the NAIC, the preferred method seems to be the rules set down by FASB 52. However, the NAIC realizes that for companies which have small overseas operations the requirements of FASB 52 might be onerous. So they allow assets and liabilities to be carried at their historical rates and one overall balancing number to be carried as a liability. This balancing number is equal to net assets, which is assets minus liabilities, times the change in foreign exchange rate. Unrealized gains or losses are direct charge to surplus and realized gains or losses should be included in net income. As far as I know, the NAIC does not specify what a realized or unrealized gain or loss is so I suppose the FASB 52 definition applies.

DIFFERENT TRANSLATION METHODS THAT COMPANIES ARE USING

One method that is presently being used by some reinsurance companies is based on the predecessor of FASB 52 -- FASB 8. FASB 8 specified a slightly different translation method than FASB 52. It was replaced by FASB 52 in December, 1982 mainly because it produced results that were not compatible with the expected economic effects of an exchange rate change. We'll look at a simple example in order to see the difference.

The big difference between FASB 52 and FASB 8 is that FASB 8 specifies that assets carried at historical cost should be translated using historical exchange rates. FASB 52 requires that all assets and liabilities be translated using the current exchange rate. Inventories are a good example of an asset carried at cost. Some reinsurance and insurance companies interpreted this ruling that losses should be translated using historical exchange rates. They fix an exchange rate for a loss at its report date. Any additional transaction with that loss will use this fixed historical exchange rate.

Let's look at an example to see where the problem comes up. Suppose we have a foreign subsidiary with FC100 in assets FC80 in losses and FC 20 in equity. Suppose the exchange rate at the beginning of the year is FC1=$1. If the exchange rate charges to FC1=$2, i.e. the dollar weakens, then we would expect the value of our existing foreign asset to increase. According to FASB 52, we would get an additional $100 gain on the assets and an additional

$80 loss on the losses for a net exchange gain of $20. This is the procedure that matches our intuition. If we follow the "FASB 8" procedure we would get an exchange gain on the assets of $100, no change for the losses for a net total exchange gain of $100. This is a good deal more than we expect.

The "FASB 8" procedure also causes problems with Schedule P and the SEC disclosures. Suppose three claims occur in 1981 and that they are reported in three different years-1981, 1982 and 1983. Let's suppose that they are all worth FC100 and that the IBNR is estimated correctly. The foreign exchange rate is going to change in the following manner: on December 31, 1980 it's FC1=$1, on December 31, 1981 it's FC1=$.50, on December 31, 1982 it's FC1= $.30 and on December 31 it's FC1=$.20. The IBNR is always translated at the current exchange rate. The Schedule P and the SEC disclosures would appear as on Exhibit 12.

Here the problem come in because the different reporting dates of the claims result in different exchange rates and because the IBNR is translated using the current exchange rates.

All in all, the "FASB 8" method does not compare very favorably to the FASB 52 method. The "FASB 8" method produces distortions in all exhibits. The one point in its favor is that the historical exhibits are less distorted then under FASB 52 because the exchange rate is fixed once it is chosen. This eliminates some of the fluctuation due to changes in foreign exchange rates.

Another exchange rate that some companies use when translating losses is the average exchange rate for each individual accident year. This is an improvement over using the exchange rate at report date but it still causes some problems. This method probably also had its basis with FASB 8.

By using each accident year's average exchange rate you get comparable loss development ratios between accident years. You cannot compare premium and loss dollar amounts from accident year to accident year since you would most likely be using different exchange rates. However, you would be able to compare all ratios between years since the different exchange rates would cancel out.

When we were talking about our simple reinsurance company we had a sequence of events that stretched over three years. Let's go back and display that example using separate average accident year exchange rates and calculate the translation reconciliation.

On Exhibit 13 we see the same information that we had in Exhibit 11 except now the Schedule P and SEC Disclosure have been translated using the 1981 accident year average exchange rate. The development patterns shown for the Schedule P exhibits are the same in both the functional currency and the translated currency. This is what we expected. We also see that when we combine loss dollars from different accident years in the SEC Disclosure Form we don't get the same results in the functional and translated currency. This is also what was expected.

Ideally, all exhibits should show reasonable results after translation. Right now FASB 52 gives good results in the balance sheet and income statements but when it come to the historical exhibits there are problems if you don't do all the work. Following the hybrid FASB 8 procedures gives very poor results in the balance sheet and income statements but slightly better results than FASB 52 in the historical exhibits. If FASB 52 gives good results all around why don't companies use it?

ARGUMENTS AGAINST FASB 52

FASB 52 implies that each year we should restate all the international information in the historical exhibits. One possible reason why companies would be reluctant to do this is that it would involve restating a large amount of historical transaction. Another possible reason is that companies feel that those prior numbers should balance against previously published statements.

There is also another reason why companies might be reluctant to restate their numbers. If we restate prior years' numbers using the current exchange rate then there is a possible scenario where a statutory reserve would have to be established because of the restatement. As the dollar weakens, prior years' earned premiums would be restated upwards. If we get enough years over $1,000,000 in premium we might have to set up a statutory reserve according to Footnote (a) of Schedule P.

To go off the topic a bit, this last paragraph brings up the whole question of how exchange rates should affect the establishment of statutory reserves. There will be certain situations where the exchange rate we choose will require a statutory reserve to be established. In my mind, this doesn't make sense. This points up a problem with the way statutory reserves are calculated rather than with exchange rates so I'm not going to dwell on it. However, it seem that a statutory reserve requirement should include the equity of the company in the trigger mechanism rather than just premiums and losses.

CLOSING WORDS

This paper has presented several different ways of translating annual statements of foreign subsidiaries into U.S. dollars. Of all the methods that we looked at, the one that comes closest to meeting the two goals that FASB sets out is the FASB 52 method.

When looking at the annual statements of companies who write a large amount of foreign business, it is important to know how they translate their results if you want to understand those results. I hope this paper makes the process a little easier.

Exhibit 1

Balance Sheet in Functional Currency

Balance Sheets
December 31

	1980		1981
Assets:			
Inventory	FC50		FC40
Cash	FC50		FC70
Total Assets	FC100		FC110
Liabilities:	FC80		FC80
Equity:			
Capital	FC10		FC10
Retained Earnings	FC20	FC10+FC10=	FC20
Total Liability and Shareholder Equity	FC100		FC110

"FC" = Functional Currency

Translated Balance Sheets

Balance Sheets
December 31

Assets:

	1980		1981
Assets:			
Inventory	$ 50		$ 20
Cash	$ 50		$ 35
Total Assets	$100		$ 55
Liabilities:	$ 80		$ 40
Equity:			
Capital	$ 10		$ 10
Retained Earnings	$ 10	$10+$7.5 =	$17.5
Translation Adjustment	$ 0		$(12.5)
Total Liability and Shareholder Equity	$100		$ 55

Exhibit 2

Statement of Income
Year Ended 1981

	Functional Currency	U.S. Dollars (FC1 = $0.75)
Revenue from sale of inventory items:	FC20	$1.5
Expenses - cost of inventory items:	FC10	$7.5
Operating Income	FC10	$7.5

Calculation of Translation Adjustments

A.	Translation Adjustment due to Net Income	
1.	Net Income for 1981	FC10
2.	Difference between the Year End Rate and the average exchange rate	($0.25)
3.	Translation Adjustment Attributable to Net Income (1) x (2)	($2.50)
B.	Translation Adjustment due to Assets & Liabilities	
4.	Total Assets on December 31, 1980	FC100
5.	Total Liabilities on December 31, 1980	FC 80
6.	Net Assets on December 31, 1980 (4) - (5)	FC 20
7.	Difference between Year End Exchange Rate	($0.50)
8.	Translation Adjustment Attributable to Net Assets (6) x (7)	($10.00)
C.	Total Translation Adjustments (3) x (8)	($12.50)

Exhibit 5

Balance Sheet in Functional Currency

Balance Sheets
December 31

	1980	1981	1982	1983
Assets:				
Cash	FC 50	FC150	FC150	FC 90
Other Assets	FC 50	FC 50	FC 50	FC 50
Total Assets	FC100	FC200	FC200	FC140
Outstanding Losses:	FC 80	FC120	FC130	FC 80
Equity:				
Capital	FC 10	FC 10	FC 10	FC 10
Retained Earnings	FC 10	FC 70	FC 60	FC 50
Total Equity	FC 20	FC 80	FC 70	FC 60
Total Liability and Equity	FC100	FC200	FC200	FC140

Calculation of Translation Adjustment 1981

A. Translation Adjustment due to Net Income

1. Net Income for 1981	FC 60
2. Difference between the Year-End Exchange Rate and the Average Rate	($0.25)
3. Translation Adjustment attributable to Net Income	($15.00)

B. Translation Adjustment due to Assets & Liabilities

4. Total Assets on December 31, 1981	FC100
5. Total liabilities on December 31, 1981	FC 80
6. Net assets on December 31, 1981 (4)-(5)	FC 20
7. Difference between year-end Exchange rates	($0.50)
8. Translation Adjustments attributable to net assets (6)X(7)	($10.00)

C. Total Translation Adjustments (3)+(8) ($25.00)

Calculation of Translation Adjustment 1983

A. Translation Adjustment due to Net Income

1. Net Income for 1983	(FC10)
2. Difference between the Year-End Exchange Rate and the Average Rate	$.125
3. Translation Adjustment attributable to Net Income (1) X (2)	($1.25)

B. Translation Adjustment due to Assets & Liabilities

4. Total Assets on December 31, 1982	FC200
5. Total liabilities on December 31, 1982	FC130
6. Net assets on December 31, 1982 (4)-(5)	FC 70
7. Difference between year-end Exchange rates	$0.25
8. Translation Adjustments attributable to net assets (6) X (7)	$17.50

C. Total Translation Adjustments	$16.25

Exhibit 7

Calculation of Translation Adjustment 1982

A. Translation Adjustment due to Net Income

1. Net Income for 1982	(FC 10)
2. Difference between the Year-End Exchange Rate and the Average Rate	($.125)
3. Translation Adjustment attributable to Net Income	$1.25

B. Translation Adjustment due to Assets & Liabilities

4. Total Assets on December 31, 1980	FC200
5. Total liabilities on December 31, 1980	FC120
6. Net assets on December 31, 1980 (4)-(5)	FC 80
7. Difference between year-end Exchange rates	($0.25)
8. Translation Adjustments attributable to net assets (6)X(7)	($20.00)

C. Total Translation Adjustments (3)+(8) ($18.75)

Exhibit 10

Translated Balance Sheet

Balance Sheet
December 31

	1980 (FC1=$1)	1981 (FC1=$.50)	1982 (FC1=$.25)	1983 (FC1=$0.50)
Assets:				
Cash	$50	$75	$37.50	$45
Other Assets	$50	$25	$12.50	$25
Total Assets	$100	$100	$50.00	$70
Outstanding Losses:	$ 80	$ 60	$32.50	$40
Equity:				
Capital	$ 10	$ 10	$10	$10
Retained Earnings	$ 10	$ 55	$51.25	$47.50
Translation Adjustments	$ 0	($ 25)	($43.75)	($27.50)
Total Equity	$ 20	$ 40	$17.50	$30.00
Total Liabilitity and Equity	$100	$100	$50.00	$70.00

Calculation of Translation Adjustment 1982

A. Translation Adjustment due to Net Income

1. Net Income for 1982	(FC 10)
2. Difference between the Year-End Exchange Rate and the Average Rate	($.125)
3. Translation Adjustment attributable to Net Income	$1.25

B. Translation Adjustment due to Assets & Liabilities

4. Total Assets on December 31, 1981	FC200
5. Total liabilities on December 31, 1981	FC120
6. Net assets on December 31, 1981 (4)-(5)	FC 80
7. Difference between year-end Exchange rates	($0.25)
8. Translation Adjustments attributable to net assets (6)X(7)	($20.00)

C. Total Translation Adjustments (3)+(8)	($18.75)

Statement of Income
Year Ended 1981

	Functional Currency	U.S. Dollar (FC1=$.75)
1. Earned Premium	FC100	$75.00
2. Incurred Losses	FC 40	$30.00
3. Net Income	FC 60	$45.00

Statement of Income
Year Ended 1982

	Functional Currency	U.S. Dollar (FC1=$.375)
1. Earned Premium	FC 0	$ 0.00
2. Incurred Losses	FC10	$ 3.75
3. Net Income	(FC10)	($3.75)

Statement of Income
Year Ended 1983

	Functional Currency	U.S. Dollar (FC1=$.375)
1. Earned Premium	FC 0	$ 0.00
2. Incurred Losses	FC10	($3.75)
3. Net Income	(FC10)	($3.75)

RESERVE REVIEW OF A REINSURANCE COMPANY

by Stephen W. Philbrick

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RESERVE REVIEW OF A REINSURANCE COMPANY

ABSTRACT

The estimation of reserves for a reinsurance company is conceptually similar to the estimation of reserves for a primary insurance company. However, due to a number of differences between primary and reinsurance companies, the actual practice of reserving for reinsurance companies involves a process which is quite distinct from methods commonly used for primary companies. In this discussion paper we hope to accomplish two goals:

- discuss a reserving approach which is appropriate for reinsurance companies, and

- provide a methodology for analyzing development patterns which is especially applicable to reinsurance companies, but also has applications for primary companies.

In this paper we will assume the reader is generally familiar with reserving practices for primary companies. We will focus on those situations which are unique or different for reinsurance companies.

I. RESERVING CONSIDERATIONS

The first step in any review of reserves for a reinsurance company is to define the scope of the project.[1] At one end of a spectrum, we may perform a cursory review of results, intended primarily to highlight potential problem areas, but not intended to produce reserve recommendations. Another possibility would be a methodology review, in which the actuary might review the procedures used by the company and the data used in these procedures for reasonableness, but with little emphasis on the bottom-line reserve. At the other end of the spectrum, a company may wish to consider a combination of portfolio transfers and commutations.[2] In this situation, the company needs more than a single, bottom-line reserve estimate; it also needs anticipated payout patterns associated with the liabilities, and possibly reserve estimates and payout patterns for individual treaties, or small groups of treaties.

In this discussion paper, we will assume that the company requires an analysis which will produce a reserve estimate for financial statement purposes, but does not require the detailed information necessary for commutation or portfolio transfer analysis.

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[1] This paper is written from the point of view of an external consultant; the discussion would also apply generally to an in-house actuary.

[2] Portfolio transfers involve transfer of liabilities forward to an unrelated party, commutations involve transfer of liabilities back to the original ceding company.

Once the project scope has been defined, it is necessary to obtain details of the company's operation and the types of business that it has written. This information will generally be gathered from several types of sources:

- Public sources - such as Best's reports, and annual statements;

- Internal company documents;

- Interviews with company personnel;

- External company documents and personnel, e.g., ceding companies or intermediaries.

There are a large number of questions which should be asked in any reserve analysis. Rather than repeat them here, the reader is referred to Berquist and Sherman [1]. In this paper, we will concentrate on those issues which are most relevant to a reinsurance company.

DATA GROUPING

In any reserve review, one of the goals is to subdivide the company into a reasonable number of pieces that can each be analyzed separately. Each piece should be as homogeneous as is practical, yet large enough that random fluctuations will not materially distort the results.

In primary companies, this division is usually performed along lines of business. The major decisions involve what level of grouping of lines is appropriate. In a reinsurance company, the situation is very different. There are a large variety of ways in which reinsurance business can be categorized:

- Location of insured exposures (country or group of countries)
- Currency[3]
- Line of Business
- Form - Quota Share, Excess of Loss, Stop Loss
- Layer - Primary, Working Excess, High layer excess, Catastrophic covers
- Facultative versus Treaty
- Accounting Basis - Earned/Incurred versus Written/Paid
- Other Special Forms - e.g., Portfolio Transfers
- Active versus Cancelled Treaties

This list is far from exhaustive. Any particular company will have its own way of categorizing business which may add to the above list. For example, the company may have formed the business into pools, for the purpose of securing retrocessional coverage. In many cases, the formation of these pools does not conform to any one of the categorizations listed above. It may

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[3] Not necessarily the same as country. For example, many reinsurance contracts are denominated in $US or £ even though the exposures may be in other countries.

be necessary to analyze reserves separately by pool to produce results which can be reported to retrocessionnaires.

It is clearly impractical (using current techniques) to maintain all categories simultaneously. This would result in a categorization encompassing at least nine dimensions. The total number of categories could easily reach several hundred, most of which are likely to contain only one or two pieces of business. (The reader may recall Longley-Cook's, [4] delightful analogy - "We may liken our statistics to a large crumbly loaf cake, which we may cut in slices to obtain easily edible helpings. The method of slicing may be chosen in different ways--across the cake, lengthwise down the cake, or even in horizontal slices--but only one method of slicing may be used at a time. If we try to slice the cake more than one way at a time, we shall be left with a useless collection of crumbs.")

Before we discuss ways to reduce the number of categories, it will be helpful to review reasons for using categories at all. Some of the reasons include:

- **Improved Estimates** - Categories which are homogeneous will exhibit more stable development patterns and more reliable estimates of ultimate losses.

- **Statutory Reporting** - Statutory rules require reporting of results along statutory lines of business.

- **Contractual Reporting** - Retrocessional agreements require reporting of results for business covered by the reinsurance contract.

- **External Relevant Data** - There may be relevant development factors from outside sources which could be used if the data is organized in a similar fashion.

Any attempt to combine or eliminate categories should be done with the above goals or requirements in mind.

Each of these possible categorizations will be discussed in turn:

Location

Reporting patterns typically vary for exposures from foreign countries due to accounting lags. It should be emphasized that the location of the broker is often as important as the location of the insured. For example, U. S. insureds may be written by a U. S. company who purchases reinsurance through a London broker. The premium and loss reporting may be subject to longer lags than if the business had been placed by a U. S. broker.

Another reason for segregating business by country is that many retrocessionnaires may restrict coverage to U. S. business (sometimes U. S. and Canadian).[4]

Development patterns also differ by country. At present, reporting and payment patterns for liability coverages in the U. S. are significantly slower than most other countries. The counterpart of workers' compensation in other countries may differ considerably in terms of coverage characteristics and reporting and payment patterns.

Currency

International reinsurance transactions involving foreign currencies will need to be converted to U. S. dollars for financial reporting purposes. This conversion process can introduce material distortions into the reserve analysis. Typically, premiums will be converted to $US at the conversion rate in effect at the time they are written, paid losses will use the conversion rate in effect at the time the loss is paid, and outstanding losses will be converted based upon the financial statement date. Note that this means that incurred losses in a financial statement will not all be converted at the same rate. In addition, there may be departures from this practice. Some companies use the conversion rate in effect on the day a case

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[4] More generally, it is common for some retrocessionnaires to impose limitations on the type of business they will accept. Typically these include certain lines of business, maximum (or minimum) attachment points, as well as location.

reserve is established, and then a more current conversion rate only on the change in reserve, not on the entire outstanding. Thus, a reserve at any point in time might be the result of several different conversion rates. Some companies use conversion rates which are spelled out in the contract.

The analysis of premium and loss development should be done with a data base that eliminates the distortions due to changing exchange rates. One method is to analyze the entire treaty in the original currency, and then apply conversion rates as a final step. A more common approach is to maintain data such that the entire incurred can be converted at a common exchange rate. This will ensure that development of premiums and losses will be true development and not due to fluctuating exchange rates. Paid losses will also have to be calculated at the conversion rate in effect at the time of payment so that financial statements will be correct. Because loss amounts may have to be available at more than one conversion rate (an historical rate for financial reporting purposes, and a current rate for reserve analysis), the analyst will have to be especially careful to use the correct values.

The advantage of converting to a common currency before analysis is that it allows combination of treaties. This may help remove one level of categorization.

Line of Business

It is rare to have line of business detail even approaching the level of detail available for most primary companies. A typical line of business breakdown for a small to medium reinsurer might include the following:

- Property
- Casualty
- Property/Casualty Combined
- Marine
- Other

Many of the larger reinsurers will have more detail available, but there usually are some significant treaties which cover broad categories of business. This makes allocation of results to statutory lines questionable at best. (This subject will be discussed in more detail later.)

Form

Reinsurance can generally be classed into two forms - either proportional or excess. Proportional forms include quota share and surplus share. Excess forms include excess of loss, aggregate excess (also called stop loss or aggregate stop loss), and catastrophe.

From a reserving point of view, policies written on a proportional form will have loss reporting and loss payment patterns similar to the patterns applicable to the underlying primary companies, although the patterns may be slower due to the accounting lag associated with the reporting of results from the primary company to the reinsurer. Excess forms will have a slower pattern because smaller losses are reported and paid faster (on average) than larger losses. Thus, the reinsured portion contains a disproportionate share of the slower reporting losses. This factor is in addition to the accounting lag which also affects excess policies.

Layer

A layer refers to the range of losses covered by the reinsurance contract. For example, if the contract covers the amount between $1,000,000 per occurrence and $6,000,000 per occurrence, this is referred to as a $5 million layer. The $1,000,000 is referred to as an attachment point. The coverage is usually written $5MM x $1MM, or $5M xs $1M. (Unfortunately, the letter M is used by some to mean thousands, by others to mean millions.)

Many combinations of layer size and attachment point are seen in practice. These can loosely be categorized into primary, working excess, and high excess layers, depending on the attachment point. Primary layers have an attachment point of zero, or some fairly small value. Working layers have attachment points over which the ceding company expects to have a number of claims each year. High excess layers are layers in which losses are not expected in every year.

Development patterns will generally be slower as the attachment point is raised. In theory, a formula might be derived which would specify the adjustment needed for any value of an attachment point but this author is not aware of any such study. Instead, business is categorized into the above rough categories and development factors are estimated for each group.

Facultative versus Treaty

Treaty business refers to a contract which provides automatic coverage of a defined portion of the business written by a company, typically all business written in specified classes such as casualty. Facultative business is written on specific individual risks.

While facultative business can be categorized into the same groups as treaty business, it is often necessary to keep facultative business separate from treaty because the pricing may be very different. In addition a block of facultative business will have a wide variety of attachment points and layers, while a particular treaty or group of treaties may have more homogeneous characteristics.

Accounting Basis

Development patterns can vary materially depending on the accounting basis of the contract. Some contracts are on a written/paid basis, meaning that premiums are remitted from the insurer to the reinsurer as they are written, and losses are received on a paid basis. Some contracts are on an earned/incurred basis, which technically means that premiums are remitted only as they are earned while losses are received on an incurred basis. In practice, losses are often still handled on a paid basis, with the outstanding being secured by a letter of credit.

Reinsurance reporting forms sometimes simply use the terms "premiums" and "losses" without further definition. The analyst must be careful to ascertain the correct nature of the data.

Other Special Forms

Portfolio transfers and commutations have become increasingly common in recent years. A portfolio transfer on a treaty or group of treaties rarely transfers the ultimate liability without limit. Thus, it will be necessary to determine if the current estimate of the ultimate liability exceeds the amounts transferred. Loss development data should be restated to eliminate any distortions that may have occurred due to the transaction. If a commutation is effected, the historical data may also have to be restated. A complicating factor is that there are differences of opinion as to the proper accounting treatment of these contracts.

The analyst should also keep in mind that portfolio transfers can be made from one policy year to the next, and some international contracts may be cancellable without runoff liability.

Active versus Cancelled Treaties

Reinsurers periodically undertake significant "reunderwriting" of their books of business. This typically includes cancellation of a large number of treaties. It may be natural for a company to track the active and cancelled treaties separately. While this may make sense in some situations, there are potential problems. In subsequent updates to data, a decision will have to be made whether the block of cancelled contracts will remain fixed. For example, if the split between active and cancelled treaties is first made on December 31, 1984, what will be done with the treaties which are cancelled during 1985? If they are included in the definition of cancelled treaties, then the runoff statistics and loss ratios may be meaningless for comparison purposes. If these treaties are not included with cancelled treaties, then we have the awkward situation that the category "active" treaties includes some cancelled treaties.

Another problem which may arise is the temptation to use the overall historical development on active treaties alone to project future development on these treaties.

It may still be appropriate to use total historical development on active treaties, depending on the reason for cancellation.

If cancellation was based on loss ratio, then it may be reasonable to assume that the non-cancelled treaties will have similar development. If cancellation was based on adverse development, then one must determine a development pattern for the active contracts. It is unlikely that sufficient historical experience exists to determine a development pattern solely based on active contracts. In addition, there is the possibility that the business underlying active contracts has changed over time.

OTHER KEY DIFFERENCES

The organization of data into groupings can be very different for reinsurance companies compared to primary companies as was shown in the previous section. There are a number of other items which are especially relevant to the reinsurance reserving analyst.

Premium Development

Premium development[5] is a critical item in reinsurance reserving for two main reasons. First, reserving techniques which use premiums (loss ratio, Bornhuetter-Ferguson) are more important in reinsurance reserving than for primary companies. Second, premium development is generally insignificant for primary companies[6] beyond 24 months. Premium development factors for reinsurance companies are often needed beyond 60 months, and often exceed 3.0 at 12 months, on an underwriting year basis.

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[5] For a more in-depth discussion of premium development, see Collins [3]. A discussion of alternative accounting treatments of premium development is included in Miccolis [6].

[6] An important exception is retro premiums, but that occurs only because they are a function of losses.

Premium development occurs primarily due to accounting lags in the reporting of premium. An original piece of business may be ceded as part of a treaty to a reinsurer, which in turn may be partially ceded to a retrocessionnaire. There often may be several levels of reinsurers. Each company will receive premium from its insureds and will pass on a portion to its reinsurer. There is typically a one quarter (3 months) lag between receiving premium and forwarding on the reinsurance premium. When several levels are involved, years can elapse.

Another reason for the long lag is due to the nature of an underwriting year. This will be discussed in the next section.

Losses are also subject to the same accounting lags in most cases. However, many treaties have provisions (typically referred to as "cash calls") which allow more rapid reporting of losses over a specified size, such as $50,000. It is not impossible for a property treaty to have premium development slower than loss development, if cash calls have occurred.

Underwriting Year

The concept of underwriting year is an important one in reinsurance reserving. An underwriting year consists of all treaties issued by a reinsurance company during the year. A typical treaty will cover the risks of a primary company. If the treaty is on a "risks attaching" basis, it will cover all policies issued by the primary company during the term of the treaty. The exposure arising from a one-year policy issued by a primary company on the last day of the treaty, for a treaty which incepts on the last day of the underwriting year, will extend three years from the beginning of the underwriting year.

More extreme examples can occur involving longer term policies, or more "layers" of reinsurers between the ultimate reinsurer and the primary company.

The actual pattern of exposures tends to vary more from the "ideal" underwriting year than is true for policy or accident years. There are several reasons for this:

- While treaties can incept on any day of the year, the most common date is January 1, followed by July 1. Typically, 50% to 75% of all treaties incept on one of these two dates.

- Three-year policies may be issued under some treaties, although this is becoming less common.

- Some treaties (particularly excess covers) are on a "losses occurring" basis, which means they cover all losses with an occurrence date during the treaty, rather than all policies with an inception date during the treaty.

- Some treaties have a provision which allows the reinsured to extend a policy period (typically up to a total of 18 months) in order to coordinate anniversary dates.

- In some cases, a treaty issued late in a year may be assigned to the subsequent underwriting year.

For the above reasons it may be necessary to determine the distribution of exposures within an underwriting year on a case-by-case basis. This distribution is helpful in determining reasonable development factors. For example, if exposures are weighted heavily toward the end of the underwriting year, loss development (and premium development) factors are likely to be higher. (In Section II, we discuss a more formal approach to the calculation of development factors.)

Lack of Data

Perhaps the most severe problem in reinsurance reserving is lack of useful data.

The shortage of data arises for several reasons.

High excess layers, catastrophe covers and clash covers are not expected to have many losses. Many treaties might expect only one or two incidents in a particular year. Traditional development factor methods depend on a relatively large number of claims to produce reasonable estimates.

Reinsurance statistical gathering is generally not as good as for primary companies. Many commercially available data processing systems for collecting statistical information are designed with primary companies in mind. These systems are often inadequate for reinsurance companies.

Information is often reported on a bordereau basis. This is typically a quarterly report which contains only summary premium and loss information. IBNR and bulk reserves as set by the reinsured are sometimes reported, but not always. Individual claim detail is seldom available. This means that many important actuarial techniques are not available - specifically, any techniques based upon counts or average claim sizes.

Another problem arises from the fact that the bulk of the claim reserving is performed at the primary company level. Reinsurers have claim departments, but they cannot afford to investigate each individual claim to the extent that a primary company can. This means that a reinsurance company's data is a mixture of results from reinsureds with a variety of case reserving methodologies and approaches.

Finally, the delayed reporting of information due to the accounting lags inherent in the system means that the data that is available tends to be much less current than for primary companies.

RESERVING METHODS

Many of the same reserving methods used by primary companies can be used by reinsurance companies. Common reserving techniques include:

- Paid Loss Development
- Incurred Loss Development
- Bornhuetter-Ferguson (can be applied on a paid or incurred basis)
- Loss Ratio

It is far easier to identify weaknesses associated with each method than to identify strengths. Paid loss development factors can become extraordinarily large due to the extremely slow payment patterns on many reinsurance books of business. Incurred loss development techniques suffer from the inconsistency of case reserving techniques and the inadequate collection of data in many cases. It is not unusual to have evaluation dates for which no change in case reserves has been received. Only cash items, such as losses paid and premiums written, may have been reported.

The loss ratio method's primary weakness is its ease of abuse. In theory, the loss ratio method can be a very reasonable and useful method. In practice, it is often used to reach a predetermined, and often optimistic, result. In many cases, the loss ratio is chosen simply by subtracting an expense loading from 100%. A more proper use of the loss ratio method is based upon actuarial analysis of the underlying pricing. In simple terms, the starting point is a base with which there is some degree of comfort regarding the overall appropriate loss ratio. This might include several older years of experience, where the losses are reaching a level of maturity, or it might be a primary layer of coverage. If adjustments are made for intervening inflation, changes in coverage and difference in limits, the results should be a reasonable current price for the coverage. This value, divided by the actual premium charged for the coverage, produces an estimated loss ratio. At early evaluation dates, this loss ratio may produce better reserves than a development technique.

The Bornhuetter-Ferguson technique represents a blend of the loss ratio and development techniques. While this combination is arguably the best combination of techniques, it is not a panacea. Erratic loss development and inappropriate loss ratios will generate unrealistic reserves no matter what technique is used. (For those unfamiliar with this technique, a brief review is provided in Appendix C.)

It is important to compare actual and expected losses for each underwriting year and to determine whether any differences indicate an inaccurate reporting pattern or initial expected loss ratio assumption. Of course, the same level of expected losses can be produced by an infinite number of loss ratio and reporting pattern assumptions. One must decide realistically whether a given level of reported losses is the result of a low loss ratio and a fast reporting pattern, or a high loss ratio and a slower reporting pattern, or something in between.

The most important statement we can make about the various methods is that one should never rely on any one of them alone. As many of these techniques should be used as possible, as well as other techniques which might be applicable depending upon data availability.

LINE OF BUSINESS

The decomposition of reinsurance experience into statutory lines of business is a valuable aid in reserve analysis. Loss development patterns are available from a variety of sources (Best's, RAA) which are segregated by major line of business. These patterns can be quite dissimilar. In some cases, loss detail will be available directly by line, but in other cases, some detective work may be needed. The reserve analyst should be cautioned not to accept loss information segregated by line blindly. In many cases, the breakdown may have been calculated by an allocation system. Nevertheless, even an approximate allocation of losses may be better than no breakdown at all. There are a number of ways to estimate the appropriate line of business breakdown.

In most cases, a ceding company or broker will supply an EPI, or estimated premium income. This is intended to provide an estimate of the amount of premium to be anticipated under the treaty. In some cases, the EPI values will be broken down by line.

In some cases, the actual premium reported can be segregated by line, even though losses cannot be similarly segregated.

The policy wording of the treaty should be helpful, as it typically states the classes of business accepted under the treaty. Unfortunately, a common description is "all classes of business written by John Doe," where John Doe is the name of an MGA or ceding company.

Underwriting files may be helpful, as they may contain a history of this treaty including some line of business detail. Direct discussions with the broker or producer of the business can sometimes produce line of business profiles. The value of this approach should not be underestimated. It is common for many reinsurers to accept a block of business based upon little more than the (presumed) reputation of the source. In these cases, the reinsurer's files may contain only limited information. Direct contact with the underwriters actually involved in the day-to-day selection of business may be the only reasonable way to understand the nature of the business.

Individual large losses may also be helpful. Although it would be dangerous to extrapolate an entire line of business profile from a few large losses, general liability losses are occasionally observed on a treaty described as property only. On the basis of that evidence, we reevaluated the line of business breakdown.

EXAMPLE

The following example illustrates a "typical" reserve review. This example is a composite of several actual reviews.

The company writes fifty treaties of varying size, but no facultative business. After reviewing each of the treaties, they are grouped into several categories:

- First, one very large treaty accounting for 30% of the company's entire business. We make arrangements to spend time at the ceding company's offices to review the business in detail.

- Next, the ten largest treaties will be reviewed in individual detail, although on-site visits will be limited to the largest treaty. These ten treaties comprise an additional 45% of the company's total earned premiums and carried reserves.

- The company participates in two large pools which are singled out for attention. Each of these pools has been subject to review by an independent actuarial consulting firm. We make arrangements to obtain copies of their most recent reports.

- Twelve other treaties are also singled out. In each case, our company is a quota-share participant in the treaty along with other companies. In each of these treaties, we have reviewed the experience on behalf of one of the other participants.

- The remaining treaties now account for less than 15% of the total business.

We now review each treaty, attempting to create a line of business profile for each one. Various methods are used as outlined in an earlier section. While this exercise is performed for every treaty, proportionately more time is spent on the larger treaties.

On the ten largest treaties we obtain historical paid and incurred development data and assemble standard triangles of data. Development factors are calculated for each one. We separately assemble "industry" loss development statistics from a variety of sources. From Best's Loss Development Reports, we put together paid and incurred patterns for auto, general liability, workers' compensation, multiple peril and medical malpractice. This is done separately for primary versus reinsurance companies. An adjustment is made for estimated inadequacy of industry results.

RAA data is also assembled by line. This data is on an accident year basis for excess of loss business. We would typically use this data to develop treaties which cover excess business. If the treaties being analyzed are on a policy year or underwriting year basis, we could make formal adjustments to the factors (as outlined in Section II). We could also make judgmental adjustments to reflect perceived differences in the layers or attachment points involved. Other sources include some special studies which produce factors for marine, property, and various components of London business. For each of the smaller treaties we assign weights to each of the possible industry patterns, based upon our line of business analysis.

We now can perform premium development, paid loss development, incurred loss development, Bornhuetter-Ferguson (both paid and incurred) and loss ratio analysis. We review the results of each method, look for consistency of loss ratios over time, and, with a heavy dose of judgment, select the final reserve estimates.

This description is far from complete. The diversity of reinsurance business produces many unique situations requiring unique approaches. However, the above approach represents a reasonable approach in the specific situations involved.

SECTION II

Loss development factors are one of the more important tools in any reserve analysis. Typically, historical data is segregated by some exposure basis. Evaluations of each group are obtained at regular intervals. Ratios of successive valuations are calculated (age-to-age development factors) and the results are averaged with various types of averages used and judgment applied when the ratios exhibit unusual patterns. The resulting factors are cumulatively multiplied together to produce age-to-ultimate development factors which can be used directly in a development factor method, or as an intermediate step in a Bornhuetter-Ferguson analysis [2].

There are a large number of variations of this method.

Historical data may be:

- Incurred Losses
- Paid Losses
- Claim Counts
- Allocated Expenses

Exposure period may be:

- Accident Year
- Policy Year
- Underwriting Year
- Report Year

Evaluation intervals may be:

- Yearly
- Quarterly
- Monthly

Of course, the above list is just illustrative; it is far from complete.

In most applications, the actuary may be working with several forms of historical data, but only a single exposure period and a single evaluation interval. In this situation, it is quite common to work with the empirical ratios as calculated. However, there are some situations where adjustments to this procedure are required. Examples of such situations include:

- Mismatched Evaluation Dates
- Tail Factors
- Exposure Period Conversion
- Evaluation Interval Conversion
- Rapid Exposure Growth

Mismatched Evaluation Dates

The evaluation dates available may not match the evaluation dates required or the data may consist of two or more subgroups with differing evaluation dates. This might occur if the anniversary date for a policy has been changed.

One way of handling this situation is interpolation. Alternatively, it may be preferable to fit a function to the development data. Several articles have discussed various functional forms, including Sherman [8] and McClenahan [5].

Tail Factors

Another reason for fitting a function to development data is to smooth erratic data points, or to project tail factors beyond the oldest available evaluation date.

Exposure Period Conversion

If the only available development factors are on an accident year basis, and the data you wish to apply it to is on a policy year basis, some conversion must be made.

Typically, the actuary adjusts the evaluation ages by the difference in average accident dates. For example, the average accident date for an accident year is six months after the beginning of the period. The average accident date for a policy year is twelve months after the beginning of the period. Thus, an accident year evaluated at 24 months is 18 months after the average accident date, which converts to 30 months after the beginning of a policy year. This conversion technique works reasonably well for evaluation dates beyond 24 months, but is less accurate at earlier periods, and is grossly inaccurate at ages less than 12 months.

Evaluation Interval Conversion

Data may be available on an accident year basis but needed on an accident quarter basis. The typical method of adjustment is similar to the previous paragraph, with similar problems at early evaluate dates.

Rapid Exposure Growth

If the exposures for a block of business have been growing rapidly, then development factors from another block of business with more even growth will not be applicable. One method of dealing with this has been described by Simon [9].

APPLICATION TO REINSURANCE

Each of the above reasons for adjustments to development factors occurs to some extent in primary insurance, but these reasons are much more important in reinsurance. In a typical reinsurance reserve review, the individual body of data being evaluated is not large enough to deserve 100% credibility. External development factors may be needed. In many cases, available external data may have different evaluation dates, or be of a different form. Typically, factors are needed on an underwriting year basis, while available factors are on an accident year or policy year basis.

The extremely long tail of reinsurance makes it more likely that tail factors will have to be estimated. The high volatility of reinsurance development, particularly excess covers, makes it more likely that a smoothing technique, such as fitting development factors to a functional form, will be needed.

Finally, it is common to have books of business that have grown very rapidly. More recently, we may have to deal with books of business which have decreased very rapidly.

For these reasons it is extremely necessary to have procedures to make adjustments to development factors. In this section, I will describe a procedure which can be extended to cover all of the above adjustments.

Underlying Adjustments

Many of the adjustments implicitly assume that a period such as an accident year can be approximated by a single point in time, typically the midpoint. Most of the functional forms used to fit development factors do not explicitly recognize that they are being fit to a period of time, rather than a point in time. An accident year of twelve months does not mean that every point in the accident year is twelve months old. The first point is twelve months old, the most recent point is zero months old, and the intermediate points are at various ages between zero and twelve.

Suppose that the development of a single point follows a particular functional form. It actually makes more sense that an exposure point would have a simple functional form than an exposure period. For example, growth within an exposure period includes the addition of new accident months as well as reevaluations of prior months. Even after the end of a period, the development contains diverse elements. For example, the 13th month of development of an accident year includes the 13th actual month of development on the first month, as well as the second month of development of the last month of the accident year.

Accident Year

Assume that development of a single exposure point can be described by a function F(t). For convenience, we will express all development factors as proportions of ultimate losses (or counts). If we represent typical age-to-ultimate factors as d_i, we will work with $p_i=1/d_i$. The development pattern of any exposure period can be expressed as an integral of F(t). In some cases, an exposure weight function will be required. For example, an accident year evaluated at age t ($t>1$) includes individual exposure points evaluated at ages ranging from t-1 to t. If we assume even exposures over the period, we can write the formula for accident year development as:

$$G(t) = \int_{t-1}^{t} F(z)dz \qquad t>1 \qquad (1)$$

If we require development factors for evaluation dates before the end of the period, the calculation is straightforward:

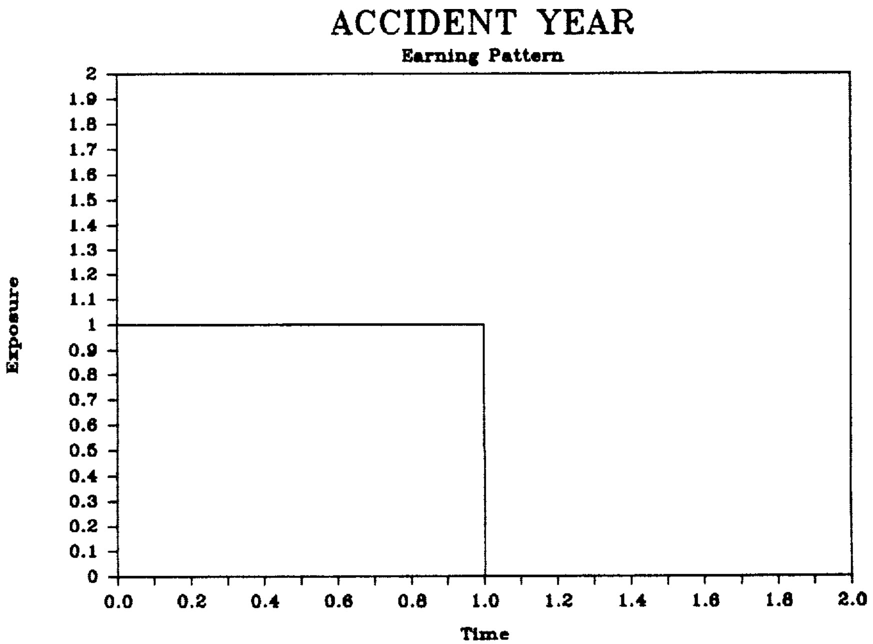
$$G(t) = \frac{\int_{0}^{t} F(z)dz}{t} \qquad t<1 \qquad (2)$$

Note that most other procedures do not work well at all in this situation. Also note carefully that the development factor does not represent the proportion of the entire period which has been reported (paid, etc.) at that point. For example, if we are interested in an accident year evaluated at six months, G(.5) represents only the proportion of the half-accident year. The percentage should be divided by two (or the corresponding age-to-ultimate factor should be doubled) to represent the correct proportion of the full year.

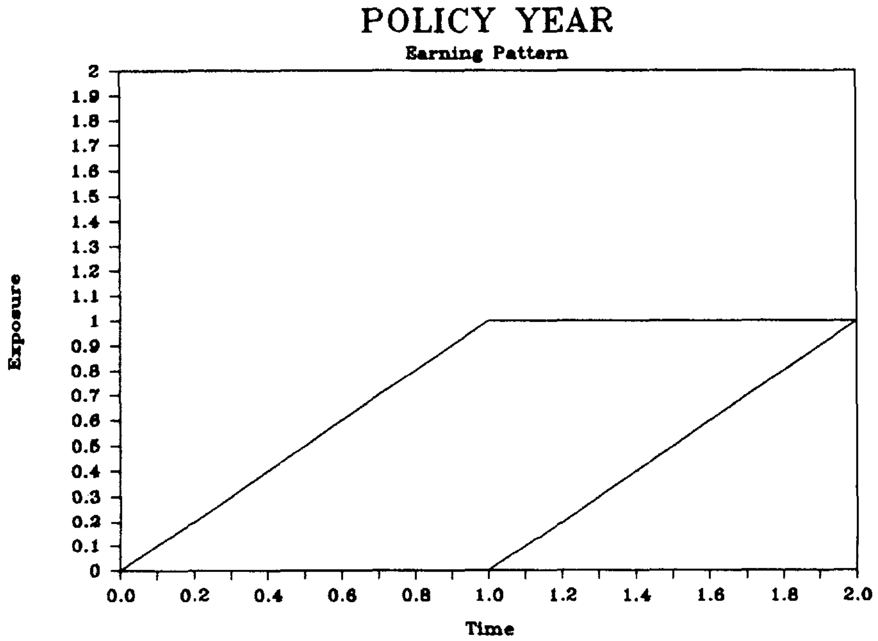
(We could rewrite the lower limit as Max (t-1, 0), the denominator as Min (t,1) and thereby use only a single equation. The reader may choose whichever notation they prefer.)

Policy Year

When we are interested in policy years, we have to introduce additional notation. Recall that an accident year can be represented as a rectangle of exposures, and a policy year can be represented as a parallelogram. Assume that we have one-year policies written evenly throughout the year. We can represent accident year exposures as:



We can represent policy year exposures as:



In the case of a single policy year, the exposure (represented by the earnings in each calendar unit of time) is not uniform over time. We can represent the exposure in each calendar point in time with the following:

$$w(t) = t \qquad 0 \le t \le 1 \tag{3a}$$

$$w(t) = 2 - t \qquad 1 < t \le 2 \tag{3b}$$

We can then write the formula for development factors for a policy year as:

$$G(t) = \int_{t-2}^{t} w(t-z)\ F(z)dz \qquad t \ge 2 \tag{4}$$

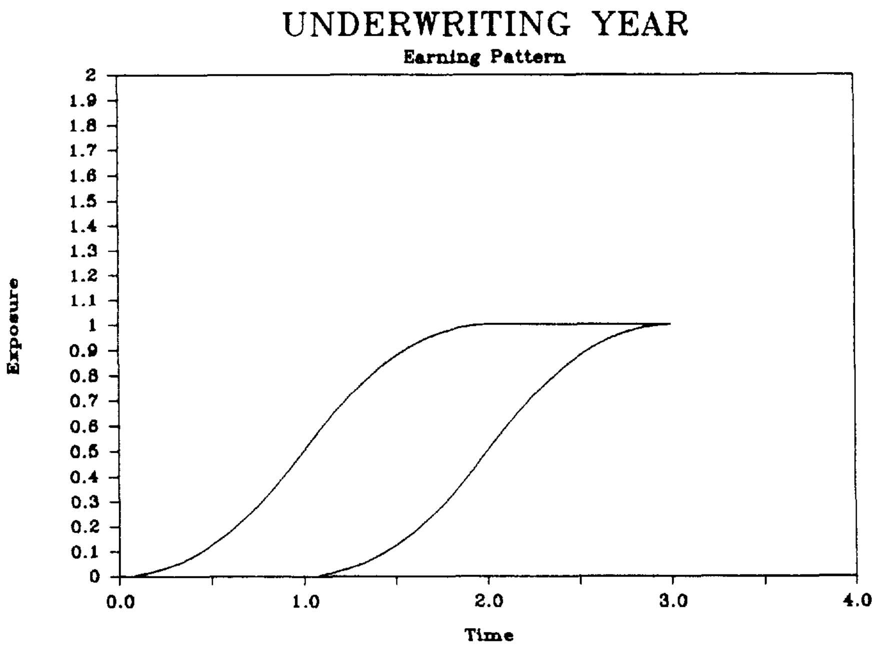
When we wish to evaluate a policy year before the end of the "year" (that is, prior to 24 months), we can use the following:

$$G(t) = \frac{\int_{0}^{t} w(t-z)\ F(z)dz}{\int_{0}^{t} w(t-z)dz} \qquad t < 2 \tag{5}$$

Note the expression in the denominator. This expression technically belongs in the denominators of the other formulations, but it simplifies in most cases.

Underwriting Years

As discussed earlier, an underwriting year consists of the exposures covered by treaties issued during a year. If we assume that treaties are written evenly throughout the year, and the treaties are written on a risks attaching basis (with underlying policies written evenly throughout the year), the pattern of earned exposures will look as follows:



Note that an underwriting year extends through three calendar years. We fully recognize that this pattern does not represent a "typical" underwriting year for several reasons:

- Treaties are not written evenly throughout the year. A large portion of all treaties incept January 1. Another large group incepts July 1. These two groups typically account for more than half of all treaties.

- Many treaties, especially excess covers, are on a "losses occurring" rather than a "risks attaching" basis.

- Many treaties cover business written by a reinsurance company, rather than a primary company. In this case, the earnings might extend more than three years after the beginning of the period.

In actual practice, the earnings pattern will have to be generated as a discrete distribution. The calculation of the development patterns will still be an integral, however, it may have to be evaluated using numerical methods. For purposes of illustration, we will use the above form for our underwriting year.

The development factors for an underwriting year can be calculated as follows:

$$G(t) = \int_{t-3}^{t} w(t-z)\, F(z)dz \qquad t \geq 3 \qquad (6)$$

$$G(t) = \frac{\int_{0}^{t} w(t-z)\, F(z)dz}{\int_{0}^{t} w(t-z)dz} \qquad t<3 \qquad (7)$$

In the case of this specific underwriting year, the exposure function can be written as follows:

$$w(t) = t^2/2 \qquad 0 \leq t \leq 1 \qquad (8a)$$

$$w(t) = -t^2 + 3t - 1.5 \qquad 1 \leq t \leq 2 \qquad (8b)$$

$$w(t) = t^2/2 - 3t + 4.5 \qquad 2 \leq t \leq 3 \qquad (8c)$$

Exposure Growth

Rapid growth in exposures (or any uneven exposure pattern) can be handled by this technique. The general form for accident year development factors is:

$$G(t) = \frac{\int_{t-1}^{t} w(t-z)\, F(z)dz}{\int_{t-1}^{t} w(t-z)\, dz} \qquad t \geq 1 \qquad (9)$$

$$G(t) = \frac{\int_{0}^{t} w(t-z)\, F(z)dz}{\int_{0}^{t} w(t-z)dz} \qquad t<1 \qquad (10)$$

If exposures are written evenly throughout the period, the function w(t) disappears. However, for uneven exposures, we simply specify the exposure growth as w(t) and integrate the more general form. If the exposure growth is a simple form, e.g. linear growth, we may be able to calculate G(t) analytically. Otherwise, we can express w(t) as a vector of exposure weights and use numerical integration.

Application

The real advantage of this procedure is not that we have a consistent notation for expressing development factors. The advantage is that with this approach, we can make adjustments to development factors that are appropriate and consistent. For example, if we are given a particular set of development factors, say, accident year factors, we can solve for the underlying F(x) and then create development factors for policy year, underwriting year, accident quarter, etc. that are consistent with the original factors. This procedure will allow us to calculate factors for evaluation dates within the period, correcting interpolate and extrapolate, and adjust for situations where exposure growth is uneven.

In actual practice, it will be desirable to create computer programs which can quickly solve the necessary equations and create the required factors, but these programs should not be particularly difficult.

Example

Suppose we are given the accident year development factors in Appendix B. Assume that the form of the underlying development is:

$$F(x) = 1 - e^{ax} \tag{11}$$

The form of the development factors will be:

$$G(t) = 1 + e^{at}/a[e^{-a} - 1] \tag{12}$$

The derivation of this result is shown in Appendix A. We can estimate the value of "a" using the Newton-Raphson (also discussed in Appendix A). The resulting estimate will be a = .5. In practice, each development factor will produce a different estimate of "a". A leasts-squares technique may be used to select a single value, or some other judgmental weighting method may be used.

If we now desire policy year development factors, we perform the integration in Equation (4). We then substitute a = .5. The resulting policy year factors are shown in Appendix B. Similarly, if we desire underwriting year factors, we perform the integration in Equation (6). After substituting a = .5, the result will be the underwriting year development factors shown in Appendix B.

This methodology is particularly useful for reinsurance problems. "Industry" development factors such as those calculated from Best's data can be converted to an underwriting year basis.

One other factor should be considered in any actual application. There is typically an accounting lag between the time a primary company receives data, and the time it is reported to a reinsurer. The methodology outlined above does not specifically recognize this lag. If the lag can be quantified, it should be a reasonably straightforward exercise to adjust for accounting lag.

Acknowledgement

I'd like to thank Doug Collins and Jerry Miccolis for numerous helpful suggestions, and John Yonkunas for his help with the mathematical derivations in Appendix A. Any errors remain the responsibility of the author.

CALCULATION OF ACCIDENT YEAR DEVELOPMENT FACTORS

Assume

$$F(x) = 1 - e^{ax}$$

We wish to evaluate G(t) where

$$G(t) = \int_{t-1}^{t} F(x)dx \qquad (1)$$

Substituting, we have

$$G(t) = \int_{t-1}^{t} [1 - e^{ax}]dx \qquad (2)$$

$$G(t) = \left[x - \frac{e^{ax}}{a} \right] \Big|_{t-1}^{t}$$

$$G(t) = \left[t - \frac{e^{at}}{a} \right] - \left[(t-1 - \frac{e^{a(t-1)}}{a} \right]$$

$$G(t) = 1 + \frac{e^{a(t-1)}}{a} - \frac{e^{at}}{a}$$

$$G(t) = 1 + \frac{e^{at}}{a}[e^{-9} - 1] \qquad (3)$$

We suggest the use of Newton-Raphson iteration to solve for the parameter "a". Restating (3) we have

$$H(t) = 1 + \frac{e^{a(t-1)}}{a} [1-e^{a}] - G(t) = 0 \qquad (4)$$

Differentiating (4) with respect to "a" yields

$$H'(t) = \frac{(t-1)e^{a(t-1)}}{a}[1 - e^{a}] - \frac{e^{a(t-1)}}{a^{2}}[1 - e^{a}] - \frac{e^{a(t-1)}e^{a}}{a} = 0$$

Simplifying,

$$H'(t) = \frac{e^{a(t-1)}}{a} \times \left[1 - e^{a}\right] \times \left[(t-1) - \frac{1}{a} - \frac{e^{a}}{1 - e^{a}}\right]$$

$$= [G(t) - 1] \left[(t-1) - \frac{1}{a} - \frac{e^{a}}{1 - e^{a}}\right] \qquad (5)$$

The nth iteration of Newton-Raphson can be stated as follows:

$$a_{n} + 1 = a_{n} - \frac{H(t)}{H^{1}(t)} \qquad (6)$$

Where H(t) and H'(t) are evaluated at a_n.

Continuing and substituting into (6)

$$a_{n+1} = a_n \frac{1 + \frac{e^{a(t-1)}}{a} \times [1 - e^a] - G(t)}{\frac{e^{a(t-1)}}{a} \times [1 - e^a] \times \left((t-1) - \frac{1}{a} - \frac{e^a}{1 - e^a} \right)} \qquad (7)$$

Formula (7) can be used to converge to a solution for "a" for a given "t" and G(t).

HYPOTHETICAL LOSS DATA

	ACCIDENT YEAR		POLICY YEAR		UNDERWRITING YEAR	
Months	% of Ultimate	Development Factor	% of Ultimate	Development Factor	% of Ultimate	Development Factor
12	22.93%	4.360	9.25%	10.814	2.95%	33.864
24	53.26%	1.878	40.61%	2.463	25.84%	3.869
36	71.65%	1.396	63.98%	1.563	54.23%	1.844
48	82.80%	1.208	78.15%	1.280	72.24%	1.384
60	89.57%	1.116	86.75%	1.153	83.16%	1.202
72	93.67%	1.068	91.96%	1.087	89.79%	1.114
84	96.16%	1.040	95.12%	1.051	93.81%	1.066
96	97.67%	1.024	97.04%	1.030	96.24%	1.039
108	98.59%	1.014	98.21%	1.018	97.72%	1.023
120	99.14%	1.009	98.91%	1.011	98.62%	1.014
132	99.48%	1.005	99.34%	1.007	99.16%	1.008
144	99.69%	1.003	99.60%	1.004	99.49%	1.005
156	99.81%	1.002	99.76%	1.002	99.69%	1.003
168	99.88%	1.001	99.85%	1.001	99.81%	1.002
180	99.93%	1.001	99.91%	1.001	99.89%	1.001
192	99.96%	1.000	99.95%	1.001	99.93%	1.001
204	99.97%	1.000	99.97%	1.000	99.96%	1.000
216	99.98%	1.000	99.98%	1.000	99.97%	1.000
228	99.99%	1.000	99.99%	1.000	99.98%	1.000
240	99.99%	1.000	99.99%	1.000	99.99%	1.000

Bornhuetter-Ferguson
Technique for Reserving

In 1972, an article was published with the title "The Actuary and IBNR" authored by Ron Bornhuetter and Ron Ferguson. The article discussed a number of ideas, but is best known for a reserving technique which now bears their names.

This technique is most easily explained if we reexamine some of the basics underlying reserving theory.

BACKGROUND

If we take a particular accident year, for example 1981, and review the loss data at some later point such as 1984, a portion of the losses will be paid and a portion will be outstanding. The sum of the paid and outstanding case basis reserves we call the case incurred. In many lines of business, we don't assume that the case basis reserves represent the ultimate incurred losses. Additional reserves are set up to account for two reasons:

1. True IBNR - additional claims reported after the evaluation date.

2. Case basis development - case basis reserves are only an estimate of the final settlement values, and the actual value can vary up or down from the estimate. Evidence shows that it is typical to expect some upward development.

If we use older years which have matured enough to provide reasonable estimates of ultimate, we can calculate the ratios of the case basis incurred to the ultimate losses at various stages of development. These ratios can be used to estimate development patterns of development factors.

EXAMPLE

Suppose, at the time the 1981 accident year is priced, the estimated losses are $10,000,000. Furthermore, assume that development patterns are examined and we expect to have 60% of the total known by 48 months of development. In other words, at 12/31/84, we expect that case incurred will be $6,000,000. If the case incurred at 12/31/84 turns out to be $6,000,000, of course we will be even more convinced that our original estimate of $10,000,000 for ultimate losses is reasonable.

Suppose, however, that case incurred at 12/31/84 is only $3,000,000. What should we conclude about the ultimate incurred? There are three types of conclusions that we can reach:

1. We can conclude that the fact that actual results differ from expected results is not sufficient evidence to change our original estimates of ultimate losses. Our reasons for rejecting the indications of actual experience might include:

   a. The volatile nature of the particular line of business.

   b. Possible inaccuracies in the presumed development pattern.

   In this case we would select $10,000,000 as the ultimate incurred, and set reserves equal to this amount less paid-to-date.

2. We could conclude that the development factors determined by the development curve are the best indicator of ultimate losses. The estimate that 60% of ultimate would be case incurred at 48 months means that a development factor of 1.67 (1 : .60) is appropriate. Accordingly, we would multiply the case incurred of $3,000,000 by 1.67 yielding an estimated ultimate of $5,000,000.

Another way of looking at this approach is that our assumption that 60% of ultimate incurred will be known at 48 months means that we expect that 40%, or $4,000,000 will be unknown (IBNR) at 48 months. To summarize:

Estimated Ultimate	$10,000,000
Expected known at 48 months	6,000,000
Expected IBNR at 48 months	$ 4,000,000

The actual known losses at 48 months were $3,000,000 or 1/2 of the expected. The development factor approach is equivalent to assuming that the better than expected actual experience will continue on a proportional basis. That is, the IBNR will only be 1/2 of the original estimate. One-half of 4,000,000 is 2,000,000. So our new estimate of ultimate incurred is:

Actual case incurred to date	$3,000,000
Revised IBNR estimate	2,000,000
Ultimate Incurred	$5,000,000

This agrees with the number calculated using the development factor.

3. The third alternative is that we assume the development patterns are reasonable. The fact that actual losses are less than the expected at this point in time is due to the volatile nature of this line. Here, we do not assume that the better than expected experience will continue into the future. In other words, we still believe that our original estimate of IBNR which would be remaining at 48 months is still reasonable. However, we are willing to recognize the difference between the actual and expected experience for the case basis portion. The current estimate of ultimate incurred would be calculated as:

Actual case incurred to date	$3,000,000
Original IBNR estimate	4,000,000
Ultimate Incurred	$7,000,000

The method described in #3 above is an application of the Bornhuetter-Ferguson approach. It allows recognition of favorable (or unfavorable) experience without creating the wide swings which would occur with the rote application of development factors.

SUMMARY

In the situations described above, we have made the following assumptions:

- Expected losses based on pricing assumptions are \$10,000,000.

- Expected known losses (case basis) at 48 months of development are \$6,000,000, hence, expected IBNR at 48 months is \$4,000,000.

- Actual known losses at 48 months are \$3,000,000.

Under these assumptions, we can summarize the reserving techniques as follows:

	Pricing Assumption	Loss Ratio Method	Bornhuetter-Ferguson Method	Development Factor Method
Known Losses	6,000,000	3,000,000	3,000,000	3,000,000
IBNR	4,000,000	7,000,000	4,000,000	2,000,000
Ultimate Incurred	10,000,000	10,000,000	7,000,000	5,000,000

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TITLE: AN ANALYSIS OF EXCESS LOSS DEVELOPMENT

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ABSTRACT: There is very little information available regarding excess loss development despite its importance in excess of loss pricing and reserving. In this study, paid and reported excess loss development patterns are estimated at various retentions for certain casualty lines of business. The effects of allocated loss adjustment expense and policy limits on excess development are discussed. The pattern of change, as development progresses, of Pareto distributions fitted to casualty loss distributions was considered in developing curve fitting methods. A method is described for determining development factors by layer. Applications to excess loss pricing, loss reserving, and increased limits factors are mentioned.

I. INTRODUCTION

Loss development patterns for both reported and paid excess losses are of fundamental importance in excess of loss pricing as well as in estimating loss reserves for excess of loss insurance and reinsurance. Excess of loss reinsurance constitutes a major portion of the business written by reinsurers and is the area involving the greatest degree of independent pricing and reserving activity.

There is a paucity of published information regarding both reported and paid excess loss development. The Reinsurance Association of America publishes a study biennially of reported excess casualty loss development patterns for certain lines of business, based on data supplied by member companies. Incurred[1] loss development patterns for Automobile Liability, General Liability, Workers' Compensation and Medical Malpractice have been described in these studies. Certain of these lines of business have well over twenty years of significant reported excess loss development, indicating that excess reporting patterns vary significantly from first

[1] "Incurred" is used in this study to mean the same as reported, i.e. it excludes IBNR.

Note: Special thanks to ISO, which provided us with a great deal of data, and to Susan Greiff, Thomas Highet, Madelyn Esposito and Francine Leong who assisted in the data processing and compilation.

dollar reporting patterns. However, in that study, excess losses in various layers are all grouped together so the data does not indicate the development patterns by line for various individual layers. Since the data indicates that excess business generally exhibits much slower reporting than that normally associated with primary business, there appears to be a relationship between the layer for which business is written and the resulting development pattern. It is this relationship that we intend to analyze in this paper for both paid and reported losses. Applications to increased limits and excess of loss pricing are also noted.

The protracted development of excess losses reflected in the RAA study suggests that the development is not only caused by late reported claims and increases in the average reported loss per claim but also by changes in the shape of the size of loss distribution at successive maturities. Accordingly, we requested and received from the Insurance Services Office various data comprising size of loss distributions at successive maturities. Specifically, included in the data provided were size of loss distributions of incurred losses for policy year evaluations up to 99 months, or the latest evaluation, for policy years 1972 through 1982. This countrywide monoline data was provided separately for OL&T, M&C and Products with each size of loss distribution containing 118 intervals.

These size of loss distributions combine data from business written at different policy limits. Thus, the data includes losses censored at each of the policy limits. While no adjustments were made to this data, the implications of using combined limits data are discussed in Appendix B.

Finally, the treatment of allocated loss adjustment expense in these distributions should be mentioned. Losses were assigned to a given size of loss interval based on loss size (Pd + O/S) excluding allocated loss adjustment expenses. The total allocated loss adjustment expense associated with the losses in each interval was given separately. As loss adjustment expense is treated in different ways in excess reinsurance, the treatment of these expenses will be discussed further in the context of deriving excess development factors.

Size of loss distributions listing paid losses and outstanding losses separately, as well as paid and outstanding allocated loss adjustment expense separately, were also provided by ISO for OL&T and M&C. The latest valuation available with this policy year data was 63 months. The RAA study provides reported loss development data for over twenty years of development for general liability and other lines on an accident year basis.

II. INCURRED EXCESS LOSS DEVELOPMENT FACTORS

In this section, we will display and discuss the incurred excess loss development factors derived from the size of loss distributions.

In developing these factors, we adjusted the retentions for policy years prior to 1982 to recognize changing levels of average cost per occurrence. For policy year 1982, the retentions used were $10,000, $25,000, $50,000, $100,000, $250,000, $500,000 and $1,000,000. For prior policy years, these retentions were multiplied by relativities reflecting the average cost per occurrence for the given policy year relative to the average cost per occurrence for the 1982 year. Thus, the relativity for 1982 was 1.00, while for a prior policy year N, it was computed by multiplying the relativity for the policy year N+1 by the ratio of the average cost per occurrence for year N to the average cost per occurrence for year N+1, based on the latest available pair of reports at the same stage of development and excluding claims closed without payment. As the resulting deflated retentions did not correspond with endpoints of the 118 size of loss intervals, the closest possible endpoints were selected.

Allocated loss adjustment expense (ALAE) is handled in different ways in excess reinsurance contracts. The three most common treatments are as follows:

1) ALAE is added to the loss amount and the total is treated as one in determining coverage.

2) ALAE is assigned to an excess layer on a pro-rata basis. That is, the ratio that the excess portion of the pure loss bears to the total loss is applied to the total ALAE to determine the excess ALAE.

3) ALAE is not included in the coverage.

Separate sets of excess loss development factors were calculated to reflect each of the above treatments of ALAE. This was done as follows:

1) All ALAE on occurrences with loss greater than a given retention was included with the excess incurred losses associated with that retention.

2) The ALAE on occurrences with loss greater than a given retention was multiplied by the ratio of the excess losses associated with that retention to the total ground up losses for occurrences with loss greater than the retention.

3) Loss experience only was used.

A discussion of the degree of accuracy of these methods of assigning ALAE can be found in Appendix A.

The factors shown in the tables below are dollar weighted averages of the factors by policy year. The retentions shown are retentions on policy year 1982 level although they actually correspond to different retentions for different policy years. By estimating the factor for the increase in average cost per occurrence from policy year 1982 to accident year 1987, for example, one could bring the retentions to accident year 1987 level.

OL&T-BI - Excess Losses Plus ALAE

Retention	27-39	39-51	51-63	63-75	75-87	87-99
10,000	1.3356	1.1799	1.1056	1.0664	1.0710	1.0118
25,000	1.3849	1.2200	1.1402	1.0877	1.0909	1.0146
50,000	1.4055	1.2549	1.1764	1.1128	1.1134	1.0167
100,000	1.4021	1.2942	1.2168	1.1506	1.1424	1.0235
250,000	1.3512	1.3517	1.2963	1.2120	1.2015	1.0383
500,000	1.2742	1.3940	1.4080	1.2787	1.2626	1.0613
1,000,000	1.0688	1.3061	1.6135	1.3662	1.3534	1.1111

OL&T-BI - Excess Losses Plus Pro-Rata ALAE

Retention	27-39	39-51	51-63	63-75	75-87	87-99
10,000	1.3437	1.1870	1.1111	1.0695	1.0729	1.0127
25,000	1.3909	1.2291	1.1483	1.0926	1.0938	1.0160
50,000	1.4098	1.2655	1.1860	1.1189	1.1172	1.0191
100,000	1.4023	1.3070	1.2287	1.1573	1.1468	1.0264
250,000	1.3563	1.3611	1.3150	1.2180	1.2077	1.0446
500,000	1.2648	1.3957	1.4292	1.2838	1.2701	1.0684
1,000,000	1.0503	1.3501	1.6417	1.3731	1.3576	1.1182

OL&T-BI - Excess Losses Only

Retention	27-39	39-51	51-63	63-75	75-87	87-99
10,000	1.3451	1.1940	1.1181	1.0735	1.0737	1.0155
25,000	1.3955	1.2389	1.1578	1.0981	1.0943	1.0193
50,000	1.4148	1.2777	1.1963	1.1249	1.1176	1.0239
100,000	1.4107	1.3191	1.2404	1.1626	1.1474	1.0319
250,000	1.3689	1.3690	1.3277	1.2199	1.2067	1.0517
500,000	1.2753	1.3981	1.4340	1.2832	1.2663	1.0740
1,000,000	1.0316	1.3888	1.6258	1.3629	1.3504	1.1197

M&C-BI - Excess Losses Plus ALAE

Retention	27-39	39-51	51-63	63-75	75-87	87-99
10,000	1.6246	1.2630	1.1100	1.0401	1.0360	1.0267
25,000	1.6816	1.2974	1.1316	1.0513	1.0449	1.0319
50,000	1.7201	1.3280	1.1509	1.0642	1.0554	1.0382
100,000	1.7528	1.3583	1.1771	1.0788	1.0724	1.0491
250,000	1.7481	1.3775	1.2214	1.1008	1.1194	1.0782
500,000	1.6110	1.3845	1.2520	1.1340	1.1898	1.1192
1,000,000	1.4056	1.5619	1.2130	1.1942	1.4206	1.2383

M&C-BI - Excess Losses Plus Pro-Rata ALAE

Retention	27-39	39-51	51-63	63-75	75-87	87-99
10,000	1.6326	1.2682	1.1128	1.0414	1.0375	1.0274
25,000	1.6909	1.3044	1.1354	1.0531	1.0475	1.0332
50,000	1.7297	1.3353	1.1556	1.0660	1.0594	1.0401
100,000	1.7689	1.3654	1.1828	1.0811	1.0789	1.0525
250,000	1.7652	1.3862	1.2306	1.1049	1.1267	1.0826
500,000	1.6093	1.4190	1.2534	1.1372	1.1993	1.1264
1,000,000	1.4064	1.5551	1.1934	1.1901	1.4891	1.2350

M&C-BI - Excess Losses Only

Retention	27-39	39-51	51-63	63-75	75-87	87-99
10,000	1.6294	1.2690	1.1136	1.0410	1.0410	1.0285
25,000	1.6933	1.3090	1.1367	1.0533	1.0519	1.0349
50,000	1.7368	1.3418	1.1587	1.0659	1.0649	1.0423
100,000	1.7835	1.3723	1.1871	1.0814	1.0858	1.0551
250,000	1.7878	1.3927	1.2346	1.1070	1.1300	1.0839
500,000	1.6334	1.4367	1.2555	1.1372	1.2014	1.1250
1,000,000	1.4010	1.5516	1.1970	1.1846	1.5060	1.2276

PRODUCTS-BI - Excess Losses Plus ALAE

Retention	27-39	39-51	51-63	63-75	75-87	87-99
10,000	1.7891	1.2906	1.1276	1.0632	1.0800	1.0293
25,000	1.9089	1.3561	1.1501	1.0776	1.0932	1.0369
50,000	1.9563	1.3844	1.1736	1.0928	1.1058	1.0405
100,000	2.0207	1.4221	1.1993	1.1165	1.1165	1.0421
250,000	2.1053	1.4790	1.2301	1.1453	1.0944	1.0440
500,000	2.3936	1.5098	1.4073	1.1660	1.1180	0.9605
1,000,000	1.8026	1.5847	1.9141	1.2074	1.2271	0.7657

PRODUCTS-BI - Excess Losses Plus Pro-Rata ALAE

Retention	27-39	39-51	51-63	63-75	75-87	87-99
10,000	1.7995	1.3065	1.1302	1.0653	1.0812	1.0311
25,000	1.8940	1.3571	1.1538	1.0805	1.0939	1.0398
50,000	1.9255	1.3847	1.1777	1.0961	1.1053	1.0443
100,000	1.9550	1.4214	1.2041	1.1203	1.1135	1.0456
250,000	1.9284	1.4790	1.2514	1.1494	1.0924	1.0302
500,000	2.1034	1.5104	1.4556	1.1520	1.1271	0.9303
1,000,000	1.7797	1.5970	1.9188	1.2199	1.2676	0.7245

PRODUCTS - Excess Losses Only

Retention	27-39	39-51	51-63	63-75	75-87	87-99
10,000	1.7291	1.2966	1.1266	1.0663	1.0758	1.0403
25,000	1.8118	1.3416	1.1505	1.0810	1.0885	1.0483
50,000	1.8340	1.3699	1.1752	1.0969	1.0993	1.0536
100,000	1.8344	1.4096	1.2034	1.1199	1.1081	1.0546
250,000	1.7100	1.4690	1.2601	1.1528	1.0942	1.0252
500,000	1.5748	1.5052	1.4556	1.1485	1.1267	0.9242
1,000,000	1.4736	1.5162	1.9311	1.2105	1.2719	0.7226

A review of the factors will show that the development is not materially affected after 39 months by the treatment of allocated loss adjustment expense. Therefore, future discussion will only deal with the case in which ALAE is included in the limit. This is probably the most common treatment in reinsurance, and corresponds to the factors for excess losses plus ALAE. It is also clear from these factors that the development increases as the retention increases. Some exceptions to this trend occur at retentions of $500,000 and $1,000,000 for individual stages of development. This is most likely due to the fact that there is a lesser amount of data at these retentions which increases the variability of the factors. Despite the exceptions, these higher retentions tend to have the largest development factors.

The excess development factors shown were all derived directly from the underlying size of loss distributions. We now use these factors to estimate curves which, in addition to smoothing the underlying factors, will generate excess development factors beyond 99 months as well as for retentions other than those previously treated. This would be necessary for computing development factors at policy year 1982 retentions which are equivalent to various retentions at accident year 1987 level, for example.

First curves are estimated to fit the excess loss development factors as functions of the retention at various stages of development. These results are then used to produce a smoothly progressing series of curves. The procedure is done separately for each line of business.

The curve selected to fit the excess development factors as a function of retention was $y = ax^b$ where x is the retention expressed as a multiple of $10,000. Thus, a is the value given by the curve for development excess of $10,000.

The use of this function was motivated by the qualities of the single parameter Pareto distribution used to model size of loss distributions. This is discussed further in Section IV.

Separate curves of the form $y=ax^b$ were fit to the excess loss development factors by retention for the following intervals of development:

27 mo. - 39 mo.

27 mo. - 51 mo.

27 mo. - 63 mo.

27 mo. - 75 mo.

27 mo. - 87 mo.

27 mo. - 99 mo.

These intervals were used rather than individual successive intervals of development in order to stabilize the curve fitting process. Also, for similar reasons, only retentions up to $250,000 were used.

The a and b values were determined from the data points x,y by fitting the values of log y and log x to a least squares line which gives:

$$\log y = \log a + b \log x$$

Thus, values for a and b were determined for each of the development intervals listed. These values were then separately fit to curves as a function of the stage of development. The method is illustrated in the exhibit below for the a values for M & C-BI.

	27-39	27-51	27-63	27-75	27-87	27-99
a values-actual	1.6401	2.0770	2.2928	2.3764	2.4395	2.4879

	27-39	39-51	51-63	63-75	75-87	87-99
a values-actual	1.6401	1.2664	1.1039	1.0365	1.0266	1.0198
(a-1)values-actual	.6401	.2664	.1039	.0365	.0266	.0198
(a-1)values-fitted		.2566	.0948	.0468	.0270	.0173

	27-39	27-51	27-63	27-75	27-87	27-99
a values-fitted	1.6401	2.0610	2.2564	2.3620	2.4258	2.4678

Thus, it is actually the values of (a-1) that are fitted to the curve $y = cx^d$ to obtain the fitted a values. Sherman[2] recommends this type of approach for fitting loss development factors. The same procedure is used to obtain fitted b values. The formulation chosen to determine fitted values of a and b dictates the nature of the tail beyond 99 months. In a few cases, an adjustment was made to an a or b value to produce a better fitting curve. The resulting fitted excess development factors by retention through 363 months of development are shown by line on the following exhibits. The corresponding actual factors derived from the data are shown at the bottom of each exhibit.

[2]Richard E. Sherman, "Extrapolating, Smoothing, and Interpolating Development Factors", PCAS, Volume LXXI, 1984, p. 123.

OL&T-BI Excess Loss & ALAE development factors

Fitted Factors

	Fitted b (values)	10,000*	25,000	50,000	100,000	250,000	500,000	1,000,000
27 - 39	.01000	1.36556	1.37813	1.38771	1.39736	1.41023	1.42004	1.42991
39 - 51	.03986	1.15206	1.19492	1.22839	1.26281	1.30978	1.34647	1.38420
51 - 63	.05066	1.08024	1.13157	1.17202	1.21390	1.27158	1.31703	1.36410
63 - 75	.03873	1.05099	1.08895	1.11858	1.14901	1.19051	1.22290	1.25617
75 - 87	.02528	1.03587	1.06014	1.07889	1.09796	1.12370	1.14356	1.16378
87 - 99	.01616	1.02691	1.04222	1.05396	1.06583	1.08173	1.09391	1.10623
99 - 111	.01055	1.02110	1.03102	1.03859	1.04622	1.05638	1.06414	1.07195
111 - 123	.00712	1.01710	1.02375	1.02882	1.03391	1.04068	1.04583	1.05100
123 - 135	.00497	1.01420	1.01882	1.02234	1.02586	1.03054	1.03409	1.03766
135 - 147	.00357	1.01203	1.01534	1.01785	1.02037	1.02372	1.02625	1.02879
147 - 159	.00263	1.01035	1.01279	1.01464	1.01649	1.01895	1.02081	1.02267
159 - 171	.00199	1.00902	1.01086	1.01226	1.01365	1.01550	1.01690	1.01830
171 - 183	.00153	1.00795	1.00937	1.01044	1.01152	1.01294	1.01401	1.01509
183 - 195	.00120	1.00708	1.00819	1.00903	1.00987	1.01098	1.01182	1.01267
195 - 207	.00096	1.00635	1.00723	1.00790	1.00857	1.00946	1.01013	1.01080
207 - 219	.00077	1.00573	1.00645	1.00699	1.00753	1.00824	1.00878	1.00933
219 - 231	.00063	1.00521	1.00579	1.00624	1.00668	1.00726	1.00770	1.00815
231 - 243	.00052	1.00476	1.00524	1.00561	1.00597	1.00646	1.00682	1.00719
243 - 255	.00044	1.00437	1.00477	1.00508	1.00538	1.00579	1.00609	1.00640
255 - 267	.00037	1.00403	1.00437	1.00463	1.00489	1.00523	1.00548	1.00574
267 - 279	.00031	1.00373	1.00402	1.00424	1.00446	1.00475	1.00497	1.00518
279 - 291	.00027	1.00347	1.00372	1.00390	1.00409	1.00434	1.00452	1.00471
291 - 303	.00023	1.00323	1.00345	1.00361	1.00377	1.00398	1.00414	1.00430
303 - 315	.00020	1.00302	1.00321	1.00335	1.00349	1.00367	1.00381	1.00395
315 - 327	.00018	1.00284	1.00300	1.00312	1.00324	1.00340	1.00352	1.00365
327 - 339	.00015	1.00267	1.00281	1.00291	1.00302	1.00316	1.00327	1.00338
339 - 351	.00014	1.00251	1.00264	1.00273	1.00282	1.00295	1.00304	1.00314
351 - 363	.00012	1.00237	1.00248	1.00257	1.00265	1.00276	1.00284	1.00293

Actual Factors

27 - 39	1.33560	1.38490	1.40550	1.40210	1.35120	1.27420	1.06880
39 - 51	1.17990	1.22000	1.25490	1.29420	1.35170	1.39400	1.30610
51 - 63	1.10560	1.14020	1.17640	1.21680	1.29630	1.40800	1.61350
63 - 75	1.06640	1.08770	1.11280	1.15060	1.21200	1.27870	1.36620
75 - 87	1.07100	1.09090	1.11340	1.14240	1.20150	1.26260	1.35340
87 - 99	1.01180	1.01460	1.01670	1.02350	1.03830	1.06130	1.11110

Cumulative Comparison

27 - 99 Actual	2.01300	2.31900	2.61400	2.97100	3.58000	4.28500	4.62700
27 - 99 Fitted	1.90000	2.24200	2.54100	2.88000	3.39900	3.85200	4.36600

* These equal the fitted a values.

M&C-BI Excess Loss & ALAE development factors

Fitted Factors

	Fitted b (values)	10,000*	25,000	50,000	100,000	250,000	500,000	1,000,000
27 - 39	.02402	1.64008	1.67658	1.70472	1.73334	1.77190	1.80165	1.83189
39 - 51	.02784	1.25665	1.28913	1.31425	1.33986	1.37449	1.40127	1.42858
51 - 63	.02666	1.09481	1.12188	1.14280	1.16412	1.19290	1.21515	1.23781
63 - 75	.02266	1.04677	1.06874	1.08566	1.10285	1.12599	1.14382	1.16193
75 - 87	.01867	1.02704	1.04476	1.05836	1.07214	1.09064	1.10484	1.11923
87 - 99	.01534	1.01728	1.03168	1.04270	1.05385	1.06876	1.08018	1.09173
99 - 111	.01270	1.01183	1.02367	1.03272	1.04185	1.05405	1.06337	1.07277
111 - 123	.01063	1.00852	1.01839	1.02592	1.03351	1.04362	1.05133	1.05911
123 - 135	.00899	1.00638	1.01471	1.02106	1.02744	1.03594	1.04242	1.04894
135 - 147	.00769	1.00493	1.01204	1.01745	1.02289	1.03012	1.03563	1.04117
147 - 159	.00665	1.00390	1.01003	1.01470	1.01938	1.02561	1.03034	1.03510
159 - 171	.00579	1.00315	1.00849	1.01255	1.01662	1.02203	1.02614	1.03027
171 - 183	.00509	1.00259	1.00728	1.01084	1.01441	1.01915	1.02276	1.02637
183 - 195	.00451	1.00216	1.00630	1.00945	1.01261	1.01680	1.01998	1.02317
195 - 207	.00402	1.00182	1.00551	1.00832	1.01113	1.01486	1.01769	1.02052
207 - 219	.00360	1.00155	1.00486	1.00737	1.00989	1.01323	1.01576	1.01830
219 - 231	.00325	1.00134	1.00432	1.00658	1.00885	1.01185	1.01413	1.01642
231 - 243	.00294	1.00116	1.00386	1.00591	1.00796	1.01068	1.01274	1.01481
243 - 255	.00267	1.00102	1.00347	1.00534	1.00720	1.00967	1.01155	1.01342
255 - 267	.00244	1.00090	1.00314	1.00484	1.00655	1.00880	1.01051	1.01223
267 - 279	.00224	1.00080	1.00285	1.00441	1.00597	1.00804	1.00961	1.01118
279 - 291	.00206	1.00071	1.00260	1.00404	1.00547	1.00738	1.00882	1.01026
291 - 303	.00190	1.00064	1.00238	1.00371	1.00503	1.00679	1.00812	1.00945
303 - 315	.00176	1.00057	1.00219	1.00342	1.00465	1.00627	1.00750	1.00873
315 - 327	.00164	1.00052	1.00202	1.00316	1.00430	1.00581	1.00695	1.00809
327 - 339	.00153	1.00047	1.00187	1.00293	1.00399	1.00539	1.00646	1.00752
339 - 351	.00142	1.00043	1.00174	1.00272	1.00371	1.00502	1.00602	1.00701
351 - 363	.00133	1.00039	1.00161	1.00254	1.00347	1.00469	1.00562	1.00655

Actual Factors

	10,000	25,000	50,000	100,000	250,000	500,000	1,000,000
27 - 39	1.62460	1.68160	1.72010	1.75280	1.74810	1.61100	1.40560
39 - 51	1.26300	1.29740	1.32800	1.35830	1.37750	1.38450	1.56190
51 - 63	1.11000	1.13160	1.15090	1.17710	1.22140	1.25200	1.21300
63 - 75	1.04010	1.05130	1.06420	1.07880	1.10080	1.13400	1.19420
75 - 87	1.03600	1.04490	1.05540	1.07240	1.11940	1.18980	1.42060
87 - 99	1.02670	1.03190	1.03820	1.04910	1.07820	1.11920	1.23830

Cumulative Comparison

	10,000	25,000	50,000	100,000	250,000	500,000	1,000,000
27 - 99 Actual	2.52000	2.79900	3.06600	3.40100	3.90800	4.21700	5.59400
27 - 99 Fitted	2.46900	2.79300	3.06800	3.36900	3.81300	4.18800	4.59900

* These equal the fitted a values.

Products-BI Excess Loss & ALAE development factors

Fitted Factors

	Fitted b (values)	10,000*	25,000	50,000	100,000	250,000	500,000	1,000,000
27 - 39	.04877	1.80564	1.88815	1.95307	2.02022	2.11254	2.18517	2.26030
39 - 51	.04373	1.27527	1.32740	1.36825	1.41036	1.46802	1.51320	1.55977
51 - 63	.02738	1.13277	1.16155	1.18381	1.20649	1.23715	1.26086	1.28502
63 - 75	.01617	1.07914	1.09525	1.10759	1.12007	1.13679	1.14960	1.16256
75 - 87	.00997	1.05298	1.06265	1.07002	1.07744	1.08733	1.09487	1.10246
87 - 99	.00650	1.03817	1.04438	1.04909	1.05383	1.06013	1.06492	1.06973
99 - 111	.00446	1.02893	1.03314	1.03634	1.03954	1.04380	1.04703	1.05027
111 - 123	.00318	1.02275	1.02574	1.02801	1.03028	1.03329	1.03557	1.03786
123 - 135	.00235	1.01841	1.02061	1.02228	1.02395	1.02616	1.02784	1.02951
135 - 147	.00179	1.01523	1.01690	1.01816	1.01943	1.02110	1.02237	1.02364
147 - 159	.00140	1.01283	1.01413	1.01511	1.01609	1.01739	1.01838	1.01937
159 - 171	.00111	1.01097	1.01200	1.01278	1.01356	1.01459	1.01537	1.01616
171 - 183	.00090	1.00950	1.01033	1.01096	1.01159	1.01242	1.01306	1.01369
183 - 195	.00074	1.00832	1.00900	1.00951	1.01003	1.01071	1.01123	1.01175
195 - 207	.00061	1.00735	1.00791	1.00834	1.00877	1.00934	1.00977	1.01020
207 - 219	.00052	1.00654	1.00702	1.00738	1.00774	1.00821	1.00858	1.00894
219 - 231	.00044	1.00587	1.00627	1.00658	1.00688	1.00729	1.00759	1.00790
231 - 243	.00038	1.00529	1.00564	1.00590	1.00616	1.00651	1.00677	1.00704
243 - 255	.00033	1.00480	1.00510	1.00533	1.00556	1.00585	1.00608	1.00631
255 - 267	.00028	1.00438	1.00464	1.00484	1.00503	1.00530	1.00549	1.00569
267 - 279	.00025	1.00401	1.00424	1.00441	1.00459	1.00481	1.00499	1.00516
279 - 291	.00022	1.00369	1.00389	1.00404	1.00420	1.00440	1.00455	1.00470
291 - 303	.00019	1.00341	1.00358	1.00372	1.00385	1.00403	1.00417	1.00430
303 - 315	.00017	1.00316	1.00331	1.00343	1.00355	1.00371	1.00383	1.00395
315 - 327	.00015	1.00293	1.00307	1.00318	1.00329	1.00343	1.00354	1.00365
327 - 339	.00014	1.00273	1.00286	1.00296	1.00305	1.00318	1.00328	1.00338
339 - 351	.00013	1.00255	1.00267	1.00276	1.00284	1.00296	1.00305	1.00313
351 - 363	.00011	1.00239	1.00250	1.00258	1.00265	1.00276	1.00284	1.00292

Actual Factors

27 - 39	1.78910	1.90890	1.95630	2.02070	2.10530	2.39360	1.80260
39 - 51	1.29060	1.35610	1.38440	1.42210	1.47900	1.50980	1.58470
51 - 63	1.12760	1.15010	1.17360	1.19930	1.23010	1.40730	1.91410
63 - 75	1.06320	1.07760	1.09280	1.11650	1.14530	1.16600	1.20740
75 - 87	1.08000	1.09320	1.10580	1.11650	1.09440	1.11800	1.22710
87 - 99	1.02930	1.03690	1.04050	1.04210	1.04400	.96050	.76570

Cumulative Comparison

27 - 99 Actual	3.07700	3.63700	3.99600	4.47700	5.01200	6.36800	6.20300
27 - 99 Fitted	3.07700	3.53900	3.93300	4.37200	5.02800	5.58800	6.21100

* These equal the fitted a values

Corresponding to the previously described method used to determine these fitted factors, the formulas for excess development factors as a function of retention are as follows. The development factors from 27 to 39 months for retentions of 10,000 x, for $x \geq 1$, were calculated using the original ax^b which was fitted to that development interval.

(OL&T-BI: a=1.3656, b=.01; M&C-BI: a=1.64008, b=.02402; Products-BI: a=1.80564, b=.04877).

For development from 27 + 12(n-1) to 27 + 12(n) months, for $n \geq 2$, $x \geq 1$, the formulas for the factors for retentions of 10,000 x follow. (We use the convention that $\prod_{y=2}^{1} F(y)=1$.)

<u>O,L&T-BI</u>

$$(1+.454n^{-1.576})\ x^{.01}\left(\prod_{y=2}^{n}(1+41.243y^{-3.371}) - \prod_{y=2}^{n-1}(1+41.243y^{-3.371})\right)$$

<u>M&C-BI</u>

$$(1+1.408n^{-2.456})\ x^{.02402}\left(\prod_{y=2}^{n}(1+4.657y^{-2.006}) - \prod_{y=2}^{n-1}(1+4.657y^{-2.006})\right)$$

<u>Products-BI</u>

$$(1+.957n^{-1.798})\ x^{.04877}\left(\prod_{y=2}^{n}(1+5.962y^{-2.733}) - \prod_{y=2}^{n-1}(1+5.962y^{-2.733})\right)$$

A simple method for converting policy year development factors to approximately equivalent accident year development factors is based on the fact that for a policy year as of 27 months the time elapsed since the average accident date is 15 months, and for an accident year as of 21 months the average time elapsed is 15 months. A policy year development factor from 27 + 12n to 27 + 12(n+1) months, for $n \geq 0$, can be estimated to be equivalent to an accident year development factor from 21 + 12n to 21 + 12(n+1) months. Accident year development factors from 24 + 12n to 24 + 12(n+1) months could then be estimated by linear interpolation or by fitting an exponential curve to the excess over one of the two adjacent factors.

Although application of calculus would yield more refined results, the accuracy of this approach improves rapidly after the estimated 24-36 month accident year factor.

As has been mentioned, the RAA Loss Development Study combines business written at various retentions. The subline mix underlying the 'General Liability Excluding Asbestos' experience is also difficult to estimate. For these reasons, as well as the fact that the RAA experience is accident year, it is difficult to make a precise comparison of our results with those of the RAA. Nevertheless, a rough comparison follows based on the following choices:

1) A retention of $250,000 is used to reflect the development characteristics of the various retentions underlying the RAA experience.

2) An equal weighting of the excess loss development factors for OL&T, M&C and Products is used to reflect the subline mix of the RAA data.

3) A weighting of 25% of the accident year factor from 12 + 12k months to 12 + 12(k+1) months and 75% of the accident year factor from 12 + 12(k+1) months to 12 + 12(k+2) months was used to estimate the policy year factor from 27 + 12k months to 27 + 12(k+1) months.

4) Dollar weighted factors are derived using the most recent five years of RAA experience.

Development Interval	Fitted ISO Data Excess $250,000	RAA
27-39	1.765	1.801
39-51	1.384	1.392
51-63	1.234	1.242
63-75	1.151	1.153
75-87	1.101	1.097
87-99	1.070	1.072
99-111	1.051	1.067
111-123	1.039	1.049
123-135	1.031	1.038
135-147	1.025	1.038
147-159	1.021	1.030
159-171	1.017	1.029
171-183	1.015	1.036
183-Ult.	1.105	1.228

The RAA data begins to show higher developments than the ISO data after 99 months. This could be due to the effects of reinsurance coverage on an aggregate basis showing up later in the development. Also, unidentified longer tailed medical malpractice losses may be present in the RAA data.

Commercial Auto Liability

The Commercial Auto Liability study was based on a total of almost $4 billion in losses from accident years 1980, 1981 and 1982. These were the only years available to us and our study is of the only available development factors: 21 to 33, 33 to 45, and 45 to 57 months.

The development factors for losses plus ALAE excess of various retentions (on accident year 1982 level) are:

Retention	21-33	33-45	45-57	21-57	33-57
- 0 -	1.084	1.031	1.011	1.130	1.042
10,000	1.137	1.044	1.012	1.201	1.057
25,000	1.152	1.050	1.014	1.227	1.065
50,000	1.159	1.053	1.016	1.240	1.070
100,000	1.172	1.058	1.013	1.256	1.072
250,000	1.177	1.030	1.043	1.264	1.074
500,000	1.444	.949	1.168	1.601	1.108

A pattern of increasing development with increasing retentions can be observed, especially in the 21-57 month factors. The factors for the $500,000 retention have limited credibility. Due to the small change in development factors from one retention to another, no curve fitting was performed.

The breakdown of premium by policy limits for accident year 1982 can be approximated at 5% at $100,000, 15% at $300,000, 60% at $500,000, and 20% at $750,000 or $1,000,000.

Accident year development factors for excess losses based on a weighted average of Reinsurance Association of America development data for the last five years for auto liabilty are:

12-24	24-36	36-48	48-60	60-72	72-84	84-ultimate
1,804	1.204	1.093	1.062	1.052	1.026	1.076

III. Excess Paid Loss & ALAE Development

In this section, ratios of excess paid losses and ALAE to excess incurred losses and ALAE were determined at policy year valuations from 27 months to ultimate for OL&T-BI and M&C-BI. (Sufficient data was not available for Products - BI). These ratios of paid to reported, in conjunction with excess incurred loss and ALAE development, will produce excess paid loss and ALAE development factors.

The procedure previously discussed which was used in developing excess incurred losses and ALAE by retention at various valuations was used for both paid and reported losses and ALAE from 27 months to 63 months of development. The resulting ratios of paid to reported are shown below for policy year 1982 cost levels.

OL&T - BI

Ratio of Paid to Reported Excess Loss and ALAE

Retention	27 mo.	39 mo.	51 mo.	63 mo.
10,000	.1937	.3587	.5041	.6356
25,000	.1616	.3217	.4634	.5964
50,000	.1518	.3080	.4469	.5754
100,000	.1585	.3210	.4519	.5838
250,000	.1852	.3616	.4919	.5640
500,000	.2269	.3103	.5106	.4205

M & C - BI

Ratio of Paid to Reported Excess Loss and ALAE

Retention	27 mo.	39 mo.	51 mo.	63 mo.
10,000	.1417	.2427	.4098	.5350
25,000	.1425	.2358	.4069	.5294
50,000	.1526	.2364	.4054	.5233
100,000	.1751	.2473	.4142	.5279
250,000	.2312	.2924	.4464	.5094
500,000	.2209	.3586	.4285	.4794

It appears that the paid to reported ratios shown for excess loss and ALAE do not vary meaningfully as a function of the retention. Accordingly, we selected the paid to reported ratios for loss and ALAE excess of $25,000 as characteristic of the various retentions shown in producing a development pattern of paid to reported ratios. It should be noted that ground up losses exhibit significantly higher paid to reported ratios than those shown for the retentions above.

The following ISO excess of $25,000 loss development data was available beyond 63 months for loss and ALAE combined.

O,L&T-BI

	(1) Excess Paid to Reported	(2) Excess Outstanding to Reported	(3) Ratio of (2) to Prior Valuation
63	.5710	.4290	-
75	.6809	.3191	.7438
87	.7768	.2232	.6995
99	.8717	.1283	.5748

M&C-BI

	(1) Excess Paid to Reported	(2) Excess Outstanding to Reported	(3) Ratio of (2) to Prior Valuation
63	.5660	.4340	-
75	.7091	.2909	.6703
87	.8019	.1981	.6810
99	.8680	.1320	.6663

In light of the column (3) ratios, and the fact that the paid to reported ratio will ultimately reach one, a factor of .67 was selected to be repeatedly applied to the outstanding to reported ratios at 63 months. The resulting patterns of paid to reported excess loss and ALAE are as follows:

Ratios of Paid to Reported Excess Loss and ALAE

OL & T BI		M & C BI	
Valuation	Ratio	Valuation	Ratio
27	.1616	27	.1425
39	.3217	39	.2358
51	.4634	51	.4069
63	.5964	63	.5294
75	.7296	75	.6847
87	.8188	87	.7887
99	.8786	99	.8585
111	.9187	111	.9052
123	.9455	123	.9365
135	.9635	135	.9574
147	.9755	147	.9715
159	.9836	159	.9809
171	.9890	171	.9872
183	.9926	183	.9914
ult.	1.0000	ult.	1.0000

Excess paid to reported ratios have been used thus far since they vary less by retention and valuation than paid development factors and they allow for the use of the more expensive reported data in estimating paid development. Excess paid loss and ALAE development factors can be determined simply by multiplying the ratio of paid to reported ratios at two valuations by the incurred loss development factor linking those same two valuations. For example, the estimated paid loss development factors for loss and ALAE excess of $100,000 are as follows:

OL & T BI		M & C BI	
27 - 39	2.7817	27 - 39	2.8682
39 - 51	1.8190	39 - 51	2.3121
51 - 63	1.5623	51 - 63	1.5146
63 - 75	1.4056	63 - 75	1.4264
75 - 87	1.2322	75 - 87	1.2351
87 - 99	1.1437	87 - 99	1.1470
99 - 111	1.0940	99 - 111	1.0985
111 - 123	1.0641	111 - 123	1.0692
123 - 135	1.0454	123 - 135	1.0504
135 - 147	1.0331	135 - 147	1.0379
147 - 159	1.0249	147 - 159	1.0293
159 - 171	1.0192	159 - 171	1.0232
171 - 183	1.0152	171 - 183	1.0188
183 - ult.	1.0872	183 - ult.	1.1152

IV. RELATION OF RESULTS TO THE SINGLE PARAMETER PARETO DISTRIBUTION

It has been seen that excess loss development increases as the retention increases. A perspective on this relationship and excess loss development in general can be obtained by considering a model which illustrates the two influences underlying loss development:

1) The reporting pattern of claims over time.

2) The changing characteristics of the size of loss distribution at successive reports.

Without the latter influence, the development factors for losses excess of different retentions would be identical.

It has been noted[3] that the single parameter Pareto distribution fits the tail of casualty loss distributions fairly well (at least if the interval of loss sizes is not too long), and that the parameter tends to decrease at successive stages of development.

If a series of Pareto distributions with parameters which are decreasing and greater than 1 were to perfectly represent a series of actual tails of loss distributions at successive development stages, the excess loss development factor from any stage m to

[3] See "A Practical Guide to the Single Parameter Pareto Distribution", by Stephen W. Philbrick, and the discussion by Kurt A. Reichle and John P. Yonkunas. Presented at May, 1985 CAS Meeting.

stage m + n (n > 0) for retention x (where x is big enough to be included in the tail) would increase as x increased, since it equals ax^b for some fixed a > 0 and b > 0. The proof follows. If k is the lower bound of the tail which is represented by a Pareto distribution with parameter q, and x represents the size of loss divided by k, then the density function $qx^{-(q+1)}$, as x ranges from 1 to infinity, represents the "normalized" (i.e. divided by k) loss distribution. The probability of a loss greater than k being between ak and bk equals $\int_a^b qx^{-(q+1)}dx$, and the losses excess of a retention ck are $nk\int_c^\infty (x-c)qx^{-(q+1)}dx$, where n is the number of losses greater than k. If the distribution of losses greater than k at i^{th} report is represented by a Pareto with parameter q_i, and at j^{th} report (j>i) by a Pareto with parameter q_j, and the numbers of losses greater than k at i^{th} and j^{th} report are n_i and n_j, then the development factor for losses excess of ck from i^{th} to j^{th} report equals

$$\frac{n_j}{n_i}\left(\frac{q_i-1}{q_j-1}\right)c^{q_i-q_j}$$

Therefore, if d is the development factor from i^{th} to j^{th} report for losses excess of k, then $dy^{q_i-q_j}$ is the development factor for losses excess of yk (for y>1).

The development factor for losses excess of x, where x>k, is thus $d\left(\frac{x}{k}\right)^{q_i-q_j}$, which equals $\frac{d}{k^{q_i-q_j}}x^{q_i-q_j}$

and $\frac{d}{k^{q_i-q_j}} > 0$ and $q_i - q_j > 0$.

This completes the proof.

The term $\frac{n_j}{n_i}$ in the expression $\frac{n_j}{n_i}\left(\frac{q_i-1}{q_j-1}\right)c^{q_i-q_j}$

represents the development due to additional reportings greater than k. The term $\frac{q_i - 1}{q_j - 1}$ represents the development arising from the change in the average excess loss above ck for occurrences greater than ck. The term $c^{q_i - q_j}$ reflects the development arising from the increased proportion of occurrences greater than k which are also greater than ck, resulting from the changing shape of the distribution. It can be seen that $c^{q_i - q_j}$ is the only term affected by a change in the retention.

As an example, let:

k = the lower bound of the tail = 25,000

x = the primary retention = 100,000

q_1 = the Pareto parameter for 1st report tail losses = 1.75

q_{10} = the Pareto parameter for 10th report tail losses = 1.25

d = the 1st to 10th development factor for losses excess of 25,000 = 2.5

Then the 1st to 10th development factor for losses excess of 100,000 is given by the formula $d\left(\frac{x}{k}\right)^{q_i - q_j}$, i.e. $2.5\ (4)^{.5} = 5.0$.

It has been noted[4] that when a Pareto is fitted to a

[4]ibid.

distribution of casualty losses greater than some amount k, the tail of the Pareto is thicker than the tail of the empirical loss distribution at very large loss sizes. Nevertheless, the effect of this error may be mitigated somewhat in using a ratio to estimate a development factor. The fact that the Pareto provides a fairly good fit over reasonably long intervals suggests the suitability of the curve ax^b for determining excess loss development factors as a function of the retention x.

V. DEVELOPMENT FACTORS BY LAYER, EXCESS LOSS RATIOS, AND INCREASED LIMITS FACTORS

The following method is used to produce development factors by layer, where the layer of losses from a to b is defined as the total of the portions between a and b of every loss. By applying the excess loss development factors to ultimate to the latest available excess losses for each retention for each policy year, we get projected ultimate excess losses for each retention for each policy year. We also have "ground-up" development factors, based on the same data, with which we project ultimate ground-up losses for each policy year. The ground-up factors to ultimate are derived by fitting a curve $1+ax^b$ to the factors through 99 months. By taking weighted averages of the ratios of ultimate excess losses to ultimate ground-up losses for all policy years for the retentions (in 000's) 10,25,50,100,250,500, and 1000, we get ratios that we call f(10), f(25), f(50), f(100), f(250), f(500) and f(1000). An exponential curve could then be fit between any two successive data points to get intermediate values of f(x). This curve gives estimates of the ratios of ultimate excess losses to ultimate ground-up losses for each retention. In order to produce the n^{th} to ultimate development factor for the layer from c to d, we first divide the curve values f(c) and f(d) by the n^{th} to ultimate development factors for losses excess of c and d, respectively, to get estimates $e_{c,n}$ and $e_{d,n}$ of the ratios of n^{th} report excess losses, for retentions c and d, to ultimate ground-up losses.

We then let the development from n^{th} to ultimate for the layer from c to d equal $(f(c)-f(d)) \div (e_{c,n}-e_{d,n})$, i.e. the estimated ultimate excess losses in the layer divided by the n^{th} report excess losses in the layer. The n^{th} to $(n+1)^{st}$ development factor for a layer is produced by dividing the n^{th} to ultimate factor by the $(n+1)^{st}$ to ultimate factor.

The values of f(x) (x is in 000's) given by the data and derived development factors for losses and ALAE are:

	OL,&T BI	M&C BI	Products BI
f(10)	.677	.802	.835
f(25)	.579	.755	.735
f(50)	.484	.674	.617
f(100)	.372	.543	.463
f(250)	.240	.319	.243
f(500)	.144	.148	.125
f(1,000)	.076	.041	.032

The O,L&T development factors for 27 months to ultimate for retentions of (in 000's) 50, 100, 250, 500 and 1,000 are 3.150, 3.668, 4.485, 5.223 and 6.081 respectively. The factors for the layers 50-100, 50-250, 50-500, and 50-1,000, using the above method follow.

Layer (in 000's)	Method and Development Factor
50 - 1,000	(.484-.076)÷((.484÷3.150)-(.076÷6.081))=2.891
50 - 500	(.484-.144)÷((.484÷3.150)-(.144÷5.223))=2.697
50 - 250	(.484-.240)÷((.484÷3.150)-(.240÷4.485))=2.437
50 - 100	(.484-.372)÷((.484÷3.150)-(.372÷3.668))=2.144

As with our unlimited development factors by retention these factors for layers are somewhat lower than the factors would be for losses uncensored by policy limits. (See Appendix B.) Since about

80% of the losses are not censored by policy limits below $500,000, the factors produced by the above method are more accurate for layers whose upper bound does not exceed $500,000. The techniques of producing different development factors by retention or layer and projecting development to ultimate could be useful in estimating ultimate uncensored excess loss ratios, which are important in reinsurance pricing. The techniques could also be used in producing increased limits factors, which are an important part of primary insurance pricing. The actual development factors and data from this study concerning excess losses by layer could provide estimates of increased limits factors up to $100,000 or possibly $250,000 limits, since the policy limits in effect have little effect on the layer up to $100,000, or even $250,000. We do not present such estimates, however.

VI. SUMMARY

The results that have been produced indicate clearly that loss and ALAE development varies significantly by retention. Accordingly, pricing and reserving estimates incorporating development factors may be substantially in error if this is not taken into account. As this applies to paid as well as reported loss development, recognition of retention is also a major factor in estimating discounted losses using paid development factors.

The protracted development of excess losses and the data limitations inherent in this study suggest a need for further study of development factors beyond 99 months. It would also be beneficial to review development by retention for other lines of business such as Medical Malpractice and Workers' Compensation.

The results are closely related to the decrease in the Pareto parameter in successive reports, and its relationship to loss development by retention. The principles employed would have relevance for other lines for which the Pareto provides a good fit.

With sufficient data, it would be very worthwhile to study excess development for uncensored losses and for higher retentions than those examined here.

APPENDICES

A. TREATMENT OF ALAE IN ESTIMATING DEVELOPMENT FACTORS

The type of occurrence excess coverage which is most common in casualty treaty reinsurance covers the amount of the loss and allocated loss adjustment expense combined in excess of the retention for each occurrence. The method of estimating the development factors for this type of reinsurance, however, was based on the development of the amount of the loss and allocated loss adjustment expense combined in excess of the retention for only those occurrences for which the pure loss exceeded the retention.

The error involved in using this approach is relatively small since the amount in excess of any retention which is produced by the losses plus ALAE for all occurrences for which the losses alone are less than the retention is small compared to the total losses plus ALAE in excess of the retention. In other words, only a small portion of the excess is missing from our development factors.

Suppose, for example, that for every occurrence, the ratio of the loss to the loss plus ALAE is a. If the tail of the "normalized" (see section IV) loss distribution is represented by the Pareto density function $qx^{-(q+1)}$, with $q>1$, then the portion of the total losses plus ALAE in excess of the retention x_0 which is produced by occurrences for which the pure loss is greater than the retention equals

$$\int_{x_0}^{\infty} q x^{-(q+1)} \left(\frac{x}{a} - x_0\right) dx \div \int_{a x_0}^{\infty} q x^{-(q+1)} \left(\frac{x}{a} - x_0\right) dx$$

which equals $\frac{q + a - qa}{a^{1-q}}$

If q=1.5 and a=.87, for example, then the above expression equals .993.

If q=1.5 and a=.87 at first report and q=1.3 and a=.85 at ultimate report, then the expression changes from .993 to .995. In this case, the estimate of the first to ultimate development factor would be 1.002 times the development that would be computed using a precise treatment of ALAE.

This problem does not apply to the development factors for losses plus pro-rated ALAE, since occurrences with pure losses below the retention are not covered by reinsurance arrangements with pro-rated ALAE. Those factors involve a different estimate - use of losses excess of a retention divided by total losses for the occurrences greater than the retention - as a multiplier for the ALAE. To be precise, the ALAE for each occurrence should be multiplied by the loss excess of the retention divided by the total loss for that occurrence. The distortion in development factors should be small, even in the product of all the development factors. For each loss and corresponding ALAE, and each retention, pro-rated ALAE = (excess loss ÷ loss) ALAE so pro-rated ALAE ÷ excess loss = ALAE ÷ loss for each loss. Since the data indicated that ALAE ÷ loss is about .15 on the average, whatever distortion there is in the estimate of the pro-rated ALAE would cause less than .15 times as much distortion in losses plus pro-rated ALAE.

B. EFFECT OF POLICY LIMITS ON DEVELOPMENT FACTORS

The general liability sublines studied had the following policy limits distributions based on policy year 1982 and policy year 1983 data:

Distribution of Premium

Policy Limit (in 000's)	O,L & T - B.I.	M & C - B.I.	Products-B.I.
25	.0043	.0034	.0018
50	.0069	.0031	.0042
100	.0366	.0347	.0248
200	.0022	.0010	.0000
250	.0013	.0032	.0025
300	.1351	.1367	.1792
500	.4161	.5334	.6464
1,000	.3609	.2464	.1354
1,500	.0043	.0027	.0005
2,000	.0191	.0136	.0019
3,000	.0132	.0218	.0033
Total	1.0000	1.0000	1.0000

As an illustration of the approximate effect of these policy limits on excess loss development factors consider the following example of their effect on an unlimited (no policy limits) loss distribution. Let 10,000 be the lower bound of a tail of unlimited losses for which the "normalized" (divided by 10,000) loss distribution is represented by the Pareto density function $qx^{-(q+1)}$.

Let q=1.6 for a policy year as of 27 months and 1.3 for a policy year at ultimate development, and let a represent the development factor from 27 months to ultimate for losses excess of $10,000.

Since $(x^{1-q})\div(q-1)$ is the formula for the normalized losses excess of x, the unlimited losses excess of $10,000, $100,000,

$300,000, $500,000 and $1,000,000 at 27 months and at ultimate development can be represented as:

Retention	Excess at 27 months	Excess at Ultimate
10,000	x	ax
100,000	.251x	.501ax
300,000	.130x	.360ax
500,000	.096x	.309ax
1,000,000	.063x	.251ax

From this, the excess losses can be divided into the following layers, by subtracting from each excess amount the amount directly below it:

Layers(in 000's)	Amount at 27 months	Amount at ultimate
100 - 300	.121x	.141ax
300 - 500	.034x	.051ax
500 - 1000	.033x	.058ax
over 1000	.063x	.251ax

Now suppose that the policy limits earned premium distribution corresponding to the time period of the losses is 20% at $300,000 (per occurrence), 60% at $500,000, and 20% at $1,000,000, instead of the losses being unlimited.

The development of the unlimited losses excess of 100,000 from 27 months to ultimate = (.501 ax) ÷ (.251 x) = 1.996 a, whereas the development of the limited losses = (.141 ax + .8(.051 ax) + .2(.058 ax)) ÷ (.121x + .8(.034x) + .2(.033x)) = 1.252a. This is a big difference, but we should consider that the development factor for the losses limited only by $ 500,000 limits = (.141ax + .051ax) ÷ (.121x + .034x) = 1.239a and that the development factor for the losses limited only by $1,000,000 limits = (141ax + .051ax + .058ax) ÷ (.121x + .034x + .033x) = 1.330a. Thus, the limited development is not that different from the development of losses limited only at \$500,000 or only at \$1,000,000. If a=3, which is not unreasonable, then 1.252a = 3.756, 1,239a = 3.717, and 1.330a = 3.990. For retentions less than \$100,000, the difference between these types of development factors is less, since the portion below \$100,000 is not affected by the limits. Similarly, the development factors for losses excess of \$300,000 from 27 months to ultimate for unlimited losses, limited losses, losses limited only at \$500,000 and losses limited only at \$1,000,000 are 2.769a, 1.559a, 1.500a, and 1.627a respectively. The development factors for losses excess of \$500,000 are the same for the given policy limit distribution as for losses limited only at \$1,000,000.

For simplicity, we have considered only one policy year rather than a series of policy years with inflation operating on both average cost per occurrence and the average policy limit. But it seems probable that the development factors for retentions up to amounts corresponding to \$500,000 on a 1982 cost level, using actual limited losses for any policy year prior to 1982, are similar to development factors for losses limited only by any single limit which is between amounts corresponding to \$500,000 and \$1,000,000 on a policy year 1982 level. The development factors for limited losses are considerably different from unlimited development factors, but only a small portion of premium is written at policy limits over \$1,000,000, so development factors for limited losses are very useful. Also, the substantial disparity between limited and unlimited losses would be expected given the excessive thickness of the Pareto tail at extremely large loss amounts.

THE OPERATIONAL ASPECTS OF OUTWARDS REINSURANCE TREATIES

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Abstract

This paper discusses several operational considerations of outwards reinsurance treaties necessary to insure that the treaties are both functioning as intended, and properly reflected in the ceding companies financial statements. Commonly used treaty provisions and their impact on financial statements are discussed. The author has seen each of these provisions mishandled and is deeply indebted to many unnamed companies for first calling his attention to the fact that a seeming innocuous treaty clause, can sometimes create a significant distortion in financial statements.

THE OPERATIONAL ASPECTS OF OUTWARDS REINSURANCE TREATIES

Proper reinsurance practices are a prerequisite for the sound operation of most insurance companies. Reinsurance practices include the design, negotiation and purchase of a reinsurance program suitable to the needs of the company, as well as certain operational matters necessary to insure that the program both actually functions as intended, and is correctly reflected in the company's financial statements.

This paper deals with the operational aspects of outwards reinsurance treaties, that is, treaties protecting the ceding company. Inwards treaties, that is, treaties assumed by the company have different considerations.

While direct policies tend to be somewhat standardized, reinsurance agreements tend to be custom drawn, reflecting a wide diversity of thought and needs. For this reason general statements about treaties cannot apply to all treaties. The examples used in this paper, while reflective of customary usage, will not apply in all cases. Treaties may apply on a written or earned basis. The unmodified word premium should be understood in this context.

Identification of Ceded Premiums

A reinsurance treaty can apply to all business that the ceding company writes, to only a particular line or lines of business, only to business written by a certain department of the ceding company, only to business produced by a given producer, or to any subset or combination of these. A particular policy may be, and frequently is, ceded to more than one treaty. Ceded premium may be based on either written or earned premium. Proper identification of ceded premiums is needed so that the reinsurer receives the right amount of premium.

When a policy is ceded to more than one treaty, the treaties generally specify the order of application. Typically, the order of application of the premium follows that of the losses. Consider a casualty book of business protected both by a quota share treaty and by an excess of loss treaty; 20% quota share and a $100,000 excess of $150,000. If there is a loss before reinsurance of $250,000, how do the treaties respond? If the quota share applies first then $50,000 (20% of $250,000) is ceded to the quota share. Of the remaining $200,000, $150,000 is retained, and $50,000 is ceded to the excess.

If the excess applies first, $100,000 (excess of $150,000) is ceded to the excess. Of the remaining $150,000, 20% or $30,000 is ceded to the quota share, and $120,000 is retained.

The amount of loss ceded to each treaty and retained by the insurer is very much dependent upon the order of precedence. It is logical that the premium should be distributed in the same way. The ceded premium to a quota share treaty is generally a percent of the policy premium, 20% in the above example. The price of excess of loss protection is also typically expressed as a percent of premium.

The general approach to premium is that the percent or rate applies to premium net to other reinsurance which inures to the benefit of the treaty. Thus, in the above example, if the rate for the excess was 10%, a $1,000 premium would be ceded as follows: When the quota share applies first, 20% of $1,000 is ceded to the quota share. The excess rate of 10% would apply to the remaining $800, and $80 would be ceded to the excess. When the excess applies first, 10% of the $1,000 is ceded to the excess, and 20% of the remaining $900 is ceded to the quota share. In this manner, premium is ceded in the same manner as losses will be.

For property books of business, surplus share and catastrophe reinsurance are a common combination presenting the same problem of precedence. There is no standard order of application of treaties. Facultative protection must also be considered, it too may apply to the net exposure of the insurer, or it may inure to the benefit of the treaties.

In developing ceded premium it is vital to properly reflect the correct order of precedence.

It is important to distinguish between subject premium and ceded premium. Subject premium is the premium of all policies to which the treaty applies, minus the premium ceded to treaties or facultative placements inuring to the benefit of the treaty. This may be written or earned premium depending upon treaty provisions. Ceded premium is the amount of premium actually given to the reinsurer. For example a quota share treaty may reinsure 20% of business classified by the ceding company as long haul trucking physical damage insurance. The premium of all policies covering long haul trucking physical damage is the subject premium of this treaty, 20% of this premium is the ceded premium. An excess of loss treaty might reinsure losses in excess of $250,000 for all losses arising from policies reported as General Liability on the ceding company's Annual Statement. The reinsurer's charge for this protection is 5% of subject premium. All General Liability premium is the subject premium, and 5% of that amount is the ceded premium.

Treaties generally provide coverage on either a losses occurring basis, or a risk attaching basis. The losses occurring basis provides coverage for losses occurring during the treaty term regardless of when the underlying policy was written. The risk attaching basis covers only policies written during the term of treaty. This may be done in two ways: only for losses occurring during the treaty term, the "cut-off" basis, or for losses occurring during the term of the underlying policies, the "run-off" basis.

The basis of coverage aligns naturally with whether the ceded premium is based upon written or earned premium. Losses occurring usually corresponds to a cession based upon earned premium. Risk attaching on a "cut-off" basis is typically ceded on a written basis excluding the unearned premium reserve. On a "run-off" basis the cession is generally on a written basis with the unearned premium reserve.

In addition to the obvious problems in deriving the proper premium to be paid the reinsurers, the distinction between written and earned can create difficulties with some financial ratios.

Consider, for example, an excess of loss treaty covering all losses occurring in 1985 regardless of when the underlying policy was written. This treaty would typically be rated as a percent of earned premium. During 1985, policies are written which will not expire until 1986. Losses occurring in 1986 are not covered by the treaty. Should the year end 1985 unearned premium reserve be carried gross or net of the treaty? Does the answer change if at year end 1985 it is known that the treaty will be renewed? This situation can be made more complex if the treaty term extends to April, 1986. Certainly, that portion of the unearned premium reserve corresponding to losses projected to occur between January 1 and March 31, 1986 should be ceded to the treaty. At year end the renewal terms are at best uncertain. How should the remainder of the unearned premium reserve be carried?

Since earned premium must be written premium plus the change in the unearned premium reserve, the method of computing net unearned will define net written. Net earned premium, and thus the income statement is unaffected. Financial ratios involving net written premium can be distorted.

Recording Minimum and Deposit Premium

When a treaty is written on a minimum and deposit basis the ceding company will for example, "pay the reinsurer a premium of 10% of the subject premium of this contract, subject to a minimum and deposit premium of $1,000,000 payable quarterly in advance". This treaty provision means that on the first day of each quarter the ceding company is to pay the reinsurer $250,000. After the end of the year the total subject premium is multiplied by 10%, and if the result is greater than $1,000,000 the difference is remitted to the reinsurer. If the total is less than $1,000,000, the ceded premium remains at $1,000,000 with no refund of premium.

The minimum and deposit premium, or M&D, should be estimated to approximate the final premium. As competitive tool, reinsurers have been known to reduce the M&D to the cash flow advantage of the ceding company. In any event, actual premium writings can be very different, either more or less, than projections. When this occurs, distortions are frequently introduced in the financial statement by improper booking of the ceded M&D premium.

Companies will often record either the M&D or the percentage rate as the ceded premium without regard to the other. In the above example, an annual premium of $10,000,000 or $2,500,000 per quarter is contemplated. This, at 10% will produce $1,000,000 ceded premium. If the company is actually writing $5,000,000 per quarter, and is recording only the M&D as ceded premium, its net premium is overstated. A company's net premium will also be overstated if it were only writing $2,000,000 per quarter, and recording 10% of $200,000 as ceded premium.

This over-statement of net premium, both written and earned, will appear in each of the company's quarterly financial statements. At the end of the year the correct ceded premium is generally computed.

Although for large multi-line companies this distortion is usually negligible, in some instances, for the smaller company, the effect can be large enough to cause unpleasant year end surprises.

The distortion can be avoided by calculating ceded premium as the higher of the year to date M&D and actual premium. Table 1 demonstrates this calculation.

Table 1

Calculation of Recorded Ceded Premium

(000) Omitted

	Cumulative			
Quarter	Written Premium	Computed Ceded Premium @ 10%	Minimum & Deposit Premium	Recorded Ceded Premium
1	$ 2,000	$ 200	$ 250	$ 250
2	5,000	500	500	500
3	9,000	900	750	900
4	11,000	1,100	1,000	1,100

It should be noted in Table 1, that the company only pays $250 per quarter to the reinsurer. The excess premium of $150 in the third quarter is kept as a reserve and not paid until the year end settlement. Ceded premium does not equate to premium paid, but rather to premium that has been or will be paid.

Identification of Ceded Losses

Losses are typically ceded to a reinsurance treaty in the same manner as premium. As discussed above, when multiple treaties cover the same loss, an order of precedence is generally specified in the treaties. There are several situations where the identification of ceded losses is less than obvious.

Excess of loss treaties can include a provision, known as the aggregate extension clause, stating that "this reinsurance will respond in the aggregate whenever the underlying policy is written in the aggregate." This provision means that when a policy is written with an aggregate limit, such as Products Liability, the excess of loss reinsurance covers when all losses under the policy exceed the retention, rather than when the retention is exceeded by a single loss. A procedure that determines ceded loss based upon claims exceeding the company's retention will not identify these aggregate claims. Many of the new policy forms to be introduced during 1986, such as the ISO Commercial General Liability policy, provide an aggregate limit on most classes of third party liability other than automobile. If the aggregate extension clause remains in reinsurance contracts, the problem of identifying aggregate losses increases.

Catastrophe treaties provide another problem in identification. Ceding loss to a catastrophe treaty involves two distinct steps: accumulating all loss arising from an event, and ascertaining if the event is a catastrophe as defined in the treaty.

Many companies determine catastrophe losses solely by relying upon source level coding by the claims examiner. This process will almost always overlook some claims that should properly be ceded. In addition it does not allow for the occasional "mini-catastrophe", that while too small to attract attention, is just big enough to pierce the company's retention.

Source level coding should be supplemented by periodic examination of losses by date, cause and location. This can be used not only to identify losses from an event, but to ascertain if the event is a catastrophe as defined by the treaty.

Catastrophe treaties generally define a covered event in terms of lines of business and dollar amount. That is, coverage is provided when an event causes losses for the included lines of business in excess of a predetermined retention. Some events are frequently defined by fixed time periods. Weather related events are usually defined as "all losses from an atmospheric disturbance occurring during a continuous 72 hour period." Civil disorder is sometimes defined in a similar fashion. The ceding company can select the period to be covered. It is anticipated that the period will be selected in the most advantageous manner to the ceding company. Date of loss analysis is necessary to do this.

Excess of loss contracts can cover losses arising from a single event regardless of the number of risks involved. Broadly written excesses, known as clash covers, will cover many if not all lines of business as well. Thus, a trucking accident involving automobile liability and Workers Compensation for the driver is subject to single retention. The identification of all such losses is a difficult process. Numerous policies written in different areas by different departments of the company may be involved. It is not hard to imagine a hotel fire with injuries creating losses under the hotel's Property policy, Workers Compensation, third party liability for the injured guests, Architects and Engineers Professional Liability, Products Liability, and even Insurance Agents Errors and Omissions if the coverages weren't placed correctly. All of these, if a single insurer were unfortunate enough to have written all the policies, would be covered by a broadly written clash excess.

Although the above example may appear far fetched, unusual aggregations of loss can and do occur. The author is not aware of a "perfect" system for accumulating these types of losses. Matching claims under various lines of business by date of loss can produce the correct answer, but it is generally prohibitively expensive for a company of size. It should be noted that the problem is most acute for a reinsurer in determining its retrocessions.

Notice of Loss, Proof of Loss, and Bordereau

In addition to identifying ceded losses for financial purposes a company must report the loss to the reinsurer so that the reinsurer can reimburse the company. This reporting is done either on a individual claim basis or bordereau basis.

Bordereau reports are typically associated with proportional treaties. The total of all losses ceded to the reinsurer is reported, often with a list of the individual claims. Details of each claim are not provided.

Excess of loss contracts usually require individual reports of loss. A notice of loss setting forth details of the claim is to be sent at the time the company is first aware of a claim potentially exceeding its retention, or for certain specified injuries, such as death claims. Notices of loss form the basis of loss reserving for the reinsurer, and if properly executed, can be a useful source of loss reserving and pricing information at the ceding company level.

The notice of loss is related to outstanding losses. It does not cause the reinsurer to pay a claim. When a claim is paid, a proof of loss is sent to the reinsurer detailing the claim payment. It is the proof of loss that triggers the reimbursement of the claim.

It is axiomatic that reimbursement cannot take place until after a proof of loss is submitted. It is, therefore, surprising how many companies have procedures that do not submit a proof until after the claim is closed. The reinsurance is due when the loss is paid. A claim is often kept open for some time after the loss has been paid for the final adjuster's or attorney's bill. These items of allocated loss adjustment expense can follow the claim payment by many months. Proofs of loss can be easily amended for subsequent payments. The lost investment income to the ceding company will usually offset the added costs of multiple proofs of loss.

Loss Sensitive Treaties

Reinsurance treaties are frequently written on a loss sensitive or retrospectively rated basis. Two types of loss sensitive plans are in common use: sliding scale commission plans where the ceding commission paid to the company is altered in response to ceded losses, and true retrospectively rated plans where the ceded premium is modified by ceded losses.

Sliding scale commission plans are generally associated with proportional contracts. A typical provision would be: "the reinsurer agrees to pay the company a provisional commission of 30%, this commission will be reduced by one-half of one percentage point for every one percentage point that the reinsurers loss ratio exceeds 65%, subject to a minimum commission of 25%; and it will be increased by one-half of one percentage point for every one percentage point that the reinsurers loss ratio exceeds 65%, subject to a maximum commission of 35%." Sliding scale commission is frequently formulated as a profit commission, where the provisional commission can only be increased.

If a company has a 20% quota share treaty with this provision, and wrote $20,000,000 of premium, it cedes $4,000,000 (20% of $20,000,000), and receives a provisional commission of $1,200,000 (30% of $4,000,000). If the company's losses are $13,000,000 it will cede $2,600,000 (20% of $13,000,000). The reinsurer's loss ratio is 65% ($2,600,000/$4,000,000), and the final commission is equal to the provisional commission. No further adjustments are made. If, however, the losses are $13,200,000, ceded losses become $2,640,000. The reinsurer's loss ratio increases to 66%, and the ceding commission is reduced to 29.5%. The company previously received a ceding commission of $1,200,000, the final commission is $1,180,000 (29.5% of $4,000,000). It owes the reinsurer $20,000. If the losses had been less than $13,000,000 the reverse would be true.

Thus, it can be seen that although the company ceded an additional $40,000, its net income is benefited by only $20,000. If the commission was a fixed percentage, the net income would have been affected by the same amount as ceded loss.

True retrospective rating is usually associated with excess of loss treaties. A typical provision would be: "the company will pay to the reinsurer a provisional premium based upon a provisional rate of 5% multiplied by subject premium. The provisional rate shall be adjusted annually based upon the current valuation of losses ceded to this treaty. The rate shall be losses ceded to this treaty, limited to $150,000 per loss, divided by subject premium, plus two percentage points for the reinsurers administration, subject to a minimum rate of 3% and a maximum rate of 9%."

If the treaty is to continue for several years, the rate may be based on the combined experience of several years or a separate rate may be established for each year.

A company with a $400,000 excess of $100,000 excess of loss treaty with this provision pays a provisional premium of $500,000 based upon a subject premium of $10,000,000. If the only large loss the company has is a $200,000 claim, then ceded losses are $50,000 and the rate would be $50,000/$10,000,000 or .5%, plus the 2% charge. This is less than the minimum so the company would pay the minimum rate of 3%. Since it had already paid a provisional premium based on 5%, the company would receive a refund.

If, however, the company had four losses of $500,000 each, ceded losses would be $1,600,000. Of this $600,000 ($150,000 per claim) would enter the retrospective formula. The final rate would be 8% ($600,000/$10,000,000 or 6% plus the 2% charge). The company would owe the reinsurer an additional $300,000 (8% of $10,000,000 less the provisional premium of $500,000).

If losses in the $150,000 excess of $100,000 layer are less than 1% of subject premium, the company pays the minimum premium. If losses in the layer exceed 7% of subject premium the company pays the maximum premium. Within these limits, the company is effectively self insured. There can be cash flow differences between retrospective rating and self insurance in that retrospective rating is usually on an incurred loss basis, while reinsurance reimbursement is on a paid basis. Therefore, the reinsurer has the use of the funds rather than the company as would be the case with self insurance.

Retrospective rating provisions have the effect of converting losses into premium. Consider a simplified example of a $400,000 excess of $100,000 excess of loss treaty where reinsurance premium is equal to ceded losses plus 2% of subject premium.

A ceding company with $10,000,000 of subject premium should record an initial ceded premium of $200,000 (2% of $10,000,000). If the company establishes a gross loss reserve of $250,000, it would record ceded outstanding loss of $150,000, and net loss reserve of $100,000. The company should then increase ceded premium by $150,000, thereby reducing net premium by that amount. The effect on net income is the same as that of a loss of $250,000 without reinsurance, however, $150,000 has been "moved" from loss to premium.

Similar distortions appear in the balance sheet. Loss reserves are carried net of reinsurance. When the additional premium is paid to the reinsurer, cash is reduced, and surplus is the same as it would have been without the reinsurance. Until the reinsurer is paid, a reserve for ceded reinsurance balances payable should be maintained.

The transfer of premium to loss inherent in loss sensitive reinsurance treaties creates difficulties in both the preparation and analysis of insurer financial statements. Loss reserves are carried net of reinsurance. The additional premium due the reinsurers as a result of those loss reserves should also be carried as a liability. This includes those additional premiums associated with IBNR.

The above example was simplified. In practice, most retrospectively rated excess of loss contracts include only losses in a particular layer in the retrospective premium formula. A $400,000 excess of $100,000 treaty may include only the first $150,000 of reinsured losses in the calculation. That is, only gross losses in the $150,000 excess of $100,000 are reflected in that retrospective formula.

IBNR calculations tend to concentrate on net IBNR, and sometimes gross IBNR. As can be seen, an IBNR between these numbers is required to properly reflect loss sensitive reinsurance treaties. This is a very difficult number for many companies to calculate.

Retrospectively rated reinsurance treaties create two distinct problems in the analysis of insurer financial statements. Losses are substantiated by several supporting schedules in the statutory Annual Statement. Ceded premium is not as well supported. Balances payable to reinsurers is not supported at all, it includes all ceded premium not yet paid to reinsurers, from fixed as well as retrospectively rated treaties. As a result, if a company ceded losses and failed to recognized the resultant ceded premium its net income and surplus would be overstated. This overstatement would be very difficult to detect. It is the author's belief that this situation is relatively common as regards the retrospective premium associated with IBNR.

The Ceded Reinsurance Report of the General Interrogatories attempts to respond to this concern. The amount of additional premium due but unaccrued on all loss sensitive treaties is estimated and reported in the interrogatory. It is the author's opinion that the difficulties inherent in estimating the amounts associated with IBNR tend to make this number suspect. Further, the complexity of many retrospectively rated reinsurance treaties is such that some companies do not respond to the interrogatories correctly.

A serious impact of the "transfer" of loss to premium is the dampening of the apparent loss development in Schedules O and P. This is particularly true for longer tail Schedule P lines when subject to a low level retention. A significant portion of the development for these lines occurs in the ceded layer. Since Schedule P is on a net basis, this development does not appear on Schedule P. Rather the development manifests itself as an increase in ceded premium. Increases in ceded premium can be attributed to causes other than retrospectively rated premium development (reinsurance rate increases for example), thus the "true" loss development of the company is obscured. Generally, the development appearing in Schedules O and P will tend to be understated as the higher development is transferred to premium.

The author is unaware of any reasonable procedures that can be instituted to overcome these difficulties. The effects of loss sensitive treaties must be taken into account in any analysis of insurer financial statements, particularly for smaller companies where the impact can be proportionally greater.

Miscellaneous Items

Excess of loss treaties are sometimes written on a deductible basis. The deductible is generally expressed as a percent of subject premium. That is, the treaty will cover for example, $400,000 excess of $100,000, subject to a deductible of 5% of subject premium. If subject premium were $10,000,000, the company would pay the first $500,000 of losses in the $400,000 excess of $100,000 layer. This must be recognized in established ceded losses, particularly those dealing with IBNR. A change in deductible must be reflected in IBNR calculations in a similar manner to a change in retention. Some treaties contain a maximum total payment for all claims which causes similar but usually less severe difficulties.

Marine and catastrophe treaties often have reinstatement provisions. These provisions are used when the treaty provides a maximum total amount of coverage. If the total is reduced or exhausted, it can be reinstated for an additional premium. The premium relates to the original premium of the treaty. The reinstatement premium may be fixed or it may be proportional to time and/or coverage. If it is proportional to coverage, a treaty providing $1,000,000 of total coverage for example, with $100,000 of ceded loss, would have reinstatement premium of 10% of the original premium; and when paid, the entire $1,000,000 limit is again available.

When the reinstatement premium is proportional to time, and nine months have elapsed on a one year treaty, the reinstatement premium is 25% of the original premium. The premium may be proportional to both time and coverage. The reinstatement may be either mandatory or at the ceding company's option. If it is mandatory, the reinstatement must be purchased and the appropriate ceded premiums should be reflected at the time of loss.

Simulating Serious Workers' Compensation Claims

Gary G. Venter
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Abstract

This paper presents a rationale for using simulation to generate samples of serious Workers' Compensation claims. It further describes choices which must be made in sources and use of data as well as procedure and interpretation of results. Components of a specific model are developed and a few conclusions drawn based on our knowledge of some actual studies using simulation.

Background

The description of size of loss distributions for any of the major lines of insurance has been a subject of much discussion in the literature of the Casualty Actuarial Society since its inception. Much of this discussion has centered on tabulating, trending, developing and fitting curves to existing empiric samples. We have come a long way in this area of research.

The need for accurate size of loss distributions in Workers' Compensation insurance is especially great. Estimating costs and consequences of purchasing or providing excess insurance/reinsurance, evaluating the effects of accident limitations in a retro plan, or as an input to the estimate of an aggregate loss distribution are some of the possible applications. One could easily imagine sundry applications to other than excess ratemaking, such as class ratemaking, or evaluation of experience rating parameters, notably D-ratios, or even reserving, that do not come under those headings.

Unfortunately, the Workers' Compensation severity distribution is especially difficult to describe analytically, much less project to some future coverage period. Samples exist only of past experience, which may not be relevant. Trend and development models are some attempts to deal with this which can be trained to work quite well, especially of the less volatile or shorter tailed lines of insurance. Workers' Compensation is subject not only to the problems of trend and development as in other long-tailed lines but the further complication of legislative changes which affect future losses and often not in a way that is proportional by size.

A further complication to the development problem is the custom of many insurers to reserve serious Workers' Compensation claims on a present value basis. This not only means a compactification of claims along the time value of money, but a discount for mortality, which is really a kind of an averaging process akin to but different from assuming everyone lives their life expectancy. Some will eventually live longer and some less, spreading out the distribution. The discount for interest is greatest in cases with longest life expectancy, usually the costliest cases, so further reducing the spread.

Since benefit provisions differ from state to state, it is difficult to determine which states can be meaningfully combined with others. Unfortunately, single states do not usually generate enough claims to confidently estimate statistics of the severity distribution. Use of more years' data can increase the number of claims but this puts greater dependence on trend and development models mentioned above. Still, it is not impossible to adjust individual claims for the effects of law amendments or even model the dispersion of claim durations using life tables; this may be useful and would incorporate many of the elements of the simulation approach to be discussed below.

It would be well to review the literature on sampling techniques before describing the simulation process. In a very real way, simulation merely produces an ersatz sample which can be - and has been - used in the same way as the empiric one.

One should perhaps look at Dunbar Uhthoff's 1950 treatise on Excess Loss Ratios but since neither of our Proceedings collections go back that far, we find Frank Harwayne's more up to date "Accident Limitations for Retrospective Rating" of 1976 to be preferable.

Harwayne looks at collections of claims by serious injury type - Fatal, Permanent Total and Major Permanent Partial to first determine excess ratios for claim amounts expressed as a ratio to average. This is a key idea and allows one to generate overall excess ratios by expressing a loss limit as a ratio to the state-wide averages by type, then weighting the appropriate three excess ratios by the relative amount of loss in each injury type. Using ratios to average in the tables of excess ratios makes it easy to recognize scale differences in size of loss distributions by state or hazard group. Differences in the shapes of the distributions, however, are still not accounted for.

Of course, the weighted excess loss ratio is still not an ELPF. Adjustments must be made for loss development, law change, multiple claim occurrences, risk and, of course, a loss to premium ratio before a usable number will be had. It is in these adjustments that the procedure is weakest, for judgement plays such a large part in the evaluation of their effects.

Still, the basic idea of a weighted excess ratio by injury type stands as a paragon for all that follows.

Directions of Research

The problem of simulating Workers' Compensation serious claims has been addressed by several actuaries, including Gary Venter and Gregg Evans at Prudential Reinsurance (the "PR" Model); the consulting firm of Liscord, Ward and Roy in their 1980 development for the Minnesota Workers' Compensation Reinsurance Association (the "LW&R" Model); Robert Sturgis of Tillinghast, Nelson and Warren in a 1984 revised model for Minnesota, (the "TNW" Model); the research team of Frank Harwayne, Charles Gruber and Michael Schwartz for NCCI in 1981 (the "NCCI" Model); and Lee Steeneck of General Reinsurance (the "GR" Model) who uses simulation to establish reserves for specific excess Workers' Compensation claims. It will be instructive to refer to some of the choices made by each as we discuss the methodology of simulation, but keep in mind the versions of the models we used are not the latest and this paper is not an analysis of the models.

An overview of their approaches will be followed by a more detailed outline of choices necessary to utilize this method.

The essential feature of these models is the creation of an ersatz sample of serious claims from which excess loss ratios can be calculated. These can be used much like the empiric samples in the traditional method described above, however, there are several aspects of the models which demand departure from the historical excess ratio approach. These follow below.

1) Simulation of only Fatal and Permanent Total Claims.

   Due credit must be given to TNW, LW&R and NCCI for attempting simulations of Major Permanent Partial claims but, to our knowledge, this is not used for pricing applications by any of the current models. The overriding influence of administration rather than statute in these cases makes modeling less reliable, the relatively small excess ratio makes it less significant, and the larger number of claims available makes it less necessary.

2) Simulation only of possible outcomes of a single claim.

   Such a strategy is used by General Reinsurance for calculating an average excess reserve for a reported serious claim.

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3) The use of trend, development and law change assumptions.

   Adjusting historic claims for these phenomena is minimized by simulating at current (or projected) levels of wages and benefits.

4) Escalation and Interest Assumptions.

   Historic claims in some states exhibit the effect of statutory adjustments for cost of living, and the reserves at each evaluation may have been discounted for some rate of interest. A proper use of this data in the empiric method should entail adjustment of these parameters for future conditions. Certainly the simulation method must project these effects to future claims. Runs of various models which involved variances of escalation assumptions have demonstrated the dramatic effect on excess pricing of this characteristic.

<u>The Simulation Procedure</u>

The beginning of the simulation procedure is the creation of a large number of individual case situations, to be administered under projected conditions. Many factors affect the size of a Workers' Compensation claim. State law will directly determine the periodic indemnity amount based on type of injury, dependency status (number and ages of dependents), and wage of worker.

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Duration probabilities of the payment stream will depend on ages of worker and/or his dependents and the propensity for widows (or widowers) to remarry. The medical portion of a loss can be as large or larger than the indemnity. Other determinants of the loss include state provisions for escalation of benefits, interest assumptions and social security offsets.

Fortunately, distributions for all these factors are available. Fratello's 1955 Proceedings article on "The Workmen's Compensation Injury Table..." contains many. Updates and newer tables have been contributed by NCCI and others.

This information can be synthesized via simulation to produce a loss size distribution. We describe below the simulation of a single claim amount which, done repeatedly, generates a distribution.

The components of loss discussed above are displayed on Exhibit 1. An example of how these can be combined to produce a single claim size follows.

1. Select Type of Claim

   The time honored method for estimating ELPF's uses sample claims to calculate excess loss ratios for the three serious claim types. With simulation, one procedure is to create discrete sets of claims for Fatal (F), Permanent Total (PT) and Major Permanent Partial (MPP) and use them like real claims. Because of difficulties mentioned above, the simulation of MPP claims is usually omitted. In this case, the simulation of F and PT claims are completed separately and the results only combined at the time a total excess ratio is computed. Another procedure is to simulate F and PT claims in a single set, with the relative probabilities of occurrence assigned to each. The resulting set of claims can be used to compute a single excess ratio without weighting. There will still be a need to estimate the effect of MPP claims in both cases, but this is usually a small adjustment.

Let us assume a Fatal claim has been selected in the sequel. The steps for PT are similar but simpler because the benefit flows to the worker and it is not necessary to track life expectancy of a flock of dependents. (It may still be necessary to use dependency status to calculate benefits; in this case, the same tables can be used.)

2. <u>Simulate Dependency Status</u>

Using appropriate injury tables, one must establish type of dependents and their ages. Table 1 is an excerpt from the NCCI injury table for dependency status, which is a 1973 update of Fratello's work. Simulation from this table will be a simple matter of selecting a random number between 1 and 11,397. The process is described in more detail below.

3. Simulate Age of Dependents

Once dependency status is established, it will be easy to use Tables 2, 3 and 4 to choose ages of widows, children or dependent parents. Tables 2 and 4 are taken from the NCCI 1973 update to the Workers' Compensation Injury Table, with the previous numbers shown in parentheses. Table 3 was built from U.S. Census data and Actuarial judgements.

In PT cases, it will be necessary to establish the age of the worker. In these cases, Table 5 from the same NCCI update may be used.

4. Simulate Wage

The wage of the worker will be needed to calculate a benefit amount. Table 6 is the 1973 Standard Wage Distribution table used by NCCI. A random number between 0 and 1 can be used to select an entry in column A, move to the corresponding R value which will be applied to the Statewide Average Weekly Wage (SAWW) to obtain a dollar amount.

5. <u>Alternate Steps 2, 3, 4</u>

It would be naive to assume dependency, age and wage are independent, which is just what has been done up to now. The use of informed judgement to combine data in a reasonable way would be more actuarial than to blithely assume independence. Dependency status, e.g., ought to imply a range of reasonableness for the ages of worker, widow and children. NCCI uses such ranges in their simulations to eliminate unrealistic combinations. Wages should also be related to worker age.

For the PR model, judgement was used to combine the information on the Standard Injury Table with information from the U.S. Bureau of the Census on husband-wife age distributions, number of children by age of mother, and wages by age to produce Tables 7 and 8. These tables were used in a way described now.

The choice of a cell in Table 7 establishes the number and type of dependents and a range of ages for the widow - if one exists - or the worker otherwise. To illustrate how random selection from the table might be done, we can imagine assigning 100,000 individuals to the cells according to the frequencies shown in the exhibit.

Then picking a random number in between 1 and 100,000 specifies a cell, namely the cell the nth individual occupies. If this picking is done enough times, the selected cells will be distributed closely to those in the exhibit. Actual age can be selected as a random draw from within the age group, assuming, e.g., a uniform distribution.

The use of Table 8 would be similar to Table 7 except previous results will determine which row of the table would be used. The non-independence of age/wage/dependency should be obvious in this procedure.

Actual wage amounts must be established by selecting a point in the range and applying it to current or projected state average weekly wage.

Recent evidence shows the average wages of F and PT victims to be significantly greater than the SAWW. It would be appropriate to increase SAWW by a factor of 1.3 or 1.5 or more when extending the tabular values to produce actual wages.

6. Simulate Ages of Children and/or Parents

Since we have established age of the widow or deceased worker, we can now utilize Table 3 to establish the age of the dependent children. The PR model selected a single age from a normal distribution with mean u, where u comes from the table, and variance σ = u/6. The NCCI model allows the children to have different ages. The PR assumption sacrifices some verisimilitude for the sake of simplicity at minimal loss of accuracy on the large cases. Parents ages can be simulated from Table 4 or taken directly as some 20 years more than the worker.

7. <u>Simulate Time Period to Death or Remarriage</u>

Tables 9 and 10 are single decrement tables for remarriage or mortality respectively. The remarriage table is based on "The 1979 NCCI Remarriage Table," by Philip Heckman (PCAS 1982, p52). In Table 10 widows use the woman's columns; children, parents and siblings use total population statistics.

To illustrate how a random draw can be made based on these tables take the case of a dependent parent of age 50. Table 8 indicates that of 100,000 births, 88,972 attain age 50. Pick a random number n from 1 to 88,972, intended to represent the nth longest lived person for this group. Finding the year attained by the nth longest lived person in the table then represents a random draw of attained age according to the distribution of lives represented by the tables. Suppose for example, n =

44,486 were the random number drawn. Then the individual would survive until age 76 but not 77, according to the table.

This is the manner in which all lifetimes are simulated, except for widows, who use the women's life table and the remarriage table. The remarriage table considers probabilities of remarriage to be a function of the widow's age and, for the first five years, the length of time widowed. For instance, out of 100,000 widowed at age 16, 93,359 would not have remarried 1 year later. Out of 83,912 widowed at age 17, 78,860 will not have remarried 1 year later. After 5 years, further increments go down the last column. Thus, of the 100,000 16 year old widows, 39,899 would be unremarried seven years later, the same as the number of 17 year old widows remaining 6 years after widowhood. Note that this table is not decremented for death but just for remarriage.

A combined table can be constructed by assuming the probability of a widow being alive and still single equals the probability of being alive times the probability of being single. A random draw from this combined distribution gives the year in which the widow's payment status fails, due to either death or remarriage, but not mentioning which. Since some states specify an additional benefit on remarriage, it must be decided whether the status failed because of death or remarriage. This is done by a random choice where the chance of remarriage is proportional to the number of such statuses that fail due to remarriage in that number of years.

8. Simulation of Medical Benefit

For fatal cases, the usual procedure is to add a flat amount. For Permanent Total, medical can be a significant amount. The PR model used a lognormal distribution with σ = .90463 and u = 10.8578 + (40 - age) $\div$ 62.5, where age means that age at injury. This gives a coefficient of variation of 1.1255 for every age and means of 107,700, 78,200, and 56,800 at ages 20, 40 and 60 respectively, based on the formulas $CV^2 = e^{\sigma^2} - 1$ and mean $= e^{\mu + \sigma^2/2}$. Much of the medical costs are of an ongoing nature, and it was felt that the younger injured worker would accumulate more of these costs. The LW&R model used a lognormal distribution with coefficient of variation 0.9, but correlated the scale with the indemnity amount.

For discounting purposes some stream of medical payments must be selected. For example, it could be assumed half the medical amount be paid the first year and the other half throughout the life of the injured worker.

A recent NCCI review of Minnesota data suggests the lognormal distribution is not heavy enough in the tail to properly fit medical amounts; a few mega losses seem to occur often enough that they should be accounted for. More work is needed in this area.

9. Social Security Offsets

Social Security can have a significant impact on proper excess pricing and must be incorporated in the model. The Actuarial Committee of the Minnesota WCRA has spent more than a little time debating possible models for this offset and noting the effects of each.

Most, but not all, pensioners are eligible for Old Age or Disability benefits. NCCI takes 90% of workers age 20 and below as eligible, graduating to 100% at age 40. This is probably an overestimate according to the Minnesota studies and later versions of the LW&R model reflect this fact.

Benefit amount must be computed based on the Average Indexed Weekly Wage (AIWW) and dependency status. The latter has been established by simulation, while some assumptions as to earnings history must be made to estimate the former from current wage.

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10. <u>Other Determinants</u>

After the above selections have been made, all the details needed to calculate indemnity benefits are present. *The benefit provisions of the relevant* jurisdiction must then be consulted to specify the payment stream.

For states with escalating benefits the indemnity payments increase periodically in proportion to some index, e.g., the state average weekly wage. By assuming a value of this index for each future year, the payment stream can be adjusted. 5 to 7% annual escalation rates are reasonable long term assumptions, but you may have a better crystal ball.

Once the payment stream has been determined, average payments, average payments excess of given retentions, discounted payments, etc. can be calculated. Discounted payments excess of given retentions can be calculated, but with care. The retention cannot simply be subtracted from the present value of the total payments. Rather the point at which the retention is pierced must be noted, and the present value of the subsequent payments determined. See Ronald Ferguson's "Actuarial Note on Workermen's Compensation Loss Reserves" in PCAS, 1971 for details.

Per Occurrence Simulations

The steps outlined thus far can be used to build a collection of individual claims which can be used much like empiric data. In the case of either, the exigency remains that most excess (re)insurance attaches on an occurrence basis. This is also the case for the application of loss limits in a retro program, hence impacting ELPF's.

There is little data available to quantify the transition from claims to occurrences. Historically, a judgement loading factor of 1.1 or more has been used to compensate for this. We suggest a more analytical method using a second stage simulation, detailed below.

We first select a distribution of fatalities per accident. We can construct multiple claimant occurrences using this distribution by adding random claim amounts from the already compiled per claim distribution according to a simulated claim count.

In the PR model, a form of the Weibull distribution was used for the number of fatalities per accident. This distribution function is $F(x) = 1 - e^{-3x^{.375}}$, discretized by considering the probability in the interval n $\pm$ .5 to be the probability of N = n accidents. More specifically for $n \geq 1$, $\Pr(N \leq n) = F(n + .5)$ is the probability of at most n claimants.

This distribution was selected largely by judgement, as data is sparse. However, the Kansas Department of Human Resources had 1978 and 1979 data indicating that about 3% of fatal work accidents involved more than one fatality, which is consistent with this model. Exhibit 2 shows some of the results of a Tillinghast study for Pennsylvania Workers' Compensation ratemaking. Our relative frequencies are higher, 52.2% for two claim occurrences, 20.8 for three, etc. to 0.7 for ten. We believe this adds a measure of risk to balance the occasionally reported 30 fatality accident.

Random number generation from a Weibull is particularly simple, since the distribution has a closed form inverse. Let q = 1 - F(x). Then $q = e^{-3x^{.375}}$ or $x = \frac{-\ln q^{8/3}}{3}$. Thus, x can be generated by picking q at random from (0, 1) and calculating x. It is slightly simpler to do this from a pick of 1 - F(x) but a similar expression could follow from a pick of F(x).

Results we have obtained using this second stage simulation have indicated the roughness of a flat 1.1 loading factor. This is probably excessive for lower retentions, even up to $100,000, but eventually inadequate, e.g. at $1-2 million, where loadings of 50% or more may be indicated.

Conclusions

Simulation has made possible more precise estimation of excess Workers' Compensation costs. Use of these models in actual pricing/reserving by WCRA, NCCI, Pru Re and Gen Re is an indication of the value of the method.

The power of the method resides not only in precision, but the ability to easily measure the effects of changes in state laws, trend and development. Our study showed the loss severity distributions in states with 1) maximum aggregate benefits, 2) no overall limit, or 3) benefits that escalate via cost of living adjustment to be respectively 1) negatively, 2) hardly, and 3) highly skewed. Other differences in laws have measurable, if not dramatic, effect on size of loss distributions.

All of the referenced studies noted differences in severity by type of claim, although treatments differ. One of the original hypotheses to be tested by the NCCI model was that it would be enough to simulate fatal claims and use that distribution for permanent partial. This would reduce the total number of simulations necessary and was demonstrably conservative, so was a practical shortcut. Experience with these models has indicated a significant difference in the permanent partial distribution and now these claims receive separate consideration.

We have tried to systematize the simulation of Workers' Compensation claims. Room for further research in this area is great and some has been cited. We believe the method is sound and its development worthwhile.

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Simulating Workers' Compensation Serious Claims

Determinants of Benefits

I. Type of Claim

- A. Permanent Total
- B. Fatal
- C. Permanent Partial
  1. Major
  2. Minor

II. Indemnity Amount

- A. State Laws
- B. Wage of Worker
- C. Dependency Status
- D. Type of Disability

III. Duration

- A. Age of Worker
- B. Ages of Dependents
  1. Wife
  2. Children
  3. Parents
  4. Siblings

IV. Termination

- A. Death
- B. Majority
- C. Remarriage

V. Medical Amount

- A. Flat
- B. Correlation with
  1. Age
  2. Indemnity
  3. Type of Accident

VI. Payment Stream

A. Interest Assumptions

B. Escalation Assumptions

C. Social Security Offsets

D. State Maximums

Notes:

1. Simulation may determine range of ages or salaries - second simulation exact age

2. Correlation between type of accident, age, dependency status, wage, medical amount, may or may not be incorporated

Exhibit 2

Relative Frequencies for Catastrophes

1972 (5th report) to 1976 (1st report) data

Number of Claims	Catastrophe Count	Relative Frequency	Smoothed Estimates
2	120	69.4%	69.0
3	27	15.6	16.0
4	11	6.4	6.5
5	5	2.9	3.0
6	4	2.3	2.0
7	2	1.2	1.5
8	1	0.6	1.0
9	2	1.2	0.5
10	1	0.6	0.5
Total	173	100.0	100.0

NATIONAL COUNCIL ON COMPENSATION INSURANCE

Accident Frequency - Fatal Cases
(According to Dependency)

Actual No. of Cases†	Type of Dependency
1,677	No Dependents
4,058	Widow Alone
1,552	Widow with 1 child
1,464	Widow with 2 children
936	Widow with 3 children
473	Widow with 4 children
248	Widow with 5 children
184	Widow with more than 5 children (Average 7)
182	1 Orphan
115	2 Orphans
81	3 Orphans
37	4 Orphans
12	5 Orphans
3	6 Orphans
142	1 Parent
191	2 Parents
13	1 Brother or Sister
1	2 Brothers or Sisters
28	One other Dependent
11,397	Total

†The above distribution was derived from actual case reports from the following states: California, Delaware, Massachusetts, New York, Pennsylvania, and Texas. Only types of dependency which occurred in the study are listed.

Table 2

Age Distribution of Widows - Fatal Disability †

Age Groups	Widow Alone		Widow with 1 Child		Widow with 2 Children		Widow with 3 Children		Widow with 4 Children		Widow with 5 Children		Widow with more than 5 Children		Total Widow with Children	
10-14	13	-	4	-	1	-	-	-	-	-	-	-	-	-	5	-
15-19	84	(84)	90	(101)	34	(19)	3	(2)	3	-	-	-	-	-	130	(122)
20-24	124	(195)	180	(375)	194	(177)	54	(70)	24	(14)	8	(6)	2	-	462	(642)
25-29	81	(225)	127	(319)	192	(342)	116	(180)	46	(86)	31	(31)	10	(25)	522	(983)
30-34	67	(216)	74	(271)	121	(360)	140	(217)	98	(94)	39	(53)	22	(49)	494	(1,044)
35-39	124	(254)	97	(259)	139	(285)	145	(185)	71	(112)	48	(48)	34	(58)	534	(947)
40-44	253	(416)	174	(273)	179	(201)	96	(118)	71	(62)	30	(37)	22	(43)	572	(734)
45-49	563	(544)	173	(231)	115	(135)	65	(53)	23	(36)	10	(9)	12	(14)	398	(478)
50-54	779	(777)	144	(166)	56	(79)	15	(33)	7	(6)	4	(5)	7	-	233	(289)
55-59	806	(669)	68	(115)	7	(33)	6	(10)	2	(4)	1	-	-		84	(163)
60-64	431	(601)	10	(32)	2	(1)	-	(1)	2	-	-		-		14	(34)
65-69	151	(347)	2	(8)	3	(1)	-		-	-	-		-		5	(9)
70-74	68	(137)	-	(2)	-		-		-		-		-		-	(2)
75-79	13	(39)	-		-		-		-		-		-		-	-
80-84	6	(6)	-		-		-		-		-		-		-	
85-89	1	-	-		-		-		-		-		-		-	
Total	3,564	(4,510)	1,143	(2,152)	1,043	(1,633)	640	(869)	347	(414)	171	(189)	109	(189)	3,453	(5,447)

†Numbers in parentheses are from the current injury table.

Childrens Mean* Ages

Widow's Age:	17-24	25-34	35-44	45-54	55-64	65 +
Number of Children						
1	5	8	10	12	14	17
2	5	7	9	11	13	16
3 or more	6	7	9	12	13	15
Worker's Age:	**17-24**	**25-34**	**35-44**	**45-54**	**55-64**	**65 +**
Number of Orphans						
1	8	10	12	13	15	17
2	8	9	11	13	15	17
3	5	7	9	11	14	16
4	5	7	9	11	14	16

*All children are taken to be the same age for a given claim. This age is generated randomly from a normal distribution with the above means and a standard deviation of 1/6th of the mean.

Table 4

NATIONAL COUNCIL ON COMPENSATION INSURANCE

Age Distribution of Parent or Parents - Fatal Disability+

Age Group	One Parent	Two Parents
25-29	2 -	0 -
30-34	4 (3)	0 (3)
35-39	1 (11)	13 (14)
40-44	6 (32)	20 (50)
45-49	11 (52)	12 (46)
50-54	11 (46)	16 (58)
55-59	11 (74)	9 (67)
60-64	12 (65)	4 (54)
65-69	12 (65)	3 (32)
70-74	12 (53)	4 (22)
75-79	8 (33)	1 (14)
80-84	9 (30)	0 (8)
85-89	2 -	0 -
Total	101 (464)	82 (368)
Average Age:		
Arithmetic	61 (61)	49 (56)
Pension	61 (61)	50 (56)
Pension (5% Escalation)	58	48
Pension (6% Escalation)	57	48

+Numbers in parentheses are from the current injury table

NATIONAL COUNCIL ON COMPENSATION INSURANCE

Age Distribution - Permanent Total Disability

Age Group	No. Of Cases†	
Under 15	4	(2)
15 - 19	128	(45)
20 - 24	307	(110)
25 - 29	410	(137)
30 - 34	494	(177)
35 - 39	571	(251)
40 - 44	697	(237)
45 - 49	771	(309)
50 - 54	794	(309)
55 - 59	818	(360)
60 - 64	621	(376)
65 - 69	187	(287)
70 - 74	95	(154)
75 - 79	35	(68)
80 - 84	7	(13)
85 - 89	3	-
Total	5,942	(2,835)

Average Age -	Arithmetic	46	(50)
	Pension	47	(50)
	Pension (5% Esc.)	44	
	Pension (6% Esc.)	43	

†Numbers in parentheses are from the current injury table.

1973 Standard Wage Distribution Table

R = Ratio to Average Wage
A = Percentage of workers receiving not more than the percentage of the average wage indicated by column R B = Percentage of wages received by the percentage
B = Percentage of wages received by the percentage of workers in column A

R	A	B	R	A	B	R	A	B
.05	.1068	.0030	2.40	98.8248	96.4991	4.75	99.9210	99.5369
.10	.3511	.0222	2.45	98.9702	96.8502	4.80	99.9245	99.5542
.15	.8384	.0845	2.50	99.1283	97.2237	4.85	99.9277	99.5700
.20	1.4357	.1903	2.55	99.2172	97.4447	4.90	99.9290	99.5762
.25	2.1432	.3483	2.60	99.3278	97.7304	4.95	99.9316	99.5881
.30	2.9058	.5629	2.65	99.3962	97.9051	5.00	99.9337	99.5984
.35	3.7375	.8393	2.70	99.4464	98.0372	5.05	99.9357	99.6093
.40	4.7328	1.2173	2.75	99.5127	98.2151	5.10	99.9390	99.6258
.45	6.1073	1.8188	2.80	99.5551	98.3291	5.15	99.9415	99.6393
.50	8.2201	2.8537	2.85	99.5867	98.4178	5.20	99.9438	99.6516
.55	11.6032	4.6692	2.90	99.6240	98.5226	5.25	99.9453	99.6594
.60	15.3290	6.7892	2.95	99.6515	98.6021	5.30	99.9483	99.6752
.65	20.5672	10.1290	3.00	99.6742	98.6709	5.35	99.9488	99.6778
.70	25.9600	13.7452	3.05	99.6888	98.7150	5.40	99.9498	99.6836
.75	32.3089	18.2868	3.10	99.7116	98.7817	5.45	99.9508	99.6892
.80	37.5110	22.2523	3.15	99.7288	98.8358	5.50	99.9539	99.7064
.85	42.9709	26.6884	3.20	99.7427	98.8809	5.55	99.9552	99.7130
.90	48.2321	31.2144	3.25	99.7614	98.9448	5.60	99.9559	99.7174
.95	53.1109	35.7149	3.30	99.7825	99.0090	5.65	99.9569	99.7228
1.00	58.4036	40.9066	3.35	99.7922	99.0422	5.70	99.9584	99.7318
1.05	62.9643	45.6459	3.40	99.7995	99.0666	5.75	99.9607	99.7447
1.10	67.1858	50.1850	3.45	99.8141	99.1161	5.80	99.9623	99.7537
1.15	70.6767	54.0985	3.50	99.8211	99.1404	5.85	99.9656	99.7730
1.20	74.0989	58.1398	3.55	99.8308	99.1747	5.90	99.9674	99.7840
1.25	77.0678	61.7560	3.60	99.8403	99.2088	5.95	99.9684	99.7903
1.30	79.9516	65.5218	3.65	99.8457	99.2272	6.00	99.9701	99.8007
1.35	82.2534	68.5701	3.70	99.8511	99.2463	6.05	99.9712	99.8069
1.40	84.5435	71.7325	3.75	99.8575	99.2701	6.10	99.9722	99.8131
1.45	86.3620	74.3294	3.80	99.8616	99.2854	6.15	99.9727	99.8161
1.50	87.9326	76.6547	3.85	99.8657	99.3029	6.20	99.9734	99.8210
1.55	89.1240	78.4667	3.90	99.8731	99.3315	6.25	99.9753	99.8315
1.60	90.4193	80.4994	3.95	99.8774	99.3499	6.30	99.9758	99.8349
1.65	91.6370	82.4738	4.00	99.8800	99.3594	6.35	99.9763	99.8380
1.70	92.4497	83.8454	4.05	99.8835	99.3739	6.40	99.9775	99.8468
1.75	93.2448	85.2260	4.10	99.8871	99.3886	6.45	99.9780	99.8504
1.80	93.9290	86.4398	4.15	99.8949	99.4207	6.50	99.9816	99.8762
1.85	94.5674	87.5957	4.20	99.8970	99.4295	6.55	99.9831	99.8855
1.90	95.1329	88.6605	4.25	99.9000	99.4429	6.60	99.9848	99.8964
1.95	95.7436	89.8715	4.30	99.9033	99.4574	6.65	99.9851	99.8978
2.00	96.2339	90.8451	4.35	99.9058	99.4689	6.70	99.9861	99.9047
2.05	96.6383	91.6662	4.40	99.9086	99.4807	6.75	99.9871	99.9118
2.10	97.1239	92.6803	4.45	99.9091	99.4831	6.80	99.9877	99.9149
2.15	97.4920	93.4767	4.50	99.9122	99.4965	6.85	99.9892	99.9259
2.20	97.8424	94.2425	4.55	99.9142	99.5052	6.90	99.9897	99.9290
2.25	98.1208	94.8736	4.60	99.9155	99.5113	6.95	99.9902	99.9321
2.30	98.3723	95.4400	4.65	99.9173	99.5197	7.00	99.9917	99.9429
2.35	98.6285	96.0369	4.70	99.9197	99.5309			

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Exhibit 1

Dependency by Age Distribution

Widow Cases by Age of Widow

	Total	17 - 24	25 - 34	35 - 44	45 - 54	55 - 64	65 + 74
Widow alone	35,235	2,925	4,017	3,453	8,597	10,817	5,426
Widow + 1 child	15,660	2,098	4,119	3,445	3,946	1,738	314
Widow + 2 children	15,660	1,801	5,873	4,745	2,694	517	30
Widow + 3 children	11,745	1,034	4,721	4,733	1,163	82	12
Subtotal Widow Cases	78,300	7,858	18,730	16,376	16,400	13,154	5,782

Non-Widow Cases by Age of Worker

?

	Total	17 - 24	25 - 34	35 - 44	45 - 54	55 - 64	65 + 74
1 orphan	1,600	102	357	342	340	291	168
2 orphans	1,000	64	223	214	213	182	104
3 orphans	700	45	156	150	149	127	73
4 orphans	400	26	89	86	85	73	41
1 parent	1,300	83	290	278	277	236	136
2 parents	1,700	109	379	364	362	309	177
other	300	19	67	64	64	55	31
none	14,700	941	3,280	3,146	3,128	2,672	1,533
Subtotal Non-Widow Cases	21,700	1,389	4,841	4,644	4,618	3,445	2,263
Total	100,000	9,247	23,571	21,020	21,018	17,099	8,045

Table 7

Exhibit 2

Wage By Age Distributions

Percent of Average Wage

Age Widow Cases	Total	≤ 16.0	16.1 to 32.1	32.2 to 48.1	48.2 to 64.2	64.3 to 80.2	80.3 to 96.2	96.3 to 120.3	120.4 to 160.4	160.5 to 200.5	> 200.5
17 - 24	10,036	474	429	634	1,816	2,292	1.670	1,549	879	205	88
25 - 34	23,921	129	277	674	2,609	4,468	4,437	5,424	4,072	1,226	605
35 - 44	20,914	70	195	444	1,782	3,222	3,425	4,707	4,354	1,668	1,047
45 - 54	20,945	93	309	604	2,066	3,414	3,462	4,402	3,995	1,528	1,072
55 - 64	16,800	107	542	904	2,419	3,125	2,719	3,003	2,367	910	704
65 + 74	7,384	76	546	823	1,697	1,484	906	862	563	221	206
Total	100,000	949	2,298	4,083	12,389	18,005	16,619	19,947	16,230	5,758	3,722
Non Widow Cases											
17 - 24	6,400	524	449	592	1,523	1,634	911	573	168	15	11
25 - 34	22,310	81	248	667	2,598	4,456	4,393	5,147	3,501	879	340
35 - 44	21,410	59	161	397	1,729	3,220	3,480	4,914	4,573	1,785	1,082
45 - 54	21,280	82	241	502	1,827	3,232	3,426	4,629	4,452	1,709	1,180
55 - 64	18,180	91	345	647	2,148	3,342	3,210	3,589	2,865	1,102	841
65 + 74	10,430	112	854	1,278	2,564	2,121	1,199	1,095	671	268	268
Total	100,000	949	2,298	4,083	12,389	18,005	16,619	19,947	16,230	5,758	3,722

Table 5

SINGLE-DECREMENT (REMARRIAGE) TABLE - SELECT PERIOD = 5 YEARS

AGE	NUMBER OF YEARS WIDOWED 0	1	2	3	4	5
16 -	100000	93359	78930	66087	56383	49379
17 -	83912	78060	67717	57551	49842	44143
18 -	71604	67696	58957	50846	44591	39899
19 -	62021	58954	52004	45450	40318	36415
20 -	54429	51990	46396	41043	36790	33514
21 -	48320	46367	41814	37399	33844	31073
22 -	43357	41763	38023	34350	31355	28995
23 -	39266	37950	34859	31770	29230	27215
24 -	35871	34790	32200	29590	27429	25605
25 -	33031	32129	29950	27737	25874	24362
26 -	30637	29800	28033	26140	24530	23213
27 -	28605	27965	26390	24761	23364	22211
28 -	26869	26325	24974	23566	22347	21334
29 -	25377	24912	23740	22524	21457	20563
30 -	24080	23680	22600	21612	20674	19883
31 -	22969	22624	21747	20812	19904	19201
32 -	21993	21695	20929	20106	19374	18748
33 -	21140	20880	20209	19484	18833	18274
34 -	20391	20164	19574	18932	18352	17852
35 -	19731	19532	19011	18442	17925	17475
36 -	19147	18972	18512	18005	17543	17139
37 -	18630	18476	18068	17616	17201	16837
38 -	18170	18035	17672	17268	14895	16567
39 -	17761	17641	17318	16956	16621	16323
40 -	17395	17289	17000	16676	16373	16104
41 -	17060	16973	16715	16424	16151	15907
42 -	16774	16690	14459	16196	15950	15720
43 -	16510	16435	16220	15991	15760	15567
44 -	16272	16205	16019	15806	15603	15420
45 -	16050	15990	15831	15630	15455	15288
46 -	15865	15811	15660	15486	15319	15167
47 -	15690	15642	15506	15340	15197	15050
48 -	15541	15498	15375	15232	15094	14968
49 -	15390	15359	15240	15110	14993	14877
50 -	15260	15233	15132	15015	14900	14795
51 -	15150	15110	15020	14921	14816	14720
52 -	15042	15014	14932	14835	14740	14651
53 -	14945	14920	14845	14757	14670	14589
54 -	14857	14834	14766	14686	14606	14532
~~55~~ -	~~14776~~	~~14756~~	~~14694~~	~~14621~~	~~14548~~	[illegible]
56 -	14703	14685	14629	14562	14496	14433
57 -	14637	14620	14569	14508	14448	14390
58 -	14576	14561	14515	14460	14404	14351
59 -	14521	14508	14466	14415	14364	14316
60 -	14471	14459	14421	14375	14328	14284
61 -	14426	14415	14380	14338	14295	14254
62 -	14385	14375	14343	14304	14265	14228
63 -	14347	14338	14310	14274	14238	14204
64 -	14314	14306	14279	14247	14214	14182
65 -	14283	14276	14252	14222	14192	14163
66 -	14256	14249	14227	14200	14172	14144
67 -	14231	14225	14205	14180	14155	14130
68 -	14209	14203	14185	14162	14139	14116
69 -	14189	14184	14167	14146	14125	14104
70 -	14171	14166	14151	14132	14112	14093
71 -	14155	14150	14137	14119	14101	14083
72 -	14140	14136	14124	14108	14091	14075
73 -	14127	14124	14112	14097	14082	14067
74 -	14115	14112	14102	14088	14074	14060
75 -	14105	14102	14093	14080	14067	14054
76 -	14095	14093	14084	14073	14061	14049
77 -	14087	14085	14077	14066	14055	14045
78 -	14080	14078	14070	14061	14051	14041
79 -	14073	14071	14064	14056	14046	14037
80 -	14067	14065	14059	14051	14043	14034
81 -	14062	14060	14054	14047	14039	14031
82 -	14057	14055	14050	14043	14036	14029
83 -	14053	14051	14046	14040	14034	14027
84 -	14049	14048	14043	14037	14031	14025
85 -	14045	14044	14040	14035	14029	14024
86 -	14042	14041	14038	14033	14029	14022
87 -	14038	14037	14034	14029	14026	14021
88 -	14037	14036	14033	14029	14025	14020
89 -	14035	14034	14031	14028	14024	14020
90 -	14033	14032	14030	14026	14023	14019
91 -	14031	14031	14028	14025	14022	14018
92 -	14030	14029	14027	14024	14021	14018
93 -	14028	14028	14026	14023	14020	14017
94 -	14027	14026	14025	14022	14019	14017
95 -	14026	14025	14024	14021	14019	14016
96 -	14025	14024	14023	14021	14018	14016
97 -	14024	14023	14022	14020	14018	14016
98 -	14023	14022	14021	14019	14017	14015
99 -	14022	14022	14071	14019	14017	14015
100 -	14022	14021	14020	14019	14017	14015
101 -	14021	14021	14020	14018	14017	14015
102 -	14020	14020	14019	14018	14016	14015
103 -	14019	14019	14018	14017	14016	14015
104 -	14019	14019	14018	14017	14016	14015
105 -	14019	14018	14018	14017	14016	14014

Table 10

1969-1971 LIFE TABLES FOR WOMEN (W) AND TOTAL POPULATION (T)

AGE	L[W]	L[T]	AGE	L[W]	L[T]	AGE	L[W]	L[T]	AGE	L[W]	L[T]
0 -	100000	100000	28 -	96724	95586	56 -	88382	84142	84 -	32921	23638
1 -	98254	97998	29 -	96636	95448	57 -	87649	83103	85 -	29538	20908
2 -	98139	97876	30 -	96544	95307	58 -	86865	81908	86 -	26206	18282
3 -	98064	97792	31 -	96445	95158	59 -	86030	80798	87 -	22940	15769
4 -	98005	97724	32 -	96339	95003	60 -	85139	79529	88 -	19801	13407
5 -	97955	97668	33 -	96224	94840	61 -	84191	78181	89 -	16858	11240
6 -	97913	97619	34 -	96101	94666	62 -	83181	76751	90 -	14160	9297
7 -	97876	97573	35 -	95966	94482	63 -	82101	75236	91 -	11715	7577
8 -	97842	97531	36 -	95821	94285	64 -	80943	73631	92 -	9523	6070
9 -	97812	97494	37 -	95662	94073	65 -	79698	71933	93 -	7595	4773
10 -	97784	97460	38 -	95490	93843	66 -	78361	70139	94 -	5943	3682
11 -	97759	97430	39 -	95302	93593	67 -	76926	68246	95 -	4565	2786
12 -	97734	97401	40 -	95097	93322	68 -	75384	66254	96 -	3443	2060
13 -	97707	97367	41 -	94876	93028	69 -	73730	64166	97 -	2553	1511
14 -	97676	97322	42 -	94637	92712	70 -	71955	61984	98 -	1864	1087
15 -	97636	97261	43 -	94379	92368	71 -	70061	59715	99 -	1342	772
16 -	97588	97181	44 -	94098	91995	72 -	68044	57360	100 -	954	542
17 -	97531	97083	45 -	93793	91587	73 -	65890	54913	101 -	669	375
18 -	97467	96970	46 -	93461	91144	74 -	63582	52363	102 -	464	257
19 -	97400	96846	47 -	93101	90662	75 -	61107	49705	103 -	318	175
20 -	97331	96716	48 -	92714	90142	76 -	58464	46946	104 -	216	117
21 -	97261	96580	49 -	92299	89579	77 -	55664	44101	105 -	145	78
22 -	97190	96438	50 -	91852	88972	78 -	52717	41192	106 -	97	52
23 -	97117	96292	51 -	91372	88315	79 -	49638	38245	107 -	64	34
24 -	97042	96145	52 -	90855	87605	80 -	46445	35285	108 -	42	22
25 -	96966	96000	53 -	90300	86838	81 -	43149	32323	109 -	27	14
26 -	96888	95859	54 -	89704	86607	82 -	39769	29375			
27 -	96807	95721	55 -	89066	85110	83 -	36344	26469			

Title: The Cost of Mixing Reinsurance

Author: Ronald F. Wiser

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Abstract:

Excess and surplus lines underwriters, and others, rely heavily on facultative reinsurance support as an important part of their underwriting function. Individual risks are often subject to multiple reinsurance transactions as a result of the underwriting process. The net retained by the underwriters for the company's account is then subject to the overall company reinsurance treaty. As a result, the final company net position has been layered in a complicated fashion. It is management's task to provide guidelines for the proper use of facultative proportional and excess reinsurance that achieves corporate risk and profitability objectives under such conditions.

This paper investigates the impact on profitability of a common reinsurance mixing situation. The impact on the stability function of excess reinsurance is quantified. General rules to guide practical use and evaluation of mixed situations are developed.

These results are equally applicable to property as well as casualty risks. The implications are valid for facultative reinsurance underwriters, and others that make heavy use of facultative proportional reinsurance arrangements.

THE COST OF MIXING REINSURANCE

INTRODUCTION

Many underwriters rely heavily on facultative reinsurance support as an important part of their underwriting function. This is especially the case in the excess and surplus lines and commercial property lines. Individual risks are often subject to multiple reinsurance transactions as a result of the initial underwriting process. The net retained by the underwriters for the company's account is then subject to the overall company reinsurance treaty. As a result, the final company net retention has been layered in a complicated fashion. This complicated net position can lead to unexpected net loss ratio and combined ratio results.

The purpose of this paper is to investigate the consequences of one such reinsurance situation - the application of an excess of loss reinsurance treaty after the placement of proportional reinsurance on the same risk - and to investigate ways of managing this situation. We will take the viewpoint of the ceding company, although the subject is also of interest to the excess reinsurer. We will assume that, in general, the mixed reinsurance situation comes about through the application of proportional facultative reinsurance on individual risks, and the retained amounts are then subject to a corporate excess of loss treaty. In the case of a portfolio of risks, we assume the aggregate effect of individual facultative cessions can be adequately modeled by an average proportional retention applying to the entire portfolio.

The consequences of this mixed reinsurance situation are twofold:

a) Magnitude of net loss ratio. The application of proportional reinsurance below an excess of loss layer reduces the excess reinsurer's loss ratio and raises the ceding company's loss ratio. The expected loss ratio on the pro-rata reinsurance is unchanged; it will always be the same as the gross loss ratio.

b) Stability of net loss ratio. While the purpose of excess of loss reinsurance is to provide stability to the net retained loss ratio, the application of proportional reinsurance under the excess of loss cover actually decreases the stability of the net loss ratio.

A heuristic argument can be given that shows that each of these effects is intuitively plausible. Actual examples will show the mechanics of both the magnitude and the stability effect. Beyond the examples, it is demonstrated that these are not isolated instances, but the effects can be shown mathematically to always hold. We will use the term "mixing reinsurance" or "mixing" to denote this scenario of applying an excess of loss reinsurance treaty after a proportional transaction.

Reasons for Mixing:

As we investigate the implications of mixing proportional and excess reinsurance, we need to keep in mind the purpose for the particular mixing situations. Since all instances of mixing will penalize the net loss ratio to different extents, management must carefully evaluate whether the cost of mixing is justified by the advantage gained. Senior management is generally heavily involved in the process of negotiation and placement of the major treaties of the company. The use of facultative reinsurance has historically been directed by lower levels of management, right down to the individual desk underwriter who places quota share facultative reinsurance on a risk as he writes it.

The premise of this paper is that the total corporate reinsurance program (not just the major corporate treaties) must be actively managed to assure that corporate objectives are met. The interaction effects of proportional and excess reinsurance in the mixed case are so significant that management must institute guidelines and controls for use of proportional reinsurance that assure the objectives intended upon placement of the corporate excess treaties are not compromised. These objectives will generally be stated in the form of expected net loss ratio, or cost of reinsurance, and protection from large swings in net loss ratio (stability).

Some common reasons for mixed reinsurance situations to occur are:

a) Capacity: An individual risk is too large to be retained net by the insurer. A proportion of the risk may be ceded on a quota share or surplus share basis to cut down its size. This is common on property risks. A mixed situation exists if the corporate property treaty is on an excess of loss basis.

b) Net Premium Targets: A corporate plan may call for a certain net premium increase that must be strictly adhered to (for instance, because of statutory income or surplus restrictions). If more gross premium is written than plan, the net target may be achieved by increased use of facultative proportional reinsurance. This strategy needs to be evaluated in light of the penalty it will impose on the net loss ratio position.

c) Protecting the Treaty: If the rate on the excess treaty is clearly not sufficient to absorb the exposure from a risk the insurer wishes to write, the excess loss potential can be scaled down by a facultative quota share placement to fit the treaty pricing. This comes about because proportional reinsurance changes the frequency and severity characteristics of the excess loss exposure. This is one case where mixing reinsurance may be the prescribed course of action to achieve the corporate objective of excess treaty perpetuation at a reasonable price.

d) Sharing of Layers: For any of the reasons above the underwriter may substitute the direct writing of a proportional share of a risk, in place of acceptance of the entire risk followed by a facultative quota share reinsurance transaction. This is, in fact, a disguised mixed reinsurance situation and is fully equivalent in its effect on net loss ratio and stability. The popularity of sharing layers increases as the facultative reinsurance market tightens. The normal operating procedure of the facultative reinsurance underwriter or the brokered treaty underwriter to accept proportional shares of an excess layer is also a mixed reinsurance situation if an excess of loss treaty protects the reinsurers net position.

e) <u>Overrides</u>: In most cases, the proportional facultative reinsurer pays a ceding commission to the ceding company. This ceding commission is meant to cover direct commission costs, plus an additional "override" commission to cover the cedent's non-commission costs. The override has the effect of reducing the net expense ratio, and can even cause a negative net commission expense in some cases. A company, or an individual underwriter, may cede large amounts of facultative proportional reinsurance to obtain this override relief to the commission expense ratio.

<u>A Simple Example</u>: The magnitude effect can be demonstrated by inspecting a very simple situation. Suppose a ceding company has a size of loss distribution that allows only two claim sizes of either $10,000 or $90,000, of equal probability. With an expected claim frequency of 48 claims per year, and an average claim size of $50,000, we have annual expected losses of $2,400,000 annually. If the company carries an excess of loss treaty with a $40,000 retention, the treaty reinsurer will have expected losses of $1,200,000 per year (24 claims @ $50,000). Assuming an 80% expected loss ratio for both companies, the excess of loss reinsurer will expect a treaty rate of 50% of subject premium.

Now assume the underwriters writing this portfolio for the company place 50% quota share facultative reinsurance on every policy as they write it. The ceding company will retain 25% of gross premium, or $750,000, after paying for treaty and facultative reinsurance. The facultative reinsurer will pay half of every loss while the excess reinsurance only responds when the ceding company's 50% share of each loss penetrates the $40,000 retention. Since there are only 24 of these large losses expected, and after the proportional reinsurance they are $45,000 each, the excess reinsurer will have an expected incurred loss of $120,000. This will give it an expected loss ratio of 16% on the $750,000 of treaty premium. The ceding company will retain $1,080,000 of expected losses, for a loss ratio of 144% on its net retained premium of $750,000.

In this simplified example the two reinsurance negotiations have a combined unfavorable effect on the company. The treaty rate was correct for placement of 100% of the risk into the treaty. Because the underwriters did not tailor the facultative cessions to coordinate with the treaty rating, the company has suffered a penalty of 64 loss ratio points. Even though the direct business was correctly priced and evaluated, the net result is a totally unacceptable combined ratio. While the example is constructed to illustrate a point, real variations on this situation can easily occur. In fact, every instance of an excess of loss reinsurance contract placed over proportional reinsurance works to the disadvantage of the net position, and thus the ceding company.

THE ROLE OF THE SIZE OF LOSS DISTRIBUTION

An inspection of a typical size of loss distribution indicates the underlying cause of mixing effects. Consider a size of loss frequency distribution of the amount of a single claim, as shown in Figure 1. The amount of loss can be read from the horizontal scale, and the relative frequency of such a loss amount from the vertical scale. Figure 1 can also be used to determine the percent of total claim counts due to claims in a given range of amounts. For instance, we can see that losses over $150,000 will represent 20% of the claims arising from this particular loss distribution. This is because the area under the size of loss curve above $150,000 represents 20% of the total area under the curve.

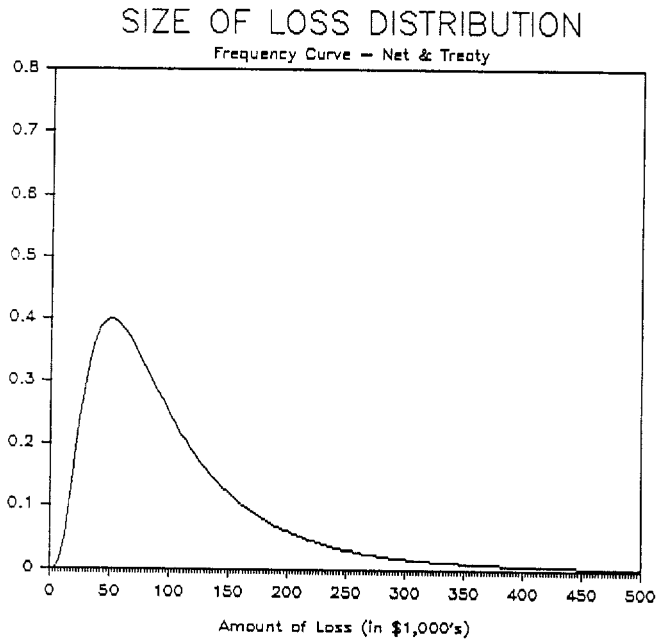

Figure 1.

The application of a 50% quota share reinsurance to this size of loss distribution essentially "shrinks" the curve horizontally, while maintaining its relative "shape", as shown in Figure 2.

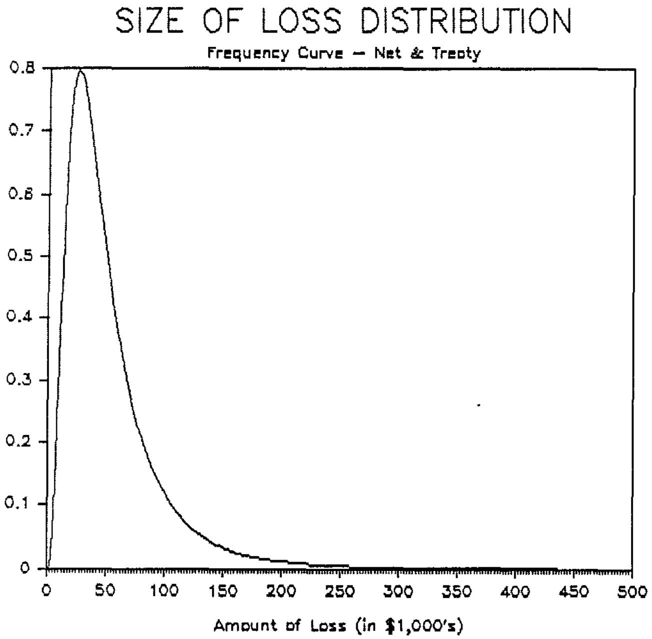

Figure 2.

Now consider the area of the "tail" of this new distribution over \$150,000. This area represented 20% of the total number of claims of the original loss distribution of Figure 1. However, the tail area of the "shrunken" distribution (Figure 2) over \$150,000 accounts for only 3.4% of total claim counts - much less than half of the original gross loss size distribution.

Thus, after the proportional "shrinking", the excess reinsurer will receive 50% of the premium that would have been received before proportional reinsurance was placed, but will experience much less penetration of its coverage layer than would have been expected in a situation without proportional reinsurance. In fact, the frequency of loss for the excess reinsurer after the 50% proportional reinsurance will be 17% (3.4% / 20%) of its original excess frequency. As a result, the excess reinsurer's expected net loss ratio after proportional reinsurance is now substantially improved over the experience before the proportional transactions.

Of course, this simply a consequence of the nonlinear nature of the size of loss distribution. It is another way of stating the fact that for large loss activity, a loss double a given size is experienced much less than half the time.

Note also that the area under the curve of Figure 2. over \$150,000 is the same as the area under the curve of Figure 1. over \$300,000 (\$150,000 / 50%). Thus the excess rate over \$150,000, after a 50% quota share placement, should be the same as the excess rate for a \$300,000 retention with no quota share, ignoring risk charge and expense components, and the effect of the upper limit on the excess layer.

In understanding the impact of proportional reinsurance on the net position and the excess reinsurer, the fundamental relationship is the simple idea illustrated above. An excess retention of M after a proportional reinsurance retention of 100a%, is equivalent to an excess retention of M/a without proportional reinsurance. This result is shown as the Mixing Price Rule below.

This relationship is also key in understanding how mixed reinsurance destabilizes net results. It seems intuitive, and can be shown mathematically (see the Appendix), that net aggregate loss results will show more stability (i.e., a lower coefficient of variation) under a $150,000 retention, than under a $300,000 retention. In general, if an entire portfolio is proportionally reinsured to retain 100a% of the total risk, with an excess of loss treaty with retention M, the stability of the portfolio's results will be identical to that of the same portfolio without proportional reinsurance and an excess loss limit of M/a. This result is shown as the Mixing Stability Rule below.

It is worth noting that the application of proportional reinsurance <u>after</u> an excess of loss treaty is applied does not change the magnitude or stability of the net loss ratio position. Hence the order of application of reinsurance is extremely important.

Some simple examples will be instructive, and show situations where a disadvantageous net position can come about in the ordinary course of business through mixing of reinsurance. This will be especially apparent if we consider the process of underwriting a single risk.

LOSS RATIO MAGNITUDE EFFECTS

A Casualty Example: Suppose an insurer is operating under an excess of loss treaty with $2,000,000 limits, excess of a retention of $250,000. The premium for this cover will be 30% of the subject premium that remains available for net and treaty; i.e. remaining after facultative placements.

The primary company underwriter writes an excess liability policy with limits of $1,000,000, excess of a self-insured retention of $100,000. He prices this at $400,000, expecting a loss ratio of 60%. He pays a commission of 15%, and his internal expenses will account for another 10% of the gross premium. This leaves him with 15% ($60,000) for profit and contingency load on this risk. This allows a 25% load on expected losses as a fluctuation margin. That is, the underwriter could suffer losses of up to $300,000, or 125% of expected losses, before he has to dip into his surplus funds.

Next, he wishes to reduce his net and treaty exposure to this risk, so he arranges a facultative quota share placement of 50% of the risk. Thus, he is left with a $500,000 exposure, net and treaty, and a subject premium for purposes of the excess treaty of $200,000.

Generally, the cedent will receive a ceding commission that will cover his direct ceding commission costs (15% in this example), plus an "override" that is meant to cover the cedent's non-commission, or fixed, expenses. The override for this example will be 10%, which is identical to the ceding reinsurer's other expense ratio.

One can analyze the underwriter's net position before his facultative quota share placement. Assume that a lognormal distribution is an adequate model (Benckert [1]) for size of loss on this risk, with a mean claim size of $30,000 and a coefficient of variation (CV) of 5.0. The following analysis of direct, reinsurance, and net results is summarized in Exhibit 1, the Mixing Cost Worksheet for this risk. Calculations on this exhibit are discussed below.

The size of loss assumption implies an average first-dollar claim severity of $270,190 in the layer of interest; hence an excess policy claim severity of $170,190. Recall that this is the expected severity for all claims greater than $100,000, but with a maximum ceding carrier liability of $1,000,000 on those claims that are greater than $1,100,000 first-dollar. Expected losses of $240,000, (60% X $400,000) imply an expected claim frequency of 1.41 claims per annum on this risk for the excess carrier ($240,000/$170,190). This analysis is displayed on Exhibit 1.4.

Now the excess of loss reinsurer would assume all loss amounts over $350,000 first-dollar, up to a maximum policy limit loss of $1,100,000 first-dollar. Thus the excess of loss reinsurer will be providing the coverage for the layer from $350,000 first-dollar to

MIXING COST WORKSHEET

Exhibit 1.

Policy:
A Casualty Example without Mixing

Input Parameters:

Direct Premium		$400,000
Policy Limits		$1,000,000
Underlying Retention		$100,000
Expected Loss Ratio		60.0%
Commission Ratio		15.0%
Other Expense Ratio		10.0%
Reinsurance:		
Percent Proportional		0.0%
Ceding Commission		25.0%
Excess Retention		$250,000
Excess Limits		$2,000,000
Excess Rate		30.0%
Ceding Commission		0.0%
Loss distribution:	Mean	$30,000
Lognormal	CV	5

Net Results:

	Gross	Proportional	Excess	Net
Loss Ratio	60.0%	NA	71.0%	55.3%
Expense Ratio	25.0%	NA	5.0%	35.7%
Combined Ratio	85.0%	NA	76..0%	91.0%
Net Underwriting Profit				$25,144
Cost of Reinsurance:				
with Mixing	$0	$0	$34,856	$34,856
Pure Excess	$0	$0	$34,856	$34,856
Additional Cost of Re	$0	$0	$0	$0
Cost of Mixing Calculation:				
Actual Cost of Excess Reinsurance			$34,856	
Cost based on Subject Premium			$34,856	
Cost of Mixing			$0	

MIXING COST WORKSHEET — Exhibit 1.1

Casualty Example
Allocation of Layer Costs &
Determination of Net Position

Policy Parameters:	(a) Gross	(b) Proportional	(c) Excess	(d) Net
1. Premium	$400,000	$0	$120,000	$280,000
2. Commission	$60,000	$0	$0	$60,000
3. Other Expenses	$40,000	$0	$6,000	$40,000
4. Expected Losses	$240,000	$0	$85,144	$154,856
5. Profit/Risk Charge	$60,000	$0	$28,856	$25,144
6. Retention	$100,000	NA	$250,000	$100,000
7. First-$ Equivalent*	$100,000	NA	$350,000	$100,000
8. Nominal layer width	1,000,000	$0	$2,000,000	$250,000
9. First-$ Equivalent*	1,100,000	NA	1,100,000	$350,000
10. Effective Layer Width	1,000,000	$0	750,000	$250,000
11. First-$ Equivalent*	1,100,000	NA	1,100,000	$350,000
12. Claim Severity	$170,192	$0	$298,113	$109,814
13. Claim Frequency	1.410	1.410	0.286	1.410
14. Commission Ratio	15.0%	25.0%	0.0%	21.4%
15. Other Expense Ratio	10.0%	3.0%	5.0%	14.3%
16. Premium rate	100.0%	0.0%	30.0%	70.0%
17. Fluctuation Loading	25.0%	NA	33.9%	16.2%
18. Expected Loss Ratio	60.0%	NA	71.0%	55.3%
19. Combined Ratio	85.0%	NA	76.0%	91.0%
20. Cost of Reinsurance	$0	$0	$34,856	$34,856

* First-Dollar Equivalent is the amount of first dollar loss needed to hit this limit.

Exhibit 1.2
Loss Distribution Table

	Loss Amount x	Number Distribution f#(x)	Amount Distribution f$(x)
Primary retention	$100,000	0.9417370	0.4069118
Reinsured's retention	$350,000	0.9881997	0.6767204
Primary policy limit	1,100,000	0.9981221	0.8627949
Effective Excess Limit	1,100,000	0.9981221	0.8627949

Distribution type: Lognormal
Distribution parameters:

Mean=	$30,000	MU=	8.6799043
CV=	5	Sigma=	1.8050198

Exhibit 1.3

Derivation of Loss Characteristics for Excess Treaty

	(a) Amounts	(b) f#(x)	(c) f$(x)
1. Primary Frequency	1.410		
First Dollar Equivalents:			
2. Primary retention	$100,000	0.94173699	0.4069118
3. Primary policy limit	$1,100,000	0.99812207	0.8627949
4. Reinsured's retention	$350,000	0.98819966	0.6767204
5. Effective Reinsurer limit	$1,100,000	0.99812207	0.8627949
6. Ratio of Excess carriers frequency to Primary frequency {1.0-(4b)}/{1.0-(2b)}	20.3%		
7. Excess layer frequency Expected claims per policy term (6)x(1)	0.286		
Severity Calculations:			
8. Mean loss (SOL)	$30,000		
9. Layer Loss Cost {(5c)-(4c)}x(8)	$5,582		
10. Limit Loss Cost (5a)x{1-(5b)}	$2,066		
11. Number of layer losses (5b)-(4b)	0.992%		
12. Number of limit losses 1.0-(5b)	0.188%		
13. Average severity of reinsured losses {(9)+(10)}/{(11)+(12)}	$648,113		
14. Less: Effective Retention	$350,000		
15. Excess layer severity (13)-(14)	$298,113		
16. Percent pro-rata reinsurance	0.0%		
17. Excess reinsurer's severity (15)x{1-(16)}	$298,113		

Exhibit 1.4

Derivation of Loss Characteristics for Primary Policy

	(a) Amounts	(b) f#(x)	(c) f$(x)
1. Expected Losses	$240,000		
First Dollar Equivalents:			
2. Primary retention	$100,000	0.94173699	0.4069118
3. Primary policy limit	$1,100,000	0.99812207	0.8627949
Severity Calculations			
4. Mean loss (SOL)	$30,000		
5. Layer Loss Cost {(3c)-(2c)}x(4)	$13,676		
6. Limit Loss Cost (3a)x{1-(3b)}	$2,066		
7. Number of layer losses (3b)-(2b)	5.639%		
8. Number of limit losses 1.0-(3b)	0.188%		
9. Average severity of primary losses {(5)+(6)}/{(7)+(8)}	$270,192		
10. Less: Retention	$100,000		
11. Primary policy severity (9)-(10)	$170,192		
12. Primary policy frequency Expected claims per policy term (1)/(11)	1.410		

$1,100,000 first-dollar for its $120,000 premium. Since 582 losses out of 10,000 exceed $100,000 first-dollar, and 118 losses out of 10,000 exceed $350,000 first-dollar, the excess of loss reinsurer's frequency will be 20% (118/582) of the direct reinsurer's frequency. Then, the reinsurer should expect 0.286 claims (1.41 X 20.3%) at an average severity of about $298,000 in the layer from $350,000 to $1,100,000 first-dollar. This implies a pure premium (expected losses) of about $85,000 (0.286 claims @ $298,113 each), and an expected loss ratio of 71% for the excess of loss reinsurer. This analysis of the excess carrier's frequency and severity is displayed on Exhibit 1.3.

The primary company underwriter retains an expected loss cost of $155,000 and a net premium of $280,000, for an expected loss ratio of 55%. This would leave $25,000 for profit and contingency load on the net position, giving a 16% loading of expected losses for a fluctuation margin.

Thus the primary company has paid 30% of its direct premium to the excess reinsurer. In return, its maximum exposure to loss from any one claim has been reduced from $1,000,000 to $250,000. However, the margin in the premium that is available to absorb fluctuations in results has also decreased from 25% to 16%. In light of this reduction in the fluctuation loading it is not immediately obvious whether the insurer is in a better position in terms of protection from random variation of results after this excess reinsurance transaction than before. However, as will be demonstrated below, excess of loss reinsurance decreases the probability of large aggregate losses to such a significant extent that this 16% risk margin actually reflects more safety than the gross position with its 25% margin.

On Exhibit 1. we have also calculated the cost of reinsurance. Of course, this is the expected cost of the reinsurance transaction. The actual cost in retrospect will vary considerably from year to year. The cost of reinsurance is simply defined as the reinsurance premium paid, less the sum of ceding commissions received and expected reinsurance recoveries. Note that since reinsurance is a service that provides value to the cedent, we should expect a positive cost of reinsurance to be the hallmark of any long term reinsurance relationship. This definition of cost of reinsurance ignores investment income lost by the ceding carrier, however this component may be required to get realistic cost estimates.

The cost of excess reinsurance in this case is $34,856, which can be expressed as a cost of $87.14 per $1,000 of premium subject to the excess treaty.

The Effect of A Proportional Cession: Now consider the net position of the ceding underwriter after a 50% proportional reinsurance transaction on this policy. As shown in Exhibits 2-2.3, $200,000 net and treaty premium remains, of which $60,000 must go to the excess of loss reinsurer. Since all losses are 50% shared before application of this excess of loss treaty, a first-dollar loss of at least $600,000 is needed before the excess of loss reinsurance responds. Since such a loss occurs for only 52 claims out of every 10,000, the excess of loss reinsurer's frequency has been cut to 9% of the reinsured's frequency by use of the proportional reinsurance (Exhibit 2.3).

The average severity of losses greater than $600,000 limited at $1,100,000 is $900,586. These losses are 50% quota shared above $100,000, so the pro-rata reinsurer and the reinsured split the layer, $500,000 excess of $100,000, evenly. Then the pro-rata reinsurer and the excess reinsurer split the next $500,000 loss layer evenly. This leaves the excess of loss reinsurer with an average claim severity of $150,293 in its layer. With a claim frequency of 0.126 claims in the excess reinsurance layer, the excess reinsurer has an expected loss cost of only about $19,000. However the reinsurer has received $60,000 of premium for the excess reinsurance, so it has now improved its expected loss ratio position to 31.4%.

Who pays for this improvement of the excess reinsurers loss ratio? Let's look at the proportional reinsurer's position. For 50% of the premium, the proportional reinsurer shares in all the gross losses equally. Thus the expected losses of the proportional reinsurer are $120,000. This indicates an expected loss ratio of 60% for the pro-rata reinsurer, the same as the gross loss ratio. In fact, the expected loss ratio of the quota share reinsurer will always be identical to that of the gross position.

MIXING COST WORKSHEET

Exhibit 2.

Policy:
A Casualty Example with Mixing

Input Parameters:

Direct Premium	$400,000
Policy Limits	$1,000,000
Underlying Retention	$100,000
Expected Loss Ratio	60.0%
Commission Ratio	15.0%
Other Expense Ratio	10.0%

Reinsurance:

Percent Proportional	50.0%
Ceding Commission	25.0%
Excess Retention	$250,000
Excess Limits	$2,000,000
Excess Rate	30.0%
Ceding Commission	0.0%

Loss distribution:	Mean	$30,000
Lognormal	CV	5

Net Results:

	Gross	Proportional	Excess	Net
Loss Ratio	60.0%	60.0%	31.5%	72.2%
Expense Ratio	25.0%	28.0%	5.0%	35.7%
Combined Ratio	85.0%	88.0%	36.5%	107.9%
Net Underwriting Profit				($11,081)
Cost of Reinsurance:				
with Mixing	$0	$30,000	$41,081	$71,081
Pure Excess	$0	$0	$34,856	$34,856
Additional Cost of Re	$0	$30,000	$6,225	$36,225

Cost of Mixing Calculation:

Actual Cost of Excess Reinsurance	$41,081
Cost based on Subject Premium	$17,428
Cost of Mixing	$23,653

MIXING COST WORKSHEET — Exhibit 2.1

Casualty Example
Allocation of Layer Costs &
Determination of Net Position

Policy Parameters:	(a) Gross	(b) Proportional	(c) Excess	(d) Net
1. Premium	$400,000	$200,000	$60,000	$140,000
2. Commission	$60,000	$50,000	$0	$10,000
3. Other Expenses	$40,000	$6,000	$3,000	$40,000
4. Expected Losses	$240,000	$120,000	$18,919	$101,081
5. Profit/Risk Charge	$60,000	$24,000	$38,081	($11,081)
6. Retention	$100,000	NA	$250,000	$100,000
7. First-$ Equivalent*	$100,000	NA	$600,000	$100,000
8. Nominal layer width	1,000,000	$500,000	$2,000,000	$250,000
9. First-$ Equivalent*	1,100,000	NA	1,100,000	$350,000
10. Effective Layer Width	1,000,000	$500,000	1,000,000	$250,000
11. First-$ Equivalent*	1,100,000	NA	1,100,000	$350,000
12. Claim Severity	$170,192	$85,096	$150,293	$71,680
13. Claim Frequency	1.410	1.410	0.126	1.410
14. Commission Ratio	15.0%	25.0%	0.0%	7.1%
15. Other Expense Ratio	10.0%	3.0%	5.0%	28.6%
16. Premium rate	100.0%	50.0%	30.0%	35.0%
17. Fluctuation Loading	25.0%	20.0%	201.3%	-11.0%
18. Expected Loss Ratio	60.0%	60.0%	31.5%	72.2%
19. Combined Ratio	85.0%	88.0%	36.5%	107.9%
20. Cost of Reinsurance	$0	$30,000	$41,081	$71,081

* First-Dollar Equivalent is the amount of first dollar loss needed to hit this limit.

Exhibit 2.2
Loss Distribution Table

	Loss Amount x	Number Distribution f#(x)	Amount Distribution f$(x)
Primary retention	$100,000	0.9417370	0.4069118
Reinsured's retention	$600,000	0.9947991	0.7755223
Primary policy limit	1,100,000	0.9981221	0.8627949
Effective Excess Limit	1,100,000	0.9981221	0.8627949

Distribution type: Lognormal
Distribution parameters:
Mean= $30,000 MU= 8.6799043
CV= 5 Sigma= 1.8050198

Exhibit 2.3

Derivation of Loss Characteristics for Excess Treaty

	(a) Amounts	(b) f#(x)	(c) f$(x)
1. Primary Frequency	1.410		
First Dollar Equivalents:			
2. Primary retention	$100,000	0.94173699	0.4069118
3. Primary policy limit	$1,100,000	0.99812207	0.8627949
4. Reinsured's retention	$600,000	0.99479906	0.7755222
5. Effective Reinsurer limit	$1,100,000	0.99812207	0.8627949
6. Ratio of Excess carriers frequency to Primary frequency {1.0-(4b)}/{1.0-(2b)}	8.9%		
7. Excess layer frequency Expected claims per policy term (6)x(1)	0.126		
Severity Calculations:			
8. Mean loss (SOL)	$30,000		
9. Layer Loss Cost {(5c)-(4c)}x(8)	$2,618		
10. Limit Loss Cost (5a)x{1-(5b)}	$2,066		
11. Number of layer losses (5b)-(4b)	0.332%		
12. Number of limit losses 1.0-(5b)	0.188%		
13. Average severity of reinsured losses {(9)+(10)}/{(11)+(12)}	$900,586		
14. Less: Effective Retention	$600,000		
15. Excess layer severity (13)-(14)	$300,586		
16. Percent pro-rata reinsurance	50.0%		
17. Excess reinsurer's severity (15)x{1-(16)}	$150,293		

Let's look at the net loss ratio, which was 60% gross, and 55% net before any facultative placement. Of the total expected loss costs of $240,000, the proportional reinsurer takes $120,000 and the excess reinsurer assumes $19,000. This leaves $101,000 of expected losses for the reinsured's net position. Since $140,000 of premium remains net, the expected net loss ratio is now 72%. This is substantially worse (17 loss ratio points) than the net loss ratio without any facultative proportional reinsurance. In addition, there is now no premium margin available for profit and contingency loading, since we are now at a combined ratio of 108%. Thus we see that use of proportional reinsurance below an excess of loss treaty simply moves loss dollars out of the excess reinsurer's account into the ceding insurer's account, without affecting the proportional reinsurer.

The Cost of Mixing: Notice that on Exhibit 2. we have calculated the Cost of Mixing. Recall that in the absence of any proportional reinsurance we calculated a cost of reinsurance of $87.14 per $1,000 of subject premium for the excess treaty. If we regard this cost as the reinsurer's price for providing an excess cover for this policy, we will hold this cost constant for any fraction of the policy that is retained after proportional reinsurance. This rate on the $200,000 of subject premium implies a cost of reinsurance $17,428 should be expected. However, the actual cost of reinsurance for the excess reinsurance in this mixed case is $41,081. We define the Cost of Mixing to be the difference of $23,653. Note that this Cost of Mixing is greater than the underwriting loss on the policy of $11,081. This implies that without the Cost of Mixing this net position should have been profitable for the ceding company. The total cost of reinsurance in the mixed situation can also be decomposed as follows:

Cost of Proportional Reinsurance	\$30,000
Cost of Excess Reinsurance	\$17,428
Cost of Mixing	\$23,653
Cost of Total Reinsurance	\$71,081

This example demonstrates a general principle that is independent of the choice of the size of loss distribution or policy parameters. That the net position after mixed reinsurance will always be worse than under a pure excess reinsurance is a corollary of the Mixing Price Rule. This Rule states that the excess loss rate for an excess retention of M after a proportional retention of 100a% must equal the loss rate for a pure excess retention of M/a.

The progressive deterioration of the loss ratio and combined ratio as the percent of proportional reinsurance increases can be seen from the table below. This table is for the casualty risk analyzed above, which has a gross expected loss ratio of 60%, with a gross combined ratio of 85%.

Percent Ceded	Net Loss Ratio	Expense Ratio	Combined Ratio
0%	55.3%	35.7%	91.0%
10%	58.0%	35.7%	93.7%
20%	61.0%	35.7%	96.7%
30%	64.3%	35.7%	100.0%
40%	68.0%	35.7%	103.7%
50%	72.2%	35.7%	107.9%
60%	77.0%	35.7%	112.7%
70%	82.6%	35.7%	118.3%
75%	85.7%	35.7%	121.4%
80%	85.7%	35.7%	121.4%
90%	85.7%	35.7%	121.4%

As the percent proportional ceded increases, losses are reduced for the excess reinsurer. These costs are shifted to the ceding company, and result in the increasing net loss ratio. Note that in the pure excess case, the loss ratio is reduced from 60% gross, to 55.3% net. However, the excess reinsurer pays no ceding commission. This increases the expense ratio, and hence the net combined ratio.

When 75% of the risk is proportionally reinsured, no losses can penetrate the excess retention. This is simply because policy limits are $1,000,000, and the 25% of each loss retained net and treaty can never be greater then the $250,000 excess treaty retention. At this point, ceding larger shares of a risk no longer affects the net loss ratio.

THE MIXING PRICE RULE

The mean value of a random variable representing the size of claim after application of proportional reinsurance and excess of loss reinsurance can be expressed analytically. This allows the calculation of the loss cost portion of the excess reinsurance rate. The risk charge and expense load components of the reinsurance rate are ignored for the purposes of this demonstration.

Let $f(x)$ be the probability density function of X, the random variable representing the amount of one claim. We will assume $f(x)$ is appropriately truncated to reflect the policy limit issued by the ceding carrier. Let a be the fraction of each loss retained by the ceding insurer after proportional reinsurance, and M the retention under the excess reinsurance program. (This notation is identical to that used in Centeno [2].)

Then, if X is the gross claim size, the amount of claim after both reinsurances apply is given by

$$X(a,M) = \mathrm{Min}\,(aX,M).$$

First, we establish the expected value of X under each single reinsurance type alone.

If only excess reinsurance applies,

$$E(\min(X,M)) = \int_0^M xf(x)dx + M\int_M^\infty f(x)dx.$$

If only proportional reinsurance applies,

$$E(aX) = a\int_0^\infty xf(x)dx.$$

It will also be useful to have an explicit formulation of the probability density of claim size under a proportional reinsurance. Let g_a be the density of x under proportional reinsurance that retains 100a% of each claim.

Then $g_a(x) = 1/a\ f(x/a)$, will yield the expected value above. (Note: This is a probability density function since

$$\int g_a(x)dx = (1/a)\int f(x/a)dx$$

Let y=ax, then dy = adx. Now we can substitute to obtain,

$$\int g_a(x)dx = (1/a)\int f(y)ady$$

$$= \int f(y)dy = 1)$$

Then applying excess of loss reinsurance to a claim after proportional reinsurance yields an expected value of

$$E(\min(aX,M) = \int_0^M xg_a(x)dx + M\int_M^\infty g_a(x)dx.$$

Again set ay=x, so that dx=ady and x=M iff y=M/a. Rewrite these integrals in terms of the variable y.

$$E(\min(aX,M)) = \int_0^{M/a} (ay)(1/a)f(y)ady + M \int_{M/a}^{\infty} (1/a)f(y)ady$$

$$= a \int_0^{M/a} yf(y)dy + M \int_{M/a}^{\infty} f(y)dy$$

$$= a[\int_0^{M/a} yf(y)dy + (M/a) \int_{M/a}^{\infty} f(y)dy]$$

$$= aE(\min(X,m/a))$$

This means that the expected value of the amount of a single loss under the combination of proportional reinsurance that retains 100a% of each claim, and excess reinsurance that retains the first M amount of each claim, is equivalent to 100a% of the expected value under an excess of loss reinsurance that retains the first M/a amount of each gross claim. This is a specific instance of the more general Mixing Moment Principle demonstrated below when we discuss stability

Excess treaty premiums are usually calculated using a rate in terms of a percent of subject premium.

Let Rate-XS(a,M) represent the excess rate for an excess retention M after a proportional retention of 100a%.

For purposes of simplifying the demonstration, recall that f(x) reflects underlying primary policy limits and assume that the excess treaty limit extends above the primary policy limits. This allows us to ignore the truncation term due to the excess layer limit.

If we consider only the loss component of the excess premium rate, then before any proportional reinsurance, the excess loss rate for limits of L over a retention of M will be

$$\text{Rate-XS}(1,M) = \frac{\int_{M}^{L+M} (x - M)\ f(x)dx + (L+M)\int_{L+M}^{\infty} f(x)dx}{\text{Subject-Premium}},$$ in the most general case.

Which simplifies to $$\text{Rate - XS}(1,M) = \frac{\int_{M}^{\infty} (X-M)\ f(x)dx}{\text{Subject-Premium}},$$ because of our assumptions.

After proportional reinsurance that retains 100a% of each claim, let XS-Rate(a,M) represent the rate. Then 100a% of the prior subject premium is now subject premium for the excess treaty, and

$$\text{Rate-XS}(a,M) = \frac{a[\int_{M/a}^{\infty} (x - M/a) f(x)dx]}{a(\text{Subject-Premium})}$$

$$= \frac{\int_{M/a}^{\infty} (x - M/a) f(x)dx}{\text{Subject-Premium}} = \text{Rate-XS}(1,M/a).$$

Thus, we can state the following:

Mixing Price Rule: The excess reinsurance loss rate for a retention M under a proportional reinsurance that retains 100a% of each loss is identical to the excess loss rate over a retention of M/a, with no proportional reinsurance.

Note one simple implication of the Mixing Price Rule. The limited mean of a distribution F under limit M is given by

$$E_M(x) = \int_0^M x \; dF + M(1 - F(M))$$

and is the "complement" of the excess loss cost $\int_M^\infty (x - M)dF$.

Then the excess reinsurance loss rate under a mixed reinsurance case must be smaller than under pure excess if and only if the limited mean of the distribution limited at M/a is larger than the limited mean at M. Thus we have the following:

<u>Mixing Loss Ratio Rule:</u> If the limited mean of a loss distribution is a strictly increasing function of the limit, then net loss ratio will always deteriorate under a mixed reinsurance case.

Only a most unusual loss distribution does not have the property of increasing limited means. Consider the following:

If $M_1 < M_2$ then

$$\int_{M_1}^\infty (x - M_1)dF = \int_{M_1}^{M_2} (x - M_1)dF + \int_{M_2}^\infty (x - M_1)dF$$

$$= \int_{M_1}^{M_2} (x - M_1)dF + \int_{M_2}^\infty (M_2 - M_1)dF + \int_{M_2}^\infty (x - M_2)dF$$

$$> \int_{M_2}^\infty (x - M_2)dF,$$

unless $\int_{M_1}^{M_2} (x - M_1)dF + \int_{M_2}^\infty (M_2 - M_1)dF = 0.$

The above sum of integrals is zero only if $dF=0$ for $x \geq M_1$.

Thus if $M_1 < M_2$, then $\int_{M_1}^\infty (x - M_1)dF > \int_{M_2}^\infty (x - M_2)dF$, hence $E_{M1} \leq E_{M2}$ with equality only if $dF=0$ for $x \geq M_1$. Practically, equality will only occur when f(x), the density associated with F, is truncated by policy limits.

We can write the full excess reinsurance rate as follows including the risk charge RC(a,M), and treaty expenses, Exp,

$$\text{XS-Rate}(a,M) = \frac{a\int_{M/a}^{\infty}(x - M/a)f(x)dx + RC(a,M) + Exp}{a(\text{Subject-Premium})}$$

Without further information about the form of the risk change, little more can be said about the excess rate. Note that Bühlman [3] has identified four premium calculation principles based on the form of the risk charge. These principles calculate the risk charge on the expected value, standard deviation, or variance of losses, or utility theory. If the premium calculation principle used in the excess rate is stated, then explicit calculations of equivalent excess rates in terms of the limit M/a are possible. This is investigated when the Mixing Stability rule is discussed.

APPLICATIONS TO PROPERTY INSURANCE

The phenomenon described in the casualty example is due to the shape of the size of loss distribution. The same deterioration of net loss ratio due to mixed reinsurance situations will occur in property situations, if the underlying size of loss distributions follow any of the accepted probability models. A study of this subject done by Shpilberg [4] indicates that a loss distribution that falls between the lognormal and Pareto distributions in its tail behavior is an adequate model for fire insurance. The Mixing Price Rule discussion shows that if the limited mean is an increasing function of the limit M, any mixture of proportional and excess of loss reinsurance worsens the net loss ratio.

As we have seen, the limited mean condition is not very restrictive. Any reasonable choice of size of loss distribution, especially the Pareto or lognormal, will satisfy this condition. Thus, the adverse consequences of mixing reinsurance will also hold for property risks.

There are, however, special characteristics of property risks that are notable. The policy limits of a property policy may be extremely large if there is a high Probable Maximum Loss level. The traditional approach to reducing this exposure to loss to a level appropriate for an excess reinsurance treaty is the use of proportional reinsurance. This can mean that a very high percentage of policy limits may be ceded, before excess reinsurance.

Thus, property risks are a particularly fertile ground for finding examples of mixed reinsurance situations. The use of facultative reinsurance on the large property risks is traditional and necessary to cut large policy limits down to net and treaty positions that are appropriate for the insurer's treaty capacity. This usage can have a great impact on the net loss ratio.

A property example will show similar net effects of proportional reinsurance as the casualty example already considered above.

Suppose the insurer has an excess of loss property treaty with $2,000,000 limits over a retention of $250,000, for this example. If a property risk that requires policy limits of $20 million is written, the underwriter must place $18 million of facultative reinsurance before he can place the remaining risk into his treaty. Most facultative property reinsurance has traditionally been on a proportional basis, so 90% of the premium must be ceded to the facultative reinsurers.

If the gross premium for the risk is $500,000, we will cede $450,000 to the facultative reinsurers, and retain $50,000 net as shown in Exhibit 3-3.4.

The results of the reinsurance can be quite different based on the type of property risk being underwritten. The differences we can attempt to model will be reflected in the Probable Maximum Loss (PML) potential, which should be closely related to the underlying size of loss distribution. The policy limits should also be based on the PML

MIXING COST WORKSHEET — Exhibit 3.

Policy:
A Property Example

Input Parameters:

Direct Premium	$500,000
Policy Limits	$20,000,000
Underlying Retention	$0
Expected Loss Ratio	60.0%
Commission Ratio	15.0%
Other Expense Ratio	10.0%

Reinsurance:

Percent Proportional	90.0%
Ceding Commission	25.0%
Excess Retention	$250,000
Excess Limits	$2,000,000
Excess Rate	30.0%
Ceding Commission	0.0%

Loss distribution:	Mean	$67,500
Lognormal	CV	10

Net Results:

	Gross	Proportional	Excess	Net
Loss Ratio	60.0%	60.0%	27.8%	73.8%
Expense Ratio	25.0%	28.0%	5.0%	35.7%
Combined Ratio	85.0%	88.0%	32.8%	109.5%
Net Underwriting Profit				($3,336)

Cost of Reinsurance:

	Gross	Proportional	Excess	Net
with Mixing	$0	$67,500	$10,836	$78,336
Pure Excess	$0	$0	$47,155	$47,155
Additional Cost of Re	$0	$67,500	($36,319)	$31,181

Cost of Mixing Calculation:

Actual Cost of Excess Reinsurance	$10,836
Cost based on Subject Premium	$4,715
Cost of Mixing	$6,121

MIXING COST WORKSHEET

Exhibit 3.1

Property Example
Allocation of Layer Costs &
Determination of Net Position

Policy Parameters:	(a) Gross	(b) Proportional	(c) Excess	(d) Net
1.Premium	$500,000	$450,000	$15,000	$35,000
2.Commission	$75,000	$112,500	$0	($37,500)
3.Other Expenses	$50,000	$13,500	$750	$50,000
4.Expected Losses	$300,000	$270,000	$4,164	$25,836
5.Profit/Risk Charge	$75,000	$54,000	$10,086	($3,336)
6.Retention	$0	NA	$250,000	$0
7.First-$ Equivalent*	$0	NA	$2,500,000	$0
8.Nominal layer width	20,000,000	$18,000,000	$2,000,000	$250,000
9.First-$ Equivalent*	20,000,000	NA	20,000,000	$250,000
10.Effective Layer Width	20,000,000	$18,000,000	20,000,000	$250,000
11.First-$ Equivalent*	20,000,000	NA	20,000,000	$250,000
12.Claim Severity	$65,577	$59,019	$310,572	$5,648
13.Claim Frequency	4.575	4.575	0.013	4.575
14.Commission Ratio	15.0%	25.0%	0.0%	-107.1%
15.Other Expense Ratio	10.0%	3.0%	5.0%	142.9%
16.Premium rate	100.0%	90.0%	30.0%	7.0%
17.Fluctuation Loading	25.0%	20.0%	242.2%	-12.9%
18.Expected Loss Ratio	60.0%	60.0%	27.8%	73.8%
19.Combined Ratio	85.0%	88.0%	32.8%	109.5%
20.Cost of Reinsurance	$0	$67,500	$10,836	$78,336

* First-Dollar Equivalent is the amount of first dollar loss needed to hit this limit.

Exhibit 3.2
Loss Distribution Table

	Loss Amount x	Number Distribution f#(x)	Amount Distribution f$(x)
Primary retention	$0	0.0000000	0.0000000
Reinsured's retention	$2,500,000	0.9970693	0.7281287
Primary policy limit	20,000,000	0.9999017	0.9423854
Effective Excess Limit	20,000,000	0.9999017	0.9423854

Distribution type: Lognormal
Distribution parameters:

Mean=	$67,500	MU=	8.8123226
CV=	10	Sigma=	2.1482831

Exhibit 3.3

Derivation of Loss Characteristics for Excess Treaty

	(a) Amounts	(b) f#(x)	(c) f$(x)
1.Primary Frequency	4.575		
First Dollar Equivalents:			
2.Primary retention	$0	0	0
3.Primary policy limit	$20,000,000	0.99990169	0.9423854
4.Reinsured's retention	$2,500,000	0.99706933	0.7281287
5.Effective Reinsurer limit	$20,000,000	0.99990169	0.9423854
6.Ratio of Excess carriers frequency to Primary frequency {1.0-(4b)}/{1.0-(2b)}	0.3%		
7.Excess layer frequency Expected claims per policy term (6)x(1)	0.013		
Severity Calculations:			
8.Mean loss (SOL)	$67,500		
9.Layer Loss Cost {(5c)-(4c)}x(8)	$14,462		
10.Limit Loss Cost (5a)x{1-(5b)}	$1,966		
11.Number of layer losses (5b)-(4b)	0.283%		
12.Number of limit losses 1.0-(5b)	0.010%		
13.Average severity of reinsured losses {(9)+(10)}/{(11)+(12)}	$5,605,719		
14.Less: Effective Retention	$2,500,000		
15.Excess layer severity (13)-(14)	$3,105,719		
16.Percent pro-rata reinsurance	90.0%		
17.Excess reinsurer's severity (15)x{1-(16)}	$310,572		

Derivation of Loss Characteristics for Primary Policy

	(a) Amounts	(b) f#(x)	(c) f$(x)
1.Expected Losses	$300,000		
First Dollar Equivalents:			
2.Primary retention	$0	0	0
3.Primary policy limit	$20,000,000	0.99990169	0.9423854
Severity Calculations			
4.Mean loss (SOL)	$67,500		
5.Layer Loss Cost {(3c)-(2c)}x(4)	$63,611		
6.Limit Loss Cost (3a)x{1-(3b)}	$1,966		
7.Number of layer losses (3b)-(2b)	99.990%		
8.Number of limit losses 1.0-(3b)	0.010%		
9.Average severity of primary losses {(5)+(6)}/{(7)+(8)}	$65,577		
10.Less: Retention	$0		
11.Primary policy severity (9)-(10)	$65,577		
12.Primary policy frequency Expected claims per policy term (1)/(11)	4.575		

potential. For instance, if the risk consists of a single large warehouse, there is a potential probability of losing the entire insured value. For the purposes of this discussion we will model this by choosing a size of loss distribution with 1 chance in 10,000 of a $20,000,000 loss. A lognormal distribution with a mean of $67,500 and a coefficient of variation of 10 is used for this size of loss. The net expected loss ratio in this case is shown in Exhibit 3 as 74%, with a combined ratio of 110%.

As expected, this net position compares unfavorably to the gross position with an 85% combined ratio. Note that this example demonstrates a capacity problem, where facultative reinsurance <u>must</u> be used before the treaty can come into use. The use of excess of loss facultative reinsurance in place of proportional may improve these net positions, if such reinsurance is available at an appropriate price. If not, the only recourse of the underwriter would be to price the gross risk appropriately to achieve his target 95% net combined ratio. A premium of $610,000 for this risk would be required to achieve a 95% combined ratio under this mixing situation with 90% proportional reinsurance. This would require pricing to a gross loss ratio of 49% and a gross combined ratio of 74% for the property. It is unlikely that the market-place will allow such pricing.

However, note one very important implication of this example. We can no longer assume the underwriter can price this risk on the basis of gross frequency and severity characteristics alone. In order to achieve combined ratio results that allow long-run survival of the ceding insurer, the gross price must be set based on gross frequency and severity, the excess reinsurance rate, the amount of proportional reinsurance needed for capacity, and the ceding commission structures.

The excess reinsurance rate must also anticipate some use of facultative reinsurance for capacity purposes. Specifically, for property risks the excess rate must be calculated anticipating a certain amount of use of proportional reinsurance. This will be the case if a loss rating approach using past experience is used to calculate the excess rate, and this past period reflects a similar use of proportional reinsurance as anticipated for the next treaty year.

OTHER MAGNITUDE EFFECT CONSIDERATIONS

The net results of both the casualty and property examples are not only a function of the percentage of proportional reinsurance used. Both the excess reinsurance rate and the ceding commission structure have an effect on the final net position. A detailed treatment of these subjects is not possible here, but some issues that relate to the magnitude effect need to be mentioned.

The Excess Reinsurance Rate: In the casualty example, an excess treaty was specified with a $2,000,000 limit over a $250,000 retention. Depending on the underlying size of loss distribution one might assume that a "correct" excess loss rate could simply be calculated from the distribution statistics. However, the policy subject to the excess reinsurance could be any one of the following.

A primary policy with policy limits of $2,250,000 that uses the entire reinsurance layer of $2,000,000.

If the primary policy limits are only $1,000,000 the rate should be substantially different.

If the $1,000,000 policy limits are excess of a self insured retention of $100,000, the appropriate rate for the excess reinsurance would again be different.

If the ceding company writes an excess policy for $1,000,000 limits over a primary policy with $500,000 limits, the correct excess reinsurance rate is again different from any of the above.

One can immediately see that with no change in the underlying risk's loss potential (as characterized by its size of loss distribution), several different, but "correct" excess reinsurance rates are possible. It becomes apparent that one cannot speak of a proper excess reinsurance rate on a portfolio without some measure of the anticipated underlying distributions of retentions and policy limits in the portfolio. Thus the excess reinsurance rate must be formulated in anticipation of a certain portfolio structure.

This point has practical implications that generate mixing situations. Suppose an excess reinsurance program has been negotiated, with the parameters agreed to for two years forward. At the time of the negotiation, management of the ceding carrier fully intended to write a book of small surplus lines SMP risks. An excess and surplus lines carrier is usually very reponsive to market opportunities; hence, six months into the program, management modifies its original marketing plan because conditions are excellent for obtaining strong rates on small casualty umbrellas. Management wants to take advantage of this opportunity. However, the original excess reinsurance rate, contemplating the SMP book, carried a provisional rate of 10%. The same calculations based on a book of small umbrella business would yield a proper rate of 35% for the excess reinsurance.

An excess reinsurance program can easily have 10 to 20 participants and have taken months of effort to place. Re-negotiating the treaty at every shift in portfolio composition is not a realistic option. Furthermore, the excess and surplus lines market depends heavily on the reinsurance market for capacity. Many such companies may cede out 50% or more of their gross writings. Thus, including this umbrella book in the treaty at an inadequate excess rate is not a viable option for a management that must be concerned about maintaining a long term presence in the market with consistent reinsurer support.

As a practical matter, the ceding underwriter has little real choice but to attempt to "protect the treaty". As we have seen, the ceding underwriter has great control over his treaty loss ratio, through his use of proportional facultative reinsurance. By altering the percent of proportional reinsurance placed on a risk, the size of loss characteristics of the net position can be fit to into the treaty rate structure.

Consider the casualty example given above to be representative of a typical umbrella policy. At a 10% rate, the excess reinsurer would receive $40,000 of premium and would have an expected loss ratio of 210% ($85,114 / $40,000), if no proportional reinsurance were placed. However, after the 50% proportional cession, the excess reinsurer would receive $20,000 of premium at the 10% rate. With expected losses of $18,853, this would yield an expected loss ratio of 94%, much better than the original 210%. Under the original scenario presented for the casualty example, the placement of 50% proportional reinsurance was not warranted. However, under this new scenario, the 50% proportional reinsurance should clearly be placed before the identical policy is placed into the excess treaty. The Cost of Mixing in this case should be paid to the excess reinsurer to bolster an inadequate treaty rate for a risk not contemplated in the original treaty price.

Thus, the situation is manageable, but becoming exceedingly complex. The underwriter must ascertain a correct price for the risk insured on a gross basis. This is no different from any underwriting situation. In addition, we again see that an essential part of the direct company's underwriting and pricing process must be the correct placement of reinsurance to achieve an acceptable net result. Even this is not enough, however. The underwriter must also balance out his net position against the results he is passing on to the excess reinsurer. He must be able to maintain long-term acceptable results for his excess reinsurance support, in the face of continuing shifts in his portfolio composition in response to market conditions.

The calculations we have made in our examples are complex and assume knowledge of the size of loss distribution underlying the policy. This is clearly an area where actuarial expertise can be applied to produce general guidelines and specific pricing procedures that aid in determining the net underwriting position. Without such pricing materials available, management will have no effective way of controlling and evaluating the proper, coordinated use of proportional and excess reinsurance.

The Gearing Factor: The existence of the override in the ceding commission has been remarked on above. The purpose of the override is to reimburse the ceding company for the non-commission expenses it incurred in writing the direct business. Unfortunately, in times of excessive reinsurance capacity the override is used as a competitive tool by reinsurers. Thus the casualty example considered above may be entitled to a 10% override based on the expense structure of the ceding carrier, however, a particularly aggressive reinsurer may offer an override of 15%. This, of course, makes the determination of the net position even less straight forward, and offers a powerful incentive to cede larger proportional reinsurance amounts.

Since the excessive override will tend to improve the combined ratio, while the mixing effect will act to worsen the combined ratio, it becomes even more imperative to calculate the net position before a risk is bound and facultative arrangements settled. For instance, the 50% proportional reinsurance on the casualty risk with a 15% override would yield the same net loss ratio of 72.2%, but an improved net combined ratio of 100.8%. The effect on the property example with 90% ceded proportional reinsurance is even more leveraged, with a net loss ratio of 73.8%, but a net combined ratio of 45.2%, much improved from the original 110%.

It can be the case that the combined effect of an excessive override and a large percent of proportional ceded reinsurance can not only cancel out the mixing penalty, but can also produce a favorable net combined ratio even when the direct risk is severely underpriced. For example, if the property risk example of Exhibit 3. were priced at a 100% gross loss ratio, the premium would be $300,000. Net retention after a 90% proportional reinsurance cession only would be $30,000 of written premium and expected losses. Expenses before ceding commission total 25% of gross premium, or $75,000. The ceding commission at a 15% override would total 30% of the $270,000 of ceded premium, or $81,000. Thus after the proportional cession the insurer would have net premium income of $30,000, and net costs as follows:

Net Incurred Losses:	$30,000
Direct Expenses:	$75,000
Ceding Commission:	($81,000)

Net Incurred Costs	$24,000

This is equivalent to a combined ratio of 80%, a substantial improvement over the direct combined ratio of 125% at which the risk was written direct. This aspect of the override in proportional reinsurance has been termed the "Gearing Factor" by Buchanan [5]. The existence of the gearing factor effect can overwhelm the unfavorable mixing effects in the transaction.

STABILITY EFFECTS

One of the less obvious effects of mixing proportional and excess of loss reinsurance types is the effect on the variation of the net loss ratio after reinsurance. The use of proportional reinsurance below an excess of loss treaty actually makes the resulting net aggregate loss costs more variable than would be the case under the excess treaty alone. This is significant because stability of net results is one of the most important benefits we are purchasing when we place an excess reinsurance treaty. Any degradation of the stability "component" of the excess treaty "product" makes the treaty worth less to us.

We will use the casualty policy example to form a small portfolio that will allow us to investigate the impact of mixing reinsurance on stability. Assume we have a portfolio of 50 policies identical to the casualty example. This means that we have a book of excess casualty business that generates $20 million of gross premium and an average of 70.5 claims annually (50 x 1.410). These claims follow the lognormal size of loss distribution specified earlier, i.e. with a mean of $30,000 and a CV of 5.0. The expected loss ratios on this book of business are identical to those on the single policy - that is, 60% gross, 55% if only the excess treaty is applied, but 72% in the mixed reinsurance case.

What does differ in the case of the portfolio from the single policy case is the distribution of the aggregate losses arising from the collection. As a simple demonstration of this, there is a substantial probability (24%) that the single policy will be loss-free. However, it is effectively impossible for the entire portfolio to be loss-free in any year (a probability of 2.4×10^{-31} of a loss-free year). The expected annual claim cost of the portfolio is $12,000,000 (70.5 claims @ $170,200) and the aggregate losses of the portfolio are distributed as shown in Figure 3. All computations of aggregate loss distributions were made using the algorithm developed by Heckman and Meyers [6].

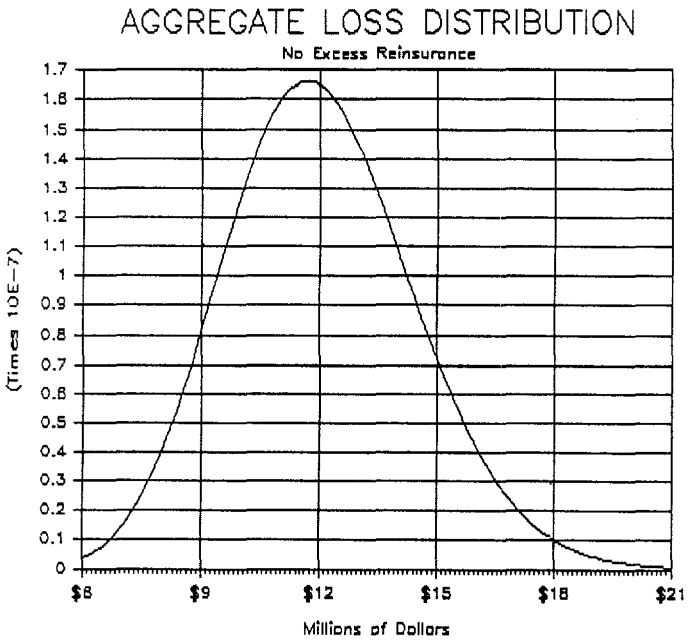

Figure 3.

In order to allow us to make comparisons between aggregate loss distributions we will normalize such distributions by setting the mean aggregate loss to 100%, and presenting the probabilities of achieving various percentages of the mean loss. This maintains the relative shape of the distribution and facilitates the comparison of different distributions with various underlying aggregate loss means. The normalized aggregate distribution of the unreinsured portfolio above can be seen as Figure 4. This distribution has a coefficient of variation of 0.2.

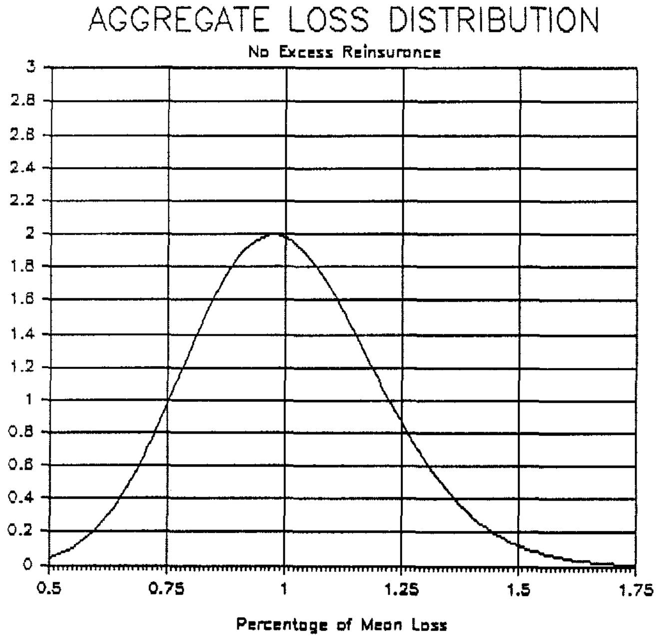

Figure 4.

After placement of the excess treaty on this portfolio the spread of the distribution is much reduced, as can be seen from Figure 5. below. Note that the probability of losses totalling over 150% of expected is substantially reduced by use of excess reinsurance, and the entire curve is distributed closer around its mean of 1.0. The coefficient of variation after excess reinsurance has reduced to 0.155.

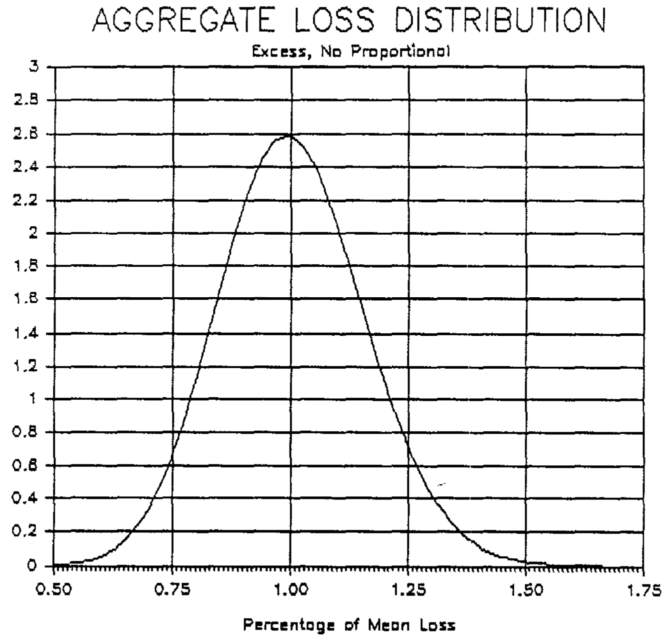

Figure 5.

Now, if the 50% proportional reinsurance is placed on each of the 50 policies in the portfolio, we obtain the aggregate loss distribution shown as Figure 6. This distribution clearly lies between the unlimited case and the pure excess case in its dispersion of possible loss amounts. Note the larger area under the curve over 150% of mean loss, for example, than under the pure excess treaty. The coefficient of variation has also increased to 0.175.

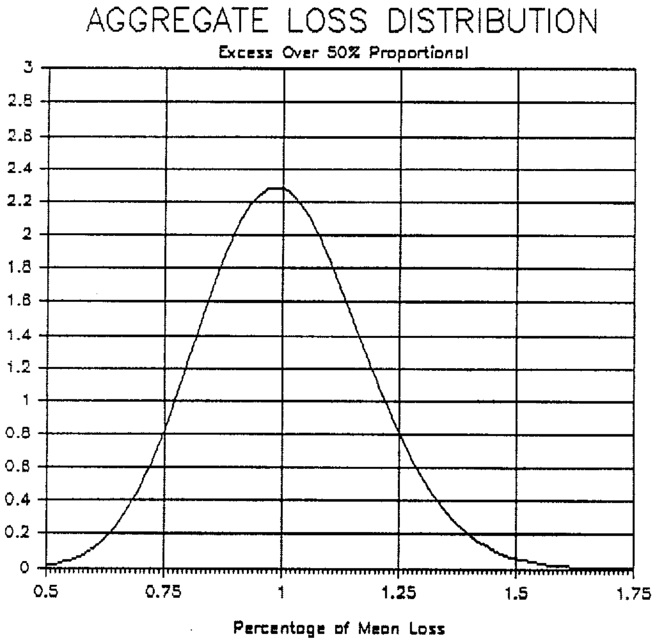

Figure 6.

Since all aggregate distributions are normalized, they can be compared on the same scale as shown in Figure 7. This chart shows that the "spread" of possible results around the mean loss in the mixed case lies in between the unlimited and pure net of excess distribution. In this sense, the stability paid for by purchase of excess reinsurance is "undone" by application of the proportional reinsurance.

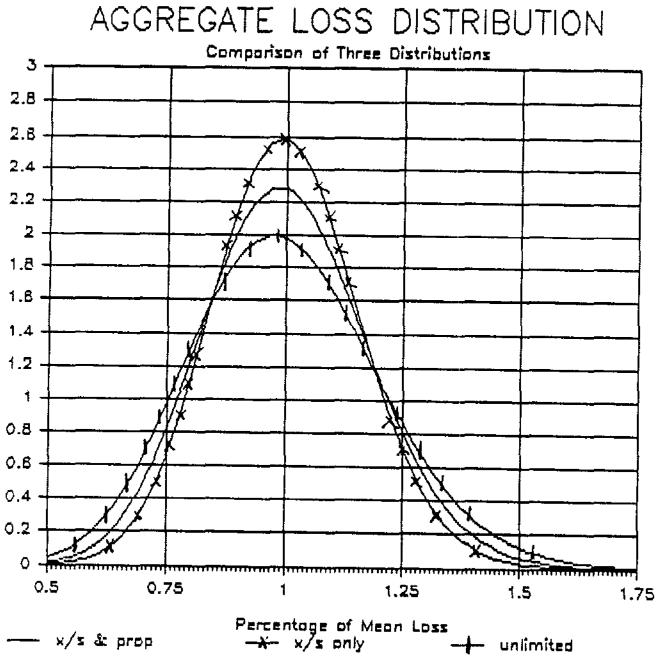

Figure 7.

In terms of the stability of the portfolio, we are most interested in the behavior of the aggregate loss distribution at the extreme right-hand tail. As shown in Figure 8., the tail behavior of the aggregate loss distribution in the mixed reinsurance case is substantially more severe than the pure excess treaty case.

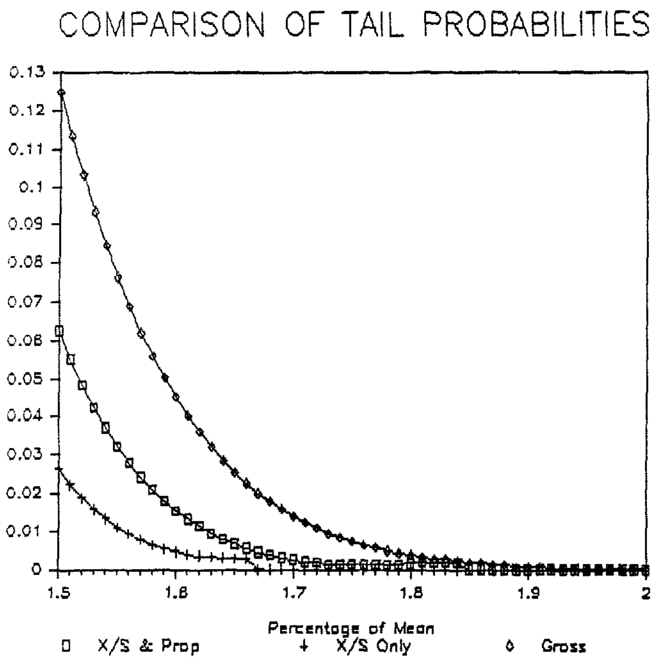

Figure 8.

The problem, of course, is that we are paying the same 30% rate of net and treaty premium for excess reinsurance protection in both the mixed reinsurance and pure excess case. As Figure 8. shows, the protection from extreme fluctuations we receive for our 30% rate is substantially less in the mixed case.

While the normalized aggregate distributions are useful for comparing aggregate loss distributions with disparate means, it is also important to focus on the bottom line - the distribution of combined ratios under the three different scenarios. The combined ratio becomes a random variable through the equation,

Combined Ratio = Expected-Loss-Ratio x Normalized-Aggregate-Loss
+ Expense Ratio.

Figure 9. shows the distribution of combined ratios for the three scenarios. Clearly, the range of alternatives under the mixed reinsurance scenario is the least desirable, not only in terms of its expected value, but also in terms of the probability of experiencing extremely adverse combined ratios. Note that there is little or no chance of a combined ratio over 120% in the case of the gross or pure excess case. However, the mixed case leaves us exposed to a substantial probability that a combined ratio over 120% will be experienced.

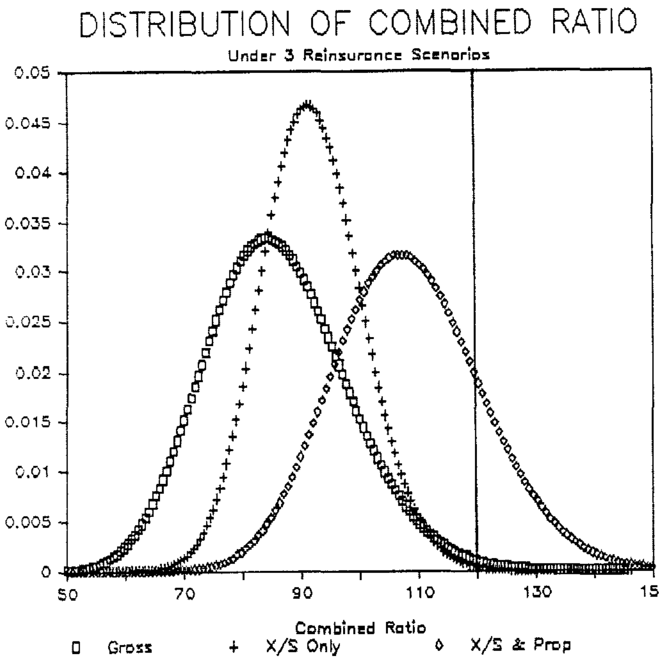

Figure 9.

Even the combined ratio comparison does not take the absolute scale into account. However, dollar magnitudes are important if we are to gauge the impact of the reinsurance programs on company surplus. An additional way of evaluating the bottom line is to simply review the distribution of statutory underwriting profit or loss. Profit can be represented as a random variable by,

Profit = Premium - Aggregate-Losses - Expenses

where Aggregate-Losses is the random variable we have been examining above, but not normalized. The resulting distribution is shown in Figure 10.

This chart is clearly of interest in evaluating ruin probabilities. Note that the gross loss distribution has a non-negligible probability of suffering an underwriting loss of over \$4 million. The pure excess reinsurance makes a loss of over \$3 million unlikely, and even the mixed case reduces the chance of suffering a \$4 million underwriting loss significantly. However, the price that must be paid for this protection in the mixed case is an expected underwriting loss. Thus the mixed case is clearly inferior in terms of both magnitude and stability of net underwriting results to pure excess reinsurance.

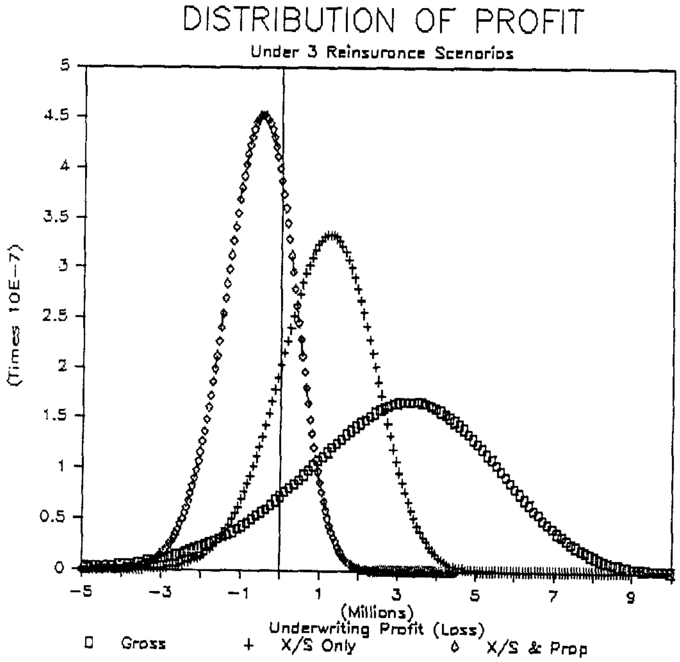

Figure 10.

A table representing the tail probabilities under the three scenarios can be useful and is presented below.

TAIL PROBABILITIES
Probabilities of Exceeding the Percent Mean

Percent of Mean	Gross	Type of Reinsurance: Excess Over Proportional	Excess Only
125%	11.07%	8.15%	5.77%
130%	7.45%	4.93%	3.09%
135%	4.85%	2.84%	1.55%
140%	3.06%	1.56%	0.73%
145%	1.87%	0.82%	0.32%
150%	1.11%	0.41%	0.14%
151%	1.00%	0.36%	0.11%
152%	0.89%	0.31%	0.09%
153%	0.80%	0.27%	0.08%
154%	0.72%	0.23%	0.07%
155%	0.64%	0.20%	0.05%
MEAN AGGREGATE LOSS	$12,000,000	$5,054,050	$ 7,742,800
NET PREMIUM	20,000,000	7,000,000	14,000,000
EXPENSES	5,000,000	2,500,000	5,000,000
EXPECTED U/W PROFIT	$ 3,000,000	$ (554,050)	$ 1,257,200

Using this table it is possible to investigate alternate scenarios, using proportional only or excess of loss only, to achieve a desired risk level with net incurred loss. For instance, suppose that the 50% proportional reinsurance was placed in order to keep the probability of an extra $3,000,000 loss at about 1% or less. From the middle column, there is about a 1% probability of a loss over 142% of mean aggregate loss in the mixed reinsurance case. This corresponds to $2.1 million dollars of loss over the expected amount of $5,054,050. Taking expenses into account, this would imply about a 1% chance of suffering an underwriting loss of $2.7 million. Note that in order to achieve this protection, the company will have an expected underwriting loss of about $500,000

Is there a more rewarding way to achieve the same risk position? There are at least two other reinsurance configurations that appear preferable. For instance, on a gross basis, there is a 1% probability of suffering loss of $18,000,000 or higher. This is equivalent to a 1% chance of an underwriting loss of $3,000,000 or more. A 10% cession of this portfolio would reduce the 1% level of loss to $2.7 million, and still leave an expected underwriting profit of $2.7 million. Even though the 90% proportional retention tail does not drop off as fast as the mixed case, the 1% level of risk is the same and expected profit is $3.2 million more.

Similarly, the 1% expected loss level for the excess of loss portfolio is 138%, of the mean, or an underwriting loss of $1.7 million. Thus, the 1% loss level is much lower than the mixed reinsurance case, and the expected value of $1.3 million is much better than the loss under the mixed case.

To summarize, at the 1% probability of loss level we have inspected three alternatives, and the mixed case is the least desirable.

	90% Quota Share	$250,000 Excess Over 50% Proportional	$250,000 Excess Only
1% level of U/W loss	($2,700,000)	($2,700,000)	($1,700,000)
Expected Profit	$2,700,000	($554,050)	$1,257,200

The above simple calculations hint at the complexity of the optimal reinsurance problem. Surprisingly, a considerable amount of work has been done by actuaries in studying this complex question. See, for instance, Beard, Pentikainen, and Pesonen [7] for a bibliography. Three related results of interest are given:

1. For a fixed amount of reinsurance premium, the optimum reinsurance (in terms of minimizing the variance of net results) is *aggregate stop loss, if one ignores risk* loadings [8].

2. If a safety loading that increases with variance is charged for reinsurance, the optimal reinsurance is proportional (quota-share) in the sense that it gives the minimum reinsurance cost for a given variance level [9].

Finally,

3. Centeno shows that with constraints on both the mean and variance, the minimal skewness of net aggregate losses, allowing mixed reinsurance treaties, is given by pure excess of loss reinsurance in most cases [10].

THE MIXING STABILITY RULE

A decrease in the amount retained after proportional reinsurance in a mixed reinsurance situation will decrease the stability of the net aggregate losses. In this sense proportional reinsurance will cancel out the major benefit of excess reinsurance.

As a measure of stability we will use the coefficient of variation of net aggregate loss results. Recall that if X is a random variable, we define

$$CV\ (X) = \frac{\text{Standard-Deviation}\ (X)}{\text{Mean}\ (X)}$$

Let X be the random variable representing the amount of one claim, and N be the random variable representing the number of claims in the experience period. Let M be amount retained under an excess of loss treaty, and 100a% be the percent retained under proportional reinsurance.

Let $X(a,M) = \min(aX,M)$ represent the net amount of one claim under both reinsurances. This is the random variable of claim amount under the mixed reinsurance situation.

Let λ_K be the k'th moment of N, the number of losses and β_K the k'th moment of X, the amount of loss. Then for any compound process Y defined by

$$Y = \Sigma_{i=1}^{N} X_i ,$$

we know that $E(Y) = \lambda_1 \beta_1$ and,

$$\mathrm{Var}\ (Y) = \lambda_1 \mathrm{Var}(X) + \mathrm{Var}(N)\beta_1^2$$ (see Miccolis [11]).

Thus, $$\mathrm{Var}\ (Y) = \lambda_1(\beta_2 - \beta_1^2) + (\lambda_2 - \lambda_1^2)\beta_1^2$$

in terms of central moments.

And, *in general*,

$$CV^2(Y) = \frac{\lambda_1\beta_2 + (\lambda_2 - \lambda_1 - \lambda_1^2)\beta_1^2}{(\lambda_1\beta_1)^2}$$

Which simplifies to

$$CV^2(Y) = \frac{\beta_2}{\lambda_1\beta_1^2} + \frac{\lambda_2 - \lambda_1 - \lambda_1^2}{\lambda_1^2} .$$

Both the mixing price and stability rules are essentially a result of of the following relationship that holds for the k'th central moment of X(a,M), denoted by $\beta_K(a,M)$.

Mixing Moment Principle: $\beta_k(a,M) = a^k \beta_k(1,M/a)$

Proof: By definition,

$$\beta_k(a,M) = \int_0^M x^k g_a(x)dx + M^k \int_M^\infty g_a(x)dx,$$

where $g_a(x) = (1/a)f(x/a)$ is the probability density of x under proportional reinsurance. If we set $ay = x$, then $ady = dx$, and $x = M$ iff $y = M/a$. Now rewrite β_k in terms of y,

$$\beta_k(a,M) = \int_0^{M/a} (ay)^k (1/a)f(y)ady + M^k \int_{M/a}^\infty (1/a)f(y)ady$$

$$= a^k \int_0^{M/a} y^k f(y)dy + M^k \int_{M/a}^\infty (y)dy,$$

$$\beta_k(a,M) = a^k[\int_0^{M/a} y^k f(y)dy + (M/a)^k \int_{M/a}^\infty f(y)dy],$$

$$= a^k \beta_k(1,M/a),$$

which proves the result.

Following notation in Centeno [2], let $Y(a,M)$ represent net aggregate loss after application of both the proportional and excess reinsurance. Then

$$Y(a,M) = \sum_{i=1}^N \min(aX_i,M).$$

We are interested in the stability of $Y(a,M)$ as a decreases. The following rule characterizes the stability of Y as a changes.

Mixing Stability Rule: The stability (coefficient of variation) of net aggregate losses after retention of 100a% under proportional reinsurance, and retention of M under an excess of loss treaty is equivalent to the stability of net aggregate losses under an excess treaty with a retention of M/a.

Proof: Write the coefficient of variation in terms of λ_i and $\beta_i(a,M)$,

$$CV(Y(a,M)) = \frac{[\lambda_1\beta_2(a,M) + (\lambda_2-\lambda_1-\lambda_1^2)\ \beta_1(a,M)^2]^{1/2}}{\lambda_1\beta_1(a,M)}$$

$$= \frac{[\lambda_1 a^2\beta_2(1,M/a) + (\lambda_2-\lambda_1-\lambda_1^2)a^2\beta_1(1,M/a)^2]^{1/2}}{\lambda_1 a\beta_1(1,M/a)}$$

$$= \frac{[\lambda_1\beta_2(1,M/a) + (\lambda_2-\lambda_1-\lambda_1^2)\beta_1(1,M/a)^2]^{1/2}}{\lambda_1\beta_1(1,M/a)}$$

$$= CV\ (Y(1,M/a))$$

which proves the result.

We would suspect that the stability of net losses decreases as the retention of the excess of loss treaty increases. That this is indeed the case is shown in the Appendix. Thus we can conclude that, in general, as the percent retained under proportional reinsurance decreases, and the excess of loss retention M remains fixed, the stability of net results of the portfolio decreases.

This shows that the situation of Figure 7. is not the result of any fortuitous choice of distributions or parameters. For any compound process, represented in general by Y(a,M), the distribution of net results after mixed reinsurance will show more "spread" than the pure excess reinsurance case but less than the gross position.

CONCLUSION

The application of an excess of loss treaty after a proportional reinsurance transaction on a policy has been shown to have a significant adverse impact on the net expected loss ratio. In addition, the stability of net results sought from the excess of loss reinsurance is also adversely affected. The Mixing Price Rule and Mixing Stability Rule allow us to evaluate these effects of the mixing situation. The Cost of Mixing Worksheet allows us to calculate the net position in a mixed reinsurance situation. These three tools should allow the underwriter to make appropriate evaluations of pricing and facultative reinsurance decisions in individual risk situations.

From a broader management perspective, the mixing of reinsurance at the individual risk level presents a difficult management control issue. In a worst case scenario, if company underwriters were to make facultative reinsurance arrangements without proper coordination and direction from management, a substantial loss ratio penalty on the entire book of business could be expected. Also the possibility of extremely adverse fluctuations in net results would result. The challenge for management is to promulgate guidelines and controls that assure individual underwriters understand enough about the overall corporate reinsurance structure and objectives to make decisions on individual risk facultative reinsurance placements that work with, not against, the excess treaty. It is hoped that the ideas developed here will give actuaries a start in attempting to explore this aspect of the underwriting and pricing process.

As actuaries become aware of the significant impact of reinsurance on net results, it becomes apparent that simply pricing a risk at a profitable direct premium is not sufficient to assure a net profit when significant amounts of different reinsurances apply. As our examples show, one can have perfect knowledge of the the direct frequency and severity characteristics of a risk, and price the risk perfectly on a direct basis, yet still have an unfavorable net combined ratio, due to facultative placements that generate high mixing costs.

On a total corporate level, the more subtle concept of probability of ruin comes into play. We have shown that unanticipated large amounts of proportional placements can destabilize net results significantly. While most insurance organizations are large enough to make the probability of ruin merely of academic interest, the chance of suffering extremely large combined ratios increases as the share retained on a proportional basis decreases. The protection paid for in the cost of the excess treaty is negated by proportional reinsurance.

Finally, most of the discussion has been from the viewpoint of the ceding company. However, the mixing cost can work both ways. The excess treaty rate is calculated anticipating a certain percent of the book will be ceded proportionally before the the treaty applies. If the ceding company finds that it can only cede a smaller than anticipated portion of its business facultatively, it will be putting larger shares of each risk into the treaty. This will result in a highly leveraged adverse loss ratio and destabilization effect on the excess treaty. This is an issue that the excess reinsurer must be sensitive to, as well as the ceding company.

Pricing actuaries on both sides of the excess reinsurance treaty transaction clearly have an interest in the mixing effects. The more use a ceding company makes of proportional reinsurance prior to the treaty, the more important the mixing effect becomes. The more we are aware of the effects of mixing, the less likely is either party to the treaty to suffer unexpected adverse consequences of mixing.

APPENDIX

Theorem: As the fraction α retained under proportional reinsurance decreases, the stability of the net aggregate losses decreases.

Proof: We wish to prove that as α decreases, the quantity $CV(Y(a,M))$ decreases. From the Mixing Stability Rule, it suffices to prove that if $M_1 < M_2$, then,

$$CV(Y(1,M_1)) < CV(Y(1,M_2)).$$

This is the case if

$$(\delta/\delta M)\ CV(Y(1,M)) > 0,$$

which is equivalent to

$$(\delta/\delta M)\ CV^2(Y(1,M)) > 0, \text{ because } CV \geq 0.$$

Let β_K represent $\beta_K(1,M)$, then

$$CV^2\ (Y(1,M)) = \frac{\lambda_1\beta_2 + (\lambda_2 - \lambda_1{}^2 - \lambda_1)\beta_1{}^2}{\lambda_1{}^2\beta_1{}^2}$$

$$= \frac{\beta_2}{\lambda_1\beta_1{}^2} + \frac{(\lambda_2 - \lambda_1{}^2 - \lambda_1)}{\lambda_2{}^2} .$$

Since only β_K is a function of M,

$$(\delta/\delta M)\ CV^2(Y(1,m)) = \frac{\lambda_1\beta_1{}^2\beta_2{}' - 2\beta_2\lambda_1\beta_1{}'\beta_1}{(\lambda_1\beta_1{}^2)^2}$$

$$= \frac{\beta_1\beta_2{}' - 2\beta_2\beta_1{}'}{\lambda_1\beta_1{}^3}$$

Thus, $(\delta/\delta M)\ (CV^2(Y(1,M)) > 0$ iff

$$\beta_1\beta_2' - 2\beta_2\beta_1' > 0.$$

Now compute β_1' and β_2',

$$(\delta/\delta M)\ \beta_1 = \delta/\delta M\ \left(\int_0^M x dF + M(1-F(M))\right)$$

$$= 1 - F(M), \text{ and}$$

$$(\delta/\delta M)\ \beta_2 = \delta/\delta M\left(\int_0^M x^2 dF + M^2\ (1-F(M))\right)$$

$$= 2M(1-F(M)).$$

$$\text{Let}\quad I_1 = \int_0^M x dF \text{ and}$$

$$I_2 = \int_0^M x^2 dF.$$

Then, $\beta_1\beta_2' = [I_1 + M(1-F(M))]\ [2M(1-F(M)]$, and

$$2\beta_2\beta_1' = 2[I_2 + M^2(1-F(M))]\ [1-F(M)].$$

So,

$$\beta_1\beta_2' - 2\beta_2\beta_1' = 2I_1M(1-F(M)) - 2I_2(1-F(M))$$

$$= 2(1-F(M))\ (MI_1-I_2)$$

$$= 2(1-F(M))\int_0^M x(M-x)dF.$$

Since $0 < x < M$ we know $M-x > 0$, hence this integral is positive, and the result is proved.

(The author thanks Professor Nasser Hadidi of the University of Wisconsin-Stout for his helpful discussions on this proof).

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