

## ACTUARIAL ASPECTS OF FINANCIAL REPORTING

by

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### Abstract

Financial reports of property/casualty insurance companies are notoriously difficult to interpret. A major reason for this difficulty is that the actuarially generated elements of those statements are usually not understood. Often they are not even identifiable.

By the use of a fairly simple model, relationships between actuarial analysis and financial statement figures can be displayed. Once the sources of data in actuarial data bases and the flow of actuarial projections into financial statements are identified at the conceptual level, progress can be made toward financial reports with which non-actuaries can feel comfortable.

The first model illustrated is called deterministic because all growth, contingency, reporting, and payment patterns are uniform. The flow of information to and from actuarial models is easily followed. Predictions can be made with a high level of comfort.

The second model is labeled stochastic. This is to clarify that some of the uniformity from the first model is relaxed. Growth rates, reporting patterns, and payment patterns are allowed to fluctuate in this model.

These models are used to identify and study the interrelationships between various actuarial projections and financial statements. Clearly, the actuarial elements of an enterprise's financial statements should be understood by as wide an audience as possible. By the use of the simple models illustrated in the paper, the interrelationships between the rating, reserving, and financial reporting functions can be examined and more fully appreciated. While the development of these relationships can be a difficult task in practice, the increased level of understanding is well worth the effort.

ACTUARIAL ASPECTS OF FINANCIAL REPORTING

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## ACTUARIAL ASPECTS OF FINANCING REPORTING

### OVERVIEW

Casualty actuaries are most often thought of as insurance professionals who perform ratemaking and reserve studies. While this may be a fairly accurate representation in general, it abstracts away much of the essence of actuarial science. It also leads to a sense of mystique about rate and reserve figures which is often unwarranted.

Three key elements of casualty actuarial work are mathematics, economics, and accounting. Because casualty actuaries are in the forefront of the struggle to evaluate the contingencies facing an insurer, they must be able to formulate algorithms and fit parameters by which to predict losses. Not only must they determine the likelihood of a loss and the amount of a loss from a given exposure, they must also determine when the loss is likely to become known by the insurer and when it will be paid.

The mathematics involved in the evaluation of casualty contingencies is formidable. The analysis includes fitting curves to frequency and severity distributions and combining them to produce an expected loss distribution for a coverage at a point in time. Predicting losses at a different point in time requires development of a growth function. These mathematical aspects of actuarial science are the least understood and most feared by non-actuaries.

A second key element of actuarial work is economics. Particularly in their pricing role, actuaries are performing an economic function. Producing an "actuarially correct" rate indication is an empty exercise if an inappropriate rate of return

results. The rate of return will be inappropriate if elasticity of demand is such that sales drop unacceptably. It will also be inappropriate if the rate algorithm used does not incorporate rate of return in an economically meaningful manner.

The accounting function of the actuary is in many ways the most important. An actuary's math can be precise, but be applied to bad data. An algorithm can be properly applied to good data, but the results can be misinterpreted or misutilized. Proper effort applied to the accounting aspect of actuarial science can assure actuarial calculations are properly applied and that the results are properly interpreted.

#### A DETERMINISTIC MODEL

It is the premise of this paper that a major reason financial statements of property/casualty insurers are difficult to interpret and utilize is that the actuarial elements impacting the figures are not well understood by the preparers and users of those statements. In addition, the financial statements into which the actuarial elements are flowing are not always understood by actuaries. As a result, a good conceptual grasp of the actuarial aspects of financial statements is hard to develop.

To illustrate the flow of actuarial elements into financial statements, a very simple model is needed. If too many complicating elements were introduced, the relationships would be difficult to trace. The idea is to see how basic reserve and ratemaking procedures impact and are impacted by financial statements.

#### Reconstructed Historical Financial Statement

Perhaps the report (not ordinarily produced) which would shed the most light on the issue is a historical financial report in which loss figures are identified

by accident year. Calendar year incurred losses are composed of payments and reserve changes on losses from a number of accident years. As such, they bury the key figures which casualty actuaries use in reserve and rate computations.

Exhibit 1A is an example of a historical financial report in which losses are broken into accident year components. The figures in this exhibit result from a "deterministic" insurance process. In other words, losses are reported and paid according to predetermined patterns, and premium and loss levels grow at predetermined rates.

A number of simplifying assumptions are made in order to illustrate the fundamental relationships in question. The model company writes in one line in one state. (Alternatively, all lines and states are aggregated for reporting, reserving, and ratemaking purposes.) No reinsurance is assumed or ceded. All transactions are on a cash basis. Investment and tax rates are aggregated.

A beginning level of written premium of \$1 million is assumed. Unearned premium is assumed to be 50% of written premium. Expenses are set at 25% of earned premiums. A more realistic approach would have expenses as a function of written and earned premiums, but for illustrative purposes the relationships have been aggregated into a single percentage.

Incurred losses are set at 75% of earned premium. Paid losses, case reserves, and IBNR reserves for a calendar-accident year combination are a function of the assumed payout and reporting patterns. Because the process is assumed to be deterministic, IBNR reserves are always accurate.

Underwriting cash flow is defined to be written premiums less paid losses less expenses. Underwriting income is earned premiums less incurred losses less expenses.

Written premium growth is a function of exposure growth, average premium growth, and mix of business effect. Exposure growth reflects change in number of policies written. Average premium growth reflects change in premium per exposure resulting from rate changes. Mix of business effect is change in premium resulting from a change in demographic makeup of policyholders toward higher or lower rated classifications. The assumed rate of growth for each growth type is, respectively, 10%, 5%, 5%.

Incurred loss growth is a function of exposure growth, frequency growth, and severity growth. Frequency represents the average number of claims per exposure unit, and severity the average cost of a claim. The growth rates for these loss elements are chosen to correspond with the growth rates for the premium elements.

The relationships chosen for this model produce a zero underwriting gain for each year. Because of the predictability of events rates keep up with losses and expenses. A zero underwriting gain is assumed to produce the target rate of return for the model company.

Investment income is a function of beginning of year assets, underwriting cash flow, and pre-tax average investment return on assets. For simplicity, it is assumed that full investment rate is earned on beginning of year assets while half the rate is earned on underwriting cash flow.

Tax is defined to be 50% of underwriting profit plus 20% of investment income. The lower rate on investment income assumes 60% of the income from the portfolio is from non-taxable instruments.

End of year assets are defined as beginning of year assets plus underwriting cash flow plus investment income less taxes. End of year liabilities are the sum of unearned premium reserves, case reserves, and IBNR reserves. Surplus is the difference between assets and liabilities. Change in surplus is after-tax underwriting plus investment gain.

Discounted calendar year incurred losses represents the sum of past payments and present value of future payments on accident years not fully paid. The present values are computed from the payment schedules in Exhibit 1B. The discounted loss reserve at any point in time is the present value of future payments on claims from accident years with claims still outstanding.

Discounted liabilities are the sum of unearned premium reserve and discounted loss reserves. Discounted surplus is the difference between assets and discounted liabilities. GAAP adjustment is defined to be 20% of unearned premium reserve. GAAP surplus is statutory surplus plus the GAAP adjustment. GAAP income is underwriting gain plus investment income less taxes plus change in GAAP adjustment. It should be noted that the fact that expenses were earlier made a function of earned premium makes it unlikely that in a real company such an adjustment would be needed to assure proper matching of income and expense.

This reconstructed historical financial statement, then, is the primary document showing the relationship between actuarial analysis and financial reporting. By

decomposing calendar year losses into their accident year components we reveal data used by actuaries in their deliberations. We are thus in a position to analyze the flow from actuarial models to financial reports.

#### Loss Development Analysis

As was indicated above, reserve estimation is one of the areas most closely identified with actuaries and their work. While very sophisticated procedures have been developed by which to estimate ultimate liabilities as of a point in time, the basic idea is quite simple. One must review liability estimates on the books and formulate a model by which to adjust those liabilities to a "best estimate" basis, assuming the booked figures are not determined to be appropriate.

The most common model for producing ultimate loss estimates is one which examines groups of accidents for historical periods and evaluates patterns by which they were paid, reported, and reserved. All other things being equal, these historical patterns are assumed to continue into the relevant future.

Exhibit 1B builds a simple reserving data base from data in the reconstructed historical financial statement. Loss figures are arranged by accident year and calendar year in the traditional manner. Very simple reserve models often utilize such data.

Because this is a deterministic model, the development patterns are totally stable. Growth in paid and reported losses from one maturity point to the next is uniform for each accident year. This is a function of the assumed uniform reporting and payment patterns mentioned previously.

Reserve estimation in such an environment is fairly routine, assuming no changes can be anticipated. Because 1.4 times as many losses are always reported in two years as were reported at one year maturity, it can be projected that the most current year's reported losses will be 40% higher next year. Likewise, it can be assumed that paid losses on the most recent accident year will be three times as high by next year.

Because loss reserving is always accurate in such a world, there is a one to one relationship between the loss figures in financial statements and those in reserve models. If such were not the case, incurred estimates for an accident year would change from one calendar year to the next as new information leads to more refined estimates. There would be a reserve table corresponding to each calendar year's financial statement, rather than a single table resulting from and feeding into a five year financial statement.

The paid and reported development factors produced in this model can be used to produce projected incremental future payments and reports by calendar year. We can thus project how historical accident years will impact results of future calendar years. Also, the projected payment schedule can be used to determine the discounted value of an accident year's loss payments at various points in time. Some elements of the relationship between an actuarial reserve model and a company's final financial statements are now becoming more obvious.

#### Rate Analysis

Ultimate incurred loss estimates produced in the reserve model for a coverage would flow into the rate level analysis for that coverage. They are often the

most critical component of a rate filing in terms of sensitivity of the rate indication. A relationship is thus established between financial statements and rate analyses.

Two critical relationships between rate analysis and financial statements involve incurred losses and rate indications. The incurred losses should tie back to financial statement figures. The rate indication, if implemented, would impact future premium levels and thereby financial statement figures. Premium and expense figures should be consistent with financial statement figures.

The rate model presented in Exhibit 1C is a simple one. The rate algorithm is peripheral to illustrating relationships between actuarial analysis and financial statements. The basic idea of any rate model, including this one, is that premiums to be collected be sufficient to produce the proper rate of return for the insuring entity. The historical figures should be consistent with those in other company reports. Financial projections should account for expected impact of the rate change, including an evaluation of demand elasticity.

#### Projected Financial Statements

Financial projections can be used for a variety of purposes. Examples are company planning, merger and acquisition, and investment strategy. As a result, it is important that these projections be as realistic as possible. They must also be understandable to non-actuarial people using them.

Exhibit 1D provides key elements of projected financial statements of a property/casualty insurer. It reproduces the historical years 1980-84 and projects results for the next five years. The primary addition to the projection model

relative to the historical model is the assumed flow through of the rate indication in 1985. Since the rate change is zero and no elasticity is assumed, the projections follow the historical figures.

The column headings in Exhibit 1D are identical to those in Exhibit 1A. Growth assumptions after 1984 are the same as those prior to 1984 except that the rate increase flows through written premium in 1985. Surplus continues to grow as investment income flows through to surplus.

This company's planning process is fairly simple. Budgets can be met by holding expenses to 25% of earned premium. Evaluating the company for merger and acquisition is also routine since net income and cash flow can be projected with a high level of comfort. Similarly, investment strategy is simplified by the fact that maturity of liabilities and taxable gains are so predictable.

#### Deterministic Model Summary

We have seen that when insurance contingencies are predictable and when complicating elements are abstracted away, the relationships between actuarial models and financial statements are fairly straightforward. As predictability of losses decreases and complications increase, these relationships become more convoluted. Nonetheless, by definition the financial statement figures of an insuring entity must ultimately be tied back to their sources, which include actuarial data bases and analyses.

#### A "STOCHASTIC" MODEL

Developing a risk theoretic model of the insurance process is beyond the scope of this paper, and would add little to the understanding of the fundamental

relationships between actuarial analyses and financial statement figures. Notwithstanding this, however, there are some random and non-normal factors which affect actuarial work and which lead to terminology which confuses non-actuaries. This confusion can lead to misinterpretation of the results of actuarial studies and misuse of the figures.

#### Reconstructed Historical Financial Statement

Exhibit 2A shows financial statement figures resulting from an insurance process which does not have uniform growth rates, payment patterns, or reporting patterns. To allow key relationships between the actuarial models and the financial statements to be easily illustrated, distortions from a deterministic process have been minimized. This model merely adds a few complicating elements and some terminology.

Actuaries often speak in terms of frequency, severity, and pure premiums. This model allows growth in frequency and severity of claims to diverge from each other and from premium growth. This leads to fluctuation in underwriting results.

This model also allows payment and reporting patterns to fluctuate from one year to the next. This opens up the possibility of changes in ultimate incurred estimates for an accident year from one calendar year to the next. Such changes would lead to reconstructed reserve models for each calendar year's development.

Written premium growth from one year to the next in this model is a function of exposure growth, growth in average gross premium, and growth in mix of business. For 1981 the respective growth rates utilized are 5%, 10%, and 5%. For 1982 through 1984 the growth rates are (5%, 10%, 5%); (5%, 5%, 5%); and (5%, 10%, 5%). Unearned premium and expense ratios are as established in the deterministic model.

Growth in incurred losses is a function of exposure growth, frequency growth, and severity growth. The respective growth rates assumed in 1981 are 5%, 5%, and 10%. Growth rates for 1982 through 1984 are (5%, 5%, 10%); (5%, 5%, 10%), and (5%, 5%, 10%). In 1984 an additional 5% growth factor is added.

Paid loss patterns are assumed to be uniform except for accident year 1981. The payment pattern for 1981 by calendar year is .2, .2, .2, .2, .2. For other years it is .1, .2, .3, .2, .2.

Reporting patterns and resulting case reserves are also uniform except for 1981. The cumulative pattern for 1981 is .4, .6, .8, .9, 1.0. For other years it is .5, .7, .8, .9, 1.0.

Other financial statement items are defined as they were in the previous model.

#### Loss Development Analysis

As was the case with the first model, the data base for loss development is extracted from the reconstructed historical financial statements. In surveying the development factors in Exhibit 2B we can see the impact of the non-uniform reporting and payment patterns for accident year 1981. To simplify the analysis, however, we have assumed that the reserve actuary for the entity was clever enough to see that 1981 was distorted. As a result, incurred estimates for each accident year as of each calendar year are the same.

The other aspects of reserve analysis for this model are analogous to those of the first model. Payments and reports are projected out and payments are discounted as of each point in time to allow for the option of discounted liabilities.

### Rate Analysis

Rate-making also becomes more complicated when non-uniformity is introduced. We see in Exhibit 2C that loss ratios fluctuate somewhat from year to year. The interaction of the various growth assumptions has led to a slight upward trend in loss ratio and the need for a rate increase. This rate increase will flow into projected financial statements.

It is interesting to note that incurred losses for accident years 1983 and 1984 in the rate analysis are different than the corresponding figure in the financial statements. The explanation for this is that the rating actuary used average incurred development factors in projecting ultimate losses. The 1981 distortion is thereby projected forward in the rate model. As a result, the loss figures and rate indication are somewhat overstated.

### Projected Financial Statements

Financial projections in this model are done under a greater degree of uncertainty. Because historical patterns have not been uniform, prediction even under the ceteris paribus assumption is more difficult. Even if future patterns can be assumed to follow those of the past, an assumption must be made as to which of the past patterns are likely to influence future figures. Exhibit 2D displays the financial projections for the stochastic model.

The years 1980-1983 are reconstructed to eliminate the premium growth anomaly in 1982. The rate of premium growth in 1985 is a function of exposure growth, average premium growth, mix of business growth, the rate increase, and zero elasticity effect. Growth rates for 1986-1989 are functions of exposure, average premium, and mix of business growth per the following: (5%, 10%, 5%); (5%, 10%, 5%); (5%, 10%, 5%); (5%, 10%, 5%).

Incurring loss growth is a function of exposure growth, frequency growth, and severity growth. The respective growth rates for 1985 are 5%, 5%, and 10%. For 1986-1989 the rates are: (5%, 5%, 10%); (5%, 5%, 10%); (5%, 5%, 10%); (5%, 5%, 10%).

Payment and reporting patterns are assumed to follow those of the historical accident years excluding 1981. Resulting payments, case reserves, and IBNR reserves therefore follow those patterns. Other elements of the projected financial statement are produced analogously to those in the historical statements.

#### Stochastic Model Summary

While the "stochastic" model added some complications, simplifying assumptions allow us to continue to trace relationships between actuarial analyses and financial reports. The more these assumptions are relaxed, and the more operating complexities added, the more abstruse these relationships become. Nonetheless, if complications are added incrementally, the relationships can continue to be observed.

## CONCLUSION

This paper has attempted to build a bridge between actuarial models and financial statement figures. Financial statements aggregate components of actuarial models. As a result, many factors which could make use and interpretation of insurance financial statements easier are not available for review. By explicitly identifying some of these factors reports can be produced which allow management to see and evaluate elements which have influenced past and may influence future results.

The models in this paper identify a few key actuarial elements and show how they interrelate with financial statement figures. The primary element allowing for the analysis is the identification of accident year components of calendar year figures. Actuaries use accident year data in producing many of the figures they provide to management.

The models here deal with two systems of relationships. The calendar year system is primarily composed of figures which show how much income was earned in a period and which show as of a point in time the volume of assets and liabilities which have arisen.

The key components of income are the premiums earned in a year, the losses which accrue, the expenses which accrue, and the investment earnings which arise. Assets and liabilities change based on the cash flow which arises and the future obligations which accrue. Much of the actuary's role involves a determination as to how loss obligations accrue over time.

The accident year system provides data which is organized in such a manner as to allow the actuary to estimate future losses based on patterns in which losses

on the books have arisen. There are two critical aspects to the loss estimation. The first aspect involves production of financial statement figures which reflect historical activity. Loss development analysis is used to estimate the extent to which losses the company is liable for at some point will differ from losses recognized by the company. Historical patterns of loss payment and loss reporting are used to determine how booked loss figures are likely to change.

The second type of loss estimation involves projection of losses likely to arise in future periods. This analysis is part of an actuary's ratemaking activity. Historical losses brought up to ultimate levels by development analysis are reviewed and compared to exposure measures to determine the rate at which losses are changing. This historical rate of change is used to predict future loss levels.

Because losses for property/casualty insurance coverages can vary in amount, it is often helpful to review trends in numbers of claims separately from trends in average claim size. These trends in frequency and severity, respectively, can be combined into a pure premium trend which measures change in loss cost per exposure unit.

The losses projected for future periods provide a basis for determining needed rate level. They can also be used to project future financial results. This process of developing losses to ultimate level and projecting them forward is one of the major functions a casualty actuary plays in the process of producing components of financial statements.

We have seen that actuarial methodology is conceptually related to financial reporting. Demonstrating this for a large multi-line, multi-state insurer would

involve enormous effort, and might be challenged on a cost/benefit basis. Working it through for a specialized entity like a one state malpractice carrier might prove both enlightening and fruitful.

Expansion of the model to encompass credibility considerations, loss distribution functions, changes in accident year incurred estimates by calendar year, reinsurance programs, and similar items, while adding complication, would increase understanding of the relationships. Only when actuaries can demonstrate how their data bases and projections relate to other aspects of company reporting can they expect non-actuaries to consistently and properly interpret financial statements of property/casualty insurance companies. Improper interpretation of those statements can lead to improper planning, improper marketing and underwriting decisions, and improper investment strategies. It can also lead to unnecessarily strained relationships with regulatory authorities.

## Technical Appendix 1

### Deterministic Model

<u>Item</u>	<u>Comment</u>
Direct Written Premium	The initial value is set at \$1 million. Values for the years 1981-1984 are a function of the assumed growth rates in exposure, average premium, and mix of business. The growth rates are 10%, 5%, and 5%. Since the rate change is 0, these same growth rates persist in projected years 1985-1989.
Unearned Premium Reserve	This value is set at 50% of written premium.
Direct Earned Premium	This is defined to be written premium less change in unearned premium reserve.
Direct Expense	This is set at 25% of earned premium.
Paid Losses	Accident year paid losses in a calendar year are a function of ultimate incurred losses for the accident year and the assumed payment pattern. This model assumes 10% of ultimate losses are paid in the first year, 20% in the second, 30% in the third, and 20% in each of the fourth and fifth.
Case Reserves	Case reserves for an accident year in a given calendar year are a function of ultimate accident year incurred losses, and the assumed reporting and payment patterns. At any point in time case reserves are reported losses less paid losses. The reporting pattern assumed is 50%, 20%, 10%, 10%, 10%. As an example, after two years an accident year will have 70% of its losses reported and 30% paid. The difference, 40%, is case reserves.
IBNR	IBNR for an accident year in a given calendar year is the difference between ultimate amount for the year and the amount reported as of the calendar year. At the end of the second year, for example, 70% of ultimate losses are reported so 30% are incurred but not reported.
Incurred Losses	This model assumes the loss elements move in correspondence with the premium elements. As a result, a 75% loss ratio is maintained. Premiums and losses grow each year by $(1.05)^2(1.10)$ .
U/W Cash Flow	Underwriting cash flow is defined as written premium less paid losses less expenses.

Deterministic Model--Continued

<u>Item</u>	<u>Comment</u>
U/W Gain	Underwriting gain is defined as earned premium minus incurred loss minus expense.
Investment Income	Investment income is the earnings rate (10%) times the sum of beginning of year assets and half the underwriting cash flow.
Tax	Tax is the tax rate (50%) times the sum of underwriting gain and 40% of investment income.
End of Year Assets	End of year assets are beginning of year assets plus underwriting cash flow plus investment income less taxes.
End of Year Liabilities	End of year liabilities are the sum of the case reserves, IBNR, and unearned premium reserves.
Surplus	Surplus is the difference between assets and liabilities. The change in surplus is the after-tax underwriting and investment gain.
Discounted Incurred Losses	Discounted incurred losses for an accident year in a particular calendar year represents the sum of past payments at that point in time plus the present value of future payments.
Discounted Loss Reserves	Discounted loss reserves in a calendar year are the sum of the present values of the payments remaining for each accident year.
Discounted Liabilities	Discounted liabilities are the sum of discounted loss reserves and unearned premium reserves.
Discounted Surplus	Discounted surplus is the difference between assets and discounted liabilities.
GAAP Adjustment	The GAAP adjustment is defined to be 20% of the unearned premium reserve.
GAAP Surplus	GAAP surplus is statutory surplus plus the GAAP adjustment.
GAAP Income	GAAP income is statutory income plus change in GAAP adjustment.
Beginning Exposure	Initial exposure is set at 1,000 units.
Exposure Trends	Annual change in exposure can be 5% or 10%.

Deterministic Model--Continued

<u>Item</u>	<u>Comment</u>
Average Premium	Initial average premium is set at \$1,000.
Average Premium Trend	Average premium may change at the rate of 5% or 10%.
Mix of Business Trend	The impact of changing mix of business is set at at unity at the beginning. Mix of business effect can be 5% or 10% per year. Thereafter the change can be a function of additional coverage being provided or a shift toward higher rated policyholders.
Frequency Trend	Initial frequency is set at 10%. The change in frequency can be 5% or 10% per year.
Severity Trend	Initial average claim size is set at \$7,500. This can increase at rates of 5% or 10% per year.

## Technical Appendix 2

### Stochastic Model

The income and balance sheet items in this model are defined in the same way as they are defined in the deterministic model. Where this model differs is in the specification of a couple of growth factors, reporting patterns, and payment patterns. To illustrate the impact of non-uniformity in some elements of the insurance process on the relationship of actuarial calculations to financial statement figures, some variations were introduced.

The first change from the uniformity of the first model is that the average premium growth rate for 1982 is reduced to zero. In 1983 it is increased so as to produce the same premium that year as was produced in the deterministic model. The second change is an additional 5% growth in losses for the 1984 accident year. The impact of these changes is most readily apparent in reviewing underwriting gain which turns negative in 1982.

The other changes introduced in this model involve payment and reporting patterns. Accident year 1981 is given payment and reporting patterns which differ from those of the other years. The impact of this is best seen in the loss development tables which show loss development factors for 1981 which differ from those of other years at the first and second points.

This model best illustrates the type of situation faced by casualty actuaries performing their rating and reserving roles. When the various factors influencing loss amounts begin to vary the degree of mathematical and professional sophistication needed to project future losses increases. The casualty actuary must often look at inconclusive historical movements and attempt to build a model which best predicts the future therefrom.

PROPERTY-CASUALTY F1  
DETERMINISTIC

I. FINANCIAL HISTORY

	TRENDS 1		TRENDS 2	
NET EXPOSURE	1000	.05	.10	
AVERAGE PREMIUM	1000	.05	.10	
RISK OF BUSINESS	1	.05	.10	
FREQUENCY	.10	.05	.10	
SEVERITY	7500	.05	.10	
PAYMENT PATTERN	.1	.2	.3	.2
CLAIM PAY PATTERN	.1	.3	.4	.0
REPORT PATTERN	.5	.2	.1	.1
CLAIM REP PATTERN	.5	.7	.0	.9

CAL YEAR	DIRECT PREMIUM	DIRECT EARNED PREMIUM		DIRECT EXPENSE	BOY ASSETS	ACC YEAR	PAID LOSSES	CASE RESERVE	UNPAID LOSSES	U/W CASH FLOW	U/W BAIN	EMERG INCOME	TAT	EDY ASSETS	EDY LIAB	EDY SURPLUS	BISC INCURRED LOSSES		BISC LOSSES RESERVES		BISC LIAB	BISC SURPLUS	GAAP ADJUST	GAAP SURPLUS	GAAP INCOME	
		UEPR	EPREMIUM														LOSS	RESERVE								
1980	1000000	500000	500000	125000	0	1980	37500	150000	187500	375000	837500	0	41875	8375	871000	837500	33500	319348	201848	781848	89452	100000	133500	133500		
1981	1212756	606375	1166375	376594	871000	1980	75000	150000	112500	375000	778178	0	126009	23702	1748965	1615678	134107	341872	706658	855037	1661407	288579	121775	255587	127087	
1982	1478763	735181	1341756	325439	1749985	1981	112500	75000	75000	375000								361519	760903							
1983	1781667	891836	1627215	406804	2676469	1982	168956	331913	248934	829781	756236	0	212810	42562	2676469	2371914	304555	856975	1404830	2180712	536257	147676	451632	196019		
1984	2163143	1081571	1973405	493351	3619376	1983	75000	37500	37500	375000								371510	799950							
						1981	248934	163956	163956	829781									922786							
						1982	201263	402527	301895	1006317	729425	0	304128	60826	3649396	3101538	567858	1039296	1911736	2803570	845826	178167	776225	274593		
						1983	122041	488164	610204	1220411																
						1980	75000	0	0	375000									375000							
						1981	163956	82978	82978	829781									422058							
						1982	301895	201263	201263	1006317									970139							
						1983	244087	488164	366123	1220411									1119108							
						1984	148005	592021	740027	1480054	734852	0	401682	80336	4705594	3836391	864704	1260106	2309968	3471539	1234035	216314	1085518	359293		

II. LOSS DEVELOPMENT TABLES

ACC YEAR	REPORTED LOSSES				
	1	2	3	4	5
1990	187500	262500	300000	337500	375000
1981	414891	580817	663825	746803	
1982	503159	704422	805054		
1983	610206	854288			
1984	740027				

ACC YEAR	PAID LOSSES				
	1	2	3	4	5
1980	37500	112500	225000	300000	375000
1981	82978	248934	497869	643825	
1982	100432	301895	603790		
1983	122041	366123			
1984	148005				

ACC YEAR	REPORTED DEVELOPMENT			
	2:1	3:2	4:3	5:4
1990	1.4000	1.1429	1.1250	1.1111
1981	1.4000	1.1429	1.1250	
1982	1.4000	1.1429		
1983	1.4000			
1984				

ACC YEAR	PAID DEVELOPMENT			
	2:1	3:2	4:3	5:4
1980	3.0000	2.0000	1.3333	1.2500
1981	3.0000	2.0000	1.3333	
1982	3.0000	2.0000		
1983	3.0000			
1984				

AVERAGE	1.4000	1.1429	1.1250	1.1111
CHOSEN	1.4000	1.1429	1.1250	1.1111
ULTIMATE	2.0000	1.4286	1.2500	1.1111

AVERAGE	3.0000	2.0000	1.3333	1.2500
CHOSEN	3.0000	2.0000	1.3333	1.2500
ULTIMATE	10.0000	3.3333	1.6667	1.2500

ACC YEAR	REPORTED LOSSES				
	1	2	3	4	5 ULTIMATE
1990	187500	262500	300000	337500	375000
1981	414891	580817	663825	746803	829781
1982	503159	704422	805054	905805	1004317
1983	610206	854288	976329	1098370	1220411
1984	740027	1036038	1184045	1332048	1480054
1985	897468	1256455	1435948	1615642	1794935
1986	1086106	1523765	1781416	1959127	2176808
1987	1319942	1847916	2111437	2375931	2639923
1988	1609781	2241097	2561254	2981410	3201567
1989	1941350	2717890	3106160	3494430	3882700

ACC YEAR	PAID LOSSES				
	1	2	3	4	5
1980	37500	112500	225000	300000	375000
1981	82978	248934	497869	643825	829781
1982	100432	301895	603790	805054	1004317
1983	122041	366123	732747	976329	1220411
1984	148005	444016	888032	1184045	1480054
1985	179494	538481	1076961	1435948	1794935
1986	217481	653042	1306085	1741416	2176808
1987	263992	781977	1563955	2111930	2639923
1988	320157	964670	1929340	2561254	3201567
1989	388270	1164810	2329620	3106160	3882700

ACC YEAR	INCREMENTAL PAID LOSSES				
	1	2	3	4	5
1980	37500	75000	112500	75000	75000
1981	82978	165956	248934	165956	165956
1982	100432	201263	301895	201263	201263
1983	122041	244082	366123	244082	244082
1984	148005	296011	444016	296011	296011
1985	179494	358987	538481	358987	358987
1986	217481	435362	653042	435362	435362
1987	263992	527985	781977	527985	527985
1988	320157	640313	964670	640313	640313
1989	388270	776540	1164810	776540	776540

ACC YEAR	DISCOUNTED PAID LOSSES				
	1	2	3	4	5
1980	319348	345872	361518	371518	375000
1981	706638	760903	799950	822050	829781
1982	856975	922786	970159	996951	1004317
1983	1019276	1119108	1176537	1209052	1220411
1984	1266406	1357199	1426845	1466278	1480054
1985	1520558	1645942	1730406	1778229	1794935
1986	1833750	1996117	2098550	2156547	2176808
1987	2248146	2426791	2545016	2615352	2639923
1988	2726430	2915814	3084849	3171769	3201567
1989	3306488	3560408	3743115	3846562	3882700

III. RATE LEV. L. ANALYSIS

ACC/CAL YEAR	WRITTEN PREMIUM	EARNED PREMIUM	RATE LEVEL	EARNED PREM AT CURR RT	EXPECTED LOSSES	DEVEL FACTOR	ULTIMATE LOSSES	EXPENSES	LOSS RATIO AT CURR RT	ANNUAL TREND	EXPENSE RATIO AT CURR RT	ANNUAL TREND	PROJ OP RATIO
1980	1800000	500000	1.00	500000	375000	1.0000	375000	125000	75.00%		25.00%		100.00%
1981	1212750	1104375	1.00	1104375	748801	1.1111	829781	276594	75.00%	100.00%	25.00%	100.00%	100.00%
1982	1470743	1341254	1.00	1341254	805034	1.2500	1006317	335439	75.00%	100.00%	25.00%	100.00%	100.00%
1983	1783447	1627215	1.00	1627215	834788	1.4286	1220411	404804	75.00%	100.00%	25.00%	100.00%	100.00%
1984	2143143	1973405	1.00	1973405	740077	2.0000	1480054	493331	75.00%	100.00%	25.00%	100.00%	100.00%
										100.00%		100.00%	

  

PROJ YEAR	LOSS RATIO	BISCUIT FACTOR	BISC LOSS RATIO	EXPENSE RATIO	RATE TND
1985	75.00%	.7786	58.40%	25.00%	1.0000

IV. FINANCIAL PROJECTIONS

	TREND 1	TREND 2		
INR EXPOSURE	1000	.05	.10	
AVERAGE PREMIUM	1000	.05	.10	
RIS OF BUSINESS	1	.05	.10	
FREQUENCY	.10	.05	.10	
SEVERITY	7500	.05	.10	
PAYMENT PATTERN	.1	.2	.3	.2
CUM PAY PATTERN	.1	.3	.6	.8
REPORT PATTERN	.5	.2	.1	.1
CUM REP PATTERN	.5	.7	.8	.9

CAL YEAR	DIRECT PREMIUM		DIRECT EARNED PREMIUM		POL ASSETS	ACC	PAID LOSSES	CASE RECOVERIES	INCURRED LMR	U/W CASH FLOW	U/W BALM	INVEST INCOME	TAX	EOY ASSETS	EOY LIAB	EOY SURPLUS	DISC INCURRED LOSSES		DISC RESERVES		GAAP ADJUST	GAAP SURPLUS	GAAP INCOME	
	YEAR	DEPM	PREMIUM	EXPENSE													DISC INCURRED	DISC LOSSES	DISC RESERVES	DISC RESERVES				
1900	1000000	500000	500000	125000	0	1900	37500	150000	187500	375000	0	41875	8375	871000	837500	33500	319348	281848	781818	89152	106000	133500	133500	
1901	1212750	606375	1104375	274574	871000	1900	75000	130000	112500	375000							343872							
						1901	82978	351913	414891	829781	778178	0	126009	25207	1749985	1615678	134307	706638	855832	1461107	288579	121275	255582	127082
1902	1470763	735381	1341754	335459	1749985	1900	112500	75000	75000	375000							361519							
						1901	163956	331913	248934	829781								768903						
						1902	100632	462527	503159	1006317	254236	0	212818	42562	2676469	2371914	304555	856975	1404830	2140212	536257	147076	451632	196049
1903	1783447	891834	1427215	404804	2674669	1900	75000	37500	37500	375000							371510							
						1901	248934	145956	145956	829781								799950						
						1902	201263	462527	301895	1006317								922786						
						1903	127041	488164	610206	1270411	229625	0	304128	60826	3649396	3101538	547850	1039296	1911736	2803570	845826	178347	226225	274593
1904	2163443	1081571	1973405	493351	3649396	1900	75000	0	0	375000							375000							
						1901	163956	82978	82978	829781								822058						
						1902	301895	201263	201263	1006317								970139						
						1903	244082	488164	344125	1270411								1119108						
						1904	148005	592021	740027	1480054	234852	0	401682	80336	4705594	3836391	8649204	1260406	2389968	3471539	1234055	216347	1085518	359293
1905	2623351	1311676	2393247	598312	4705594	1901	163956	0	0	829781							829781							
						1902	201263	100632	100632	1006317								996751						
						1903	366123	244082	244082	1270411								1176537						
						1904	296011	592021	444016	1480054								1357198						
						1905	179494	717974	897468	1794935	816192	0	511369	102278	5930882	4452583	1278299	1528550	2898434	4210109	1720772	262335	1540634	455116
1906	3181449	1590735	2907418	725603	5930882	1902	201263	0	0	1006317							1006317							
						1903	244082	122041	122041	1270411								1209052						
						1904	444016	296011	296011	1480054								1426845						
						1905	358987	717974	538481	1794935								1645942						
						1906	217681	878723	1088404	2176808	989837	0	442580	128516	7434782	5647420	1792363	1653758	3515075	5105810	2328972	318147	2110510	564876
1907	3858327	1929163	3519898	879974	7434782	1903	244082	0	0	1270411							1270411							
						1904	296011	148005	148005	1480054								1484278						
						1905	358481	358987	358987	1794935								1730486						
						1906	435362	878723	653042	2176808								1996117						
						1907	263992	1055969	1519962	2639923	1200425	0	803499	160706	9278007	6842844	2935162	2248146	4262908	6192971	3085936	385831	2828995	710485
1908	4679186	2339593	4268756	1047189	9278007	1904	296011	0	0	1480054							1480054							
						1905	358987	179494	179494	1794935								1778279						
						1906	453042	435362	435362	2176808								2048550						
						1907	527985	1055969	791977	2639923								2428991						
						1908	270157	1280427	1600788	3201567	1455815	0	1000591	200118	11524295	8298659	3235636	2726438	5169841	7509434	4024861	467919	3703554	882599
1909	5676682	2837341	5176934	1294233	11524295	1905	358987	0	0	1794935							1794935							
						1906	435362	217681	217681	2176808								2136547						
						1907	791977	527985	527985	2639923								2545016						
						1908	640313	1280427	964476	3201567								2935814						
						1909	388270	1553080	1961350	3882700	1765540	0	1261706	248361	14293200	10064199	8229001	3304488	6269225	9107066	5186134	567668	4784669	1092915

PROPERTY-CASUALTY FINANCIAL MODEL  
STOCHASTIC MODEL

I. FINANCIAL HISTORY

	TREND 1	TREND 2
BEG EXPOSURE	1000	.05 .10
AVERAGE PREMIUM	1000	.05 .10
MILE OF BUSINESS	1	.05 .10
FREQUENCY	.10	.05 .10
SEVERITY	7500	.05 .10
PAYMENT PATTERN	.1	.2 .3 .2 .2
CUM PAY PATTERN	.1	.3 .6 .8 1.0
REPORT PATTERN	.5	.2 .1 .1 .1
CUM REP PATTERN	.5	.7 .8 .9 1.0

YEAR	DIRECT CAL WRITTEN PREMIUM	DIRECT EARNED PREMIUM	DIRECT EXPENSE	ADJ ASSETS	ACC YEARS	PAID LOSSES	CASE RESERVES	UNPAID LOSSES	U/W CASH FLOW	U/W GAIN	INVEST INCOME	TAX	EQY ASSETS	EQY LIAB	EQY SURPLUS	DISC LOSSES	DISC RESERVES	DISC LIAB	DISC SURPLUS	GAAP ADJUST	GAAP SURPLUS	GAAP INCOME		
1980	1000000	500000	500000	125000	0	1980	37500	150000	102500	375000	837500	0	41015	8175	81000	817500	33500	319100	261000	89152	100000	117500	135000	
1981	1212730	606375	1106375	276574	871000	1980	75000	150000	322500	375000						341072								
						1981	165956	165956	497669	879781	695200	0	121060	24372	1663880	1533700	110900	717692	783108	1389463	774205	121275	252263	118763
1982	1340096	670048	1276473	319106	1063088	1980	112500	75000	75000	375000						361519								
						1981	165956	165956	331913	879781							764765							
						1982	109632	402527	503159	1006317	641902	-49000	198464	15193	2468661	2223002	265259	836925	1325714	1995762	493099	150010	399269	143006
1983	1703667	891834	1561601	390470	248861	1980	75000	37500	37500	375000						371510								
						1981	165956	165956	165956	879781							799950							
						1982	201263	402527	301895	1006317							522780							
						1983	122041	688184	610206	1220411	828936	-49000	290333	33567	5574564	5101536	474025	1039206	1911716	2803570	330994	176367	614372	152123
1984	2163743	1081871	1977403	493151	3574564	1980	75000	0	0	375000						375000								
						1981	165956	82976	82978	879781							822098							
						1982	301895	201263	201263	1006317							370139							
						1983	244062	488184	366123	1220411							1119108							
						1984	152406	674623	277026	455056	227452	-74003	393829	41764	4654780	3900591	751067	2103427	2445568	3527159	1126921	216374	96400	100009

ACC YEAR	REPORTED LOSSES				
	1	2	3	4	5
1980	187500	262500	300000	337500	375000
1981	331913	497869	663825	746803	
1982	503159	704422	805054		
1983	610206	854288			
1984	777028				

ACC YEAR	PAID LOSSES				
	1	2	3	4	5
1980	37500	112500	225000	300000	375000
1981	165956	331913	497869	663825	
1982	100632	301895	603790		
1983	122041	366423			
1984	155406				

ACC YEAR	REPORTED DEVELOPMENT			
	2:1	3:2	4:3	5:4
1980	1.4000	1.1429	1.1250	1.1111
1981	1.5000	1.3333	1.1250	
1982	1.4000	1.1429		
1983	1.4000			
1984				

ACC YEAR	PAID DEVELOPMENT			
	2:1	3:2	4:3	5:4
1980	3.0000	2.0000	1.3333	1.2500
1981	2.0000	1.5000	1.3333	
1982	3.0000	2.0000		
1983	3.0000			
1984				

AVERAGE	1.4250	1.2063	1.1250	1.1111
EMOSEN	1.4000	1.1429	1.1250	1.1111
ULTIMATE	2.0000	1.4286	1.2500	1.1111

AVERAGE	2.7500	1.8333	1.3333	1.2500
EMOSEN	3.0000	2.0000	1.3333	1.2500
ULTIMATE	10.0000	3.3333	1.6667	1.2500

ACC YEAR	REPORTED LOSSES					5 ULTIMATE
	1	2	3	4	5	
1980	187500	262500	300000	337500	375000	375000
1981	331913	497869	663825	746803	829781	629781
1982	503159	704422	805054	905205	1006317	1006217
1983	610206	854288	976329	1098370	1220411	1220411
1984	777028	1087839	1243245	1398651	1554056	1554056
1985	897468	1256455	1435948	1615542	1794935	1794935
1986	1088404	1523765	1741446	1957127	2176808	2176808
1987	1319962	1843966	2111939	2375931	2639923	2639923
1988	1606784	2241097	2561254	2881410	3201567	3201567
1989	1941350	2717890	3106160	3494430	3882200	3882200

ACC YEAR	PAID LOSSES					5
	1	2	3	4	5	
1980	37500	112500	225000	300000	375000	375000
1981	165956	331913	497869	663825	829781	829781
1982	100632	301895	603790	805054	1006317	1006317
1983	122041	366423	732847	976329	1220411	1220411
1984	155406	466217	932434	1243245	1554056	1554056
1985	179494	538401	1076801	1435948	1794935	1794935
1986	217681	652042	1304085	1741446	2176808	2176808
1987	263992	791977	1583954	2111939	2639923	2639923
1988	320157	960470	1920940	2561254	3201567	3201567
1989	388270	1164810	2329620	3106160	3882200	3882200

INCREMENTAL PAID LOSSES					
1	2	3	4	5	
37500	75000	112500	75000	75000	
165956	165956	165956	165956	165956	
100632	201263	301895	201263	201263	
122041	244082	366123	244082	244082	
155406	316811	466217	316811	316811	
179494	358987	538401	358987	358987	
217681	435362	652042	435362	435362	
263992	527985	791977	527985	527985	
320157	640313	960470	640313	640313	
388270	776540	1164810	776540	776540	

DISCOUNTED PAID LOSSES					
1	2	3	4	5	
319348	343872	561519	371510	375000	
717682	764265	799950	872028	879781	
656975	922256	970139	956951	1006317	
1039296	1119198	1176537	1206652	1220411	
1321427	1422028	1494947	1529592	1554056	
1528558	1645542	1740406	1778229	1794935	
1851758	1996117	2098520	2156347	2176808	
2248146	2420791	2545016	2615352	2639923	
2726430	2935614	3066469	3171769	3201567	
3306488	3500498	3743315	3846562	3882200	

III. RATE LEVEL ANALYSIS

ACC./CM. YEAR	WRITTEN PREMIUM	EARNED PREMIUM	RATE		REPORTED LOSSES	EXPECTED LEVEL IN RATE		LOSS RATIO AT		EXPENSE RATIO AT		ANNUAL TREND	PROJ OF RATIO
			INDEX	CURR RT		DEVELOP	LOSSES	CURR RT	TREND	CURR RT	TREND		
1980	1000000	500000	1.00	500000	375000	3.4000	375000	125000	75.002		25.002		112.281
1981	1212750	1106375	1.00	1106375	746005	1.1111	829781	276594	75.001	100.002	25.002	100.002	109.678
1982	1340096	1276423	1.00	1276423	805054	1.2500	1006317	319106	78.842	105.172	25.002	100.001	111.352
1983	1781667	1561881	1.00	1561881	654788	1.5079	1268212	390470	82.481	104.672	25.002	100.001	112.642
1984	2163143	1973405	1.00	1973405	777078	2.1488	1669686	493351	84.612	102.582	25.002	100.002	112.222
										103.062		100.002	

  

PROJ YEAR	LOSS RATIO	DISCOUNT FACTOR	DISC		RATE IMP
			LOSS RATIO	EXPENSE RATIO	
1985	87.272	.7786	67.912	75.001	1.1222

IV. FINANCIAL PROJECTIONS

	TREND 1	TREND 2		
RES EXPOSURE	1000	.05	.10	
AVERAGE PREMIUM	1000	.05	.10	
MIX OF BUSINESS	1	.05	.10	
FREQUENCY	.10	.05	.10	
SEVERITY	.7500	.05	.10	
PAYMENT PATTERN	.1	.2	.3	.2
CUM PAY PATTERN	.1	.3	.6	.8
REPORT PATTERN	.5	.2	.1	.1
CUM REP PATTERN	.5	.7	.6	.9

YEAR	DIRC		P		BEN	SEC	PAID		CASE	M		I		GISA		DISC	GAP	GAP	GAP							
	W/	ITEM	BLFR	FREEMG			EX/EASE	LOSS		RES	INFORMED	CASH	U/M	INVEST	TOT					ASSETS	LIAB	NET	DISC	ASSETS	LIAB	DISC
1980	1000000	500000	500000	500000	125000	0	1500	37500	150000	187500	575000	875000	0	41875	8375	871000	875000	33500	519348	281048	781048	89157	100000	155000	135000	
1981	1212750	600375	1100375	776554	871000		1980	70000	150000	112500	375000								543872							
							1961	165956	165956	497869	879781	895200	0	121860	24372	1603688	1532700	150008	717692	763108	1384461	274005	121005	752005	118761	
1982	1470765	735381	1341750	135439	1003698		1969	112500	25000	25000	375000								561519							
							1961	165956	165956	331913	679781									764765						
							1962	100632	402527	501159	1006317	756236	0	204181	40836	2583268	2206956	29435	636975	1325014	2000005	502173	140005	600005	105000	
1983	1783667	891834	1627215	400804	2563256		1980	75000	37500	37500	375000								771516							
							1961	165956	165956	165956	829781									294450						
							1962	201263	402527	301895	1006317									962786						
							1963	126041	488164	610006	1260411	812603	0	268957	59791	3635036	3100508	553496	1000096	1941756	1000000	851466	1700000	1000000	1000000	1000000
1984	2163143	1081571	1979005	497351	3035076		1969	75000	0	0	375000								375000							
							1961	165956	829781	829781	629781									824095						
							1962	301895	201263	201263	1006317									770137						
							1963	244062	488164	360123	1260411									1000000						
							1964	155406	621623	779026	1554056	727452	74003	399876	42974	4719591	3000000	816396	1500000	2445960	5529156	1192231	216000	1000000	1000000	1000000
1985	2941755	1470877	2553669	638767	4719731		1961	165956	0	0	829781								825081							
							1962	201263	100632	100632	1006317									996931						
							1963	306173	244062	244062	1260411									1176537						
							1964	310611	621623	466217	1554056									1425058						
							1965	179494	717974	897469	1794935	1081780	120164	526028	165289	6161990	4664666	1297303	1508558	2944093	4415996	1450919	294375	1500000	556991	
1986	3570067	1785043	3750044	814235	4361909		1962	201263	0	0	1006317								1006317							
							1963	244062	122041	122041	1220411									1200052						
							1964	466217	310611	310611	1554056									1496187						
							1965	358987	717974	538491	1794935									1645942						
							1966	217681	870773	1068494	2176808	1767621	265898	679572	268863	7840239	5866339	1973510	1853758	3542016	5327600	2513180	352000	2500000	739706	
1987	4329676	2164838	3948655	987464	7000205		1981	244062	0	0	1220411								1220411							
							1961	310611	155406	155406	1554056									1515992						
							1965	358987	358987	358987	1794935									1730406						
							1966	435362	870773	653042	2176808									1992117						
							1967	260992	1055969	1319962	2639923	1549431	322468	861496	333533	8917633	7050000	2674446	2246046	4277620	6441831	1475800	430000	1500000	526304	
1988	5250756	2625375	4750750	1197547	5917633		1964	310611	0	0	1554056								1554056							
							1965	358987	179494	179494	1794935									1738009						
							1966	653042	435362	435362	2176808									2098050						
							1967	527985	1055969	791977	2639923									2620091						
							1968	370157	1280627	1600784	3701567	1882722	391073	1085874	417211	12473016	8584442	3888576	2726438	5169641	7795216	4677861	525005	1410000	1156349	