ANALYSIS OF RESULTS, FORECASTING AND CORPORATE PLANNING

1985
DISCUSSION PAPER PROGRAM

Presented May 8-11, 1985
Boca Raton, Florida

CASUALTY ACTUARIAL SOCIETY
ORGANIZED 1914

ANALYSIS OF RESULTS, FORECASTING AND CORPORATE PLANNING

These papers have been prepared in response to a call for papers by the Casualty Actuarial Society to provide discussion material for its Spring meeting, May 8-11, 1985, in Boca Raton, Florida.

Actuaries have been becoming increasingly involved in all aspects of corporate planning and are often called upon to explain results to people without technical or insurance backgrounds. It is hoped that these papers will serve as the basis for a stimulating discussion at the Spring meeting while establishing a foundation for further research in these areas.

Committee on Continuing Education

NOTICE

TABLE OF CONTENTS

TITLE: CORPORATE PLANNING -- AN APPROACH FOR AN EMERGING COMPANY

AUTHORS: Irene K. Bass is Vice President of the U.S. Fire Insurance Company and Actuarial Department head for Crum and Forster Personal Insurance. Prior to joining Crum and Forster, Miss Bass held positions at Commercial Union and Hanseco. She is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. She holds a B.A. in German from Bowling Green State University and an M.S. in mathematics from Northeastern University.

Larry D. Carr is Senior Vice President and Chief Financial Officer of the U.S. Fire Insurance Company with responsibility for the Underwriting, Actuarial, and Financial and Planning areas of Crum and Forster Personal Insurance . He serves as a member of the Board of Directors and Executive Committee of the U. S. Fire Insurance Company. Mr. Carr had extensive experience at Allstate Insurance Company prior to joining CFPI in 1983. He holds a B.S. in finance from the University of Illinois.

ABSTRACT: A major property-casualty insurance group recently created a separate profit center for personal lines, thereby signaling a new emphasis on these lines. Since the group had been a carrier with primary emphasis on commercial business, the planning process at the personal lines profit center became similar to that of an emerging company: there was little relevant history available to use in order to plan effectively for a very different future.

Because planning at insurance companies is too often separated from field operations -- the very people who must make the plan happen -- we installed a process that is operationally driven.

We therefore developed an approach to premium and loss planning which did not rely on a simple projection of last year's results. The major benefit of this planning process is its lack of dependence on historical information and its explanation of current results in terms of specific components of the plan. We offer to share and discuss this approach to personal lines planning.

INTRODUCTION

Not too long ago, our insurance group reorganized and created, along with several commercial lines profit centers, a separate profit center for personal lines. Traditionally, the group's personal lines volume was a relatively minor part of its total business, representing about one-fourth of total premium. As such, there was no special attention paid to it in the corporate planning process. The profit center's new management, with its singular responsibility for personal lines, initiated a planning process with five basic goals in mind:

1. To build an operationally driven planning system from which the financial plan would follow

2. To isolate measurements which are controlled -- or at least influenced -- by line management

3. To insure commitment to the planning process by involving field management in selecting planned levels of performance

4. To create a plan optimal for personal lines which would work in an environment where the historical results would not necessarily be an accurate predictor of the future

5. To blend the annual operating and financial plans into the profit center's strategic plan.

Although the personal lines volume is substantial, this new profit center is an emerging company in the sense that the past will probably not be representative of the future. It is also an emerging company in terms of appointment of agents separately from the remainder of the corporation, planned growth within the American agency system, and independently defined goals, objectives, and marketing strategies.

This paper is intended to illustrate a premium and loss planning process for a personal lines company in a changing or emerging environment. Expense planning is an important phase of all planning, but the scope of this paper will be limited to premiums and losses only. Our planning process is fairly straightforward in terms of the mathematical concepts employed. It is not simply a matter of determining last year's results and then making some estimates for the coming year. Rather, each component of the premium is analyzed down to the basic elements, beginning with number of agents and ending with premium. Likewise, losses are analyzed in their elementary components of frequency and severity.

In this paper we first offer some background on the difficulties of planning for an emerging company and obtaining the proper source data to do a zero-base analysis. Then we discuss the premium and loss planning methods. Finally, we evaluate what was accomplished in the planning process.

BACKGROUND

The operational plan is segmented into three separate but connected pieces:

- Agent Plan
- Production/Premium Plan
- Losses Plan

These represent the major planning decision areas in which baseline values and the impact of operational changes must be established. Charts of the detailed statistical flow are contained in Exhibits 1-5. Once a conceptual understanding of this flow is achieved, four things are required to plan:

1. Base Data -- the explicit values of the variables in the equations to get from number of agents to premiums to incurred losses.

2. Operational Plans -- the expected changes in operating philosophy, approach, and execution. This information is based on the continuing analysis of programs and their results and on management's evaluation of areas where current performance requires improvement.

3. Quantification of Plans -- the specific numerical ramifications of operational plans. If operational plans are carefully

prepared within a structured framework and are based on objective evaluation of data, this quantification is often completed in the operational analysis process. If not, basic analysis to quantify operational changes is required. This is a critical step in that it allows management to see whether planned activity will achieve desired results. This is the fundamental reason for planning.

4. <u>External or Extraordinary Factors</u> -- the impact of any anticipated changes in external factors. These factors must also be isolated and considered. Extraordinary weather-related claim volume in the historical data is an example of this kind of influence.

The planning model -- composed of the variables in the planning equations and the equations themselves -- is fundamentally important in that it allows for measuring the impact of planned actions on operating results.

Because this profit center is a personal lines operation emerging from a predominantly commercial lines environment, it is not surprising that the kind of data we sought was not available from readily accessible sources. Therefore, the acquisition of base data was the most difficult part of the process and certainly the most time-consuming. Annual Statement and Insurance Expense Exhibit data do not provide sufficient detail for an operationally driven plan. New and renewal production, frequency, and severity data just do not exist in these sources. At our company, it also required compromises regarding the desired breakdown of data by line of business.

It became obvious that ideal information was not to be found, so we settled on the following compromise prioritization.

1. Necessary data would become available over time, so the most important aspect was the integrity of the statistical flow. When a source of net written premium by desired line of business did not have a companion policy count report, we chose a summary level that preserved the desired statistical flow. More important than planning by ideal product line, it was essential to begin installing the operational discipline, involving field management in the planning process, and planning in a fundamental step-by-step method that would permit isolating the sources of variance from plan.

2. Because of the need to focus on markets, results, constraints, and opportunities on a state-by-state basis, higher levels of product summarization were accepted than was desired. For example, homeowners was handled as one line rather than separating it into owner, renter, and condo business.

3. When all else failed and source data was not to be found, prior experience, judgment, and estimates were often used.

We often used sources for purposes other than their original intent. Agent counts were developed from a monthly report used to check validity of mailing lists. New business production and renewal ratios were derived by an elaborate manipulation of in-force policy counts and cancellation activity. New business and renewal premiums were generated from reports originally prepared to measure processing through the underwriting department. Claim counts -- and, ultimately, frequency and severity -- were completed using claim department workload reports.

STATISTICAL MODEL

The premium planning process is outlined on Exhibits 1-4. Loss planning is outlined on Exhibit 5.

Agents

The process begins with the agent plan and develops into premium on a line of business basis.

Because of the radically changing environment in the company, we could not begin with last year's premiums and project forward. We knew we would be cancelling personal lines contracts for many agents who were commercial lines oriented and appointing new agents for personal lines only. Since this activity is part of regional management's objectives, it made sense to involve them in the quantification of the number of active agents and to have this item become a measurable variable in the plan. This is an example of field management involvement and of the isolation of measurements that are controlled or influenced by them.

Production

Average number of new policies per agent was calculated by line of business from prior years, and an estimate for the ensuing year was based on several assumptions. This number would be very different from prior years because now we would be dealing with agents who are primarily personal lines agents and who would use our company as a major market. Agency management would be directed at personal lines only, and major changes in pricing would shift our competitive position.

By multiplying average number of agents by number of new policies we obtained the total number of new policies issued for each line (Exhibit 1). But renewals also had to be calculated. The prior year's policies (which become available for renewal this year) were multiplied by a renewal ratio to obtain the number of renewal policies issued (Exhibit 2). Careful analysis of the renewal ratio was necessary since the expected termination of some agents and the planned re-underwriting programs would most likely cause this ratio to drop. At the same time, improved policy service and changes in the mix of business would tend to increase the renewal ratio.

Premium

The next step starts with number of policies issued and ends with the net written premium (Exhibit 3). New policies must be handled separately from renewal policies when estimating the average written premium by line of business. New business, given the thrust to appoint primarily personal lines agents and to penetrate a different market sector, should have a significantly different average premium from prior period business that will be renewing. For example, an effort to write many more homes of high value will cause the average premium to be much higher.

Even the average renewal premium might be very different from prior years. If, as in the prior example, an effort to penetrate the high-value homeowners market is coupled with competitive pricing in that segment, changes in homeowners relativity curves may greatly increase the premiums charged for low-valued dwellings. Re-underwriting and a campaign to insure to value may also increase these averages. In our plan, field management input along with information from the pricing actuaries was needed to quantify this variable.

Gross written premium is obtained by multiplying the number of policies issued by the average premium separately for new and renewal policies (Exhibit 3). Endorsement premium is loaded by means of a factor and the remainder of the steps leading to net written premium are straightforward.

Earned premium and policies in force are important by-products of the statistical flow (Exhibit 4). Neither is planned directly: they flow from the numbers being generated by the internal relationships of the model. By developing a historical relationship between formula earned premium (1/24 of current month's written premium plus 1/12 of prior eleven months' written, plus 1/24 of twelfth prior month's written) and actual monthly earned premium, a clear pattern should develop which will establish an appropriate relationship between formula and actual earned premium (Exhibit 4).

At this point in the flow, developing the earned premium involves only selecting an appropriate earned premium compensating factor and doing the arithmetic. Policies in force (new and renewal separately) are arrived at by accumulating all of the policies that could be in force from the prior 12 months' policies issued and applying an appropriate termination rate.

Losses

Once the premium plan is complete, the generation of incurred losses essentially flows out of a continuation of the logic. Policies in force become the base against which frequency ratios (new business separately from renewals) are applied to arrive at claim counts (Exhibit 5). It is important that the assumptions about changes in the agency force, market focus, and underwriting rules be carried through to frequency selection. Otherwise, loss ratios will be distorted.

Claim counts can then be multiplied by an appropriate severity to arrive at losses (Exhibit 5). A number of different approaches are possible here but, essentially, historical levels are modified to reflect expected changes in average claim costs due to inflation, changes in mix of business, limits, deductible, etc. The effectiveness of the claims department must also be reflected. IBNR reserve changes can then be added to incurred losses before arriving at the final loss ratio.

It is important to recognize that the various operational action plans are quantified in very different ways in the development of the plan. A careful evaluation of the loss ratio will help insure that the impact of various operational plans is consistently assessed in determining both premium-related and loss-related base data.

EVALUATION

A number of legitimate questions seem obvious. With all the compromises in data, did the process really accomplish anything? Was the cost in time and effort appropriate in the completely manual environment? In short, was it worth the effort? Our answer is an unequivocal, "Yes," because several very important baselines were established, as discussed below.

1. A base of information was developed.

2. Management became more in touch with the company's expected results more quickly than it would have without the planning process.

3. An operationally based planning process was installed. Accepting that the first plan would be the worst, we decided it was important to install the process in order to begin its evolution. We could not have waited another year to begin establishing those disciplines and thought processes.

4. The planning process clearly established the relationship between functional management and results. Operational actions -- the changes, refinements, and corrections -- are necessary if results are to change.

5. A set of performance benchmarks were established. They could have been established in a number of other less time-consuming ways. But the advantage of this approach is that when actual net written premiums or incurred losses are different from plan, the source of the difference can be specifically identified and evaluated. From an informed perspective, management can then decide to accept the variance or take action to correct it.

CONCEPTUAL MODEL

Critical questions to ask in an operationally based planning process include:

Market Direction -- Will there be any changes in products, geographic focus, or target market segments? Will there be changes in the relationship with agents such as new profit-sharing agreements, increased or decreased leverage, or an agency force restructuring? All of these areas could impact assumptions about agents, productivity, renewal trends, average premium, retention, losses, and expenses.

Pricing Philosophy -- Are there going to be changes in price, competitiveness, or rating schemes to attract certain risks? These will also impact nearly all parts of the statistical flow.

Underwriting Approach -- Will any changes occur in underwriting rules which could impact production and average premiums as well as losses? Changes in emphasis on policy limits, deductibles, or sale of optional coverages should also be carried through to the numbers selected. It is important to evaluate the impact of past underwriting decisions that could be "cycling through" their period of impact, e.g., a re-underwriting program begun in the middle of the prior year.

Level of Service -- Strong correlations exist between retention and the level of policyholder/claim service. What will be the impact of anticipated changes in level of service which should be considered in developing renewal and retention levels and changes?

Claims Handling Practices -- How will opening and closing practices, different emphasis on case reserve adequacy, clearing backlogs, etc., influence both frequency and severity measurements?

Once a consensus on these five areas is achieved, only one conceptual step remains: environmental issues need to be examined. Inflation, unemployment, housing starts and car sales, trends in miles driven, gasoline prices, and so on, all impact key planning variables. So, too, do the actions of competitors, regulators, and legislators as their collective modifications of the environment influence the effectiveness of our actions.

At this point, the truly difficult part of planning is finished. The remaining work involves, simply, a translation of these operational and environmental conclusions into statistical impact. It is at this point that the planner needs extraordinary discipline. There will be times when, after all the work is completed, the results are unsatisfactory -- management can't live with the bottom line.

An easy way to correct this situation is simply to change a number. If this happens, the entire planning process is invalidated and displeasure with planned performance quickly becomes dismay over actual results. When unacceptable results are projected in the planning process, only two valid actions can be taken. First, the translation of concepts to numbers should be rechecked. Any mistakes should be corrected and the numbers recalculated. If this does not correct the problem of unacceptable results, management must return to its basic assumptions about operations and modify them to achieve acceptable results.

Only in this way is it possible to arrive at financial plans based on sound operating decisions. Only with this detailed approach can the future sources of variance be isolated and corrected.

Discussion Questions

1. If results are different from plan, does this method allow one to identify the cause of the variation? Does it pinpoint the cause sufficiently for management to take proper corrective action?

2. Is this approach adaptable to planning in the commercial lines environment?

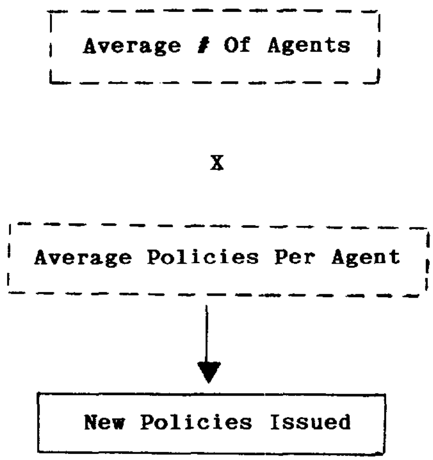
3. Is an operationally focused, detailed plan really needed in the insurance industry?

4. What sort of controls are necessary to insure the integrity of the plan?

5. Will this approach be effective for an emerging company or does it require an established company "culture"?

Exhibit 1

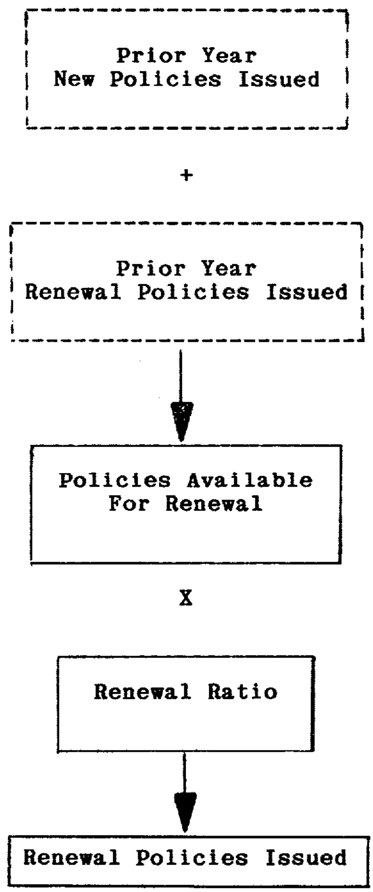
NEW PRODUCTION



Prepared by line of business

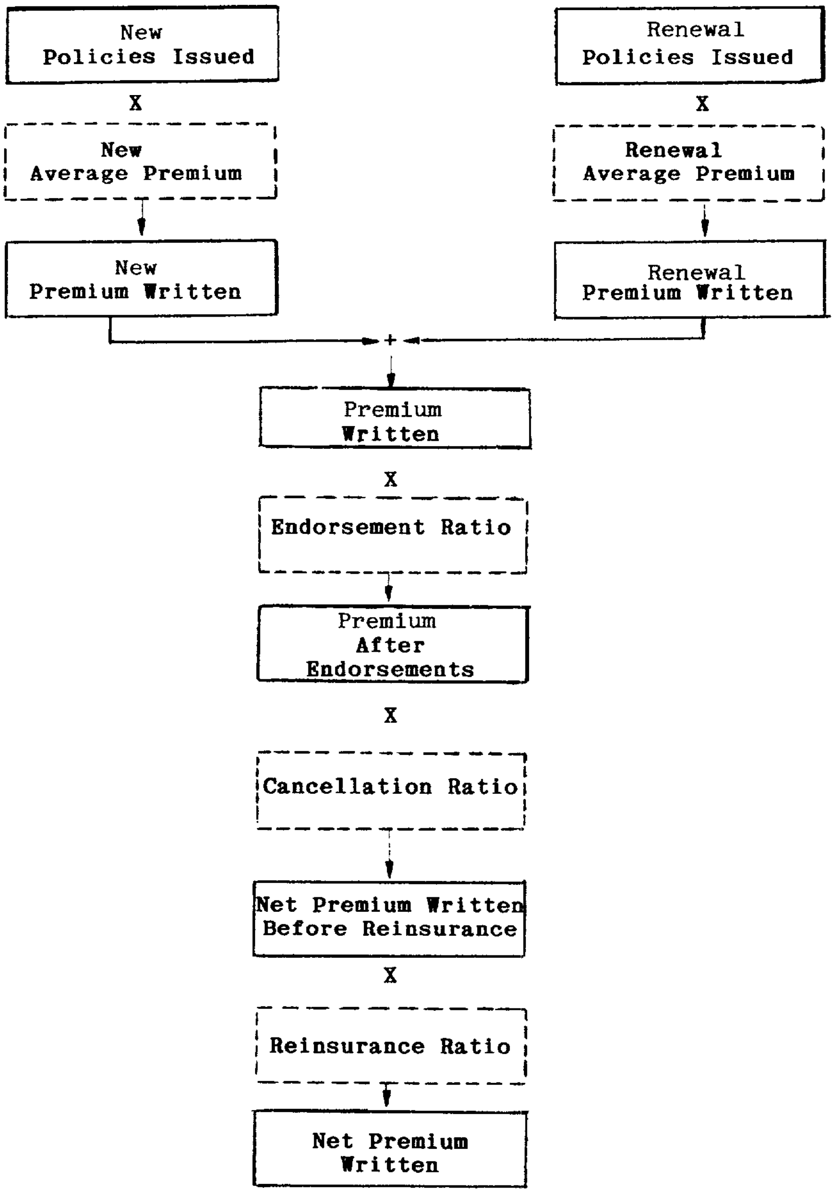
Exhibit 2

RENEWAL PRODUCTION



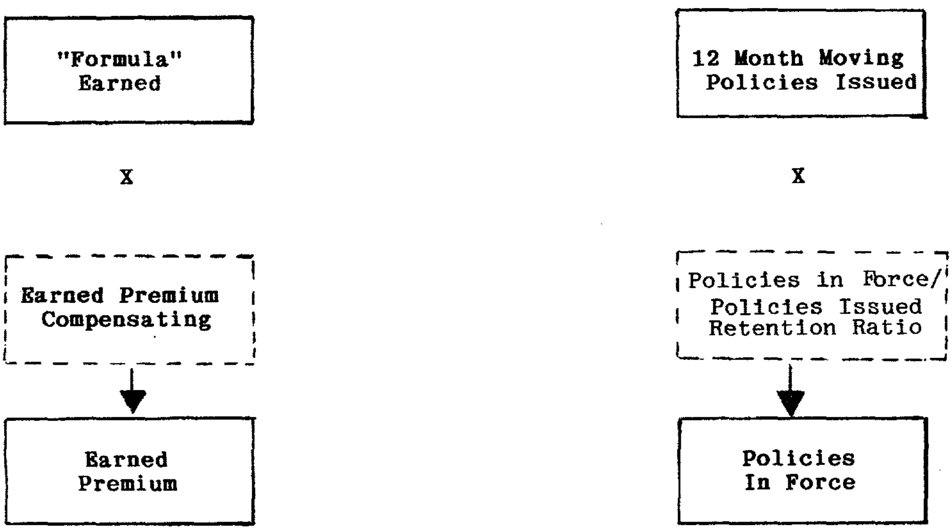
Prepared by line of business

PREMIUM **Exhibit 3**



Prepared by line of business

EARNED PREMIUM
AND
POLICIES IN FORCE*

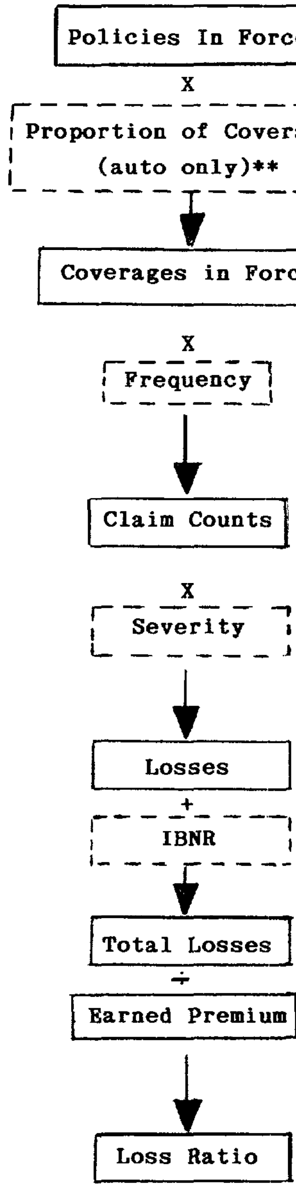


Prepared by line of business

*Policies in force calculated separately for new and renewal

LOSSES

Exhibit 5



Prepared by Line of Business

**Prepared by Coverage for Auto

BRANCH OFFICE PROFIT MEASUREMENT FOR PROPERTY-LIABILITY INSURERS

Robert P. Butsic

Biographical Sketch

Current employer is Fireman's Fund Insurance Co.; prior affiliation was with CNA Insurance from 1969-1979. Earned BA in Mathematics in 1967 and MBA (Finance) in 1978, both from the University of Chicago. Associate in Society of Actuaries, 1975; Member of American Academy of Actuaries. Wrote papers for previous CAS Call Paper programs in 1979 ("Risk and Return for Property-Casualty Insurers") and 1981 ("The Effect of Inflation on Losses and Premiums for Property-Liability Insurers").

Abstract

Effective measurement of financial performance for individual branch offices is hindered by two major problems. The first is an appropriate definition of profit; this is addressed through an economic-value accounting method which minimizes distortions due to the timing of income recognition. Return on equity is the basic profitability gauge used to compare results between profit centers. The second problem is that, in comparing results between branches, different levels of risk will produce an uneven chance of error in measuring true performance vs. reported results. This problem is addressed through techniques which equalize systematic risk (by implying an equity value) and non-systematic risk (through internal reinsurance), for each branch. To develop the internal reinsurance concept, a Poisson claim frequency and a Pareto claim severity model is constructed. Finally, in order to recognize the credibility of each profit center's actual experience, a compromise is made to the equal-variance principle. The analysis concludes that branch office profit measurement is best served when the branch network has minimal variation in size and product mix.

BRANCH OFFICE PROFIT MEASUREMENT FOR PROPERTY-LIABILITY INSURERS

INTRODUCTION

The insurance industry is presently becoming less regulated, creating an increasingly competitive long-term environment. To effectively meet the challenge posed by this new climate, insurers must strengthen their marketing and get closer to the consumer. Consequently, a strong branch office network is needed in order to cope with the variety and complexity of local conditions.

A major factor in the development of a viable branch office organization is the principle that each branch is completely responsible for its own contribution to corporate profits. Hence, the profit center concept. Given the objective to drive corporate profits from the sub-units which are held accountable, an appropriate tool for measuring performance must be used.

The purpose of this paper, then, is to outline a general approach for measuring the performance of individual profit centers comprising a property-casualty company. The methods presented here could apply to line of business or regional definitions of "profit center", but it will be most useful to think of a profit center as a branch office.

The paper focuses on the risk aspects of profit measurement and presents methods which equalize both systematic and purely random variation in profit center results. Other important aspects of profit measurement, including the accounting treatment, are developed in lesser detail. Many of the thoughts presented here have evolved over time at the writer's own company and are now being brought to a practical application in its management reports.

PROFIT MEASUREMENT IN PROPERTY-CASUALTY INSURANCE

Corporate Profitability

Before addressing the profitability measurement of individual profit centers one must first define an appropriate yardstick for the corporation as a whole. For this important concept we will choose return on equity, or ROE for short. This measure is commonly used for other industries, and represents the return to an investor in the corporation (for further background on this topic, see references [4], [8] and [10]).

Although ROE is a simple concept, it must be carefully defined. The accounting conventions used must be suitable for performance measurement over a reasonable time frame such as a month, quarter or year. Stated differently, management reports should encourage behavior which will tend to maximize the value of the firm.

The return on equity measure can be separated into two components: net income (the numerator) and equity (the denominator).

Net income must reflect, to the extent possible, the current impact of all future transactions related to the premium earned in the current period. This means that:

1. Accident-period accounting is used, with all losses, premiums, expenses and dividends continually being restated as better estimates of their ultimate values become known.

2. All future investment income earned on cash flows arising from the current period must be recognized in the period. The usual device for collapsing an income stream is net present value.

3. The future cash flows are taken to present value using a market interest rate, and not the portfolio rate. It should be noted that the appropriate rate for corporate performance measurement may differ from that for profit centers, due to the separation of responsibility for investment and underwriting risk.

Equity, in a similar fashion, must be adjusted to reflect the above timing of income:

1. All assets must be evaluated at market prices and all liabilities must be discounted to present value (i.e., the market price in a portfolio reinsurance transaction).

2. Otherwise, normal GAAP accounting for equity should be used.

The preceding concepts attempt to recognize what is known in the accounting literature (see Lev [5]) as economic income -- the anticipated consequences of current decisions are directly reflected in current earnings. The notion of economic income can be further explained by a numerical example. Assume that a miniature "company" is formed under the following circumstances:

1. An annual policy of $100 is written on January 1, 1985.

2. A single claim of $68.20 occurs on January 1, 1985 and is paid over three years, as indicated in the following cash flow schedule.

3. Cash transactions occur on January 1 of each year; the last loss payment occurs in 1987.

4. Cash is invested at a yield of 10% per year.

5. All cash flows are certain.

6. There are no income taxes applicable.

7. Capital of $50 cash (initial equity) is available on December 31, 1984.

8. Expenses of $35 are paid on January 1, 1985.

The cash flows are shown below:

Cash Flow Schedule

	1/1/85	1/1/86	1/1/87	Total	1/1/85 Present Value
Premium	100			100	100
Losses	0	-44	-24.2	-68.2	-60
Expenses	-35			-35	-35
Total	65	-44	-24.2	-3.2	5

Here we see that the value of the policy to us at the time the premium was written is $5.00. Under economic-value accounting, the balance sheet would look like:

Balance Sheet: Economic Value

	1984	1985		1986		1987
	12/31	1/1	12/31	1/1	12/31	1/1
Assets	50.00	115.00	126.50	82.50	90.75	66.55
Loss Reserve	0	60.00	66.00	22.00	24.20	0
Equity	50.00	55.00	60.50	60.50	66.55	66.55
- Pres. Value (1/1/85)	50.00	55.00	55.00	55.00	55.00	55.00

The loss reserve here equals the present value of unpaid losses. Notice that beginning 1/1/85, the present value of equity is always $55.00, because no additional income is generated by the future cash flows arising from the insurance operation (as opposed to re-investment of equity):

Income Statement: Economic Value

	1985	1986	Total
Underwriting Gain	-1.00	-2.20	-3.20
Investment Income - Loss Reserves	6.00	2.20	8.20
Net Income - Insurance Operation	5.00	0	5.00
Investment Income - Equity	5.50	6.05	11.55
Net Income - Total	10.50	6.05	16.55

This tabulation clearly shows that all income arising from the policy is recorded in 1985, when the premium is earned. For comparison, the traditional accounting method would give the following balance sheet and income statement:

Balance Sheet: Traditional Accounting

	1984	1985		1986		1987
	12/31	1/1	12/31	1/1	12/31	1/1
Assets	50.00	115.00	126.50	82.50	90.75	66.55
Loss Reserve	0	68.20	68.20	24.20	24.20	0
Equity	50.00	46.80	58.30	58.30	66.55	66.55
- Pres. Value (1/1/85)	50.00	46.80	53.00	53.00	55.00	55.00

Income Statement: Traditional Accounting

	1985	1986	Total
Underwriting Gain	-3.20	0	-3.20
Investment Income - Loss Reserves	6.82	2.42	9.24
Net Income - Insurance Operation	3.62	2.42	6.04
Investment Income - Equity	4.68	5.83	10.51
Net Income - Total	8.30	8.25	16.55

Notice that by the end of 1986 (a moment before the last cash transaction on 1/1/87) the accumulated equity is equivalent under either accounting method. However, the timing of income recognition differs dramatically.

The preceding example illustrates some fundamental ideas which can be developed more fully. The first is the concept of the total profit margin, or TPM. In economic valuation, the issuance of the $100 policy created an instant "profit" of $5; we are indifferent to selling the policy or accepting $5 in cash. This total profit brought to present value is 5% of premium, hence a 5% total profit margin.

The second is the economic return on equity. We started with $50 of equity and one year later the economic value of the mini-enterprise is $60.50, yielding a 21% ROE.

More compactly, the ROE can be represented by

(1) $1 + R = (1 + i)(1 + km)$,

where R denotes the (economic) return on equity, i the market (riskless) interest rate, k the premium divided by initial equity, and m the total profit margin. To verify the preceding example, we get $1 + R = 1.1[1 + 2(.05)] = 1.21$.

Although traditional accounting may provide a reasonable means of aggregate performance measurement for an insurer under conditions of stable growth, interest rates and product mix, it can fail miserably at the individual profit center level due to more severe timing distortions of income recognition (for example, individual case reserve changes on prior period losses can be dramatic for a single branch).

PRODUCT LINE PROFIT MEASUREMENT

Having established the economic return on equity evaluation approach as a viable aggregate profitability measurement, its application must be extended to individual product lines.

Relative Risk

Although ROE is a good profit measure, there are problems associated with its use in comparing insurance (and other types of) companies or product lines. These difficulties arise because the amount of risk associated with an ROE measure varies significantly by line of business. For example, given the option of buying stock in a medical malpractice insurer with an expected ROE of 20% or in a personal lines company of the same size having a 15% ROE, it is unclear what choice to make. The medical malpractice insurer would be considered to have a riskier return. The returns must be adjusted to equalize risk between various types of coverage.

Here risk means systematic or process risk, which cannot be reduced (relative to premium) by increasing the number of individual exposures. Examples of systematic risk for property-liability insurance include:

1. uncertainty of ultimate losses due to length of time from occurrence to final settlement,

2. uncertainty of ultimate loss due to future costs (inflation) being higher than anticipated in pricing, and

3. uncertainty of loss costs arising from low-frequency events, such as in municipal bond or nuclear reactor coverages.

Further discussion of systematic risk can be found in references [2], [4] and [9]. Also, Appendix I provides a more rigorous treatment of this topic, showing the difference between systematic and non-systematic (random) risk.

To adjust the ROE measure for risk, consider Equation (1) with the ROE and TPM as random variables:

(2) $1 + \tilde{R} = (1 + i)(1 + k\tilde{m}).$

Here "~" denotes a random variable. The interest rate, being riskless, is not a random variable. Also, the premium/equity ratio k is a constant since it represents a known quantity. The variance and standard deviation of the ROE are given by

(3a) $\mathrm{Var}(\tilde{R}) = (1 + i)^2\, k^2\, \mathrm{Var}(\tilde{m})$ and

(3b) $\sigma(\tilde{R}) = (1 + i)\, k\sigma(\tilde{m}).$

In order to proceed further, the ROE risk will be defined as being equal to its standard deviation. This is commonly used in financial theory (see Sharpe [9], for example) and has the intuitively appealing and important property that it is independent of the scale of operation: two companies identical in all other respects but size will have the same risk, measured in terms of variation from expected ROE. This definition is equivalent to that of systematic risk, which, being independent of the number of exposures, is constant for a particular product line, regardless of size.

For two different product lines (denoted by subscripts) one can equate the ROE risk using (3b);

(4a) $$\sigma(\tilde{R}_1) = \sigma(\tilde{R}_2) = k_1\sigma(\tilde{m}_1) = k_2\sigma(\tilde{m}_2), \text{ or}$$

(4b) $$\sigma(\tilde{m}_1)/\sigma(\tilde{m}_2) = k_2/k_1.$$

In other words, two product lines have identical ROE risk when their premium/equity ratios are inversely proportional to their respective measures of systematic risk.

Risk Equalization

How do we measure systematic risk for property-liability lines of business? Unfortunately, there is no objective way to measure some of the risk components, such as the uncertainty of low-frequency events. Nevertheless, even a judgemental approach is better than to assume that all lines have equal risk.

A suggested method for subjectively balancing risk for various product lines is:

1. Select a product line, say Commercial Multiple Peril, with an average perceived risk. Assign to it an arbitrary premium/equity ratio in the neighborhood of the long-term industry average premium/equity ratio for all lines; e.g., 2.5-to-1.

2. Select another line, compare it to the standard line (CMP) and set a premium/equity ratio at which you would be indifferent to writing this line compared to the standard line. For example, Fire (having a fast loss payout and a relatively complete pricing data base) at a 4-to-1 premium/equity ratio might be considered equally risky as CMP at 2.5-to-1.

3. Repeat the process for all applicable product lines. Of course, the method can be extended to sublines or even new types of insurance.

This procedure, or one which actually attempts to measure the relative systematic risk (see Fairley [1]), will produce imputed equity values for each line based upon the respective premiums written. The aggregate all-lines imputed equity need not equal the "actual" equity reported externally, since our intent is to measure relative profitability between lines without having to be concerned about their different absolute levels of risk.

Returning to the earlier quandary (medical malpractice at 20% ROE vs. personal lines at 15% ROE), suppose that the medical malpractice ROE arises from a 5% TPM having a standard deviation of 3%, while the personal lines TPM is 2% with a standard deviation of 1%. The interest rate is 10%.

Applying Equation (2), the medical malpractice premium/equity ratio is 1.818 and the personal lines ratio is 2.273. Equation (3b) gives a 6% medical malpractice ROE standard deviation and a 2.5% value for personal lines. To equalize the ROE risk, we increase the personal lines premium/equity ratio to 2.273(6/2.5) = 5.455 (conversely, we could decrease the malpractice premium/equity ratio by a factor of 2.5/6), yielding an ROE standard deviation of 6%. However, the expected personal lines ROE now increases to 1.1[1 + 5.455(.02)] - 1 = 22%. Therefore the personal lines return would be superior to that of the medical malpractice insurer.

The preceding calculations are summarized in the following table:

			Before Equalization			After Equalization		
	$E(\tilde{m})$	$\sigma(\tilde{m})$	k	$E(\tilde{R})$	$\sigma(\tilde{R})$	k	$E(\tilde{R})$	$\sigma(\tilde{R})$
Medical Malpractice	5.0%	3.0%	1.82	20.0%	6.0%	1.82	20.0%	6.0%
Personal Lines	2.0	1.0	2.27	15.0	2.5	5.45	22.0	6.0

PROFIT CENTER RISK EQUIVALENCE

Having developed the basis for equalizing systematic risk between product lines, it is now possible to apply the method to a composite of various lines, namely the profit center.

Systematic Risk Equivalence Between Profit Centers

For the composite of two (or more) lines, there is an additional element which tends to increase systematic risk — correlation between total profit margins. Suppose that the implied equity for each line has been determined to equalize the respective ROE risk. Let f_1 and f_2 represent the respective proportions of the total ($f_1 + f_2 = 1$) implied equity for each line, ρ the correlation coefficient between $\tilde{m}_1$ and $\tilde{m}_2$, and the subscript t the results for the branch total. The variance of the branch ROE is

(5) $$\text{Var}(\tilde{R}_t) = \text{Var}(\tilde{R}_1)[1 - 2(1-\rho)f_1 f_2].$$

Appendix I derives this result. Notice that $\text{Var}(\tilde{R}_1) = \text{Var}(\tilde{R}_2)$ since the ROE risk has been equalized.

The following numerical example illustrates the preceding result:

Line(i)	Premium	k_i	Equity	$E(\tilde{m}_i)$	$\sigma(\tilde{m}_i)$	$E(\tilde{R}_i)$	$\sigma(\tilde{R}_i)$	ρ
1	100	4.0	25	1.5%	1.0%	16.60%	4.4%	-
2	150	2.0	75	4.0%	2.0%	18.80%	4.4%	-
Total	250	2.5	100	3.0%	1.6% 1.3%	18.25%	4.4% 3.5%	1 0

Notice that the standard deviation of the ROE for this branch will be reduced (down to 3.5%) if the lines have total profit margins which are not fully correlated.

Several observations can be made from the analysis so far:

1. To the extent that lines within a profit center are not perfectly correlated in their TPM's, the overall ROE risk is reduced.

2. For a given correlation structure, the profit center ROE risk reduction is a function of the line mix.

3. Maximum risk reduction is attained when the line mix is such that the implied equity amount is equal for each line (i.e., $f_1 = f_2$ in Equation (5)).

For comparing results between two different profit centers, the theoretically correct procedure would adjust the implied equity for each branch due to risk reduction from line mix. However, this would be a formidable computational task due to the large size of the relevant correlation matrix and the difficulty of estimating the line correlation coefficients from empirical data. A more practical approach is to assume that the line mix among branches is such that there will be a negligible variation in ROE risk due to intercorrelation.

Non-Systematic Risk: Poisson-Pareto Model

We have now reached the point where each profit center can be evaluated on the basis of its own ROE given the equalization of systematic risk. Ideally, we would like to remove all sources of chance variation, systematic or otherwise, in order to ascertain whether the measured result is the "true", or inherent result. However, as discussed earlier, systematic risk by its very nature is difficult to reduce since it is independent of the size of operation (loss ratio reinsurance would work somewhat, but at the expense of a lower return). On the other hand, non-systematic (or NS) risk can be trimmed more readily through internal reinsurance. Increasing the number of exposures will also reduce NS risk, but in practice the size of a profit center is severely constrained.

A major source of random risk for a profit center is large losses due to single occurrences. Here we define a large loss as arising from a single insured, to distinguish natural (i.e., ISO serial-number) catastrophes, which will be treated later.

For a network of profit centers, the NS risk arising from large claims can be formulated readily using some simplifying assumptions:

1. A large loss is denoted by the random variable $\tilde{X} \geq r$, where r is a reference point above which we are willing to establish an internal excess loss reinsurance pool. Losses below r are assumed to be fixed in number and amount, and do not contribute to the variance of total losses for the profit center. For simplicity of presentation we will henceforth assume that these losses are zero.

2. All individual losses arise from a single product line and have the same size distribution; however, the expected number N_i of large losses varies by branch i.

3. The number of large losses $\tilde{N}_i$ has the Poisson distribution with parameter N_i.

4. The loss amount $\tilde{X}$ has the Pareto distribution, with parameter $\underline{a}$ (Patrik[7] discusses the applicability of this assumption). Other functions, such as the log-normal, are less computationally tractable, besides being unsuitable for fitting the tail of loss size distributions. Basically, the $\underline{a}$ parameter indicates the "tail thickness" of the loss size distribution: the higher the value of $\underline{a}$, the less likely a loss will occur at a higher level relative to a lower level. Empirical evidence indicates that $\underline{a}$ varies between about 1.5 and 3, with low values for liability lines and high values for property coverages. Appendix II gives more detail regarding the Pareto distribution.

5. All losses have the same payment pattern. Thus, the present value of the loss $\tilde{X}$ can be represented by $d\tilde{X}$, where d is a constant.

6. The total premium for the profit center is collected when written and is proportional to the expected losses. Since the average loss size is the same for all branches, the premium, denoted by PN_i, is proportional to the expected number of large losses.

7. There are no expenses, only losses and premiums.

The total value of all losses in a particular branch i is (subscript i removed for clarity)

$$(6) \quad \tilde{S} = \tilde{X}_1 + \tilde{X}_2 + \ldots + \tilde{X}_{\tilde{N}} ,$$

where the number of losses $\tilde{N}$ is also a random variable with expectation N. Because $\tilde{S}$ is a compound Poisson distribution it has mean and variance (see Appendix III for derivation):

(7a) $E(\tilde{S}) = NE(\tilde{X}) = Nra/(a-1)$ and

(7b) $Var(\tilde{S}) = NE(\tilde{X}^2) + N^2C = Nr^2a/(a-2) + N^2C$,

where $C = Cov(\tilde{X}_k,\tilde{X}_j)$ is the covariance between two separate losses $\tilde{X}_k$ and $\tilde{X}_j$.

The total profit margin for the profit center is

(8) $\tilde{m} = (PN - d\tilde{S})/PN$.

The variance of the TPM is, from (7b) and (8),

(9) $Var(\tilde{m}) = \frac{d^2}{P^2}\left[\frac{E(\tilde{X}^2)}{N} + C\right]$.

Notice that as N becomes infinite, the variance of the branch TPM is proportional to the covariance of individual losses; i.e., only systematic risk is present. This result is consistent with the basis for selection of the implied premium/equity ratios for different product lines. However, in the large loss model here, we have assumed a single line and therefore the covariance C is the same for all branches as is the premium/equity ratio. Consequently, equating the ROE risk for two branches implies that $E(\tilde{X}^2)/N$ must be the same for the branches.

Because the loss size distribution is the same for all profit centers, but the expected number of losses N may vary (in fact, N defines the size of the branch), the size of loss distribution must be transformed so that the second moment $E(\tilde{X}^2)$ can vary by branch. The common mechanism for achieving this goal is excess reinsurance.

To do this, we select a retention $br > r$, where b is a scaling factor. Now let $\tilde{Y} = \tilde{X}$ for $r \leq \tilde{X} \leq br$ and $\tilde{Y} = br$ for $\tilde{X} > br$. In other words, the loss is "stopped" at a value of br. For this protection, we charge the branch an amount such that its expected total losses remain equal to $N_i E(\tilde{X})$. As shown in Appendix II, the expected portion of a single loss retained in the interval r to br is

$$(10) \quad (1-b^{1-a})E(\tilde{X}) = (1-b^{1-a})\ ar/(a-1),$$

and the expected ceded amount above br is

$$(11) \quad b^{1-a}E(\tilde{X}) = b^{1-a}\ ar/(a-1).$$

Also shown in Appendix II, under the Pareto distribution, the second moment of the retained loss size becomes

$$(12a) \quad E(\tilde{Y}^2) = r^2(a - 2b^{2-a})/(a-2) \quad \text{for } a \neq 2,$$

$$(12b) \quad E(\tilde{Y}^2) = r^2[1 + 2\ln(b)] \quad \text{for } a = 2.$$

To determine the relative retentions which will equalize the NS risk for two branches, set $E(\tilde{Y}_1^2)/N_1 = E(\tilde{Y}_2^2)/N_2$, where the subscripts denote the respective branches. Letting $K = N_2/N_1$ be the ratio of the expected number of losses for the two branches, Equation (12) can be solved to produce the relationship between retentions b_1 and b_2:

$$(13a) \quad b_2 = [Kb_1^{2-a} - a(K-1)/2]^{\frac{1}{2-a}} \quad \text{for } a \neq 2,$$

$$(13b) \quad b_2 = b_1^K e^{(K-1)/2} \quad \text{for } a = 2.$$

The Frequency Problem

The above results have some interesting implications. From Equation (7b) we see that the non-systematic component of the variance of total losses is $NE(\tilde{X}^2)$ with no reinsurance protection. With excess reinsurance protection and a >2, the non-systematic variance ranges from $NE[\tilde{Y}^2] = Nr^2$ when b = 1, to $NE[\tilde{Y}^2] = NE[\tilde{X}^2]$ $= Nr^2\ a/(a-2)$ when b is infinite. Thus, the non-systematic variance can only be reduced by a factor of (a-2)/a.

However, the NS variance needs to be reduced by a ratio of N_1/N_2 if $N_2 > N_1$. Consequently, when a is large (indicating a thin-tail loss size distribution), the excess reinsurance program will be insufficient. One way to further reduce variance is to stop the number of large claims in addition to (or instead of) limiting individual loss amounts.

To further illustrate this point, we can separate the total NS loss variance into frequency and severity components. The frequency component is obtained by setting the loss equal to its expected value; the severity component arises from fixing the number of losses. This decomposition is derived in Appendix III with the following result for the Poisson-Pareto model:

Frequency	Severity	Total
$\frac{a\ (a-2)}{(a-1)^2}$	$\frac{1}{(a-1)^2}$	1

For example, if a = 4, an excess reinsurance program can only reduce the NS variance by 50%. Thus the maximum spread of branch sizes is 2-to-1 for risk equalization. However, since the frequency component of the variance is 89% of the total, it is possible to allow up to a 9-to-1 range of branch sizes by holding the number of claims at the expected value while allowing unlimited loss sizes.

Notice that if a <2, then the NS variance is infinite, but can be made finite with excess reinsurance. In this case the retention scaling factors b_i can be determined for any range of branch sizes.

Numerical Example

To illustrate the NS variance-equalizing choice of retentions, suppose that the lower limit r of the loss size distribution is $50,000 and the branch sizes range from 10 to 80 expected large losses. The following table shows two (of many possible) sets of equivalent retentions for three different values of a:

Equal-Variance Retentions (1,000's)

Branch Size (Exp. No. of Losses)	a					
	1.5		2.0		3.0	
10	50	100	50	100	50	100
20	78	216	82	165	100	-
40	153	580	224	448	-	-
80	378	1,838	1,656	3,312	-	-

For a = 3, it is not possible to find equal-variance retentions beyond a range of 3-to-1 in branch sizes. Notice that the range of retention amounts can be greater than the range of branch sizes if that range is large enough. Another observation is that, if a >2, setting the lowest retention above r further reduces the range of branch sizes which will equalize NS variance. In the example, when a = 3 and the lowest retention is 100,000 (instead of 50,000), the maximum range is 1.5-to-1.

The Credibility Problem

We have determined that the excess loss approach will equalize NS variance across branches provided that a is low enough or the spread of branch sizes is narrow enough. If not, limiting the number of losses will be required. But for now, assume that the excess method works. The preceding analysis has indicated that various sets of retentions will equalize variance. How do we choose the right set?

On one hand, we would like to minimize the NS variance for a particular branch. On the other hand, we would like to measure the true performance of each profit center to the extent that it differs from the average of all branches. This is the classical credibility problem.

Using the model developed in the preceding section, we can specify the problem more precisely. For a branch i, let N_i represent the true (but unknown) number of large losses (greater than r). Let $\hat{N}_i$ be the estimate of the true number. Recall that a is assumed to be the same for all branches, so that the expected loss size is identical by profit center.

Above the retention br, the reinsurance cost to the branch is

(14) $\hat{N}_i \, b^{1-a} \, E(\tilde{X})$

when $\tilde{X}$ is a Pareto variable (from Equation (11)).

With no reinsurance, the expected true branch total loss cost is $N_i E(\tilde{X})$. Hence, the expected cost below the retention br is, from Equation (10)

(15) $N_i(1-b^{1-a})E(\tilde{X})$,

and total expected cost is

(16) $[N_i + (\hat{N}_i-N_i)\ b^{1-a}\ ]E(\tilde{X})$.

The <u>error</u> introduced by the reinsurance program is the expected total cost with reinsurance (Equation (16)) minus the true expected total losses of $N_i E(\tilde{X})$. Therefore the error for branch i is

(17) $(\hat{N}_i-N_i)b^{1-a}\ E(\tilde{X})$.

Since N_i is an unknown parameter, it has a prior probability distribution. Let $h(N_i)$ denote the prior distribution. We want to minimize the variance of the error in this process as a function of b. Thus we integrate the square of equation (17) over all possible values of the parameter N_i:

(18) $$EV(b) = \int_0^\infty (\hat{N}_i-N_i)^2 h(N_i) b^{2-2a}\ E^2(\tilde{X}) dN_i.$$

Bayesian credibility methods can be used to determine the $\hat{N}_i$ which will minimize the error variance for a fixed b. Since N_i is assumed to be a Poisson variable, a Gamma distribution could be used as the conjugate prior distribution (see Mayerson [6] for further information). Developing the optimal $\hat{N}_i$ estimate is an important practical problem, and needed for pricing internal reinsurance, but will be left for subsequent analysis.

The error variance, EV(b), is a decreasing function of the retention, and is zero with no reinsurance (infinite b). However, the NS variance due to randomness is an increasing function of b, being proportional to $a - 2b^{2-a}$. To illustrate the trade-off involved, we can scale both the NS and the error variance so that their maximum value is 1,000 units:

	a = 1.5		a = 2.0		a = 3.0	
b	EV	NS	EV	NS	EV	NS
1	1,000	120	1,000	194	1,000	333
2	500	320	250	463	63	667
4	250	602	63	731	4	833
8	125	1,000	16	1,000	0	917
	0	1,000	0	1,000	0	1,000

Actually, the NS variance is infinite for $a \leq 2$, so in the above table the maximum value of 1,000 has been set to occur at b = 8.

The non-systematic vs. error variance trade-off can be specified directly. First, we assume that the function of N_i in Equation (18) is proportional to N_i^2. This generally would be true when the number of large claims N_i equals an unknown parameter λ_i, having the same distribution for all branches, times an exposure measure for branch i. Thus the error variance becomes

$$(19) \quad EV(b) = N_i^2 c b_i^{2-2a} E^2(\tilde{X}),$$

where c is a constant, equal for all branches. Next, assume that we are willing to trade v units of NS variance for one unit of error variance. This condition produces an objective function T_i which is a linear combination of the NS variance $N_i E(\tilde{Y}^2)$ and Equation (19):

$$(20) \quad T_i = N_i r^2(a - 2b_i^{1-a})/(a-2) \quad + \quad vcN_i^2 b_i^{2-2a} r^2 a^2/(a-1)^2.$$

By setting the derivative with respect to b_i equal to zero, the optimal b_i minimizing T_i is

$$(21) \quad b_i^* = [N_i vca^2/(a-1)]^{1/a}.$$

This result implies that $b_2/b_1 = (N_2/N_1)^{1/a} = K^{1/a}$, a more elegant form compared to Equation (13).

The objective function T_i may be thought of as a "credibility-weighted" NS variance. Therefore the ratio T_i/N_i^2 is equivalent to the NS variance ratio $E(\tilde{X}^2)/N_i$ in Equation (9). However, we no longer want to equalize the NS variance across branches since doing so would not give proper weight to each profit center's true result based upon N_i. Instead of equalizing ROE variance by branch we minimize the <u>sum</u> of the individual branch credibility-weighted ROE variances. Since the T_i for each branch are independent, separately minimizing each of them (by choosing the b_i^*) minimizes the sum of the branch variances.

Equation (21) can be used to establish a set of retentions without directly specifying the v and c parameters. We merely choose (judgementally), for the smallest branch, a retention which would provide a reasonable balance between NS variance reduction and credibility of actual losses incurred. Retentions for the remaining branches are then determined easily.

Catastrophe Losses

Natural catastrophe losses are inherently unpredictable at the profit center level due to their low frequency and high severity (empirical evidence indicates an a value near 1.0). Even at the corporate level, these losses have a high variance. Consequently, there is almost no information regarding the true catastrophe loss expectation in a profit center's own experience for a single year. Since there is virtually no credibility in branch experience and the variance of catastrophe losses is high, it is necessary to remove variance, rather than to equalize it.

A reasonable method of handling catastrophe losses is to charge each profit center with its annual aggregate catastrophe loss expectation, as a percentage of earned premium. All actual catastrophe losses incurred are absorbed by the internal reinsurance pool. Notice that this protection is an extreme form of reinsurance since the variance of losses charged to the profit center is zero.

SUMMARY

A workable approach to measuring profits in property-liability insurance is the return on equity concept, with income defined such that timing differences are minimal. For measuring profits at the branch office level, it is important to equalize the ROE variance between branches. Otherwise the effect of measurement error is not uniform by profit center and a haphazard incentive system may result.

The preceding analysis has shown, in general terms, how both the systematic and non-systematic risk components can be equalized for profit centers. However, when the credibility of branch results is considered, some equalization of non-systematic risk must be forgone. Based upon the complexity of the risk-equalizing problem, a key observation emerges: As the range of branch sizes expands, the difficulty of equitably measuring profit center results increases dramatically. Also, the difficulty is compounded if the branches have widely varying product mixes.

Practical applications of these risk-equalizing and profit-measuring techniques will require additional, more specific assumptions and much empirical work. Appendix IV provides an example of a profit center income statement which might arise from these efforts.

APPENDIX I: SYSTEMATIC RISK

Assume that a product line has N identically distributed exposures with profit margins $\tilde{m}_i$ for $i = 1$ to N. For simplicity, let each exposure have a premium of one unit. The profit margin for the line is

$$(1.1) \quad \tilde{M} = (\sum \tilde{m}_i)/N,$$

where the limits of summation are 1 to N. The variance of the line profit margin is

$$(1.2) \quad Var(\tilde{M}) = Var[(\sum \tilde{m}_i)/N] = \Big[\sum Var(\tilde{m}_i) + \sum_{i \neq j} Cov(\tilde{m}_i, \tilde{m}_j)\Big]/N^2$$

$$= [NV^2 + (N^2 - N)\gamma V^2]/N^2 = V^2[\gamma + (1-\gamma)/N]$$

where $V^2 = Var(\tilde{m}_i)$ and γ is the correlation coefficient between two different $\tilde{m}_i$ and $\tilde{m}_j$. As the number of exposures becomes infinite, $Var(\tilde{M}) = \gamma V^2$. This is called systematic risk because it cannot be reduced by the law of large numbers. The remainder of the profit margin variance, $(1 - \gamma)V^2/N$, becomes smaller as N increases. This portion is called the non-systematic, or diversifiable risk; i.e., adding more exposures to a portfolio will reduce overall variance. The classical risk theory model assumes that $\gamma = 0$.

Variance of ROE for Combination of Two Product Lines

For two separate product lines, denoted by subscripts 1 and 2, the return on equity for a composite of the two lines is

(1.3) $\tilde{R}_t = (1 + i)(1 + k_t\tilde{m}_t)$.

The composite premium/equity ratio k_t equals $f_1k_1 + f_2k_2$ and the composite TPM is $\tilde{m}_t = (f_1k_1\tilde{m}_1 + f_2k_2\tilde{m}_2)/k_t$. Thus

(1.4) $\tilde{R}_t = (1 + i)(1 + f_1k_1\tilde{m}_1 + f_2k_2\tilde{m}_2)$.

Now assume that each line has an infinite number of exposures so that only sytematic risk is present. The variance of $\tilde{R}_t$ is

$$\begin{aligned}(1.5)\quad \mathrm{Var}(\tilde{R}_t) &= (1 + i)^2[f_1{}^2k_1{}^2\mathrm{Var}(\tilde{m}_1) + f_2{}^2k_2{}^2\mathrm{Var}(\tilde{m}_2) + 2f_1f_2k_1k_2\mathrm{Cov}(\tilde{m}_1,\tilde{m}_2)] \\ &= (1 + i)^2k_1{}^2[f_1{}^2\mathrm{Var}(\tilde{m}_1) + f_2{}^2\mathrm{Var}(\tilde{m}_1) + 2f_1f_2\,\rho\mathrm{Var}(\tilde{m}_1)] \\ &= (1 + i)^2k_1{}^2\mathrm{Var}(\tilde{m}_1)[(f_1 + f_2)^2 - 2f_1f_2(1-\rho)] \\ &= \mathrm{Var}(\tilde{R}_1)[1 - 2f_1f_2(1-\rho)].\end{aligned}$$

Here ρ is the correlation coefficient between $\tilde{m}_1$ and $\tilde{m}_2$. Notice that $k_1{}^2\mathrm{Var}(\tilde{m}_1) = k_2{}^2\mathrm{Var}(\tilde{m}_2)$ by definition of the premium/equity ratios.

APPENDIX II: THE PARETO DISTRIBUTION

The Pareto distribution in its simplest form is useful for fitting the tails of loss size distributions. Its cumulative function is

(2.1) $F(x) = 1 - (r/x)^a$ (where $a \geq 1$ and $x \geq r$)

with a density

(2.2) $f(x) = ar^a x^{-a-1}$.

The mean and second moment are

(2.3) $\mu = \int_r^\infty x f(x)dx = ar/(a-1)$

(2.4) $\mu_2 = \int_r^\infty x^2 f(x)dx = ar^2/(a-2)$.

Notice that the mean is infinite if $a \leq 1$ and the variance ($\mu_2 - \mu^2$) is infinite if $a \leq 2$. The expected portion of loss in the interval from r to br is

(2.5) $\int_r^{br} x f(x)dx = ar(1-b^{1-a})/(a-1)$, and

the remaining segment of expected loss above br is

(2.6) $\int_{br}^\infty x f(x)dx = arb^{1-a}/(a-1)$.

The second moment of loss limited to a retention br is

$$(2.7) \qquad \int_r^{br} x^2 f(x)dx + \int_{br}^{\infty} b^2 r^2 f(x)dx = r^2(a - 2b^{2-a})/(a-2) \text{ for } a \neq 2,$$

$$= r^2[1 + 2\cdot \ln(b)] \qquad \text{for } a = 2.$$

APPENDIX III: RANDOM SUMS

A common stochastic model for the claim-generating process is the random sum. This is discussed at length in probability theory (see Feller [3]). The total value of all losses occurring in a fixed time period is

$$(3.1)\quad \tilde{S}_N = \tilde{X}_1 + \tilde{X}_2 + \ldots + \tilde{X}_{\tilde{N}},$$

where the random variables $\tilde{X}_i$ are identically distributed and the number of claims $\tilde{N}$ is also a random variable independent of any $\tilde{X}_i$. Usually the $\tilde{X}_i$ are assumed to be independent of each other, but we will assume that correlation exists. The conditional expectation of the random sum given a fixed number of claims N is

$$(3.2)\quad E(\tilde{S}_N|N) = E(\sum_{i=1}^{N}\tilde{X}_i) = NE(\tilde{X})$$

The unconditional expectation is therefore

$$(3.3)\quad E(\tilde{S}_N) = \sum_{N=0}^{\infty} NE(\tilde{X})f(N) = E(\tilde{X})\, E(\tilde{N}),$$

where f(N) is the density function of N.

The conditional second moment, given a fixed N is

$$(3.4)\quad E(\tilde{S}_N{}^2|N) = E\Big[\sum_{i=1}^{N}\tilde{X}_i{}^2 + \sum_{i \neq j}\tilde{X}_i\tilde{X}_j\Big] = NE(\tilde{X}^2) + (N^2 - N)E(\tilde{X}_i\tilde{X}_j)$$

for $i \neq j$. The unconditional second moment becomes

$$(3.5)\quad E(\tilde{S}_N{}^2) = \sum_{N=0}^{\infty} [NE(\tilde{X}^2) + (N^2 - N)E(\tilde{X}_i\tilde{X}_j)]f(N)$$

$$= E(\tilde{N})E(\tilde{X}^2) + [E(\tilde{N}^2) - E(\tilde{N})]\ [Cov(\tilde{X}_i,\tilde{X}_j) + E^2(\tilde{X})]$$

since $Cov(\tilde{X}_i,\tilde{X}_j) = E(\tilde{X}_i,\tilde{X}_j) - E(\tilde{X}_i)E(\tilde{X}_j)$. The variance of the random sum is

$$(3.6)\quad Var(\tilde{S}_N) = E(\tilde{S}_N{}^2) - E^2(\tilde{S}_N)$$
$$= E(\tilde{N})Var(\tilde{X}) + E^2(\tilde{X})Var(\tilde{N}) + [Var(\tilde{N}) + E^2(\tilde{N}) - E(\tilde{N})]Cov(\tilde{X}_i,\tilde{X}_j),$$

after some manipulation of terms. If $\tilde{N}$ is a Poisson variable, then

$Var(\tilde{N}) = E(\tilde{N})$ and (3.6) simplifies to

$$(3.7)\quad Var(\tilde{S}_N) = E(\tilde{N})E(\tilde{X}^2) + E^2(\tilde{N})Cov(\tilde{X}_i,\tilde{X}_j).$$

Letting the premium charge to cover the aggregate losses be PE(N), and letting ρ represent the correlation coefficient between $\tilde{X}_i$ and $\tilde{X}_j$, the variance of the loss ratio becomes

$$(3.8)\quad Var[\tilde{S}_N/PE(\tilde{N})] = Var(\tilde{S}_N)/P^2E^2(\tilde{N}) = \frac{1}{P^2}\left[\frac{E(\tilde{X}^2)}{E(\tilde{N})} + \rho\, Var(\tilde{X})\right].$$

Frequency vs. Severity Components of Variance

Equation (3.6) can be separated into distinct frequency and severity components by alternately fixing $\tilde{N}$ and $\tilde{X}$ at their respective means (variances of the fixed variables are zero):

(3.9) $V[\tilde{S}_N | \tilde{X} = E(\tilde{X})] = E^2(\tilde{X})Var(\tilde{N})$ (Frequency Variance)

(3.10) $V[\tilde{S}_N | \tilde{N} = E(\tilde{N})] = E(\tilde{N})Var(\tilde{X}) + [E^2(\tilde{N}) - E(\tilde{N})]\rho\, Var(x)$ (Severity Variance)

When $\rho = 0$, the sum of the frequency and severity variances equals the total variance $Var(\tilde{S}_N)$. For a Poisson $\tilde{N}$ and a zero covariance between claim amounts, we find that the ratios of the variance components to the total variance are a sole function of the loss size distribution:

(3.11) Frequency Ratio: $\dfrac{E^2(\tilde{X})}{E(\tilde{X}^2)}$ Severity Ratio: $\dfrac{Var(\tilde{X})}{E(\tilde{X}^2)}$.

APPENDIX IV: PROFIT CENTER INCOME STATEMENT

Illustrative Example

Branch A: Second Quarter 1985 ($1,000's)

	Amount	%*	Comments
Premium Written	$1,150	8.2	Growth %
Premium Earned	1,000	5.4	Growth %
Gross Losses	650	65.0	Before internal reinsurance
Excess Losses	-45	-4.5	Amount recovered
Reinsurance Charge	40	4.0	For excess reinsurance
Catastrophe Charge	25	2.5	Gross losses exclude catastrophes
Net Losses	670	67.0	
Allocated Loss Expense	50	5.0	
Commissions	150	15.0	
Taxes and Fees	30	3.0	
Branch Overhead Expense	95	9.5	Actual branch costs
Home Office Overhead Exp.	65	6.5	Allocated as a fixed % of earned premium
Underwriting Result	-60	-6.0	
Total Profit	25	2.5	After income tax
Return on Equity	--	15.5	Implied; based on required equity

*of premium earned, except for premium

REFERENCES

1. Fairley, William B., "Investment Income and Profit Margins in Property-Liability Insurance: Theory and Empirical Results," The Bell Journal of Economics, X (Spring, 1979), 192-210.

2. Fama, Eugene F., and Miller, Merton H. The Theory of Finance, Hinsdale, Il.: Dryden Press, 1972.

3. Feller, William, An Introduction to Probability Theory and Its Applications, Vol. I., New York: John Wiley & Sons, Inc., 1968.

4. Kahane, Yehuda, "The Theory of Insurance Risk Premiums--A Re-examination in the Light of Recent Developments in Capital Market Theory," The ASTIN Bulletin, Vol. 10, Part 2, 223-239.

5. Lev, Baruch, Financial Statement Analysis: A New Approach, Englewood Cliffs, N.J.: Prentice-Hall, 1974.

6. Mayerson, Allen L., A Bayesian View of Credibility," Proceedings of the Casualty Actuarial Society, LI, 85-104.

7. Patrik, Gary, "Estimating Casualty Insurance Loss Amount Distributions," Proceedings of the Actuarial Society, LXVII, 57-109.

8. Plotkin, Irving H., Rates of Return in the Property and Liability Insurance Industry: 1955-1967, Cambridge: Arthur D. Little, Inc., 1969.

9. Sharpe, William R., Portfolio Theory and Capital Markets, New York: McGraw-Hill, 1970.

10. Williams, C. Arthur, Jr., "Regulating Property and Liability Insurance Rates Through Excess Profits Statutes," Journal of Risk and Insurance, L, No. 3, 445-472.

A FORMAL APPROACH TO CATASTROPHE RISK ASSESSMENT AND MANAGEMENT

Karen M. Clark

The author is a Research Associate for Commercial Union Insurance Company where she has worked on pricing and reinsurance issues as well as catastrophe risk assessment. She holds an M.B.A. and an M.A. in Economics from Boston University.

ABSTRACT

Insurers paid $1.9 billion on property claims arising from catastrophes in 1983. Researchers have estimated that annual insured catastrophe losses could exceed $14 billion. Certainly, the financial implications for the insurance industry of losses of this magnitude would be severe; even industry losses much smaller in magnitude could cause financial difficulties for insurers who are heavily exposed to the risk of catastrophic losses.

The quantification of exposures to catastrophes, and the estimation of expected and probable maximum losses on these exposures pose problems for actuaries. This paper presents a methodology based on Monte Carlo simulation for estimating the probability distributions of property losses from catastrophes and discusses the uses of the probability distributions in management decision-making and planning.

INTRODUCTION

There were 33 named catastrophes in 1982, and they resulted in an estimated $1.5 billion of insured property damage. Most of these catastrophes were natural disasters such as hurricanes, tornadoes, winter storms, and floods. In 1983, hurricane Alicia caused over $675 million of insured losses; the December storms caused insured damage of $510 million.[1]

Hurricane Alicia barely rated a three on a severity scale ranging from one to five, and destruction from hurricanes increases exponentially with increasing severity. A hurricane that rated a four hit New York and New England in 1938; 600 people died and wind speeds of 183 mph caused hundreds of millions of dollars of damage.

If this storm were to strike again, dollar losses to the insurance industry could amount to six billion given the current insured property values on Long Island and along the New England coast. Estimates of the dollar damages that will result if a major earthquake occurs in Northern or Southern California are even larger in magnitude.

A very severe hurricane or earthquake would produce a year of catastrophic loss experience lying in the upper tail of the probability distribution of annual losses from catastrophes, and it is the opinion of the author that the 1982 catastrophe loss figure lies in the lower end of this distribution. However, the determination of the shape and the estimation of the parameters that describe this distribution are tasks that are not easily performed by standard actuarial methodologies. Yet since insurers need the knowledge of their exposures to catastrophes and the probability distributions of annual catastrophic losses to make pricing, marketing, and reinsurance decisions, the estimation of the distribution and the expected and probable maximum losses pose problems for actuaries.

Standard statistical approaches to estimation involve the use of historical data to forecast future values of variables. However, models based on time series of past catastrophe losses are not appropriate for estimating future losses. Catastrophes are rare events so that the actual loss data are sparse and their accuracy is questionable; average recurrence intervals are long so that many exogenous variables change in the time periods between occurrences. In particular, changing population distributions, changing building codes, and changing building repair costs alter the annual catastrophe loss distribution.

Since most catastrophes are caused by natural hazards and since most natural hazards have associated with them geographical frequency and severity patterns, the population distribution impacts the damage producing potentials of these hazards. A natural disaster results when a natural hazard occurs in a populated area. Changing population patterns necessarily alter the probability distribution of catastrophic losses. Since the average recurrence intervals of natural hazards in any particular area are long, patterns of insured property values may vary between occurrences to an extent that damage figures of historical occurrences have no predictive power. For example, if hurricane Alicia had struck in 1950, dollar damages would have been significantly lower even after adjustment for inflation because of the smaller number of insured residential and commercial structures in the Houston area at that time.

It is primarily the influence of the geographic population distribution that renders time series models inadequate although changing building codes also alter the loss producing potentials of natural hazards. As time passes, building materials and designs change, and new structures become more or less vulnerable to particular natural hazards than the old structures. Of course, changes in building repair costs also affect the dollar damages that will result from catastrophes.

The above issues do not render the estimation problem intractable, but they do produce a need for an alternative methodology to approaches which employ historical catastrophe losses adjusted for inflation to approximate the probability distribution of losses. Even models which adjust these losses for population shifts can give only very rough approximations of expected and probable maximum losses.

This paper presents a methodology based on Monte Carlo simulation, and it focuses on property damage from natural disasters. Part I discusses Monte Carlo simulation and the natural hazard simulation model. A windstorm example is employed to illustrate the approach. Part II outlines the ways in which management may use the model and its output for decision-making and strategy formulation. It discusses how knowledge of the probability distribution of property losses due to catastrophes enables management to make risk versus return trade-offs in marketing, pricing, and reinsurance decisions.

PART I: ESTIMATING THE PROBABILITY DISTRIBUTION OF CATASTROPHE LOSSES

Monte Carlo Simulation

Dramatic decreases in computing costs have led to the increased use of computer simulation in the analysis of a wide variety of problems. Many of these problems involve solutions that are difficult to obtain analytically. For example, computer simulation may be employed to evaluate complex integrals or to determine one or more attributes of complex systems. Law and Kelton state that "Most complex, real-world systems ... cannot be accurately described by a mathematical model which can be evaluated analytically. Thus, a simulation is often the only type of investigation possible." [8, p.8]

The simulation approach is very basically the development of computer programs that describe the particular system under study. All of the system variables and their interrelationships are included. A high speed computer then "simulates" the activity of the system and outputs the measures of interest.

Simulation models may be deterministic or stochastic. Monte Carlo simulation models are stochastic models with random variables from stable probability distributions; they are static, i.e. not time dependent, models.

A Monte Carlo simulation model is an excellent tool for performing sensitivity analyses of the system of interest. Alternative values of input variables may be given; the system may be resimulated and new output produced. This type of simulation may be employed to analyze a variety of insurance related problems. Arata described five areas in which actuaries may employ the simulation approach; one of these is the pricing of difficult or catastrophic exposures. [1]

The Natural Hazard Simulation Model

The natural hazard simulation model is a model of the natural disaster "system". As stated in the introduction, models based on historical catastrophe losses are not appropriate for forecasting future losses.

Standard statistical approaches are found lacking for three reasons. First, since the losses are caused by rare events, there is not much historical loss data available and those that are available are imprecise. Parameters estimated from the historical loss distribution will be subject to much uncertainty because of the small sample size. Secondly, the shape of the distribution itself is not clearly discernible. Finally, the distribution is not stable since many factors that influence it change with time.

The Monte Carlo model described below simulates natural hazards so that the primary variables are meteorological or geophysical in nature. These variables are random variables that have stable probability distributions, and although the historical data on these variables are sparse as are the loss data, their probability distributions may be supplemented with the knowledge of authoritative meteorologists and geophysicists.

This is, therefore, a stochastic yet stable system. The variables that change with time, i.e. the geographic distribution of exposure units, the insured property values, and the building construction types, are inputs into the model and the probability distribution of losses from natural hazards given these inputs is the model output. These inputs may be changed to see how the loss distribution is altered.

The model variables may also be classified as frequency or severity variables. The frequency variables indicate the expected number of occurrences of the particular events within a given time period. Severity variables represent the physical components of natural hazards and they do not have a time dimension. Severity variables account for a hazard's force, size, and duration.

A year of natural disasters is simulated thousands of times to generate the probability distribution of annual losses. For each model iteration and for each natural hazard, the following is performed:

1. The annual number of occurrences is generated from the frequency distribution.

2. The exact location of each occurrence is generated.

3. For each occurrence, values for the force and size are generated from the severity distributions.

4. The movement of the event across the affected area is simulated, and dollar damages are calculated and accumulated.

The average of all iterations is the model-generated expected loss estimate; a higher percentile loss is the probable maximum loss estimate.

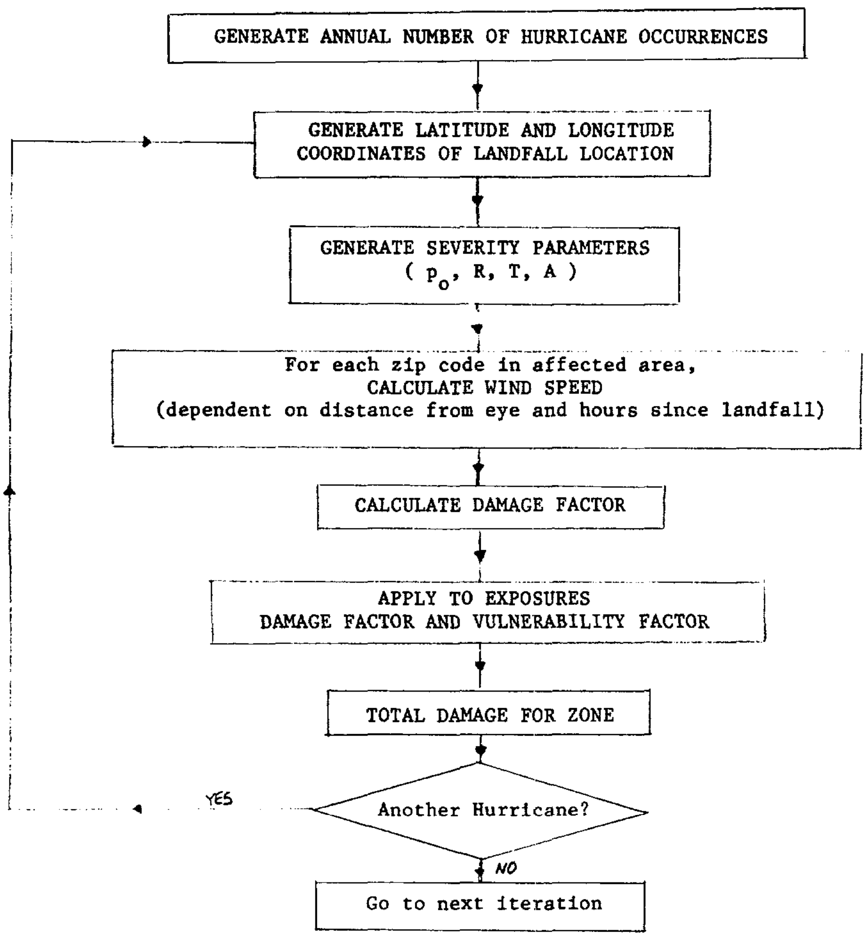
A Windstorm Example

A model of the hurricane hazard has been developed, and this model will be used to illustrate the Monte Carlo simulation approach. Exhibit 1 is a flowchart of the computer model.

All of the storm data used in the development of the model were obtained from the U.S. Department of Commerce. The data had been collected and analyzed by various agencies of the National Weather Service, and they included seventy-nine years of history spanning the period from 1900 to 1978. Complete and accurate data were available for most of the hurricanes that struck the U.S in this time period.

EXHIBIT 1

MODEL FLOWCHART



A hurricane is a closed atmospheric circulation which develops over tropical waters and in which winds move counterclockwise around a center of pressure lower than the surrounding area. It is a severe tropical storm in which the center of pressure is less than or equal to 29 (in.) which causes sustainable wind speeds of 74 mph or more. One hundred and twenty-eight hurricanes either approached and bypassed or entered the U.S. during the sample period.

Referring back to exhibit 1, the first step of the model is the generation of the annual number of landfalling hurricanes. Table 1 shows the numbers of years in which the number of occurrences was 0, 1, 2, and so on. The exhibited data fit a Poisson distribution with mean and variance equal to 1.8, and the model generates the annual frequency from this distribution.

Table 1

ANNUAL NUMBER OF HURRICANES LANDFALLING IN U.S
1900-1978

No. storms per year	Observed occurrence
0	25
1	25
2	14
3	8
4	5
5	1
6	1

The next step of the model is the determination of the landfall location of each storm. Hurricanes enter the U.S. from the Gulf and East Coasts. The map in exhibit 2 shows the U.S. coastline from Texas to Maine divided into thirty-one smoothed 100 nautical mile segments.[2] The number of hurricanes that entered through each segment during the sample period is also shown.

The numbers seem to indicate that there are variations in locational frequencies. In this case, it would not be appropriate to generate the landfall location from a distribution which assigns equal probabilities to all values, i.e. a uniform distribution. However, the limited amount of data precludes one from ascertaining statistically whether there are true frequency differences or whether the variations are caused by randomness within the small samples.

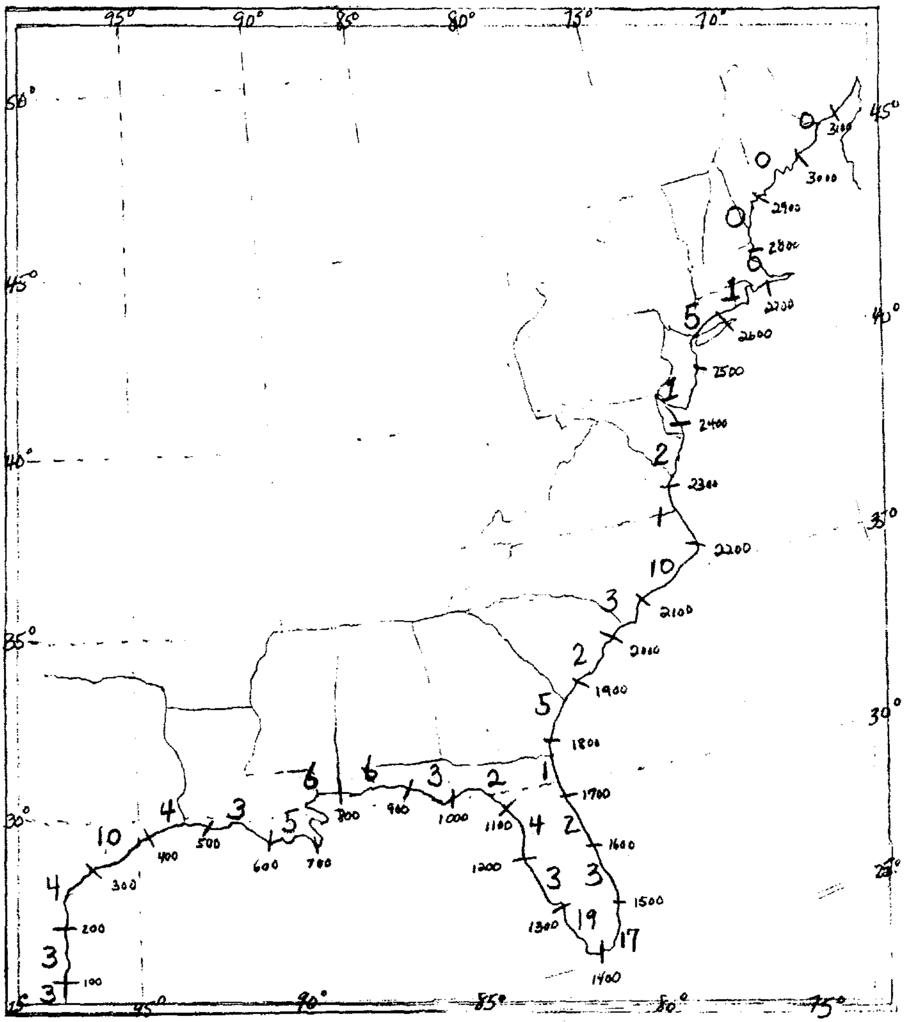
The actual number of storm occurrences within each segment is not employed by the model to develop the relative frequency distribution. It is not clear, first of all, if 100 nautical mile segments are the appropriate lengths of coastline to use for the calculations. Additionally, although several segments are completely free of historical storm occurrences, it is not clear that the probability of hurricane landfall is zero in these areas.

The relative frequency may be estimated by correlating it with another variable for which the value is known or may be estimated for each segment. Alternatively, the causal relationship between a variable(s) and the frequency of landfall may be employed if such a relationship

[2]The coastline is smoothed for irregularities such as inlets and bays.

Exhibit 2

HURRICANES ENTERING THE U.S. 1900-1978



exists. Of course, if one knew all of the conditions favoring landfall, one could assign probabilities based on the existence or absence of these conditions at each coastal location.

The way in which hurricanes are formed as well as the process by which energy is supplied to the circulating winds determine the likely paths of these storms. To illustrate, "hurricanes obtain kinetic energy from latent heat from the condensation and precipitation of water vapor. Therefore, hurricanes develop over warm tropical ocean areas where evaporation rates are very high and vast quantities of water vapor are stored in the atmosphere. The general movement of air over most of the Tropics is from the east while in higher latitudes it is usually from the west. Consequently, most hurricanes move initially to the west and may drift slightly northward. However, as they continue to drift toward higher latitudes, they come under the influence of westerly winds and recurve to the east" [4, p.3]. Wind patterns, therefore, provide an explanation for the lower frequencies at higher latitudes.

To derive the model locational frequency distribution, the following approach was adopted. First, the hurricane data were supplemented with data on all tropical storms. Tropical storms are closed atmospheric circulations with less intense winds than those of hurricanes. The assumption here is that the atmospheric conditions that favor the occurrence of a tropical storm are the same conditions that favor the occurrence of a hurricane. The additional data eliminate the problem of long coastal segments with no historical occurrences.

Next, the raw data on numbers of occurrences were smoothed using a procedure that was selected on the basis of its ability to capture turning points in the data while smoothing slight variations. The coastline was redivided into 50 nautical mile segments, and the number of occurrences for each segment was set equal to the weighted average of 11 successive data points centered on that segment. The smoothed frequency values are obtained as follows:

$$F_i = \frac{\sum_{n=-5}^{5} W_n C_{i+n}}{\sum_{n=-5}^{5} W_n}$$

where C_i = the number of historical hurricane occurrences for the ith segment

F_i = the smoothed frequency value for the ith segment

W_n = .300, .252, .140, .028, -.04, -.03

for n = 0, ±1, ±2, ±3, ±4, ±5, respectively

This is the preferred smoothing procedure in climatological analyses because the weighting scheme maintains the frequency and phase angle of the original series of numbers. The endpoints of the series were approximated so that each segment of the coast was assigned a relative frequency. The landfall location of each storm is generated from the thus derived locational frequency distribution.

Step three of the model is the generation of values for the severity variables. There are four primary variables which account for hurricane severity. These variables are: the minimum central pressure, the radius of maximum winds, the forward speed, and the wind inflow angle.

Central pressure (p_o) is defined as the sea-level pressure at the hurricane center or eye. This variable is the most important for computing hurricane windspeeds, and it is a universally accepted index of hurricane intensity. All else being equal, the square of the wind speed varies directly with Δp ($\Delta p = p_w - p_o$) where p_w is the peripheral pressure, i.e. the sea level pressure at the periphery of the storm.

The radius of maximum winds (R) is the radial distance from the hurricane center to the band of strongest winds. Forward speed (T) refers to the rate of translation of the hurricane center from one geographical point to another. Track direction (A) is the path of forward movement along which the hurricane is traveling and is measured clockwise from north.

The empirical data on each severity variable cannot be fit to standard theoretical distributions as were the annual frequency data. There appears to be a geographical hurricane severity pattern as there was a locational frequency pattern so that the probability density functions of the severity variables vary by location, and there are not enough data points at each location to estimate these functions. Additionally, the available data indicate that the severity variables are not independent. Linear correlation coefficients are positive between most pairs of variables. However, it is not possible to test the significance of the correlation coefficients unless it is assumed that pairs of variables form bivariate normal distributions.

If the variables are not independent, their correlations must be explicitly formulated within the model since the correlations will impact the variance of the model output, i.e. the estimated hurricane loss distribution.

The strongest correlations seem to be between the severity variables and latitude. In general, as latitude increases, average hurricane severity decreases as does frequency. When a hurricane moves over cooler waters, its primary source of energy is reduced so that the intensity of circulation decreases in the absence of outside forces. "The reasons for the increase in central pressure[3] from south to north include: the inability of hurricanes to carry their warm, moist, tropical atmosphere into temperate latitudes and the entrance of colder and drier air at low levels, which ... decreases the amount of energy available to the storm." [7, p.39]

The data, however, indicate a more direct relationship between severity and latitude than that between frequency and latitude, and the mathematical expressions which describe the relationships between the hurricane severity components and latitude were estimated and employed

[3] Central pressure is inversely related to severity so that high central pressures result in less severe storms.

by the simulation model in the following manner: Given the latitude and longitude coordinates of the landfall location, the latitude coordinate is entered into the equations to obtain initial values of the severity variables. Stochastic elements are added to the initial values and the sums become the simulated values. The stochastic elements are generated from the distribution of the error term for each equation.

Linear transformations of exponential, power, hyperbolic, and other special functions were fit to the empirical data for each severity variable using the ordinary least squares estimation procedure. Simple linear equations provided the best fits of the relationships between R and latitude and T and latitude.

Exhibit 3 shows a plot of the latitude, radius of maximum winds pairs for the 128 hurricanes in the data sample. Exhibit 4 shows the linear regression residuals plotted against latitude. Although the dispersion of the residuals is wide, i.e. the standard deviation is 10.10, the errors are distributed normally with expected value equal to zero. This statistical distribution is employed to generate the values e_i for the following equation:

$$R_i = a + b(L_i) + e_i$$

where R_i = the ith simulated value for R

L_i = the latitude coordinate for the ith hurricane

a,b are the estimated regression coefficients

Exhibit 3

LATITUDE VS. RADIUS OF MAXIMUM WINDS

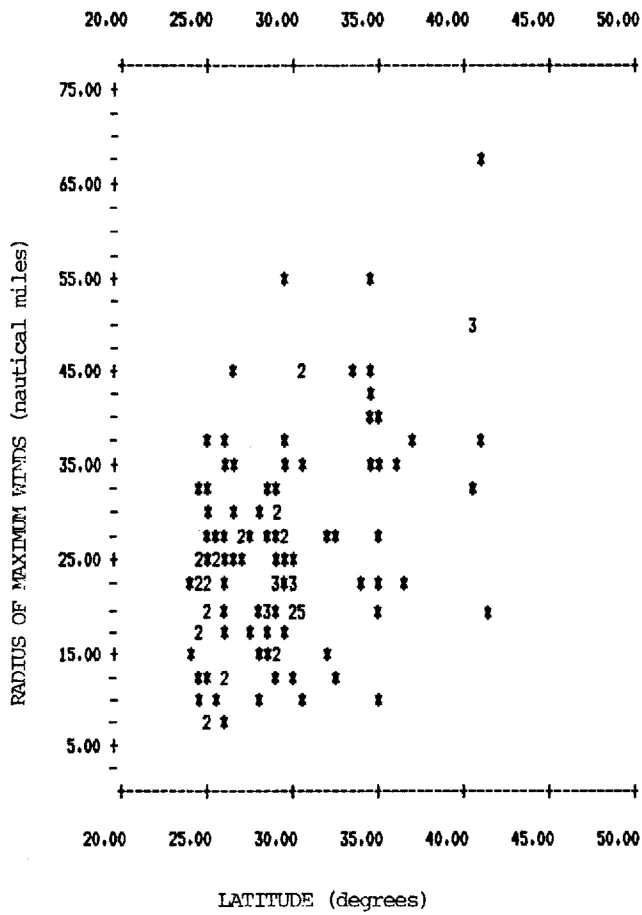
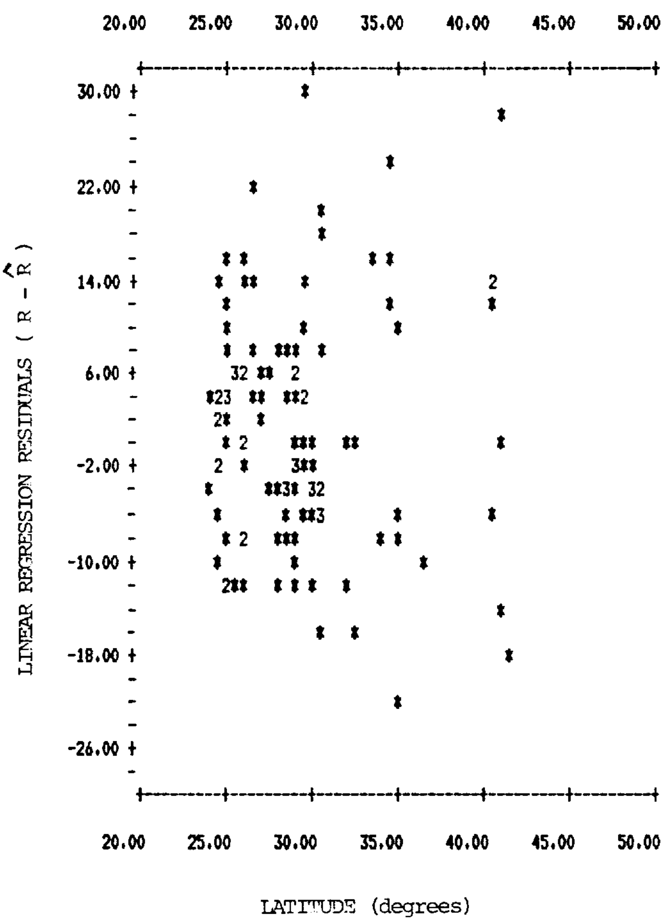


Exhibit 4

LATITUDE VS. LINEAR REGRESSION RESIDUALS ($R - \hat{R}$)



The distribution of the simulated values of R is then bounded by meteorological estimates of lowest and highest possible values.

The strength of the linear relationship between latitude and forward speed is even greater than that between latitude and the radius of maximum winds as shown by exhibit 5. However, the regression residuals shown in exhibit 6 seem to be heteroskedastic, i.e. the variance of the residuals increases with latitude. A basic assumption of the linear regression model is that the distribution of the error term has a constant variance, and the violation of this assumption leads to least squares estimators that are not efficient, i.e. minimum variance, or asymtotically efficient.

For the simulation model, it is also important that the distribution of the error term from which values are generated is stable for all values of latitude. If this is not the case, the simulated values of the particular variable will not form probability distributions that match the true underlying distributions, and the model-generated probability distribution of losses will not provide an accurate estimate of the true probability distribution of losses.

Corrections for heteroskedasticity were made by assuming that the variance of the error term is proportional to latitude. The re-estimated regression equation residuals are shown in exhibit 7; they form a normal distribution with mean equal to zero and standard deviation equal to 4.9.

Exhibit 5

LATITUDE VS. FORWARD SPEED

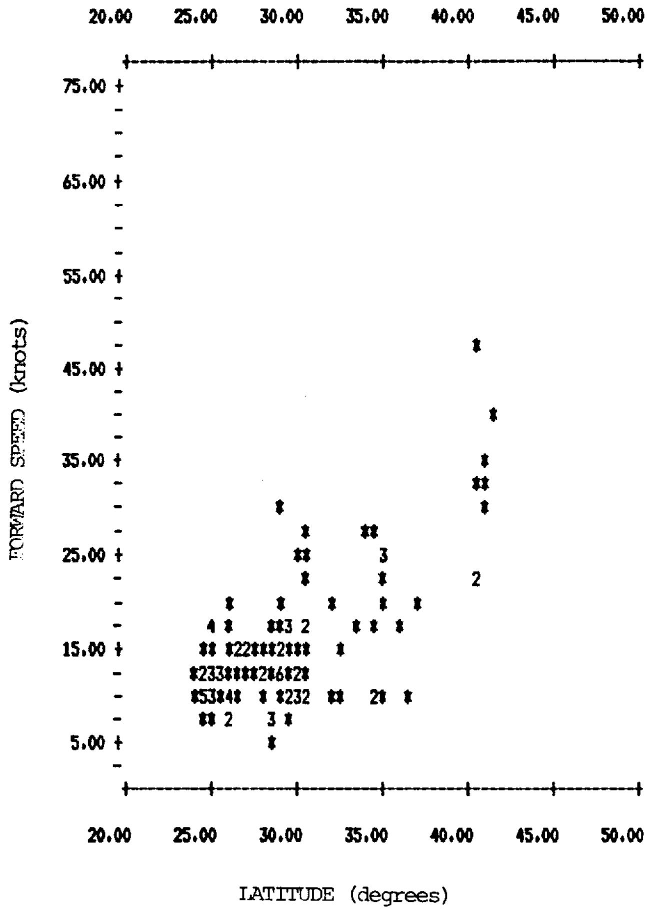


Exhibit 6

LATITUDE VS. LINEAR REGRESSION RESIDUALS ($T - \hat{T}$)

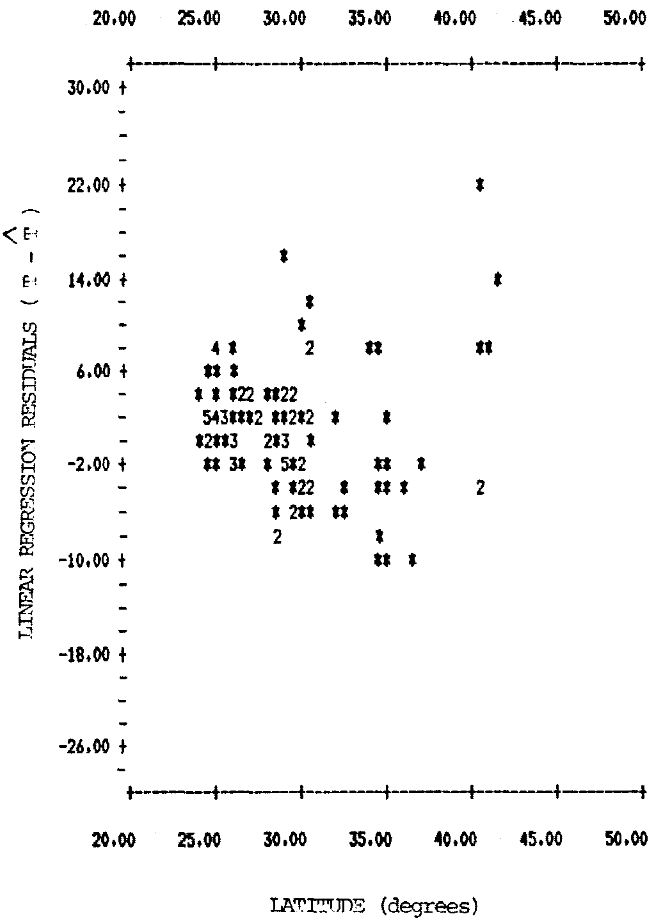
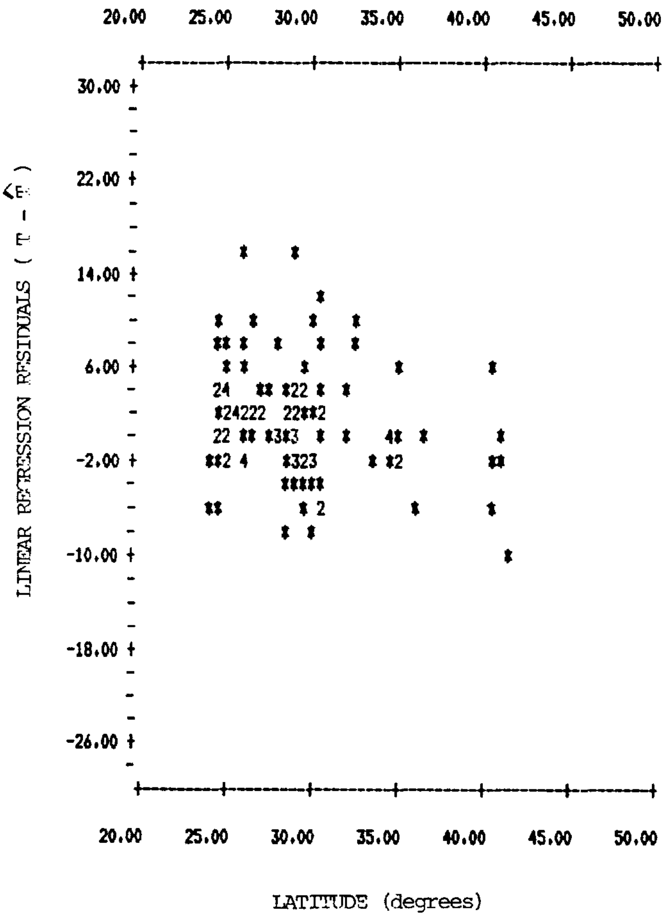


Exhibit 7

LATITUDE VS. LINEAR REGRESSION RESIDUALS ($T - \hat{T}$)
AFTER CORRECTION FOR HETEROSKEDASTICITY



Although hurricane central pressure and track direction are both correlated with latitude, the relationships between p_o and latitude and A and latitude were more difficult to estimate statistically. For these variables, the simulated values are generated from the empirical distributions. Outliers are first removed so that the simulated values for each coastal location are within the lower and upper bounds that have been developed by meteorologists.

The movement of the storm is next simulated by the computer model, and the property damage inflicted by the circulating winds is calculated for each geographic location. The particular geographical unit for which the damages are accumulated is determined by the model input. Insured property values are input along with the construction types and ages of the insured buildings and locational information such as zipcodes and counties. Wind speeds and dollar damages are calculated for each zipcode, but the damages may be accumulated by larger units to provide more meaningful output.

The dollar damages are calculated by applying damage and vulnerability factors to the dollar amounts of liability. The damage factors are based on the results of engineering studies of the relationship between wind speeds and structural damage. The vulnerability factors account for the variability in inflicted damage due to construction type and age. The dollar damages are accumulated by the selected geographical units.

Two thousand years of hurricane experience are simulated by the model. These two thousand model iterations provide a complete probability distribution of annual hurricane losses from which the expected loss and probable maximum loss estimates are derived.

Exhibit 8 shows the expected losses as well as the 80%, 90%, 95%, and 99% confidence level losses calculated as the 80th, 90th, 95th, and 99th percentile losses, respectively, for a given geographical distribution of exposures. These confidence level losses may be interpreted in two ways. A given confidence level loss shows the loss amount for which the probability of experiencing losses above that amount is 1.0 minus the particular confidence level. For the loss distribution in exhibit 8 the probability of experiencing losses above $10 million is .20. The confidence level loss also shows the loss amount for which losses greater than that amount will be experienced on average once in every 1.0/(1.0 - confidence level) years. Again, from exhibit 8 losses greater than $10 million will be experienced once in every five years on average. The loss distribution is highly skewed with a median value which is much below the mean and a high proportion of zero values.

The model output provides management with information that may be used in the formulation of pricing, marketing, and reinsurance strategies. Before the uses of the model output are discussed, the next section will summarize the Monte Carlo simulation approach.

Exhibit 8

MODEL-GENERATED LOSS ESTIMATES

LIABILITIES	EXPECTED LOSS	CONFIDENCE LEVEL LOSSES 80%	90%	95%	99%
7,170,753,024	9,011,808	10,003,715	24,179,636	44,827,623	117,946,980

Summary of the Methodology

The Monte Carlo simulation approach to the estimation of the probability distribution of catastrophe losses involves the development of models to simulate catastrophes. Each model is developed around the probability distributions of the random variables of the loss-producing "system."

Ideally, the model builder will have an a priori theory on the shape of the probability distribution underlying each random variable. For the results of the Monte Carlo simulation to be valid, the underlying model assumptions must be true. The empirical distribution formed by the raw data may be compared to standard statistical distributions using appropriate goodness-of-fit tests, and if the data do fit a well-known probability distribution, the moments of the distribution may be estimated and employed by the simulation model. In the windstorm example, the Poisson distribution was used to generate the annual number of hurricanes.

Alternatively, the expressions which describe the relationships between model variables may be estimated and employed by the model to generate simulated values of variables. This approach was adopted for some of the hurricane severity components.

Finally, the empirical distribution may be employed for the generation of values for a particular model variable. This procedure, however has a few drawbacks. First, since the sample is a collection of random data, a different sample could yield a very different empirical distribution. Secondly, the generation of random variables from an empirical distribution precludes the possibility of generating a value of the variable outside of the observed range, and the observed range may not include all possible values of the variable. Finally, the generation of values from empirical distributions is, in general, less efficient from the standpoint of computing time than the generation of values from theoretical distributions. Nevertheless, in some cases generation from the empirical distribution is either necessary or preferred for various reasons, and in these cases, the empirical distribution can be programmed into the model.

The testimony of experts may be employed along with the statistical data to build the model. Physical scientists who have studied extensively the phenomena of interest can provide information on the ranges of possible values of particular variables as well as on the most likely value or values. This information may enable the model builder to substitute theoretical distributions for empirical distributions, to identify outliers in the data, and/or to determine appropriate points at which to bound the probability distributions.

Once the model is built, i.e. the important variables have been identified and their probability distributions and interrelationships have been programmed into the computer, the system is simulated many times to provide a range of all possible annual loss amounts. There is no standard formula that gives the number of model iterations necessary to produce output with a given level of precision for this type of Monte

Carlo simulation model. The necessary number of iterations is endogenous, i.e. model-dependent.

Given that the variable of interest is annual dollar losses from catastrophes, one hopes to derive accurate estimates of annual expected losses and maximum probable losses. Assuming that the model has been specified correctly, the expected loss estimate will converge to the true expected loss and the model generated loss distribution will converge to the true loss distribution as the number of iterations increases. Very basically,

$$E(X) = \lim_{n \to \infty} \frac{\sum_{i=1}^{n} X_i}{n}$$

where $E(X)$ = expected annual loss

X_i = annual loss from ith model iteration

and,

$$F(X) = \lim_{n \to \infty} F_n(X)$$

where $F(X)$ = the distribution function of annual losses

$F_n(X)$ = distribution function of n model generated annual loss figures.

The larger the variance of the probability distribution of annual losses, the larger the value of n needed to produce loss estimates with a given level of precision. The variability of the model generated annual losses is determined by the variability of the model variables, i.e. the frequency and severity variables, and their correlations. If the model variables are positively correlated, the variance of the loss distribution will be greater than it would be in the absence of this correlation.

Although there is no straightforward procedure for calculating the value of n needed for specified precision levels, there are a few procedures that may be employed to develop confidence intervals for E(X) if certain assumptions are made. These procedures will not be discussed here, and the interested reader is referred to Chapter 8 of Law and Kelton. [9]

The recommendation of this author is to perform at least 1000 model iterations if possible. This should not present a problem given the low costs of computing time on high speed computers; however, development time may be well spent on writing efficient computer programs that minimize the computing time, particularly if the model is to be run frequently. If each iteration is still expensive, certain variance reduction techniques may be employed to reduce the number of iterations needed to reach convergence. The model builder may perform tests to see how quickly the loss distribution is converging. Iterations may be performed in groups of 100 so that the changes in the loss distribution may be monitored. A stable loss distribution indicates that convergence has been reached.

Validation of simulation models often presents a problem if there are no historical data on the variable that the model is designed to measure. In the case of natural hazard simulation models, the historical data are sparse. Past occurrences may be simulated, nevertheless, if the geographical distribution of exposures that is input to the model corresponds precisely to the geographical distribution of exposures that existed at the time of the occurrence. Insured values, construction types, and ages of exposure units should also match precisely. Values for the variables which account for the severity components of the hazard are input to the model, the model is simulated, and the model-generated loss estimate is output. This estimate is compared to actual dollar damages to test the validity of the model and its underlying assumptions.

There are several advantages of the simulation approach. First of all, it is able to capture the effects on the loss distribution of changes in variables over time. Secondly, this estimation procedure provides management with a complete picture of the probability distribution of losses rather than just estimates of expected and probable maximum losses. And finally, the Monte Carlo simulation approach provides a framework for performing sensitivity analyses and "what-if" studies. The model uses will be described in the following sections.

PART II: MANAGING EXPOSURE TO CATASTROPHES

A methodology for estimating the probability distribution of annual catastrophe losses given a particular geographical distribution of exposures was described and illustrated in Part I. Knowledge of the probability distribution of losses enables insurers to manage their exposures to catastrophes. With respect to these exposures, management has several options:

1. Write no business in catastrophe prone areas.

2. Exclude coverage for losses caused by natural hazards.

3. Plan to recover losses after a catastrophe occurs by retrospective pricing.

4. Spread property business so that it is not concentrated in catastrophe prone areas.

5. Add loadings to premiums and build up reserves to cover catastrophe losses when they occur.

6. Reinsure property business.

Option 1 does not present a very viable strategy since most areas of the continental U.S. are prone to natural disasters of at least one type.

For example, the Gulf and East Coast states are prone to hurricanes while the Great Plains and Midwestern states are highly prone to tornadoes. Earthquakes are natural hazards with the greatest damage producing potential in California, Nevada, Washington, and parts of Indiana, Missouri, Tennessee, Arkansas, South Carolina, and Massachusetts. [3]

Option 2 may also not be feasible. If an industry-wide attempt is made to exclude coverage for losses resulting from a particular hazard, legislation may be passed to prevent effective exclusion. Recent legislation in California concerning concurrent causation is a case in point. On the other hand, if a single company or group of companies attempt to exclude coverage, business will certainly be lost to competitors who do provide coverage unless the policy premiums are reduced sufficiently.

The insurance industry has traditionally priced its products retrospectively since expected costs are estimated from past costs. Policy premiums are determined by the most recent historical loss experience so that larger than expected losses in year t will lead to higher prices in year t+1. As long as the individual firm's loss experience is better than or equal to the industry average, the firm may set premiums in relation to its own loss experience (in the absence of regulatory barriers.) However, if the individual firm's loss experience is worse than the industry average, competition will force the firm to price below its costs.

Retrospective pricing cannot be used to recover losses from catastrophes. If an individual firm experiences a disproportionate share of total industry losses from the occurrence of a catastrophic event in year t, competition will prevent the firm from increasing its rates enough to recover all of its losses in year t+1. Additionally, the industry as a whole is prevented from increasing rates dramatically after the occurrence of a catastrophe by the threat of new entry.

The barriers to entry into the insurance industry are high enough to allow retrospective pricing of normal insurance covers; however, a financial need of existing firms to raise prices by a significant amount would provide a competitive advantage for new entrants that are free of the financial burden.

Accordingly, option 3 is an inferior strategy as were options 1 and 2. The last three alternatives, however, are all viable strategic options, and each one will be discussed in turn under the headings of marketing, pricing, and reinsurance.

Marketing

The windstorm simulation model output as illustrated in exhibit 8 shows the probability distribution of annual countrywide losses from the hurricane hazard. For marketing purposes, however, it may be more useful to divide the country into zones so that the specific areas of high windstorm risk are clearly identifiable.

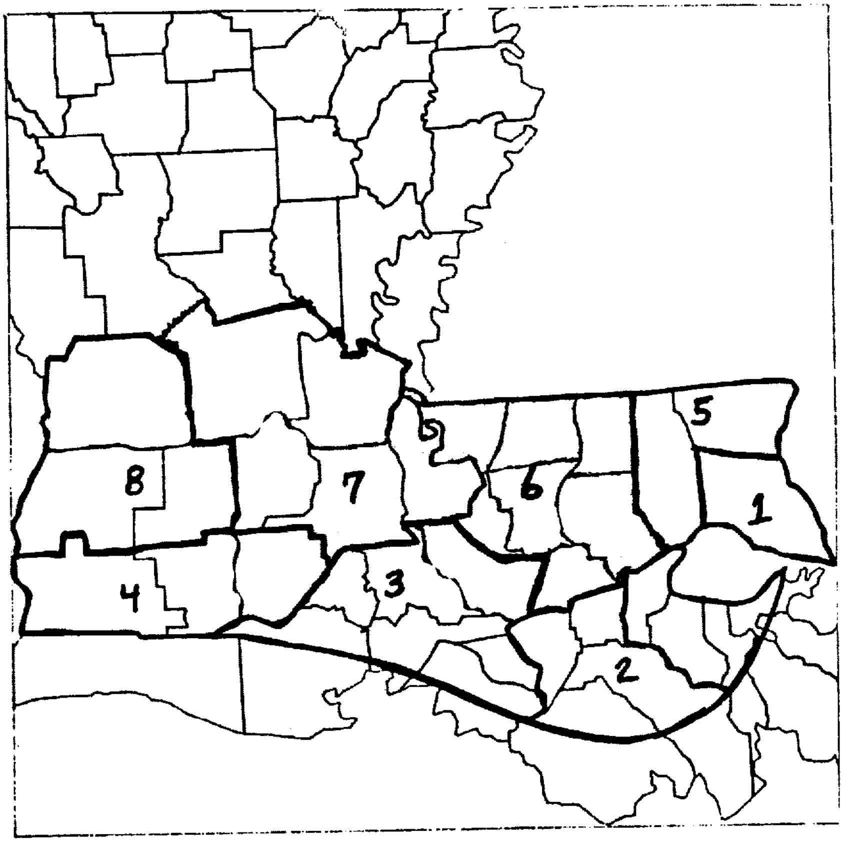
The computer model may be designed to accumulate dollar damages by state, by county, or by any other geographical configuration. Exhibit 9 shows the state of Louisiana divided into eight zones. The dollars of liability, i.e. exposure, the expected loss, and various confidence level losses[4] are shown for each zone. The loss figures show clearly that the higher risk areas are the coastal zones. The hurricane is at maximum force just as it crosses overland; as it travels inland, the storm dissipates because of the elimination of its primary energy source i.e. kinetic energy derived from the sea, and because of frictional effects.

Because all natural hazards have associated with them geographical frequency and severity patterns, they will produce gradations of damage or pockets of high risk and low risk. Management will want to avoid concentrations of property exposures in high risk areas, and the model output enables the development of marketing plans that are based on the long term profit potentials of various markets.

Property business in high risk areas may be very profitable in years of no natural hazard occurrences. As years pass and no catastrophes occur, insurers may begin to compete for the business in a high risk area. The competition may drive the profits as well as the catastrophe loading to zero so that there are no resources available to cover the catastrophic losses when they occur. Knowledge of the probability distributions of losses from natural hazards in these areas enables insurers to resist the temptation to write business based on the very recent loss experience in these areas.

[4]It is interesting to note that for small geographic areas, the confidence level losses may be zero since the frequencies of hurricances in specific locations are low.

Exhibit 9



Louisiana Windstorm Zones

zone		$ exp	expected loss	confidence level losses 80%	90%	95%	99%
LOUIS	1	90417112.	256512.	0.	276770.	1947396.	4938375.
LOUIS	2	9210113.	25540.	0.	12932.	213693.	537371.
LOUIS	3	56674660.	94866.	0.	31306.	653101.	2098500.
LOUIS	4	50672900.	71042.	0.	0.	234088.	1722377.
LOUIS	5	79796656.	80965.	0.	0.	547837.	2021005.
LOUIS	6	176149552.	231604.	0.	0.	598946.	6823092.
LOUIS	7	40664716.	47598.	0.	0.	193227.	1309985.
LOUIS	8	33114748.	16552.	0.	0.	5991.	772278.

The natural hazard simulation model is an excellent tool for evaluating the exposure to natural hazards resulting from alternative marketing plans. If marketing plans alter the geographic distribution of exposures, the alternative distributions of exposures may be input to the model and new loss distributions generated.

Pricing

The model-generated expected loss figures may be used to calculate appropriate catastrophe premium loadings. The loadings may be expressed as percentages of insured values by dividing the expected loss figures by the dollars of liability for each of the established zones, i.e. the geographical units into which the country is divided.

Theoretically, if an insurer establishes a reserve for catastrophe losses and makes annual contributions equal to the annual expected losses, the insurer will break even with respect to catastrophe losses over the long run. The model-generated output enables management to fine tune the catastrophe loadings in particular locations. Presumably, premiums charged in catastrophe prone areas include loadings for catastrophe losses, but these loadings may be subjective and may not correspond closely with expected catastrophe losses. Since the model can be programmed to accumulate dollar damages by any geographical configuration, expected loss estimates may be derived for any unit of area, and premiums that are in line with costs may be established.

Clearly, competitive factors dictate the amount of freedom that management has to set prices. If demand is very elastic, small increases in price will lead to large decreases in market share. Price changes may be tempered to result in the desired distribution of premium volume.

An additional caveat is that pricing in accordance with expected loss does not eliminate the risk of large losses since it is possible that catastrophes will occur when the loss fund is at a level that is not sufficient to cover all of the losses. The losses could then lead to financial difficulties for the insurer. Insurers may, however, transfer part or all of this risk through reinsurance agreements.

Reinsurance

To evaluate alternative reinsurance proposals, management needs the following:

1. An estimate of the probability distribution of losses for which the reinsurance contracts are to provide cover.

2. Knowledge of the reinsurance market and the types of contracts that are available.

3. A methodology for performing risk versus return trade-offs and obtaining preference orderings.

Part I of this paper provided a methodology for estimating the probability distribution of property losses from catastrophes. From the cumulative distribution function, one may determine the probability of experiencing losses in excess of any dollar amount.

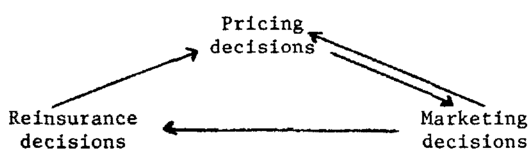
There are two broad categories of reinsurance contracts: proportional and nonproportional. Each type of treaty performs certain functions for the reinsured. Proportional or quota share treaties provide capacity and financing as well as reductions in the variance of the loss distribution. Non proportional or excess-of-loss treaties provide catastrophe and stop loss covers.

Borch [2] has shown that the "most efficient" reinsurance contract from the viewpoint of the ceding company is the stop loss contract. The type of treaty leads to the greatest reduction in variance for a given price if the premium paid to the reinsurer is proportional to the expected loss of the ceded portfolio and not to its variance. From the viewpoint of the reinsurer, the quota share treaty that gives a ceded portfolio with the same expected loss is superior because the variance of the ceded portfolio will be smaller.

In general, the reinsurer will charge a premium that compensates for the variability as well as the expected loss of the ceded portfolio. Accordingly, the premium will be lower for a quota share treaty that gives the reinsurer a portfolio with the same expected loss as the excess-of-loss treaty. The specific premium that the reinsurer will charge for a particular contract depends on the risk profile of the company.

The estimated probability distribution of losses shows the benefits that will be derived from particular reinsurance agreements, and these benefits may be compared to the costs. The reinsuring company will rank order the alternatives that are available in the reinsurance market using its own risk profile. The derivation of the risk profile relies on utility theory and will not be discussed here.

The pricing, marketing, and reinsurance decisions are not independent and as such should be evaluated simultaneously in the planning process. Obviously, pricing policies impact marketing plans which influence the geographical distribution of property exposures. This is a two-way relationship since marketing decisions also impact pricing decisions. The geographical distribution of property exposures will affect the probability distribution of catastrophe losses which in turn will influence the price of reinsurance since the reinsurer will demand a higher premium to cover exposures in high risk areas. Finally, the reinsurance covers influence the loss distribution and change the expected losses which drive the catastrophe loadings. The diagram below illustrates the decision triangle.



Summary

Catastrophic events can affect significantly the results of property and casualty insurance companies. Since the losses resulting from the occurrences of catastrophes could affect adversely the financial condition of a company, management must plan for these events. The first part of this paper described an estimation methodology based on Monte Carlo simulation. A windstorm example illustrated the approach and its primary advantages. These advantages are: It estimates the full probability distribution of losses, it captures the effects on this distribution of changes in population patterns, building codes, and repair costs, and it may be used to perform sensitivity analyses. The second part of the paper outlined how knowledge of the probability distribution of losses enables management to evaluate the effects on the probabilities of severe losses of alternative marketing, pricing, and reinsurance strategies.

REFERENCES

[1] Arata, David A. "Computer Simulation and the Actuary: A Study in Realizable Potential", Proceedings of the Casulaty Actuarial Society, Volume LXVIII, 1981.

[2] Borch, Karl. The Mathematical Theory of Insurance, Lexington: D.C. Heath and Company, 1974.

[3] Conning & Company. Natural Catastrophes, Exposures and Reinsurance October, 1983.

[4] Cry, George W. "Tropical Cyclones of the North Atlantic Ocean," U.S. Department of Commerce, Weather Bureau, Technical Paper No. 55.

[5] Friedman, Don G. Computer Simulation in Natural Hazard Assessment, University of Colorado: Program on Technology, Environment and Man, Monograph #NSF-RA-E-15-002, 1975.

[6] Haggerty, Alfred G. "Earthquake Insurance Sales Nearing Limit", National Underwriter, June 29, 1984.

[7] Ho, Francis P., Schwerdt, Richard W. and Goodyear, Hugo V. "Some Climatological Characteristics of Hurricanes and Tropical Storms, Gulf and East Coast of the United States," U.S. Department of Commerce, National Weather Service, NOAA Technical Report HW515, May 1975.

[8] Insurance Information Institute. Insurance Facts, 1983.

[9] Law, Averill M. and Kelton, W. David. Simulation Modeling and Analysis, New York: McGraw-Hill, 1982.

[10] Rubinstein, Reuben Y. Simulation and the Monte Carlo Method, New York: John Wiley & Sons, 1981.

[11] Schwerdt, Richard W., Ho, Francis P. and Watkins, Roger R. "Meteorological Criteria for Standard Project Hurricane and Probable Maximum Hurricane Windfields, Gulf and East Coasts of the United States," U.S. Department of Commerce, National Oceans and Atmospheric Administration, NOAA Technical Report NWS 23, September 1979.

TITLE: MEASURING THE IMPACT OF UNREPORTED PREMIUMS ON A REINSURERS' FINANCIAL RESULTS

AUTHOR: Douglas J. Collins

Mr. Collins is a Vice President of Tillinghast, Nelson & Warren, Inc. He is a graduate of Rensselaer Polytechnic Institute, with a Bachelor of Science degree in mathematics. He became a Fellow of the Casualty Actuarial Society and a member of the American Academy of Actuaries in 1977. He has served on the Examination Committee of the C.A.S.

ABSTRACT: A reinsurers' financial results are materially influenced by unreported premiums, and the losses and expenses related to these premiums. When unreported premiums and losses are estimated using underwriting year experience, an approximation is required to separate premiums earned and losses incurred before the reserve date from those earned and incurred after the reserve date. This paper presents a technique for approximating the earned portion of unreported premiums. It also demonstrates that financial results can vary significantly depending on the treatment of the earned and unearned portion of unreported premiums. A series of examples based on hypothetical data are used to show alternative estimates of underwriting income.

MEASURING THE IMPACT OF UNREPORTED PREMIUMS ON A REINSURERS' FINANCIAL RESULTS

INTRODUCTION

The analysis of reinsurers' financial results is complicated by the fact that both premiums and losses are subject to significant reporting lags. A further complication is the difficulty in quantifying the earned portion of unreported premiums. Reinsurers do not generally have sufficient information to calculate this quantity exactly. Thus, accounting practices vary as to the methodology for reflecting unreported premiums in financial statements.

This paper will present a technique for approximating unreported earned premiums. Several examples will then be reviewed in which calculations of unreported (IBNR) loss reserves are a function of expected losses. These examples will provide a comparison of ultimate loss estimates using alternative measures of unreported premiums in the expected loss calculation. The financial impact of the various assumptions will then be reviewed.

We will focus on the use of underwriting year experience to measure financial results. While accident year loss reserve estimates are required for Schedule P of the statutory annual statement, the use of accident year data has several disadvantages that can create problems for a reinsurer seeking to accurately measure its financial position.

Accident year loss estimates are usually compared to premiums reported and earned during the accident year. Due to premium reporting lags, this comparison does not match premiums and losses from the same policies. Thus, accident year loss ratios do not always provide a meaningful measure of underwriting results. Another disadvantage is that reinsurers can not always identify the proper accident year of a claim payment or reserve, since this information may not be supplied by ceding carriers. Despite these problems, an estimated accident year allocation of statistics is often the basis for reinsurers' loss reserves, due to the reporting requirements for Schedule P. However, if loss reserves are to be matched with an appropriate measure of unreported premiums, a supplement to calendar-accident year statistics is required.

A policy year type exposure period is used quite often by reinsurers since it provides an appropriate matching of premiums and losses, and also provides the data needed for calculating unreported premiums. Reinsurers usually use the term underwriting or contract year, rather than policy year, to reflect the difference between reinsurance and primary insurance contracts. Since some reinsurance contracts cover underlying policies written throughout the contract period (i.e., they cover the reinsured's policy year), an underwriting year normally includes parts of three accident years (or more if policy terms exceed twelve months). For example, a reinsurance contract written on July 1, 1983 might cover underlying policies written through June 30, 1984 which would cover losses occurring through June 30, 1985. All of the premiums and losses from this contract would be included in underwriting year 1983.

The measurement of unreported premium reserves can be accomplished by multiplicative projection techniques using historical underwriting year premium development patterns. This unreported premium may include the following components.

1. Premiums which have been written with effective dates prior to the reserve date, but which have not yet been reported. These premiums have an unearned and an earned portion. The earned but not reported premium is often referred to as EBNR.

2. Premiums which will be written with effective dates after the reserve date but within the exposure period being projected. For example, policies written after 6/30/84, which might be included in a projection of ultimate premiums for underwriting year 1984 as of 6/30/84.

The following section will describe a simple procedure for estimating the component portions of an unreported premium reserve.

UNREPORTED PREMIUM RESERVES

Exhibit 1 shows the underlying data for Sebago Re, a hypothetical growing young reinsurance company. We project that Sebago Re will ultimately collect premiums of $15 million for underwriting years 1980 to 1984. As of 6/30/84, they have collected written premiums of $9 million. Thus, their total unreported premiums are $6 million. Of course, the portion of these unreported premiums that will be written with effective dates after 6/30/84, should not influence Sebago Re's 6/30/84 financial statement. The first task is to estimate this portion. The assumption that policies are written uniformly throughout the year is usually not appropriate for a reinsurer, but the exact distribution of policy effective dates for underwriting year 1984 is unknown. As an estimate Sebago Re could sample the distribution of premiums by effective month in prior underwriting years. Such an analysis would typically produce a premium distribution such as the following:

Policy Effective Month	Portion of Premiums Written
January - March	57%
April - June	18
July - September	21
October - December	4
	100

Assuming that this is the distribution that Sebago Re finds, then 25% of the ultimate premium for underwriting year 1984, or $1.25 million, has not yet been written.

The remaining $4.75 million of unreported premium contains an earned and an unearned portion. Calculating these portions exactly would require that Sebago Re know the ultimate premium for each policy that it has written with an effective date of 6/30/84 or prior. Based on each policy's coverage provisions, Sebago Re could then estimate the earned portion. However, it is not usually feasible for a reinsurer to calculate reasonable ultimate premiums for each policy. As an alternative, the earned portion can be estimated on an aggregate basis.

The earning of premium on a reinsurance contract can follow different patterns, depending on the type of contract. The determining factor is the provision for coverage of losses. The following are common types of coverage found in reinsurance.

1. Occurrence - The reinsured is indemnified for a defined share of all claims occurring during the reinsurance contract period.

2. Claims-made - The reinsured is indemnified for a defined share of all claims reported to it during the reinsurance contract period.

3. Claims-paid - The reinsured is indemnified for a defined share of all claims paid during the reinsurance contract period.

4. Risks attaching - The reinsured is indemnified for a defined share of all claims occurring on policies that it writes during the reinsurance contract period.

Premiums are usually earned on a pro rata basis over the policy term for types 1 through 3 above. The underlying assumption is that the occurrence, reporting, and payment of losses are evenly distributed throughout the year. For type 4, if the policies written by the reinsured are earned on a pro-rata basis, the reinsurer's contract would be earned according to the parallelogram rule. That is, the earning of premium on a twelve month reinsurance contract would take

24 months, as the underlying policies are written and earned. If the reinsured's policies include coverage on a risks-attaching basis, the premium earning would be extended further. For simplicity we will assume that Sebago Re's policies are earned either by the pro-rata or parellelogram rules. A sampling of the contract provisions could easily be used to determine premium volumes written by contract type. Preferably, reserving data should be segregated so that ultimate premiums and losses are projected separately for each type. In order to simulate a mixed book of business for this example, we will combine the two types into one calculation.

Exhibit 2, Sheet 2, shows the earning patterns that would result from the pro-rata and parallelogram rules. In both cases we have assumed that all policy terms are twelve months. Assuming that 50% of Sebago Re's business is in each type, their average earning pattern is shown in column (4). On Sheet 1 of Exhibit 2, the earning pattern is combined with Sebago Re's monthly distribution of policy effective dates to calculate a weighted average earned factor for each underwriting year. These factors represent the earned portion of ultimate written premiums for each underwriting year as of 6/30/84. Averages could be calculated for any other reserving date using a similar procedure.

The top half of Exhibit 3 derives the earned portion of the unreported premium. We now have the following measures of premiums for Sebago Re.

1.	Reported written premium as of 6/30/84	$ 9,000,000
2.	Reported earned premium as of 6/30/84	7,950,000
3.	Reported unearned premium as of 6/30/84 (1-2)	1,050,000
4.	Ultimate written premium	15,000,000
5.	Portion of 4. written after 6/30/84	1,250,000
6.	Adjusted ultimate written premium (4-5)	13,750,000
7.	6/30/84 earned ultimate premium	10,395,700
8.	Unreported earned premium (7-2)	2,445,700
9.	Unreported unearned premium (6-7-3)	2,304,300

Thus, policies written through 6/30/84 are estimated to ultimately bring in premiums of $13,750,000. Of this total $10,395,700 represents exposure that has been earned as of 6/30/84. The remaining $3,354,300 is unearned at that date. The $6 million of unreported premium consists of: $2,445,700 earned, $2,304,300 unearned, and $1,250,000 unwritten. The results of these calculations are sensitive to the hypothetical amounts of written, earned, and ultimate premiums. The selected figures are believed to be reasonable, but the breakdown of unreported premiums will vary significantly among reinsurers and at different points in time.

IBNR RESERVES

In order to complete the example (and Exhibit 3) we need to calculate Sebago Re's IBNR reserves. We have selected a technique described by

Bornhuetter and Ferguson[1] that is well suited to long-tailed lines such as reinsurance. For this technique, two parameters are required to calculate the IBNR reserve: an initial expected loss ratio and an expected loss reporting pattern. The IBNR reserve is equal to the product of expected ultimate losses and the expected percentage unreported as of the reserve date.

As shown on Exhibit 1, reported losses to date total approximately $4.3 million. Applying appropriate development factors to these losses would indicate an ultimate loss level of $14 million. However, due to the large development factors the resulting ultimate loss ratios are not consistent with our expectations. The Bornhuetter-Ferguson technique might be chosen in this case to reduce the inconsistency. There are four alternative premium bases that could be used in this calculation.

Ultimate Reserves

Exhibit 4 shows a reserve calculation based on projected ultimate premiums. The initial expected loss ratios in Column 3 will be used in each example that follows. Their selection may be assumed to have been based on reasonable actuarial judgments. The loss reporting pattern is based on the development factors shown with the underlying data (e.g., in Column (5): 83.33% = 1.00 ÷ 1.20). The estimated ultimate losses Column (10) reflect the combination of actual reported losses and expected unreported (IBNR) losses.

[1] Bornhuetter, R. L. and Ferguson, R. W. "The Actuary and IBNR". Proceedings of the Casualty Actuarial Society, LIX, 1972, p. 181.

This calculation results in an overstated IBNR reserve since it reflects premiums written and losses incurred after 6/30/84. It is shown for comparison with the projections of underlying data in Exhibit 1.

Adjusted Ultimate Reserves

Exhibit 5 includes an adjustment of underwriting year 1984 premiums to reflect only those written as of 6/30/84. The expected percentage of losses reported for 1984 is also adjusted to reflect writings as of 6/30/84. That is, since only 75% of the ultimate premiums have been written as of June, the indicated percent reported is divided by .75. The purpose of matching the reporting pattern and premium adjustments is to maintain the same expected reported losses. Note that Column (7) will remain unchanged in each example.

This example still results in an overstated IBNR since it reflects premiums earned and losses incurred after 6/30/84. However, it does provide a meaningful estimate of Sebago Re's financial position after all of its current underwriting commitments have run off.

Ultimate Earned Reserves

The 6/30/84 ultimate earned premiums derived in Exhibit 3 are used as the base to calculate IBNR on Exhibit 6. The loss reporting pattern requires adjustment for underwriting years 1982 to 1984 to reflect the difference between ultimate earned and ultimate written premium. This adjustment is similar to that described above. In this case, the percent reported is divided by the ratio of ultimate earned premiums to ultimate written premiums. Thus, the percentage reported for underwriting year 1984 is 2.00% ÷ ($973,000 ÷ $5,000,000), or 10.28%. The other percentages in Column (5) are calculated similarly.

Theoretically, this example provides IBNR reserves reflecting exposures earned and incurred prior to 6/30/84. In this respect, it would be the appropriate figure for Sebago Re to carry in its statutory financial statements.

Current Earned Reserves

Exhibit 7 shows a reserve calculation based on the reported earned premiums as of 6/30/84. The initial expected loss ratios and the loss reporting pattern are the same as before, with similar adjustments to the reporting pattern.

The calculation on Exhibit 7 results in an understated IBNR because it does not reflect losses relating to premiums that are earned but unbooked as of 6/30/84. Sebago Re might use this IBNR in their financial statements assuming that the understatement of liabilities would be offset by an understatement of assets. This assumption requires that the unreported premium is equal to the unreported losses and expenses relating to that premium. In times of poor underwriting results, the unbooked liability will exceed the unbooked asset. The extent of this shortfall for Sebago Re is estimated on Exhibit 3.

Returning to the lower portion of Exhibit 3, we have assumed that expenses associated with unreported premiums will average 30%. This figure is intended to include only those expenses, such as commissions, brokerage, and taxes, that would be directly incurred as a result of receiving the premium. The net additional liability in Column (11) is actually the combination of one asset and three liabilities: unreported written premium, unreported unearned premium, and loss and expense liabilities associated with the unreported earned premium. This net liability represents the difference between financial results calculated on a current earned basis and financial results calculated on an ultimate earned basis.

A comparison of inception to date underwriting results under three alternative IBNR calculations is shown below. The premiums and losses are from Exhibits 5-7; the total expense ratio is assumed to be 35%.

	Adjusted Ultimate	Ultimate Earned	Current Earned
Premiums	$13,750,000	$10,395,700	$7,950,000
Underwriting Expenses	4,812,500	3,638,495	2,782,500
Ultimate Losses	13,006,666	9,652,696	7,261,666
Underwriting Profit	(4,069,166)	(2,895,491)	(2,094,166)
Combined Ratio	129.6%	127.9%	126.3%

This comparison indicates the impact of poor underwriting results on these calculations. The range of combined ratios spans 3.3%. However, the underwriting loss varies by almost $2 million, which would be 10% to 20% of surplus, assuming that Sebago Re is a small reinsurer with surplus of $10 million to $20 million.

The same comparison for a reinsurer with more acceptable underwriting results would have a smaller range. If Sebago Re's reported losses and IBNR are reduced by 25%, the underwriting results would be as follows.

	Adjusted Ultimate	Ultimate Earned	Current Earned
Ultimate Losses	$9,755,000	$7,239,522	$5,446,250
Underwriting Profit	(817,500)	(482,317)	(278,750)
Combined Ratio	105.9%	104.6%	103.5%

The combined ratio swing is reduced to 2.4%, and the underwriting loss range is only $538,750, or 2.7% to 5.4% of the surplus assumptions mentioned above. However, this difference could still be material, especially to a reinsurer that writes at a more typical premium to surplus ratio of 1:1 or higher.

If further improvements in underwriting results were assumed, the range would continue to decline. A reinsurer that writes consistently at a 100% combined ratio would have zero underwriting profit using each method. Thus, the unreported premium would have no impact on the income or surplus of such a reinsurer. Reinsurers writing below 100% would be understating income and surplus by ignoring unreported premiums.

CONCLUSIONS

Loss reserves are intended to reflect all occurrences prior to the reserve date. When underwriting year data is used in reserving, an approximation is required to estimate the portion of each year's losses that have occurred. Similarly, an approximation of the unreported premium reserve is required, to determine the amount that is earned prior to the reserve date. We have seen that financial estimates which exclude the unreported premium and those which include the unearned premium can differ materially from estimates based on the ultimate earned.

The alternative of using accident year data may reduce the uncertainty concerning losses which occur prior to the reserve date. However, calendar-accident year data provides no means for calculating unreported premiums, and also can produce distorted measures of loss ratios due to mismatching of premiums and losses.

It should be noted that if financial results are measured over a complete underwriting cycle, the impact of unreported premiums is less severe than at the peaks and troughs. However, the impact will almost always be material.

SEBAGO RE

UNDERLYING DATA

Underwriting Year	Earned Premiums As of 6/30/84	Written Premiums As of 6/30/84	Development Factor to Ultimate	Estimated Ultimate Written Premiums (3) x (4)
(1)	(2)	(3)	(4)	(5)
1980	$1,000,000	$1,000,000	1.000	$ 1,000,000
1981	2,000,000	2,000,000	1.000	2,000,000
1982	2,450,000	2,500,000	1.200	3,000,000
1983	2,100,000	2,500,000	1.600	4,000,000
1984	400,000	1,000,000	5.000	5,000,000
Total	7,950,000	9,000,000		15,000,000

Underwriting Year	Incurred Losses As of 6/30/84	Development Factor to Ultimate	Estimated Ultimate Incurred Losses (6) x (7)	Estimated Ultimate Loss Ratio (8) / (5)
(1)	(6)	(7)	(8)	(9)
1980	$ 583,333	1.200	$ 700,000	.700
1981	1,333,333	1.500	2,000,000	1.000
1982	1,200,000	2.000	2,400,000	.800
1983	1,100,000	4.000	4,400,000	1.100
1984	90,000	50.000	4,500,000	.900
Total	4,306,666		14,000,000	.933

Exhibit 2
Sheet 1

SEBAGO RE

Average Earned Premium Factors
By Underwriting Year

Policy Effective Month (1)	Estimated Premium Distribution (2)	Earned Premium Factor* (3)
1/82 - 7/82	93%	1.0000
8/82	2	.9983
9/82	1	.9931
10/82	2	.9844
11/82	1	.9722
12/82	1	.9566
	100	.9989
1/83	54	.9375
2/83	1	.9149
3/83	2	.8889
4/83	11	.8594
5/83	2	.8264
6/83	5	.7899
7/83	18	.7500
8/83	2	.6684
9/83	1	.5903
10/83	2	.5156
11/83	1	.4444
12/83	1	.3767
	100	.8565
1/84	54	.3125
2/84	1	.2517
3/84	2	.1944
4/84	11	.1406
5/84	2	.0903
6/84	5	.0434
7/84 - 12/84	25	.0000
	100	.1946

*Assumes policies effective on the 1st day of each month. Subtotals are weighted averages using Column (2) as weights.

Exhibit 2
Sheet 2

SEBAGO RE

Earned Premium Factors

No. of Months From Policy Effective Date (1)	Pro-Rata Earning (2)	Parallelogram Earning* (3)	Average [(2)+(3)] ÷ 2.0 (4)
1	.0833	.0035	.0434
2	.1667	.0139	.0903
3	.2500	.0313	.1406
4	.3333	.0556	.1944
5	.4167	.0868	.2517
6	.5000	.1250	.3125
7	.5833	.1701	.3767
8	.6667	.2222	.4444
9	.7500	.2813	.5156
10	.8333	.3472	.5903
11	.9167	.4201	.6684
12	1.0000	.5000	.7500
13	1.0000	.5799	.7899
14	1.0000	.6528	.8264
15	1.0000	.7188	.8594
16	1.0000	.7778	.8889
17	1.0000	.8299	.9149
18	1.0000	.8750	.9375
19	1.0000	.9132	.9566
20	1.0000	.9444	.9722
21	1.0000	.9688	.9844
22	1.0000	.9861	.9931
23	1.0000	.9965	.9983
24	1.0000	1.0000	1.0000

*Assumes underlying policies are evenly distributed throughout the year. Figures are calculated by the following formula:

Earned Factor $= M^2 \div 288$, for $M \leq 12$

$= 1 - [(24 - M)^2 \div 288]$, for $M > 12$

Where M = Number of months from policy effective date.

SEBAGO RE

CALCULATION OF NET ADDITIONAL LIABILITY

Underwriting Year	Ultimate Premium	6/30/84 Earned Factor	6/30/84 Ultimate Earned Premium (2) X (3)	6/30/84 Reported Earned Premium	6/30/84 Unreported Earned Premium (4) - (5)
(1)	(2)	(3)	(4)	(5)	(6)
1980	$ 1,000,000	1.0000	$ 1,000,000	$1,000,000	$ 0
1981	2,000,000	1.0000	2,000,000	2,000,000	0
1982	3,000,000	.9989	2,996,700	2,450,000	546,700
1983	4,000,000	.8565	3,426,000	2,100,000	1,326,000
1984	5,000,000	.1946	973,000	400,000	573,000
Total	15,000,000		10,395,700	7,950,000	2,445,700

Underwriting Year	Expense Ratio	Initial Expected Loss Ratio	Unreported Expenses (6) x (7)	Additional Unreported Losses (6) x (8)	Net Additional Liability (9)+(10)-(6)
(1)	(7)	(8)	(9)	(10)	(11)
1980	.30	.800	$ 0	$ 0	$ 0
1981	.30	.850	0	0	0
1982	.30	.900	164,010	492,030	109,340
1983	.30	1.000	397,800	1,326,000	397,800
1984	.30	1.000	171,900	573,000	171,900
Total			733,710	2,391,030	679,040

Exhibit 4

SEBAGO RE

Ultimate Written Premium Basis

Projected Ultimate Losses Using Bornhuetter-Ferguson Technique
On Incurred Losses
as of 6/30/84

Underwriting Year	Ultimate Premium	Initial Expected Loss Ratio	Initial Expected Losses (2) x (3)	Expected Percentage Reported	Expected Percentage Unreported
(1)	(2)	(3)	(4)	(5)	(6)
1980	$ 1,000,000	.800	$ 800,000	83.33	16.67
1981	2,000,000	.850	1,700,000	66.67	33.33
1982	3,000,000	.900	2,700,000	50.00	50.00
1983	4,000,000	1.000	4,000,000	25.00	75.00
1984	5,000,000	1.000	5,000,000	2.00	98.00
Total	15,000,000		14,200,000		

Underwriting Year	Expected Reported Losses (4) x (5)	Actual Reported Losses	Expected Unreported Losses (4) x (6)	Estimated Ultimate Losses (8) + (9)	Estimated Ultimate Loss Ratio (10) / (2)
(1)	(7)	(8)	(9)	(10)	(11)
1980	$ 666,667	$ 583,333	$ 133,333	$ 716,666	.717
1981	1,133,333	1,333,333	566,667	1,900,000	.950
1982	1,350,000	1,200,000	1,350,000	2,550,000	.850
1983	1,000,000	1,100,000	3,000,000	4,100,000	1.025
1984	100,000	90,000	4,900,000	4,990,000	.998
Total	4,250,000	4,306,666	9,950,000	14,256,666	.950

SEBAGO RE

Adjusted Ultimate Written Premium Basis

Projected Ultimate Losses Using Bornhuetter-Ferguson Technique
On Incurred Losses
as of 6/30/84

Underwriting Year	Adjusted Ultimate Premium	Initial Expected Loss Ratio	Initial Expected Losses (2) x (3)	Expected Percentage Reported	Expected Percentage Unreported
(1)	(2)	(3)	(4)	(5)	(6)
1980	$ 1,000,000	.800	$ 800,000	83.33	16.67
1981	2,000,000	.850	1,700,000	66.67	33.33
1982	3,000,000	.900	2,700,000	50.00	50.00
1983	4,000,000	1.000	4,000,000	25.00	75.00
1984	3,750,000	1.000	3,750,000	2.67	97.33
Total	13,750,000		12,950,000		

Underwriting Year	Expected Reported Losses (4) x (5)	Actual Reported Losses	Expected Unreported Losses (4) x (6)	Estimated Ultimate Losses (8) + (9)	Estimated Ultimate Loss Ratio (10) / (2)
(1)	(7)	(8)	(9)	(10)	(11)
1980	$ 666,667	$ 583,333	$ 133,333	$ 716,666	.717
1981	1,133,333	1,333,333	566,667	1,900,000	.950
1982	1,350,000	1,200,000	1,350,000	2,550,000	.850
1983	1,000,000	1,100,000	3,000,000	4,100,000	1.025
1984	100,000	90,000	3,650,000	3,740,000	.997
Total	4,250,000	4,306,666	8,700,000	13,006,666	.946

SEBAGO RE

Ultimate Earned Premium Basis

Projected Ultimate Losses Using Bornhuetter-Ferguson Technique
On Incurred Losses
as of 6/30/84

Underwriting Year	Ultimate Earned Premium	Initial Expected Loss Ratio	Initial Expected Losses (2) x (3)	Expected Percentage Reported	Expected Percentage Unreported
(1)	(2)	(3)	(4)	(5)	(6)
1980	$ 1,000,000	.800	$ 800,000	83.33	16.67
1981	2,000,000	.850	1,700,000	66.67	33.33
1982	2,996,700	.900	2,697,030	50.06	49.94
1983	3,426,000	1.000	3,426,000	29.19	70.81
1984	973,000	1.000	973,000	10.28	89.72
Total	10,395,700		9,596,030		

Underwriting Year	Expected Reported Losses (4) x (5)	Actual Reported Losses	Expected Unreported Losses (4) x (6)	Estimated Ultimate Losses (8) + (9)	Estimated Ultimate Loss Ratio (10) / (2)
(1)	(7)	(8)	(9)	(10)	(11)
1980	$ 666,667	$ 583,333	$ 133,333	$ 716,666	.717
1981	1,133,333	1,333,333	566,667	1,900,000	.950
1982	1,350,000	1,200,000	1,347,030	2,547,030	.850
1983	1,000,000	1,100,000	2,426,000	3,526,000	1.029
1984	100,000	90,000	873,000	963,000	.990
Total	4,250,000	4,306,666	5,346,030	9,652,696	.929

SEBAGO RE

Reported Earned Premium Basis

Projected Ultimate Losses Using Bornhuetter-Ferguson Technique On Incurred Losses as of 6/30/84

Underwriting Year	Reported Earned Premium	Initial Expected Loss Ratio	Initial Expected Losses (2) x (3)	Expected Percentage Reported	Expected Percentage Unreported
(1)	(2)	(3)	(4)	(5)	(6)
1980	$1,000,000	.800	$ 800,000	83.33	16.67
1981	2,000,000	.850	1,700,000	66.67	33.33
1982	2,450,000	.900	2,205,000	61.22	38.78
1983	2,100,000	1.000	2,100,000	47.62	52.38
1984	400,000	1.000	400,000	25.00	75.00
Total	7,950,000		7,205,000		

Underwriting Year	Expected Reported Losses (4) x (5)	Actual Reported Losses	Expected Unreported Losses (4) x (6)	Estimated Ultimate Losses (8) + (9)	Estimated Ultimate Loss Ratio (10) / (2)
(1)	(7)	(8)	(9)	(10)	(11)
1980	$ 666,667	$ 583,333	$ 133,333	$ 716,666	.717
1981	1,133,333	1,333,333	566,667	1,900,000	.950
1982	1,350,000	1,200,000	855,000	2,055,000	.839
1983	1,000,000	1,100,000	1,100,000	2,200,000	1.048
1984	100,000	90,000	300,000	390,000	.975
Total	4,250,000	4,306,666	2,955,000	7,261,666	.913

TITLE: BANK ACCOUNTS AS A TOOL FOR RETROSPECTIVE ANALYSIS OF EXPERIENCE ON LONG-TAIL COVERAGES

AUTHORS: Claudia S. Forde

Ms. Forde is a Consulting Actuary for Tillinghast, Nelson & Warren, Inc. Her eight years of experience in the casualty actuarial field include four years with Tillinghast and over four years with a major personal lines writer. Ms. Forde received her ACAS in 1983 and is a Member of the American Academy of Actuaries. She holds a B.S. degree in Mathematics from Emory University.

W. James MacGinnitie

Mr. MacGinnitie is the Managing Principal of the Casualty Division of Tillinghast, Nelson & Warren, Inc. He has twenty-three years of experience in actuarial work including ten at Tillinghast. Immediately prior to joining Tillinghast, Mr. MacGinnitie was Director of the Master of Actuarial Science Program at the University of Michigan's Graduate School of Business Administration. He is a Past President of the CAS, a Fellow of the CAS and a Fellow of the Society of Actuaries.

ABSTRACT: This paper presents a model for the retrospective analysis of experience on long-tail coverages. The model chosen is a "bank account" model which considers separately the profitability of each exposure period. The model treats premium income and interest earnings as bank account deposits, and loss payments, expense payments and interest charges as withdrawals.

The exposure period results are calculated on three different bases: traditional underwriting profit/loss, net operating result at current value, and net operating result at exposure period value. Results are also displayed graphically for an effective presentation of the profitability/unprofitability of the exposure period.

A model of this type is a valuable tool for communicating financial results to management and others in an effective and straightforward manner. It is particularly helpful in the evaluation of long-tail lines of business.

BANK ACCOUNTS AS A TOOL FOR RETROSPECTIVE ANALYSIS OF EXPERIENCE ON LONG-TAIL COVERAGES

As interest rates have risen and claim settlement patterns have lengthened, actuaries have increasingly been called upon to take the time value of money into account when evaluating profitability. When analyzing historic results, this sometimes requires assumed rates of return and payment patterns, because actual historic data is not readily available. Where the data is available, however, it is possible to produce an analysis which can be communicated to management and other interested parties in an effective and straightforward manner.

The model chosen is a "bank account". It considers each exposure period as a separate bank account. The account receives an initial deposit (premiums collected less expenses) which is reduced by withdrawals (loss and loss expense payments), increased by interest earned and reduced by interest charges when the balance goes negative.

The model will seem straightforward, even simple, to anyone who has mastered compound interest. It is presented here only because of its utility in communicating results, particularly to important but less experienced audiences.

By looking at each exposure period separately, the model enables us to estimate profit or loss individually by exposure period. It provides a hindsight look at what an appropriate rate level would have been in each exposure period. The bank account model is particularly helpful in evaluating long tail lines. The attached exhibits show the application of the model to the medical malpractice line of business.

In long tail lines such as medical malpractice, investment income is a crucial variable in the ultimate profitability or unprofitability of the insurance enterprise. Care should be taken in the selection of interest rates for the bank account model. The over or understatement of investment income can obviously have a distortive effect on the exposure period results. In the example which is attached, we have used actual portfolio average rates of return for those calendar years in which this information was available to us (1974 through 1983) and one-year Treasury Bill rates as a reasonable approximation of available rates of return for the earlier years (1959 through 1973).

Although real world considerations introduce complexities to this simple illustration, the basic concepts are still applicable. Expenses include unallocated loss adjustment expenses in addition to taxes, underwriting and acquisition expenses, if any. No Federal income tax implications have been considered. However, consideration has been given to the interest charge which is made when the exposure period or report period account balance turns negative. This charge represents interest income lost because the particular exposure period account is overdrawn and "borrowing from the bank." In the case of a multi-line/state company, the bank represents one of three sources of funds: surplus, profits from other states or lines of business, or future policy years (i.e., other "bank accounts"). For a one-line, one-state company, there are only two sources of funds: surplus and future income. Obviously, the borrowing of funds from the future to pay the losses of the past implies both that those funds will not be available to pay future losses and that the expected interest return on those funds (which may be anticipated in the premium structure) will not be collected.

Beneath the account's annual transactions is shown a Summary of Results. This summary consolidates the transactions over time and also includes the unpaid loss liability for the account, both reported and estimated unreported.

In the Summary of Results table, results are shown on three different bases. The first is traditional underwriting profit/loss, which is simply the amount of funds available for losses, less loss and allocated loss adjustment expenses paid and remaining to be paid. This measure does not consider any investment income.

The second measure reflects imputed investment income received through the evaluation date (including the charge for borrowing if the balance is negative), as well as discount on unpaid losses and loss expenses at some assumed rate and payment pattern. This is shown as the net operating result at 12/31/83 value.

The third measure is the net operating result at the middle of the initial exposure period. If negative, it is the additional amount which, if it had been in the account at the time that premiums were collected, would have resulted in no net gain or loss after all claims were paid. If the account is profitable, it represents the "profit at issue".

In order to clarify how the bank account analogy works, we can review Exhibit I, which shows the 1964 policy year account for Employers Insurance of Wausau physicians and surgeons professional liability business in New York. The initial deposit (premium paid) in 1964 was $4,325,000. This was reduced for expenses of $700,000, resulting in the initial balance of $3,625,000. It should be noted that, although this amount is shown as the balance at January 1, 1964, the interest calculation in the initial calendar year assumes that this amount is not available until July 1.

Withdrawals (loss and allocated expense payments) for calendar year 1964 were $31,000, while interest income on the average fund at the rate of 3.89% yielded $69,000. In subsequent years, withdrawals (loss and allocated expense payments) increase sharply, exceeding the interest income earned and reducing the account balance dramatically. In 1971 the withdrawal exceeds the account balance making the balance as of January 1, 1972 $-282,000. From this point on, the account is charged, rather than credited, with interest. In calendar years 1972-1983, loss and expense payments total $4,643,000 and imputed interest charged is $3,312,000, creating an account balance of $-8,237,000 on January 1, 1984.

The Summary of Results shows an underwriting loss of $6,836,000, which is $4,325,000 of premium income, less $700,000 of expense, $9,615,000 of paid loss and allocated expense, and $846,000 of unpaid loss and allocated expense (of which $676,000 is reported and $171,000 is unreported). The net operating result valued at 12/31/83 is a loss of $8,862,000. This reflects discount, at the rate of 12%, on the unpaid loss and allocated loss expense, in the amount of $221,000. The net operating result at 1964 value is a loss of $2,851,000, which is to say that Employers needed $2,851,000 of additional funds at the time that premiums were being collected in 1964 in order to come out with a breakeven result when the last claim is paid.

The bank account information can also be shown in graphical format, as displayed on Exhibit II. The solid line shown as "Available Funds" represents premium income, reduced by expenses, then augmented by interest income. The dashed line represents paid losses and allocated loss adjustment expenses, while the dotted line shows reported losses and allocated expenses (paid plus case basis reserves). The available funds line stops at the point at which paid losses and allocated loss adjustment expenses exceed available funds. We have included a "dollars per doctor" scale on the right-hand side of the graph, as well as reported loss and allocated expense information (not included in the bank account on Exhibit I), to enhance the value of the graph as a communications tool.

Where a good series of historical data is available, it is possible to prepare separate accounts for each exposure period. Accident years or policy years can be used, with appropriate adjustments in the computation of interest earned in the initial year. If the experience has been consistently profitable or unprofitable, the cumulative communi-

:ative impact of a series of graphs can be quite effective. It is also easy to produce exhibits which account for an entire period of years to illustrate the cumulative experience. Exhibit III shows the cumulative experience of fifteen and one-half years of medical malpractice writings and Exhibit IV shows the same experience graphically. In Exhibits III and IV, no interest was charged on the cumulative balance after it turned negative. (Interested readers can obtain the complete set of bank accounts and graphs which underlie these cumulative exhibits from the authors.)

There are a number of simplifications in the present model, as described earlier, which could be removed for a more in-depth analysis. Expenses could be spread over time rather than charged to the initial calendar year. Consideration could be given to Federal income taxes. Finally, there are numerous adjustments that could be made to the method of allocating investment income. Investment year methods, for example, could keep track of funds by year originally received, and could reflect the actual term structure and reinvestment results. In this regard, our life insurance colleagues have already investigated several methodologies for handling the complex accounting and allocation procedures. Consistency in the choice of either new money yields or embedded yields is obviously desirable.

The advantages of the bank account model for presenting exposure period experience are mainly in the area of better communication. It provides an effective way of looking at past underwriting results which is particularly helpful when explaining results to non-actuaries or the nonfinancially sophisticated. We have found it particularly useful in the medical malpractice field, where specialty companies formed in the middle 1970's are cash rich but still need regular rate increases because of the long term trend in pure premiums on the order of 20% per annum. These rate increases must be decided upon by boards of directors composed primarily of medical professionals with little insurance or financial expertise. It has also been useful in dealing with arguments advanced by the plaintiff's bar, generally to the effect that investment income has been inadequately taken into account.

The bank account model is not without its disadvantages as well. Although it is a valuable tool in explaining financial results to the non-sophisticated, it should not be used in place of actuarial analysis. It can in fact be a dangerous weapon in the hands of someone unfamiliar with its limitations. Chief among its limitations is the fact that the bank account model can not be used to make rates prospectively, nor to set adequate reserves. It is, however, a very useful method for analyzing and communicating financial results by exposure period.

We have programmed the model in APL, but it could be easily undertaken in spreadsheet software as well. Contributions to the development of the model were also made by Jim Hurley, Dean Anderson, and Terry Biscoglia. The data used in the attached exhibits was provided by Employers Insurance of Wausau.

NEW YORK PHYSICIANS AND SURGEONS

EMPLOYERS INSURANCE OF WAUSAU

1964 POLICY YEAR

($000)

PREMIUM INCOME = $ 4325
LESS EXPENSES(A) = 700

AVAILABLE FOR LOSSES(B) = $ 3625

CALENDAR YEAR	BALANCE AT JAN. 1(C)	WITH-DRAWALS (PMTS)(B)	RATE OF RETURN(D)	INTEREST INCOME(E)
(1)	(2)	(3)	(4)	(5)
1964	$ 3625	$ 31	3.89 %	$ 69
1965	3663	61	4.23	157
1966	3759	211	5.34	200
1967	3748	568	4.94	175
1968	3355	671	5.78	180
1969	2863	916	7.28	182
1970	2130	1654	6.94	95
1971	571	860	4.90	7
1972	-282	1176	5.01	-44
1973	-1503	478	7.54	-136
1974	-2117	319	5.47	-128
1975	-2563	679	5.68	-169
1976	-3411	266	5.53	-201
1977	-3879	119	5.63	-228
1978	-4226	213	5.72	-255
1979	-4693	75	5.88	-286
1980	-5054	134	6.55	-346
1981	-5534	72	6.89	-397
1982	-6003	738	7.36	-485
1983	-7227	374	8.26	-637
1984	-8237			
TOTAL		$ 9615		$ -2247

SUMMARY OF RESULTS

FUNDS AVAILABLE FOR LOSSES			$ 3625
PAID LOSS AND ALLOCATED LAE			9615
UNPAID LOSS AND ALLOCATED LAE			846
REPORTED	$	676	
UNREPORTED(F)		171	
UNPAID LOSS AND ALAE DISCOUNTED AT 12%			625
UNDERWRITING PROFIT/LOSS			-6836
INTEREST INCOME/LOSS			-2247
NET OPERATING RESULT AT 12/31/83 VALUE			-8862
NET OPERATING RESULT AT 1964 VALUE			-2851

Exhibit I
Sheet 2

Notes to Exhibit I, Sheet 1

(a) Expenses include administrative and unallocated loss adjustment expenses. Employers Insurance of Wausau figures have been extracted from figures they have provided to the New York Insurance Department (Exhibit IA).

(b) Includes allocated loss adjustment expense.

(c) In the initial year, the balance is only available for a half-year.

(d) Yields for calendar years 1959-1973 are based on one-year Treasury bill yield rates. Yields for subsequent years are the actual calendar year yields achieved by Employers Insurance of Wausau, as defined by net investment gain or loss (including realized capital gain or loss) divided by mean total assets.

(e) Assumes all transactions occur uniformly throughout the year.

(f) Provision for unreported losses is based on the projected ultimate pure premiums developed in Part I of the Medical Malpractice Insurance Association of New York June 1984 physicians and surgeons professional liability rate filing.

NEW YORK PHYSICIANS AND SURGEONS
EMPLOYERS INSURANCE OF WAUSAU
1964 POLICY YEAR

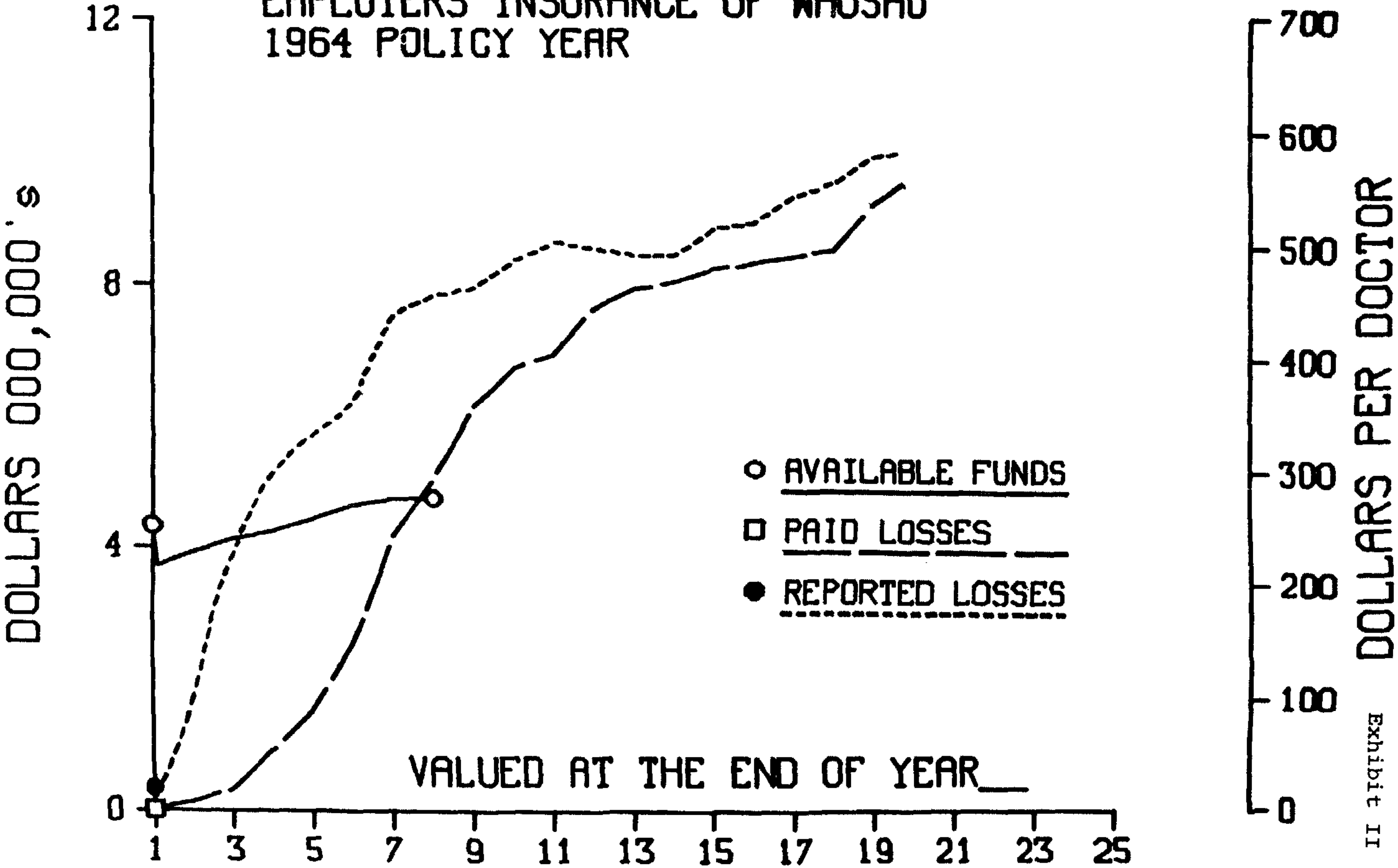


Exhibit II

Exhibit III
Sheet 1

NEW YORK PHYSICIANS AND SURGEONS

Employers Insurance of Wausau
Financial Results for Policy Years 1959 - 1974(a)
at 12/31/83
($000)

Calendar Year (1)	Premium Income (2)	Expenses(b) (3)	Loss and ALAE Paid(c) (4)	Insurance Cash Flow(d) (5)	Interest Yield(e) (6)	Interest Income(f) (7)	Funds at Year-End (8)
1959	$ 2,442	$ 544	$ 76	$ 1,822	4.71%	$ 42	$ 1,864
1960	2,895	642	183	2,070	3.55	104	4,038
1961	3,269	610	499	2,160	2.89	149	6,347
1962	3,790	655	633	2,502	3.10	238	9,087
1963	4,121	671	825	2,625	3.41	359	12,071
1964	4,325	700	1,491	2,134	3.89	519	14,724
1965	4,428	701	1,884	1,843	4.23	674	17,241
1966	5,004	769	3,385	850	5.34	966	19,057
1967	5,906	910	3,547	1,449	4.94	997	21,503
1968	8,948	1,359	4,306	3,283	5.78	1,370	26,156
1969	11,939	1,657	4,778	5,504	7.28	2,164	33,824
1970	18,964	2,297	8,204	8,463	6.94	2,707	44,994
1971	33,256	3,060	7,130	23,066	4.90	2,811	70,871
1972	38,760	3,989	13,410	21,361	5.01	4,160	96,392
1973	36,903	4,157	19,297	13,449	7.54	8,008	117,849
1974	18,594	3,524	25,108	(10,038)	5.47	6,536	114,347
1975	-	-	30,162	(30,162)	5.68	5,810	89,995
1976	-	-	25,990	(25,990)	5.53	4,385	68,390
1977	-	-	30,976	(30,976)	5.63	3,073	40,487
1978	-	-	42,904	(42,904)	5.72	1,137	(1,280)
1979	-	-	51,930	(51,930)	5.88	0	(53,210)
1980	-	-	48,044	(48,044)	6.55	0	(101,254)
1981	-	-	39,460	(39,460)	6.89	0	(140,714)
1982	-	-	37,372	(37,372)	7.36	0	(178,086)
1983	-	-	30,383	(30,383)	8.26	0	(208,469)

Loss and ALAE Case Reserve @ 12/31/83	$ 76,583
Loss and ALAE IBNR @ 12/31/83 (g)	109,894
Total Unpaid Loss and ALAE @ 12/31/83	$186,477
Unpaid Loss and ALAE Discounted at 12%	$101,997

Exhibit III
Sheet 2

Notes to Exhibit III, Sheet 1

(a) Based on Employers' experience for policy years 1959-1974. Policy year 1974 is a partial policy year, reflecting writings from 1/1/74 - 6/30/74.

(b) Expenses include administrative and unallocated loss adjustment expenses. Employers' figures have been extracted from figures they have provided to the New York Insurance Department (Exhibit 1a).

(c) Includes allocated loss adjustment expense.

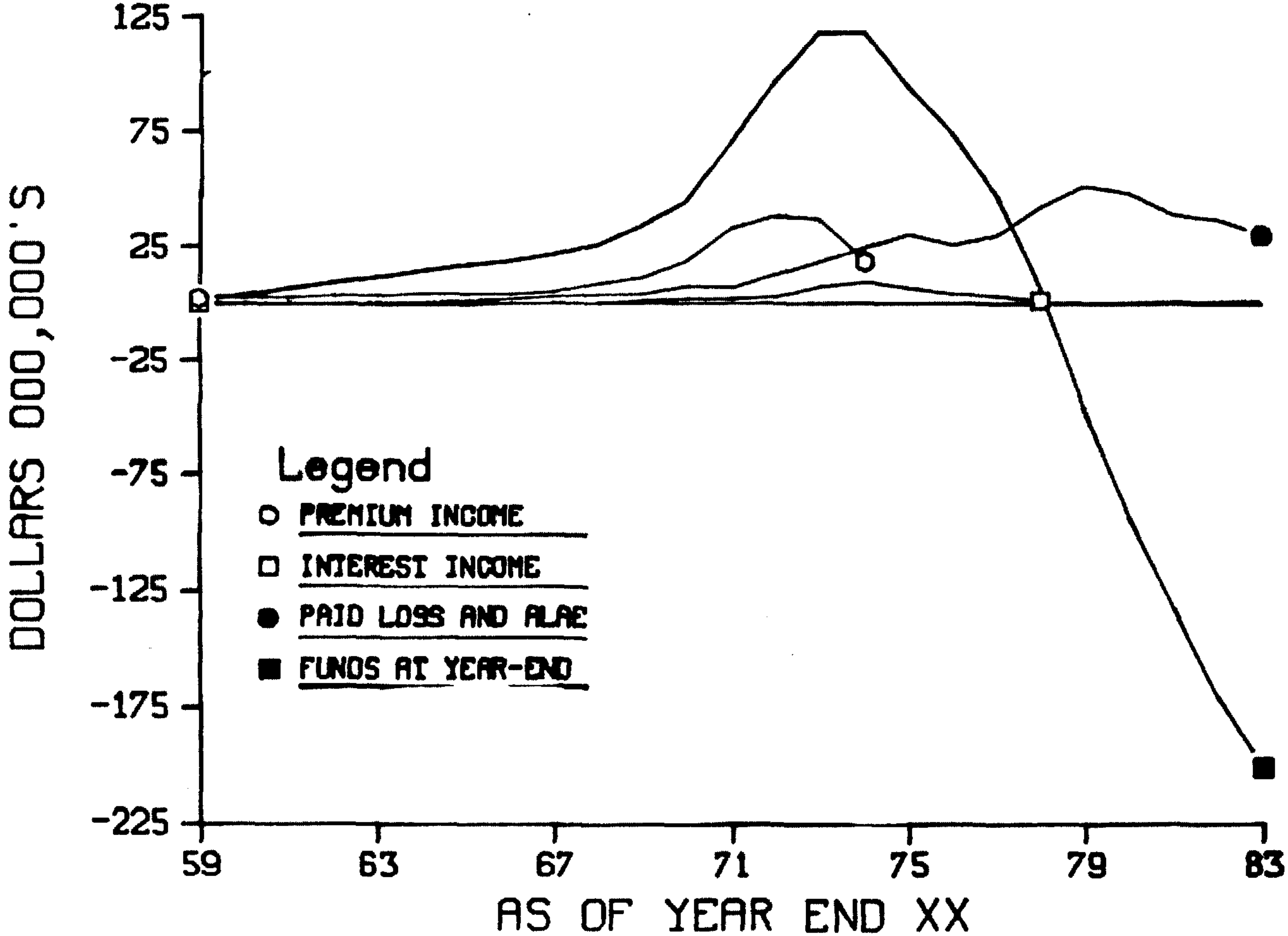
(d) Insurance Cash Flow [Column (5) = Columns (2)-(3)-(4)].

(e) Yields for calendar years 1959-1973 are based on one-year Treasury bill yield rates. Yields for subsequent years are the actual calendar year yields achieved by Employers, as defined by net investment gain or loss (including realized capital gain or loss) divided by mean total assets.

(f) Assumes all transactions occur uniformly throughout the year.

(g) Provision for unreported losses is based on the projected ultimate pure premiums developed in Part I of the Medical Malpractice Insurance Association of New York June 1984 physicians and surgeons professional liability rate filing.

NEW YORK PHYSICIANS AND SURGEONS
EMPLOYERS INSURANCE OF WAUSAU
POLICY YEARS 1959-1974 COMBINED



Projections of Surplus for Underwriting Strategy

William R. Gillam

Mr. Gillam is an Actuarial Associate at North American Reinsurance. His previous experience was at Prudential Reinsurance and Insurance Services Office. He has an M.S. from Rutgers University (1976) and a B.A. from Wesleyan University (1971), both in mathematics. He became an Associate of the CAS in 1982.

Abstract

The paper presents a specific modeling approach to the projection of surplus.

The model uses assumptions on growth, underwriting results, underwriting cash flow, interest and tax to simulate the operating results of an insurance company. Investible assets are incremented by cash flow and surplus by after tax income on an iterative basis for the years of the projection.

Results of several possible underwriting strategies of a multi-line company are compared according to several financial tests.

The vehicle for this model is an APL program, whose specifications are part of the paper.

It is hoped that the value of a model will be evidenced by this exposition. In addition, some of the components of this particular model may be useful in themselves, and conclusions I have made at least thought provoking.

William R. Gillam

Projections of Surplus for Underwriting Strategy

William R. Gillam

I. Need for a Projection of Insurance Company Surplus

Of course, anyone connected with the management of an insurance company would like to be able to predict income or surplus for the years of the foreseeable future. This is more than just wishing for a crystal ball; good estimates of future income may be necessary for several purposes. Among them are the following:

1. Part of a valuation
2. Planning for management
3. Reports to stockholders or a parent company
4. An aid to underwriting strategy

This paper is primarily about the last, although the methods presented have been used for item 3. and could easily be used for 1. or 2..
I have included an example of the prediction of surplus for a fictitious multiline company, and some conclusions about underwriting strategies the company may adopt. The input data for this company is hypothetical, but the cash flow patterns resemble those of a casually oriented reinsurer, an example which should best exhibit the distortions inherent in statutory accounting.

Specifications of the program are listed in III and hard copies of programs and outputs appear in Exhibits C and D.

II. Need for a Computer Model

I probably need not defend the use of modeling to the actuarial community. Power and flexibility to handle a variety of situations are a primary consideration. Ease of use and modification should also characterize a good model. APL, which I have used for this particular model, allows satisfaction of both criteria. This model could also probably be executed on a spread sheet package such as LOTUS 1-2-3, although the reader will have to write his/her own.

Another advantage of a model which is not so obvious is that it forces the creator to be more aware of each of his assumptions, its effect, or whether it is even necessary. In my model, future growth, underwriting results, interest rates, and transaction cash flow patterns will be assumed. The computer does the accounting. The computer is no better at predicting the future than the user, but for testing the effect of varying some selection of input parameters while holding others fixed, it excels.

III Specifications of the Model

For the purposes of this paper, I have studied a hypothetical company with eight years of underwriting experience, 1977 to 1984. The growth rate of the company during those years was greater than one would be willing to project into the next 22 years. Underwriting results during the eight experience years are spotty, but show clear deterioration from 1980 to 1984, so resembling reality.

I have separated company business into two groups, Property and Casualty, distinguished by respective faster and slower loss payment patterns, premium volumes and underwriting results. I would probably want to use more groups to better model a particular company, but these should suffice for the tests I wish to make.

It should be noted that the calendar year results are taken to be the same as the accident year, or, put another way, reserving is perfect.

A. The entries in the output matrix are either input directly or computed per the specifications which follow. Some other input items are as follows:

   1. The cash flow pattern of each of the groups was selected based on my experience in portfolio reinsurance and my study of some Annual Statements. These patterns appear on the output as percents of the total paid by calendar year, and are shown below.

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th
Premium Collection	55	35	10									
Expense Payout	50	40	5	5								
Property Loss	25	35	20	10	5	5						
Casualty Loss	5	18	12	10	10	10	8	7	5	5	5	5

2. Surplus and investible assets for 1984, $100 and $300 thousand respectively. For the purposes of this paper "assets" will mean investible assets, unless otherwise noted.

3. Effective interest rates by year. I selected +10% by year, starting 1985. For a cyclical pattern, I selected a ten year cycle, which starting in 1984 is

   11, 10, 10, 8, 8, 8, 9, 10, 11, 12%

4. Tax rate of 46%, ignoring any surtax exemptions.

5. An arbitrary portion (20%) of investment income was designated tax free. This could be due to tax free bonds and/or the 85% dividends received deduction. In practice, it would be adjusted to better fit a company's results and investment portfolio strategy.

6. A provision for a tax loss carry forward of seven years and a paid tax recovery of three.

B. Items 1 thru 9 are calculated separately for each group as follows:

1. Premiums Written PW_i, year i	These are entered exactly for (presumably) historic years and as annual growth factors for years to be projected.
2. Accident Year Loss Ratio LR_i	These are input as a ratio to premium earned.

Projections of Surplus for Underwriting Strategy

3. Expense Ratio

ER_i

These are ratios to premium written, as entered.

4. Premium Earned

$PE_i = \alpha PW_i + (1-\alpha) PW_{i-1}$

Where

$$\alpha = \frac{\sum_2^H PE_j - PW_{j-1}}{\sum_2^H PW_j - PW_{j-1}}$$

H = number of historic years

These are entered for historic years and calculated for the years of the projection, based on an average premium earnings pattern.

5. Premium Collected

$PC_i = \sum_{j=1}^{5} CF_j \ PW_{1+i-j}$

CF_j = input collection factor

This is a sum of the respective percents of the present and four prior years' written premium collected.

6. Expense Incurred

$E_i = ER_i \cdot PW_i$

Extension of premium written by expense ratio.

7. Expense Paid

$EX_i = \sum_{j=1}^{5} EF_j \cdot E_{1+i-j}$

EF_i = input expense payment factor

A sum of respective portions of present and four prior years' incurred expenses.

8. Losses Incurred

$L_i = LR_i \cdot PE_i$

Extension of premium earned by loss ratio.

9. Losses Paid

$LP_i = \sum_{j=1}^{15} LF_j \cdot L_{1+i-j}$

LF_j = input loss payment factor

A sum of respective portions of present and 14 prior years' losses incurred.

Items 10 and 11 are calculated for all groups combined.

10. Underwriting Profit

$U_i = PE_i - (L_i + E_i)$

Premium earned less losses and expense incurred.

Projections of Surplus for Underwriting Strategy

11. Underwriting Cash Flow — Premium collected less loss and expense paid.

$$UC_i = PC_i - (EX_i + LP_i)$$

Items 12 thru 20 are calculated sequentially by year for total business, starting at a selected "last historic" year.

12. Investment Income — Extension of prior year's ending invested assets plus one half of underwriting cash flow by rate of interest.

$$I_i = (A_{i-1} + 0.5\, UC_i) \times R_i$$

Where R_i is the annual effective rate

13. Gross Operating Income — Total underwriting profit and investment income.

$$GI_i = U_i + I_i$$

14. Taxable Income — Underwriting profit plus taxable portion of investment income.

$$TI_i = U_i + \beta I_i$$

15. Effective Taxable Income — This is adjusted income on which tax is actually paid, after loss carry forward or prior paid tax recovery.

ATI_i

(see APL)

16. Tax Paid — Per input tax rate on effective taxable income.

$$TP_i = TR \times ATI_i$$

17. Dividends or other decrements to income. — As input

D_i

Projections of Surplus for Underwriting Strategy

18. Ending Investible Assets

$$A_i = A_{i-1} + UC_i + I_i - (TP_i + D_i)$$

Previous year ending assets plus U/W cash flow and investment income minus tax paid and dividends.

19. Ending Surplus

$$S_i = S_{i-1} + U_i + I_i - (TP_i + D_i)$$

Previous surplus plus U/W and investment profit minus tax and dividends paid.

20. Discounted Surplus

$$DS_i = S_i + \prod_{H+1}^{i} \frac{1}{1+R_j}$$

At a rate 1/10 greater than the rate of interest income, to reflect an arbitrary premium for risk.

21. GAAP Adjusted Surplus

$$S_i + 0.15 \sum_{1}^{i} PW_j - PE_j$$

Ending surplus from 19. plus 15% of an approximation for the unearned premium reserve, this being cumulative written less earned premium.

IV Comparison of Some Underwriting Scenarios

A. General characteristics of the projection.

These comments pertain to the information graphed in Exhibit A. Each scenario 1 through 6 is a combination of growth vs. no growth in premium volume and underwriting results which remain poor, improve, or follow a cyclical pattern.

1. Flat writing, retain high loss and expense ratios.

   Probably a worst case result for underwriting would be continued writing at a combined ratio almost as high as the worst year, 1984, and not increase volume. The ultimate combined ratios are 106 and 115% for Property and Casualty respectively.

   Surplus and assets both increase steadily and immediately, at a reasonable rate which is, however, less than the 11% target rate of discount.

   The ratio of premiums written to prior surplus drops steadily to less than 1.1 in 1995, so a company in such a position could pay substantial dividends or expect a takeover by a capital hungry purchaser.

   The leverage ratio (assets-surplus) ÷ surplus also drops steadily, again indicating unused capital. (Investible assets minus surplus is used as a somewhat imperfect measure of liabilities).

   Statutory and GAAP surpus are parallel after a year or so.

Projections of Surplus for Underwriting Strategy

2. Flat writing, decreasing loss ratio.

This may be an actual short-term goal of some insurance company management. The ultimate combined ratios I have projected are 102 and 108% for Property and Casualty respectively, which may be as good as we can hope for.

The increase in surplus is steady and immediate as for Scenario 1, but the rate of increase flirts with 11% for more than 10 years. After 1997, discounted surplus drops consistently as returns to equity become more and more disappointing.

The premiums written to surplus ratio drops precipitously as does the leverage ratio (Assets-Surplus) ÷ Surplus. This is very inefficient use of capital, and the same comments as for Scenario 1 apply.

3. Growth, retaining high loss and expense ratio.

What would happen if a company continues to chase cash flow by increasing volume at the expense of effecting any underwriting control?

Surplus and assets both eventually increase, but surplus does decrease for a year after the particularly poor 1984 underwriting year. It is not until 1990 that surplus increases at a rate greater than the target 11%.

Projections of Surplus for Underwriting Strategy

The test ratio Written Premium ÷ Prior Surplus is high, especially in 1986, but remains less than three. This would not be the case for much higher growth rates than the selected +8% for Casualty and +6% for Property.

The leverage ratio (Assets-Surplus) ÷ Surplus remains fairly constant and as such seems within control, although this favorable appearance may be a distortion due to my approximation for liabilities.

Surplus increases at a rate greater than 11% after 1989, but GAAP surplus is already growing faster than +11% in 1987.

4. Steady growth, decreasing loss ratio.

This would be an ideal situation. Recovery from the exceptional 1984 year really only begins in 1986, but after this growth and vital signs all appear good. The leverage ratio steadily decreases and a payment of dividends would be in order.

Somehow, I feel we do not need to spend much time admiring this scenario.

5. Growth, with cyclical underwriting results.

Surplus and GAAP surplus increase unevenly, as might be expected.

Premiums to surplus is almost three in 1986, coming off the bad underwriting year 1984. Our leverage ratio is worst in 1987.

After this year, most vital signs are under control and surplus increases at a rate alternating above and below the target 11%.

In general, we observe a 2-3 year lag for financial results following underwriting results.

6. Growth, with cyclical underwriting results, which lag a cyclical pattern of interest rates by two years.

   This may or may not resemble reality, and I have included it for curiosity's sake. My comments are nearly the same as those for Scenario 5 above, except results are worse due to lesser investment income. It is interesting that surplus discounted along rates which follow the fluctuations (plus the premium) remains significantly worse than discounted surplus in the case of a uniform 10% rate, seen in Scenario 5. This may be the result of continuing to write to a high combined ratio when interest rates are falling, just as in current industry results.

B. Some comparisons of the projections.

These comparisons are graphed in Exhibit B.

1. Flat premium writings with improving underwriting vs. growth at expense of continued high loss ratio, A2. vs. A3.

   Writing to a lower loss and expense ratio at the expense of growth has some bad characteristics mentioned above, but the effect on statutory surplus is quite desirable. In this case, the strategy produces higher surplus for 15 years than that for continued growth at a high combined ratio.

This comparison holds even for GAAP adjusted surplus, which recognizes equity in the unearned premium for commissions paid but not yet earned, so the regulatory strategem of curtailment of premium writings for too rapidly growing companies may make more sense than it seems, at least if underwriting results improve.

2. Flat premium writings, no improvement in underwriting vs. growth with no improvement in underwriting, A1. vs. A3.

Even when the curtailment of premium writings does not result in better underwriting, statutory surplus will be better in the case of no growth than in the case of growth for some eight years. The effect is not great, but it even occurs in GAAP surplus for five years.

The retention of a high leverage ratio in the growth case ultimately leads to greater income, but with a significant time delay if underwriting results are poor.

The effect on statutory surplus of no growth, even if underwriting does not improve, may still justify regulatory procedure.

Projections of Surplus for Underwriting Strategy

3. Growth with poor loss and expense ratio vs. growth with cyclical underwriting results, A3. vs. A5.

Surplus may be expected to vary more if underwriting results are cyclical than if they are steady. So it is here, where the surplus under cyclical underwriting snakes around the surplus of the steady case.

It may be observed from the graph that the surplus from the cyclical underwriting case averages higher than that for the stable scenario. This is in spite of the fact that the premium weighted 20 year average loss and expense ratios for the cyclical case, 80.1 and 31.4%, are higher than those in the steady state, 80.0 and 31.1. I would be a little hesitant to recommend this as a strategy, but if positive cash flow can be maximized at time preceding high interest rates, the cycle may make economic sense.

V Conclusions

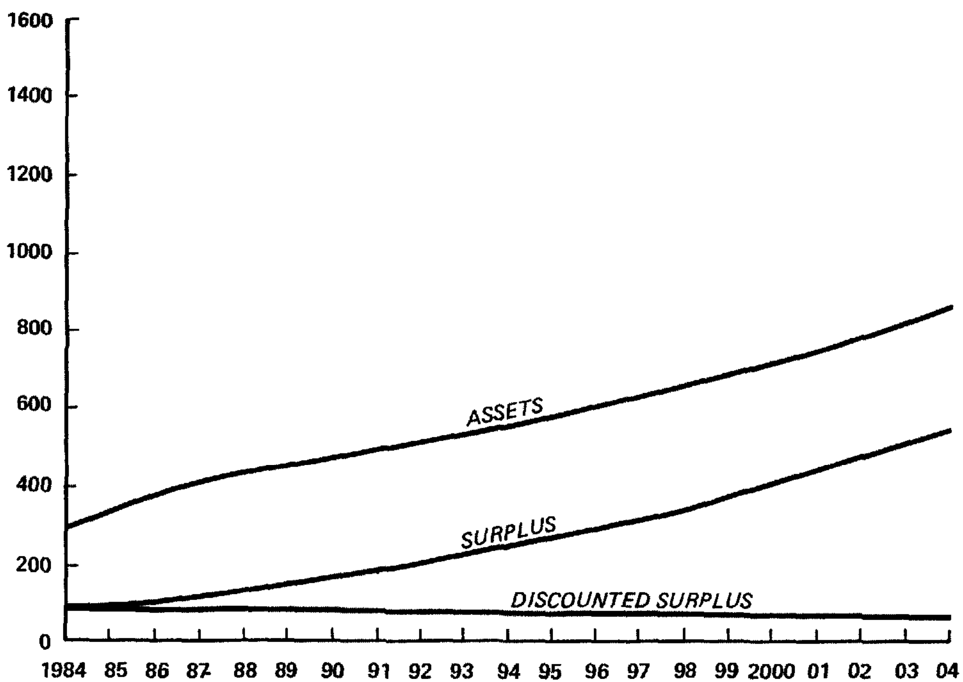
The following conclusions are based on a fairly simple computer model and a set of assumptions which may not be fully justified, so are open to debate. I hope: 1. the shortcomings of the model are minimized when comparing results of different ingoing strategies under the same assumptions, and; 2. a debate is actually opened, resulting in improved modeling techniques. It may be the case the conclusions of my research are no more than what is common knowledge of financial planners, but even at that, the viability of a model will have been demonstrated.

A. Conservative underwriting, even at the expense of growth, results in higher income than growth with continued poor underwriting. This is even true for GAAP adjusted surplus.

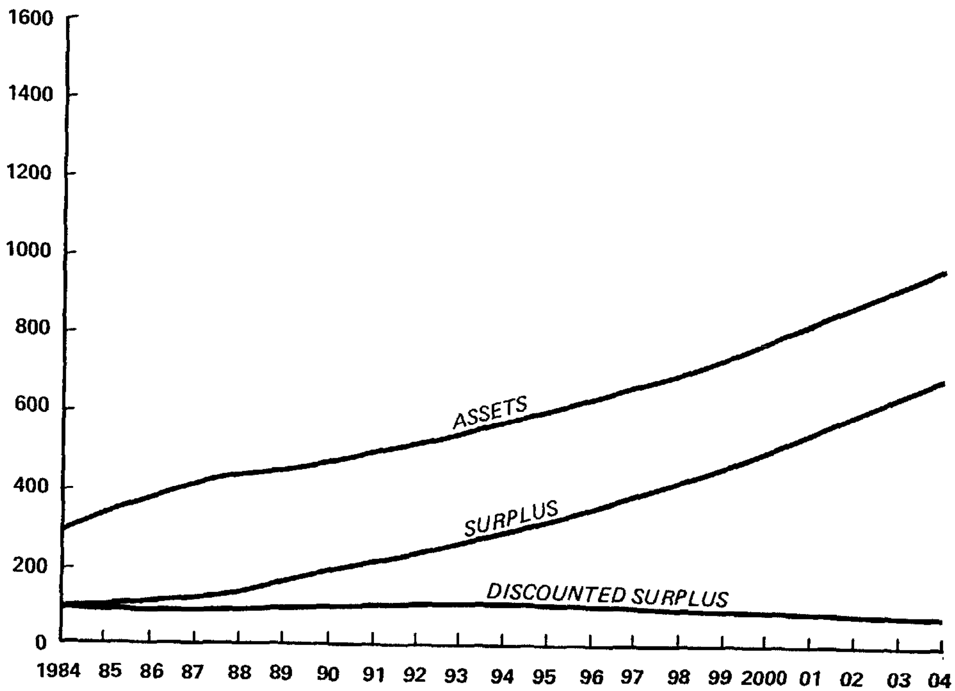
B. There is a need for other than a statutory approach for measurement of results. My simple proxy for a GAAP adjustment suggests even GAAP may hide the true picture. I believe discounting reserves and better monitoring of cash flow would be parts of such an approach, as well as a true GAAP adjustment.

C. A cyclical underwriting pattern may have desirable effect on ultimate income. This is especially true, of course, if cash flow can be timed to take full advantage of changes in interest rates. It is also known that the underwriting cycle may well have adverse effects on down years' financial results, which this model shows lag poor underwriting years by 1-3 years. Management may find reporting such results undesirable, even when there need be no cause for alarm.

1. FLAT WRITINGS CONTINUED POOR LOSS RATIO



2. FLAT WRITINGS, IMPROVING LOSS RATIO



3. GROWTH, RETAIN HIGH LOSS RATIO

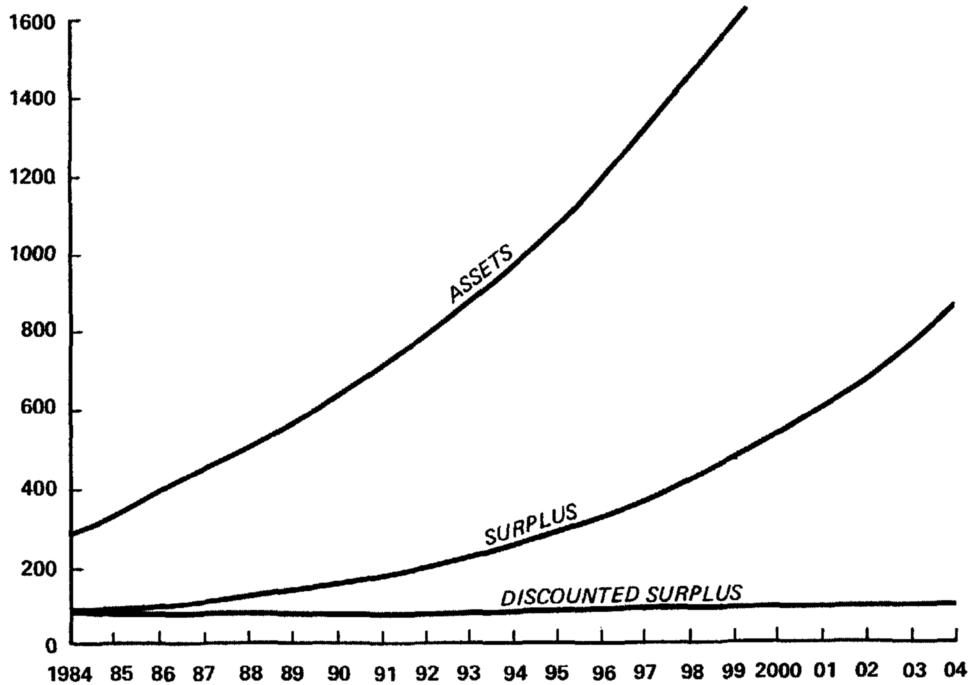
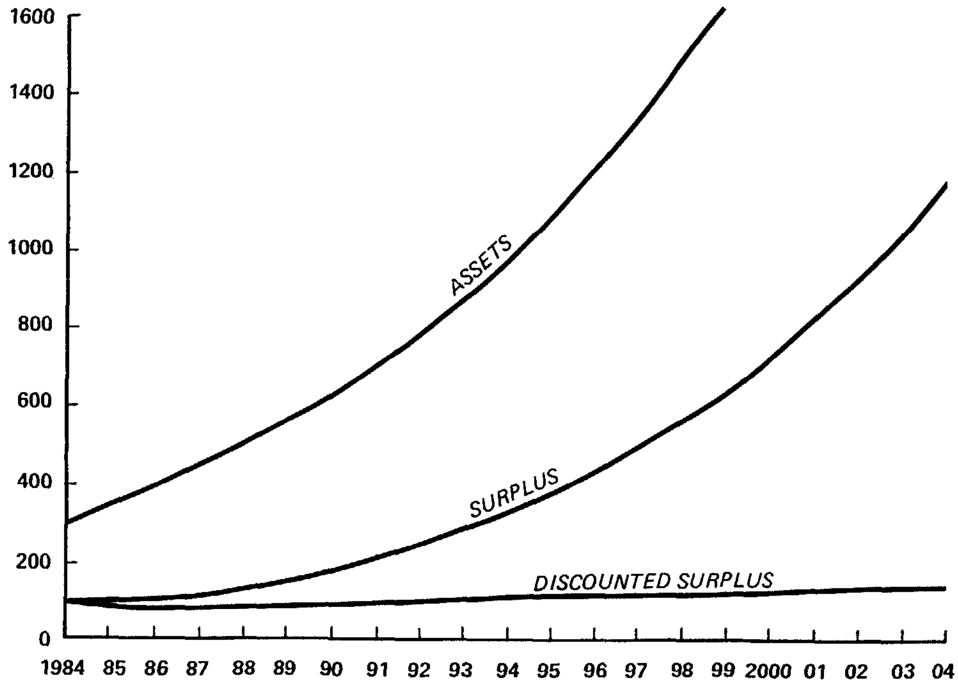
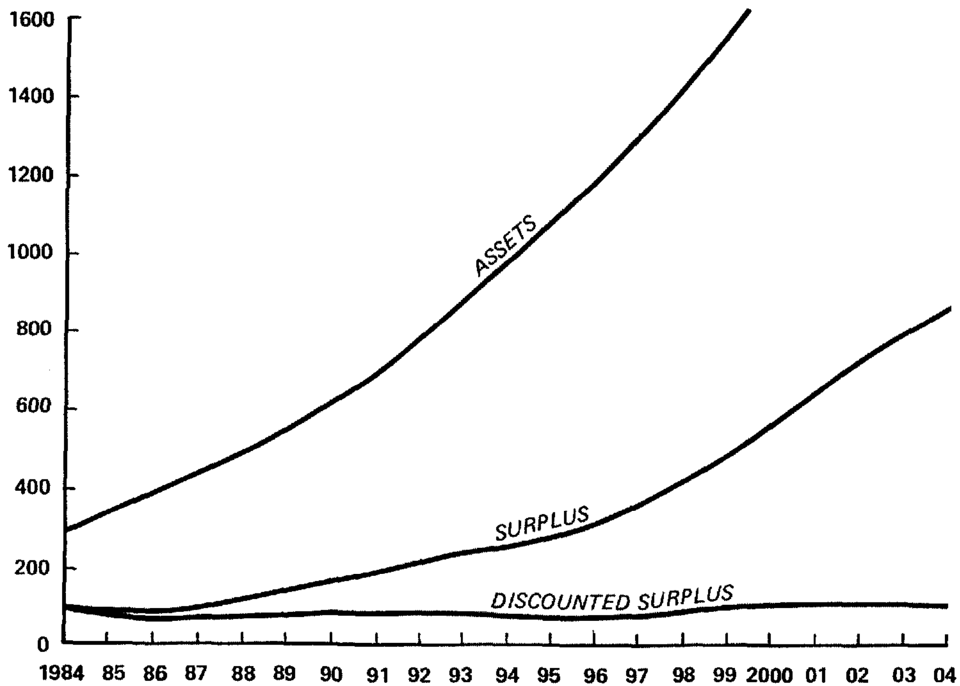


EXHIBIT A, p.4

4. GROWTH, IMPROVING LOSS RATIO



5. CYCLICAL UNDERWRITING RESULTS



6. CYCLICAL UNDERWRITING, FOLLOWING INTEREST CHANGES

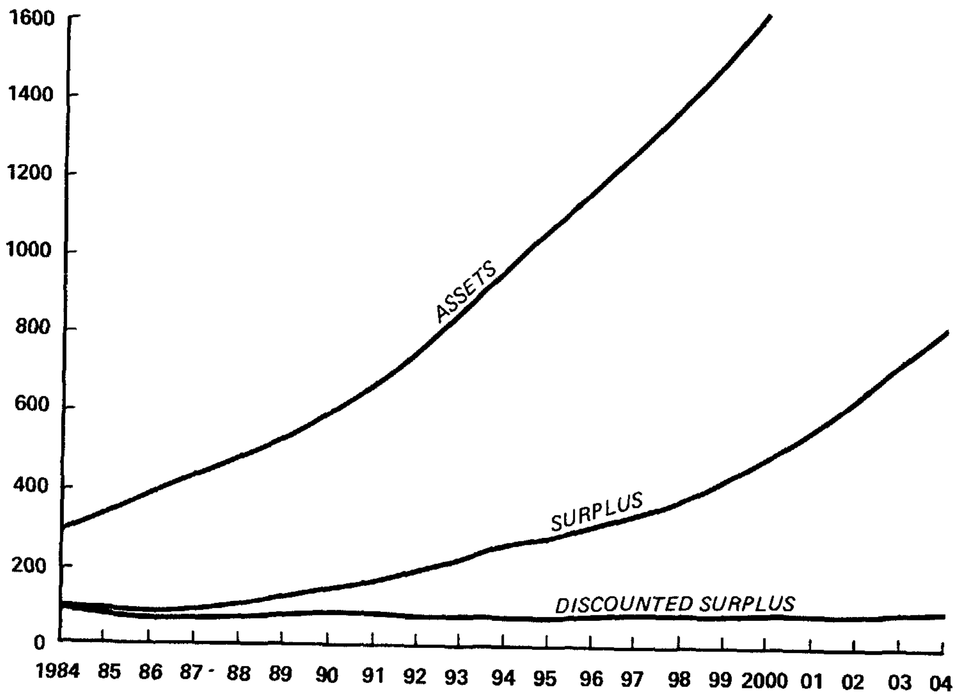
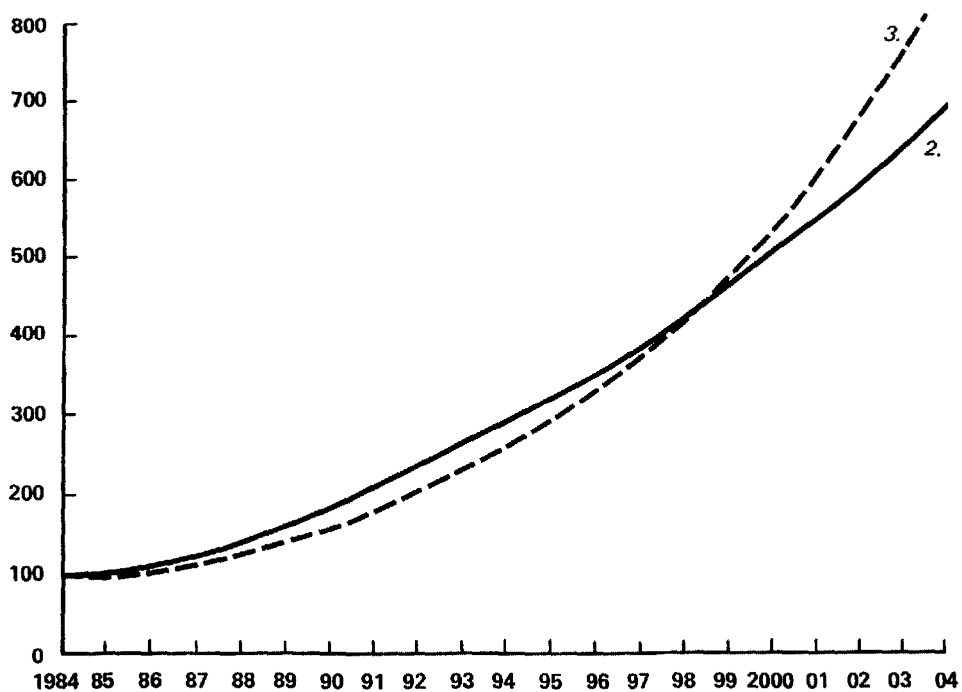


EXHIBIT B. p.1

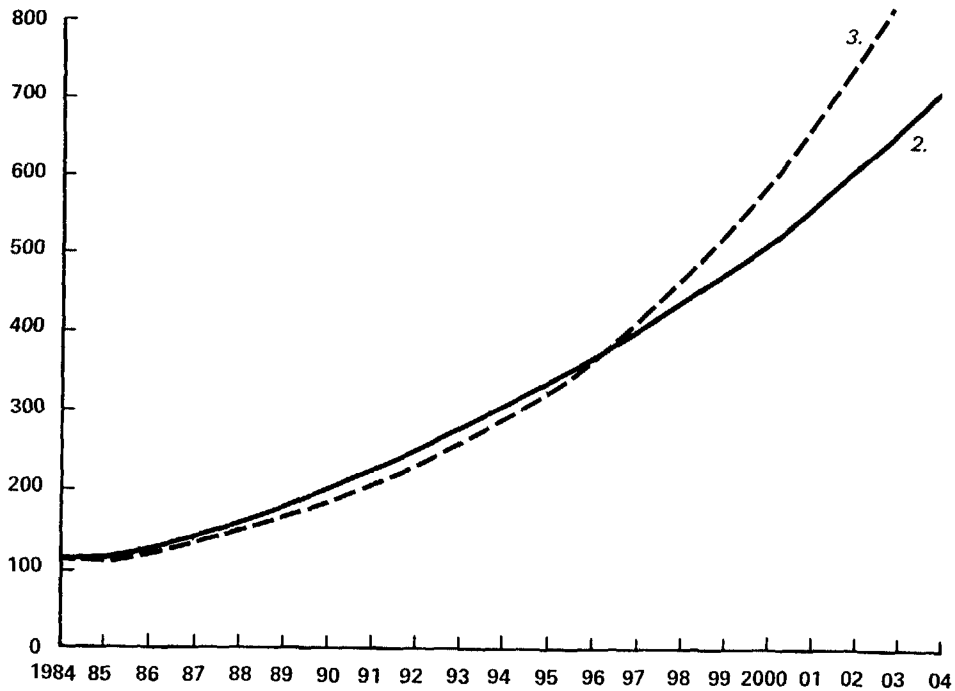
SURPLUS COMPARISON

2. Flat Writings, Improving Loss Ratio
vs
3. Growth, Retain High Loss Ratio



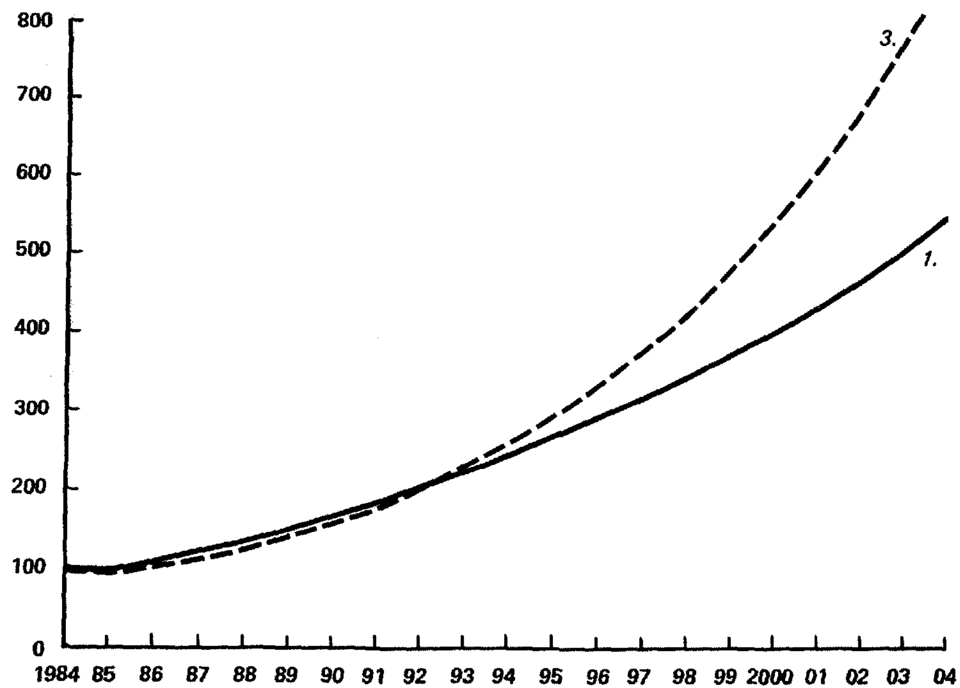
GAAP SURPLUS COMPARISON

2. Flat Writings, Improving Loss Ratio
vs
3. Growth, Retain High Loss Ratio



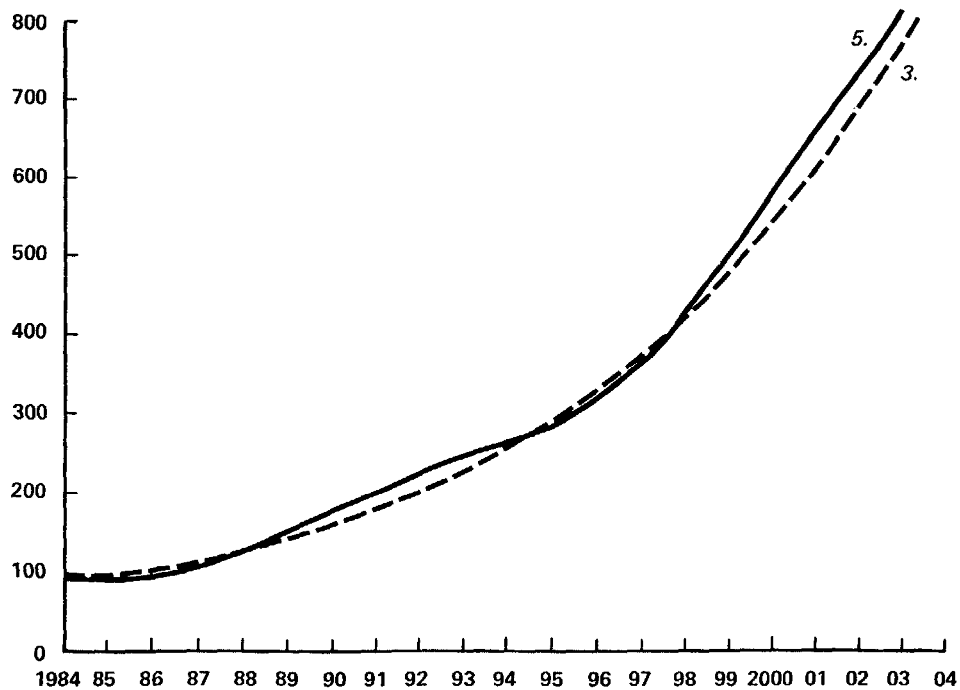
SURPLUS COMPARISON

1. Flat Writings, High Loss Ratio
vs
3. Growth, High Loss Ratio



SURPLUS COMPARISON

3. Growth, High Loss Ratio
vs
5. Cyclical Underwriting Results



```
∇MASTERSP[⎕]∇

    ∇ MASTERSP;STRT;BSC;NAMES;LAG;ALPHA;RG;RATES;SA;SS;LOCAL;ANFG;TXR
[1]   'ENTER STARTING YEAR OF U/W RESULTS'
[2]   STRT←⎕ -1
[3]   'ENTER THE MATRIX OF GROUPS'' HISTORIC AND PROJECTED UNDERWRITING RESULTS'
[4]  ⍝THIS SHOULD BE IN THE FORM GROUPS×4×YRS, WITH FOUR ROWS FOR WRT PREM, L/R, E/R, ERND PREM
[5]  ⍝GROUPS ARE CHARACTERIZED BY COLLECTION AND PAYOUT LAG PATTERNS AS WELL AS THE PARAMETERS IN THE ABOVE 4 ROWS
[6]  ⍝YRS SHOULD BE 10, 20, OR 30.  THE NUMBER OF GROUPS IS VARIABLE WITHIN REASON.
[7]  ⍝WRT PREM MAY APPEAR AS FACTORS FOR ANNUAL INCREASE, STARTING AT ANY YEAR IN THE PROJECTION
[8]  ⍝ERND PREM NEED BE ENTERED FOR HISTORIC YEARS ONLY.  THE REST WILL BE CALCULATED.
[9]   BSC←⎕
[10]  'ENTER A GRPS×25 MATRIX OF 5 COLLECT LAGS,5 EXPENSE PAYOUT LAGS, AND 15 LOSS PAYOUT LAGS FOR EACH GROUP'
[11]  LAG←⎕
[12]  'ENTER THE RESPECTIVE NAMES OF THE GROUPS IN QUOTES, AS  /GP1/GP2/  ,,ETC'
[13]  NAMES←(⍳0)ROWNAMES⎕
[14]  'ENTER YEAR AT WHICH ENDING ASSETS AND SURPLUS WILL BE INITIALIZED'
[15] ⍝THIS WOULD PRESUMABLY BE THE LATEST YEAR OF HISTORICAL DATA
[16]  ANFG←⎕
[17]  'ENTER ENDING (INVESTIBLE) ASSETS FOR THAT YEAR'
[18]  SA←⎕
[19]  'ENTER ENDING SURPLUS FOR THAT YEAR'
[20]  SS←⎕
[21]  'ENTER TWO ROW MATRIX OF INTEREST RATES (AS DECIMALS) AND PAID DIVIDEND AMOUNTS BY CALENDAR YEAR'
[22] ⍝THESE SHOULD BE EFFECTIVE OR COMPOUNDED RATES
[23]  RATES←⎕
[24]  'ENTER THE TAX RATE AS A DECIMAL'
[25]  TXR←⎕
[26]  'ENTER THE DECIMAL PORTION OF INVESTMENT INCOME WHICH IS TAX FREE'
[27]  ALPHA←1-⎕
[28]  'ENTER A NAME FOR THE GLOBAL VARIABLE IN WHICH THE TOTAL SURPLUS PROJECTION MATRIX WILL BE STORED'
[29]  'AFTER ENTERING,YOU SHOULD DELIBERATELY TURN TO THE TOP OF A NEW PAGE'
[30]  GLOBAL←⎕
[31]  LOCAL←RATES SURPROJ RG←LAG CASH BSC
[32]  ⍎GLOBAL,'←LOCAL'
    ∇
```

```
      ∇CASH[⎕]∇

     ∇ Z←A CASH B;M;N;AEP;YRSEP;RATE;I;TEM;Y;EPF;SZ;G
[1]   ⍝CASH SPRANG PARTIALLY GROWN FROM THE FOREHEAD OF G. VENTER
[2]   ⍝A IS AN ARRAY OF 5 COLLECT LAGS,  5 EXPENSE PAYOUT LAGS, AND 15 LOSS PAYOUT LAGS FOR EACH CATEGORY
[3]   ⍝B IS AN ARRAY  WITH ROWS FOR WP, L/R, E/R, EP, A COLUMN FOR EACH YEAR, AND  PLANES FOR EACH GROUP
[4]    Y←((G←(1↑⍴B)),11,M←¯1↑⍴B)⍴0
[5]    SZ←⌊0.5+M÷10
[6]    Y[; 1 2 3 ;]←B[; 1 2 3 ;]
[7]    N←0
[8]   TOP:N←N+1
[9]   ⍝FIRST FILL IN WRITTEN PREM USING ANNUAL INCREASES
[10]   TEM←(TEM>0)/TEM←(⍳M)×Y[N;1;]<10
[11]   Y[N;1;TEM]←Y[N;1;(¯1+⌊/TEM)]××\Y[N;1;TEM]
[12]  ⍝FOLLOWING IS THE EXTENTION OF EARNED PREM USING HISTORIC ERND TO WRTN RATIO EPF
[13]   YRSEP←+/(B[N;4;]≠0)
[14]   AEP←YRSEP↑B[N;4;]
[15]   EPF←(+/B[N;4;(1↓⍳YRSEP)]-B[N;1;(¯1↓⍳YRSEP)])÷+/B[N;4;(1↓⍳YRSEP)]-B[N;4;(¯1↓⍳YRSEP)]
[16]  ⍝EARNED PREMIUM
[17]   Y[N;4;]←(EPF×Y[N;1;])+(1-EPF)×0,¯1↓Y[N;1;]
[18]   Y[N;4;⍳YRSEP]←AEP
[19]  ⍝PREMIUM COLLECTED
[20]   Y[N;5;]←+⌿((⍳M)∘.≤⍳M)×(-0,⍳M-1)⌽(M↑5↑A[N;])∘.×Y[N;1;]
[21]  ⍝EXPENSES INCURRED
[22]   Y[N;6;]←Y[N;1;]×Y[N;3;]÷100
[23]  ⍝EXPENSES PAID
[24]   Y[N;7;]←+⌿((⍳M)∘.≤⍳M)×(-0,⍳M-1)⌽(M↑5↑5↓A[N;])∘.×Y[N;6;]
[25]  ⍝LOSSES INCURRED
[26]   Y[N;8;]←Y[N;4;]×Y[N;2;]÷100
[27]  ⍝LOSSES PAID
[28]   Y[N;9;]←+⌿((⍳M)∘.≤⍳M)×(-0,⍳M-1)⌽(M↑10↓A[N;])∘.×Y[N;8;]
[29]   →(N<G)/TOP
[30]  ⍝UNDERWRITING PROFIT
[31]   Y[;10;]←Y[;4;]-+/[2]Y[; 6 8 ;]
[32]  ⍝UNDERWRITING CASH FLOW
[33]   Y[;11;]←Y[;5;]-Y[;7;]+Y[;9;]
[34]   I←0
[35]  LP:I←I+1
[36]   I PRINTC Y
[37]   →(I<G)/LP
[38]   Z←+⌿Y
[39]   Z[2;]←100×÷⌿Z[8 4 ;]
[40]   Z[3;]←100×÷⌿Z[6 4 ;]
     ∇
```

```
∇SURPROJ[⎕]∇

    ∇  C←R SURPROJ UW;IND;TI;LCF
[1]    ⍝PROGRAM DEVISED BY R. GILLAM WITH  ENCOURAGEMENT AND TECHNICAL ADVICE FROM J. STANARD
[2]    ⍝PROGRAM LOOPS THRU CALANDAR YEARS, INCREMENTING SURPLUS BY INCOME LESS TAX AND DIVIDENDS
[3]     C←(10 0 +⍴UW)↑UW
[4]     IND←ANFG-STRT
[5]     C[18;IND]←SA
[6]     C[19 20 ;IND]←SS
[7]     C[21;IND]←SS+0.15×-/+/C[1 4 ;⍳IND]
[8]    ⍝INITIALIZE LOSS CARRY FORWARD
[9]     LCF←0
[10]   LOOP:IND←IND+1
[11]   ⍝INVESTMENT INCOME
[12]    C[12;IND]←(C[18;IND-1]+0.5×C[11;IND])×R[1;IND]
[13]   ⍝GROSS OPERATING INCOME
[14]    C[13;IND]←+/C[10 12 ;IND]
[15]   ⍝TAXABLE INCOME
[16]    TI←C[14;IND]←C[10;IND]+ALPHA×C[12;IND]
[17]   ⍝EFFECTIVE INCOME ON WHICH TAX IS PAID, AFTER LOSS CARRY FORWARD OR PREVIOUSLY PAID TAX RECOVERY
[18]    C[15;IND]←ETI←((0⌈TI+¯1↑LCF)×TI≥0)+(0⌊TI⌈-+/¯3↑C[15;⍳IND-1])×TI<0
[19]   ⍝TAX PAID
[20]    C[16;IND]←TXR×ETI
[21]    LCF←¯7↑(-1↓LCF)+LCF←0⌊(((LCF,¯1↑LCF)+TI)×TI≥0)+(LCF,((TI-ETI)+¯1↑LCF))×TI<0
[22]   ⍝DIVIDENDS OR OTHER DETRIMENTS TO INCOME
[23]    C[17;IND]←R[2;IND]
[24]   ⍝ENDING INVESTED ASSETS
[25]    C[18;IND]←C[18;IND-1]+(+/C[11 12 ;IND])-+/C[16 17 ;IND]
[26]   ⍝ENDING SURPLUS
[27]    C[19;IND]←C[19;IND-1]+(+/C[10 12 ;IND])-+/C[16 17 ;IND]
[28]   ⍝DISCOUNTED SURPLUS
[29]    C[20;IND]←C[19;IND]÷×/1+1.1×R[1;⍳(IND-ANFG-STRT)]
[30]   ⍝GAAP ADJUSTED SURPLUS
[31]    C[21;IND]←C[19;IND]+0.15×-/+/C[1 4 ;⍳IND]
[32]    →(IND<(¯1↑⍴UW))/LOOP
[33]    PRINTP C
    ∇
```

```
∇PRINTC[⎕]∇

  ∇  P PRINTC L;II
     II←¯1
     2 2 ⍴' '
     ((⌊(130-⍴NAMES[P;])÷2)⍴' '),NAMES[P;]
    PLNS:II←II+1
     2 2 ⍴' '
     ('YEAR                    '), 10 0 ⍕STRT+(10×II)+⍳10
     ' '
     ROWTITLES,(11 4 ⍴' '), 10 0 ⍕Y[P;;(10×II)+⍳10]
     2 2 ⍴' '
     →(II<⌊¯0.5+N÷10)/PLNS
     'COLLECTION FACTORS;          ', 6 2 ⍕5↑A[P;]
     'EXPENSE PAYOUT FACTORS;      ', 6 2 ⍕5↑5↓A[P;]
     'LOSS PAYOUT FACTORS;         ', 6 2 ⍕10↓A[P;]
     10 10 ⍴' '
  ∇


∇PRINTP[⎕]∇

  ∇  PRINTP PR
     4 2 ⍴' '
     (64⍴' '),'TOTAL'
     2 2 ⍴' '
     K←¯1
    CIR:K←K+1
     ('YEAR                    '), 10 0 ⍕STRT+(10×K)+⍳10
     ' '
     XX\RWTTLS,(21 4 ⍴' '), 10 0 ⍕PR[;(10×K)+⍳10]
     3 2 ⍴' '
     →(K<⌊¯0.5+(¯1↑⍴PR)÷10)/CIR
  ∇
```

PROPERTY LINES

YEAR	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986
PREMIUMS WRITTEN	45000	65000	80000	105000	110000	125000	130000	150000	150000	150000
CAL/ACC YR LOSS RATIO	53	63	58	64	70	75	78	80	78	76
EXPENSE RATIO	25	25	25	28	28	28	30	30	30	30
PREMIUMS EARNED	35000	50000	70000	90000	107000	115000	127000	140000	150000	150000
PREMIUMS COLLECTED	24750	51500	71250	92250	105250	117750	126250	140500	148000	150000
EXPENSE INCURRED	11250	16250	20000	29400	30800	35000	39000	45000	45000	45000
EXPENSE PAID	5625	12625	17063	24075	28973	32290	36510	41390	44200	44700
LOSS INCURRED	18550	31500	40600	57600	74900	86250	99060	112000	117000	114000
LOSS PAID	4638	14368	24885	36765	51083	65860	79298	92321	103512	109814
U/W PROFIT	5200	2250	9400	3000	1300	¯6250	¯11060	¯17000	¯12000	¯9000
U/W CASH FLOW	14488	24507	29302	31410	25195	19600	10442	6789	288	¯4514

YEAR	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
PREMIUMS WRITTEN	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
CAL/ACC YR LOSS RATIO	76	76	76	76	76	76	76	76	76	76
EXPENSE RATIO	30	30	30	30	30	30	30	30	30	30
PREMIUMS EARNED	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
PREMIUMS COLLECTED	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
EXPENSE INCURRED	45000	45000	45000	45000	45000	45000	45000	45000	45000	45000
EXPENSE PAID	45000	45000	45000	45000	45000	45000	45000	45000	45000	45000
LOSS INCURRED	114000	114000	114000	114000	114000	114000	114000	114000	114000	114000
LOSS PAID	112266	113453	114050	114150	114000	114000	114000	114000	114000	114000
U/W PROFIT	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000
U/W CASH FLOW	¯7266	¯8453	¯9050	¯9150	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000

YEAR	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
PREMIUMS WRITTEN	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
CAL/ACC YR LOSS RATIO	76	76	76	76	76	76	76	76	76	76
EXPENSE RATIO	30	30	30	30	30	30	30	30	30	30
PREMIUMS EARNED	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
PREMIUMS COLLECTED	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
EXPENSE INCURRED	45000	45000	45000	45000	45000	45000	45000	45000	45000	45000
EXPENSE PAID	45000	45000	45000	45000	45000	45000	45000	45000	45000	45000
LOSS INCURRED	114000	114000	114000	114000	114000	114000	114000	114000	114000	114000
LOSS PAID	114000	114000	114000	114000	114000	114000	114000	114000	114000	114000
U/W PROFIT	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000
U/W CASH FLOW	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000	¯9000

COLLECTION FACTORS; 0.55 0.35 0.10 0.00 0.00
EXPENSE PAYOUT FACTORS; 0.50 0.40 0.05 0.05 0.00
LOSS PAYOUT FACTORS; 0.25 0.35 0.20 0.10 0.05 0.05 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

CASUALTY LINES

YEAR	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986
PREMIUMS WRITTEN	20000	40000	50000	70000	70000	75000	80000	90000	90000	90000
CAL/ACC YR LOSS RATIO	120	105	78	80	85	87	87	92	88	85
EXPENSE RATIO	20	25	25	30	30	30	31	33	31	30
PREMIUMS EARNED	15000	33000	45000	60000	70000	72000	78000	85000	90000	90000
PREMIUMS COLLECTED	11000	29000	43500	60000	68000	72750	77250	85000	89000	90000
EXPENSE INCURRED	4000	10000	12500	21000	21000	22500	24800	29700	27900	27000
EXPENSE PAID	2000	6600	10450	16200	20025	21325	23500	26945	28195	27385
LOSS INCURRED	18000	34650	35100	48000	59500	62640	67860	78200	79200	76500
LOSS PAID	900	4973	10152	14676	21092	28377	35023	41934	49327	55395
U/W PROFIT	¯7000	¯11650	¯2600	¯9000	¯10500	¯13140	¯14660	¯22900	¯17100	¯13500
U/W CASH FLOW	8100	17428	22898	29124	26883	23048	18727	16121	11478	7221

YEAR	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
PREMIUMS WRITTEN	90000	90000	90000	90000	90000	90000	90000	90000	90000	90000
CAL/ACC YR LOSS RATIO	85	85	85	85	85	85	85	85	85	85
EXPENSE RATIO	30	30	30	30	30	30	30	30	30	30
PREMIUMS EARNED	90000	90000	90000	90000	90000	90000	90000	90000	90000	90000
PREMIUMS COLLECTED	90000	90000	90000	90000	90000	90000	90000	90000	90000	90000
EXPENSE INCURRED	27000	27000	27000	27000	27000	27000	27000	27000	27000	27000
EXPENSE PAID	27180	27045	27000	27000	27000	27000	27000	27000	27000	27000
LOSS INCURRED	76500	76500	76500	76500	76500	76500	76500	76500	76500	76500
LOSS PAID	60477	65265	68841	71263	73435	74799	75595	76288	76720	76635
U/W PROFIT	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500
U/W CASH FLOW	2344	¯2310	¯5841	¯8263	¯10435	¯11799	¯12595	¯13288	¯13720	¯13635

YEAR	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
PREMIUMS WRITTEN	90000	90000	90000	90000	90000	90000	90000	90000	90000	90000
CAL/ACC YR LOSS RATIO	85	85	85	85	85	85	85	85	85	85
EXPENSE RATIO	30	30	30	30	30	30	30	30	30	30
PREMIUMS EARNED	90000	90000	90000	90000	90000	90000	90000	90000	90000	90000
PREMIUMS COLLECTED	90000	90000	90000	90000	90000	90000	90000	90000	90000	90000
EXPENSE INCURRED	27000	27000	27000	27000	27000	27000	27000	27000	27000	27000
EXPENSE PAID	27000	27000	27000	27000	27000	27000	27000	27000	27000	27000
LOSS INCURRED	76500	76500	76500	76500	76500	76500	76500	76500	76500	76500
LOSS PAID	76500	76500	76500	76500	76500	76500	76500	76500	76500	76500
U/W PROFIT	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500
U/W CASH FLOW	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500	¯13500

COLLECTION FACTORS; 0.55 0.35 0.10 0.00 0.00
EXPENSE PAYOUT FACTORS; 0.50 0.40 0.05 0.05 0.00
LOSS PAYOUT FACTORS; 0.05 0.18 0.12 0.10 0.10 0.10 0.08 0.07 0.05 0.05 0.05 0.05 0.00 0.00 0.00

TOTAL

YEAR	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986
PREMIUMS WRITTEN	65000	105000	130000	175000	180000	200000	210000	240000	240000	240000
LOSS RATIO (ACC.YR.)	73	80	66	70	76	80	81	85	82	79
INCURRED EXPENSE RATIO	30	32	28	34	29	31	31	33	30	30
PREMIUMS EARNED	50000	83000	115000	150000	177000	187000	205000	225000	240000	240000
PREMIUMS COLLECTED	35750	80500	114750	152250	173250	190500	203500	225500	237000	240000
EXPENSE INCURRED	15250	26250	32500	50400	51800	57500	63800	74700	72900	72000
EXPENSE PAID	7625	19225	27513	40275	48998	53615	60010	68335	72395	72085
LOSS INCURRED	36550	66150	75700	105600	134400	148890	166920	190200	196200	190500
LOSS PAID	5538	19340	35037	51441	72175	94237	114321	134255	152839	165208
UNDER. PROFIT	¯1800	¯9400	6800	¯6000	¯9200	¯19390	¯25720	¯39900	¯29100	¯22500
UNDER. CASH FLOW	22588	41935	52200	60534	52078	42648	29169	22910	11766	2707
INVESTMENT INCOME	0	0	0	0	0	0	0	0	30588	34371
TOTAL INCOME	0	0	0	0	0	0	0	0	1488	11871
TAXABLE INCOME	0	0	0	0	0	0	0	0	¯4629	4997
INCOME AFTER LCF OR PLR	0	0	0	0	0	0	0	0	0	367
TAX PAID	0	0	0	0	0	0	0	0	0	169
DIVIDENDS	0	0	0	0	0	0	0	0	0	0
ENDING INVESTED ASSETS	0	0	0	0	0	0	0	300000	342355	379263
ENDING SURPLUS	0	0	0	0	0	0	0	100000	101488	113190
DISCOUNTED SURPLUS	0	0	0	0	0	0	0	100000	91431	91868
GAAP ADJ SURPLUS	0	0	0	0	0	0	0	116950	118438	130140

YEAR	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
PREMIUMS WRITTEN	240000	240000	240000	240000	240000	240000	240000	240000	240000	240000
LOSS RATIO (ACC.YR.)	79	79	79	79	79	79	79	79	79	79
INCURRED EXPENSE RATIO	30	30	30	30	30	30	30	30	30	30
PREMIUMS EARNED	240000	240000	240000	240000	240000	240000	240000	240000	240000	240000
PREMIUMS COLLECTED	240000	240000	240000	240000	240000	240000	240000	240000	240000	240000
EXPENSE INCURRED	72000	72000	72000	72000	72000	72000	72000	72000	72000	72000
EXPENSE PAID	72180	72045	72000	72000	72000	72000	72000	72000	72000	72000
LOSS INCURRED	190500	190500	190500	190500	190500	190500	190500	190500	190500	190500
LOSS PAID	172742	178718	182891	185413	187435	188799	189595	190288	190720	190635
UNDER. PROFIT	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500
UNDER. CASH FLOW	¯4922	¯10763	¯14891	¯17413	¯19435	¯20799	¯21595	¯22288	¯22720	¯22635
INVESTMENT INCOME	37680	40312	42612	44725	46745	48722	50717	52763	54882	57118
TOTAL INCOME	15180	17812	20112	22225	24245	26222	28217	30263	32382	34618
TAXABLE INCOME	7644	9750	11590	13280	14896	16478	18073	19710	21406	23194
INCOME AFTER LCF OR PLR	7644	9750	11590	13280	14896	16478	18073	19710	21406	23194
TAX PAID	3516	4485	5331	6109	6852	7580	8314	9067	9847	10669
DIVIDENDS	0	0	0	0	0	0	0	0	0	0
ENDING INVESTED ASSETS	408503	433570	455960	477163	497621	517964	538772	560180	582496	606309
ENDING SURPLUS	124854	138182	152963	169079	186472	205114	225017	246213	268748	292697
DISCOUNTED SURPLUS	91292	91024	90776	90397	89816	89004	87965	86712	85269	83665
GAAP ADJ SURPLUS	141804	155132	169913	186029	203422	222064	241967	263163	285698	309647

YEAR	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
PREMIUMS WRITTEN	240000	240000	240000	240000	240000	240000	240000	240000	240000	240000
LOSS RATIO (ACC.YR.)	79	79	79	79	79	79	79	79	79	79
INCURRED EXPENSE RATIO	30	30	30	30	30	30	30	30	30	30
PREMIUMS EARNED	240000	240000	240000	240000	240000	240000	240000	240000	240000	240000
PREMIUMS COLLECTED	240000	240000	240000	240000	240000	240000	240000	240000	240000	240000
EXPENSE INCURRED	72000	72000	72000	72000	72000	72000	72000	72000	72000	72000
EXPENSE PAID	72000	72000	72000	72000	72000	72000	72000	72000	72000	72000
LOSS INCURRED	190500	190500	190500	190500	190500	190500	190500	190500	190500	190500
LOSS PAID	190500	190500	190500	190500	190500	190500	190500	190500	190500	190500
UNDER. PROFIT	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500
UNDER. CASH FLOW	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500	¯22500
INVESTMENT INCOME	59506	62052	64758	67636	70696	73949	77407	81084	84994	89151
TOTAL INCOME	37006	39552	42258	45136	48196	51449	54907	58584	62494	66651
TAXABLE INCOME	25105	27141	29307	31609	34057	36659	39426	42367	45495	48820
INCOME AFTER LCF OR PLR	25105	27141	29307	31609	34057	36659	39426	42367	45495	48820
TAX PAID	11548	12485	13481	14540	15666	16863	18136	19489	20928	22457
DIVIDENDS	0	0	0	0	0	0	0	0	0	0
ENDING INVESTED ASSETS	631767	658834	687611	718207	750737	785322	822094	861189	902755	946948
ENDING SURPLUS	318155	345221	373999	404595	437124	471710	508481	547577	589143	633336
DISCOUNTED SURPLUS	81929	80090	78167	76182	74151	72088	70006	67918	65832	63757
GAAP ADJ SURPLUS	335105	362171	390949	421545	454074	488660	525431	564527	606093	650286

Title: INTERACTION OF TOTAL RETURN PRICING AND ASSET MANAGEMENT IN A PROPERTY/CASUALTY COMPANY

Author: Owen M. Gleeson

Biography: Currently Assistant Vice President and Actuary at General Reinsurance Corporation; previously employed at USFG Corporation in a variety of capacities including Actuary in Corporate Actuarial Department and Vice President -- Financial Planning. Member of the Casualty Actuarial Society (ACAS-1975; FCAS-1978) and American Academy of Actuaries (1978). Education: BA, MS and PhD in Mathematics - St. Louis University.

Abstract: This paper proposes to show that it is not possible for a property-casualty company to price on a total return basis and achieve the targeted return without the aid of a detailed flow of funds statement. It is demonstrated that, for a company to achieve a targeted total rate of return, it is imperative that a company position itself so that funds can be invested at the assumed rate. As part of the demonstration an example of a hypothetical company is presented and the flow of funds constructed and analyzed. Projections of proforma statements of sources and uses of funds are employed to show that a company so situated must pay for "old losses" with "new money." Data is presented suggesting that many companies in the industry are positioned in a similar fashion. The cause of the problem is identified and a tentative solution offered. Additional data is compiled from Annual Statements of a sample of companies indicating that assets maturing in a given year are insufficient to meet current payments on losses incurred in prior years. Finally, solutions to some of the problems in the area of planning and forecasting are suggested.

1. Introduction

Changes in the economic structure of the United States over the past two decades have had significant impact on all segments of the financial services industry. Some segments have been quicker to react than others and, even within a given segment, such as the property-casualty insurance business, the response to the changing conditions has shown a wide variation.

In order to operate in an environment characterized by sharp changes it is necessary that the management of an enterprise be able to react to the unpredictable events. One such event is the sharp fluctuations in investment rates. Effective response to any one of a range of events requires a degree of sophistication in planning that is unprecedented in the industry. Of course part of the planning process is the objective analysis of the current financial condition of the company and the identification of the opportunities and constraints.

The intention of this note is to examine the problems that can occur in the situation in which interest rates move up for a number of years and in which the maturity of assets significantly exceeds the maturity of liabilities. Foremost among the problems is the lack of flexibility to respond to changes, and in particular the difficulty in implementing a strategy of pricing on a total return basis. It will be seen that a program such as this can only be effectively carried out when the planning process involves the functions of pricing, planning and investment. In addition to the discipline and coordination that is required it is also necessary that the proper tools be available to analyze the current situation and to control the process of implementing the operational plan. The tool that will be examined here is the Statement of Sources and Uses of Funds both on actual and proforma bases.

It is conventional to construct Statements of Sources and Uses of Funds by starting with accounting statements, the income statements and the balance sheet, and construct the flow of funds statement through a series of adjustments. This can be a cumbersome process and will not be employed in this presentation. In the highly simplified example that follows, it is very easy to calculate the inflow and outflow of funds directly and for this reason the exposition will dispense with the distracting intermediate steps.

The example constructed here hypothesizes a highly idealized economic scenario rather than drawing on past data. Motivation for this approach stems from a desire to focus on the structure of the situation and to keep details simple enough so that the reader does not become bogged down in nuances that are beside the point. It will also obviate the need for parenthetical explanations of random and nonrandom deviations from the norms in actual economic events.

Of course no business operates in the world of contrived examples but rather in world in which deviations from the expected are to the anticipated. Therefore at the conclusion of this paper some space will be devoted to examination of the range of results that might obtain and in particular, how the actuary can play a significant role in quantifying risks associated with given strategies.

In spite of the fact that the following analysis focuses on a situation in which a company is using total return pricing in determining strategy, this focus should not be construed as necessarily recommending this methodology. Nor should the problems that will be brought to light be interpreted as arguments against the use of this technique. Approaches to the conduct of the business of a given company will depend upon its situation and circumstances as well as the attitude of management towards required results.

2. Financial Profile of Hypothetical Company

As indicated in the introduction, the example which will be constructed will be very simple so that the model does not become enmeshed in unnecessary detail.

A. History of Premiums and Losses

Year	Written Premiums	Earned(1) Premiums	Incurred Losses(2)
1980	140,000		
1981	150,000	145,000	116,000
1982	160,000	155,000	124,000
1983	170,000	165,000	132,000
1984	180,000	175,000	140,000

(1) Earned Premiums = (1/2)(Prior year's written premiums) + (1/2)(Current year's written premiums).

(2) The ratio for losses and loss adjustment expense is assumed to be 80% in each year.

B. Assumptions

1. Payout on Incurred Losses

The payout rate on accident year losses is given by the schedule:

Calendar Year	Percent
Current	40%
1st Following	30%
2nd Following	15%
3rd Following	10%
4th Following	5%

2. Expense Components

Category	Percent
Commission	15%
Premium Tax	3%
General Expense	8%

3. Agent's Balances

The assumption made here is that there is a delay of about 36 days i.e., 1/10 of a year in the remittance of premiums by agents to the company.

4. Dividends

The companys anticipates paying dividends to stockholders in the amount of $5,000 during 1985.

C. Balance Sheet

To the history and assumptions above is appended the additional supposition that the ratio of premium to surplus at year end 1984 is 3 to 1. The following simplified balance sheet results.

Balance Sheet

Assets		Liabilities	
Bonds	$282,700	Loss Reserves	$148,000 (2)
Agents Balances	15,300 (1)	Unearned Premium	90,000 (3)
	$298,000	Surplus	60,000
			$298,000

(1) 15,300 = (1/10)(1 - .15)(180,000)

(2) 148,000 = (.60)(140,000)+(.30)(132,000)+(.15)(124,000)+ (.05)(116,000)

(3) 90,000 = ½(180,000)

D. Asset Maturity Schedule

The following maturity schedule has not been constructed in a way that is intended to mirror a profile of the industry or any company within the industry. Varying levels of cash flow, widely fluctuating interest rates and other economic events over the recent past have been such that the yield by duration of an actual company would not show the smoothness of progression presented in the example. The only aspect of the maturity schedule which is similar to that of the industry is that the average maturity of the assets is greater than the average maturity of liabilities.

Year	Amount	Yield Rate	Yield
1985	28,270	.0533	1,507
1986	28,270	.0667	1,886
1987	28,270	.0711	2,010
1988	28,270	.0733	2,072
1989	28,270	.0747	2,112
1990	28,270	.0756	2,137
1991	28,270	.0762	2,154
1992	28,270	.0767	2,168
1993	28,270	.0770	2,177
1994	28,270	.0773	2,185
	282,700		20,408

The invested assets are assumed to be bonds and it is further assumed that the bonds are carried at par value. As a result if the company chooses to liquidate the bonds in a period of high interest rates it will suffer an accounting loss and a decrement to its statutory surplus.

3. Planning in a Vacuum

Now suppose that the company embarks on its planning process for the upcoming year (1985) and finds that the current and projected rates for year range from 8 to 12 percent depending upon maturity of the assets. Its targeted return on equity is 15%. Another factor entering into the analysis are that the company wishes to continue to write at approximately a 3:1 ratio. The company does not feel constrained to selecting early maturities and hence assumes that funds can be invested at the maximum rate of 12%. It has adopted a philosophy of total return pricing and so seeks the loss ratio that will result in the 15% return on equity. The analysis that follows is, by itself, neither unusual nor unreasonable.

Losses will be paid out as indicated in Section 2 and the funds invested at 12% so that the discount factor is .8414. This naturally is obtained as: $(.40)/(1.12)^{1/2} + (.30)/(1.12)^{3/2} + (.15)/(1.12)^{5/2}$

$$+ (.10)/(1.12)^{7/2} + (.05)/(1.12)^{9/2} = .8414$$

The ratio of premiums to surplus for the company is 3 to 1 so that the profit as related to surplus is 3 times that of the profit margin in each dollar of premium. In addition to the income from premiums, the company is earning investment income on the assets equal to the surplus. The rate of return on these assets will be assumed to be the same as that of the portfolio as a whole. Therefore, the target loss ratio is the solution to the equation $[1-(.8414 \times LR + .26)] \times 3 + .0722 = .15$ and is 84.87% which will be rounded to 85%. At this point, it will be assumed that the company is satisfied with a target combined of 111% which will produce an economic return on equity of 15% although both statutory accounting and GAAP results will be poorer than this due to the lag in the earning of investment income and that fact that loss reserves are not discounted.

The question that will now be investigated is whether this target can be achieved given the constraints and financial condition of the company. To answer this it is necessary to turn to a proforma statement of sources and applications of funds for the year 1985.

4. Sources and Applications of Funds

With the exception of newly formed companies, each property-casualty company is, to a certain extent, constrained by the past in its ability to react to new situations. The most significant constraints in the context of the problem under discussion is the requirement to pay losses in the current and future years which are the result of coverages issued in past years and the results of the investment policies of those years. Because of this it is necessary to start with a study of the application of funds. It is assumed that the company writes premiums in the amount of 190,000 in 1985 so that the earned premium figure is 185,000. The incurred losses for 1985 will be $(\frac{1}{2})(180{,}000)(.80) + (\frac{1}{2})(190{,}000)(.85) = 152{,}750$. Application of funds is calculated as follows:

Paid Losses	141,100 (1)
Paid Expenses	20,900 (2)
Dividends	5,000
	167,000

(1) $141{,}100 = (.40)(152{,}750) + (.30)(140{,}000) + (.15)(132{,}000) + (.10)(124{,}000) + (.05)(116{,}000)$

(2) $20{,}900 = (.11)(190{,}000)$

Note that commissions are not included in paid expenses but that this will not cause a problem in that the inflow of paid premium will be net of commissions.

Sources of funds include not only those from continuing operations but also from maturing assets. These will be projected separately then totaled.

Inflow from Operations:

Paid Premiums	\$160,650	(1)
Investment Income	22,309	(2)
	\$182,959	

(1) 160,650 = 15,300 + (9/10)(.85)(190,000)

(2) 22,309 = 20,408 - (1/2)(.0533)(28,270) + (1/2)(.12)(28,270) + 1/2(15,959)(.12)

When funds from maturing assets are added to the inflow/outflow difference the total funds available for investment at 12% amounts to 44,229 = 182,959 - 167,000 + 28,270. It is assumed here that the company will not liquidate bonds maturing in 1986 and following because of the fact that they were purchased in a time when interest rates were lower than the current level and hence sale at market would cause a decrement to surplus.

It can now be seen that, given this last listed constraint and the prior history of the company, it is not possible to implement the strategy of writing at a combined ratio of 111% and obtaining at 15% return on equity. The reason for this of course is that for the strategy to be successful an amount equal to the losses incurred on the premiums written in 1985 must be invested at the new money rate of 12% as indicated in Section 3. But this amount is 157,250 and the funds available for investment at 12% total only 44,229 for a shortfall of 113,021.

An estimate of the difference between statement and market value is developed in Appendix A. This Appendix also presents some evidence of the magnitude of the decline in bond prices over the last 25 years and the industry condition with respect to difference in the market/statement values of bonds.

Appendix B is devoted to the study of a sample of property-casualty companies with a view towards determining the relation between maturing assets and demands on funds resulting from prior commitments.

5. Analysis of Achieved Results

The funds assigned to the 1985 losses can be segregated into two portions, first the funds newly invested at 12% and secondly funds from the incoming portfolio. As an expedient it will be assumed that the rate on the required additional funds will be that of the average of the portfolio, that is 7.43% = (20,408 - 1,507)/(282,700 - 28,270). This gives a weighted rate of the portfolio supporting the losses incurred in 1985 of approximately 8.71% where [(44,229)(.12) + (.0743)(113,021)]/157,250 = 8.71%.

The discount factor using this rate of return is .879 so that the return on premiums written during 1985 is actually 1 - [(.879)(.85) + .26] = 1 - 1.0072 = -.0072, that is a slight loss rather than the anticipated gain. This results in a return on equity of approximately 5.1% rather than the originally targeted 15%.

The company would not only fail to achieve the targeted rate of return on equity but may not even obtain a sufficient addition to surplus to maintain a 3 to 1 ratio. In this example the harm is not great as shown in the following analysis.

Surplus (12/31/84):	60,000
Income Statement	
Earned Premiums	185,000
Incurred Losses	152,750
Expenses	49,400
Underwriting Gain	-17,150
Investment Income	22,309
Net Income	5,159
Dividends	5,000
Surplus (12/31/85)	60,159

Premium to Surplus Ratio = 190,000/60,159 = 3.158

Although the company only increases its premium to surplus ratio slightly and no real problem is generated, it would be easy to construct a situation in which the resulting decline in surplus came as a great shock to the company and caused real difficulties. Unpleasant surprises should not befall companies with access to a reasonably accurate financial planning model -- other than those resulting from overly optimistic assumptions.

6. Solutions

There are many actions that a company might take in the effort to avoid the pitfalls demonstrated in the previous example. Only two will be suggested here -- one dealing with revision of asset management and the other dealing with a more effective use of a sources and applications of funds statement.

The problem presented here resulted from a mismatch of liabilities and assets combined with a shift to higher rates. Volatility of interest rates has now become a fact of life and it is incumbent on those charged with the successful conduct of the affairs of a company to recognize the need to accomodate this aspect of the economic environment in the planning. In order to respond to challenges and changes a company must maintain a significant degree of flexibility and this includes flexibility in the assets that it manages. It is tempting to reach for the last few basis points by going further out on the yield curve but, as can be seen from this example, this causes a loss of ability to respond to changing conditions and to take advantage of new opportunities.

One observation that should be made is that, all things being equal, newly capitalized companies have an advantage over older companies with ill-positioned assets. The latter are forced to make the choice between, on the one hand, realizing losses by selling assets resulting in a weakening of the statutory balance sheet and, on the other hand, paying for "old losses" with "new money." This situation will allow the new companies to be more competitive and still receive an acceptable economic return. However, in the case where the older companies have followed a program of matching assets and liabilities this threat should not be a concern.

Another action that the company in this example might have taken centers on the use of a more realistic interest rate. In this situation the company was positioned in such a way that it could not invest the new money fully at the 12% rate. The analysis indicated that rate available for the assets to be matched with the losses incurred in 1985 was 8.71%. This information could be used as input into the pricing policy to determine that target loss ratio. As seen previously the discount factor was .879 so that the target loss ratio could be determined from the equation

$$[1 - (.879 \times LR + .26)] \times 3 + .0722 = .15$$

The solution is LR = .812 which is rounded to 81% with a combined ratio of 107%.

7. The Actuary's Role

Important items of the analysis is the example include the level of loss reserves and the payout on losses. Both of these have been used without reference to the variation experienced in each. The actuary is particularly well suited to provide management with estimates of the variation in these elements and thereby play a vital role in analyzing the current position of the company, its constaints and its range of opportunities.

Pricing also is substantially within the purview of the actuary and when a company prices on the base of total return it is absolutely essential that the company have not only a good estimate of the ultimate cost but also reasonably accurate projections of the cash flows associated with premiums, losses, expenses and investment income. For many actuaries this is already part and parcel of their work and for others an extension of their current functions.

These are but two of the obvious applications of the actuary's expertise in the areas of implementing the use of proforma sources and application of funds statements and total return pricing. In addition the actuary is particularly well suited to the assessment of risk associated with various strategies and to the application of optimization techniques to determine the best strategies.

8. Summary and Conclusion

The example presented in this paper has been shown to be representative of many companies in the insurance industry -- at least in the aspect of the mismatch between liabilities and assets. Difficulties of planning and strategy determination have been explored with particular reference to total return pricing. It has been shown that, to put it very simply, a company cannot fully take advantage of high interest rates and apply these to its pricing if the funds generated cannot be entirely invested at the prevailing rates. Better analysis and planning can be achieved through the use of proforma statements of the source and application of funds.

In a rapidly changing environment a company needs to remain flexible. Deployment of assets in such a way that reduces the range of use of these assets can be detrimental to a company. In particular the purchase of bonds with extremely distant maturities in order to take advantage of the additional yield commits a company to holding these securities to maturity in the event that interest rates rise and under the circumstances where a company cannot afford the reduction in stated worth resulting from sale. This limited study would suggest that companies should invest in bonds with shorter maturities. It is still an open question as to whether pure asset/liability matching is necessary.

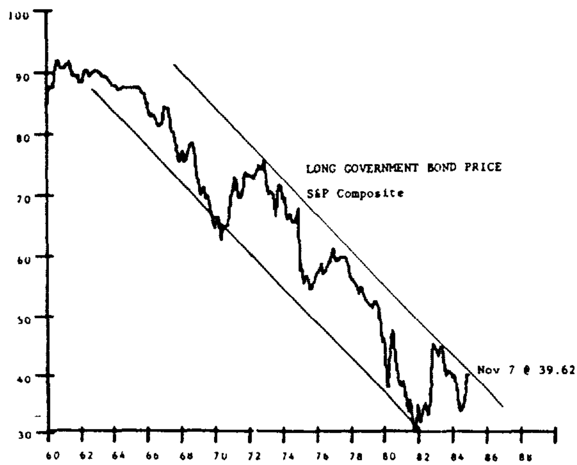
There are many who feel that the long uptrend in interest rates is in the process of being reversed. This does not mean that the concepts explored in this paper will not apply in this event. A substantial portion of the bonds owned by property casualty insurance companies were purchased at a time when rates were extremely low by today's standards. A cessation of increase in interest rates will not provide an immediate solution to the problem of bonds with statement values in excess of market values. Nor will a small decrease in the level of interest rates. Therefore, if companies are constrained not to sell bonds at a loss because of the resulting capacity problem, they should find proforma statements of sources and applications of funds of vital use in planning -- at least for the next few years.

Finally, it should be clear that the planning, pricing and investment functions cannot operate independently in the current environment if a company hopes to achieve satisfactory results. A process of coordination and control must be introduced that brings together the different skills so that the parties involved not only have a thorough understanding of the impact of their actions on the company but also so that they work together towards objectives which are mutually consistent.

Appendix A - Book/Market Differential in Bond Holdings

The company is holding a portfolio of bonds on which the coupon yield is less than that currently available according to the hypothesized economic scenario. The long general decline in the bond market over the past 25 years resulting from the increase in interest rates is well known and graphically illustrated in Figure 1.

Figure 1



This coupled with the industry practice of investing long although the liabilities are fairly short has led to a condition in the industry where the market values of bonds are substantially below the statement value. For a number of years First Boston Corporation has issued a study on this important aspect of property casualty insurers' financial condition. Part of the 1984 study which exemplifies the magnitude of the problem is reproduced on the following page.

Table 1

Property-Casualty Insurance Operations
Statutory Surplus Adjusted For Unrealized Bond Losses
The Relationship of Writings to Surplus, 12/31/83
($ in millions)

	1983 Premiums Written (1)	12/31/83 Statutory Surplus (2)	Property-Casualty Bond Portfolio 12/31/83 Book (3)	Market (4)	Unrealized Losses (5)	12/31/83 "Adjusted" Surplus (Column 2 Less 72% of Col. 5) (6)	Premiums Written to Policyholders' Surplus (Column 1 Div. by Col. 2) (7)	(Column 1 Div. by Col. 6) (8)
Aetna Life & Casualty	$ 4,416	$1,645	$6,158	$4,985	$1,173	$800	2.7x	5.5x
Chubb	1,241	401	1,376	1,211	165	282	3.1	4.4
CIGNA	3,535	1,314	5,161	4,333	828	718	2.7	4.9
Continental	2,401	936	3,311	2,841	470	598	2.6	4.0
General Re	902	836	2,462	2,180	282	633	1.1	1.4
Kemper	858	354	853	718	135	257	2.4	3.3
Ohio Casualty	843	426	994	793	201	281	2.0	3.0
Progressive	245	95	234	232	2	93	2.6	2.6
SAFECO	888	494	928	916	12	485	1.8	1.8
St. Paul	1,744	783	2,785	2,457	328	547	2.2	3.2
Travelers	2,945	1,057	4,708	3,587	1,121	250	2.8	11.8
USF&G	1,989	779	2,151	1,993	158	665	2.6	3.0
Total	$22,007	$9,117				$5,608		
Weighted Average							2.4x	3.9x

Source: The First Boston Corporation

It has been asserted earlier that the hypothetical company would suffer a decrement to surplus if forced to sell bonds. As is the case with other sections of this highly idealized situation, determination of the amount will depend on assumptions and methods which greatly simplify the situation. However, the results are acceptably accurate for the purposes of example and planning. The redemption value of the bond and the statement value are assumed to be equal with the statement value having been given earlier in the Asset Maturity Schedule. Yields also have been given so that all that is needed for a rough estimate of the market value of the bonds is the currently prevailing interest rates by maturity. The data is given on the following page.

Years to Maturity	Redemption Value	Yield	Current Rate
1	28,270	1,507	8.00%
2	28,270	1,886	10.00%
3	28,270	2,010	10.67%
4	28,270	2,072	11.00%
5	28,270	2,112	11.20%
6	28,270	2,137	11.33%
7	28,270	2,154	11.43%
8	28,270	2,168	11.50%
9	28,270	2,177	11.56%
10	28,270	2,185	11.60%

The method used here to estimate the market value of the bonds is the yield-to-maturity method. Then the present value of the bond -- all things being equal -- is given by the formula,

Present Value = (Redemption Value)/$(1 + r)^n$ + (Annual Yield) $a_{\overline{n|}\, r}$

where r is the yield to maturity, n represents the number of years to maturity and the coupons are assumed to be paid annually. For example, the bonds maturing in 5 years have a present value of $(28{,}270)/(1.112)^5 + (2{,}112)\ a_{\overline{5|}\ .1112}$ which equals 16,626 + 7,767 = 24,393 so that sale of the bonds would result in a decrease in surplus of $3,877. The results of this method of estimating the market value of the bonds versus the statement value of the bonds is given on the following page.

Year of Maturity	Statement Value	Market Value	Discount
1985	28,270	27,571	699
1986	28,270	26,637	1,633
1987	28,270	25,796	2,474
1988	28,270	25,051	3,219
1989	28,270	24,393	3,877
1990	28,270	23,803	4,467
1991	28,270	23,264	5,006
1992	28,270	22,794	5,476
1993	28,270	22,358	5,912
1994	28,270	21,984	6,286
	282,700	243,651	39,049

The stated surplus of the company under study is 60,000 with premiums writings of 180,000 for a 3 to 1 ratio. If the surplus is adjusted to reflect market value of bonds, the adjusted surplus is 20,951 with a resultant ratio of approximately 8.59 to 1.

This ratio is considerably higher than most of the ratios in Column (8) of the table reproduced from the study conducted by David Seifer. The difference is due in part to methodology. The reader should note that the "Adjusted Surplus" in the study is obtained by subtracting 72% of the unrealized bond losses from the stated surplus thereby reflecting the potential impact of capital gains tax. If the unrealized losses had not been reduced by 28% the ratios of premiums to adjusted surplus would have been substantially higher with the weighted average rising to 5.2 from 3.9 as calculated in the table.

Use of yield to maturity has some drawbacks that have been covered in the book "Inside the Yield Book" by Sidney Homer and Martin Leibowitz. However, the technique produces a useful approximation which can be of great value in assessing the projected condition of a company under a variety of scenarios associated with a range of economic events and strategies.

Appendix B - Maturing Assets versus Paid Losses from Prior Years: An Industry Sample

The example constructed in this paper presents a picture of a company with maturing assets far lower than those needed to meet the obligations of the past. Specifically, the losses from 1984 and prior that are projected to be paid in 1985 amount to 80,000 while the maturing assets show only 28,270. This results in a ratio of paid losses from prior years to funds from maturing assets of about 2.8 to 1.

The reader might ask whether this is a strawman or whether this is somehow representative of industry conditions. Some data has been drawn from the Annual Statements of eight companies to examine this question. The companies were selected randomly and include not only stock and mutual companies but companies of varying size. Column (1) is the amount of losses and loss adjustment expense paid in 1983 from accident years 1982 and prior. The figures were compiled using Schedules O and P from the 1982 and 1983 Annual Statements. Column (2) is the amount of funds available to the companies from assets held at year end 1982 and maturing in 1983. The data was obtained using Schedule D-Part 1A and Lines 6.1 and 6.2 of Page 2(Assets) -- Cash and Short-term Investments, respectively.

Company	(1) Paid in 1983 * on 1982 and prior	(2) Assets Maturing* in 1983	(3) (1)/(2)	(4) (2)/(1)
A	1,024,315	239,396	4.28	.234
B	672,908	246,958	2.73	.367
C	644,214	210,042	3.07	.326
D	409,654	120,245	3.41	.294
E	348,573	44,276	7.87	.127
F	210,204	28,003	7.51	.133
G	10,410	3,027	3.44	.291
H	3,522	3,250	1.08	.923
	3,323,800	895,197	3.71	.269(w)
			4.17	.240(u)

*Figures in thousands
(w) - weighted
(u) - unweighted

This exhibit is certainly not to be construed as representing an exhaustive study of industry conditions but rather to show that the hypothetical example is not totally at odds with prevailing conditions. As a matter of fact, if the exhibit is truly representative, then the hypothetical company is in somewhat better shape -- at least in terms of the ability to pay for old losses with maturing assets.

One interpretation of Column (4) is that the sample companies are only able to cover about 27% of obligations from prior years through use of maturing assets. This means that 73% of the obligations coming due must be met through the use of new funds. The figures for the hypothetical company are 35% and 65% respectively.

AN ECONOMETRIC MODEL OF PRIVATE PASSENGER LIABILITY UNDERWRITING RESULTS

Richard M. Jaeger and Christopher J. Wachter

Richard Jaeger is currently Actuarial Director in the Actuarial Development Department of Insurance Services Office. He is a Fellow in the Casualty Actuarial Society and a Member of the American Academy of Actuaries. He is serving on the CAS Syllabus and Examination Committees. He holds a BA degree in Mathematics from Boston College and an MS degree in Mathematics from the University of Notre Dame.

Christopher Wachter is currently Senior Actuarial Assistant in the Actuarial Development Department of Insurance Services Offices. His responsibilities include econometric modeling. He received his BS degree in Actuarial Science from Lebanon Valley College in 1983 and is currently pursuing associateship.

This paper presents an econometric model of private passenger liability underwriting results. The model, fitted on data from 1954 to 1983, is used to forecast results from 1984, 1985 and 1986. Premiums, losses, and expenses are modelled separately, with the loss model based on two sub-models (severity and traffic accidents). The paper covers the process of model building from initial *a priori* analysis, through forecasting. The paper also attempts to provide a general framework useful in the modeling of other lines.

PURPOSE

Our objective in undertaking the research presented here was to forecast industry combined ratios for private passenger automobile liability. While actuaries have always been concerned with trending, projecting and forecasting there is little in the actuarial literature on forecasting industry results. Some of the papers in the actuarial literature (Alff and Nikstad, James, Lommele and Sturgis) are listed in the Bibliography of this paper. Although such forecasting may be several steps removed from the actuary's day-to-day work, senior executives, insurance regulators and financial analysts are all interested in what the results will be. The actuary has the training and experience to help. A secondary objective of this paper is to indicate a general approach that can be used to model other lines of insurance as well.

ORGANIZATION OF THE PAPER

This paper follows a chronological format showing the development of the model from initial *a priori* analysis through forecasting. The presented models, therefore, appear after the section on the model fitting process and before the section on *a posteriori* analysis. The paper contains two appendices and a brief bibliography. The appendices contain a glossary of useful econometric terms, a list of data sources, and graphs displaying the fit of the presented models.

A PRIORI ANALYSIS

The importance of *a priori* analysis cannot be overemphasized. In an ideal, perfectly efficient world the researcher will analyze the situation to be modeled or forecasted, determine the relevant forces, select the appropriate variables, specify the form of the mathematical relationship, indicate the signs (and perhaps the magnitudes) of each variable in the relationship, and only then test the hypothesis against the data. In the real world one tries to follow this approach while coping with disappointing test results, new ideas that come to mind after the first results, and the nagging question - where can the model be improved?

It is important to use the data to test the a priori hypothesis, rather than to search for a model which fits the data well, and then derive an a posteriori "hypothesis" from the model. We want assurance that it is a good model, not just that a good fit results from much trial and error. We, of course, want the model to fit well in addition to agreeing with the a priori hypothesis.

This is especially important when there is limited data. Everyone is familiar with the inadvisability of explaining the variation in ten data points by using ten independent variables, or even six independent variables. The same effect can occur when the best-fitting model is chosen after testing too many sets of variables using several model forms, even if all of the variables and forms are reasonable.

There are ways to mitigate this problem. One way is to use part of the data for fitting and another part for testing. Any partition that is expected to yield the same model on the subsets could be used. Some possibilities are first and third quarters separate from second and fourth quarters, seasonally adjusted as necessary; a geographical partition, by state or region; and stock companies separate from mutual companies. Another way is ex post testing in which we try to forecast the latest points after fitting to the data excluding those points.

The first assurance, however, of having a good model comes from the model's agreement with _a priori_ analysis. _A priori_ analysis provides an intuitively reasonable explanation of the actual situation. Steps to improve the model should be governed by attempts to improve the _a priori_ analysis. This insures that the resultant model will be sound on a conceptual basis.

As stated earlier the purpose of the model is to forecast combined ratios. We decided early, however, not to model the combined ratios directly, but to model the losses and premiums separately and even to attempt to decompose losses and premiums into separate factors. We attempted to model losses as volume times frequency times severity and premiums as expected losses times a "pricing factor."

One reason for this approach is to reduce the problem to relatively bite-size pieces, each with a more manageable number of possible causal factors. Another reason is to make more efficient use of relatively few data points. Separating losses from premiums creates, in effect, twice as many points as using just the combined ratios.

The most important reason, however, for decomposition is to guide efforts to improve the model. If the premium model behaves better than the loss model, then attention can be directed to the loss model. If frequency is the loss factor showing the most unusual behavior, then frequency can be investigated before the other factors. A related reason is that it is easier to confirm whether a proposed improvement has the expected effect on the proper component.

One consideration in the _a priori_ analysis is that the model is primarily intended for forecasting rather than explanation of the changes in the historical period. The independent variables selected should be easily forecastable or already forecasted in a satisfactory way.

Losses

The initially selected form of decomposition for incurred losses was volume times frequency times severity. At this point there were two ways of proceeding. One way was to seek sources of standard insurance data for each element in the decomposition, such as, earned car-years for volume, and incurred or paid claim frequency for frequency. Each element could then be modeled separately. The other way was to build a precise decomposition from a reasonable starting point. An example of such a decomposition is to start with the number of registered vehicles (VRCAR) for volume and then, using the number of traffic accidents (TRAFACC), define frequency as TRAFACC/VRCAR and severity as incurred losses/TRAFACC. Both VRCAR and TRAFACC are forecasted by Data Resources, Inc. (DRI). Severity would have to be modeled.

The advantages of the first way arise from the fact that the elements in the decomposition are standard insurance concepts.

- Prior knowledge of these concepts can be applied directly to the analysis. If the elements have already been modeled, then much of the work is already done.
- If there are strong judgmental reasons to expect particular changes in the elements, these judgmental values can be used directly in the model to obtain a forecast.

The advantages of the second way are:

The decomposition is precise, that is, the product of the factors exactly equals the variable of interest. There is no need to adjust the product for such differences as absolute scale (because of using a subset of industry data) or paid rather than incurred data.

If there is a reasonable starting point that is already modeled or forecasted, then part of the work is already done.

We chose the second approach to decomposition because of the above advantages plus a very practical additional advantage. We wanted to have at least 20 years of data for modeling, so that several underwriting cycles and a variety of economic conditions would be present in the data. TRAFACC was available back to 1950 and VRCAR even farther back. Insurance time series for a decomposition would have been more difficult to obtain for a comparable length of time.

This second approach still leaves open the possibility of comparing elements of the decomposition to available insurance time series for reasonableness.

Early work with the decomposition based on VRCAR and TRAFACC led to the conclusion that TRAFACC showed very anomalous behavior, described more fully later in the paper.

Frequency

A number of factors were identified that might influence frequency. The principle of simplicity and the 80-20 rule were applied. (Keep it simple, and 80% of the effect comes from 20% of the causes.) Factors expected to have considerable effect were demographic shifts (notably changes in the proportion of young drivers), changes in the incidence of reporting traffic accidents (as repair costs go up while reporting thresholds tend to remain fixed), and energy crises (gas shortages). Selection of the first factor was influenced by work that had been done on other automobile insurance frequency data. The ratio of population aged 16-28 to the number of vehicle drivers licenses was selected to represent the demographic shift. A CPI-based measure of automobile repair costs, BODYWORK, was selected as a variable corresponding to the second factor. Eventually, it was decided to represent energy crises by the variable vehicle miles traveled (VMTCAR), and to recast the decomposition as losses equals accidents times severity, where TRAFACC is modeled as VMTCAR times some factor. This is equivalent to substituting VMTCAR for VRCAR in the initial loss decomposition.

Severity

Inflation should be the major force driving the loss severity model. Loss severity is a combination of bodily injury and property damage severities. To represent inflation we created an index that is a weighted sum of various CPI component indices expected to be related to automobile liability severity. The weights were judgmentally selected in the *a priori* stage, with the expectation that the exact weighting would not be critical.

Social inflation, as distinct from the purely economic underlying inflation, may also be a factor. We expected that social inflation would be reflected in the fitted quantitative relationship between severity and economic inflation, and therefore did not represent it by a separate variable.

Small cars are likely to provide less protection to occupants in an accident. They may also tend to be damaged more severely. To represent the proportion of small cars to total cars, we selected the ratio of foreign new car sales to total new cars sales. We realized, however, that a possible future refinement would use this ratio averaged over several years.

It is possible that the introduction and subsequent modification of no-fault laws affected severity. We did not expect a strong effect, however, and did not attempt to represent this factor in the model.

Premiums

Premiums may be influenced by a large number of factors. Some of the factors are exposure volume, inflation, recent loss experience, recent profitability, competition, supply and demand, capacity, investment yields and the future expected value of several of these factors. We finally selected several variables that represent reasonably distinct factors. The number of vehicle registrations was selected to represent exposure volume. The GNP deflator was selected to represent inflation. The incurred losses of the current year and the prior year were selected to represent the ratemaking process, recent profitability, and management's expectation of future losses. Real surplus (using the GNP deflator) was selected to represent supply and capacity and as a proxy for competition. These variables should be positively related to premiums, except for real surplus. A high real surplus should have a downward effect on premiums due to over-capacity and consequent increased competition.

We intended to model written premiums as above and to produce earned premiums by a simple linear model using the current year and prior year written premiums.

Expenses

The expense ratio should be inversely related to deflated written premium, since there are fixed expenses which do not vary with written premiums. We used written premiums deflated by the GNP deflator to model the expense ratio and obtained a reasonable model.

Model Form

The final stage in a priori analysis is selection of the model form. The selection of model form is significant, but is somewhat less important than the earlier stages of a priori analysis. We selected a logarithmic form for the written premium and severity models for the following reasons:

1. We expected a multiplicative relationship between the component independent variables.

2. The coefficients are elasticities, rather than absolute magnitudes. The effect of a 1% change in an independent variable is the coefficient times 1%. Thus, the relative contribution of each component variable can be easily determined.

3. Inflation-sensitive time-series are transformed from an exponential form to a linear form.

4. Heteroscedasticity is minimized, since inflation will not cause residuals to grow as large with time if a logarithmic form is used.

5. The fit is more robust, since outliers tend to have less of an effect on parameter estimation.

This completes the initial *a priori* analysis and prepares us for fitting and testing actual models.

THE MODEL FITTING PROCESS

The first stage in the model fitting process is the selection of data. Ideally, data should come from recognized and reliable sources, and be available for a significant length of time. We used insurance premium, loss, expense and surplus data from A.M. Best's *Aggregates and Averages*, and various economic time series from DRI's data banks.

The second stage in the model fitting process was to splice data together. Prior to 1974, for example, auto liability was not split into private passenger liability and commercial automobile liability. We applied a splicing factor of 0.89 to the pre-1974 automobile liability data (stock and mutual only) to extend backwards the 1974-83 private passenger liability data (stock, mutual and reciprocal). The splicing factor was based on the observed ratios from 1974 to 1983 which were very stable and averaged 0.89. The advantage to splicing is that we are able to use 30 years of data (1954-83) rather than only 10 years (1974-83).

The third step in our model fitting was to handle a problem variable, TRAFACC. TRAFACC is a time-series which measures reported traffic accidents. It is also a variable whose definition has changed over the historical period. Prior to 1968, the ratio of TRAFACC to highway fatalities is remarkably stable, indicating that TRAFACC for that period may have been defined by multiplying highway fatalities by a constant. We decided, therefore, to model TRAFACC over the period 1968 to 1983, and use the fitted values produced by the model over the full period 1954 to 1983 in place of the original TRAFACC series. We later decided, when a reasonable severity model had been fitted, to create a final series representing traffic accidents called TRAFACC', because of the problems noted above with TRAFACC. We felt that a reasonable model for severity would be better than incurred losses divided by fitted TRAFACC, and therefore would probably create a more accurate decomposition.

The fourth stage in the model fitting process was to decompose the incurred losses into severity and traffic accidents. We defined severity as incurred losses divided by fitted TRAFACC (described in the previous paragraph). We selected a deflator judgmentally, and chose a variable to proxy for small cars (the proportion of imported car sales to total car sales) to model severity. The resulting model fit reasonably well, and we therefore tentatively accepted the severity model.

The fifth stage in our model fitting process was to use the fitted values generated by the severity model to create TRAFACC'. We defined TRAFACC' as incurred losses divided by fitted severity. We modeled TRAFACC' using the same variables that we had used to model TRAFACC, and decided tentatively to accept the traffic accident model. Multiplying the two models (severity and TRAFACC') we were able to construct our incurred loss model.

After fitting the loss series, we fixed our attention on the premiums. We selected several variables based on our a priori analysis corresponding to incurred losses, capacity, volume, inflation, investment yields and recent profitability. We looked at the relationship between changes in written premiums and changes in potential explanatory variables at various lags to determine a reasonable lag structure. We determined, for example, that change in surplus lagged two years was related to change in written premiums more strongly than if it were lagged 0, 1, 3, or 4 years.

We fitted a preliminary model, and decided to discard some variables which did not appear significant (had t-statistics of less than 2). We fitted a second model, checked for and corrected for multicollinearity (the independent variables should behave independently of each other, and not show strong correlation), and tentatively accepted the premium model.

After tentatively accepting the premium and loss models, we tested the models to determine if they were acceptable. There were four final tests:

1. We determined that the final models agreed with our a priori analysis. Specifically, we checked the coefficient of every independent variable and confirmed that each coefficient had the expected sign and had reasonable magnitude.

2. We determined that the model's error was acceptable (standard errors of 2.3% for premiums and 2.9% for losses) and that each dependent variable was statistically significant at the 5% level (as determined by the t-statistic).

3. We performed an ex-post test. An ex-post test involves fitting the model over a shorter historical period (we used 1954-80) and then "forecasting" the latest values (1981-83) using actual values for the independent (input) variables. The ex-post forecast errors were deemed acceptable, and are summarized below:

EX-POST ERRORS*

Year	Premium Model	Loss Model
1981	-2.85%	+1.72%
1982	+1.92%	-0.17%
1983	+0.66%	+2.79%

* Error = (Actual-Forecasted)/Actual

4. We analyzed the residuals (errors) of the models for violations of least square assumptions.

Potential Violation	Correction needed (if any)
1. There are outliers.	A dummy variable was incorporated into the severity model for a 1974 outlier.
2. The residuals are correlated.	Autocorrelation corrections were implemented as deemed appropriate.
3. The variance of the residuals is not constant. (heteroscedasticity)	The residuals were examined, and no indication of heteroscedasticity was found.
4. The independent variables are strongly correlated. (multicollinearity)	Multicollieartty was found in an early premium model and corrected for. Whatever multicollinearity remains does not appear substantial based on the observed correlations between the independent variables.
5. The relationship between dependent and independent variables is unstable.	Ex-post testing shows stable parameter estimation. When the models were refitted over the period 1954-80, the parameters did not substantially change from the model fitted over 1954-83.

On the basis of these four tests, we decided to accept the premium and loss models.

THE MODELS

The private passenger liability models are as follows:

1. Written premiums

$$\text{Log }(NPW_t) = 0.664 \text{ Log }((A_t + A_{t-1})/2) + 1.315 \text{ Log }(VRCAR_t)$$

$$- .115 \text{ Log }(Surplus_{t-2}/PGNP_{t-2}) + .884 \text{ Log }(PGNP_t) + .573 \text{ AR1}$$

Where:

NPW	=	Net premiums written
VRCAR	=	Vehicle Registrations (in thousands)
PGNP	=	GNP deflator (1972 = 1.000)
Surplus	=	Surplus (in thousands)
A	=	Incurred Losses/ (VRCAR x PGNP)
AR1	=	Autoregressive term of order 1.

t-statistics are 6.77, 18.08, -3.02, 27.24 and 3.27, respectively, for the five coefficients.

The model has a normalized standard error of 2.3%.

2. Incurred Losses

Incurred Losses equal TRAFACC' times severity

$$TRAFACC' = TRAFACC_{78} \cdot VMTCAR_t \left(.4575 + .4933 \frac{YOUTH_t}{VDL_t} + .0812\ BODYWORK_t\right)$$

Where:

$TRAFACC_{78}$ = Number of traffic accidents in 1978 (original TRAFACC series)

VMTCAR = Vehicle miles traveled by cars

YOUTH = Population aged 16-28

VDL = Vehicle Drivers Licenses

BODYWORK = CPI for auto bodywork (prior to 1978, CPI for auto repair and maintenance)

Note: all variables are normalized to 1.000 in 1978 except for TRAFACC' and $TRAFACC_{78}$.

t-statistics are 3.131, 3.301 and 5.301 for the three coefficients.

$$\text{Log}(SEVERITY_t) = .7321\ \text{Log}(DEFLATOR_t) + .1008\ \text{Log}(PROP_t) - .0955\ Dummy_t + 6.3108 + .9419\ AR1$$

Where:

DEFLATOR = .35 MEDCARE + .35 WAGE + .20 BODYWORK + .10 PC

MEDCARE = .67 CPIU for Hospitals (CPIU for Hospital Rooms before 1978) + .28 CPIU for Physician's Services + .05 CPIU for Medical Commodities. (All components indexed to 1.000 in 1978)

WAGE = Average Hourly Earning Index for Production Workers (1978 = 1.000)

PC = Implicit Price Deflator for Personal Consumption Expenditures (1978 = 1.000)

PROP = Foreign New Car Sales/Total New Car Sales

DUMMY = 1 in 1974, 0 otherwise

t-statistics are 8.96, 3.94, - 3.70, 64.34 and 12.20 for the five coefficients.

The incurred loss model has a normalized standard error of 2.9%.

3. <u>Expense Ratio and Earned Premium</u>

$$EP_t = .668\ WP_t + .336\ WP_{t-1}$$

$$\text{Log}(ER_t) = 2.458 - .240\ \text{Log}(NPW_t/PGNP_t) + 1.096AR1 - .690AR2$$

A POSTERIORI ANALYSIS

After the model fitting stage is completed, the a posteriori analysis stage begins. The function of a posteriori analysis is to examine the accepted model and attempt to explain any unusual features of the model. This is useful, because possible refinements to the model are identified for future research. It is important to realize that model building is an on-going process and that models should be monitored and updated as additional data becomes available.

We have identified three unusual features in our model:

1. The elasticity of VRCAR, a volume measure, is greater than one in the premium model.

2. The elasticity of DEFLATOR, an inflation measure, is less than one in the loss model.

3. The lag structure of the incurred losses in the premium model is shorter than might be expected.

Possible explanations for these features (which represent deviations from what might be expected) are:

1. A rise in the proportion of insured vehicles over the historical period (1954-83) would impact the elasticity of VRCAR.

2. The "real" severity may be declining somewhat due to safer automobiles and roads.

3. The incurred loss term may combine two separate components: expected losses (IL_t) and "fast-track" experience (IL_{t-1}). If this is, in fact, the case, a longer distributed lag structure (using IL_{t-2} and even further back) for the "expected loss" component may be more appropriate.

This _a posteriori_ analysis could serve as an input to the _a priori_ analysis stage of future model-building efforts. We feel that the present models are sound and useful for forecasting, but that the _a posteriori_ analysis indicates some areas for future research.

FORECASTS

After the analysis, model fitting and testing, the model can be used. The model has two main applications: explanation and forecasting. To the extent the model explains the mechanisms underlying industry written premiums and incurred losses, alternative "what-if" scenarios can be devised and forecasts made for these scenarios. Three potentially interesting scenarios are:

1. The banks enter the insurance industry injecting significant amounts of capital.

2. The campaign against drunk driving significantly reduces accident frequency.

3. Inflation surges upward again.

The actual scenario design and forecasts based on alternative scenarios are beyond the scope of this paper. Our forecasts, based on insurance data through 1983 and on DRI control scenario forecasts (using the July 1984 forecast) are as follows:

	1984	1985	1986
Written Premiums	24,763,224	26,048,890	28,039,674
Earned Premiums	24,391,568	25,721,102	27,482,929
Incurred Losses	21,080,953	22,606,582	24,082,622
Loss Ratio	0.864	0.875	0.876
Expense Ratio	0.246	0.239	0.233
Dividend Ratio (selected)	0.01	0.01	0.01
Combined Ratio	1.120	1.124	1.119

FUTURE RESEARCH

The research to develop these models has raised some questions for further investigation. Potential topics for research include:

1. Incorporating investment yield into the current premium model. There are many investment yield statistics, and also a variety of time frames to select (current yield, recent yield, expected yield, and embedded yield). In addition, it is possible that investment yield may be significant over a small subset of the historical period.

2. Incorporating changes in the proportion of insured motorists in the total driving population. The chief problem is to locate a source of data over the historical period.

3. Incorporating a variable representing increased safety of roads and automobiles. The major task is to find a valid time series which can proxy

for automobile safety, and which is available over the entire historical period.

4. Selecting other measures to proxy for industry price competition.

The authors will, as time permits, research these areas further, and would welcome the insights, suggestions and research of other people involved in this area of actuarial/econometric research.

APPENDIX A - GLOSSARY

Autocorrelation: The correlation between residuals and the residuals lagged a certain number of periods, called the order. An assumption of least squares regression is that autocorrelation is not present.

Autoregressive Term: A term (in a model equation) used to correct for autocorrelation when the analysis of the residuals of a model indicate the presence of autocorrelation. In this paper ARi denotes an autoregressive term of order i.

Decomposition: The breaking of a problem into smaller, more easily handled problems. The solutions to the small problems are combined to form a solution to the overall problem.

Dummy Variable: A variable that takes on two values, 0 and 1. The dummy variable is used to account for abnormal real world conditions (energy crises, price controls, wars, etc.) or to remove the effects of obvious outliers.

Elasticity: A measure of the relationship between two variables.

Heteroscedasticity: Heteroscedasticity exists when the variance of model's residuals is not constant over the entire range of data. Least squares regression assumes heteroscedasticity does not exist.

Lag: The length of time between the effect on an independent variable and the effect on the dependent variable. If several lags of an independent variable are combined into are term, we say that the term represents a distributed lag structure.

Multicollinearity: The degree of correlation between the independent variables. Least squares regression assumes that the independent variables are independent of each other.

Normalized Standard Error: The standard deviation of the error of a model expressed as a proportion of the dependent variable.

Outlier: A data point that is questionable due to an abnormally large deviation from its expected value. Outliers bias regression results, sometimes quite substantially.

Proxy: A variable used as a measure for something that is not readily quantifiable.

Residual: The difference between an actual observation and the expected value of that observation based upon model.

Robustness: The degree to which a model is stable and unresponsive to outliers.

Splicing: The combination of two similar time series covering differing time periods into a unified series.

T-statistic: The ratio of a coefficient to the standard error of that coefficient. Generally a T-statistic with absolute value greater than 2 indicates a significant relationship between an independent variable and the dependent variable.

APPENDIX B - Sources of Data

Insurance data was obtained from A.M. Best's Aggregates and Averages.

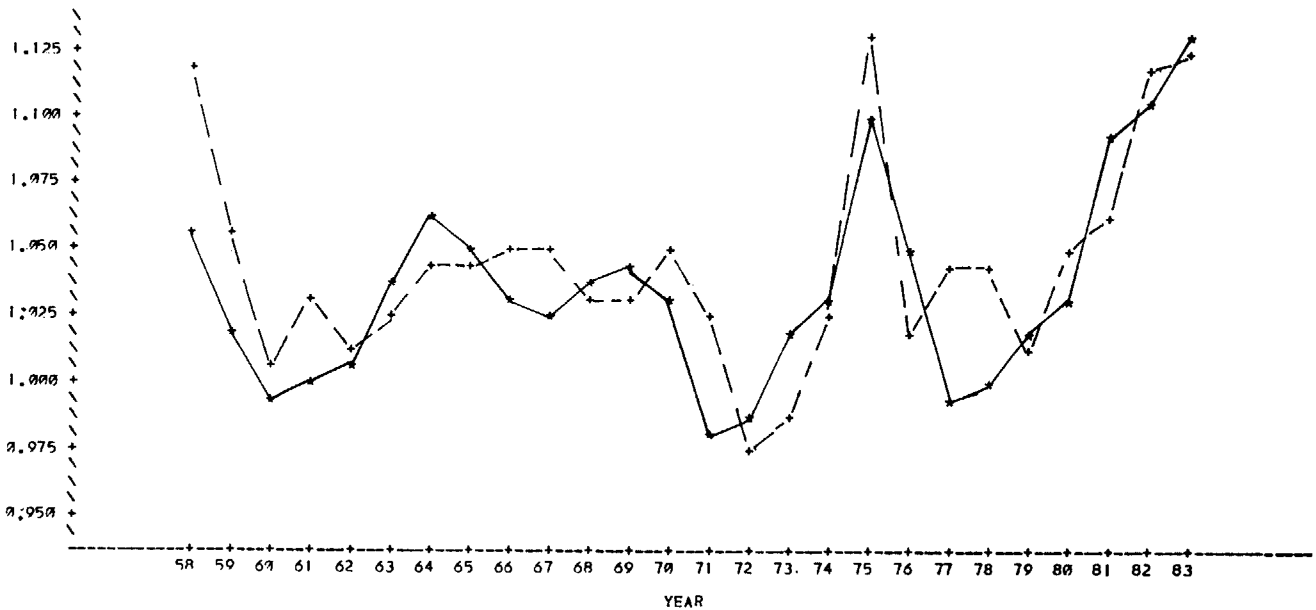
The following time series were obtained from stock, mutual and reciprocal companies combined:

Net premiums written, Net premiums earned, Incurred losses, Expense ratio, Surplus.

Economic data and forecasts were obtained from Data Resources, Inc. The following time series were obtained:

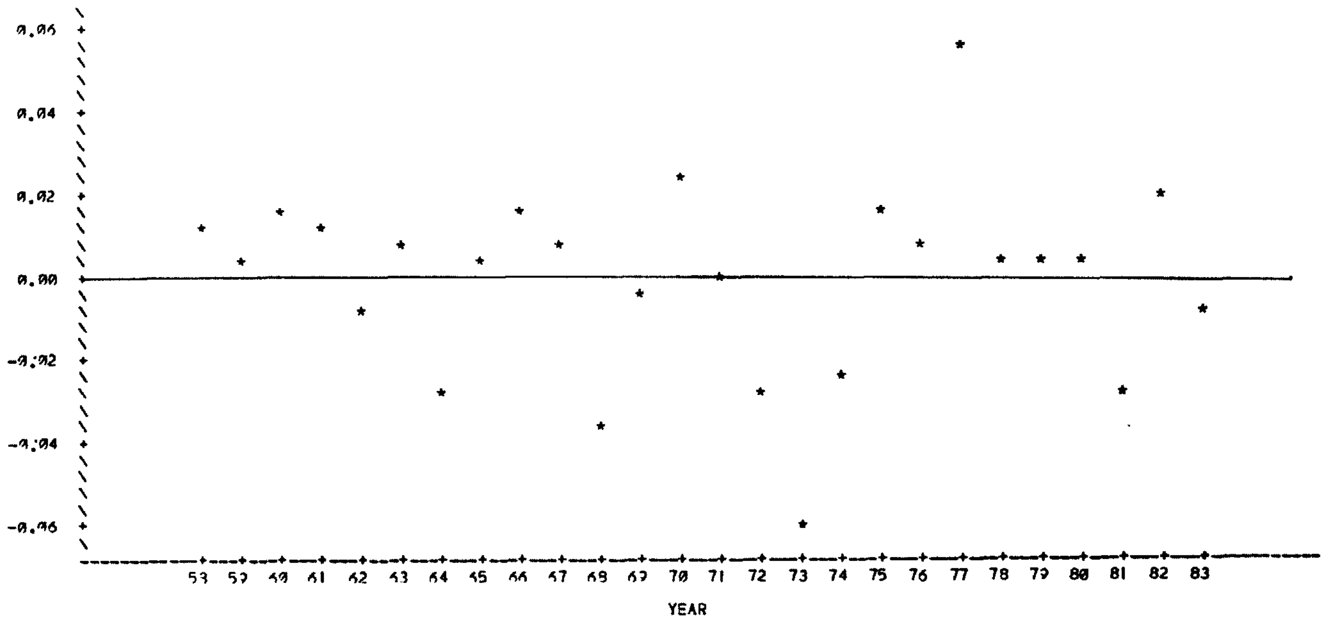
Primary source:	Bureau of the Census Population aged 16 through 28
Primary source:	Bureau of Economic Analysis, Department of Commerce Gross national product deflator, Personal consumption deflator, Retail sales, imported passenger cars, Retail sales, new cars.
Primary source:	Bureau of Labor Statistics, Department of Labor Index of average hourly earnings of non-farm production workers, Consumer Price Indices: Auto bodywork, Auto repair and maintenance, hospital and other medical services, hospital room, medical commodities, physicians services.
Primary source:	Federal Highway Administration, Department of Transportation Vehicle driver licenses (estimated), vehicle miles traveled-passenger cars, vehicle registrations - automobiles.
Primary source:	Insurance Information Institute Traffic accidents.

FITTED VS. ACTUAL COMBINED RATIOS



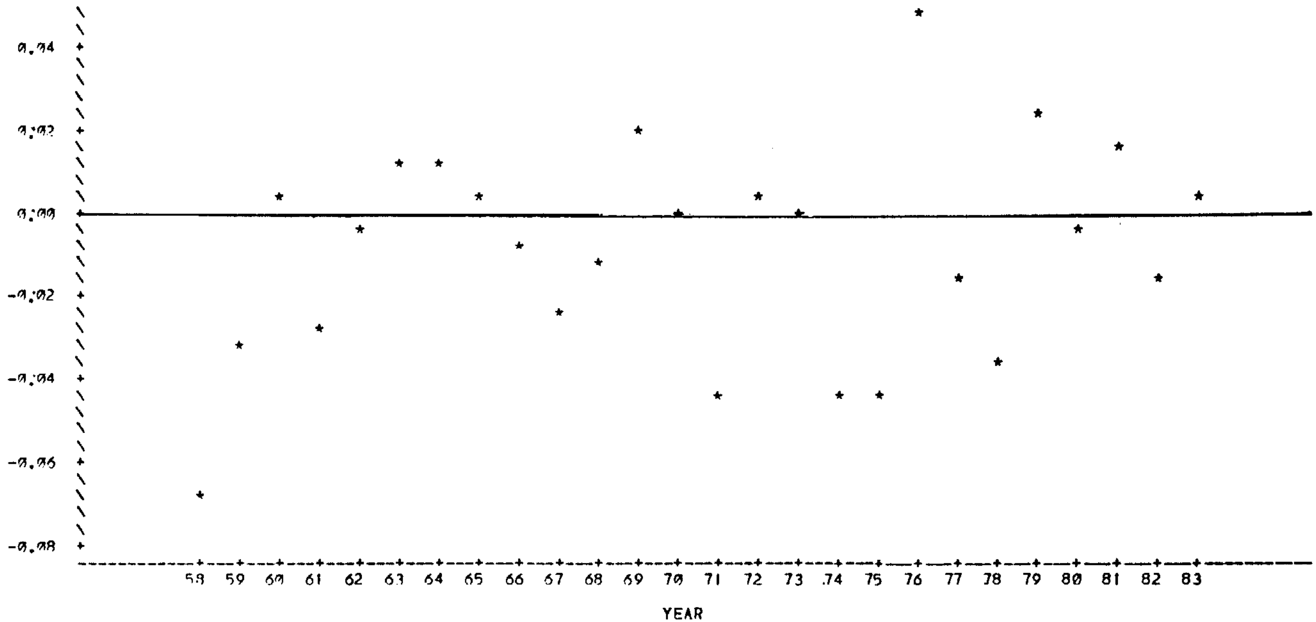
NOTE: Solid lines connect actual combined ratios,
Dashed lines connect fitted combined ratios.

RELATIVE ERRORS - PREMIUM MODEL



$$\text{Relative Error} = \frac{\text{Actual Minus Fitted}}{\text{Actual}}$$

RELATIVE ERRORS-LOSS MODEL



$$\text{Relative Error} = \frac{\text{Actual Minus Fitted}}{\text{Actual}}$$

BIBLIOGRAPHY

Alff, Gregory N. and Nikstad, James R. "A Model of General Liability Net Written Premiums," PCAS LXIX, pp. 30-45.

Armstrong, J. Scott. Long Range Forecasting, New York: John Wiley & Sons, 1978.

James, Kaye D. "Underwriting Cycles in the Property-Casualty Insurance Industry," Inflation Implications For Property-Casualty Insurance, 1981 CAS Discussion Paper Program

Lomele, Jan A. and Sturgis, Robert W. "An Econometric Model of Workmen's Compensation," PCAS LXI, pp. 170-189.

CONTINGENCY MARGINS IN RATE CALCULATIONS

Steven G. Lehmann

The author is a 1970 graduate of the University of Illinois with a Bachelor of Science degree in Actuarial Science. He is a Fellow of the CAS (1979), a Fellow of the Society of Actuaries (1974) and a Member of the Amercian Academy of Actuaries. He is Vice-Chairman of the Education (Syllabus) Committee of the CAS. He is employed as an Actuary for the State Farm Mutual Automobile Insurance Company, Bloomington, Illinois.

Abstract

Most insurance rating laws require consideration of "a reasonable margin for underwriting profit and contingencies" as one of the factors in establishing insurance rates. The purpose of this paper is to examine the contingency margin. A "contingency" is defined to be an uncertain, unexpected or unforeseen event. Evidence of the existence of "contingencies" can be seen by examination of industry underwriting results over the last 30 years. These results show a consistent shortfall between the anticipated, or target, underwriting results and the actual results. A practical example of how the contingency provision may be calculated for a hypothetical company is discussed in detail. Other methods of calculating the contingency margin are also discussed. In conclusion, for a variety of reasons, contingencies do occur and result in significant shortfalls between expected and actual results. It is essential that anyone undertaking a determination of insurance rates take this factor into account as part of the ratemaking process.

CONTINGENCY MARGINS IN RATE CALCULATIONS

I. INTRODUCTION

Actuaries are directed by most state insurance laws to carefully consider "a reasonable margin for underwriting profit and contingencies" as one of the factors in establishing insurance rates. This paper has been written with the sole purpose of investigating the "contingency" element. Historically, the term has been given limited attention, being taken by many as merely part of the "profit" factor. Actually it encompasses a host of events which must be recognized by the ratemaker if he/she is to establish adequate insurance rates.

The discussion of "contingencies" in this paper will be entirely in the context of the profit and contingency margin to be considered in establishing insurance rates.

II. DEFINITION

A contingency may be defined as an uncertain, unexpected, or unforeseen event. In the insurance context, we are specifically concerned with events which impact, or may impact, an insurance company's underwriting results. These events do not occur with predictable regularity, yet do occur from time to time and have resulted in consistent shortfalls between the target underwriting profit allowance in the rates and the actual results.

The following are examples of things which would be included in this definition of a contingency:

- Adverse court decisions
- Legislative changes
- Dramatic increase in inflation
- Regulatory delay or reduction of the rate filing
- Inadequate residual market rates
- Catastrophic events not sufficiently recognized in the normal ratemaking process

The general definition of contingency which many courts agree upon is that it is something that may or may not happen. Butler vs. Attwood, C.A. Mich, 369F 2d 811. One court has interpreted "contingency" as implying the possibility of happening and as something that may or may not happen, not something that cannot happen. Commissioner of Corporations and Taxation vs. Bullard, 46 N. E. 2d 557, 313 Mass. 72.

A legitimate case could be made for including investment risk in the definition of a contingency. However, in this paper, I will restrict the definition to contingencies which affect the underwriting results.

Another possible example of a contingency might be the downward pressure on rates caused by competition and other market forces. A company may have an indicated rate need of 10%, but implement only 5% due to competitive pressure. The problem with including this as a "contingency" is that for any particular filing, the ratemaker will know whether or not such competitive adjustments are necessary. Therefore it would not seem to be a contingency at the time of the filing.

Using the above definition one might ask if it is possible to have a "negative contingency", i.e. an improvement in the results brought about by positive unforeseen events? The answer is that while this is theoretically possible, it is extremely unlikely to occur in practice. Despite the best intentions of the ratemaker or the rate approving regulators, there are just too many forces at work keeping premiums at the lowest possible levels. In addition, while it is possible for a court to interpret policy language in a more restrictive sense than was drafted and priced, this is a pretty remote possibility, and unlikely to be brought into court in the first place. Most court cases involve situations where a more liberal interpretation of the policy contract is sought.

III. EVOLUTION OF THE CONTINGENCY MARGIN

Despite the importance of the margin for underwriting profit and contingencies, there is very little literature dealing directly with the contingency margin. The recent NAIC Investment Income Task Force Report speculates that the contingency provision began as a catastrophe allowance, and cites the 3% conflagration allowance which was to be added to the 5% profit provision in the original "1921 Standard Profit Formula". This conflagration allowance was subsequently reduced to 1% and, according to the report, has now been eliminated in most cases.

However, it is clear that as early as 1934, some actuaries recognized the need for a contingency margin in the rates. In a paper published in the Proceedings of the Casualty Actuarial Society[1], James Cahill described the need for a contingency loading in Workers' Compensation Insurance. The purpose was to ensure that, over a period of years, there would be neither an underwriting loss nor an underwriting profit. The mechanics of the calculation dealt with a comparison between target and actual results over a period of years. The contingency provision was subject to a maximum of 5% and a minimum of 0%.

Little mention of the provision was made for the next several years and the procedure was apparently discontinued after the war due to technical problems with the calculation formula.

[1] "Contingency Loading - New York Workmen's Compensation Insurance" James M. Cahill. PCAS Vol. XXVI, Part 1, 1939.

A recent paper by Mike Walters on Homeowners Insurance Ratemaking[2] discusses the need for an "extra contingency loading" in the profit and contingency factor for the catastrophe hazard because

> "no amount of actuarial smoothing or averaging of past loss data for prospective ratemaking purposes has any influence on the inherent risk of loss. Since profit is essentially a reward for risk taking, increased risk can be reflected in the profit provision independently of the average loss provision however calculated, i.e. through either long term averaging or no averaging."

Most recently, contingency provisions have been discussed in the NAIC Investment Income Task Force Report and in the Advisory Committee Report to that Task Force. Further discussion of issues raised in these reports will be taken up in a later section of this paper. The Florida Insurance Department has issued a rule regarding the contingency provision in Florida automobile insurance rates. This rule will be discussed in Section V.

IV. INDUSTRY RESULTS--THE SHORTFALL PROBLEM

Evidence of the existence of "contingencies" can be seen by an examination of industry results over the last 30 years. Attached Exhibit I shows the underwriting results for the Property-Casualty Lines, all insurance companies combined. As can be seen from this Exhibit, for the latest ten year period (1971-1980), the industry had a statutory underwriting loss of 0.1%, despite a target profit provision in the rates during that period of approximately 5%. Similarly, the underwriting results over the entire 30 year period aggregate to +0.3%, again despite a target of 5%.

---

[2] Homeowners Insurance Ratemaking, Michael A. Walters, PCAS Volume LXI, 1974 p.28.

The above results are calculated on a statutory accounting basis. The Advisory Committee to the NAIC Investment Income Task Force recalculated the latest 10 year results using the Generally Accepted Accounting Principles (GAAP) basis on page 70 of the Appendices to their Report. The result was an underwriting loss of -0.7% as compared to the -0.1% statutory underwriting loss. Thus the adjustment to a GAAP basis has little effect on the shortfall in underwriting results.

Examination of Massachusetts results provides further insight. The attached Exhibit II is a comparison of the target underwriting profit with actual underwriting results for private passenger automobile insurance in the State of Massachusetts from 1978 to 1983. Over the six year period, actual results have been consistently worse than the target underwriting results. For the entire period, the actual underwriting profit was 6.1% worse than the target underwriting profit. Similarly, Exhibit III is a comparison of the permissible loss ratio to the actual loss ratio for workers' compensation insurance in Massachusetts from 1971 to 1980. Again, the shortfall is consistent, and averages over 14% for the ten year period.

Plotkin in his statement to the NAIC Investment Income Task Force cites Workers' Compensation experience in Minnesota demonstrating the shortfall phenomenon[3].

V. THE FLORIDA INSURANCE DEPARTMENT RULE

The Florida Insurance Department has recently issued a rule dealing directly with the contingency provision in automobile insurance rates. The rule states:

[3]Report of the Advisory Committee to the NAIC Task Force on Profitability and and Investment Income, Volume II. Statement by Irving H. Plotkin, page 3.

> "All provisions for contingencies shall be derived utilizing reasonable actuarial techniques, and appropriate supporting material shall be included in the rate filing. Provisions for contingencies greater than 1.5% of premium are prima facie excessive and unreasonable until actuarially supported by clear and convincing evidence. Provisions for contingencies shall be added to the underwriting profit allowance, as determined under subsection (7) of this rule, in order to produce the percentage factor included in the rate filing for profit and contingencies."

As can be seen, this rule provides for a provision for contingencies in the rate calculated using "reasonable actuarial techniques". We shall next examine some reasonable methods of making this calculation. It is significant that the rule makes it clear that the contingency margin is a separate, identifiable element to be added to the underwriting profit allowance in determining the combined margin for "profit and contingencies".

VI. MEASUREMENT

How do you measure the "unmeasurable"? Some may argue that measurement of the contingency factor is impossible because, by their very nature, contingencies are events which are not susceptible to treatment in the normal ratemaking approach--things you cannot plan for. This school of thought would suggest that rather than measure the contingency element, you should add some reasonable safety loading into the rates to take care of the various adverse contingencies which may occur. Of course, this brings you right back to the question of what is reasonable.

One measure of the contingency factor can be derived by examining the industry results cited in Section IV of this paper. For example, based on the Massachusetts private passenger data, a contingency factor of 6.1% is indicated. The countrywide, all industry results indicate a contingency factor of approximately 5%.

Another approach would be to examine individual company results to determine a reasonable contingency provision. A practical example will illustrate this calculation.

In this example, we will compare the average anticipated, or target, provision for underwriting profit plus contingencies in the implemented rates with the average underwriting profit actually realized. As shown in Exhibit IV, over the last 10 years, the hypothetical XYZ Company had an average target provision for underwriting profit and contingencies of 2.4% in the state of Florida. Please note that the target has been adjusted for differences between the profit and contingency provision in the indicated rates as calculated by the XYZ Company, and the profit and contingency provision implicit in the rate changes actually implemented. For example, in 1975 the company implemented a smaller increase than was indicated. The target underwriting profit has therefore been reduced to account for this.

Of course, it may be that the reason that the XYZ Company implemented a smaller rate increase than indicated was that it planned some management action, such as reducing expenses, which the company felt would allow it to realize its filed target profit provision even with the reduced rate change. However, for the purpose of this example, it is assumed that no major changes in company operations were contemplated or implemented and that the lower rate change was selected for competitive reasons only.

As summarized in Exhibits IV and V, for the latest 10 years the company had an average profit/contingency target of +2.4% and actually realized +0.2%. The indicated contingency factor for the XYZ Company is therefore 2.2%.

A number of issues arise from the calculation. For example:

(1) What experience period should be used? The experience period selected should be long enough to eliminate short term fluctuations in the results from year to year. In this example a 10-12 year period is recommended (roughly two underwriting cycles).

(2) Should the actual results be before or after policyholder dividends? It has been argued that policyholder dividends are the result of voluntary action by the company, and not the result of any "contingency" or unforeseen event. On the other hand the dividend may have been "uncertain or unexpected" by the ratemaker at the time of the rate calculations and therefore fall within the definition of a contingency. Also, the exclusion of dividends would tend to "bias" the company's results downwards, i.e. dividends would reduce the profit in good years, with no compensating increase in the results during bad years. Dividends may also be paid in order to comply with excess profits statutes and therefore, perhaps, should be treated in the same fashion as excess profits refunds (see item (4) below). This example has been calculated before dividends.

(3) Should the calculation be made based on an arithmetic average of the 10 year results or should the average be weighted by the premium volume in each year? Of course, for a growing company the use of a weighted average would place more emphasis on the most current periods results. It seems inappropriate to give additional weight (or lesser weight for a company with declining business) to a contingency which occurred last year as compared to one which occurred five years ago. The better approach would therefore seem to be an unweighted or arithmetic average.

(4) Should the calculations be adjusted for any "excess profits" refunds? The Florida "excess profits" statute requires insurers who earn a profit during the test period in excess of the filed profit and contingencies provision, plus 5%, to refund such "excess profits" to policyholders. I have constructed a simple example, Exhibit VI, of how an excess profits provision might work for the ABC Company which had fluctuating results over a 12 year period. Please note that over the 12 year period, the results average out, before adjustment, to exactly the 0% target. As can be seen, the effect of the excess profits statute is to reduce the actual underwriting results to -0.3%. (For simplicity, a one year test period for application of the excess profits test has been assumed). Thus even though the rates were, on the average, correct in producing the target profit provision, the company actually realizes a lesser result due to excess profits refunds. Failure to adjust for excess profits refunds would bias the results downward.

An interesting point is raised with regard to this excess profits adjustment. If, in the above example, the excess profits adjustment causes the profit and contingency margin in the rates to be increased from 0% to 0.3%, this adjustment will result in moving the excess profits "threshold" up from 5.0% to 5.3%, presumably resulting in slightly less in excess profits refunds than was originally contemplated. Thus, it may be argued that such an excess profits adjustment is not appropriate.

The flaw in this argument is that if such an adjustment is made to the contingency provision, the rates would be increased by 0.3%, in effect raising the entire chart by 0.3%. Yes, the threshold is raised by 0.3%, but so is each year's actual profit, all other things being equal. To put it another way, if a 0.3% contingency margin had been built into the rates for the entire 12 year period, the net result would be that the 0% target would, in fact, have been *exactly achieved, no more and no less, after payment of the excess* profits refunds.

(5) Should Florida or countrywide data be used? Most companies would not have sufficient data on a statewide basis to be credible. This would, of necessity, require a countrywide calculation. Where sufficient, credible data exists on a state basis, it should be reviewed along with the countrywide data in determining a reasonable contingency provision.

(6) Should the target profit be tested against calendar year results, accident year results or policy year results? In theory, the most accurate test would be a policy year. However, when the test is made over a sufficiently long period of time, any of the three bases should yield similar results. Calendar year results are usually the most readily available and have been used in the example. Please note that if a rate filing with a revised profit/contingency provision is made during the year, the profit target for the year should be pro-rated based on earned premium in order to test against calendar year results.

VII. MEASUREMENT--ANOTHER APPROACH

The previous section examined approaches for determining the indicated contingency provision by comparing actual results with target results. Another approach, which has been particularly favored by European actuaries, is to apply risk theory principles to analyze the "probability of ruin" for an insurance company. Contingency margins ("safety loadings") are then derived in order to minimize the probability that the company will become insolvent due to adverse underwriting results.

Although a detailed discussion of these techniques is beyond the scope of this paper, they do provide additional evidence of the need for a contingency margin in insurance rates.

A special case occurs in states which limit the maximum profit an insurance company may realize via excess profits statutes. The following describes a calculation of an additional contingency margin for these circumstances.

Indicated Loading for the Capping of Profits

Assume that underwriting profits are normally distributed with Mean μ and Standard Deviation σ. If profits are limited to $\mu + k\sigma$, then the following loadings are needed to ensure the capped profits still average μ over the long run.

$$\text{Loading} = \int_{\mu+k\sigma}^{\infty} \frac{(x-\mu-k\sigma)}{\sigma\sqrt{2\pi}} \, e^{-\frac{(x-\mu)^2}{2\sigma^2}} \, dx$$

$$= \int_{k}^{\infty} \frac{\sigma(t-k)}{\sqrt{2\pi}} \, e^{-t^2/2} \, dt$$

$$= \sigma \left\{ \frac{e^{-k^2/2}}{\sqrt{2\pi}} - k\left(1-N(k)\right) \right\}$$

$$\text{where} \quad N(k) = \int_{-\infty}^{k} \frac{1}{\sqrt{2\pi}} \, e^{-t^2/2} \, dt$$

which can be evaluated from Tables of the Standardized Normal Distribution. Setting $k\sigma$ = .050, the following values are derived for the necessary loading.

σ	Loading
.050	.004
.100	.020
.150	.038
.200	.057

Thus, it may be seen that when profits are capped through an excess profits statute, an additional contingency loading is needed. This contingency loading varies with the standard deviation of the underwriting results. This loading is in addition to the normal contingency loading since it assumes that actual results vary around the mean μ, which typically is less than the target underwriting profit.

Applying this calculation to the ABC Company results previously referred to (Exhibit VI, page 2) gives an indicated additional contingency loading of approximately 0.4%. Of course, if the excess profits statute uses a rolling three year average for the test period, as does Florida's, underwriting results would have to be grouped in performing this calculation. This would tend to reduce the needed excess profits contingency loading somewhat.

VIII. THE NAIC INVESTMENT INCOME TASK FORCE REPORT

The NAIC Investment Income Task Force Report was adopted by the National Association of Insurance Commissioners in June of 1984. This report contains several references to the contingency margin in rate calculations.

On page 7 it is stated that:

> "An important point to make in connection with target returns based upon relative risk is that the total risk of the enterprise is reflected in the target. No additional provision for contingencies is necessary."

There are several problems with this rather simplistic view. Basically, this statement says that somehow you should take the contingency provision into account when you select the target, and since the target is selected using the relative risk of the enterprise, it will automatically be taken into account. Unfortunately there is no generally accepted method of determining the relative risk of the enterprise or the insurance industry. The Task Force Report acknowledged this on page 7 where it stated:

> "All of the techniques reviewed by the task force were subject to question and gave divergent views of the relative risk of the industry."

Even if you were to somehow determine the relative risk of the enterprise, this only accounts for the variation in earnings from year to year and not a consistent shortfall between the target and the actual results. The Task Force Report notes this problem later on in the Report and suggests that

> "If the estimate of losses and expenses is a priori biased one way or another, the method used to estimate losses and expense should be changed to remove that bias."

In effect, this would require an additional loading in the losses and expenses for shortfall bias. While such an approach would be feasible, it seems more appropriate to include this directly in the rates through the contingency loading.

The Report also suggests on page 25 that although the "shortfall" between target results and actual results has been significant, the indicated rates have been calculated using a 5% profit allowance, but lesser rates have been implemented. Discussion of this point was included in Section VI. Section VI also presents a method for adjusting the target for differences between the indicated rates and the rates actually implemented. Finally, this statement is inconsistent with the Massachusetts data, cited in Section IV where the industry did, in fact, implement the indicated rates, yet the shortfall still occurred.

In Vol. II of the "Report of the Advisory Committee to the NAIC Task Force on Profitability and Investment Income", Messrs. Hunter and Wilson discuss the contingency question on page 99. They suggest that target results might differ from actual results because a company might loosen its underwriting rules or become less efficient. The obvious flaw in this reasoning is that the industry as a whole couldn't "loosen its underwriting rules", so this fails to account for the industry shortfall demonstrated earlier in Section IV of this paper. It is also difficult to conceive of an individual company loosening its underwriting practices indefinitely over time. The efficiency problem also seems unrealistic. In a competitive industry, there is every incentive to increase efficiency and therefore become more profitable. If there were a question about a company's efficiency, this could be evaluated by comparing the company's expense ratio during the test period to see if there is any consistent trend.

IX. CONCLUSION

Any thorough study of industry results over the last 30 years will document the existence of a shortfall between anticipated underwriting results and the actual results. For a variety of reasons, contingencies do occur and produce these shortfalls. It is essential that anyone undertaking a determination of insurance rates take this factor into account as part of the ratemaking process.

Exhibit I

Combined Property-Casualty Lines

Underwriting Results

1951-1980

(Amounts in Millions)

Year	Earned Premium	Underwriting Gain or Loss	%	Year	Earned Premium	Underwriting Gain or Loss	%
1951	$ 6,928	$ 216.6	3.1	1966	$20,272	$ 343.7	1.7
1952	7,765	418.8	5.4	1967	21,975	156.7	0.7
1953	8,738	627.9	7.2	1968	23,895	- 173.6	-0.7
1954	9,214	715.8	7.8	1969	26,571	- 506.8	-1.9
1955	9,672	543.3	5.6	1970	31,164	93.4	0.3
1956	10,271	66.4	0.6	1971	33,867	1409.1	4.2
1957	11,116	-143.5	-1.3	1972	37,561	1793.9	4.8
1958	11,863	175.6	1.5	1973	40,838	778.2	1.9
1959	12,884	380.0	2.9	1974	43,665	-1893.2	-4.3
1960	13,914	422.0	3.0	1975	47,829	-3623.6	-7.6
1961	14,590	439.3	3.0	1976	57,119	-1571.9	-2.8
1962	15,331	316.3	2.1	1977	68,823	1883.0	2.7
1963	15,835	-175.6	-1.1	1978	78,686	2508.4	3.2
1964	16,999	-338.3	-2.0	1979	86,855	- 25.7	-0.0
1965	18,415	-363.4	-2.0	1980	93,676	-1743.1	-1.9

		Earned Premium	Underwriting Gain or Loss	%
Totals:	1951-1960	$102,365	$ 3,422.9	3.3
	1961-1970	205,047	- 208.3	-0.1
	1971-1980	588,919	- 484.9	-0.1
	Grand Total	896,331	2,729.7	0.3

Source: Best's Aggregates and Averages, Property-Casualty, 1952-1981 Editions, Stock, Mutual and since 1970 Reciprocal Companies Combined, Statutory Basis, Before Dividends to Policyholders.

Exhibit II

Massachusetts Private Passenger Automobile Insurance

Comparison of Target Underwriting Profit with Actual Underwriting Results

1978-1983

Policy Year	Target Underwriting Profit	Actual Underwriting Profit	Shortfall
1978	+0.2%	- 2.5%	- 2.7%
1979	-2.5%	-13.7%	-11.3%
1980 (Remand)	-1.9%	- 9.6%	- 7.8%
1981	-2.0%	-12.9%	-11.0%
1982	-2.3%	- 7.5%	- 5.3%
1983	-7.7%	- 6.3%	+ 1.4%
Six Year Average	-2.7%	- 8.8%	- 6.1%

Source: Massachusetts Automobile Rating Bureau Underwriting Results.

Exhibit III

Massachusetts Workers' Compensation Insurance

Comparison of Permissible Loss Ratio to Actual Loss Ratio During Policy Year

(1) Policy Year	(2) Permissible Loss Ratio	(3) Actual Loss Ratio	(4) (2) - (3) Loss Ratio Deficiency
a. 1971	.640	.632	+.008
b. 1972	.640	.708	-.068
c. 1973	.640	.740	-.100
d. 1974	.640	.771	-.131
e. 1975	.622	.754	-.132
f. 1976	.610	.792	-.182
g. 1977	.610	.773	-.163
h. 1978	.666	.906	-.240
i. 1979	.685	.916	-.231
j. 1980	.733	.903	-.170

XYZ Company

Profit and Contingency Target

Year	Profit and Contingency Target	Comments
1973	5.0%	
1974	5.0%	
1975	2.8%	Implemented smaller increase than indicated. Pro-rata effect on profit/contingency target = -2.2%, so pro-rated target = 2.8%.
1976	5.0%	
1977	5.0%	
1978	5.0%	
1979	2.8%	Financial needs change; underwriting profit/ contingency target changes from 5.0% to 2.0%. Pro-rated target for the year is 2.8%.
1980	2.0%	
1981	-2.0%	Implemented smaller increase than indicated. Pro-rated target is lowered from +2.0% to -2.0%.
1982	-7.0%	Implemented smaller increase than indicated. Pro-rated target is lowered to -7.0%.
Average	2.4%	

Exhibit V

XYZ Company

Ten Year Profit Summary

Florida

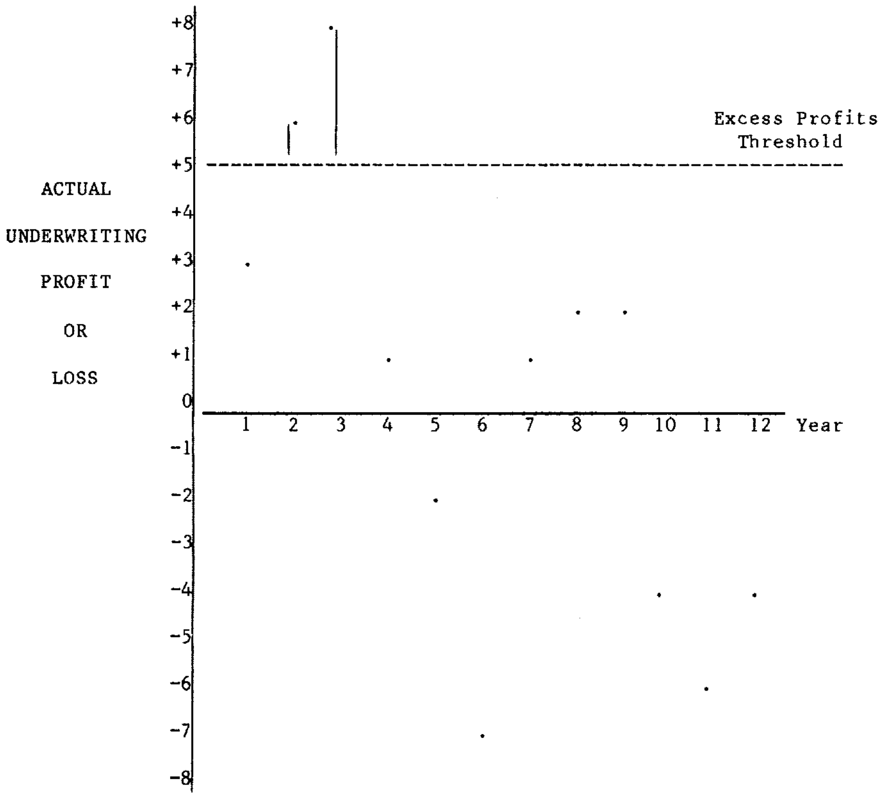
Year	Earned Premium (000)	Adjusted Profit or Loss* $ (000)	%
1973	$101,914	-11,547	-11.3
1974	103,378	-17,349	-16.8
1975	122,749	-18,895	-15.4
1976	156,129	14,017	9.0
1977	179,952	60,907	33.8
1978	198,501	53,352	26.9
1979	218,112	14,750	6.8
1980	247,362	- 6,422	- 2.6
1981	289,560	-49,203	-17.0
1982	349,103	-40,366	-11.6
Average			+ 0.2

*Profit or Loss is before policyholder dividends and after excess profits refunds.

Exhibit VI
Page 1

ABC COMPANY

Target Underwriting Profit	---	0%
12 Year Average Underwriting Profit (without Excess Profit Adjustment)	---	0%
12 Year Average Underwriting Profit (with Excess Profit Adjustment)	---	-0.3%



NOTE: See page 2 for actual numbers

ABC COMPANY

Underwriting Profit or Loss

Year	Before Excess Profits Adjustment	After Excess Profits Adjustment
1	3%	3%
2	6%	5%
3	8%	5%
4	1%	1%
5	-2%	-2%
6	-7%	-7%
7	1%	1%
8	2%	2%
9	2%	2%
10	-4%	-4%
11	-6%	-6%
12	-4%	-4%
Average	0.0%	-0.3%

TITLE: BUDGET VARIANCES IN INSURANCE COMPANY OPERATIONS

AUTHOR: Mr. George M. Levine

Mr. Levine is employed by the National Council on Compensation Insurance. He received his B.S. degree from the University of Pennsylvania's Wharton School, with a double major in Actuarial Science and Accounting. George received his ACAS in 1982 and is a member of the American Academy of Actuaries.

ABSTRACT:

This paper attempts to provide the actuary with a methodology for monitoring the price and quantity of insurance for budgeting purposes. The paper discusses and defines cost accounting concepts and relates them to casualty actuarial work. The technique entitled "Analysis of Budget Variances" is applied to budgeted figures and actual results displayed on a net income statement prepared using the contribution method of allocating expenses. Although this process is shown to have applications for the assignment of responsibility for budget variances, its main contribution is to provide a separation of the variances of components of the net income statement into their price and quantity variances.

The need for explaining variances from budgeted results is a concern for casualty actuaries in insurance companies. Often, the method of presentation is the determination of an "indication," which shows the rate change necessary to balance the actual historical loss ratio with the expected, or budgeted, loss ratio.

The "indicated rate change" evaluates the price adequacy of the insurance product. However, the economic equation, "Price times Quantity equals Revenue," implies that only one half of the total revenue component of the net income statement is being examined by the indication. A technique is needed which evaluates the variances of the actual results from those expected for both the price of insurance (rates) and the quantity of insurance written (exposures).

This paper presents a methodology for monitoring these elements through the application of the cost accounting technique "Analysis of Budget Variances."

THE COST ACCOUNTANT AND THE ACTUARY

Cost accounting has been defined as "ways of accumulating historical costs and tracing them to units of output and to departments, primarily for purposes of providing the inventory valuations used in balance sheets and income statements."[1] In some ways, the role of the cost accountant is performed by the actuary. The reserving actuary accumulates losses (historical costs) and traces them to premiums (units of output) and to departments, providing reserve evaluations (inventory valuations) for the balance sheet and the income statement. Similarly, the pricing actuary accumulates incurred losses (historical costs) and traces them to premiums (units of output), providing the proper rate evaluation for the future balance sheet and net income statement.

The reserving and pricing actuaries may discover that cost accounting techniques, however, are not appropriate for their actuarial work. Due to the elements of risk and uncertainty inherent in insurance, historical loss patterns and loss costs are only considered the best estimates of loss reserves and pure premiums after appropriate actuarial analyses. Also, regulatory constraints in various jurisdictions, such as legislation or judicial decisions which prohibit recoupment, preclude a pure historical cost accounting analysis as a basis for ratemaking.

In an insurance company, actuaries often perform other duties besides those responsibilities of the pricing or reserving actuary. Before the beginning of a fiscal period, actuaries may participate in the corporate planning of budgeted goals for the forthcoming period. After the close of the period, a system of measurement is necessary to evaluate the performance of the respective departments in attaining their goals.

The "Analysis of Budget Variances" can be adapted to the planning activities of a casualty insurance company. Although other firms, such as manufacturing concerns, use this technique primarily to assign responsibility to various departments for variances from budgeted goals, its primary value for corporate management of an insurance company is the separation of the variances of expense components into price and quantity variances. This analysis provides the corporate planning actuary with a more detailed evaluation of a company's expense allocation system, which could be of value to the pricing actuary as well.

COST ACCOUNTING TERMINOLOGY

Before presenting examples of the budget analysis, some cost accounting terminology must be introduced.

Expense Allocation--The Contribution Method

The contribution method for the allocation of expenses is introduced in Roger Wade's paper "Expense in Ratemaking and Pricing."[2] This method of expense allocation separates and classifies the different expense components of the net income statement by product and line of business, as opposed to the traditional full absorption method of expense classification which details expenses by function.

A net income statement, prepared using both methods of expense allocation, is shown in Appendix A. Wade implies that the primary value of the contribution method is to evaluate alternate policies in a marginal situation through the maximization of the line of business contribution margin.[3] Another benefit of this expense allocation is an explicit separation of fixed and variable costs for expense analysis purposes. This cost component division is necessary to analyze budget variances.

The Budget

A budget is defined as a "detailed plan showing how resources will be acquired and used over some specific time interval,"[4] representing "a plan for the future expressed in formal quantitative terms."[5]

The pricing actuary recognizes the permissible or expected loss ratio as the budgeted expected losses as a percentage of one dollar of premium. The indicated rate change that the pricing actuary develops is a budget analysis of the adequacy of rates; the budgeted expenses (and profit)[6] are the complement of the expected loss ratio, while the actual incurred losses adjust the budget for the purposes of balancing the anticipated premium collected from the budgeted rates with the expected losses and budgeted expenses. Because losses are the most volatile portion of the premium dollar, the actuary maintains the other expenses as the budgeted fraction of the premium dollar, and shows how the historic adjusted losses compare with the budgeted losses.

For the underwriter, the budget is often expressed as total dollars of premium to be written at a future time. If the pricing actuary has accepted responsibility for the pricing budget, then the underwriter provides recommendations regarding the quantity of insurance to be written. This budgeted quantity, expressed as units of exposure, is obtained by dividing the total dollars of budgeted premium by the budgeted rate.

Standard Costs

A standard cost is defined as "the budgeted cost for one unit of product."[7] Different standard costs can have different measurement bases, with the appropriate base depending on the expense item being examined. For the total premium dollar, the standard cost base is the exposure unit, chosen as a medium which should vary with the hazard of loss, but is practical and preferably already in use.[8]

The exposure unit, however, may not be the medium that varies most directly with the level of expenses incurred. For example, a more appropriate standard cost measure to analyze the budget variances for salary might be number of hours worked rather than exposure units. Therefore, in the example shown in Appendix B, hours worked is the salary standard cost base applied due to accuracy considerations, although for practical purposes the exposure unit may be substituted.

Overhead Costs

Overhead costs, which are also known as indirect costs for an insurance company, are all costs not directly associated with the selling costs of an insurance product. Appendix A shows that overhead costs can be classified as variable overhead costs, such as product promotion, underwriting, marketing or actuarial, or fixed overhead costs, such as administration, marketing management, and building and maintenance.

THE ANALYSIS OF BUDGET VARIANCES

With the cost accounting terminology introduced, the "Analysis of Budget Variances" technique is presented.

Budget Variances--Variable Expenses

The total variance of actual results from expected results for variable expenses can be divided into price and quantity variances. Although cost accounting textbooks present this concept in terms of manufacturing companies,[9] this paper adapts the technique for a service industry such as insurance. The following example introduces an analysis method for the variable expenses through the loss component, which is the most significant cost that varies directly with the earned premium of an insurance company.

Example

Insurance Company Management (ICM) has outlined a Master Budget for the year 1984. Based on discussions with the Underwriting Department, $1,400 of premium is planned to be written on January 1, 1984. The Actuarial Department, basing its recommendation on the rate indication, has budgeted a "standard price" for the rate at $1.00 per exposure. The actuaries have also agreed that the standard cost for losses, or expected loss ratio, for the 1,400 planned exposures ($1,400 ÷ $1.00 per exposure) is $.650, which will allow budgeted losses of $910.

After the close of the year, 1984 calendar year results show that 1,200 exposures were written at $.833 per exposure, for $1,000 of written and earned premium. The incurred losses have been posted at $700 for the year. The results follow:

Exhibit I
Total Variance Comparison

	(1) Master Budget	(2) Actual Results	(3) Variance*
Earned Premium	$1,400	$1,000	$400 U
Incurred Losses	910	700	210 F
Variable Gross Profit	$ 490	$ 300	$190 U

* The following notation is used throughout the paper:

U designates an Unfavorable Variance
F designates a Favorable Variance

The "flexible budget" has been developed by cost accountants to provide more information than the information presented in Exhibit I. Exhibit II presents the flexible budget, shown as Column (3) of Exhibit II, and several variable cost variances for the earned premium and incurred losses of this example. The foundation for the flexible budget, the concept of "standard exposures," is presented below. Once this concept is understood, the remaining variances are formulas which can be plugged to measure the price and quantity variances. Appendix C contains a graph of the variances.

Exhibit II
Analysis of Variable Cost Variances

Premium

(1) Actual Exposures at Actual Prices	(2) Actual Exposures at Standard Prices	(3) Flexible Budget: Standard Exposures at Standard Prices	(4) Master Budget: Original Exposures At Standard Prices
$1,000 =1,200 Exposures @$.833/Exposure	$1,200 =1,200 Exposures @1.00/Exposure	$1,000 =1,000 Exposures @$1.00/Exposure	$1,400 =1,400 Exposures @$1.00/Exposure

Price Variance =$200 U

Quantity Variance =$200 F

Budget Adjustment Variance = $400 U

Flexible Budget Variance = $0

Overall Variable Cost Variance = $400 U

Losses

(1) Actual Exposures at Actual Prices	(2) Actual Exposures at Standard Prices	(3) Flexible Budget: Standard Exposures at Standard Prices	(4) Master Budget: Original Exposures At Standard Prices
$700 =1,200 Exposures @$.583/Exposure	$780 =1,200 Exposures @.650/Exposure	$650 =1,000 Exposures @$.650/Exposure	$910 =1,400 Exposures @$.650/Exposure

Price Variance =$80 F

Quantity Variance =$130 U

Budget Adjustment Variance = $260F

Flexible Budget Variance = $50 U

Overall Variable Cost Variance = $210 F

Standard Exposures - Exhibit II

Originally, 1,400 exposures have been budgeted through the master budget to be written at $1.00 per exposure. However, since $1,000 of premium is the final amount of premium written, the expected number of written exposures associated with the actual premium, at the original budgeted price of $1.00, is 1,000 exposures (not 1,400). Restated, one would expect that the number of exposures written would have been 1,000, if the original budgeted price of $1.00 had been actually charged for the $1,000 of premium actually written. Therefore, the concept of "standard exposures" gives the budget some flexibility, because the quantity of standard exposures adjusts to the level of premium actually written.

The formula for standard exposures shows the flexibility:

Standard Exposures = Actual Premium ÷ Standard Price (1)

In this example,

1,000 Standard Exposures	= $1,000 of Actual Written Premium	÷ $1.00 per Exposure, the Budgeted Rate

The original, master budgeted quantity of 1,400 exposures can be considered an original, independent assessment of the quantity to be written, while 1,000 exposures is the flexible budget's standard quantity, dependent on the actual premium written.

Price Variance - Exhibit II

The general formula for the price variance is:

Price Variance = Actual Exposures x (Actual Price - Standard Price) (2)

The specific formulas for premium and losses are as follows:

Price Variance for Premiums = Actual Exposures x (Actual Changed Rate - Budgeted Rate) (2a)

and

Price Variance for Losses = Actual Exposures x (Actual Loss Ratio - Expected Loss Ratio) (2b)

The price variance, therefore, is the revenue variance due to the difference in actual and expected prices (or costs) while holding the quantity constant at the level of actual quantity written.

Quantity Variance - Exhibit II

The general formula for the quantity variance is:

Quantity Variance = Standard Price x (Actual Exposures - Standard Exposures) (3)

The specific formulas for premium and losses are as follows:

Quantity Variance for Premiums = Budgeted Rate x (Actual Exposures - Standard Exposures) (3a)

Quantity Variance for Losses = Expected Loss Ratio x (Actual Exposures - Standard Exposures) (3b)

The quantity variance, likewise, is the revenue variance due to the difference in actual and standard quantity while holding the price (or cost) constant at the price (or cost) level originally budgeted.

Budget Adjustment Variance - Exhibit II

The general formula for the budget adjustment variance is:

(4) Budget Adjustment Variance = Standard Price x (Standard Exposures - Original Exposures)

This variance is the revenue variance due to the difference in the standard quantity, flexibly adjusted for actual premium written, and the original budgeted quantity, while holding the price (or cost) constant at the price (or cost) level originally budgeted. It is also the difference between the flexible budget's revenue components and the master budget's revenue components.

The specific formulas for premiums and losses are as follows:

$$\text{Budget Adjustment Variance for Premiums} = \text{Budgeted Rate} \times (\text{Standard Exposures} - \text{Original Exposures}) \quad (4a)$$

$$\text{Budget Adjustment Variance for Losses} = \text{Expected Loss Ratio} \times (\text{Standard Exposures} - \text{Original Exposures}) \quad (4b)$$

Flexible Budget Variance - Exhibit II

The flexible budget variance is the net effect of the price and quantity variances, obtained as follows:

$$\text{Flexible Budget Variance} = \underset{(2)}{\text{Price Variance}} + \underset{(3)}{\text{Quantity Variance}} \quad (5)$$

Overall Variable Cost Variance - Exhibit II

The overall variable cost variance is as follows:

$$\text{Overall Variable Cost Variance} = \underset{(5)}{\text{Flexible Budget Variance}} + \underset{(4)}{\text{Budget Adjustment Variance}} \quad (6)$$

Uses of the Flexible Budget

The flexible budget is a budget tailored to actual results, built to a level using standard costs. The flexible budget, column (3), is the primary benchmark for performance appraisal, while the budget adjustment variance, which reflects the master budget, can be considered a measure of the effectiveness of the operation. ICM's failure to reach the attainable level of $1,400 of premium written, targeted by the master budget, shows ineffective operation. The extent of the ineffectiveness of the operation is indicated through the $400 unfavorable budget adjustment variance for the premium. However, once the actual level of premium written is accepted, the efficiency of the operation may be considered favorable as indicated by the favorable quantity variance ($200) associated with the premium.

Exhibit II shows that the flexible budget concept for variable costs can be adapted for revenue, although cost accounting textbooks do not display revenue in this manner. Due to the nature of the flexible budget, price and quantity variances for premium will always net to zero. This fact does not render that exercise useless, as the variable gross profit variances, obtained by subtracting the loss variance from the premium variance, may be a valuable tool in explaining results to non-actuaries.

The separation of the flexible budget variance into price and quantity variances can offer additional insight into cash-flow underwriting practices. Cash-flow underwriting, through the Analysis of Budget Variances, is considered profitable when a favorable quantity variance, with its guaranteed positive variable gross profit, combines with an attractive investment environment to compel the profit seeker to overlook the unfavorable price variance which is likely to occur. Exhibit II shows that the $80 favorable price variance for losses could not overcome the $200 unfavorable price variance associated with the premium, which was necessary in order to attract the additional business.

The separation of the flexible budget variance into price and quantity components does not imply the absence of a price and volume relationship. Economic theory, through the ideas of supply, demand and elasticities, demonstrates that the price and quantity of a product are related. The flexible budget is a method to measure the sensitivity of the price and volume trade-offs, which Wade discusses in his paper.[10]

As explained above, the assignment of responsibilities for the price and quantity variances is the primary use of this system for manufacturing firms. Although the assignment of responsibility for some expenses may be realistic, the applicability of the responsibility assignment for the loss component of the price variance is questionable. First, an unfavorable variance may not necessarily be "bad"; reserve strengthening may produce an unfavorable variance but be warranted. Secondly, the multitude of forces that impact incurred losses, such as claims awarding, loss control, and reserving practices, obviate the assignability of a variance to one certain department or person. For the loss component of the price variance, its complex nature compels a more detailed investigation into its nature before the assignment of responsibility.

<u>Pure Price Variance</u>

Exhibit II shows the price variance as the difference between columns (1) and (2). The price variance, in addition, can be divided into two more variances. Exhibit III presents the price variances of Exhibit II, separated into additional variances. Appendix D contains a graph of these variances.

Exhibit III
Analysis of Price Variance

(1) (Actual Exposures minus Standard Exposures) at Actual Prices	(2) (Actual Exposures minus Standard Exposures) at Standard Prices	(3) Standard Exposures at Actual Prices	(4) Flexible Budget: Standard Exposures at Standard Prices
Premium			
\$167 =(1,200-1,000)x\$.833	\$200 =(1,200-1,000)x\$1.00	\$833 =1,000x\$.833	\$1,000 =1,000x\$1.00
Losses			
\$117 =(1,200-1,000)x\$.583	\$130 =(1,200-1,000)x\$.650	\$583 =1,000x\$.583	\$650 =1,000x\$.650

Premium

Joint Price-Quantity Variance=\$33 U (columns 1–2)

Pure Price Variance =\$167 U (columns 3–4)

Overall Price Variance =\$200 U

Losses

Joint Price-Quantity Variance=\$13 F (columns 1–2)

Pure Price Variance =\$67 F (columns 3–4)

Overall Price Variance =\$80 F

The general formula for the price variance, as stated in formula (2), is:

$$\text{Price Variance} = \text{Actual Exposures} \times (\text{Actual Price} - \text{Standard Price}) \quad (2)$$

A pure price variance can be calculated as follows:

$$\text{Pure Price Variance} = \text{Standard Exposures} \times (\text{Actual Price} - \text{Standard Price}) \quad (7)$$

A joint price-quantity variance is defined as:

$$\text{Joint Price-Quantity Variance} = \left\{\text{Actual Exposures} - \text{Standard Exposures}\right\} \times \left\{\text{Actual Price} - \text{Standard Price}\right\} \quad (8)$$

The sum of formulas (7) and (8) equal the price variance, formula (2), which is apparent from the formulas.

The pure price variance, therefore, is the revenue variance due to the difference in actual and expected prices (or costs), while holding the quantity constant at the level of expected quantity written. This variance is more pure than the price variance, which holds the quantity at the level of actual quantity written.

This additional procedure may be unnecessary, as the overall price variance is a method of recognizing that the actual exposures written will impact the price of a product through supply and demand elasticities.

The remaining specific formulas for premium and losses, which produce Exhibit III, are as follows:

$$\text{Pure Price Variance for Premium} = \text{Standard Exposures} \times \left\{ \text{Actual Charged Rate} - \text{Budgeted Rate} \right\} \quad (7a)$$

$$\text{Pure Price Variance For Losses} = \text{Standard Exposures} \times \left\{ \text{Actual Loss Ratio} - \text{Expected Loss Ratio} \right\} \quad (7b)$$

$$\text{Joint Price-Quantity Variance for Premium} = \left\{ \text{Actual Exposures} - \text{Standard Exposures} \right\} \times \left\{ \text{Actual Charged Rate} - \text{Budgeted Rate} \right\} \quad (8a)$$

$$\text{Joint Price-Quantity Variance for Losses} = \left\{ \text{Actual Exposures} - \text{Standard Exposures} \right\} \times \left\{ \text{Actual Loss Ratio} - \text{Expected Loss Ratio} \right\} \quad (8b)$$

Budget Variances--Fixed Expenses

The fixed costs are budgeted and monitored through a different analysis of variance technique than the procedure described for the variable costs. The specific technique, called fixed-overhead application, requires the development of a fixed-overhead rate which will be used to monitor the fixed costs throughout the budget period. This rate is computed by dividing the budgeted dollar level of fixed costs by the best measure of capacity over the budget period. This measure is called the denominator level.[11]

One insurance definition of capacity is "the total premium volume a single multiple-line insurer can write for all lines of insurance."[12] As long as a Kenney-type rule of the ratio of net written premium to policyholders' surplus is followed, the choice for an appropriate denominator level is facilitated.

Example

ICM's master budget for 1984 includes $140 of fixed costs. Corporate management has chosen to adhere to the Kenney rule, which states that capacity equals twice the level of policyholders' surplus.[13] At December 31, 1983, policyholders' surplus is $1,750, producing a denominator level of $3,500. The fixed overhead rate is set at .04($140 ÷ $3,500) per dollar of capacity.

On December 31, 1984, the net income statement shows $150 of fixed costs were incurred, and $1,000 of premium was written. Exhibit IV shows the Fixed Costs Analysis of Budget Variances.

Exhibit IV

Fixed Costs Analysis of Budget Variances

(1) Actual Fixed Costs Incurred	(2) Flexible Budget Based on Premium	(3) Fixed Overhead Applied
$150	$140 (same regardless of volume level)	$100 =$2,500*of standard capacity x.04 per dollar of capacity

Spending Variance** =$10 U

Denominator Variance =$40 U

Underapplied Overhead = $50 U

*Since $1,400 of premium was allowed in the master budget for $3,500 of capacity, then $1,000 of "good output" of premium actually written produces standard capacity of $2,500 (($1,000 ÷ $1,400) x $3,500 = $2,500).

**The spending variance is the budget variance.

Uses of Fixed-Overhead Analysis

The fixed costs variance analysis does not have an explicit quantity variance, as fixed costs are presumed to be constant over a range of volume levels. Column (2) is called a flexible budget because the $140 was selected as the best flexible measure of fixed costs over that range of volume levels.

The denominator variance, which replaces the quantity variance for fixed costs analysis purposes, is an approximate measure of the efficiency of production. This firm has been inefficient in its production, as the amount of premium actually written is on the low end of the range of volume levels.

Wade warns that one of the potential misapplications of the contribution method of allocation of expenses is in the treatment of the fixed costs.[14] The contribution method is an appropriate technique to compare alternate policies in a marginal situation only when fixed costs truly remain "fixed" over the analysis period.

The spending variance, and the causes for its balance, should be examined to discover the true reason for any observed changes in fixed costs. Although Wade indicates that changes in the volume of business can affect the level of fixed costs, other factors such as inflation or poor cost estimation methods can produce unanticipated fixed cost differences.

Committed fixed costs, including depreciation, real estate taxes, and insurance,[15] are likely to be independent of short-term changes in volume. For example, an unanticipated increase in property tax assessments could produce an unfavorable spending variance, but would not likely be produced due to a change in volume. Discretionary costs, which are budgeted fixed costs due to short-term decisions, are more likely to be incurred due to growth reasons. Here, a recent surge in premium writings might encourage a company to undertake a management development program which it might not have afforded in the absence of the change in volume.

These above examples illustrate that the contribution method of allocation of expenses is not rendered an inappropriate comparison measure of alternatives, if the fixed-overhead budget analysis reveals that the variances occurred for reasons other than expanding capacity.

CONCLUSION

This paper has presented a technique which can evaluate the variances of the actual results from those expected for all the components of the net income statement. Price and quantity variances, which can be produced for the premium and variable cost components, may have some applications for the assignment of responsibilty. The fixed costs analysis provides a more detailed evaluation of a company's expense allocation system, which could be of value to the pricing actuary as well as the corporate planning actuary.

APPENDIX A
Net Income Statement Comparison[16]

Full Absorption Method

Earned Premiums		$200
Incurred Losses	$100	
Loss Adjustment Expenses Incurred	$ 30	
Commissions Incurred	$ 24	
Other Acquisition Expenses Incurred	$ 8	
General Expenses Incurred	$ 20	
Taxes, Licenses and Fees Incurred	$ 6	
Net Income		$ 12

Contribution Method

Earned Premiums		$200
Loss and Loss Adjustment Expenses Incurred (Variable Cost of Goods Sold)	$130	
Variable Gross Profit		$ 70
Commissions Incurred	$ 24	
Other Acquisition Expenses Incurred	$ 28	
Premium Taxes	$ 5	
Other Variable Costs Associated with Product (10% of General Expenses)	$ 2	
Variable Profit (Distribution Contribution Margin)		$ 31
Variable Overhead Expenses* (50% of General Expenses)	$ 10	
Line of Business Contribution Margin		$ 21
Fixed Overhead Expenses** (40% of General Expenses)	$ 8	
Other Taxes	$ 1	
Line of Business Profit/Net Income		$ 12

* Indirect Costs - Variable and Not Directly Associated with Product.
** Indirect Costs - Fixed and Not Directly Associated with Product.

APPENDIX B

This Appendix contains examples of standard cost bases other than exposure units to measure variable cost budget variances.

Examples--Hourly Wages and Number of Policies

A data processing department of an insurer has a clerical staff which is paid an hourly wage. In order to monitor the budget for clerical salaries, a standard cost system based on hourly wages is maintained.

This same insurer is also concerned with the General Expenses of the Other Underwriting Expenses shown on Part 1 of the Investment Income Exhibit. In particular, Items 3 through 17 are the itemized expenses to be monitored. All of these expenses have been deemed variable overhead by this insurer. The clerical salaries, to be examined in another standard cost base, are removed from these expenses. The standard cost for this group of expenses is number of policies written.

For 1984, the clerical salaries are budgeted for $100,000, composed of 20,000 hours at $5.00 per hour. The Other Underwriting Expenses are budgeted for $150,000, with 500 policies planned at a cost of $300 per policy. Budgeted earned premium for 1984 is $1,000,000.

Actual 1984 results show that 480 policies were written and $900,000 of earned premium was posted. Union negotiations have raised the clerical hourly wage to $5.20, and 17,000 hours have been worked by the clerical staff. The Other Underwriting Expenses actually incurred total $139,200.

Exhibits B-I and B-II show the Analysis of Budget Variances for clerical salaries and Other Underwriting Expenses, respectively.

Exhibit B-I

Salaries: Analysis of Budget Variances

(1) Actual Exposures at Actual Prices	(2) Actual Exposures at Standard Prices	(3) Flexible Budget: Standard Exposures at Standard Prices	(4) Master Budget: Original Exposures at Standard Prices
$88,400 =17,000 hours @$5.20 per hour	$85,000 =17,000 hours @$5.00 per hour	$90,000 =18,000* hours at @$5.00 per hour	$100,000 =$20,000 hours at @$5.00 per hour

Price Variance =$3,400 U

Efficiency Variance =$5,000 F

Flexible Budget Variance = 2,600 F

Budget Adjustment Variance = $10,000 F

*18,000 hours = $1 of clerical salaries allowed for $10 of earned premium ("good output"), so $900,000 of earned premium allows $90,000 of clerical salaries, and $90,000 ÷ $5.00 per hour = 18,000 hours.

Exhibit B-II

Other Underwriting Expenses: Analysis of Budget Variances

(1) Actual Exposures at Actual Prices	(2) Actual Exposures at Standard Prices	(3) Flexible Budget: Standard Exposures at Standard Prices	(4) Master Budget: Original Exposures at Standard Prices
$139,200 =480 policies at $290 per policy	$144,000 =480 policies at $300 per policy	$135,000 =450* policies at $300 per policy	$150,000 =500 policies at $300 per policy

Price Variance =$4,800 U

Efficiency Variance =$9,000 F

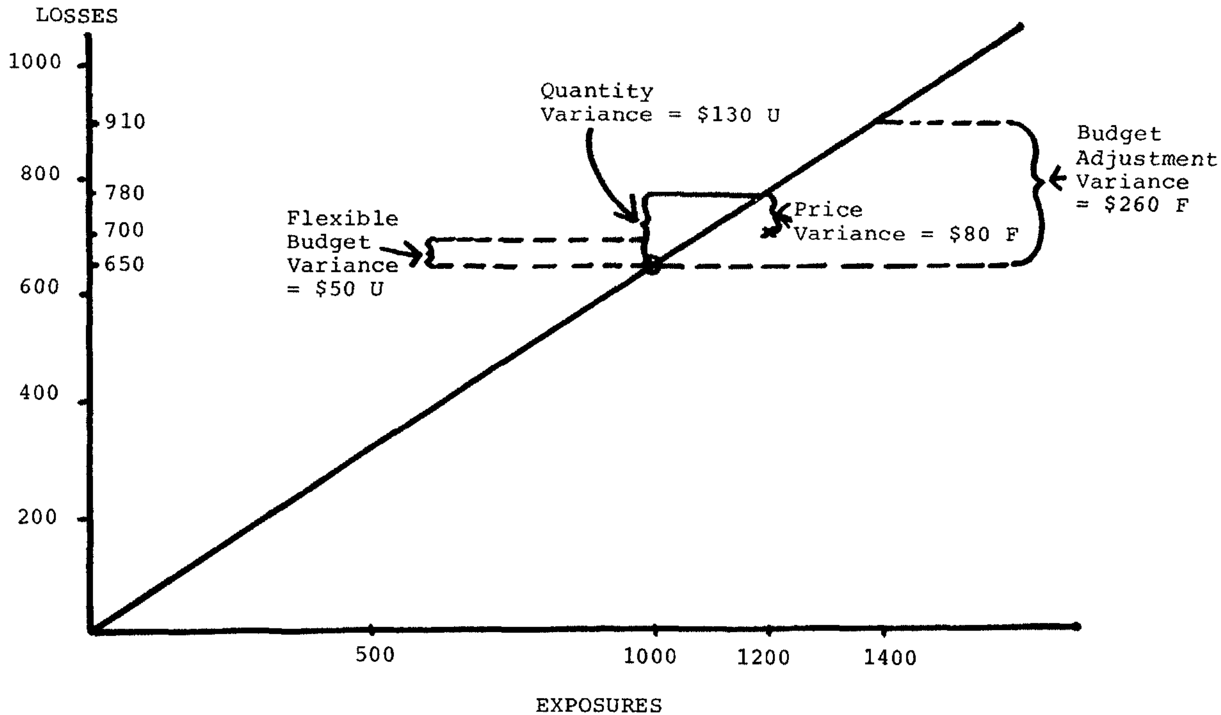
Flexible Budget Variance = 4,200 U

Budget Adjustment Variance = $15,000 F

*450 policies = $15 of Other Underwriting Expenses allowed for $100 of earned premium ("good output"), so $900,000 earned premium allows $135,000 of Other Underwriting Expenses as a standard cost, and $135,000 ÷ $300 per policy = 450 policies.

APPENDIX C

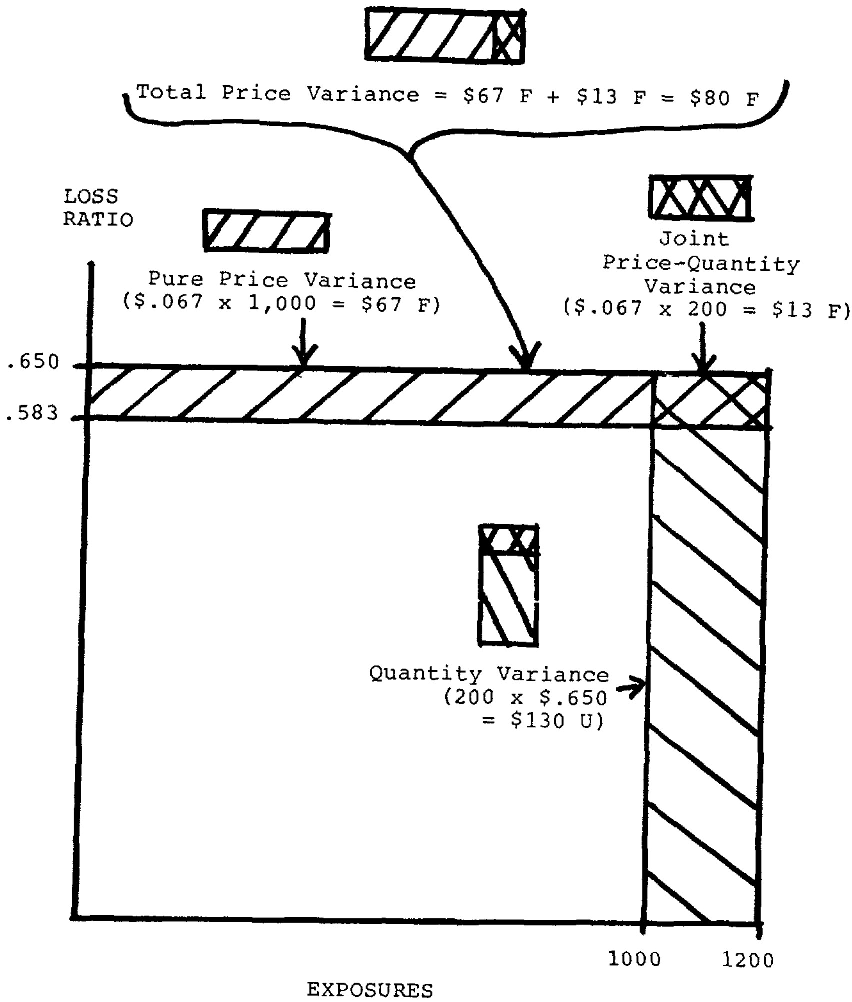
Graph of Variable Cost Variances



x = Actual Losses Point of $700
o = Standard Losses Point of $650

APPENDIX D

Graph Illustrating Pure Price Variance



FOOTNOTES

1. Charles T. Horngren, Cost Accounting: A Managerial Emphasis, 4th ed. (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1977), p.4. Much of the general techniques for this paper are found in chapters 7 through 10 of this reference.

2. Roger C. Wade, "Expense Analysis in Ratemaking and Pricing," PCAS Vol. LX, 1973.

3. Ibid., p.1.

4. Ray H. Garrison, Managerial Accounting, 3rd ed. (Plano, Texas: Business Publications, Inc., 1982), p. 288. Additional material for this paper, very similar to Horngren's, was gathered in chapters 8-10.

5. Ibid.

6. Explicit considerations for investment income and underwriting profit can embellish this method, but are ignored for the purposes of this paper.

7. Garrison, p.347.

8. Paul Dorweiler, "Notes on Exposures and Premium Bases," PCAS Vol. LVIII, 1971, p.60.

9. See Footnotes [1] and [4].

10. Wade, p.6.

11. Horngren, p.265

12. Robert I. Mehr, "Insurance Capacity: Issues and Perspectives," Issues in Insurance, Volume 1 (Malvern, PA: American Institute for Property and Liability Underwriters, 1981), p.318.

13. Ibid., p.321.

14. Wade, p.9.

15. Garrison, p.154.

16. This example was based on Wade's of p.5.

TITLE: PRICING, PLANNING AND MONITORING OF RESULTS: AN INTEGRATED VIEW

AUTHOR: Stephen P. Lowe

Mr. Lowe is a Vice President and Principal of Tillinghast, Nelson & Warren, Inc. in Hartford, Connecticut. He holds a Bachelor of Science degree in Mathematics from Union College, and is currently pursuing a Master of Arts Degree in Economics at Trinity College.

Mr. Lowe is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. He is a past president of the Casualty Actuaries of New England.

ABSTRACT: The paper discusses current approaches to financial planning, and argues that planning and monitoring of results can be improved by integrating them with the pricing process. Traditional insurance accounting information is rejected as not being meaningful. In its stead the author argues that products should be priced, planned and monitored either on an issue year or an exposure year basis.

Most major property/casualty insurance companies (and many of the smaller ones) are now actively engaged in business planning. The centerpiece of this activity is a financial forecast of operating results over some future time horizon, usually ranging from one to five years.

Inputs to the financial forecast include estimates of future premium growth based on market analyses and production objectives; projections of future loss ratios reflecting actuarial, claim and underwriting input; as well as expense assumptions based on fairly detailed budgets and staffing projections. As a final step the investment and tax areas overlay their projections onto the underlying forecast of the insurance area.

These efforts are by no means the only type of planning activity that takes place at most companies. In addition to the financial planning described above, a great deal of operational planning is underway. Marketing, claim, underwriting, and investment are all devising strategies and operational plans designed to accomplish their objectives.

Operational planning is important; however, the unfortunate truth is that it usually takes a back seat to financial planning where the focus is on results.

Once the financial plan has been constructed, it is then used as a benchmark against which actual results are measured. At most companies considerable effort is expended in the analysis of planned versus actual results, particularly in explaining significant variances between actual and plan (especially adverse variances).

CURRENT APPROACHES TO PLANNING & MONITORING OF RESULTS

The proper construction of a property/casualty financial planning exercise is the key to its success. Unfortunately, at many companies the construction is not well conceived, and suffers as a result. The defects in the construction of the financial planning exercise usually stem from the historical traditions of the property/casualty industry and fall into two main (but related) categories:

- An overemphasis on calendar year accounting results
- Separation of underwriting results from investment results

This situation creates a straightjacket for any analysis, severely limiting its effectiveness. Management is focusing on the wrong numbers, configured in the wrong manner for intelligent decision-making.

Calendar year accounting results are not a meaningful measure of an insurer's profitability. This is true whether these results are presented on a statutory or a GAAP basis; and also whether or not the reserves are discounted or undiscounted.

Calendar year results reflect the change in the inventory of accumulated profits and losses on all past policies issued.

- They are distorted by changing reserve margins on past issues.

- They reflect current investment income on cash flows generated by current and prior issues, with all funds commingled.

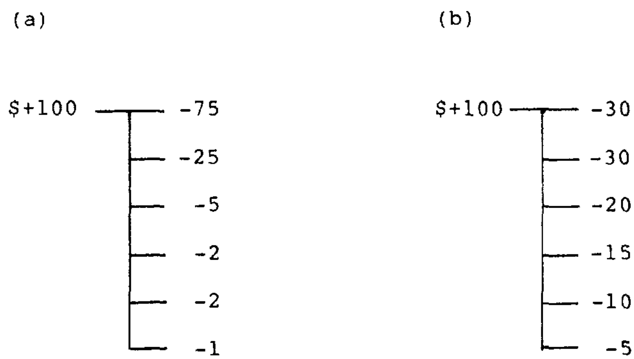
- Investment income derived from the insurance cash flow is often commingled with investment income on retained surplus.

- Underwriting results on current business are almost entirely an estimate.

Data configured in this manner is largely useless in assessing the current profitability (or unprofitability) of products being sold. The discussion above also implies that attempts to forecast calendar year results directly are doomed to failure.

Related to the problem of reliance on calendar year results is the traditional separation of underwriting and investment income. Interest rates have been high enough for long enough that almost everyone has developed an appreciation for the time value of money (although I am still surprised to hear of underwriters who do not consider deferred premium plans to be a form of price cutting). However, this appreciation has not extended to the traditional income statement.

Consider the following two cash flows:



Cash flows (a) and (b) are both insurance products with combined ratios of 110%, but with distinctly different cash flow characteristics. Cash flow (a) is attractive only when interest rates exceed 26%. On the other hand, cash flow (b) is attractive if interest rates are above 7%.

Clearly the traditional underwriting result, which is the same for both products, does not tell us enough to compare their performance. By the underwriting results standard both products are losers. Yet in reality, if interest rates are 10% (and stay at 10%) one product is actually a winner.

And if interest rates are 10%, what is the magnitude of our true loss on product (a) and gain on product (b)?

At a 10% interest rate, cash flow (a) has a net present value of $-5.35, while cash flow (b) has a net present value of $+4.99. Regardless of the accounting treatment (and ignoring taxes) these are the true economic profits on these two transactions, given the interest rate.

At least for internal management purposes, we should be measuring our performance in a manner that more directly reflects this fact.

MACRO-PRICING

As the title of this paper suggests, the author believes that effective planning can only be accomplished by integrating financial planning with pricing. Before proceeding further, it is necessary to define pricing in this context. To do so, a distinction must be drawn between macro-pricing and micro-pricing of property/casualty insurance products.

Micro-pricing is concerned with individual rates for specific states, classes, territories and coverage limits. This activity is traditional at large companies and at the bureaus, where armies led by actuaries wage warfare with insurance departments using the rate filing as their weaponry.

In contrast, macro-pricing is concerned with the overall cash flow characteristics of a product line, and the resulting return on equity that it produces. This kind of activity needs to be done by all companies, regardless of whether they make their own rates or rely on ISO or NCCI.

Macro-pricing takes as its starting point, the aggregate ratemaking statistics for the product. This includes exposure, premium, loss and expense data, configured to reflect both the <u>timing</u> as well as the amount of each item.

Using traditional pricing techniques (loss development, trend, etc.), coupled with a cash-flow model it is possible to calculate the overall premium revenue necessary during some future period, to produce any desired rate of return. This is the essence of macro-pricing: the calculation produces an indication of future rate level need for the product line as a whole.

The macro-pricing indication is the benchmark by which micro-pricing decisions are measured. In fact, an operational planning issue becomes the design of a strategy to obtain via the individual micro-pricing decisions the overall macro-pricing objective.

Alternatively, if the macro-pricing objective is unattainable (due to market or regulatory constraints) the company might consider withdrawing the product from the market, altering the product, or curtailing the growth in its sales.

Inevitably, when the subject of rates of return arise the conversation turns to the question of what constitutes an adequate return. It is argued that total return pricing is an exercise in futility, without an answer to this question. A further impediment to total return pricing is the question of how much capital is required to support the line.

I will offer no answers to these related questions.

However, I would argue that the lack of concrete answers to these questions doesn't mean that we ought to reject the thought process embedded in them. Finance and economics strongly suggest that this is the right way to look at things. The supply of answers to these questions will not increase until it is spurred by a rise in the demand for them.

MONITORING OF RESULTS

In order to analyze the results for a product and measure its profitability, either historically or currently, it is necessary to accumulate all of the associated premium, loss, and expense transactions together in an orderly and consistent manner.

Fundamentally, there are only two approaches to accomplish this objective.

- Accumulate all transactions that relate to a particular exposure period.
- Accumulate all transactions that relate to a particular set of contracts, such as those issued during a set time interval.

The reader should recognize that for traditional applications, and at least for losses, these two approaches are synonymous with accident period and policy period (and if the policies are annual we have the traditional accident year and policy year.) I have utilized the exposure and issue terminology to imply a more generalized concept, capable of handling specialized products without a traditional policy term, and also to emphasize that the approach applies not only to losses, but also to exposures, premiums, and expenses.

All insurance transactions are assignable or allocable both to an issue period and an exposure period.

- Losses and allocated loss adjustment expenses are directly assignable to both issue and exposure period.

- Premiums, commissions, state premium taxes, and other excise taxes that are a direct function of premium are directly assignable to issue periods; they are traditionally allocated pro-rata over the term of the exposure.

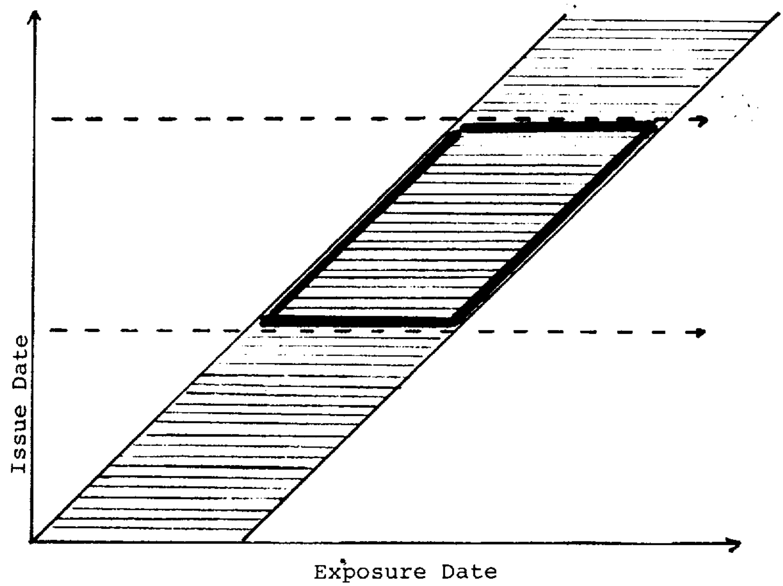
- All other expenses are overhead, and must be allocated in some reasonable manner both to issue periods and exposure periods.

Ideally, all allocations of costs should be based on functional cost and time studies. What is more important is that the methods of allocation be consistent, particularly with pricing assumptions.

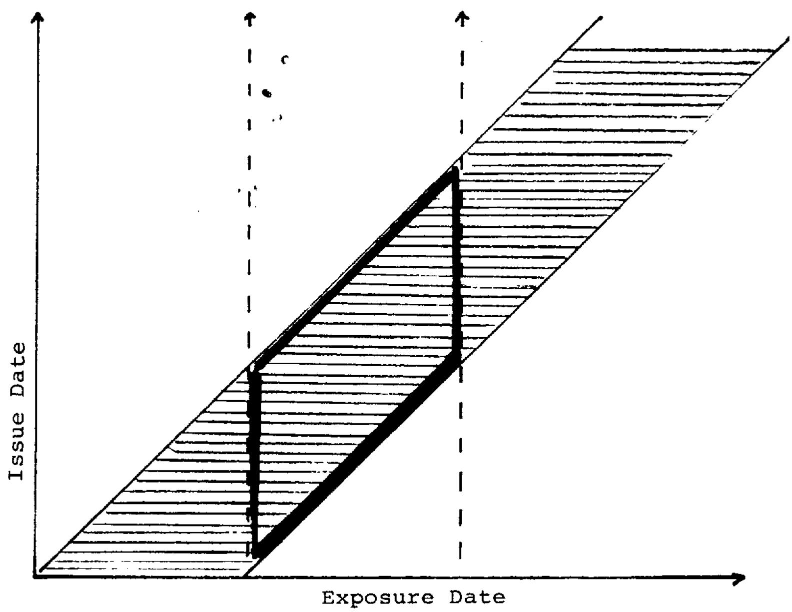
Exhibit I diagrams the traditional configuration of issue year versus exposure year for contracts with an annual term. (Note that both the policy year <u>and</u> the accident year are conceptualized as parallelograms). Each approach has advantages and disadvantages, most notably:

EXHIBIT I - ISSUE YEAR VERSUS EXPOSURE YEAR

Issue Year Configuration of Experience:



Exposure Year Configuration of Experience:



- Exposure year loss data matures more quickly than issue year loss data, hence its greater acceptance in ratemaking.

- Premiums, commissions and taxes are easier to assign to issue year, because all transactions associated with a given policy are assigned to the same issue year. Correct allocation to exposure year requires systems efforts not always easy to obtain. Individual transactions must be split between exposure periods; the pro-ration is intuitively simple, but quickly becomes very complex in the real world of endorsements, audit adjustments and error corrections.

- Policies are priced at the time of their issue. Similarly, production goals are established by issue period. The issue year approach has the advantage of relating directly to these and other related marketing decisions and objectives.

For this last reason, the author prefers the issue year approach for planning and monitoring of results.

INVESTMENT INCOME

In discussing the allocation of transactions to issue year and exposure year, some readers may have noted that allocation of investment income was absent from the discussion.

This was intentional, not because investment income is not important, but instead because the author does not believe that actual investment income should be utilized in pricing, planning and monitoring of property/casualty insurance results.

Instead, the investment department should guarantee to the insurance area spot and forward interest rates to be used in the pricing process. These rates should be used in the pricing of the product. Investment income for the insurance area should be based on these rates, as if they were actually the rates being earned.

In other words, the insurance operation ought to loan its available cash to the investment operation at negotiated, fixed, and guaranteed rates. In essence the investment department should "pay" for the cash flow that it obtains from the insurance area.

The performance of the investment area should be based on its ability to earn investment returns in excess of that which it is paying for the funds that it has obtained. The investment department becomes a true banking operation.

Some might argue that the artificial book-keeping entries required to accomplish what I am describing are not worth the effort. I would disagree. The approach I am suggesting may be the only way to establish the missing dialogue between the two areas about such topics as durations and the risks associated with mis-matching maturities.

This approach will also facilitate meaningful consideration of investment income in the pricing of these products.

The rates guaranteed to the insurance area should be at or near market interest rates, with any differences reflecting risk margins. They should be the rates that management is willing to concede to the buyer in the pricing of a product. For example, the pricing of alternative payment plans should use these rates, so that the seller is indifferent to the payment plan selected.

The concession of market interest rates in the pricing of the product does not imply a reduction in the <u>overall</u> target profit margin of the product. It merely causes that profit margin to be consolidated into a single number, rather than being split between an underwriting and an investment component.

Consolidating the profit margin into a single number is critical to the pricing process. The decision to reduce the profit margin to meet competitive pressures becomes an explicit one, and not an implicit one based on the amount of an indicated rate increase "left on the table".

A FINANCIAL PLANNING SYSTEM

Financial planning, including macro-pricing requires a specialized management information system to house both actual and projected experience. The system should consist of one or more databases capable of sorting and summarizing the various data elements in a variety of ways.

Exhibit II displays such a system in schematic form. The system consists of three inter-related databases:

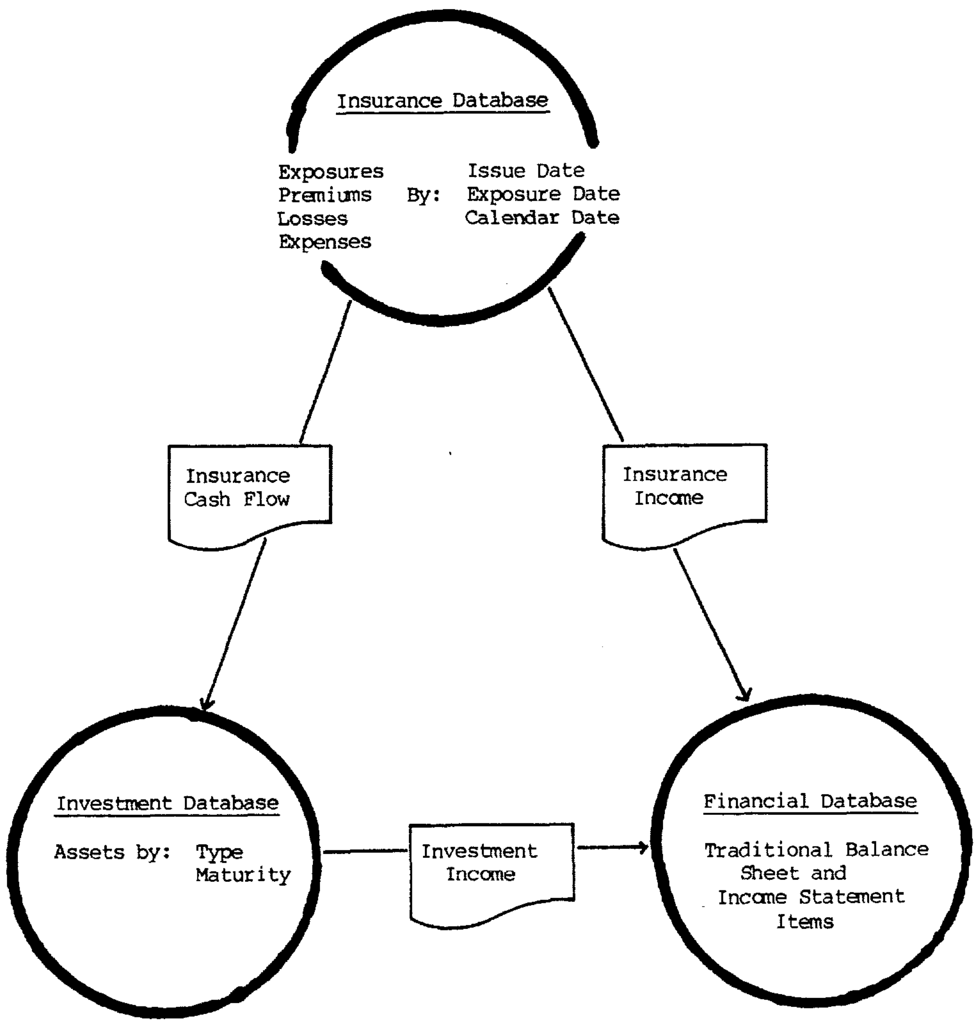
- An insurance database containing actual and projected exposures, premiums, losses and expenses, for each product line.

- An investment database containing actual and projected assets by type (taxable bond, tax-exempt bond, common stock, etc.) and maturity.

- A financial database containing traditional balance sheet and income statement items.

These three databases would interact: actual and forecast results from one database would serve as input to the others.

EXHIBIT II - A COMPREHENSIVE PLANNING SYSTEM



The insurance financial planning function (including macro-pricing and monitoring) is housed entirely within the uppermost database in Exhibit II. The analysis of actual results, and the forecast of future results would be by issue year or exposure year, (or both), depending on the specific design of the forecasting model for the line. (For major lines, it may be desirable to maintain greater than annual detail, e.g., issue quarter and/or exposure quarter).

The main objective of the planning exercise is the generation of the insurance cash flow, which is the principal input to the investment department for investment planning purposes. The investment department's plan involves the selection of a maturity profile appropriate to the cash flow forecast.

Both the investment and insurance areas can generate accrual forecasts in addition to cash flows. These can be fed into the financial database that produces forecasts of traditional accounting results. While ancillary to the first two steps, this last step is probably necessary, at least for tax planning purposes.

Given these three databases and modern database management techniques reports in a near-infinite variety of formats can be generated. Those reports should be designed to support either the macro-pricing, financial planning or monitoring of the line.

SUMMARY

In any business, decisions must be supported by adequate management information. The financial planning exercise is the support tool by which management assesses the current and future prospects of each product being sold by the company. It is imperative that the financial planning exercise be well defined, and reflect the true economics of the enterprise. Decisions should not be driven by either statutory or GAAP reporting requirements, either in their current or future forms. These requirements should, instead, be viewed on a constraint that must be satisfied. (Admittedly, an important constraint, but nonetheless a constraint).

Pricing, financial planning and reserving all involve forecasts of future transactions of the insurance company (hence they are <u>all</u> actuarial). Differences between the forecasts stem from timing differences. None of the differences suggest that the fundamental approaches to each forecast should be different.

The author has argued that pricing and financial planning should be performed on an issue year, or alternatively on an exposure year basis. Forecasts should be made that reflect both the timing and the amount of future cash flows. Within the insurance operation, investment income should be based on simple, fixed interest rate assumptions consistent with those used in pricing the product.

Finally, the monitoring of insurance results should not be on the basis of calendar year accounting results. Rather, comparisons between actual and planned results should be maintained by issue period or exposure period.

THE CASH FLOW OF A RETROSPECTIVE RATING PLAN

by

Glenn Meyers

Glenn Meyers is an associate professor in the Department of Statistics and Actuarial Science at the University of Iowa. He holds a Ph.D. in Mathematics from SUNY at Albany, M.A. in Mathematics from Oakland University, and B.S. from Alma College in Mathematics and Physics. Glenn is a Fellow of the Casualty Actuarial Society. He currently serves the CAS on the Committee on Theory of Risk, the Committee on Review of Papers and the Examination Committee. Previous papers include: "An Analysis of Retrospective Rating;" "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions" (with Philip Heckman); "Parameter Uncertainty in the Collective Risk Model" (with Nathaniel Schenker) and "Empirical Bayesian Credibility in Workers' Compensation Classification Ratemaking."

Abstract

With current methodology, the parameters of a retrospective rating plan are calculated to place the plan in balance on an underwriting basis. This paper provides a way of calculating the present value of the retrospective premium. Using this methodology, one can compare the expected profitability of various retrospective rating plans on a dicounted or operating basis. This includes paid loss retros. It is also possible to determine the parameters of a plan that will yield a predetermined operating profit.

1. Introduction

In recent years, the state of the property and casualty insurance industry could be characterized by three highs: high combined ratios; high interest rates; and a high degree of competition. Insurance company managers know that a great deal of investment income can be made by writing insurance, and they are willing to lower prices in order to do this.

The question to be asked, then, is how much can rates be lowered in order to still maintain an acceptable overall profit? It should be noted that in practice, actuaries do not have complete control of the pricing process. Underwriting and marketing personnel have considerable input. If actuaries do not calculate the contribution of investment income to the profitability of a line of insurance, someone else will. And the resulting "calculation" may amount to no more than a reaction to competitive pressures.

The question is not whether to use investment income in the calulation of rates. Instead the question is how to use investment income in the calculation of rates.

This paper considers the effect of investment income in choosing the parameters of a retrospective rating plan. With current methodology, the parameters of a retrospective rating plan are chosen to place the plan in balance on a nominal, or underwriting basis. By this we mean that the expected retrospective premium is equal to the sum of the losses, expenses and the anticipated profit. However it is possible for different plans to have the same expected premium and have different cash flows.

For example, a plan with a no maximum will have premium flowing in as long as losses develop, while a plan with a low maximum will stop producing premium as the insured breaks the maximum. Not all insureds will break the maximum, but there will, on average, be a faster premium flow because of the higher basic and the increased number of insureds who do break the maximum.

Other factors, such as the loss conversion factor and the minimum premium factor will also affect the cash flow of a retrospective rating plan.

This paper will provide a way of calculating the present value of the retrospective premium. Using this methodology, one can compare the profitability of various retrospective rating plans on a discounted or operating basis. This includes paid loss retros. It is also possible to calculate parameters of a plan that will yield a predetermined operating profit.

The principal tool used will be the collective risk model. Excess pure premiums will be calculated for the insured at various stages of development. One can then calculate the expected retrospective premium at each stage, and obtain the present value of the retrospective premium.

This technique will enable the insurer to offer a standard incurred loss retro which is competitive with a paid loss retro. This could help relieve some of the pressure that the Internal Revenue Service is putting on paid loss retros. In addition it will become possible to properly price a retro with loss development factors. This will minimize the size of retrospective adjustments as time passes.

We begin by first defining the parameters of a retrospective rating plan.

2. The Parameters Defined

The retrospective premium, R, for an insured is given by the following formula[1].

$$R = (B + c \cdot E + c \cdot L) \cdot t$$

R is subject to a maximum of G and a minimum of H.

B is the basic premium. Traditionally B covers general expenses, profit and the insurance charge (i.e. the net cost of the minimum and maximum premium provisions). There is no particular reason why B has to be set equal to these cost provisions. In its pure form, B is simply an amount that is used to determine the retrospective premium.

The factor c is called the loss conversion factor. Traditionally c covers the loss adjustment expenses. Again, there is no reason why it has to be set equal to a loss adjustment factor. In its pure form, c is simply a factor used to determine the retrospective premium.

Many retrospective rating plans provide that no claim amount over a specified loss limit shall be used to calculate the retrospective premium. In this case, the expected loss resulting from this provision must be added to the retrospective premium. This amount is denoted by E.

L represents the actual losses incurred under the plan.

Premium taxes are provided for by the factor t.

In order to keep this paper as simple as possible, we will not consider the effect of loss limits and premium taxes until the end of the paper. We shall also ignore the minimum premium. This results in a simplified formula for the retrospective premium.

$$R = B + c \cdot L$$

subject to the maximum, G.

The timing of the retrospective premium payments is of particular importance. Recall that some claims are open a long time before final settlement. Thus incurred losses are necessarily estimates of the final claims costs. Experience has shown these estimates are usually low, and so one should expect the retrospective premium to increase over time. The first calculation is based on losses reported eighteen months after the effective date of the policy. Subsequent calculations are performed on a yearly basis. Payments typically lag three months behind the retrospective premium calculations.

It is usually required to make some sort of premium payment before the first retrospective adjustment. Traditionally, this payment has been equal to the standard premium due on the effective date of the policy. More recently, the trend has been to pay an amount totaling less than the standard premium in installments.

We will be following a single hypothetical insured throughout this paper. The loss and expense information for this insured is given in the following table.

Table 1

	Nominal	Present Value at 8%
Expected Incurred Losses	1000000	820000
Expected Loss Adj. Exp.	100000	87000
Other Expenses	57500	55000
Total	1157500	962000

The expected incurred losses for each retrospective adjustment period are given in the following table.

Table 2

Retrospective Adjustment	Expected Incurred Losses
#1 @ 18 months	833333
#2 @ 30 months	946970
#3 @ 42 months	975610
#4 @ 54 months	986193
#5 @ 66 months	991080
#6 @ 78 months	996016
#7 @ 90 months	1000000

In order to calculate the average retrospective premium, one needs to have tables of excess pure premiums which correspond to each retrospective adjustment. These tables are provided in Exhibit I. The Heckman-Meyers algorithm[2] was used to generate these tables. While the input for this algorithm could be provided, it seems just as easy to assume the tables are given.

These tables provide excess pure premiums for loss amounts in increments of 10000. Linear interpolation can be used to calculate excess pure premiums for loss amounts that are not a multiple of 10000.

The average retrospective premium is calculated in the following manner[3]. Define the effective maximum to be equal to (G - B)/c, and let X be the excess pure premium for losses over the effective maximum. Then the average retrospective premium is given by:

$$E[R] = B + c\cdot(E[L] - X).$$

As an example, assume B = 232450, G = 1500000, c = 1.1, and E[L] = 1000000. Then the effective maximum = 1152320. By linear interpolation on Exhibit I (90 months), we find X = 131775, and E[R] = 1187500.

3. The Standard Incurred Loss Retro

We first calculate the expected underwriting profit for a standard incurred loss retro. We need only consider the seventh (final) retrospective adjustment for this calculation.

Table 3

Basic	232450
l.c.f.	1.1
Maximum	1500000
E[R] @ 90 mths.	1187500
Loss & Expense	1157500
Underwriting Profit	30000

This plan was designed to yield approximately the 2.5% underwriting profit that is budgeted in standard Workers' Compensation rate filings.

Next we calculate the expected operating profit for the same plan assuming an effective annual interest rate of 8%. That is to say, for example, that a payment due in three months is discounted at a rate of $1.08^{0.25}$. A deposit premium of 960000 is to be payable in six quarterly installments of 160000. The present value of the deposit premium is 915410. Additional amounts of premium due to retrospective adjustments are assumed to be paid three months after the calculation of the retrospective premium.

Table 4

Basic	232450
l.c.f.	1.1
Maximum	1500000
Deposit	960000
E[R] @ 18 mths.	1078380
@ 30 mths.	1155720
@ 42 mths.	1173210
@ 54 mths.	1179480
@ 66 mths.	1182340
@ 78 mths.	1185200
@ 90 mths.	1187500
P.V. Retro Premium	1103720
P.V. Loss & Expense	962000
Operating Profit	141720

In this example we see that the standard rating method yields an operating profit of nearly 12% of the ultimate average retrospective premium. This is fine if the competition will allow it. If not, the insurance company management must decide what operating profit to seek.

Suppose they decide to seek an operating profit of 100000. Perhaps there is a vague notion that an underwriting profit of 30000 already anticipates a certain amount of investment income, and is not appropriate for an operating profit. Anyway, the question becomes one of selecting the basic premium that yields the desired operating profit. This can be done by repeating the calculations of Table 4 on a trial and error basis, although a numerical method may yield the desired solution more quickly[4]. The results of this process are in the following Table.

Table 5

Basic	167150
l.c.f.	1.1
Maximum	1500000
Deposit	960000
E[R] @ 18 mths.	1024100
@ 30 mths.	1106410
@ 42 mths.	1125210
@ 54 mths.	1131970
@ 66 mths.	1135050
@ 78 mths.	1138140
@ 90 mths.	1140620
P.V. Retro Premium	1062000
P.V. Loss & Expense	962000
Operating Profit	100000

Having demonstrated how to select the basic premium which yields a predetermined operating profit, it should be pointed out that it is possible to fix the basic premium and select the loss conversion factor which yields a predetermined operating profit.

Certain other cash flow provisions of a retrospective rating plan are often subject to negotiation between insurer and insured. Thus it seems appropriate that we show how to properly account for them.

4. Retro Development Factors

An optional provision of most retrospective rating plans is to adjust the incurred losses to their ultimate value by means of a loss (or retro) development factor. An advantage to the insured is that the retrospective premium is close to its ultimate value at the first retrospective adjustment. A disadvantage is that the insured must pay the premium sooner. To overcome this disadvantage, the insurer can offer to lower either the basic premium or the loss conversion factor.

In the following table we consider the latter option. The deposit premium is to be paid in installments as before. Although several retrospective adjustments are made, the contribution of the later adjustments is assumed to be negligible. The final table of excess pure premiums (evaluated at 90 months) was used to calculate the average retrospective premium at the first adjustment.

Table 6

Basic	167150
l.c.f.	1.0775
Maximum	1500000
Deposit	960000
E[R] @ 18 mths.	1127730
P.V. Retro Premium	1062000
P.V. Loss & Expense	962000
Operating Profit	100000

The results of this calculation should be directly comparable with the previous calculation (Table 5). The introduction of retro development factors caused about a

1.1% decrease in the average retrospective premium on a nominal basis.

The accuracy of this calculation depends upon our ability to calculate the proper loss development factors. Even if we get the correct overall loss development factors, changes in the shape of the aggregate loss distribution over time will affect the average retrospective premium. The author suspects that the result, over time, will be a thicker tail for the aggregate loss distribution, a higher excess pure premium and a slight decrease in the average retrospective premium. Losses which are revalued upward will be limited by the maximum premium, while losses which are valued downward will be unaffected. A full treatment of this effect is beyond the scope of this paper.

5. Paid Loss Retros

A very popular rating plan in recent years has been the so called "paid loss retro." While the details of the financial transactions may vary, a typical plan could work as follows. A basic premium is paid, possibly in installments. The retrospective premium based on paid losses is continuously paid from a special fund set up by the insured. At some point in time, usually 54 months after the effective date, the plan switches over to an ordinary incurred loss retro.

The continuous adjustment of the retrospective premium presents a technical problem. There is always the possibility that the insured will break the maximum on paid losses before the 54 month switchover. This could, in theory, require daily tables of excess pure premiums. In practice, the possibility of breaking the maximium before the switchover is considered remote, and is ignored in the following calculations. The average retrospective premium can then be estimated using ordinary loss payout patterns.

The effect of this simplifying assumption would be to overstate the average retrospective premium before the switchover. It will be corrected at the 54 month adjustment. The end result will be to overstate the present value of the average retrospective premium by the amount of interest earned on the excess pure premium before the switchover. This should be a negligibile amount.

Let us assume that our hypothetical insured is expected to have paid 800000 in losses by the switchover time, and that the present value of these payments is 720000. Let us also assume that the basic premium is paid on the effective date of the plan. The following table describes the plan in detail.

Table 7

Basic	215170
l.c.f.	1.1
Maximum	1500000
E[Paid R]	1095170
E[R] @ 54 mths.	1167130
@ 66 mths.	1170050
@ 78 mths.	1172980
@ 90 mths.	1175320
P.V. E[Paid R]	1007170
P.V. Retro Premium	1062000
P.V. Loss & Expense	962000
Operating Profit	100000

The results of this calculation should be directly comparable to the straight incurred loss retro (Table 5). The paid loss provision caused about a 3% increase in the average retrospective premium on a nominal basis.

5. Excess Loss Premium and Tax Multiplier

We did not consider the excess loss premium or the tax multiplier in the above calculations. The intent was to keep the discussion as simple as possible. We now show how to modify the calculation to take these into account.

On the premium side of the calculation, the only adjustment needed to handle the loss limit is to input a limited claim severity distribution into the Heckman-Meyers algorithm. No adjustment is needed on the loss and expense side. Make note that the present value of the unlimited losses is still used.

A wrinkle in the above adjustment occurs when the excess layer is reinsured and one wants to incorporate the cost of reinsurance in the pricing. In this case one takes the sum of the present value of the limited losses, and the cost of the reinsurance. This sum is used in place of the present value of the unlimited losses. A note of caution: the payout pattern for limited losses is faster than that of unlimited losses.

Premium taxes are paid on the basis of written premium. One should note that retrospective adjustments are also adjustments in written premium. The present value of the premium taxes can be calculated by using the average retrospective premium at each adjustment.

The following question should be asked at this point. Do we really need to have separate factors in the retrospective rating plan for excess losses and premium taxes?

Tax multipliers are not used in guaranteed cost plans, so why use them for retrospective rating? Rates for other guaranteed cost plans reflect premium taxes, and so could the basic premium and the loss conversion factor.

Skurnick[5] put the excess premium into the basic premium for the California Table L, and there is no reason why this could not be done for all retrospective rating plans.

What really matters is that the present value of the retrospective premium is equal to the profit plus the present value of the losses and expenses. This can be accomplished by a proper selection of the basic premium and the loss conversion factor. The result will be a simpler formula for retrospective rating.

6. Conclusion

This paper is written under the premise that an explicit calculation of investment income is superior to the implicit recognition of investment income that some say is in many present rating formulas. We do not attempt to determine the proper operating profit. This task belongs to insurance company management and/or regulators. It does not belong to some ratemaking formula based on underwriting profit.

We have provided a methodology for finding the expected operating profit for a retrospective rating plan. This methodology is presently in use at a major insurance company.

The author suspects that the more complicated versions of retrospective rating, such as paid loss retros, arose because the present plan does not allow for investment income. Now that the various versions of retrospective rating can be rated on a comparable basis, it is hoped that the more complicated versions will no longer be necessary. Retrospective rating can be made simple.

7. Acknowledgments.

This paper is an outgrowth of a project which I directed during my final year at CNA Insurance Companies. I worked very closely with John Meeks and Steve Maguire in developing the conceptual basis for what we called the "Account Pricing System." Steve and Ron Swanstrom wrote a program which made these ideas very workable in a production environment.

Exhibit I - Excess Pure Premiums

LOSSES VALUED AT 18 MONTHS			LOSSES VALUED AT 30 MONTHS		
EXPECTED LOSSES = 833333			EXPECTED LOSSES = 946970		
LOSS AMOUNT	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM	LOSS AMOUNT	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM
900000	0.6508	129345	900000	0.5469	196000
910000	0.6594	125896	910000	0.5561	191516
920000	0.6678	122532	920000	0.5653	187123
930000	0.6760	119251	930000	0.5742	182820
940000	0.6840	116051	940000	0.5831	178607
950000	0.6919	112930	950000	0.5918	174481
960000	0.6996	109887	960000	0.6003	170442
970000	0.7071	106920	970000	0.6088	166487
980000	0.7144	104028	980000	0.6170	162616
990000	0.7216	101208	990000	0.6252	158827
1000000	0.7286	98459	1000000	0.6332	155119
1010000	0.7355	95780	1010000	0.6410	151490
1020000	0.7422	93168	1020000	0.6487	147939
1030000	0.7488	90623	1030000	0.6563	144464
1040000	0.7552	88143	1040000	0.6638	141064
1050000	0.7614	85726	1050000	0.6711	137739
1060000	0.7675	83371	1060000	0.6782	134485
1070000	0.7735	81076	1070000	0.6853	131303
1080000	0.7793	78840	1080000	0.6922	128190
1090000	0.7850	76662	1090000	0.6989	125145
1100000	0.7906	74540	1100000	0.7056	122168
1110000	0.7960	72473	1110000	0.7121	119256
1120000	0.8013	70459	1120000	0.7185	116409
1130000	0.8065	68498	1130000	0.7247	113625
1140000	0.8115	66588	1140000	0.7309	110903
1150000	0.8165	64728	1150000	0.7369	108241
1160000	0.8213	62917	1160000	0.7427	105639
1170000	0.8260	61153	1170000	0.7485	103095
1180000	0.8306	59435	1180000	0.7542	100609
1190000	0.8350	57763	1190000	0.7597	98178
1200000	0.8394	56135	1200000	0.7651	95802
1210000	0.8436	54550	1210000	0.7704	93479
1220000	0.8478	53007	1220000	0.7756	91209
1230000	0.8519	51505	1230000	0.7807	88991
1240000	0.8558	50043	1240000	0.7857	86823
1250000	0.8597	48620	1250000	0.7906	84704
1260000	0.8634	47235	1260000	0.7954	82634
1270000	0.8671	45887	1270000	0.8001	80611
1280000	0.8707	44576	1280000	0.8046	78635
1290000	0.8742	43300	1290000	0.8091	76703
1300000	0.8776	42058	1300000	0.8135	74816

Exhibit I - Excess Pure Premiums

LOSSES VALUED AT 42 MONTHS

EXPECTED LOSSES = 975610

LOSS AMOUNT	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM
900000	0.5218	214600
910000	0.5311	209865
920000	0.5403	205223
930000	0.5494	200672
940000	0.5584	196210
950000	0.5672	191838
960000	0.5759	187553
970000	0.5844	183355
980000	0.5928	179241
990000	0.6011	175211
1000000	0.6093	171263
1010000	0.6173	167396
1020000	0.6252	163608
1030000	0.6330	159899
1040000	0.6406	156267
1050000	0.6481	152711
1060000	0.6555	149229
1070000	0.6627	145820
1080000	0.6698	142483
1090000	0.6768	139216
1100000	0.6837	136019
1110000	0.6904	132889
1120000	0.6970	129826
1130000	0.7035	126829
1140000	0.7099	123895
1150000	0.7161	121025
1160000	0.7222	118216
1170000	0.7282	115468
1180000	0.7341	112779
1190000	0.7399	110149
1200000	0.7455	107576
1210000	0.7511	105058
1220000	0.7565	102596
1230000	0.7618	100188
1240000	0.7670	97832
1250000	0.7722	95528
1260000	0.7772	93274
1270000	0.7821	91070
1280000	0.7869	88915
1290000	0.7916	86808
1300000	0.7962	84747

LOSSES VALUED AT 54 MONTHS

EXPECTED LOSSES = 986193

LOSS AMOUNT	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM
900000	0.5127	221641
910000	0.5221	216815
920000	0.5313	212081
930000	0.5404	207440
940000	0.5493	202888
950000	0.5582	198426
960000	0.5669	194051
970000	0.5755	189763
980000	0.5840	185560
990000	0.5923	181442
1000000	0.6005	177406
1010000	0.6086	173452
1020000	0.6166	169578
1030000	0.6244	165782
1040000	0.6321	162065
1050000	0.6397	158423
1060000	0.6471	154857
1070000	0.6544	151365
1080000	0.6616	147945
1090000	0.6686	144596
1100000	0.6756	141317
1110000	0.6824	138106
1120000	0.6891	134963
1130000	0.6956	131887
1140000	0.7021	128875
1150000	0.7084	125927
1160000	0.7146	123042
1170000	0.7207	120218
1180000	0.7266	117454
1190000	0.7325	114749
1200000	0.7382	112103
1210000	0.7438	109513
1220000	0.7494	106978
1230000	0.7548	104499
1240000	0.7601	102073
1250000	0.7653	99700
1260000	0.7704	97378
1270000	0.7754	95106
1280000	0.7803	92884
1290000	0.7851	90711
1300000	0.7898	88585

Exhibit I - Excess Pure Premiums

LOSSES VALUED AT 66 MONTHS

EXPECTED LOSSES = 991080

LOSS AMOUNT	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM
900000	0.5086	224922
910000	0.5179	220054
920000	0.5271	215279
930000	0.5362	210595
940000	0.5452	206002
950000	0.5540	201499
960000	0.5628	197083
970000	0.5714	192754
980000	0.5799	188510
990000	0.5883	184351
1000000	0.5965	180275
1010000	0.6046	176280
1020000	0.6126	172366
1030000	0.6204	168531
1040000	0.6282	164774
1050000	0.6358	161094
1060000	0.6432	157489
1070000	0.6506	153957
1080000	0.6578	150499
1090000	0.6649	147112
1100000	0.6718	143796
1110000	0.6787	140548
1120000	0.6854	137368
1130000	0.6920	134255
1140000	0.6985	131207
1150000	0.7048	128223
1160000	0.7110	125302
1170000	0.7172	122443
1180000	0.7232	119645
1190000	0.7291	116906
1200000	0.7348	114225
1210000	0.7405	111601
1220000	0.7460	109034
1230000	0.7515	106522
1240000	0.7568	104063
1250000	0.7621	101658
1260000	0.7672	99304
1270000	0.7723	97001
1280000	0.7772	94748
1290000	0.7820	92544
1300000	0.7868	90388

LOSSES VALUED AT 78 MONTHS

EXPECTED LOSSES = 996016

LOSS AMOUNT	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM
900000	0.5044	228254
910000	0.5137	223345
920000	0.5229	218528
930000	0.5320	213803
940000	0.5410	209168
950000	0.5499	204622
960000	0.5586	200165
970000	0.5673	195795
980000	0.5758	191510
990000	0.5842	187310
1000000	0.5924	183193
1010000	0.6006	179158
1020000	0.6086	175203
1030000	0.6164	171328
1040000	0.6242	167532
1050000	0.6318	163812
1060000	0.6393	160167
1070000	0.6467	156597
1080000	0.6539	153100
1090000	0.6611	149675
1100000	0.6681	146321
1110000	0.6749	143036
1120000	0.6817	139818
1130000	0.6883	136668
1140000	0.6948	133584
1150000	0.7012	130564
1160000	0.7075	127607
1170000	0.7136	124712
1180000	0.7197	121879
1190000	0.7256	119105
1200000	0.7314	116390
1210000	0.7371	113732
1220000	0.7427	111131
1230000	0.7482	108585
1240000	0.7536	106094
1250000	0.7588	103656
1260000	0.7640	101270
1270000	0.7691	98936
1280000	0.7741	96651
1290000	0.7789	94416
1300000	0.7837	92229

Exhibit I - Excess Pure Premiums

LOSSES VALUED AT 90 MONTHS

EXPECTED LOSSES = 1000000

LOSS AMOUNT	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM
900000	0.5010	230957
910000	0.5103	226014
920000	0.5195	221163
930000	0.5287	216405
940000	0.5377	211736
950000	0.5465	207157
960000	0.5553	202667
970000	0.5640	198263
980000	0.5725	193945
990000	0.5809	189712
1000000	0.5892	185562
1010000	0.5973	181494
1020000	0.6053	177508
1030000	0.6132	173600
1040000	0.6210	169771
1050000	0.6286	166020
1060000	0.6362	162344
1070000	0.6436	158742
1080000	0.6508	155214
1090000	0.6580	151758
1100000	0.6650	148373
1110000	0.6719	145057
1120000	0.6787	141810
1130000	0.6853	138630
1140000	0.6919	135516
1150000	0.6983	132467
1160000	0.7046	129481
1170000	0.7108	126558
1180000	0.7168	123696
1190000	0.7228	120894
1200000	0.7286	118151
1210000	0.7344	115466
1220000	0.7400	112837
1230000	0.7455	110265
1240000	0.7509	107747
1250000	0.7562	105283
1260000	0.7614	102871
1270000	0.7665	100511
1280000	0.7715	98201
1290000	0.7765	95941
1300000	0.7813	93729

Notes

1. National Council on Compensation Insurance, Retrospective Rating Plan D.

2. Heckman, P. E. and Meyers, G.G. "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions," PCAS LXX, 1983, p. 22.

3. Fiebrink, M. E. Discussion of "An Analysis of Retrospective Rating" (by Glenn Meyers), PCAS LXVIII, 1981, p. 113.

4. Burden, R. L., Faires, J. D., and Reynolds, A. C. Numerical Analysis, 2nd Edition, Prindle, Weber & Schmidt, 1981, Ch. 2.

5. Skurnick, D. "The California Table L," PCAS LXI, 1974.

Title: APPLICATION OF PRINCIPLES, PHILOSOPHIES AND PROCEDURES OF CORPORATE PLANNING TO INSURANCE COMPANIES

Author: Mary Lou O'Neil

Biography: The author is the Chief Actuary for Property/Casualty and Life/Health Insurance at the New Jersey Department of Insurance and was previously Vice President and Assistant Actuary at the Prudential Property and Casualty Insurance Company. The author is a Fellow of the Casualty Actuarial Society, and a member of the American Academy of Actuaries and the Casualty Actuaries of New York. The author holds a B.S. in Mathematics and an M.A. in Statistics, both from The Pennsylvania State University. The author is a Contributing Editor to the Actuarial Review, a member of the External Communications Committee, a member of the American Academy Committee on Property Liability Insurance; and a prior contributor to the Discussion Paper Program.

Abstract: The purpose of this paper is to illustrate the need for development of unique applications of general corporate planning concepts to insurance companies. This is accomplished by briefly reviewing the principles, philosophies and procedures of corporate planning as applied to corporations in general, the unique aspects of insurers and the insurance business, and, finally presenting examples of the unique application of general corporate planning concepts to insurers.

I. INTRODUCTION AND PURPOSE

In researching the subject of corporate planning, I found a multitude of well developed material providing detailed principles, philosophies and procedures of corporate planning for corporate entities in general. Although these broad concepts apply to insurers as corporate entities, insurers and their businesses possess unique characteristics which require correspondingly unique applications of these general concepts. The purpose of this paper is to demonstrate this need by illustrating some of these unique applications in the context of the broad corporate planning concepts. This paper is, however, only a beginning in this endeavor and is not intended to present a complete identification of all aspects of the planning process unique to insurers.

II. CORPORATE PLANNING

As background for exploring the applicability of corporate planning principles, philosophies and procedures to insurers, this section presents a brief review of the principles, philosophies and procedures of corporate planning currently applied to corporate entities in general.

A. Need for Planning

The first question often asked is why plan at all? Basically, business planning parallels personal planning. In our everyday activity we anticipate the outcomes of various actions so that we can select the action which will result in the most desirable outcome. So too for business. Forecasts and plans are made to illuminate today's decisions -- to provide a process to make decisions now about what action(s) to take in the future.[1]

B. Definition of Planning and Planning Terminology

In reviewing many definitions of planning, I found that there were four common elements to these definitions. These are:

- A company must determine its goals and objectives.

- Planning is a way to chart a path so that the company can achieve those goals.

- Planning involves decision making before a crisis occurs -- the future is anticipated and alternative courses of action are evaluated in a structured way.

- Planning is a multifaceted process involving the entire organization.

These elements were best encompassed in the following definition:

> "Planning is an analytical process which encompasses an assessment of the future, the determination of desired objectives in the context of that future, the development of alternative courses of action to achieve such objectives, and the selection of a course (or courses) of action from among these alternatives."[2]

For ease of reference plans may be classified into three categories: (1) Subject area, referring to plans in terms of observable specs such as marketing plans, financial plans and production plans, (2) Scope, referring to the range of influence and amount of detail of the plans such as goals or policies and (3) Time, referring to the duration of the plans (long-range, mid-range, short-range). Certain common planning terms such as policies, strategies and tactics, overlap the scope and time categories and may be distinguished as follows. Policies are overall general guides to action or basic long term precepts of company philosophy.[3] Strategies, however, represent the overall plans of action set by top management of the company[4] while tactics are relatively short term plans set by lower levels of management in an attempt to implement strategies.[5]

In developing corporate plans the organization first sets its long term policies and strategies and then develops appropriate tactics and short range operational plans necessary to achieve these overall goals. This represents the essence of the corporate planning process described in the next section.

C. The Corporate Planning Process

The corporate planning process is generally recognized to consist of the following steps:

- Establish objectives or goals.
- Develop basic planning assumptions or premises.
- Identify alternative courses of action.
- Evaluate alternative courses of action.
- Implement the plan (select a course(s) of action).[6, 7, 8]
- Control[9]

For interested readers these steps are described in more detail below.

1. Establishment of Goals and Objectives

This stage of the process can be viewed narrowly by defining objectives so that the only objective of any company is to make a profit.[10] Or from a broader view, objectives are defined as long range desired states or outcomes such as desired market shares[11] while goals are shorter range and are considered to be milestones along the path toward achievement of objectives. In this phase of the planning process management must first determine its overall objectives and their translation into goals such as target profit and then provide operational definitions of each goal and a way to measure progress toward their achievement.[12] This is probably the most important phase of the planning process since it charts the company's long range course of activity and is generally completed at the executive level.

2. Development of Planning Assumptions

In this phase the company must try to explicitly identify the universe of conditions which may influence its operation or affect the achievement of objectives. These assumptions should reflect noncontrollable conditions such as business cycles, semicontrollable conditions such as market share or internal pricing policy, and controllable conditions such as whether to change product line or pricing strategy.[13]

3. Identification of Alternative Courses of Action

Based on the planning assumptions, the company then, forecasts its expected results given no new actions are to be taken. This is generally achieved using a system of linked together models, representing various aspects of the firm such as the firm itself, supply, sales, consumers, competition, environment and financial. Using variations of the original planning assumptions, based on possible factors which may influence results, the company can identify the results associated with different actions. The purpose of this phase of the process is to identify the range of actions available to the company, the expected results associated with each, and an assessment of the risk or margin of error inherent in each projection.[14]

4. Evaluate Alternative Courses of Action

The company must evaluate its array of possible actions identified in phase three in light of its moral policies, resources or other constraints to action.[15]

5. Implement the Plan

Once the company has selected its course of action it must be translated into specific policies, programs, procedures and practices by which the desired objectives and goals may be achieved.[16] This stage is generally considered the short range or operational phase of the process and is carried out by managers thruout the organization.[17]

6. Control

In order to assure achieving its selected plans, the company must establish check points to indicate whether it is on course. This requires continual monitoring and correction in the form of revised plans whenever significant deviations occur.[18] In fact, the planning process may appropriately be considered an iterative one -- continually changing as the need arises. The required monitoring is most easily achieved via a management information system designed to parallel the corporate planning model.

D. Organization for Planning

As noted above, certain aspects of the planning process are properly carried out by top management -- setting objectives, policies and strategies, while others are properly carried out by individual managers -- setting short term operational goals. The key to success of the planning process is participation thruout the organization.[19] The ideal organization for planning consists of an individual corporate planner coordinating the plans of operational managers -- there is no corporate planning department or specialized group of planners.[20]

III. THE APPLICATION OF CORPORATE PLANNING TO INSURANCE COMPANIES

The preceding section presented an overview of corporate planning as it currently relates to corporations in general. Several authors illustrate the need to conform these basic concepts to individual industries and individual companies within an industry using the results of a study of the motor freight industry from which they found that regular long-range planning had little correlation with profitability. They concluded that the reason for this was the regulated nature of the industry -- markets, prices, and labor policies were stringently controlled leaving only day-to-day operations to management control. Thus, these companies could get by with ad hoc planning for acquisition or rate increases. The authors further point out that planning needs differ among different organizations and among the various levels in an organization. For example, a young company may require only a simple plan while a mature company in the same industry will need a formal planning function, or a small high-tech firm may desparately need plans while a large firm in a static industry may have less need. The planning needs of any organization are individual and situational.[21]

What does this mean for the role of planning for insurance companies? In order to answer this question, the next sections identify the unique characteristics of the insurance industry and its companies, and the consequent unique application of general corporate planning concepts to insurers.

IV. UNIQUE CHARACTERISTICS OF THE INSURANCE INDUSTRY

The insurance industry shares many characteristics with other industries but is also quite unique in many respects. Both types of characteristics are described below.

Insurers are companies engaged in the sale of the product, insurance. They share the characteristics of corporations in other industries such as a corporate organizational structure including shareholders (stock companies), board of directors, and officers. However, at the same time, they differ greatly from similar sized companies in other industries. The primary sources of these differences are in the uniqueness of the insurance product and the significant role of regulation on nearly all aspects of an insurer's operation.

The insurance product differs from most products in two main ways, it is intangible and its cost of production is unknown until, often times, long after its sale. The intangible nature of the product likens insurers to other service industries. However, the uncertainty with respect to product cost remains a significant difference.

The other, and probably most important difference between insurers and other corporate entities is close regulation of many aspects of the business. This includes requirements for licensure to sell various insurance products, requirements to retain specified minimum capital and surplus, specification of the kinds and amounts of investment holdings, limitations on the amount of business which may be sold, rules regarding to whom the business must be sold (such as mandatory insurance or "take-all comers" laws and maximum premium to surplus ratios), rules regarding estimation of the final cost of the product, rules specifying the product itself (such as mandatory contract language), price regulation, and, finally limitation on rate of return. Because of these stringent guidelines the industry is often compared to the utility industry. However, this comparison is defective because that industry is provided a guaranteed rate of return in exchange for its regulation while insurers are limited only in the maximum return they may earn with no minimum.

In addition to regulation, insurers are set apart from other businesses because of their unique jargon (e.g. frequency, severity, pure premium, etc.) and unique accounting practices (e.g. deduction of pre-paid expenses and unpaid claims, etc.).

Insurers further differ among themselves in their organizational structure (stock vs. mutual), sales organization structure (direct writers, independent agency system), product mix (personal, commercial, reinsurance, etc.), etc.

However, despite the constraints of regulation, there is intense competition, particularly in property/casualty insurance. This competition is the result of the maturity of the insurance product reflected in the saturation of the major insurance markets (more than 90% of individuals are covered by life insurance and more than 96% of workers are covered by group life and health insurance) and relatively low growth (1 to 3% per year) in the number of insurable objects such as people, cars, houses, businesses, etc., per year.[22]

V. CORPORATE PLANNING -- AND ITS APPLICATION TO INSURANCE COMPANIES

The earlier sections of this paper presented a brief overview of corporate planning -- its definition, its need, the process and its steps. This section illustrates some of the unique applications of general corporate planning concepts to insurance companies.

A. Need for Planning

Is planning needed at all by the insurance industry?

I believe that there is a role for planning in insurance companies. For example, despite the constraints of regulation, there is a need for the firm to establish its target profit or rate of return, to choose objectives such as market dominance, market (or product) emphasis, geographic emphasis, prices, attitude toward employees and clients, attitude toward social goals and insurance, and reactions to changes in regulatory or legal climate, etc. Wherever there is room for choice or change, there is room for planning.

Stated more strongly, insurance executives will be forced to reckon with significant changes in the forseeable future requiring the ability to innovate and respond quickly.[23] It is no longer possible to rely on the cyclical nature of the business and operational strategies to pull them through.[24]

B. Definition of Planning and Planning Terminology

Is the definition of planning provided in section II. B appropriate for insurers? Is there any unique meaning of the common planning terminology for insurers? What are the appropriate subject areas for insurer planning? What should the scope and planning horizon be? What should the level of detail of planning be?

The appropriateness of the definition of planning may be ascertained by examining each of the common elements of the various definitions of planning. The first element requires a company to identify its goals and objectives. I believe it is fundamental for insurers to identify goals and objectives particularly its target rate of return -- otherwise the company has no direction. The second element of the definition asserts that planning is a way for the company to chart a path towards its objectives. Of course objectives are worthless if the organization cannot identify a way to achieve them -- insurer or not. Third, planning is said to provide an opportunity to make decisions in advance. Insurers can benefit from this insulated decision making as well as other corporate entities. Finally, the definition identifies planning as a process. This would be true regardless of the entity employing it. Thus, the definition of planning applies to insurers without modification.

The categorization of plans into subject area, scope and time as well as the terminology associated with these planning categories may be taken as generic and hence applicable to insurers. Subject areas for insurance planning include, as for corporations in general, marketing plans, financial plans, and production plans.

The planning horizon will depend on the company's chosen lines of business and areas of operation. In determining the planning horizon for a property/casualty insurer it must be remembered that as of January 1 the results for the ensuing year are largely already determined because the book of business which will create those results has already been written. With restrictions on cancellations or non-renewals, there is a long period of time between action and result for a property/casualty insurer. Thus, the preparation of a one-year plan is a useful exercise which will point out the work to be accomplished over a longer term, say five years.[25] Thus, I believe there is an appropriate role for both short and long term plans, each designed to meet the specific needs of the individual organization. And, these two types of plans should be interdependent.

Plan scope can range from the broad statements of objectives to detailed plans regarding every aspect of the business (number of claims in a given geographic area in a given period of time for a given coverage, etc.) Planning must be done in sufficient detail in order to provide a basis to monitor and control results. There must be a balance between the detail used and the detail required or the cost versus the benefit to the specific organization.

C. The Planning Process -- Establishing Objectives and Goals

Section II. C.1. defines and distinguishes objectives and goals for corporations in general. What objectives and goals may be relevant to an insurer? Who should set the objectives and goals? What should be included in the objectives and goals? What should the level of detail of the objectives and goals be?

I believe that the objectives and goals relevant to an insurer, must certainly parallel those of most corporations, in general, to make a profit and to survive, and then, specifically, to the individual company, to gain certain market share, etc. The detailed specification of these goals may take on a different nomenclature due to the uniqueness of insurance jargon. For example, the goals of controlling claim costs or improving renewal ratios parallel goals for other businesses such as decrease production costs or maintain current customer accounts. Objectives and goals could, as for corporations, be completed either by top executives or by individual managers. I believe that the objective and goal formulation phase of the planning process establishes the path of the organization and, hence, should remain with top executives. Individual managers must be given a framework within which to establish short term operational goals.

Objectives and goals can be broad statements of direction or specific measurable achievements. I believe insurers have a need for both. Long range policy statements and overall broad organizational goals provide the framework for specific measurable goals to be completed by each manager.

The level of detail of objectives and goals should be sufficient to determine whether or not the desired result has been achieved.

D. The Planning Process -- Establishing Planning Assumptions

Section II. C.2. defines planning assumptions for corporations in general. What should these assumptions include for insurers? Who should prepare the assumptions?

Planning assumptions were defined to include all those conditions which may confront the organization. For insurers, general conditions of importance include inflation, interest rates, and regulatory and legislative changes. Dutter enumerates six key forces which will confront insurers in the foreseeable future: erratic swing in rates of inflation or interest, decline in attractiveness of other business combined with ease of entry to insurance, dramatic advances in EDP, unbundling of commercial property/casualty, group life/health, and pensions, and greater distribution efficiency.[26] Howland provides an extensive list of external influences which must be considered, which is reproduced as Exhibit I.[27] These fall into the category of noncontrollable forces.

Insurers should further enumerate controllable factors such as the number of employees, its anticipated market share, product mix, geographic emphasis, etc. These assumptions may take the form of numerical projections (discussed below) of new business sales, claim counts, etc., and provide the basis for input into the company's modeling system.

E. The Planning Process --Identification of Alternative Courses of Action

Section II. C.3. presented the process of identification of alternative courses of action for corporations in general. What is the proper role for a management information system in an insurance company? What aspects of the insurance firm should be modeled?

As noted in Section II. C.3, corporations generally create models reflecting various aspects of the firm linked together to form an overall model. Hylas states that for insurers, computer planning models may be used to systematize the planning process, create "what if" analyses, measure the possible effects of inflation on future earnings, evaluate the impact of state and other regulations, evaluate the effect of recent court decisions, for tax planning, projecting cash flow, structuring competitive rates, and determining the best mix of business.

He goes on to state that a successful planning model must be designed to meet the specific planning needs of the organization. There is no universal all purpose planning model. He does, however, provide a simplified outline of the basic elements which might be common to most insurer modeling systems. These are reproduced as Exhibit II.

He also asserts that the overall model consists of convenient submodels which are unique to the individual company. His possible input/output variables for each submodel are displayed on Exhibit III. The key to model design is to be sure that the model meets the user's needs.[28] Verrengia concurs with these thoughts, suggesting that for a medium size insurer models should focus on earned premiums, loss ratios, profit and loss planning and ratemaking.[29]

Hence, models are appropriate as well as useful tools as part of the planning process. I believe computerized models are key to the effectiveness of the planning process.

F. The Planning Process -- Evaluation of Alternative Courses of Action

Section II. C.4. noted that corporations must evaluate alternative courses of action in light of its constraints. Similarly, what criteria should be included in the action selection process for insurers?

An insurer, which sells a product affected by the public interest, must clearly identify its moral policies and first evaluate each action on this basis discarding those which are unacceptable. Other constraints might include available capital (premium to surplus ratio), maximum rate levels, or prohibited underwriting standards. I believe that each insurer must develop its own criteria for evaluation of its alternative actions. These criteria should be established before they are needed, and should be consistent with the company's other policies and philosophies. To the extent possible, the criteria should be objective and measurable.

G. The Planning Process -- Implementation

Section II. C.5. discussed implementation of the planning process for corporations in general. How should the plan be implemented by insurers?

This phase of the process refers to the implementation of the actions selected in the previous phase and applies to insurers as well as corporations. Policies, programs, procedures and practices must be established to provide individual managers with the actions which each must accomplish in order to achieve overall goals.

H. The Planning Process -- Control

Section II. C.6. discussed the need for control of implemented plans for corporations. What kind of analysis of results should be done, by whom, when? How does the plan provide for control?

As noted for corporations in general, without control, there was basically no reason to plan i.e. there would be no way for the company to know whether the actions it implemented have put it on its desired course toward achievement of its objectives or not. Reuter described the controlling process in his insurance company as consisting of two items: an objective or standard which is clearly stated in measurable terms and a timely assessment or measure of actual performance or results. These two items are incorporated into control reports at the individual manager level.[30]

I believe that control includes both of Reuter's items as well as a constant monitoring of key variables via a management information system designed to parallel the plan models, so that timely action may be taken in response to deviations -- positive or negative.

I. The Planning Process -- Organization for Planning

Section II. D. discussed the proper organization for planning. Who should plan? Should there be a Corporate Planning Department in insurance companies? What should be the involvement of individual managers in the planning process?

The planning process works best when the number of participants is as great as possible. Certain forecasts can begin in the field, such as production, loss ratio and development by line of business and then progress thru the organization.

Participation in the planning process should involve all of management via specific measurable goals. Overall objectives, however, should be set by top management.

VI. OVERALL GUIDELINES IN DESIGNING AN INSURER PLANNING SYSTEM

The preceding sections presented various concepts related to an effective planning system. These are highlighted here.

- The company must develop a "plan", i.e. choose a set of identifiable measurable actions designed to achieve certain identifiable measurable results not just a "forecast" which is merely a numerical prediction of what results might be under one specific set of assumed actions.

- The planning process must balance detail with needs i.e. cost must be justified by benefits realized.

- The company must set overall objectives and policies. Executives should assume the responsibility of setting the company's direction and make it well known throughout the organization.

- Those individuals responsible for results should also be responsible for the plans. Participation in the planning process should be as broad as possible. There should be no need for a separate corporate planning department or a separate group of "planners".

- Plans should be structured so that the results are measurable and specific accountability should be assigned throughout the organization -- rewards should be tied to achievement of plan goals.

- The planning process should be a dynamic ongoing flexible activity responding to both favorable and unfavorable deviations as they arise.

- The plan should include models and a management information system specifically structured for the individual insurer's needs. Models and processes suitable for other firms should not be transplanted.

VII. SUMMARY AND CONCLUSIONS

The preceding sections presented an overview of the planning process for corporations in general and identified some unique areas of application of the process to insurers. This identification represents only a beginning. There is a need to more fully develop these unique areas of application for insurers in general and finally individual insurers must carry the process into their own firms.

Exhibit I

External Influences Influencing the Business Climate of Insurers[31]

I. For Property-Casualty Companies

- Extreme competition, especially for commercial business
- Cash-flow underwriting
- Mandated subsidies in involuntary markets
- Growing company/agent computer interfacing
- Financial service marketing
- Multiple types of marketing outlets
- Nonrisk-bearing insurance services
- Loss of market share by independent agents
- Continued inflation in medical expenses and auto repairs
- Deterioration of smoke stack industries
- High energy costs
- Growth of self-insurance and captives
- Open competition and requirement of independent pricing
- Possible increased taxes

II. For Life and Health Companies

- Increasing medical expenses
- Investment-oriented life policies
- Agent retention
- Possible increased taxes

Exhibit II

Basic Elements of An
Insurance Planning Model[32]

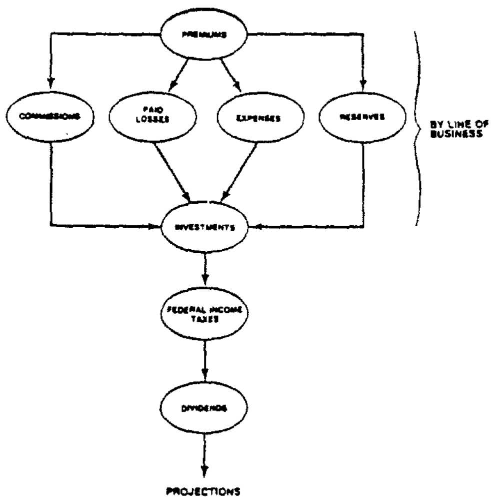


Exhibit III

Examples of Input/Output Variables for an Insurance Planning Model[33]

MODEL COMPONENT	INPUT VARIABLES Outside Variables	Policy Variables	OUTPUT VARIABLES
Premiums	o Employment Levels o Demographic Trends o Competition o Government Regulation o Interest Rates o Other Economic Indicators	o Premium Rates o Levels of Advertising o Type of Marketing Techniques o Types of Policies o Ceded Premiums	o Direct Premium Written o Return Premiums o Additional Premiums o Earned Premiums o Unearned Premiums o Premiums In-Force o Net Premiums
Commission	o Prevailing Commission Rates	o Agency Agreement o Commission Rates	o Commissions o Agents Current Account
Paid Losses	o Historical Experience o Government Regulations o Mortality Rates o Catastrophic Events o Judicial Decisions o Inflation Rates	o Underwriting Policies o Claim Adjustment Policies	o Paid Losses o Reported Claims o Incurred but not Reported Losses o Outstanding Losses o Other Reserves
Expenses	o Salary and Wage Rates o Cost Accounting Data o Historical Experience for not Reported Losses o Inflation Rates	o Personnel Policies o Location of Company Facilities o Company Organization o Administrative Support Functions, Number and Types of Personnel o Union contracts	o Expense Reserves o Loss Adjusting Expenses o Unallocated Expenses o Operating Expenses o Expense Ratio
Investments	o Current Interest Rates o Riskiness of Various Investment Opportunities	o Maturities on Current Investments o Types of Investments o Mix of business: Long-tail vs. short-tail losses	o Investment Portfolio o Yield on Investments
Income Taxes	o Federal Income Tax Code o State Income Tax Code o International Income Tax Code	o Types of Policies Qualifying for Special Exemptions o Accounting Policies o Capital Expenditures	o Federal, State, International, Income Tax Expense o Deferred Taxes

Footnotes

1. John Argenti, Corporate Planning A Practical Guide (Homewood, Illinois: Dow Jones - Irwin, Inc., 1969), p. 204.

2. Brian W. Scott, Long-Range Planning in American Industry (American Management Association, 1965), p. 21.

3. George A. Steiner and John B. Miner, Management Policy and Strategy Text, Readings, and Cases (New York: Macmillan, 1977), p. 26-27.

4. Steiner, p. 19.

5. Steiner, p. 23-24.

6. William R. Osgood, Basics of Successful Business Planning (American Management Association, 1982), p. 20.

7. Scott, p.22.

8. Argenti, p. 17.

9. Russell L. Ackoff, A Concept of Corporate Planning (New York: John Wiley & Sons, Inc., 1970), p. 6.

10. Argenti, p. 28.

11. Ackoff, p. 24.

12. Leon Reinharth, H. Jack Shapiro, and Ernest A. Kallman, The Practice of Planning Strategic, Administrative, and Operational (New York, Van Nostrand Reinhold Company, 1981), p. 31.

13. Osgood, p. 20-21.

14. Argenti, p. 110.

15. Argenti, p. 131-135.

16. Ackoff, p. 42.

17. David Dewan, Planning and Control for Managers (American Management Association Extension Institute, 1975), p. 4-5.

18. Reinharth, Shapiro, and Kallman, p. 38.

19. Ackoff, p. 129-131.

20. Argenti, p. 227.

21. Reinharth, Shapiro, and Kallman, p. 3-4.

22. William A. Sherden, "Stategic Planning: Do the Concepts Apply", Best's Review (October, 1983), 29.

23. Philip H. Dutter, "Successful Management in a Changing Environment", Best's Review (December, 1983), 33-34.

24. James E. Morey, "Planning for Survival", Best's Review (September, 1983), 128.

25. Wilfred G. Howland, "Corporate Planning -- Prescription for Survival", United States Banker (May, 1984), 121.

26. Dutter, p. 33-34.

27. Howland, p. 121.

28. Robert E. Hylas, "An Introduction to Planning Models for Insurance Companies", Insurance Accounting and Statistical Proceedings, 1982, p. 624-626.

29. Robert A. Verrangia, "Computer Modeling/Planning", Insurance Accounting and Statistical Association Proceedings, 1981, p. 469.

30. Robert E. Reuter, "Corporate Planning", Insurance Accounting and Statistical Association Proceedings, 1981, p. 193.

31. Howland, p. 120.

32. Hylas, p. 626.

33. Hylas, p. 627.

MEASURING DIVISION OPERATING PROFIT

by David Skurnick, FCAS

David Skurnick, FCAS 1970, (Chairman, Examination Committee 1980-81; Woodward Fondiller Prize 1975) MAAA, CPCU. Senior Vice President, Argonaut Insurance Company, formerly Associate Actuary, Insurance Company of North America 1968-73 and Actuary at California Inspection Rating Bureau 1973-75. B.S. in mathematics, University of Chicago 1963, M.A. University of California, Berkeley 1965, Ph.C. 1968.

ABSTRACT

We have developed a "Company Return" of operating results by division office and line. Company Return actuarially reflects loss development, retrospective rating plans, dividend plans, reinsurance, cash flow plans, and investment income. Losses are on an accident year basis. A retro accrual is deducted from the premium for retrospective returns paid or anticipated. A similar adjustment is made for dividends to policyholders. Large audits appearing in the wrong year are adjusted to the proper year. Reinsurance ceded is deducted from premiums and losses.

Division investment income is split into two components. Investment income reflecting the fact that losses are paid out over a period of time is handled by an incurred loss discount factor, which varies by line of business. The investment income gained or lost based on the speed with which the premium is collected is measured by a so-called Cash Collection Adjustment. Our top management uses the Company Return as the primary measure of division profitability.

MEASURING DIVISION OPERATING PROFITABILITY

Insurance companies traditionally measure their division office profitability in an accounting sense. Premiums, losses, and expenses are shown on a calendar year basis. Dividends to policyholders are either ignored or shown on a paid or declared basis. Retrospective return reserves and IBNR reserves are calculated countrywide, then distributed to divisions using the "meat axe" method. No adjustment is made for investment of unearned premiums or loss reserves. This accounting type report will accurately tie to the company totals, but is inadequate for management of a division.

It is said than an actuary is content to be approximately right, while an accountant would rather be exactly wrong. We have developed an "Argonaut Return" of operating results by division office and line (Exhibit 1). Argonaut Return actuarially reflects loss development, retrospective rating plans, dividend plans, reinsurance, cash flow plans, and investment income. Losses are on an accident year basis. A retro accrual is deducted from the premium for retrospective returns paid or anticipated. A similar adjustment is made for dividends to policyholders. Large audit premiums appearing in the wrong year are adjusted to the proper year. Reinsurance ceded is deducted from premiums and losses.

Division investment income is split into two components. Investment income reflecting the fact that losses are paid out over a period of time is handled by an incurred loss discount factor, which varies by line of business. The investment income gained or lost based on the speed with which the premium is collected is measured by a so-called Cash Collection Adjustment. Our top management uses the Argonaut Return as the primary measure of division profitability.

The starting point is accident year loss development by line, by division office. An example is shown in Exhibit 2. Since the investment income is explicitly credited to the divisions, there is no cushion for adverse loss development. Therefore, loss development factors must be fully adequate. We develop losses to 10 years, with an additional factor to a 20 year ultimate. This development also provides an IBNR balance by division, which is used in internal calendar year reports.

In order to improve the accuracy of the loss development factors, we use a weighted average of division LDF's and countrywide (Total) LDF's. (See Exhibit 3). A credibility weighted LDF is selected, where credibility is:

$$Z = \sqrt{\frac{\text{\# of division claims in last 5 years}}{10{,}623}}$$

In the example shown, the division is large enough to receive full credibility.

An adjustment factor is used when it appears that the formula derived loss development factors may not be appropriate. These adjustment factors are somewhat judgmental. The most common reason for an adjustment would be a change in average severity, shown in Exhibit 4.

For example, the 1984 adjustment was derived by comparing the average value at age 1 (8761) with a projection based on the five prior values. These earlier values were increased corresponding to a change in workers' compensation benefits and trended for inflation, producing a projected 1984 value of 8612. Presumably the actual value is higher than the projected value because the 1984 case reserves are stronger than they were during the period used for deriving the LDF's. Hence we adjust the 1984 LDF by a factor of .983 (8612 ÷ 8761).

Another situation in which an LDF adjustment would be made would be a $1 million (policy limits) liability claim in a division with only $2 million of incurred liability loss, for an immature accident year. We would reduce the LDF, since this large claim could not develop adversely.

Adjusted Net Premium Earned, shown on line 7 of Exhibit 1, is the amount we expect to retain after after retrospective returns, dividends to policyholders, and reinsurance. Line 1 of Exhibit 1 is the calendar year direct earned premium. Line 2, Audit Adjustments, gives the actuary an opportunity to correct the premium for large final audits or coding errors that have transferred premium from one year to another. The sum of lines (1) and (2) corresponds more closely to the accident year losses than line (1) above.

The dividend accrual on line 4 of Exhibit 1 represents the dividends to policyholder paid or anticipated, by <u>accident</u> year. In order to estimate this number, we compute dividends paid and dividend reserves by <u>policy</u> year. The accrual (paid and reserve) for a more recent year is estimated from the amounts paid in older years, taking into account changes in the dividend plans used by the division. The accident year accrual rates are weighted averages of the policy year accrual rates, based on the distribution of premiums by policy month.

Retro accruals are handled in a similar fashion. Policy year retro returns follow the Berry method with individual input by division.[1]. The accident year retro accrual rate is a weighted average of policy year retro accrual rates. The use of accident year retro and dividend accrual ratios provides much more stability than the use of calendar year retro and dividend returns.

[1] C.H. Berry, "A Method for Setting Retro Reserves," <u>PCAS</u> LXVII 1980, p. 226

Expenses shown are the same as the calendar year expenses done by the accountants.

Line 13 of Exhibit I shows an accident year underwriting profit or loss. However, in today's insurance world there are several reasons to take investment income into account. First, the true operating profit of a company is significantly due to its investment income, especially as relates to casualty lines. Also, the division management has the power to affect the rate at which premium is collected. In some cases, the full premium may be collected at policy inception. Alternatively, the premium may be paid in monthly or quarterly installments and the deposit percentage can vary. With cash flow retro policies, a substantial percentage of the premium may be deferred until the first retro adjustment. In a paid loss retro plan, the company collects only the retro basic and the paid losses, with the reimbursement for loss reserves deferred to the fifth retro adjustment or even later.

We decided to handle investment income in two pieces. The investment income on the loss reserves is measured prospectively by discounting incurred losses. We discount the loss payment patterns for our various lines of business at an assumed interest rates. As a result, workers' compensation losses were discounted at 20% in most states. (We chose to discount incurred loss rather than apportion interest to loss reserves in order to encourage prompt claims settlement. Also, we preferred to reflect estimated future investment income on the current accident year rather than actual current investment income on past accident years.)

The investment income on the premium is measured by comparing the collected premium to the earned premium. If the all-time collected premium is greater than the all-time earned premium, the division receives interest on the difference, currently .9% per month. If the all-time collected premium is less than the earned premium, the division is charged at the same rate.

The difference between the all-time collected premium and the all-time earned premium equals the unearned premium reserve plus the dividend reserve plus the retrospective returns minus the Agents' Balances.[2] Fortunately, our company calculates each of these reserve balances by division, so the calculation of our Cash Collection Adjustment is straight-forward.

The Cash Collection Adjustment properly penalizes the divisions for lost investment income when they sell cash flow policies. It also rewards them for prompt premium collection or large deposits. A policy with a large anticipated retro or dividend return will generate extra Cash Collection Adjustment, reflecting the period the company holds the premium until the return is paid.

Not only is the Cash Collection Adjustment a part of Argonaut Return, but its display also calls attention to the speed of collecting premium. It serves as a management barometer of timely policy issuance, deposit adequacy, speed of audit adjustments, and promptness of collections. One can see the improvement made by the Division shown in Exhibit I, an improvement encouraged by the company's use of the Cash Collection Adjustment.

Never before has division management had a greater opportunity to control their own profitability. Divisions have enormous pricing flexibility as well as the ability to select

[2] (Written Premium) - (Earned Premium) = (Unearned Premium Reserve) + (Retro Reserve) + (Dividend Reserve)

(Written Premium) - (Collected Premium) = (Agents' Balances) or reject accounts. They also control the rate at which premium is collected and the commission rates. It is essential that we have a measured of bottom line profit which is stable and accurate. The Argonaut Return provides division management with a convenient and realistic measurement of the operating profit of their business.

In the past, lacking a meaningful measure of operating income, management has not always focused on the key items. A low expense ratio might be rewarded while a high loss ratio was considered bad luck or a timing problem. The appearance of proper management took priority over the substance of profitable results. The use of Argonaut Return has helped us combine all the factors and work toward achieving profit for each division, and thus for the entire company.

Dly-26

Argonaut Return
Evaluated at 9/84

Division or Branch Name:
Workers' Compensation

	Calendar and Accident Year ($000)					
	1982		1983		1984 (9 months)	
1. Direct Premium Earned	$38,056	128.5%	$39,217	130.0%	$33,000	143.4%
2. Final Audit Adjustment			1,326	4.4	-1,326	-5.8
3. Retro Accrual	-1,699	5.7	-2,109	-7.0	-2,259	-9.8
4. Dividend Accrual	-6,621	22.4	-8,137	-27.0	-6,135	-26.7
5. Adjusted Direct Earned Premium	29,736	100.4	30,297	100.4	23,280	101.2
6. Ceded Premium	-126	-0.4	-129	-0.4	-268	-1.2
7. Adjusted Net Earned Premium	29,610	100.0	30,168	100.0	23,012	100.0
8. Direct AY Loss & ALAE	23,498	79.4	29,199	96.8	21,349	92.8
9. Ceded Loss & ALAE	-564	-1.9	-701	-2.3	-512	-2.2
0. Net AY Loss & ALAE	22,934	77.5	28,498	94.5	20,837	90.5
1. ULAE	1,175	4.0	1,460	4.8	1,067	4.6
2. Net Underwriting Expense	8,090	27.3	7,744	25.7	6,540	28.4
3. Adjusted Net Underwriting Income	-2,589	-8.7	-7,534	-25.0	-5,432	-23.6
4. Loss Discount	4,587	15.5	5,700	18.9	4,167	18.1
5. Cash Collection Adjustment	-1,034	-3.5	-602	-2.0	149	0.6
6. Company Return	964	3.3	-2,436	-8.1	-1,116	-4.8

Exhibit 1

ACCIDENT YEAR LOSS DEVELOPMENT REPORT
PRODUCT LINE: WORKERS' COMPENSATION
AS OF SEPTEMBER 30, 1984

DIVISION NAME:

ACCIDENT YEAR	ADJ. DIRECT EARNED PREMIUM	LOSS DEVELOPMENT - YEARS * (000-OMITTED)											LOSS DEVELOPMENT FACTOR	GROSS ULTIMATE LOSS	IBNR
		1	2	3	4	5	6	7	8	9	10	CURRENT			
All Prior	-	245,340	254,268	258,878	264,339	265,774	266,592	266,871	269,700	269,802	271,072	271,691	1.029	279,570	7,879
1975	31,043	18,369	20,111	21,124	20,503	20,795	20,716	20,512	20,760	21,097	21,336	21,336	1.060	22,616	1,280
1976	29,808	16,829	19,163	21,381	21,560	21,726	21,350	21,406	21,375	21,445		21,445	1.071	22,968	1,523
1977	42,162	22,196	24,762	25,811	26,083	26,399	26,537	26,528	27,035			27,035	1.075	29,063	2,028
1978	44,994	24,269	28,367	29,812	30,640	31,141	30,903	31,069				31,069	1.082	33,617	2,548
1979	46,906	25,082	28,341	30,759	30,710	29,890	29,897					29,897	1.075	32,139	2,242
1980	30,413	18,099	21,176	22,081	22,420	22,463						22,463	1.064	23,900	1,437
1981	32,253	18,079	22,203	23,818	23,704							23,704	1.071	25,387	1,683
1982	29,736	17,055	21,214	21,479								21,479	1.094	23,498	2,019
1983	30,297	21,532	25,886									25,886	1.128	29,199	3,313
1984	23,280	15,558										15,558	1.372	21,349	5,791
												511,563		543,306	31,743

* Cumulative dollar incurred losses at yearly intervals past each accident year.

IBNR = Gross Ultimate Loss - Current Incurred Loss

Exhibit 2

ACC YEAR	DIVISION LDF	CNTRYWDE LDF	CRD-WTD* LDF	ADJUSTMENT FACTOR	APPLIED LDF
1974	1.029	1.029	1.029	1.000	1.029
1975	1.060	1.060	1.060	1.000	1.060
1976	1.071	1.070	1.071	1.000	1.071
1977	1.075	1.061	1.075	1.000	1.075
1978	1.082	1.070	1.082	1.000	1.082
1979	1.075	1.067	1.075	1.000	1.075
1980	1.064	1.068	1.064	1.000	1.064
1981	1.071	1.072	1.071	1.000	1.071
1982	1.094	1.109	1.094	1.000	1.094
1983	1.174	1.174	1.174	0.961	1.128
1984	1.396	1.403	1.396	0.983	1.372

* 100.0% X DIVISION LDF + 0.0% X COUNTRYWIDE LDF

Exhibit 3

ACCIDENT YEAR	AVERAGE SEVERITY - EXCLUDING M.O.'s, CWP's, & CZP's								* NOTE 2	
	1	2	3	4	5	6	7	8	9	10	CURRENT
All Prior											
1975	4,073	3,957	4,141	3,930	3,943	3,918	3,880	3,923	3,984	4,030	
1976	3,708	3,660	3,840	3,829	3,850	3,779	3,780	3,776	3,787		
1977	4,098	4,014	4,141	4,164	4,204	4,221	4,216	4,291			
1978	3,977	4,104	4,272	4,385	4,450	4,412	4,433				
1979	4,737	4,719	5,107	5,075	4,922	4,924					
1980	4,997	5,118	5,275	5,342	5,350						
1981	5,193	5,665	6,003	5,952							
1982	5,938	6,454	6,454								
1983	7,892	8,435									
1984	8,761										

* NOTE 2

Excludes from claim count and incurred loss
--Medical only (M.O.) claims
--Claims closed without payment (CWP)
--Claims closed with zero loss payment, but with allocated expense payment (CZP)

Exhibit 4

ACTUARIAL ASPECTS OF FINANCIAL REPORTING

by

Lee M. Smith

Lee M. Smith has a Bachelor's degree in Finance from Northern Illinois University and a Master's degree in Economics from Trinity College. He is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. He has served on CAS Examination Committees (Parts 3 and 8) and on the CAS Risk Classification Committee. He has been Chief Actuary for the State of Michigan, Director of Planning at Aetna Insurance Company, President of his own consulting firm, and is currently a consultant with Ernst & Whinney. He has taught actuarial science at the Universities of Connecticut and Michigan.

Abstract

Financial reports of property/casualty insurance companies are notoriously difficult to interpret. A major reason for this difficulty is that the actuarially generated elements of those statements are usually not understood. Often they are not even identifiable.

By the use of a fairly simple model, relationships between actuarial analysis and financial statement figures can be displayed. Once the sources of data in actuarial data bases and the flow of actuarial projections into financial statements are identified at the conceptual level, progress can be made toward financial reports with which non-actuaries can feel comfortable.

The first model illustrated is called deterministic because all growth, contingency, reporting, and payment patterns are uniform. The flow of information to and from actuarial models is easily followed. Predictions can be made with a high level of comfort.

The second model is labeled stochastic. This is to clarify that some of the uniformity from the first model is relaxed. Growth rates, reporting patterns, and payment patterns are allowed to fluctuate in this model.

These models are used to identify and study the interrelationships between various actuarial projections and financial statements. Clearly, the actuarial elements of an enterprise's financial statements should be understood by as wide an audience as possible. By the use of the simple models illustrated in the paper, the interrelationships between the rating, reserving, and financial reporting functions can be examined and more fully appreciated. While the development of these relationships can be a difficult task in practice, the increased level of understanding is well worth the effort.

ACTUARIAL ASPECTS OF FINANCIAL REPORTING

Overview

Deterministic Model

- Reconstructed Historical Financial Statement
- Loss Development Analysis
- Rate Analysis
- Projected Financial Statements
- Deterministic Model Summary

Stochastic Model

- Reconstructed Historical Financial Statement
- Loss Development Analysis
- Rate Analysis
- Projected Financial Statements
- Stochastic Model Summary

Conclusion

ACTUARIAL ASPECTS OF FINANCING REPORTING

OVERVIEW

Casualty actuaries are most often thought of as insurance professionals who perform ratemaking and reserve studies. While this may be a fairly accurate representation in general, it abstracts away much of the essence of actuarial science. It also leads to a sense of mystique about rate and reserve figures which is often unwarranted.

Three key elements of casualty actuarial work are mathematics, economics, and accounting. Because casualty actuaries are in the forefront of the struggle to evaluate the contingencies facing an insurer, they must be able to formulate algorithms and fit parameters by which to predict losses. Not only must they determine the likelihood of a loss and the amount of a loss from a given exposure, they must also determine when the loss is likely to become known by the insurer and when it will be paid.

The mathematics involved in the evaluation of casualty contingencies is formidable. The analysis includes fitting curves to frequency and severity distributions and combining them to produce an expected loss distribution for a coverage at a point in time. Predicting losses at a different point in time requires development of a growth function. These mathematical aspects of actuarial science are the least understood and most feared by non-actuaries.

A second key element of actuarial work is economics. Particularly in their pricing role, actuaries are performing an economic function. Producing an "actuarially correct" rate indication is an empty exercise if an inappropriate rate of return results. The rate of return will be inappropriate if elasticity of demand is such that sales drop unacceptably. It will also be inappropriate if the rate algorithm used does not incorporate rate of return in an economically meaningful manner.

The accounting function of the actuary is in many ways the most important. An actuary's math can be precise, but be applied to bad data. An algorithm can be properly applied to good data, but the results can be misinterpreted or misutilized. Proper effort applied to the accounting aspect of actuarial science can assure actuarial calculations are properly applied and that the results are properly interpreted.

A DETERMINISTIC MODEL

It is the premise of this paper that a major reason financial statements of property/casualty insurers are difficult to interpret and utilize is that the actuarial elements impacting the figures are not well understood by the preparers and users of those statements. In addition, the financial statements into which the actuarial elements are flowing are not always understood by actuaries. As a result, a good conceptual grasp of the actuarial aspects of financial statements is hard to develop.

To illustrate the flow of actuarial elements into financial statements, a very simple model is needed. If too many complicating elements were introduced, the relationships would be difficult to trace. The idea is to see how basic reserve and ratemaking procedures impact and are impacted by financial statements.

Reconstructed Historical Financial Statement

Perhaps the report (not ordinarily produced) which would shed the most light on the issue is a historical financial report in which loss figures are identified by accident year. Calendar year incurred losses are composed of payments and reserve changes on losses from a number of accident years. As such, they bury the key figures which casualty actuaries use in reserve and rate computations.

Exhibit 1A is an example of a historical financial report in which losses are broken into accident year components. The figures in this exhibit result from a "deterministic" insurance process. In other words, losses are reported and paid according to predetermined patterns, and premium and loss levels grow at predetermined rates.

A number of simplifying assumptions are made in order to illustrate the fundamental relationships in question. The model company writes in one line in one state. (Alternatively, all lines and states are aggregated for reporting, reserving, and ratemaking purposes.) No reinsurance is assumed or ceded. All transactions are on a cash basis. Investment and tax rates are aggregated.

A beginning level of written premium of $1 million is assumed. Unearned premium is assumed to be 50% of written premium. Expenses are set at 25% of earned premiums. A more realistic approach would have expenses as a function of written and earned premiums, but for illustrative purposes the relationships have been aggregated into a single percentage.

Incurred losses are set at 75% of earned premium. Paid losses, case reserves, and IBNR reserves for a calendar-accident year combination are a function of the assumed payout and reporting patterns. Because the process is assumed to be deterministic, IBNR reserves are always accurate.

Underwriting cash flow is defined to be written premiums less paid losses less expenses. Underwriting income is earned premiums less incurred losses less expenses.

Written premium growth is a function of exposure growth, average premium growth, and mix of business effect. Exposure growth reflects change in number of policies written. Average premium growth reflects change in premium per exposure resulting from rate changes. Mix of business effect is change in premium resulting from a change in demographic makeup of policyholders toward higher or lower rated classifications. The assumed rate of growth for each growth type is, respectively, 10%, 5%, 5%.

Incurred loss growth is a function of exposure growth, frequency growth, and severity growth. Frequency represents the average number of claims per exposure unit, and severity the average cost of a claim. The growth rates for these loss elements are chosen to correspond with the growth rates for the premium elements.

The relationships chosen for this model produce a zero underwriting gain for each year. Because of the predictability of events rates keep up with losses and expenses. A zero underwriting gain is assumed to produce the target rate of return for the model company.

Investment income is a function of beginning of year assets, underwriting cash flow, and pre-tax average investment return on assets. For simplicity, it is assumed that full investment rate is earned on beginning of year assets while half the rate is earned on underwriting cash flow.

Tax is defined to be 50% of underwriting profit plus 20% of investment income. The lower rate on investment income assumes 60% of the income from the portfolio is from non-taxable instruments.

End of year assets are defined as beginning of year assets plus underwriting cash flow plus investment income less taxes. End of year liabilities are the sum of unearned premium reserves, case reserves, and IBNR reserves. Surplus is the difference between assets and liabilities. Change in surplus is after-tax underwriting plus investment gain.

Discounted calendar year incurred losses represents the sum of past payments and present value of future payments on accident years not fully paid. The present vaues are computed from the payment schedules in Exhibit 1B. The discounted loss reserve at any point in time is the present value of future payments on claims from accident years with claims still outstanding.

Discounted liabilities are the sum of unearned premium reserve and discounted loss reserves. Discounted surplus is the difference between assets and discounted liabilities. GAAP adjustment is defined to be 20% of unearned premium reserve. GAAP surplus is statutory surplus plus the GAAP adjustment. GAAP income is underwriting gain plus investment income less taxes plus change in GAAP adjustment. It should be noted that the fact that expenses were earlier made a function of earned premium makes it unlikely that in a real company such an adjustment would be needed to assure proper matching of income and expense.

This reconstructed historical financial statement, then, is the primary document showing the relationship between actuarial analysis and financial reporting. By decomposing calendar year losses into their accident year components we reveal data used by actuaries in their deliberations. We are thus in a position to analyze the flow from actuarial models to financial reports.

Loss Development Analysis

As was indicated above, reserve estimation is one of the areas most closely identified with actuaries and their work. While very sophisticated procedures have been developed by which to estimate ultimate liabilities as of a point in time, the basic idea is quite simple. One must review liability estimates on the books and formulate a model by which to adjust those liabilities to a "best estimate" basis, assuming the booked figures are not determined to be appropriate.

The most common model for producing ultimate loss estimates is one which examines groups of accidents for historical periods and evaluates patterns by which they were paid, reported, and reserved. All other things being equal, these historical patterns are assumed to continue into the relevant future.

Exhibit 1B builds a simple reserving data base from data in the reconstructed historical financial statement. Loss figures are arranged by accident year and calendar year in the traditional manner. Very simple reserve models often utilize such data.

Because this is a deterministic model, the development patterns are totally stable. Growth in paid and reported losses from one maturity point to the next is uniform for each accident year. This is a function of the assumed uniform reporting and payment patterns mentioned previously.

Reserve estimation in such an environment is fairly routine, assuming no changes can be anticipated. Because 1.4 times as many losses are always reported in two years as were reported at one year maturity, it can be projected that the most current year's reported losses will be 40% higher next year. Likewise, it can be assumed that paid loses on the most recent accident year will be three times as high by next year.

Because loss reserving is always accurate in such a world, there is a one to one relationship between the loss figures in financial statements and those in reserve models. If such were not the case, incurred estimates for an accident year would change from one calendar year to the next as new information leads to more refined estimates. There would be a reserve table corresponding to each calendar year's financial statement, rather than a single table resulting from and feeding into a five year financial statement.

The paid and reported development factors produced in this model can be used to produce projected incremental future payments and reports by calendar year. We can thus project how historical accident years will impact results of future calendar years. Also, the projected payment schedule can be used to determine the discounted value of an accident year's loss payments at various points in time. Some elements of the relationship between an actuarial reserve model and a company's final financial statements are now becoming more obvious.

Rate Analysis

Ultimate incurred loss estimates produced in the reserve model for a coverage would flow into the rate level analysis for that coverage. They are often the most critical component of a rate filing in terms of sensitivity of the rate indication. A relationship is thus established between financial statements and rate analyses.

Two critical relationships between rate analysis and financial statements involve incurred losses and rate indications. The incurred losses should tie back to financial statement figures. The rate indication, if implemented, would impact future premium levels and thereby financial statement figures. Premium and expense figures should be consistent with financial statement figures.

The rate model presented in Exhibit 1C is a simple one. The rate algorithm is peripheral to illustrating relationships between actuarial analysis and financial statements. The basic idea of any rate model, including this one, is that premiums to be collected be sufficient to produce the proper rate of return for the insuring entity. The historical figures should be consistent with those in other company reports. Financial projections should account for expected impact of the rate change, including an evaluation of demand elasticity.

Projected Financial Statements

Financial projections can be used for a variety of purposes. Examples are company planning, merger and acquisition, and investment strategy. As a result, it is important that these projections be as realistic as possible. They must also be understandable to non-actuarial people using them.

Exhibit 1D provides key elements of projected financial statements of a property/casualty insurer. It reproduces the historical years 1980-84 and projects results for the next five years. The primary addition to the projection model relative to the historical model is the assumed flow through of the rate indication in 1985. Since the rate change is zero and no elasticity is assumed, the projections follow the historical figures.

The column headings in Exhibit 1D are identical to those in Exhibit 1A. Growth assumptions after 1984 are the same as those prior to 1984 except that the rate increase flows through written premium in 1985. Surplus continues to grow as investment income flows through to surplus.

This company's planning process is fairly simple. Budgets can be met by holding expenses to 25% of earned premium. Evaluating the company for merger and acquisition is also routine since net income and cash flow can be projected with a high level of comfort. Similarly, investment strategy is simplified by the fact that maturity of liabilities and taxable gains are so predictable.

Deterministic Model Summary

We have seen that when insurance contingencies are predictable and when complicating elements are abstracted away, the relationships between actuarial models and financial statements are fairly straightforward. As predictability of losses decreases and complications increase, these relationships become more convoluted. Nonetheless, by definition the financial statement figures of an insuring entity must ultimately be tied back to their sources, which include actuarial data bases and analyses.

A "STOCHASTIC" MODEL

Developing a risk theoretic model of the insurance process is beyond the scope of this paper, and would add little to the understanding of the fundamental relationships between actuarial analyses and financial statement figures. Notwithstanding this, however, there are some random and non-normal factors which affect actuarial work and which lead to terminology which confuses non-actuaries. This confusion can lead to misinterpretation of the results of actuarial studies and misuse of the figures.

Reconstructed Historical Financial Statement

Exhibit 2A shows financial statement figures resulting from an insurance process which does not have uniform growth rates, payment patterns, or reporting patterns. To allow key relationships between the actuarial models and the financial statements to be easily illustrated, distortions from a deterministic process have been minimized. This model merely adds a few complicating elements and some terminology.

Actuaries often speak in terms of frequency, severity, and pure premiums. This model allows growth in frequency and severity of claims to diverge from each other and from premium growth. This leads to fluctuation in underwriting results.

This model also allows payment and reporting patterns to fluctuate from one year to the next. This opens up the possibility of changes in ultimate incurred estimates for an accident year from one calendar year to the next. Such changes would lead to reconstructed reserve models for each calendar year's development.

Written premium growth from one year to the next in this model is a function of exposure growth, growth in average gross premium, and growth in mix of business. For 1981 the respective growth rates utilized are 5%, 10%, and 5%. For 1982 through 1984 the growth rates are (5%, 10%, 5%); (5%, 5%, 5%); and (5%, 10%, 5%). Unearned premium and expense ratios are as established in the deterministic model.

Growth in incurred losses is a function of exposure growth, frequency growth, and severity growth. The respective growth rates assumed in 1981 are 5%, 5%, and 10%. Growth rates for 1982 through 1984 are (5%, 5%, 10%); (5%, 5%, 10%), and (5%, 5%, 10%). In 1984 an additional 5% growth factor is added.

Paid loss patterns are assumed to be uniform except for accident year 1981. The payment pattern for 1981 by calendar year is .2, .2, .2, .2, .2. For other years it is .1, .2, .3, .2, .2.

Reporting patterns and resulting case reserves are also uniform except for 1981. The cumulative pattern for 1981 is .4, .6, .8, .9, 1.0. For other years it is .5, .7, .8, .9, 1.0.

Other financial statement items are defined as they were in the previous model.

Loss Development Analysis

As was the case with the first model, the data base for loss development is extracted from the reconstructed historical financial statements. In surveying the development factors in Exhibit 2B we can see the impact of the non-uniform reporting and payment patterns for accident year 1981. To simplify the analysis, however, we have assumed that the reserve actuary for the entity was clever enough to see that 1981 was distorted. As a result, incurred estimates for each accident year as of each calendar year are the same.

The other aspects of reserve analysis for this model are analogous to those of the first model. Payments and reports are projected out and payments are discounted as of each point in time to allow for the option of discounted liabilities.

Rate Analysis

Ratemaking also becomes more complicated when non-uniformity is introduced. We see in Exhibit 2C that loss ratios fluctuate somewhat from year to year. The interaction of the various growth assumptions has led to a slight upward trend in loss ratio and the need for a rate increase. This rate increase will flow into projected financial statements.

It is interesting to note that incurred losses for accident years 1983 and 1984 in the rate analysis are different than the corresponding figure in the financial statements. The explanation for this is that the rating actuary used average incurred development factors in projecting ultimate losses. The 1981 distortion is thereby projected forward in the rate model. As a result, the loss figures and rate indication are somewhat overstated.

Projected Financial Statements

Financial projections in this model are done under a greater degree of uncertainty. Because historical patterns have not been uniform, prediction even under the cet par assumption is more difficult. Even if future patterns can be assumed to follow those of the past, an assumption must be made as to which of the past patterns are likely to influence future figures. Exhibit 2D displays the financial projections for the stochastic model.

The years 1980-1983 are reconstructed to eliminate the premium growth anomaly in 1982. The rate of premium growth in 1985 is a function of exposure growth, average premium growth, mix of business growth, the rate increase, and zero elasticity effect. Growth rates for 1986-1989 are functions of exposure, average premium, and mix of business growth per the following: (5%, 10%, 5%); (5%, 10%, 5%); (5%, 10%, 5%); (5%, 10%, 5%).

Incurred loss growth is a function of exposure growth, frequency growth, and severity growth. The respective growth rates for 1985 are 5%, 5%, and 10%. For 1986-1989 the rates are: (5%, 5%, 10%); (5%, 5%, 10%); (5%, 5%, 10%); (5%, 5%, 10%).

Payment and reporting patterns are assumed to follow those of the historical accident years excluding 1981. Resulting payments, case reserves, and IBNR reserves therefore follow those patterns. Other elements of the projected financial statement are produced analogously to those in the historical statements.

Stochastic Model Summary

While the "stochastic" model added some complications, simplifying assumptions allow us to continue to trace relationships between actuarial analyses and financial reports. The more these assumptions are relaxed, and the more operating complexities added, the more abstruse these relationships become. Nonetheless, if complications are added incrementally, the relationships can continue to be observed.

CONCLUSION

This paper has attempted to build a bridge between actuarial models and financial statement figures. Financial statements aggregate components of actuarial models. As a result, many factors which could make use and interpretation of insurance financial statements easier are not available for review. By explicitly identifyng some of these factors reports can be produced which allow management to see and evaluate elements which have influenced past and may influence future results.

The models in this paper identify a few key actuarial elements and show how they interrelate with financial statement figures. The primary element allowing for the analysis is the identification of accident year components of calendar year figures. Actuaries use accident year data in producing many of the figures they provide to management.

The models here deal with two systems of relationships. The calendar year system is primarily composed of figures which show how much income was earned in a period and which show as of a point in time the volume of assets and liabilities which have arisen.

The key components of income are the premiums earned in a year, the losses which accrue, the expenses which accrue, and the investment earnings which arise. Assets and liabilities change based on the cash flow which arises and the future obligations which accrue. Much of the actuary's role involves a determination as to how loss obligations accrue over time.

The accident year system provides data which is organized in such a manner as to allow the actuary to estimate future losses based on patterns in which losses on the books have arisen. There are two critical aspects to the loss estimation. The first aspect involves production of financial statement figures which reflect historical activity. Loss development analysis is used to estimate the extent to which losses the company is liable for at some point will differ from losses recognized by the company. Historical patterns of loss payment and loss reporting are used to determine how booked loss figures are likely to change.

The second type of loss estimation involves projection of losses likely to arise in future periods. This analysis is part of an actuary's ratemaking activity. Historical losses brought up to ultimate levels by development analysis are reviewed and compared to exposure measures to determine the rate at which losses are changing. This historical rate of change is used to predict future loss levels.

Because losses for property/casualty insurance coverages can vary in amount, it is often helpful to review trends in numbers of claims separately from trends in average claim size. These trends in frequency and severity, respectively, can be combined into a pure premium trend which measures change in loss cost per exposure unit.

The losses projected for future periods provide a basis for determining needed rate level. They can also be used to project future financial results. This process of developing losses to ultimate level and projecting them forward is one of the major functions a casualty actuary plays in the process of producing components of financial statements.

We have seen that actuarial methodology is conceptually related to financial reporting. Demonstrating this for a large multi-line, multi-state insurer would involve enormous effort, and might be challenged on a cost/benefit basis. Working it through for a specialized entity like a one state malpractice carrier might prove both enlightening and fruitful.

Expansion of the model to encompass credibility considerations, loss distribution functions, changes in accident year incurred estimates by calendar year, reinsurance programs, and similar items, while adding complication, would increase understanding of the relationships. Only when actuaries can demonstrate how their data bases and projections relate to other aspects of company reporting can they expect non-actuaries to consistently and properly interpret financial statements of property/casualty insurance companies. Improper interpretation of those statements can lead to improper planning, improper marketing and underwriting decisions, and improper investment strategies. It can also lead to unnecessarily strained relationships with regulatory authorities.

Technical Appendix 1

Deterministic Model

Item	Comment
Direct Written Premium	The initial value is set at $1 million. Values for the years 1981-1984 are a function of the assumed growth rates in exposure, average premium, and mix of business. The growth rates are 10%, 5%, and 5%. Since the rate change is 0, these same growth rates persist in projected years 1985-1989.
Unearned Premium Reserve	This value is set at 50% of written premium.
Direct Earned Premium	This is defined to be written premium less change in unearned premium reserve.
Direct Expense	This is set at 25% of earned premium.
Paid Losses	Accident year paid losses in a calendar year are a function of ultimate incurred losses for the accident year and the assumed payment pattern. This model assumes 10% of ultimate losses are paid in the first year, 20% in the second, 30% in the third, and 20% in each of the fourth and fifth.
Case Reserves	Case reserves for an accident year in a given calendar year are a function of ultimate accident year incurred losses, and the assumed reporting and payment patterns. At any point in time case reserves are reported losses less paid losses. The rpeortring pattern assumed is 50%, 20%, 10%, 10%, 10%. As an example, after two years an accident year will have 70% of its losses reported and 30% paid. The difference, 40%, is case reserves.
IBNR	IBNR for an accident year in a given calendar year is the difference between ultimate amount for the year and the amount reported as of the calendar year. At the end of the second year, for example, 70% of ultimate losses are reported so 30% are incurred but not reported.
Incurred Losses	This model assumes the loss elements move in correspondence with the premium elements. As a result, a 75% loss ratio is maintained. Premiums and losses grow each year by $(1.05)^2(1.10)$.
U/W Cash Flow	Underwriting cash flow is defined as written premium less paid losses less expenses.

Deterministic Model--Continued

Item	Comment
U/W Gain	Underwriting gain is defined as earned premium minus incurred loss minus expense.
Investment Income	Investment income is the earnings rate (10%) times the sum of beginning of year assets and half the underwriting cash flow.
Tax	Tax is the tax rate (50%) times the sum of underwriting gain and 40% of investment income.
End of Year Assets	End of year assets are beginning of year assets plus underwriting cash flow plus investment income less taxes.
End of Year Liabilities	End of year liabilities are the sum of the case reserves, IBNR, and unearned premium reserves.
Surplus	Surplus is the difference between assets and liabilities. The change in surplus is the after-tax underwriting and investment gain.
Discounted Incurred Losses	Discounted incurred losses for an accident year in a particular calendar year represents the sum of past payments at that point in time plus the present value of future payments.
Discounted Loss Reserves	Discounted loss reserves in a calendar year are the sum of the present values of the payments remaining for each accident year.
Discounted Liabilities	Discounted liabilities are the sum of discounted loss reserves and unearned premium reserves.
Discounted Surplus	Discounted surplus is the difference between assets and discounted liabilities.
GAAP Adjustment	The GAAP adjustment is defined to be 20% of the unearned premium reserve.
GAAP Surplus	GAAP surplus is statutory surplus plus the GAAP adjustment.
GAAP Income	GAAP income is statutory income plus change in GAAP adjustment.
Beginning Exposure	Initial exposure is set at 1,000 units.
Exposure Trends	Annual change in exposure can be 5% or 10%.

Deterministic Model--Continued

Item	Comment
Average Premium	Initial average premium is set at $1,000.
Average Premium Trend	Average premium may change at the rate of 5% or 10%.
Mix of Business Trend	The impact of changing mix of business is set at at unity at the beginning. Mix of business effect can be 5% or 10% per year. Thereafter the change can be a function of additional coverage being provided or a shift toward higher rated policyholders.
Frequency Trend	Initial frequency is set at 10%. The change in frequency can be 5% or 10% per year.
Severity Trend	Initial average claim size is set at $7,500. This can increase at rates of 5% or 10% per year.

Technical Appendix 2

Stochastic Model

The income and balance sheet items in this model are defined in the same way as they are defined in the deterministic model. Where this model differs is in the specification of a a couple of growth factors, reporting patterns, and payment patterns. To illustrate the impact of non-uniformity in some elements of the insurance process on the relationship of actuarial calculations to financial statement figures, some variations were introduced.

The first change from the uniformity of the first model is that the average premium growth rate for 1982 is reduced to zero. In 1983 it is increased so as to produce the same premium that year as was produced in the deterministic model. The second change is an additional 5% growth in losses for the 1984 accident year. The impact of these changes is most readily apparent in reviewing underwriting gain which turns negative in 1982.

The other changes introduced in this model involve payment and reporting patterns. Accident year 1981 is given payment and reporting patterns which differ from those of the other years. The impact of this is best seen in the loss development tables which show loss development factors for 1981 which differ from those of other years at the first and second points.

This model best illustrates the type of situation faced by casualty actuaries performing their rating and reserving roles. When the various factors influencing loss amounts begin to vary the degree of mathematical and professional sophistication needed to project future losses increases. The casualty actuary must often look at inconclusive historical movements and attempt to build a model which best predicts the future therefrom.

PROPERTY-CASUALTY FI
DETERMINISTIC

1. FINANCIAL HISTORY

		TREND 1	TREND 2		
NEG EXPOSURE	1000	.05	.10		
AVERAGE PREMIUM	1000	.05	.10		
MIX OF BUSINESS	1	.05	.10		
FREQUENCY	.10	.05	.10		
SEVERITY	7500	.05	.10		
PAYMENT PATTERN	.1	.2	.3	.2	.2
CUM PAY PATTERN	.1	.3	.6	.8	1.0
REPORT PATTERN	.5	.2	.1	.1	.1
CUM REP PATTERN	.5	.7	.8	.9	1.0

CAL YEAR	DIRECT WRITTEN PREMIUM	UEPR	DIRECT EARNED PREMIUM	DIRECT EXPENSE	BOY ASSETS	ACC YEAR	PAID LOSSES	CASE RESERVES	IBNR	INCURRED LOSSES	U/W CASH FLOW	U/W GAIN	INVEST INCOME	TAX	EOY ASSETS	EOY LIAB	EOY SURPLUS	DISC INCURRED LOSSES	DISC LOSS RESERVES	DISC LIAB	DISC SURPLUS	GAAP ADJUST	GAAP SURPLUS	GAAP INCOME
1980	1000000	500000	500000	125000	0	1980	37500	150000	187500	375000	837500	0	41875	8375	871000	837500	33500	319348	281848	781848	89152	100000	133500	133500
1981	1212750	606375	1106375	276594	871000	1980	75000	150000	112500	375000								343872						
						1981	82978	331913	414891	829781	778178	0	126009	25202	1749985	1615678	134307	706638	855032	1461407	288579	121275	255582	122082
1982	1470763	735381	1341756	335439	1749985	1980	112500	75000	75000	375000								361319						
						1981	165956	331913	248934	829781								760903						
						1982	100632	402527	503159	1006317	756236	0	212810	42562	2676469	2371914	304555	854975	1404830	2140212	536257	147076	451632	196049
1983	1783667	891834	1627215	406804	2676469	1980	75000	37500	37500	375000								371510						
						1981	248934	165956	165956	829781								799950						
						1982	201263	402527	301895	1006317								922786						
						1983	122041	488164	610206	1220411	729625	0	304120	60826	3649396	3101538	547858	1039296	1911736	2803570	845826	178367	726225	274593
1984	2163143	1081571	1973405	493351	3649396	1980	75000	0	0	375000								375000						
						1981	165956	82978	82978	829781								822058						
						1982	301895	201263	201263	1006317								970139						
						1983	244082	488164	366123	1220411								1119108						
						1984	148005	592021	740027	1480054	734852	0	401682	80336	4705594	3836391	869204	1260406	2389968	3471539	1234055	216314	1085518	359293

Exhibit 1A

II. LOSS DEVELOPMENT TABLES

ACC YEAR	REPORTED LOSSES 1	2	3	4	5
1980	187500	262500	300000	337500	375000
1981	414891	580847	663825	746803	
1982	503159	704422	805054		
1983	610206	854288			
1984	740027				

ACC YEAR	PAID LOSSES 1	2	3	4	5
1980	37500	112500	225000	300000	375000
1981	82978	248934	497869	663825	
1982	100632	301895	603790		
1983	122041	366123			
1984	148005				

ACC YEAR	REPORTED DEVELOPMENT 2:1	3:2	4:3	5:4
1980	1.4000	1.1429	1.1250	1.1111
1981	1.4000	1.1429	1.1250	
1982	1.4000	1.1429		
1983	1.4000			
1984				
AVERAGE	1.4000	1.1429	1.1250	1.1111
CHOSEN	1.4000	1.1429	1.1250	1.1111
ULTIMATE	2.0000	1.4286	1.2500	1.1111

ACC YEAR	PAID DEVELOPMENT 2:1	3:2	4:3	5:4
1980	3.0000	2.0000	1.3333	1.2500
1981	3.0000	2.0000	1.3333	
1982	3.0000	2.0000		
1983	3.0000			
1984				
AVERAGE	3.0000	2.0000	1.3333	1.2500
CHOSEN	3.0000	2.0000	1.3333	1.2500
ULTIMATE	10.0000	3.3333	1.6667	1.2500

ACC YEAR	REPORTED LOSSES 1	2	3	4	5	ULTIMATE
1980	187500	262500	300000	337500	375000	375000
1981	414891	580847	663825	746803	829781	829781
1982	503159	704422	805054	905685	1006317	1006317
1983	610206	854288	976329	1098370	1220411	1220411
1984	740027	1036038	1184043	1332048	1480054	1480054
1985	897468	1256455	1435948	1615442	1794935	1794935
1986	1088404	1523765	1741446	1959127	2176808	2176808
1987	1319962	1847946	2111939	2375931	2639923	2639923
1988	1600784	2241097	2561254	2881410	3201567	3201567
1989	1941350	2717890	3106160	3494430	3882700	3882700

ACC YEAR	PAID LOSSES 1	2	3	4	5
1980	37500	112500	225000	300000	375000
1981	82978	248934	497869	663825	829781
1982	100632	301895	603790	805054	1006317
1983	122041	366123	732247	976329	1220411
1984	148005	444016	888032	1184043	1480054
1985	179494	538481	1076961	1435948	1794935
1986	217681	653042	1306085	1741446	2176808
1987	263992	791977	1583954	2111939	2639923
1988	320157	960470	1920940	2561254	3201567
1989	388270	1164810	2329620	3106160	3882700

INCREMENTAL PAID LOSSES 1	2	3	4	5
37500	75000	112500	75000	75000
82978	165956	248934	165956	165956
100632	201263	301895	201263	201263
122041	244082	366123	244082	244082
148005	296011	444016	296011	296011
179494	358987	538481	358987	358987
217681	435362	653042	435362	435362
263992	527985	791977	527985	527985
320157	640313	960470	640313	640313
388270	776540	1164810	776540	776540

DISCOUNTED PAID LOSSES 1	2	3	4	5
319348	343872	361519	371510	375000
706638	760903	799950	822050	829781
856975	922786	970139	996951	1006317
1039296	1119108	1176537	1209052	1220411
1260406	1357198	1426845	1466278	1480054
1528550	1645947	1730406	1778229	1794935
1853758	1996147	2098550	2156547	2176808
2248146	2420791	2545016	2615352	2639923
2776438	2935814	3086469	3171769	3201567
3306488	3560408	3743115	3846562	3882700

Exhibit 1B

III. RATE LEVEL ANALYSIS

ACC/CAL YEAR	WRITTEN PREMIUM	EARNED PREMIUM	RATE LEVEL INDEX	EARNED PREM AT CURR RT	REPORTED LOSSES	EXPECTED DEVEL FACTOR	ULTIMATE LOSSES	EXPENSES	LOSS RATIO AT CURR RT	ANNUAL TREND	EXPENSE RATIO AT CURR RT	ANNUAL TREND	PROJ OP RATIO
1980	1000000	500000	1.00	500000	375000	1.0000	375000	125000	75.00%		25.00%		100.00%
1981	1212750	1106375	1.00	1106375	746801	1.1111	829781	276594	75.00%	100.00%	25.00%	100.00%	100.00%
1982	1470743	1341756	1.00	1341756	805054	1.2500	1006317	335439	75.00%	100.00%	25.00%	100.00%	100.00%
1983	1783667	1627215	1.00	1627215	854788	1.4286	1220411	406804	75.00%	100.00%	25.00%	100.00%	100.00%
1984	2163143	1973405	1.00	1973405	740027	2.0000	1480054	493351	75.00%	100.00%	25.00%	100.00%	100.00%
										100.00%		100.00%	

PROJ YEAR	LOSS RATIO	DISCOUNT FACTOR	DISC LOSS RATIO	EXPENSE RATIO	RATE IND
1985	75.00%	.7786	58.40%	25.00%	1.0000

IV. FINANCIAL PROJECTIONS

		TREND 1	TREND 2		
NUM EXPOSURE	1000	.05	.10		
AVERAGE PREMIUM	1000	.05	.10		
MIX OF BUSINESS	1	.05	.10		
FREQUENCY	.10	.05	.10		
SEVERITY	7500	.05	.10		
PAYMENT PATTERN	.1	.2	.3	.2	.2
CUM PAY PATTERN	.1	.3	.6	.8	1.0
REPORT PATTERN	.5	.2	.1	.1	.1
CUM REP PATTERN	.5	.7	.8	.9	1.0

CAL YEAR	DIRECT WRITTEN PREMIUM	UEPR	DIRECT EARNED PREMIUM	DIRECT EXPENSE	BOY ASSETS	ACC YEAR	PAID LOSSES	CASE RESERVES	IBNR	INCURRED LOSSES	U/W CASH FLOW	U/W GAIN	INVEST INCOME	TAX	EOY ASSETS	EOY LIAB	EOY SURPLUS	DISC INCURRED LOSSES	DISC LOSS RESERVES	DISC LIAB	DISC SURPLUS	GAAP ADJUST	GAAP SURPLUS	GAAP INCOME
1980	1000000	500000	500000	125000	0	1980	37500	150000	187500	375000	837500	0	41875	8375	871000	837500	33500	319348	281848	781848	89152	100000	133500	133500
1981	1212750	606375	1106375	276594	871000	1980	75000	150000	112500	375000								343872						
						1981	82978	331913	414891	829781	778178	0	126009	25207	1749985	1615678	134307	704638	855032	1461407	288579	121775	255582	172082
1982	1470763	735381	1341756	335439	1749985	1980	112500	75000	75000	375000								361519						
						1981	165956	331913	248934	829781								760903						
						1982	100632	402527	503159	1006317	754236	0	212810	42562	2676469	2371914	304555	856975	1404830	2140212	536257	147076	451632	196049
1983	1783447	891834	1627215	406804	2676469	1980	75000	37500	37500	375000								371510						
						1981	248934	165956	165956	829781								799950						
						1982	201263	402527	301895	1006317								922786						
						1983	122041	488164	610206	1220411	729625	0	304128	60826	3649396	3101538	547858	1039296	1911736	2803570	845826	178367	726225	274593
1984	2163143	1081571	1973405	493351	3649396	1980	75000	0	0	375000								375000						
						1981	165956	82978	82978	829781								822058						
						1982	301895	201263	201263	1006317								970139						
						1983	244082	488164	366123	1220411								1119108						
						1984	148005	592021	740027	1480054	734852	0	401682	80336	4705594	3836391	869204	1260406	2389968	3471539	1234055	216314	1085518	359293
1985	2623351	1311676	2393247	598312	4705594	1981	165956	0	0	829781								829781						
						1982	201263	100632	100632	1006317								996951						
						1983	366123	244082	244082	1220411								1176537						
						1984	296011	592021	444016	1480054								1357198						
						1985	179494	717974	897468	1794935	816192	0	511349	102274	5930882	4652583	1278299	1528558	2898434	4210109	1720772	262335	1540634	455116
1986	3101469	1590735	2902410	725603	5930882	1982	201263	0	0	1006317								1006317						
						1983	244082	122041	122041	1220411								1209052						
						1984	444016	296011	296011	1480054								1426845						
						1985	358987	717974	538481	1794935								1645942						
						1986	217681	870723	1088404	2176808	989837	0	642580	128516	7434782	5642420	1792363	1853758	3315075	5105810	2328972	318147	2110510	569876
1987	3858327	1929163	3519898	879974	7434782	1983	244082	0	0	1220411								1220411						
						1984	296011	148005	148005	1480054								1466278						
						1985	538481	358987	358987	1794935								1730486						
						1986	435362	870723	653042	2176808								1996117						
						1987	263992	1055969	1319962	2639923	1200425	0	803499	160700	9278007	6842844	2435162	2248146	4262908	6192071	3085936	385835	2820995	710485
1988	4679186	2339593	4268756	1067189	9278007	1984	296011	0	0	1480054								1480054						
						1985	358987	179494	179494	1794935								1778229						
						1986	653042	435362	435362	2176808								2098550						
						1987	527985	1055969	791977	2639923								2420791						
						1988	320137	1280427	1600784	3201567	1455815	0	1000591	200118	11534295	8298659	3235636	2726438	5169841	7509434	4024861	467919	3703554	882559
1989	5674682	2837341	5176934	1294233	11534295	1985	358987	0	0	1794935								1794935						
						1986	435362	217681	217681	2176808								2156547						
						1987	791977	527985	527985	2639923								2545016						
						1988	640313	1280427	960470	3201567								2935814						
						1989	388270	1553080	1941350	3882700	1765540	0	1241706	248341	14293200	10064199	4229001	3306488	6269725	9107066	5186134	567468	4796469	1092915

Exhibit 1D

PROPERTY-CASUALTY FINANCIAL MODEL
STOCHASTIC MODEL

I. FINANCIAL HISTORY

		TREND 1	TREND 2		
REG EXPOSURE	1000	.05	.10		
AVERAGE PREMIUM	1000	.05	.10		
MIX OF BUSINESS	1	.05	.10		
FREQUENCY	.10	.05	.10		
SEVERITY	7500	.05	.10		
PAYMENT PATTERN	.1	.2	.3	.2	.2
CUM PAY PATTERN	.1	.3	.6	.8	1.0
REPORT PATTERN	.5	.2	.1	.1	.1
CUM REP PATTERN	.5	.7	.8	.9	1.0

CAL YEAR	DIRECT WRITTEN PREMIUM	UEPR	DIRECT EARNED PREMIUM	DIRECT EXPENSE	BOY ASSETS	ACC YEAR	PAID LOSSES	CASE RESERVES	IBNR	INCURRED LOSSES
1980	1000000	500000	500000	125000	0	1980	37500	150000	187500	375000
1981	1212750	606375	1106375	276594	871000	1980	75000	150000	112500	375000
						1981	165956	165956	497869	829781
1982	1340096	670048	1276423	319106	1663688	1980	112500	75000	75000	375000
						1981	165956	165956	331913	829781
						1982	100632	402527	503159	1006317
1983	1703667	891834	1561601	390470	2468661	1980	75000	37500	37500	375000
						1981	165956	165956	165956	829781
						1982	201263	402527	301895	1006317
						1983	122041	488164	610206	1220411
1984	2163143	1081571	1973405	493351	3574564	1980	75000	0	0	375000
						1981	165956	82978	82978	829781
						1982	301895	201263	201263	1006317
						1983	244082	488164	366123	1220411
						1984	155406	621623	777028	1554056

CAL YEAR	ACC YEAR	U/W CASH FLOW	U/W GAIN	INVEST INCOME	TAX	EOY ASSETS	EOY LIAB	EOY SURPLUS	DISC INCURRED LOSSES	DISC LOSS RESERVES	DISC LIAB	DISC SURPLUS	GAAP ADJUST	GAAP SURPLUS	GAAP INCOME
1980	1980	837500	0	41875	8375	871000	837500	33500	319348	261848	261848	89152	100000	133500	133500
1981	1980								343872						
	1981	695200	0	121860	24372	1663688	1532700	130988	717692	783108	1389463	274205	121275	252263	118763
1982	1980								361519						
	1981								764765						
	1982	641907	-49000	198464	15193	2468661	2203402	265259	856975	1325714	1995762	493099	134010	399269	147006
1983	1980								371510						
	1981								799950						
	1982								922786						
	1983	828936	-49000	290333	33567	3574564	3101539	473025	1039296	1911736	2803570	770994	178367	651392	252125
1984	1980								375000						
	1981								822058						
	1982								970139						
	1983								1119108						
	1984	727452	-74003	393829	41764	4654080	3902993	751087	1323427	2445568	3527159	1126921	216314	967401	160009

Exhibit 2A

ACC YEAR	REPORTED LOSSES 1	2	3	4	5
1980	187500	262500	300000	337500	375000
1981	331913	497869	663825	746803	
1982	503159	704422	805054		
1983	610206	854288			
1984	777028				

ACC YEAR	PAID LOSSES 1	2	3	4	5
1980	37500	112500	225000	300000	375000
1981	165956	331913	497869	663825	
1982	100632	301895	603790		
1983	122041	366123			
1984	155406				

ACC YEAR	REPORTED DEVELOPMENT 2:1	3:2	4:3	5:4
1980	1.4000	1.1429	1.1250	1.1111
1981	1.5000	1.3333	1.1250	
1982	1.4000	1.1429		
1983	1.4000			
1984				
AVERAGE	1.4250	1.2063	1.1250	1.1111
CHOSEN	1.4000	1.1429	1.1250	1.1111
ULTIMATE	2.0000	1.4286	1.2500	1.1111

ACC YEAR	PAID DEVELOPMENT 2:1	3:2	4:3	5:4
1980	3.0000	2.0000	1.3333	1.2500
1981	2.0000	1.5000	1.3333	
1982	3.0000	2.0000		
1983	3.0000			
1984				
AVERAGE	2.7500	1.8333	1.3333	1.2500
CHOSEN	3.0000	2.0000	1.3333	1.2500
ULTIMATE	10.0000	3.3333	1.6667	1.2500

ACC YEAR	REPORTED LOSSES 1	2	3	4	5	ULTIMATE
1980	187500	262500	300000	337500	375000	375000
1981	331913	497869	663825	746603	829781	829781
1982	503159	704422	805054	905685	1006317	1006317
1983	610206	854288	976329	1098370	1220411	1220411
1984	777028	1087839	1243245	1398651	1554056	1554056
1985	897468	1256455	1435948	1615442	1794935	1794935
1986	1088404	1523765	1741446	1959127	2176808	2176808
1987	1319962	1847946	2111939	2375931	2639923	2639923
1988	1600784	2241097	2561254	2881410	3201567	3201567
1989	1941350	2717890	3106160	3494430	3882700	3882700

ACC YEAR	PAID LOSSES 1	2	3	4	5
1980	37500	112500	225000	300000	375000
1981	165956	331913	497869	663825	829781
1982	100632	301895	603790	805054	1006317
1983	122041	366123	732247	976329	1220411
1984	155406	466217	932434	1243245	1554056
1985	179494	538481	1076961	1435948	1794935
1986	217681	653042	1306085	1741446	2176808
1987	263992	791977	1583954	2111939	2639923
1988	320157	960470	1920940	2561254	3201567
1989	388270	1164810	2329620	3106160	3882700

INCREMENTAL PAID LOSSES 1	2	3	4	5
37500	75000	112500	75000	75000
165956	165956	165956	165956	165956
100632	201263	301895	201263	201263
122041	244082	366123	244082	244082
155406	310811	466217	310811	310811
179494	358987	538481	358987	358987
217681	435362	653042	435362	435362
263992	527985	791977	527985	527985
320157	640313	960470	640313	640313
388270	776540	1164810	776540	776540

DISCOUNTED PAID LOSSES 1	2	3	4	5
319348	343872	361519	371516	375000
717692	764765	799950	822058	829781
656975	922756	970139	996951	1006317
1039296	1119108	1176537	1209052	1220411
1323427	1425058	1492147	1539592	1554056
1528558	1645547	1730406	1778229	1794935
1851758	1996117	2098554	2156547	2175808
2248146	2420791	2545016	2615352	2639923
2726438	2935614	3086469	3171769	3201567
3306488	3560408	3743115	3846562	3882700

Exhibit 2B

III. RATE LEVEL ANALYSIS

ACC/CAL YEAR	WRITTEN PREMIUM	EARNED PREMIUM	RATE LEVEL INDEX	EARNED PREM AT CURR RT	REPORTED LOSSES	EXPECTED DEVEL FACTOR	ULTIMATE LOSSES	EXPENSES	LOSS RATIO AT CURR RT	ANNUAL TREND	EXPENSE RATIO AT CURR RT	ANNUAL TREND	PROJ OF RATIO
1980	1000000	500000	1.00	500000	375000	1.0000	375000	125000	75.00%		25.00%		112.28%
1981	1212750	1106375	1.00	1106375	746603	1.1111	829781	276594	75.00%	100.00%	25.00%	100.00%	109.67%
1982	1340096	1276423	1.00	1276423	805654	1.2500	1006317	319106	78.84%	105.12%	25.00%	100.00%	111.35%
1983	1781667	1561881	1.00	1561881	654286	1.5079	1268212	390470	82.48%	104.62%	25.00%	100.00%	112.64%
1984	2163143	1973405	1.00	1973405	777028	2.1488	1669686	493351	84.61%	102.58%	25.00%	100.00%	112.22%
										103.06%		100.00%	

PROJ YEAR	LOSS RATIO	DISCOUNT FACTOR	DISC LOSS RATIO	EXPENSE RATIO	RATE IND
1985	87.22%	.7786	67.91%	25.00%	1.1222

IV. FINANCIAL PROJECTIONS

		TREND 1	TREND 2		
BEG EXPOSURE	1000	.05	.10		
AVERAGE PREMIUM	1000	.05	.10		
MIX OF BUSINESS	1	.05	.10		
FREQUENCY	.10	.05	.10		
SEVERITY	2500	.05	.10		
PAYMENT PATTERN	.1	.2	.3	.2	.2
CUM PAY PATTERN	.1	.3	.6	.8	1.0
REPORT PATTERN	.5	.2	.1	.1	.1
CUM REP PATTERN	.5	.7	.8	.9	1.0

Exhibit 2C

CAL YEAR	DIREC WRITTEN PREMIUM	UEPR	Y EARNED PREMIUM	DIRECT EXPENSE	BOY ASSETS	ACC YEAR	PAID LOSSES	CASE RESERVES	IBNR	INCURRED LOSSES	W CASH FLOW	U/W GAIN	INVEST INCOME	TAX	EOY ASSETS	EOY LIAB	EOY SURPLUS	I INCURRED LOSSES	DISC LOSS RESERVES	DISC LIAB	DISC SURPLUS	GAAP ADJUST	GAAP SURPLUS	GAAP INCOME
1980	1000000	500000	500000	125000	0	1980	37500	150000	187500	375000	837500	0	41875	8375	871000	837500	33500	319348	281048	781648	89157	100000	133500	133500
1981	1212750	606375	1106375	276594	871000	1980	75000	150000	112500	375000								343877						
						1981	165956	165956	497869	829781	695200	0	121860	24372	1663688	1532700	130988	737692	783108	1389483	274205	121275	[illegible]	118763
1982	1470763	735381	1341756	335439	1663688	1980	112500	75000	75000	375000								361519						
						1981	165956	165956	331913	629781								764765						
						1982	100632	402527	503159	1006317	756236	0	204181	40836	2583268	2286936	294332	856975	1325714	2061095	522173	147076	441405	108514
1983	1783667	891834	1627215	406804	2563256	1980	75000	37500	37500	375000								371519						
						1981	165956	165956	165956	829781								799950						
						1982	201263	402527	301895	1006317								922786						
						1983	122041	488164	610206	1220411	812603	0	298957	59791	3635036	3101538	533496	1039296	1911736	2601540	851466	[illegible]	[illegible]	[illegible]
1984	2163143	1081571	1973405	493351	3635036	1980	75000	0	0	375000								375000						
						1981	165956	82978	82978	829781								822055						
						1982	301695	201263	201263	1006317								976139						
						1983	244082	488164	366123	1220411								1115195						
						1984	155406	621623	777028	1554056	727452	-74003	399876	42974	4719391	3902953	816396	1525477	2445560	3527159	1192231	216314	[illegible]	326847
1985	2943795	1471897	2553469	638362	4719391	1981	165956	0	0	829781								829781						
						1982	201263	100632	100632	1006317								996951						
						1983	366123	244082	244082	1220411								1176537						
						1984	310611	621623	466217	1554056								1425058						
						1985	179494	717974	897468	1794935	1081780	120166	526028	165289	6161909	4664606	1297303	1528558	2944093	4415996	1745919	294375	1591663	556971
1996	3570087	1785043	3256941	814235	6161909	1982	201263	0	0	1006317								1006317						
						1983	244082	122041	122041	1220411								1209052						
						1984	466217	310611	310811	1554056								1496187						
						1985	358987	717974	538481	1794935								1645947						
						1986	217681	870723	1088404	2176808	1767621	265898	679572	268863	7840239	5866330	1973910	1853758	3542016	5327600	2513180	357009	2330919	719276
1987	4329623	2164811	3949655	987464	7840239	1983	244082	0	0	1220411								1220411						
						1984	310811	155406	155406	1554056								1539592						
						1985	538481	358987	358987	1794935								1730406						
						1986	435362	870723	653042	2176808								1996117						
						1987	263992	1055969	1319962	2639923	1549431	322468	861496	333533	9917633	7093293	2824340	2246146	4227020	6441831	3475802	432962	[illegible]	926384
1988	5250756	2625375	4790186	1197547	9917633	1984	310811	0	0	1554056								1554056						
						1985	358987	179494	179494	1794935								1778229						
						1986	653042	435362	435362	2176808								2058550						
						1987	527985	1055969	791977	2639923								2470791						
						1988	320157	1280627	1600784	3201567	1882222	391073	1085874	412711	12473018	8564442	3888576	2726438	5169841	7795216	4677801	525075	[illegible]	1156349

Exhibit 2D

TITLE: PROJECTING CALENDAR PERIOD IBNR AND KNOWN LOSS USING RESERVE STUDY RESULTS

AUTHORS: Edward W. Weissner
Arthur J. Beaudoin

Edward W. Weissner is currently an Actuarial Director with Prudential Reinsurance Company. His current interests include Reinsurance reserving, reinsurance database design, applications of mathematics and statistics to actuarial problems and presentation of results in a format that clearly defines any problems and suggests solutions. He holds a Ph.D (1970) in mathematical statistics from the University of North Carolina and is a Fellow of the Casualty Actuarial Society (1980).

Arthur J. Beaudoin is currently an Actuarial Assistant with Prudential Reinsurance Company. He has tackled assignments in reinsurance reserving, reinsurance pricing and actuarial systems. He earned a Ph.D (1982) in philosophy from Northwestern University and is striving to become an FCAS.

ABSTRACT:

Periodically, our reinsurance company does a time consuming, indepth reserves study of each of its underwriting areas. These studies generate detailed information on exposure, market factors, report delay patterns, ultimate expected loss ratios, et cetera, for each homogeneous group of contracts in the underwriting area. While these studies enable the company to periodically check the adequacy of its reserve levels in each underwriting area, they, by themselves, do not yield

1) interim IBNR for each future calendar month until the next study,
2) projected future calendar period IBNR and known loss for company planning based on our current book of business and future writings,
3) a comparison of actual "future" calendar period known loss experience with projected "future" calendar period known loss experience,
4) the comparison in (3) by homogeneous group of contracts and by accident period.

The purpose of this paper is to show how the detailed group information from our reinsurance study is used by our company to address (1) - (4) above. It is hoped that in presenting our methodology the reader will be able to abstract general principles that will allow him to develop a similar system based on his reserve study and its output. The methodology to be discussed here is currently in use and programmed on an IBM-PC. A small teaching example is included.

INTRODUCTION

The purpose of this paper is to show how the detailed information from our reinsurance reserve studies can be used to determine the monthly change to IBNR for the interim months until the next reserve study and determine the change to IBNR and the known loss for future calendar periods of interest. It is hoped that in presenting our methodology that the reader will be able to "see how" to develop his own system based on his company's reserving formulae and on the information contained in his company's reserve studies.

Our reinsurance company does an indepth reserve study of each of its underwriting areas as often as possible. Given our management's committment to the production of quality reserve analyses and our difficulties in obtaining useful data from our system along with the usual problems in grouping and analyzing reinsurance contracts, an individual study can take three to four months. While the size of a reserve study prevents us from doing more than one study a year for each underwriting area, it follows, given the enormous effort put into these studies, that they do generate valuable detailed information on exposure, market factors, report delay patterns, ultimate expected loss ratios, et cetera, for each homogeneous group of contracts in the underwriting area. However, while these studies enable the company to periodically check the adequacy of its reserve levels in each underwriting area, they, by themselves, do not yield

1) interim IBNR for each future calendar month until the next study,

2) projected future calendar period IBNR and known loss for company planning based on our current book of business and future writings,

3) a comparison of actual "future" calendar period known loss experience with projected "future" calendar period known loss experience,

4) the comparison in (3) by homogeneous group of contracts and by accident period.

For some time we have been looking for a way to use much of the key information in our latest reserve study to address the concerns in (1) to (4) above. In addition, management wanted us to be able to generate results overnight and to be able to run varying senarios for (2) to (4) above. Of course the obvious solution in light of today's information is to load <u>all</u> the information from your latest reserve study into an IBM-PC with a hard disk and to program it to generate for each group the IBNR and known loss figures you need to answer (1) to (4) above and then to add up the results over all the groups. This is precisely what we did. The inherant speed and storage capacity of the IBM-PC makes it feasible to literally do all kinds of calculations for several hundred groups in a very short time.

We now present our methodology for extending a reserve study into the future. Again, our purpose here is to show our system in the hope that it will help the reader to see more clearly how he could design a similar system for his company based on their reserve techniques and the information in their latest reserve study.

While we focus on reinsurance for our examples we believe the concepts are applicable to primary companies also.

PLAN OF THE PAPER

In presenting our methodology, we shall follow the following format. First, we shall give a brief overview of our reserves methodology and of the information found in our reserve studies. Then, so we can illustrate our concepts later, we present the "information" from a reserve study at Example Reinsurance Company. This is followed by a discussion of how we extend the results of a reserve study to obtain calendar period IBNR. Next, we address the determination of IBNR for future calendar periods. We will then be ready to discuss the calculation of expected known losses for current and future calendar periods. From there we move on to a brief discussion of the comparison of actual and expected known loss by calendar period. This is followed by a section on general issues which is followed by a final section on implementation of this methodology on an IBM-PC. We conclude with a list of references.

OVERVIEW OF A RESERVE STUDY

Before we can discuss how we would use the results of our reserve study to project IBNR and known loss for future calendar periods, we need to briefly review the essence of our reserve methodology and procedures. These have been discussed extensively in John (1982), Patrik (1978), and Weissner (1981).

We begin each reserve study by sorting our contracts into homogeneous groups. These groups can be based on coverage (casualty, marine, property, ...), category type (working, catastrophe, retro rated, ...), pro rata or excess, line (automobile, general liability, workers' compensation, fire, SMP, ...), retention, size of contract, ... (see John (1982 p.129-130)). For facultative business some of the typical groups could be casualty certificates with low retentions, property certificate with high retentions, et cetera. For treaty business some of the typical groups could be property pro rata contracts, casualty working contracts, crop hail contracts, funded covers, the large ABC contract, the large DEF contract, et cetera. One of our underwriting areas has over 70 distinctive and credible reserve groups. More typically, an underwriting area has approximately 10 to 30 reserve groups.

Next, we develop the case reserve supplements (they can be positive or negative) which will bring the known reserves associated with each group to a level adequate to pay the ultimate liability. The analysis to do this is based on a review of the pertinent report period loss development triangles. Through an allocation rule based on the known outstanding reserves by accident year, the case reserve supplements for each group are assigned to individual accident years.

Simultaneously, we evaluate the "earned but unreported" premium associated with each group. This is a very important figure since it can be quite large for a reinsurer and since we use full earned premium as a measure of exposure. Our evaluation is based on the premium reporting practices of each contract. It could also be developed by studying the premium development triangles associated with collections of similarly sized contracts.

We also estimate the underlying distribution of the report lags associated with the claims of each group. This is discussed in detail in John (1982, p.130-154), Weissner (1978), and Weissner (1981, p.287-292). Generally, we have found that the underlying distributions of report lags can be adequately described by an exponential model, a Weibull model or a log normal model. For our purposes here we need only know that a report lag distribution has been selected for each reserve group and that given the distribution and its parameters, we can, for any "accident month", say m, determine the proportion of claims yet to be reported relative to the ultimate number. Since this proportion is the area under the report lag density and to the right of the largest "observable" (not observed) report lag relative to the evaluation date of the study, we will refer to it as a "tail probability" and label it TP_m

We are now almost ready to determine the IBNR for each group. First, let us review our inputs. We have earned premiums by calendar/accident year (these can be allocated to calendar/accident month) along with a good estimate of earned premium which is "unreported". Since a reinsurer has no good measure of exposure like car - years, most tend to use earned premium or written premium as a general measure of exposure.

Since we use earned premium for exposure, it is important to us to have a good estimate of our ultimate earned premium.

Of course, with shifts in rate adequacy even earned premium can prove to be a poor exposure base. To overcome this, our underwriters, based on reviews of prices, retentions, limits, shares, and coverage, provde us with "market adjustment factors", labelled MAF's. These factors record the shifts in rate adequacy from year to year. More specifically, you select any year as a base year and set the MAF for that year as 1. Also, you select a "typical risk" and its price for that year. The MAF for any other year is the price you got or would get for that "typical risk" divided by the base year price. This definition implies that the factors can be above and below one. (If you selected 1980 as your base year, most reinsurers would be exhibiting MAF factors below 1 that decrease by year through 1983 due to the so called "soft market".)

Dividing the calendar/accident year earned premiums by the respective MAF's we have a "better" exposure base. An example may help. Assume you have $200 of earned premium in 1980 and $100 of earned premium in 1981. If a typical $1 risk in 1980 generated only 50 cents of premium in 1981, then the MAF for 1981 is .5 if 1980 is the base year with MAF of 1. It follows that the exposure premium (earned premiums divided by MAF) for 1980 is 200 (200 divided by 1) and for 1981 is 200 (100 divided by .5). Given the underlying pricing assumptions it appears that our exposure premiums are a reasonable exposure base.

In addition to exposure premium by calendar/accident year, we also have loss experience by accident year. If we combine the known losses with our case reserve supplements, then we can be confident that our reserves for known losses are adequate to pay ultimate losses.

We are now ready to evaluate the IBNR for each group. The formula is discussed in detail in Weissner (1981). Basically underlying our formula are two relationships. They are:

$$IBNR_m = (EP_m \% MAF_m) \times XLR_p \times TP_m$$

$$XLR_p = (\sum_{m \epsilon p} IBNR_m + L_p) \% (\sum_{m \epsilon p} (EP_m \% MAF_m))$$

where $IBNR_m$ = the IBNR for accident month m

EP_m = earned premium for calendar/accident month m

MAF_m = the market adjustment factor for month m

XLR_p = the ultimate exposure loss ratio for an accident period p of many months m (note that this ratio refers to "exposure premium" and not premium. Hence the XLR is not the same as the ultimate loss ratio)

L_p = the known losses set to ultimate pay out for the accident period p

TP_m = the proportion of claims yet to be reported for accident month m

m = a specific accident month

p = a specific accident period, usually 2 to 5 years in length, for which the XLR_p is assumed to be constant. (see Weissner

(1981, p.278))

In reviewing the above formulas, it should become clear that all of variables are known for a period p except the XLR and the IBNR. Since there are two relationships and two unknowns, simple algebra yields solutions for the IBNR and XLR (see Weissner (1981, p.293-294)).

It follows from all of the above that as a result of a reserve study we have for each reserve group, either from inputs or outputs, for each month

EP	earned premiums
MAF	market adjustment factors
XLR	ultimate exposure loss ratios

along with a distribution of report lags that can be used to generate TP. Also, though it was not mentioned above we usually have a claim severity, labelled SEV, which is by accident month and which can be used to obtain IBNR counts by the obvious division, that is

$$\text{IBNR Counts}_m = \text{IBNR}_m \div \text{SEV}_m$$

AN EXAMPLE

Since the purpose of this paper is to show how we use the results of a reserve study to project calendar period IBNR and known loss, we thought it would be best to illustrate the concepts with an example. Since a realistic example would prove to be unwieldly, we include a very simplified example that is completely artificial. None of the numbers are real; in fact they were selected more for illustration than reality.

Let us suppose that "Example Reinsurance Company" which writes only domestic facultative business did a reserve study as of 6/30/84. Suppose that study followed the methods of the prior section. Further, suppose the homogeneous reserve groups were Property, Casualty and Marine (remember we want to keep this simple).

Exhibit 1 shows for each group the results (inputs and outputs by month) of that 6/30/84 study. For each variable the values are given in vector notation with the 6/84 value as the first component and the 1/80 value as the last component. The symbol ρ , an APL character, is the reshape character. When you see something like 12 ρ .50, you can replace it with 12 .5's. The only exception to the vector notation is the LAG variable. Here we store the general shape of report lag distribution (1 = exponential, 2 = log normal, 3 = Weibull, ...) and its two parameters.

According to exhibit 1 the Property group parameters for October, 1983 are:

EP = 46000

MAF = .73

XLR = .60

SEV = 50

Also the underlying report lag distribution is log normal (2) with parameters 2.24 and .86.

Finally, for groups like Marine where there is zero earned premium in some months, we will put "filler" numbers in the other variables just to keep the vectors consistent. This will be useful later.

Exhibit 2 shows the IBNR and IBNR counts associated with each group as of 6/30/84. These figures come directly from the reserve study. They can also be regenerated by using the first basic relationship.

$$IBNR_m = (EP_m \ \% \ MAF_m) \times XLR_p \times TP_m$$

$$IBNR\ Count_m = IBNR_m \ \% \ SEV_m$$

and then summing over the months in each year. If you wish to verify these figures, let

$$TP_m = Pr[Lag \geq \text{"6/84 - m"} + .5]$$

and the distribution functions be:

1) exponental: $F(x) = 1 - exp\ (-t(x-s))$

2) log normal: $F(x) = \Phi\ ((log\ x - m)\ \%\ v)$

where the respective parameters are t,s and m, v^2.

EXTENDING THE RESULTS OF A RESERVE STUDY

Once a reserve study is completed we would like to use the results of that study to keep the IBNR current until the next reserve study and to project IBNR and known losses for future calendar periods. According to our IBNR formula, if we can get earned premium by calendar/accident month and by group, then we only need to develop a rule for projecting market adjustment factors, MAF's, ultimate exposure loss ratios, XLR's, and severities, SEV's, into the months after a study. This assumes, of course, that our report lag distribution remains stable.

Let's assume that we can get earned premiums by month and group, that the report lag distribution is stable and that for a "few months" after a study the "most recent" values of MAF, XLR, and SEV should be continued. This seems very plausible.

Exhibit 3 shows the values of our variables by group through 9/30/84. Again, the values are in vector format except here the lead values are for 9/84. Note that the first four values of MAF, XLR, and SEV are the same; that is the latest three months carry the 6/84 value. All prior (for EP, too) values are as they were in Exhibit 1. Also note that Example Re continues to have zero earned premium for Marine.

Using our basic formula, we can now easily calculate the IBNR as of 9/30/84 for Example Re. Exhibit 4 shows the IBNR as of 9/30/84; Exhibits 5-7 show the IBNR calculation in detail for each group. Note that all the variable values below the dotted line came directly from the reserve study. Only the latest 3 months of MAF, XLR and SEV come from our projection rule of "no change". (We are still assuming the earned premiums are real, actual values.)

Unfortunately, while we have the cumulative IBNR as of 9/30/84, we need the September calendar month change in IBNR. To obtain it, we must subtract the August, 1984 cumulative IBNR from the September cumulative IBNR. While this creates no problem mathemetically, it does confuse some non-actuarial people who are very comfortable with calendar month changes and very uncomfortable with changes to cumulatives.

While it is indeed informative to see the cumulative IBNR associated with a group of contracts as of a certain date split by accident year or even by accident month as in Exhibits 5-7, underwriters and management tend to be more interested with calendar period results. They tend to focus on how much are we writing this year and how much IBNR will be added?

We have found it to be much more productive to discuss the change in IBNR for a calendar month and to show how it can be divided into various pieces due to current and past accident months or years. We now introduce the formula for the calendar month increase to IBNR. The presentation underscores some basic concepts that underwriters and management feel comfortable with; the formula is, of course, equivalent to taking the difference of the cumulative IBNR as of the end of this month and that as of the end of the prior month.

Before we develop the formula for the monthly increase to IBNR, let us comment on notation. Further, let us restrict our interest to a specific group. Clearly, the results for the month are just the sum of the various group results.

For a specific accident month, m, let P' be the proportion of accident month, m, claims that will be reported, according to the underlying report lag distribution, in the calendar month of interest.

That is,

$$P_m = Pr[Lag = \text{"calendar month of interest - m"}]$$
$$= Pr[\text{"cal. mon. - m"} - .5 < Lag < \text{"cal. mon. -m"} + .5]$$

where all report lags are measured in months. It should be noted that

$$P_m = TP_{m-1} - TP_m$$

since the probability that a claim from accident month m will be reported in this calendar month is the same as the probability that a claim from this accident month m will be reported this month or in the future less the probability that a claim from this accident month m will be reported in the future. The second probability in the difference is exactly TP_m ; the first probability is equivalent to the probability that a claim from the accident month just after accident month m will be reported in the future, i.e., TP_{m-1} where "m-1" means just after m. This formula for P_m gives us an easy way to calculate it.

In discussing the formula we will also be interested in the ultimate expected loss associated with an accident month m. In general this ultimate loss would be the earned premium for the period times the ultimate loss ratio for the period. Recall however that in our reserve study earned premiums have been converted to exposure earned premiums (EP % MAF) and the loss ratio is an ultimate exposure loss ratio. Hence,

$$\text{Ultimate Expected Loss}_m = (EP_m \ \% \ MAF_m) \times XLR_p$$

We now discuss the formula for a group. Let us separate the discussion between prior accident months and the current accident month. For the current month, the change (increase) to calendar month IBNR due to it is the ultimate expected loss associated with the exposure for the accident month less the expected losses associated with the exposure for the accident month that should have been reported. Since the expected reported losses equal the ultimate expected loss times the expected proportion of losses to be reported, P_m , we have using the prior notation,

$$\text{Change to IBNR}_m = \text{Ultimate Expected Loss}_m \text{ less Expected Losses}_m \text{ Reported this Month}$$

$$= \text{Ultimate Expected Loss}_m - \text{Ultimate Expected Loss}_m \times \text{Percent Reported}_m$$

$$= [(EP_m \ \% \ MAF_m) \times XLR_p] - [(EP_m \ \% \ MAF_m) \times XLR_p] \times P_m$$

For a prior accident month, the change to calendar month IBNR due to it is simply a take down for the expected losses associated with the specific prior accident month that should have been reported. Observe that IBNR for the ultimate expected losses due to this prior accident month would have been included in the prior accident month's calendar month. Hence, as time moves on, we need only reduce the IBNR associated with each prior accident month based on expected reported losses. Following the above, for a prior accident month m, we have:

$$\text{Change to IBNR}_m = -[(EP_m \ \% \ MAF_m) \times XLR_p] \times P_m$$

To clarify this formula let us return to our example. Exhibits 8, 9, and 10 provide the details of the group calculations for the September, 1984 Monthly Increase to IBNR. Note that the first four columns of Exhibit 8-10 are identical to the first four columns of Exhibits 5-7 respectively. Also, in Exhibits 8-10 the fifth column contains the P values whereas in Exhibit 5-7 the fifth column contains the TP values. Note that successive differences of the TP values yield the P values. Columns 6 and 7 show the current month expected increase to IBNR (the ultimate loss due to the current month's exposure) and the current month expected decrease to IBNR (the expected reported losses due to each accident month). Finally, column (8) contains each accident month's contribution to the monthly increase to IBNR. Column (9) shows each accident year's contribution. Again, observe also that all the parameters from the reserve study are enclosed by the dotted lines.

We have found exhibits like this to be very useful in discussing monthly IBNR. It is easy to see the increase in the monthly IBNR due to the current month's new exposure and the decreases in the monthly IBNR due to the prior month's exposures. It is easy to see the "length of the tail" or the number of prior months that still effect the IBNR. Further, for groups like Marine, see Exhibit 10, the scheduled reduction of IBNR due to expected loss patterns is clear to see.

Exhibit 11 summarizes the results of Exhibits 8-10. These summaries of the monthly change to IBNR by group and by contributing accident year seem to be more than adequate for monthly reports.

Of course, some people will want to see both IBNR's - the cumulative IBNR as of month end and the change in IBNR for the month. Exhibits 12A and 12B do just that. In addition, they show the year to date change to IBNR which is simply the sum of the monthly changes to IBNR for each of this year's months to date. A formula much like the monthly change to IBNR formula can be developed to generate this figure directly.

Exhibit 13 contains a summary of the month's IBNR for August and September, 1984. From these you can verify that the monthly formula is equivalent to taking the difference of the cumulative IBNR figures.

The above methodology and computer sheets describe how we can move the results of a reserve study forward in time to set monthly changes to IBNR. Of course, we have assumed that we would receive earned premium each month by group and that for a "few months" anyway we could extend our factors by using the latest study factors. We'll discuss these assumptions in more detail later.

PROJECTED IBNR FOR FUTURE CALENDAR PERIODS

As soon as the September, 1984 IBNR, both cumulative as of 9/30/84 and monthly change for September, has been reported to the Comptroller, Management wants to know (1), how much more IBNR will we have to book by year end and (2) how much IBNR will we book next year. The first question is usually motivated by the need to evaluate the year-end results early and often so strategic planning can take place. The second question is usually related to next year's budget and planning process.

Since our best information is contained in the most recent reserve study parameters, it seems only natural to use these parameters with a rule for recent month's parameters along with some good estimates of earned premium for each group.

Let's tackle question one first. Here we must extend our parameters another three months, i.e., to October, November and December 1984. We have already moved the MAF, XLR, and SEV forward in time by assuming that July through September, 1984 have the same values as June, 1984. Let's continue this rule and assign the June, 1984 value for MAF, XLR, and SEV to the values for October through December 1984. (Of course, if there is good reason to increase or decrease values one should do it - more later.) Further, lets assume the earned premium projections for October through December, 1984 are 55,750 and 27,583 per month for the Property and Casualty groups respectively.

Exhibit 14 shows the reserving parameters we shall use to make our year-end projection. They are based on our rule for moving parameters forward in time and on the study earned premiums prior to 6/84, the actual earned premiums for 7/84 to 9/84 and the projected earned premiums for 10/84 to 12/84. Again, the first component of each vector is 12/84.

Using our IBNR formulas, we obtain the projected December, 1984 IBNR results. Exhibit 15 shows the results by group and by accident year. Clearly the expected increase to IBNR over the next three months (10/84 to 12/84) is 120,960, that is 1,538,836 (the all groups cumulative IBNR as of 12/84 - see Exhibit 15) less 1,417,876 (the all groups cumulative IBNR as of 9/84 - see Exhibit 12). One could also take the difference of the respective year-to-date changes to IBNR, that is 491,986 less 371,026.

To determine the change in IBNR for 1985 we need to again move our parameters forward in time. Let's assume that the earned premiums for the next year are projected to be 50,175 and 24,825 per month respectively for the Casualty and Property groups. Marine continues to get zero earned premium. (If you know your earned premium varies by season, you could enter the seasonally adjusted projected earned premium.) Lets also assume that the XLR and SEV can remain at the 6/84 value. However, let's assume that due to increases in prices, the MAF values in 1985 will be 20% higher than at the end of 1984. This means that 7/84 - 12/84 have the 6/84 MAF value but 1/85 - 12/85 have the 6/84 MAF value times 1.2 (a more realistic approach, given that prices have suddenly jumped 20% in a month, would be to let the MAF value in each successive month be approximately 1.02 times the prior month MAF beginning with the 1/85 MAF.)

Exhibit 16 shows the reserving parameters through year-end 1985. They are based on the above rules for setting parameter values. The vector begins, of course, with the 12/85 value.

Again, using our formulas, we obtain the projected December, 1985 IBNR results. Exhibit 17 shows the results by group and accident year. Clearly, the expected increase to IBNR for 1985 is 145,745, the all groups year to date IBNR increase less the IBNR decrease, i.e., 894,225 - 748,480. You could also subtract the all groups cumulative IBNR figures as of 12/85 and 12/84 (see Exhibit 15) i.e., 1,684,581 - 1,538,836.

Clearly different rules to move the parameters forward in time could have been used. Moreover, in doing projections, you may want to run various senarios. Hopefully, the above illustrates how projections can be developed.

EXPECTED KNOWN LOSSES

In addition to current and future IBNR questions, management also has current and future known loss questions. More specifically, what does the latest reserve study imply about the expected known losses for the latest calendar months? Can we use the expected known loss figures to monitor our actual experience? What does the reserve study imply about the expected calendar period known losses for next year or for the rest of the current year?

Under certain regularity assumptions, the answers to these questions are already contained in our exhibits. In fact, if your losses are reserved to ultimate on the day they are first received and you therefore have no case development to consider, the expected known loss for a calendar period is exactly equal to the "decrease" part of the IBNR for the calendar period. That is, the expected known loss is precisely the expected reported losses for the calendar period. It follows then that the expected known loss for the September, 1984 calendar month is 56,214 (see Exhibit 11, all groups, the total in column 3) and that the expected number of claims is 781.2. The expected known loss for the remaining three months of 1984 evaluated at 9/30/84 is 177,113 which is the 12/31/84 year to date expected known loss of 627,266 (see Exhibit 15, all groups, the total in column 8) less the 9/30/84 year to date expected known loss of 450,153 (see Exhibit 11, all groups, the total in column 8). The expected number of claims is 1450 which is 8,710.7 less 6,260.7. Finally, the expected known loss for the 1985 calendar year is 748,480 which is the 12/31/85 year to date expected loss figure on exhibit 17 (see all groups, the total in column 8).

Of course, the no case development assumption is definitely an unrealistic assumption. However, if one can assume that case development patterns are stable and that new claims are entering the loss process as fast as old claims are being closed so that the mix of losses in various stages of development is stationary, then the expected known losses for a calendar period still equals the expected reported losses for the calendar period.

To see this we need to discuss a number of concepts. First, the known loss for a calendar period equals the development on claims known at the beginning of the period (i.e., the change in incurred over the period) plus the value as observed at the end of the period of all the newly reported claims in the period. Since claims develop in this senario, the value of a newly reported claim at the end of the period is usually not its ultimate value or value after development. If we restate the known loss for a calendar period using ultimate values for new claims, we have

Known Loss = Case development on old claims
+ Current value on new claims

= Case development, for the period, on old claims
+ Ultimate value on new claims
− Case development, all time, on new claims

Recall that the IBNR take down (decrease) for expected reported losses in a calendar period is precisely the ultimate value of the expected new claims in the period. If we can now show that the case development on the old claims for the period is equal to the <u>full</u> case development over all time for the new claims, then the calendar period Known Loss is exactly the "take down piece of the monthly IBNR".

Let us now show that the case development for the calendar period on the old claims is precisely the all time case development on the newly reported claims. We are assuming, of course, that the mix of claims is stationary. The following illustration will be helpful in visualizing the concepts.

Assume that each calendar year we get $100 of new claims and that the incurred pattern of development for every calendar year as of each year-end is $100, $130, $110, $100, $95, $95,... Then as of the end of 1983 the report period incurred development pattern is:

Calendar Year	12	24	36	48	60	72
1978	100	130	110	100	95	95
1979	100	130	110	100	95	
1980	100	130	110	100		
1981	100	130	110			
1982	100	130				
1983	100					

Now, given stable development and constant dollars of new claims the incurred development triangle at the end of 1984 looks like the prior triangle with one older row, i.e.,

Calendar Year	12	24	36	48	60	72	84
1978	100	130	110	100	95	95	95
1979	100	130	110	100	95	95	
1980	100	130	110	100	95		
1981	100	130	110	100			
1982	100	130	110				
1983	100	130					
1984	100						

It follows that the 1983 and 1984 legs of the report period incurred development triangle are:

Calendar Year	12	24	36	48	60	72	84	Calendar Losses (1984)
1978						95	95	0
1979					95	95		0
1980				100	95			-5
1981			110	100				-10
1982		130	110					-20
1983	100	130						+30
1984	100							100
								95

The calendar 1984 known losses are $95. Note however that the $95 is precisely the ultimate payout on the $100 of new claims and that the year by year one year development losses correspond precisely to the all time development pattern for the $100. That is, the development pattern for our $100 of losses is $100, $30,

$-20, $-10, $-5, $0 on a change basis. Thus the 30, -20, -10, -5, 0 is the development pattern for the individual year's case development of old losses or for the all time case development of the new claims.

Thus, if you are willing to assume a level book of claims with an homogeneous mix of development stages, we can use our IBNR exhibits to estimate the known loss for a calendar period. Of course, if you have an expanding book of claims, the suggested procedure would probably misstate the known losses. But even in that case, the expected known losses for our exhibits might serve as a useful guide until you can do something better.

MONITORING ACTUAL KNOWN LOSSES

Since all of our IBNR and known loss figures have been calculated at the group level (recall the groups are those of the latest reserve study) and are available by accident year within group, we have everything we need to monitor actual known loss experience by calendar period. To compare actual versus expected known loss experience for a calendar period, we could first make a comparison at the total level. If a large difference existed, we could do comparisons by group. When the group or groups that generated the difference were found, we could do the comparisons by accident year.

In making these comparisons where the expected known loss was based on projected earned premiums, one should also compare the actual earned permium to the projected earned premium. Since our formulas use earned premiums as an exposure base, any big deviation from the projected earned premiums can itself be the cause of differences between actual and expected known loss.

SOME GENERAL ISSUES

Before concluding this paper, we would like to briefly discuss some of the underlying, unstated assumptions that may cause problems. The issues to be discussed include the allocation of earned premium to group, earned but unrecorded premium, accident month versus calendar month earned premiums, extension rules for the parameters, asbestos, and contract exclusions.

In this presentation we assumed that the monthly earned premiums received from the Comptroller could be split by group. In our company the earned premiums are not split by group. The comptroller delivers to us each month the calendar month earned premium for each underwriting area. We then allocate this premium to group based on the prior year's distribution of earned premium to group. To the extent that our mix of business is constant this should be reasonable; if the underwriters decide to change the mix of business by group, the allocation will be incorrect. To monitor changes in the mix, we continually ask the underwriters about their plans, about new big treaties, about major cancellations, et cetera. Note that in our example the Marine group had no earned premium since early 1983. Thus an allocation for 1984 would assign zero to Marine. If the underwriters were about to start writing Marine business again, we would have to adjust our allocation ratios.

We also assumed here that the monthly earned premiums were accident month earned premiums. Clearly the comptroller only has calendar month earned premiums to give us. These can be quite different concepts for a reinsurer since many premiums are sent to us after they are earned. This happens for instance on working treaties which report activity after the quarter, on retro-rated covers which could be sending us premium "on schedule" ten years after the exposure, and on certificates with audits. If we can assume that we "knew" that these earned but unrecorded premiums were coming, then our past accident months have an estimate of earned but unrecorded premiums. If we replace the past accident months estimated earned but unrecorded premiums dollar for dollar with this calendar month's contributions to the past accident months and then set up an earned but unrecorded premium for this accident month to reflect the delays in premium reporting, it seems to us that the increases and decreases to this month's calendar earned premium for earned but unrecorded premiums offset each other, if you are writing a constant premium volume. Hence, the comptroller's calendar month earned premium is for all practical purposes equivalent to the "full" accident month's earned premium.

Let us now discuss our extension rules for moving parameters forward in time. In most of our presentation we either continued the most recent value (i.e., the 6/84 value) or made an across the groups change (i.e., the MAF in 1985 jumped by 20%). It seems to us that within six months of a study the continuation rule is best. If you want another rule you're free to use it. However in dealing with hundreds of groups we found that the rules for extension should be kept simple, should be applied uniformly to all groups and should apply for several months. While the updating of the parameters is difficult if it is complex, the task is miniscule in comparison to the task of getting underwriters to help you update the parameters group by group.

Finally, we have found it necessary to treat some contracts and issues outside the formula. We do not allow the earned premium for large reserve transfers, funded covers, or certain financial guarantee covers to enter the formula. The IBNR and known loss for these are handled outside the formula. Further, because of the issues surrounding asbestos, and other mass action claims, we handle the IBNR and known losses for them outside the formula.

SYSTEMS IMPLEMENTATION OF THIS METHODOLOGY

In this section we include some comments that might help the reader if he desires to develop a system like ours on a personal computer. We will comment on our hardware, creation of the database, changing the database, monthly runs for financial results, output options, and documentation.

Currently our system is programmed in APL and run on an enhanced IBM PC with 512K internal memory, a hard disk and two disk drives. Our system covers eight underwriting areas. The largest has two cost centers with approximately seventy groups per cost center. The smallest has one cost center with five groups.

In creating the database of reserving parameters we decided to set up one file for each group's parameters. In fact, the group parameters in exhibit 1 are copies of the group files. If you can download these parameters from another system you can save a lot of time. We load the parameters, including earned premium, from a reserve study by hand. The parameters from the reserve study of the largest underwriting area can be set up in three days. Since we intend to update or extend these parameters forward in time you must leave space in the files for growth. At times, our need to hold up to 20 years of earned premiums has caused storage space problems. We store the data on diskettes and keep the programs on the hard disk.

To update or extend the parameters each month we found that we needed many options. We developed various extension rules, i.e., repeat the prior factor, multiply the prior factor by a selected input, use a new factor that is being input, truncate the prior three factors and update the file using one of the above options for the next four factors, et cetera. Further, we found that we needed to be able to change selected parameters in selected groups. Sometimes a special review of certain large contracts caused us to change the associated group parameters. Also we found that we needed to store the reserving parameters associated with future projections on separate disks from the so called official parameters used for the monthly IBNR.

To obtain the monthly change to IBNR we must obtain from the comptroller the calendar month earned premium for each underwriting area, then allocate it to group and then calculate. We can do all the updating for a month for all the areas and run the results in a day. Usually we set the programs to run over night. We like to do that because then we can print the results by group, by cost center, by treaty or facultative, et cetera. We have various levels of print options depending on how much detail we need. We also have a snapshot option that allows you to look at the output of a group anytime.

In addition to the various print options mentioned above, we have developed several parameter summaries for quick review of the group inputs, and created several summary output reports that help us internally to quickly review the results. Also we have begun to store for historical reference the group results and the above summaries even though they can be recalculated. Management likes quick responses to questions.

Finally a comment or two on documentation. Of course you need to have documentation that explains how to run the system. But you also need a way to keep track of all the extension rules and changes that have been applied to the database. You need to keep track of all the various copies of the database and their reason for being. And you need a production record of all the runs and their use. Finally, make two backup copies of your data disks. If you only have one back up and two disk drives, a new person can erase your database and cause you to have to reload the data.

CONCLUSION

In this paper we tried to show how we use detailed group information from our reinsurance reserve study to determine monthly IBNR changes for interim months until the next reserve study and determine the expected change to IBNR and the expected known loss for future calendar periods of interest. Hopefully you gained some insight into how you could do something similar based on your reserve studies and reserves methodology. If so, our goal has been achieved.

REFERENCES

John, Russell T. (1982), "Report Lag Distributions and IBNR," 1982 Casualty Loss Reserve Seminar Transcript, p.124-165.

Patrik, Gary S. (1978), "An Actuarial Procedure for Estimating a Reinsurance Company's IBNR," Proceedings of the Insurance Accounting and Statistical Association, vol.1978, p.531-534.

Weissner, Edward W. (1978), "Estimation of the Distribution of Report Lags By The Method of Maximum Likelihood," Proceeding of the CAS, vol.65, p.1-9.

Weissner, Edward W. (1981), "Evaluation of IBNR on a Low Frequency Book Where The Report Development Pattern is Still Incomplete," 1981 Casualty Loss Reserve Seminar Transcripts, p.273-294.

EXAMPLE REINSURANCE COMPANY
Reserve Study as of June 30, 1984
Group Parameters

GROUP: PROPERTY

EP = 54000,53000,52000,51000,50000,49000,48000,47000,46000,45000,44000,43000,42000,41000,40000,39000,38000,37000,36000,35000,34000,33000,32000,31000,30000,29000,28000,27000,26000,25000,24000,23000,22000,21000,20000,19000,18000,17000,16000,15000,14000,13000,12000,11000,10000,9000,8000,7000,6000,5000,4000,3000,2000,1000
MAF = (6ρ0.66),(12ρ0.73),(12ρ0.81),(12ρ0.9),(12ρ1)
XLR = (18ρ0.6),(24ρ0.55),(12ρ0.5)
SEV = (54ρ50)
LAG = 2 2.24 0.86

GROUP: CASUALTY

EP = (19ρ24000),23000,22000,21000,20000,19000,18000,17000,16000,15000,14000,13000,12000,11000,10000,9000,8000,7000,6000,5000,4000,3000,2000,1000,(12ρ0)
MAF = (6ρ0.51),(12ρ0.64),(12ρ0.8),(24ρ1)
XLR = (42ρ0.9),(12ρ0)
SEV = (42ρ1000),(12ρ1)
LAG = 1 0.02 0

GROUP: MARINE

EP = (18ρ0),1000,2000,3000,4000,5000,6000,7000,8000,9000,10000,11000,(14ρ12000),11000,10000,9000,8000,7000,6000,5000,4000,3000,2000,1000
MAF = (54ρ1)
XLR = (54ρ1.2)
SEV = (18ρ1),(36ρ100)
LAG = 1 0.05 0

1) 'ρ' can be defined as follows: '5ρb' means b,b,b,b,b.
2) In each vector of monthly parameters, the first component is 6/84 and the final component is 1/80.

EXHIBIT 2

EXAMPLE REINSURANCE COMPANY
Cumulative IBNR as of June 30, 1984
by Group, by Accident Year

GROUP: PROPERTY

CUMULATIVE IBNR

AS OF:6/84

ACC YR	CUM $ IBNR	CUM # IBNR
84	247,162	4,943.2
83	179,535	3,590.7
82	41,236	824.7
81	10,617	212.3
80	1,698	34.0
TOTAL	480,249	9,605.0

GROUP: CASUALTY

CUMULATIVE IBNR

AS OF:6/84

ACC YR	CUM $ IBNR	CUM # IBNR
84	239,459	239.5
83	319,344	319.3
82	156,903	156.9
81	35,506	35.5
80	0	0.0
TOTAL	751,212	751.2

GROUP: MARINE

CUMULATIVE IBNR

AS OF:6/84

ACC YR	CUM $ IBNR	CUM # IBNR
84	0	0.0
83	0	0.0
82	26,006	260.1
81	28,991	289.9
80	9,404	94.0
TOTAL	64,401	644.0

GROUP: ALL

CUMULATIVE IBNR

AS OF:6/84

ACC YR	CUM $ IBNR	CUM # IBNR
84	486,621	5,182.7
83	498,879	3,910.0
82	224,146	1,241.7
81	75,114	537.8
80	11,102	128.0
TOTAL	1,295,862	11,000.2

EXAMPLE REINSURANCE COMPANY
Reserving Parameters as of Sept 30, 1984 *
Based on Reserve Study as of June 30, 1984
by Group

GROUP: PROPERTY

EP = 54858,52182,53520,54000,53000,52000,51000,50000,49000,48000,47000,46000,45000,44000,43000,42000,41000,40000,39000,38000,37000,36000,35000,34000,33000,32000,31000,30000,29000,28000,27000,26000,25000,24000,23000,22000,21000,20000,19000,18000,17000,16000,15000,14000,13000,12000,11000,10000,9000,8000,7000,6000,5000,4000,3000,2000,1000
MAF = (9ρ0.66),(12ρ0.73),(12ρ0.81),(12ρ0.9),(12ρ1)
XLR = (21ρ0.6),(24ρ0.55),(12ρ0.5)
SEV = (57ρ50)
LAG = 2 2.24 0.86

GROUP: CASUALTY

EP = 27142,25818,26480,(19ρ24000),23000,22000,21000,20000,19000,18000,17000,16000,15000,14000,13000,12000,11000,10000,9000,8000,7000,6000,5000,4000,3000,2000,1000,(12ρ0)
MAF = (9ρ0.51),(12ρ0.64),(12ρ0.8),(24ρ1)
XLR = (45ρ0.9),(12ρ0)
SEV = (45ρ1000),(12ρ1)
LAG = 1 0.02 0

GROUP: MARINE

EP = (21ρ0),1000,2000,3000,4000,5000,6000,7000,8000,9000,10000,11000,(14ρ12000),11000,10000,9000,8000,7000,6000,5000,4000,3000,2000,1000
MAF = (57ρ1)
XLR = (57ρ1.2)
SEV = (21ρ1),(36ρ100)
LAG = 1 0.05 0

1) 'ρ' can be defined as follows: '5ρb' means b,b,b,b,b.
2) In each vector of monthly parameters, the first component is 9/84 and the final component is 1/80.

* Extension rule: Parameters for months after 6/84 are set at the 6/84 value (i.e. MAF, XLR, SEV).
Earned premiums after 6/84 are from the Comptroller.

EXHIBIT 3

EXHIBIT 4

EXAMPLE REINSURANCE COMPANY
Cumulative IBNR as of Sept 30, 1984
Based on Reserving Parameters as of Sept 30, 1984 (see Exhibit 3)
by Group, by Accident Year

GROUP: PROPERTY

CUMULATIVE IBNR

AS OF:9/84

ACC YR	CUM $ IBNR	CUM # IBNR
84	336,537	6,730.7
83	138,390	2,767.8
82	33,546	670.9
81	8,956	179.1
80	1,466	29.3
TOTAL	518,894	10,377.9

GROUP: CASUALTY

CUMULATIVE IBNR

AS OF:9/84

ACC YR	CUM $ IBNR	CUM # IBNR
84	361,600	361.6
83	300,747	300.7
82	147,766	147.8
81	33,438	33.4
80	0	0.0
TOTAL	843,551	843.6

GROUP: MARINE

CUMULATIVE IBNR

AS OF:9/84

ACC YR	CUM $ IBNR	CUM # IBNR
84	0	0.0
83	0	0.0
82	22,384	223.8
81	24,953	249.5
80	8,094	80.9
TOTAL	55,430	554.3

GROUP: ALL

CUMULATIVE IBNR

AS OF:9/84

ACC YR	CUM $ IBNR	CUM # IBNR
84	698,137	7,092.3
83	439,137	3,068.5
82	203,696	1,042.5
81	67,347	462.1
80	9,559	110.2
TOTAL	1,417,876	11,775.7

EXAMPLE REINSURANCE COMPANY
Cumulative IBNR Calculation
Domestic Facultative - PROPERTY
As of Sept 30,1984

ACC MON	EP	MAF	XLR	TP	CUMULATIVE IBNR ACCIDENT MONTH	CUMULATIVE IBNR ACCIDENT YEAR (see Exhibit 4)
SEPT 84	54858	.66	.600	.9992	49832	336537
AUG 84	52182	.66	.600	.9760	46302	
JULY 84	53520	.66	.600	.9233	44921	
JUNE 84	54000	.66	.600	.8565	42045	
MAY 84	53000	.66	.600	.7863	37884	
APR 84	52000	.66	.600	.7181	33946	
MAR 84	51000	.66	.600	.6543	30337	
FEB 84	50000	.66	.600	.5959	27086	
JAN 84	49000	.66	.600	.5429	24184	
DEC 83	48000	.73	.600	.4951	19534	138390
NOV 83	47000	.73	.600	.4522	17469	
OCT 83	46000	.73	.600	.4136	15639	
SEPT 83	45000	.73	.600	.3790	14018	
AUG 83	44000	.73	.600	.3479	12580	
JULY 83	43000	.73	.600	.3198	11304	
JUNE 83	42000	.73	.600	.2946	10169	
MAY 83	41000	.73	.600	.2718	9158	
APR 83	40000	.73	.600	.2511	8256	
MAR 83	39000	.73	.600	.2324	7451	
FEB 83	38000	.73	.600	.2155	6729	
JAN 83	37000	.73	.600	.2000	6083	
DEC 82	36000	.81	.550	.1860	4545	33546
NOV 82	35000	.81	.550	.1731	4114	
OCT 82	34000	.81	.550	.1614	3726	
SEPT 82	33000	.81	.550	.1506	3375	
AUG 82	32000	.81	.550	.1408	3059	
JULY 82	31000	.81	.550	.1317	2772	
JUNE 82	30000	.81	.550	.1234	2513	
MAY 82	29000	.81	.550	.1157	2278	
APR 82	28000	.81	.550	.1086	2065	
MAR 82	27000	.81	.550	.1020	1871	
FEB 82	26000	.81	.550	.0960	1695	
JAN 82	25000	.81	.550	.0904	1534	
DEC 81	24000	.90	.550	.0852	1249	8956
NOV 81	23000	.90	.550	.0803	1129	
OCT 81	22000	.90	.550	.0758	1020	
SEPT 81	21000	.90	.550	.0716	919	
AUG 81	20000	.90	.550	.0677	828	
JULY 81	19000	.90	.550	.0641	744	
JUNE 81	18000	.90	.550	.0607	668	
MAY 81	17000	.90	.550	.0575	598	
APR 81	16000	.90	.550	.0546	534	
MAR 81	15000	.90	.550	.0518	475	
FEB 81	14000	.90	.550	.0492	421	
JAN 81	13000	.90	.550	.0467	371	
DEC 80	12000	1.00	.500	.0444	267	1466
NOV 80	11000	1.00	.500	.0423	233	
OCT 80	10000	1.00	.500	.0403	201	
SEPT 80	9000	1.00	.500	.0384	173	
AUG 80	8000	1.00	.500	.0366	146	
JULY 80	7000	1.00	.500	.0349	122	
JUNE 80	6000	1.00	.500	.0333	100	
MAY 80	5000	1.00	.500	.0318	79	
APR 80	4000	1.00	.500	.0303	61	
MAR 80	3000	1.00	.500	.0290	43	
FEB 80	2000	1.00	.500	.0277	28	
JAN 80	1000	1.00	.500	.0265	13	

RESERVE STUDY PARAMETERS

Note: 1) Col(6) = (Col(2) ÷ Col(3)) × Col(4) × Col(5).
2) TP is based on a log normal distribution with parameters 2.24,0.86. It is defined as specified in the section 'An Example'.

EXAMPLE REINSURANCE COMPANY
Cumulative IBNR Calculation
Domestic Facultative - CASUALTY
As of Sept 30,1984

ACC MON	EP	MAF	XLR	TP	CUMULATIVE IBNR ACCIDENT MONTH	CUMULATIVE IBNR ACCIDENT YEAR (see Exhibit 4)
SEPT 84	27142	.51	.900	.9900	47421	361600
AUG 84	25818	.51	.900	.9704	44215	
JULY 84	26480	.51	.900	.9512	44450	
JUNE 84	24000	.51	.900	.9324	39490	
MAY 84	24000	.51	.900	.9139	38708	
APR 84	24000	.51	.900	.8958	37941	
MAR 84	24000	.51	.900	.8781	37190	
FEB 84	24000	.51	.900	.8607	36454	
JAN 84	24000	.51	.900	.8437	35732	
DEC 83	24000	.64	.900	.8270	27910	300747
NOV 83	24000	.64	.900	.8106	27357	
OCT 83	24000	.64	.900	.7945	26816	
SEPT 83	24000	.64	.900	.7788	26285	
AUG 83	24000	.64	.900	.7634	25764	
JULY 83	24000	.64	.900	.7483	25254	
JUNE 83	24000	.64	.900	.7334	24754	
MAY 83	24000	.64	.900	.7189	24264	
APR 83	24000	.64	.900	.7047	23783	
MAR 83	24000	.64	.900	.6907	23312	
FEB 83	24000	.64	.900	.6771	22851	
JAN 83	24000	.64	.900	.6637	22398	
DEC 82	24000	.80	.900	.6505	17564	147766
NOV 82	23000	.80	.900	.6376	16499	
OCT 82	22000	.80	.900	.6250	15469	
SEPT 82	21000	.80	.900	.6126	14473	
AUG 82	20000	.80	.900	.6005	13511	
JULY 82	19000	.80	.900	.5886	12581	
JUNE 82	18000	.80	.900	.5769	11683	
MAY 82	17000	.80	.900	.5655	10816	
APR 82	16000	.80	.900	.5543	9978	
MAR 82	15000	.80	.900	.5434	9169	
FEB 82	14000	.80	.900	.5326	8388	
JAN 82	13000	.80	.900	.5220	7635	
DEC 81	12000	1.00	.900	.5117	5526	33438
NOV 81	11000	1.00	.900	.5016	4966	
OCT 81	10000	1.00	.900	.4916	4425	
SEPT 81	9000	1.00	.900	.4819	3903	
AUG 81	8000	1.00	.900	.4724	3401	
JULY 81	7000	1.00	.900	.4630	2917	
JUNE 81	6000	1.00	.900	.4538	2451	
MAY 81	5000	1.00	.900	.4449	2002	
APR 81	4000	1.00	.900	.4360	1570	
MAR 81	3000	1.00	.900	.4274	1154	
FEB 81	2000	1.00	.900	.4190	754	
JAN 81	1000	1.00	.900	.4107	370	
DEC 80	0	1.00	.000	.4025	0	0
NOV 80	0	1.00	.000	.3946	0	
OCT 80	0	1.00	.000	.3867	0	
SEPT 80	0	1.00	.000	.3791	0	
AUG 80	0	1.00	.000	.3716	0	
JULY 80	0	1.00	.000	.3642	0	
JUNE 80	0	1.00	.000	.3570	0	
MAY 80	0	1.00	.000	.3499	0	
APR 80	0	1.00	.000	.3430	0	
MAR 80	0	1.00	.000	.3362	0	
FEB 80	0	1.00	.000	.3296	0	
JAN 80	0	1.00	.000	.3230	0	

RESERVE STUDY PARAMETERS

Note: 1) Col(6) = (Col(2) ÷ Col(3)) × Col(4) × Col(5).
2) TP is based on an exponential distribution with parameters 0.02,0. It is defined as specified in the section 'An Example .

EXAMPLE REINSURANCE COMPANY
Cumulative IBNR Calculation
Domestic Facultative - MARINE
As of Sept 30,1984

ACC MON	EP	MAF	XLR	TP	CUMULATIVE IBNR ACCIDENT MONTH	CUMULATIVE IBNR ACCIDENT YEAR (see Exhibit 4)
SEPT 84	0	1.00	1.200	.9753	0	0
AUG 84	0	1.00	1.200	.9277	0	
JULY 84	0	1.00	1.200	.8825	0	
JUNE 84	0	1.00	1.200	.8395	0	
MAY 84	0	1.00	1.200	.7985	0	
APR 84	0	1.00	1.200	.7596	0	
MAR 84	0	1.00	1.200	.7225	0	
FEB 84	0	1.00	1.200	.6873	0	
JAN 84	0	1.00	1.200	.6538	0	
DEC 83	0	1.00	1.200	.6219	0	0
NOV 83	0	1.00	1.200	.5916	0	
OCT 83	0	1.00	1.20[illegible]	.5627	0	
SEPT 83	0	1.00	[illegible]	.5353	0	
AUG 83	0	1.00	1.200	.5092	0	
JULY 83	[illegible]	1.00	1.200	.4843	0	
JUNE 83	0	1.00	1.200	.4607	0	
MAY 83	0	1.00	1.200	.4382	0	
APR 83	0	1.00	1.200	.4169	0	
MAR 83	0	1.00	1.200	.3965	0	
FEB 83	0	1.00	1.200	.3772	0	
JAN 83	0	1.00	1.200	.3588	0	
DEC 82	1000	1.00	1.200	.3413	410	22384
NOV 82	2000	1.00	1.200	.3247	779	
OCT 82	3000	1.00	1.200	.3088	1112	
SEPT 82	4000	1.00	1.200	.2938	1410	
AUG 82	5000	1.00	1.200	.2794	1677	
JULY 82	6000	1.00	1.200	.2658	1914	
JUNE 82	7000	1.00	1.200	.2528	2124	
MAY 82	8000	1.00	1.200	.2405	2309	
APR 82	9000	1.00	1.200	.2288	2471	
MAR 82	10000	1.00	1.200	.2176	2611	
FEB 82	11000	1.00	1.200	.2070	2732	
JAN 82	12000	1.00	1.200	.1969	2836	
DEC 81	12000	1.00	1.200	.1873	2697	24953
NOV 81	12000	1.00	1.200	.1782	2566	
OCT 81	12000	1.00	1.200	.1695	2441	
SEPT 81	12000	1.00	1.200	.1612	2322	
AUG 81	12000	1.00	1.200	.1534	2208	
JULY 81	12000	1.00	1.200	.1459	2101	
JUNE 81	12000	1.00	1.200	.1388	1998	
MAY 81	12000	1.00	1.200	.1320	1901	
APR 81	12000	1.00	1.200	.1256	1808	
MAR 81	12000	1.00	1.200	.1194	1720	
FEB 81	12000	1.00	1.200	.1136	1636	
JAN 81	12000	1.00	1.200	.1081	1556	
DEC 80	12000	1.00	1.200	.1028	1480	8094
NOV 80	11000	1.00	1.200	.0978	1291	
OCT 80	10000	1.00	1.200	.0930	1116	
SEPT 80	9000	1.00	1.200	.0885	956	
AUG 80	8000	1.00	1.200	.0842	808	
JULY 80	7000	1.00	1.200	.0801	672	
JUNE 80	6000	1.00	1.200	.0762	548	
MAY 80	5000	1.00	1.200	.0724	435	
APR 80	4000	1.00	1.200	.0689	331	
MAR 80	3000	1.00	1.200	.0655	236	
FEB 80	2000	1.00	1.200	.0623	150	
JAN 80	1000	1.00	1.200	.0593	71	

RESERVE STUDY PARAMETERS

Note: 1) Col(6) = (Col(2) ÷ Col(3)) × Col(4) × Col(5).
2) TP is based on an exponential distribution with parameters 0.05,0. It is defined as specified in the section 'An Example'.

EXAMPLE REINSURANCE COMPANY
Monthly Increase to IBNR Calculation
Domestic Facultative - PROPERTY
For Sept , 1984

					CURRENT CALENDAR MONTH EXPECTED		MONTHLY IBNR INCREASE DUE TO	
ACC MON	EP	MAF	XLR	P	ULTIMATE LOSS	REPORTED LOSSES	ACC MON	ACC YR
SEPT 84	54858	.66	.600	.0008	49871	39	49832	28308
AUG 84	52182	.66	.600	.0232	0	1099	-1099	
JULY 84	53520	.66	.600	.0528	0	2568	-2568	
JUNE 84	54000	.66	.600	.0668	0	3279	-3279	
MAY 84	53000	.66	.600	.0702	0	3382	-3382	
APR 84	52000	.66	.600	.0682	0	3223	-3223	
MAR 84	51000	.66	.600	.0638	0	2956	-2956	
FEB 84	50000	.66	.600	.0584	0	2656	-2656	
JAN 84	49000	.66	.600	.0530	0	2360	-2360	
DEC 83	48000	.73	.600	.0478	0	1884	-1884	-12317
NOV 83	47000	.73	.600	.0429	0	1659	-1659	
OCT 83	46000	.73	.600	.0386	0	1458	-1458	
SEPT 83	45000	.73	.600	.0346	0	1281	-1281	
AUG 83	44000	.73	.600	.0311	0	1126	-1126	
JULY 83	43000	.73	.600	.0280	0	990	-990	
JUNE 83	42000	[illegible]	.600	.0253	0	872	-872	
MAY 83	41000	.73	.600	.0228	0	769	-769	
APR 83	40000	.73	.600	.0206	0	678	-678	
MAR 83	39000	.73	.600	.0187	0	599	-599	
FEB 83	38000	.73	.600	.0170	0	530	-530	
JAN 83	37000	.73	.600	.0154	0	470	-470	
DEC 82	36000	.81	.550	.0141	0	344	-344	-2344
NOV 82	35000	.81	.550	.0128	0	305	-305	
OCT 82	34000	.81	.550	.0117	0	271	-271	
SEPT 82	33000	.81	.550	.0108	0	241	-241	
AUG 82	32000	.81	.550	.0099	0	214	-214	
JULY 82	31000	.81	.550	.0091	0	191	-191	
JUNE 82	30000	.81	.550	.0083	0	170	-170	
MAY 82	29000	.81	.550	.0077	0	151	-151	
APR 82	28000	.81	.550	.0071	0	135	-135	
MAR 82	27000	.81	.550	.0065	0	120	-120	
FEB 82	26000	.81	.550	.0061	0	107	-107	
JAN 82	25000	.81	.550	.0056	0	95	-95	
DEC 81	24000	.90	.550	.0052	0	76	-76	-515
NOV 81	23000	.90	.550	.0048	0	68	-68	
OCT 81	22000	.90	.550	.0045	0	60	-60	
SEPT 81	21000	.90	.550	.0042	0	54	-54	
AUG 81	20000	.90	.550	.0039	0	48	-48	
JULY 81	19000	.90	.550	.0036	0	42	-42	
JUNE 81	18000	.90	.550	.0034	0	37	-37	
MAY 81	17000	.90	.550	.0032	0	33	-33	
APR 81	16000	.90	.550	.0030	0	29	-29	
MAR 81	15000	.90	.550	.0028	0	25	-25	
FEB 81	14000	.90	.550	.0026	0	22	-22	
JAN 81	13000	.90	.550	.0024	0	19	-19	
DEC 80	12000	1.00	.500	.0023	0	14	-14	-73
NOV 80	11000	1.00	.500	.0022	0	12	-12	
OCT 80	10000	1.00	.500	.0020	0	10	-10	
SEPT 80	9000	1.00	.500	.0019	0	9	-9	
AUG 80	8000	1.00	.500	.0018	0	7	-7	
JULY 80	7000	1.00	.500	.0017	0	6	-6	
JUNE 80	6000	1.00	.500	.0016	0	5	-5	
MAY 80	5000	1.00	.500	.0015	0	4	-4	
APR 80	4000	1.00	.500	.0014	0	3	-3	
MAR 80	3000	1.00	.500	.0013	0	2	-2	
FEB 80	2000	1.00	.500	.0013	0	1	-1	
JAN 80	1000	1.00	.500	.0012	0	1	-1	

RESERVE STUDY PARAMETERS

Note: 1) Col(6) = (Col(2) ÷ Col(3)) × Col(4) (for current month only).
2) Col(7) = Col(6) × Col(5).

EXAMPLE REINSURANCE COMPANY
Monthly Increase to IBNR Calculation
Domestic Facultative - CASUALTY
For Sept , 1984

RESERVE STUDY PARAMETERS

ACC MON	EP	MAF	XLR	P	CURRENT CALENDAR MONTH EXPECTED: ULTIMATE LOSS	CURRENT CALENDAR MONTH EXPECTED: REPORTED LOSSES	MONTHLY IBNR INCREASE DUE TO: ACC MON	MONTHLY IBNR INCREASE DUE TO: ACC YR
SEPT 84	27142	.51	.900	.0100	47898	477	47421	41074
AUG 84	25818	.51	.900	.0196	0	893	-893	
JULY 84	26480	.51	.900	.0192	0	898	-898	
JUNE 84	24000	.51	.900	.0188	0	798	-798	
MAY 84	24000	.51	.900	.0185	0	782	-782	
APR 84	24000	.51	.900	.0181	0	766	-766	
MAR 84	24000	.51	.900	.0177	0	751	-751	
FEB 84	24000	.51	.900	.0174	0	736	-736	
JAN 84	24000	.51	.900	.0170	0	722	-722	
DEC 83	24000	.64	.900	.0167	0	564	-564	-6075
NOV 83	24000	.64	.900	.0164	0	553	-553	
OCT 83	24000	.64	.900	.0161	0	542	-542	
SEPT 83	24000	.64	.900	.0157	0	531	-531	
AUG 83	24000	.64	.900	.0154	0	520	-520	
JULY 83	24000	.64	.900	.0151	0	510	-510	
JUNE 83	24000	.64	.900	.0148	0	500	-500	
MAY 83	24000	.64	.900	.0145	0	490	-490	
APR 83	24000	.64	.900	.0142	0	480	-480	
MAR 83	24000	.64	.900	.0140	0	471	-471	
FEB 83	24000	.64	.900	.0137	0	462	-462	
JAN 83	24000	.64	.900	.0134	0	452	-452	
DEC 82	24000	.80	.900	.0131	0	355	-355	-2985
NOV 82	23000	.80	.900	.0129	0	333	-333	
OCT 82	22000	.80	.900	.0126	0	312	-312	
SEPT 82	21000	.80	.900	.0124	0	292	-292	
AUG 82	20000	.80	.900	.0121	0	273	-273	
JULY 82	19000	.80	.900	.0119	0	254	-254	
JUNE 82	18000	.80	.900	.0117	0	236	-236	
MAY 82	17000	.80	.900	.0114	0	218	-218	
APR 82	16000	.80	.900	.0112	0	202	-202	
MAR 82	15000	.80	.900	.0110	0	185	-185	
FEB 82	14000	.80	.900	.0108	0	169	-169	
JAN 82	13000	.80	.900	.0105	0	154	-154	
DEC 81	12000	1.00	.900	.0103	0	112	-112	-676
NOV 81	11000	1.00	.900	.0101	0	100	-100	
OCT 81	10000	1.00	.900	.0099	0	89	-89	
SEPT 81	9000	1.00	.900	.0097	0	79	-79	
AUG 81	8000	1.00	.900	.0095	0	69	-69	
JULY 81	7000	1.00	.900	.0094	0	59	-59	
JUNE 81	6000	1.00	.900	.0092	0	50	-50	
MAY 81	5000	1.00	.900	.0090	0	40	-40	
APR 81	4000	1.00	.900	.0088	0	32	-32	
MAR 81	3000	1.00	.900	.0086	0	23	-23	
FEB 81	2000	1.00	.900	.0085	0	15	-15	
JAN 81	1000	1.00	.900	.0083	0	7	-7	
DEC 80	0	1.00	.000	.0081	0	0	0	0
NOV 80	0	1.00	.000	.0080	0	0	0	
OCT 80	0	1.00	.000	.0078	0	0	0	
SEPT 80	0	1.00	.000	.0077	0	0	0	
AUG 80	0	1.00	.000	.0075	0	0	0	
JULY 80	0	1.00	.000	.0074	0	0	0	
JUNE 80	0	1.00	.000	.0072	0	0	0	
MAY 80	0	1.00	.000	.0071	0	0	0	
APR 80	0	1.00	.000	.0069	0	0	0	
MAR 80	0	1.00	.000	.0068	0	0	0	
FEB 80	0	1.00	.000	.0067	0	0	0	
JAN 80	0	1.00	.000	.0065	0	0	0	

Note: 1) Col(6) = (Col(2) ÷ Col(3)) × Col(4) (for current month only).
2) Col(7) = Col(6) × Col(5).

EXAMPLE REINSURANCE COMPANY
Monthly Increase to IBNR Calculation
Domestic Facultative - MARINE
For Sept , 1984

RESERVE STUDY PARAMETERS

ACC MON	EP	MAF	XLR	P	CURRENT CALENDAR MONTH EXPECTED: ULTIMATE LOSS	CURRENT CALENDAR MONTH EXPECTED: REPORTED LOSSES	MONTHLY IBNR INCREASE DUE TO: ACC MON	MONTHLY IBNR INCREASE DUE TO: ACC YR
SEPT 84	0	1.00	1.200	.0247	0	0	0	0
AUG 84	0	1.00	1.200	.0476	0	0	0	
JULY 84	0	1.00	1.200	.0452	0	0	0	
JUNE 84	0	1.00	1.200	.0430	0	0	0	
MAY 84	0	1.00	1.200	.0409	0	0	0	
APR 84	0	1.00	1.200	.0389	0	0	0	
MAR 84	0	1.00	1.200	.0370	0	0	0	
FEB 84	0	1.00	1.200	.0352	0	0	0	
JAN 84	0	1.00	1.200	.0335	0	0	0	
DEC 83	0	1.00	1.200	.0319	0	0	0	0
NOV 83	0	1.00	1.200	.0303	0	0	0	
OCT 83	0	1.00	1.200	.0289	0	0	0	
SEPT 83	0	1.00	1.200	.0274	0	0	0	
AUG 83	0	1.00	1.200	.0261	0	0	0	
JULY 83	0	1.00	1.200	.0248	0	0	0	
JUNE 83	[illegible]	1.00	1.200	.0236	0	0	0	
MAY 83	0	1.00	1.200	.0225	0	0	0	
APR 83	0	1.00	1.200	.0214	0	0	0	
MAR 83	0	1.00	1.200	.0203	0	0	0	
FEB 83	0	1.00	1.200	.0193	0	0	0	
JAN 83	0	1.00	1.200	.0184	0	0	0	
DEC 82	1000	1.00	1.200	.0175	0	21	-21	-1148
NOV 82	2000	1.00	1.200	.0166	0	40	-40	
OCT 82	3000	1.00	1.200	.0158	0	57	-57	
SEPT 82	4000	1.00	1.200	.0151	0	72	-72	
AUG 82	5000	1.00	1.200	.0143	0	86	-86	
JULY 82	6000	1.00	1.200	.0136	0	98	-98	
JUNE 82	7000	1.00	1.200	.0130	0	109	-109	
MAY 82	8000	1.00	1.200	.0123	0	118	-118	
APR 82	9000	1.00	1.200	.0117	0	127	-127	
MAR 82	10000	1.00	1.200	.0112	0	134	-134	
FEB 82	11000	1.00	1.200	.0106	0	140	-140	
JAN 82	12000	1.00	1.200	.0101	0	145	-145	
DEC 81	12000	1.00	1.200	.0096	0	138	-138	-1279
NOV 81	12000	1.00	1.200	.0091	0	132	-132	
OCT 81	12000	1.00	1.200	.0087	0	125	-125	
SEPT 81	12000	1.00	1.200	.0083	0	119	-119	
AUG 81	12000	1.00	1.200	.0079	0	113	-113	
JULY 81	12000	1.00	1.200	.0075	0	108	-108	
JUNE 81	12000	1.00	1.200	.0071	0	102	-102	
MAY 81	12000	1.00	1.200	.0068	0	97	-97	
APR 81	12000	1.00	1.200	.0064	0	93	-93	
MAR 81	12000	1.00	1.200	.0061	0	88	-88	
FEB 81	12000	1.00	1.200	.0058	0	84	-84	
JAN 81	12000	1.00	1.200	.0055	0	80	-80	
DEC 80	12000	1.00	1.200	.0053	0	76	-76	-415
NOV 80	11000	1.00	1.200	.0050	0	66	-66	
OCT 80	10000	1.00	1.200	.0048	0	57	-57	
SEPT 80	9000	1.00	1.200	.0045	0	49	-49	
AUG 80	8000	1.00	1.200	.0043	0	41	-41	
JULY 80	7000	1.00	1.200	.0041	0	34	-34	
JUNE 80	6000	1.00	1.200	.0039	0	28	-28	
MAY 80	5000	1.00	1.200	.0037	0	22	-22	
APR 80	4000	1.00	1.200	.0035	0	17	-17	
MAR 80	3000	1.00	1.200	.0034	0	12	-12	
FEB 80	2000	1.00	1.200	.0032	0	8	-8	
JAN 80	1000	1.00	1.200	.0030	0	4	-4	

Note: 1) Col(6) = (Col(2) ÷ Col(3)) × Col(4) (for current month only).
2) Col(7) = Col(6) × Col(5).

EXAMPLE REINSURANCE COMPANY
Monthly Increase to IBNR for September 1984
Based on Reserving Parameters as of Sept 30, 1984 (see Exhibit 3)
by Group, by Accident Year

GROUP: PROPERTY

ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.
84	49,871	21,563	997.4	431.3
83	0	12,317	0.0	246.3
82	0	2,344	0.0	46.9
81	0	515	0.0	10.3
80	0	73	0.0	1.5
TOTAL	49,871	36,812	997.4	736.2

EXPECTED NET $ & # IBNR INCREASE
CAL MONTH = 13,059 261

GROUP: CASUALTY

ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.
84	47,898	6,823	47.9	6.8
83	0	6,075	0.0	6.1
82	0	2,985	0.0	3.0
81	0	676	0.0	0.7
80	0	0	0.0	0.0
TOTAL	47,898	16,559	47.9	16.6

EXPECTED NET $ & # IBNR INCREASE
CAL MONTH = 31,338 31

GROUP: MARINE

ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.
84	0	0	0.0	0.0
83	0	0	0.0	0.0
82	0	1,148	0.0	11.5
81	0	1,279	0.0	12.8
80	0	415	0.0	4.1
TOTAL	0	2,842	0.0	28.4

EXPECTED NET $ & # IBNR INCREASE
CAL MONTH = -2,842 -28

GROUP: ALL

ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.
84	97,769	28,387	1,045.3	438.1
83	0	18,393	0.0	252.4
82	0	6,477	0.0	61.3
81	0	2,470	0.0	23.8
80	0	488	0.0	5.6
TOTAL	97,769	56,214	1,045.3	781.2

EXPECTED NET $ & # IBNR INCREASE
CAL MONTH = 41,555 264

EXHIBIT 11

EXHIBIT 12A

EXAMPLE REINSURANCE COMPANY
IBNR Review as of Sept 30, 1984
Based on Reserving Parameters as of Sept 30, 1984 (see Exhibit 3)
by Group, by Accident Year

GROUP: PROPERTY

EXPECTED IBNR INCREMENT

FOR CALENDAR MONTH: 9/84					YEAR TO DATE: 1/84 TO 9/84					CUMULATIVE IBNR AS OF:9/84		
ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	ACC YR	CUM $ IBNR	CUM # IBNR
84	49,871	21,563	997.4	431.3	84	426,873	90,336	8,537.5	1,806.7	84	336,537	6,730.7
83	0	12,317	0.0	246.3	83	0	162,227	0.0	3,244.5	83	138,390	2,767.8
82	0	2,344	0.0	46.9	82	0	31,146	0.0	622.9	82	33,546	670.9
81	0	515	0.0	10.3	81	0	6,334	0.0	126.7	81	8,956	179.1
80	0	73	0.0	1.5	80	0	853	0.0	17.1	80	1,466	29.3
TOTAL	49,871	36,812	997.4	736.2	TOTAL	426,873	290,896	8,537.5	5,817.9	TOTAL	518,894	10,377.9

EXPECTED NET $ & # IBNR INCREASE (i.e. increase - decrease)
CAL MONTH = 13,059 261 YEAR TO DATE = 135,977 2,720

GROUP: CASUALTY

EXPECTED IBNR INCREMENT

FOR CALENDAR MONTH: 9/84					YEAR TO DATE: 1/84 TO 9/84					CUMULATIVE IBNR AS OF:9/84		
ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	ACC YR	CUM $ IBNR	CUM # IBNR
84	47,898	6,823	47.9	6.8	84	394,306	32,706	394.3	32.7	84	361,600	361.6
83	0	6,075	0.0	6.1	83	0	59,313	0.0	59.3	83	300,747	300.7
82	0	2,985	0.0	3.0	82	0	29,142	0.0	29.1	82	147,766	147.8
81	0	676	0.0	0.7	81	0	6,595	0.0	6.6	81	33,438	33.4
80	0	0	0.0	0.0	80	0	0	0.0	0.0	80	0	0.0
TOTAL	47,898	16,559	47.9	16.6	TOTAL	394,306	127,755	394.3	127.8	TOTAL	843,551	843.6

EXPECTED NET $ & # IBNR INCREASE (i.e. increase - decrease)
CAL MONTH = 31,338 31 YEAR TO DATE = 266,551 267

EXHIBIT 12A

EXHIBIT 12B

EXAMPLE REINSURANCE COMPANY
IBNR Review as of Sept 30, 1984
Based on Reserving Parameters as of Sept 30, 1984 (see Exhibit 3)
by Group, by Accident Year

GROUP: MARINE

EXPECTED IBNR INCREMENT										*	CUMULATIVE IBNR		
FOR CALENDAR MONTH: 9/84					*	YEAR TO DATE: 1/84 TO 9/84				*	AS OF:9/84		
ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	* ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	*	ACC YR	CUM $ IBNR	CUM # IBNR
84	0	0	0.0	0.0	* 84	0	0	0.0	0.0	*	84	0	0.0
83	0	0	0.0	0.0	* 83	0	0	0.0	0.0	*	83	0	0.0
82	0	1,148	0.0	11.5	* 82	0	12,721	0.0	127.2	*	82	22,384	223.8
81	0	1,279	0.0	12.8	* 81	0	14,181	0.0	141.8	*	81	24,953	249.5
80	0	415	0.0	4.1	* 80	0	4,600	0.0	46.0	*	80	8,094	80.9
TOTAL	0	2,842	0.0	28.4	* TOTAL	0	31,502	0.0	315.0	*	TOTAL	55,430	554.3

EXPECTED NET $ & # IBNR INCREASE (i.e. increase - decrease)
CAL MONTH = -2,842 -28 YEAR TO DATE = -31,502 -315

GROUP: ALL

EXPECTED IBNR INCREMENT										*	CUMULATIVE IBNR		
FOR CALENDAR MONTH: 9/84					*	YEAR TO DATE: 1/84 TO 9/84				*	AS OF:9/84		
ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	* ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	*	ACC YR	CUM $ IBNR	CUM # IBNR
84	97,769	28,387	1,045.3	438.1	* 84	821,179	123,042	8,931.8	1,839.4	*	84	698,137	7,092.3
83	0	18,393	0.0	252.4	* 83	0	221,540	0.0	3,303.9	*	83	439,137	3,068.5
82	0	6,477	0.0	61.3	* 82	0	73,009	0.0	779.3	*	82	203,696	1,042.5
81	0	2,470	0.0	23.8	* 81	0	27,110	0.0	275.1	*	81	67,347	462.1
80	0	488	0.0	5.6	* 80	0	5,452	0.0	63.1	*	80	9,559	110.2
TOTAL	97,769	56,214	1,045.3	781.2	* TOTAL	821,179	450,153	8,931.8	6,260.7	*	TOTAL	1,417,876	11,775.7

EXPECTED NET $ & # IBNR INCREASE (i.e. increase - decrease)
CAL MONTH = 41,555 264 YEAR TO DATE = 371,026 2,671

EXHIBIT 12B

EXHIBIT 13

EXAMPLE REINSURANCE COMPANY
Calendar Month IBNR Summary
for August and September, 1984
Based on Reserving Parameters as of Sept 30, 1984

UNDERWRITING AREA	COST CENTER	@MONTHLY NET EP	@MONTHLY INCREASE TO IBNR	>@YEAR-TO-DATE INCREASE TO IBNR	*CUMULATIVE IBNR	CURR
FOR: August 1984						
FACULTATIVE	ALL	78,000	38,253	329,471	1,376,321	US$
FOR: September 1984						
FACULTATIVE	ALL	82,000	41,555	371,026	1,417,876	US$

* EXACT, OFFICIAL FIGURE @ NOT EXACT DUE TO ROUNDING
> NOTE: DOES NOT EQUAL ACTUAL YTD TOTAL, UNLESS THE PARAMETERS CURRENTLY IN USE WERE EMPLOYED THROUGHOUT THIS YEAR
THIS FORMULA EXCLUDES IBNR FOR: CASE RESERVE DEVELOPMENT

EXHIBIT 13

EXAMPLE REINSURANCE COMPANY
Reserving Parameters as of Dec 31, 1984 *
Based on Reserve Study as of June 30, 1984
by Group

GROUP: PROPERTY

EP = (3ρ55750),54858,52182,53520,54000,53000,52000,51000,50000,49000,48000,47000,46000,45000,44000,43000,42000,41000,40000,39000,38000,37000,36000,35000,34000,33000,32000,31000,30000,29000,28000,27000,26000,25000,24000,23000,22000,21000,20000,19000,18000,17000,16000,15000,14000,13000,12000,11000,10000,9000,8000,7000,6000,5000,4000,3000,2000,1000
MAF = (12ρ0.66),(12ρ0.73),(12ρ0.81),(12ρ0.9),(12ρ1)
XLR = (24ρ0.6),(24ρ0.55),(12ρ0.5)
SEV = (60ρ50)
LAG = 2 2.24 0.86

GROUP: CASUALTY

EP = (3ρ27583),27142,25818,26480,(19ρ24000),23000,22000,21000,20000,19000,18000,17000,16000,15000,14000,13000,12000,11000,10000,9000,8000,7000,6000,5000,4000,3000,2000,1000,(12ρ0)
MAF = (12ρ0.51),(12ρ0.64),(12ρ0.8),(24ρ1)
XLR = (48ρ0.9),(12ρ0)
SEV = (48ρ1000),(12ρ1)
LAG = 1 0.02 0

GROUP: MARINE

EP = (24ρ0),1000,2000,3000,4000,5000,6000,7000,8000,9000,10000,11000,(14ρ12000),11000,10000,9000,8000,7000,6000,5000,4000,3000,2000,1000
MAF = (60ρ1)
XLR = (60ρ1.2)
SEV = (24ρ1),(36ρ100)
LAG = 1 0.05 0

1) 'ρ' can be defined as follows: '5ρb' means b,b,b,b,b.
2) In each vector of monthly parameters, the first component is 12/84 and the final component is 1/80.

*Extension rule: Parameters for months after 6/84 are set at the 6/84 value (i.e. MAF, XLR, SEV).
Earned premiums for July, Aug, Sept, 1984 are actual figures from the Comptroller, and for Oct 1984 through Dec 1984 are based on Underwriter projections.

EXHIBIT 14

EXHIBIT 15A

EXAMPLE REINSURANCE COMPANY
Projected IBNR Review as of Dec 30, 1984
Run at Sept 30, 1984
Based on Reserving Parameters as of Dec 30, 1984 (see Exhibit 14)
by Group, by Accident Year

GROUP: PROPERTY

EXPECTED IBNR INCREMENT												CUMULATIVE IBNR		
FOR CALENDAR MONTH: 12/84						YEAR TO DATE: 1/84 TO 12/84						AS OF: 12/84		
ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	*	ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	*	ACC YR	CUM $ IBNR	CUM # IBNR
84	50,682	28,202	1,013.6	564.0	*	84	578,918	168,630	11,578.4	3,372.6	*	84	410,288	8,205.8
83	0	9,051	0.0	181.0	*	83	0	192,400	0.0	3,848.0	*	83	108,217	2,164.3
82	0	1,826	0.0	36.5	*	82	0	37,107	0.0	742.1	*	82	27,585	551.7
81	0	419	0.0	8.4	*	81	0	7,682	0.0	153.6	*	81	7,608	152.2
80	0	61	0.0	1.2	*	80	0	1,047	0.0	20.9	*	80	1,272	25.4
TOTAL	50,682	39,560	1,013.6	791.2	*	TOTAL	578,918	406,866	11,578.4	8,137.3	*	TOTAL	554,970	11,099.4

EXPECTED NET $ & # IBNR INCREASE (i.e. increase - decrease)
CAL MONTH = 11,122 222 YEAR TO DATE = 172,052 3,441

GROUP: CASUALTY

EXPECTED IBNR INCREMENT												CUMULATIVE IBNR		
FOR CALENDAR MONTH: 12/84						YEAR TO DATE: 1/84 TO 12/84						AS OF: 12/84		
ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	*	ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	*	ACC YR	CUM $ IBNR	CUM # IBNR
84	48,676	9,253	48.7	9.3	*	84	540,334	58,061	540.3	58.1	*	84	482,273	482.3
83	0	5,722	0.0	5.7	*	83	0	76,827	0.0	76.8	*	83	283,233	283.2
82	0	2,811	0.0	2.8	*	82	0	37,747	0.0	37.7	*	82	139,161	139.2
81	0	636	0.0	0.6	*	81	0	8,542	0.0	8.5	*	81	31,491	31.5
80	0	0	0.0	0.0	*	80	0	0	0.0	0.0	*	80	0	0.0
TOTAL	48,676	18,422	48.7	18.4	*	TOTAL	540,334	181,177	540.3	181.2	*	TOTAL	936,157	936.2

EXPECTED NET $ & # IBNR INCREASE (i.e. increase - decrease)
CAL MONTH = 30,253 30 YEAR TO DATE = 359,157 359

EXHIBIT 15A

EXHIBIT 15B

EXAMPLE REINSURANCE COMPANY
Projected IBNR Review as of Dec 30, 1984
Run at Sept 30, 1984
Based on Reserving Parameters as of Dec 30, 1984 (see Exhibit 14)
by Group, by Accident Year

GROUP: MARINE

EXPECTED IBNR INCREMENT														
FOR CALENDAR MONTH: 12/84						YEAR TO DATE: 1/84 TO 12/84						CUMULATIVE IBNR AS OF:12/84		
ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	*	ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	*	ACC YR	CUM $ IBNR	CUM # IBNR
84	0	0	0.0	0.0	*	84	0	0	0.0	0.0	*	84	0	0.0
83	0	0	0.0	0.0	*	83	0	0	0.0	0.0	*	83	0	0.0
82	0	988	0.0	9.9	*	82	0	15,839	0.0	158.4	*	82	19,266	192.7
81	0	1,101	0.0	11.0	*	81	0	17,657	0.0	176.6	*	81	21,477	214.8
80	0	357	0.0	3.6	*	80	0	5,727	0.0	57.3	*	80	6,966	69.7
TOTAL	0	2,446	0.0	24.5	*	TOTAL	0	39,223	0.0	392.2	*	TOTAL	47,709	477.1

EXPECTED NET $ & # IBNR INCREASE (i.e. increase - decrease)
CAL MONTH = -2,446 -24 YEAR TO DATE = -39,223 -392

GROUP: ALL

EXPECTED IBNR INCREMENT														
FOR CALENDAR MONTH: 9/84						YEAR TO DATE: 1/84 TO 9/84						CUMULATIVE IBNR AS OF:9/84		
ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	*	ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	*	ACC YR	CUM $ IBNR	CUM # IBNR
84	99,358	37,456	1,062.3	573.3	*	84	1,119,252	226,691	12,118.7	3,430.7	*	84	892,561	8,688.0
83	0	14,773	0.0	186.8	*	83	0	269,227	0.0	3,924.8	*	83	391,450	2,447.6
82	0	5,625	0.0	49.2	*	82	0	90,693	0.0	938.3	*	82	186,012	883.5
81	0	2,156	0.0	20.0	*	81	0	33,881	0.0	338.8	*	81	60,576	398.4
80	0	418	0.0	4.8	*	80	0	6,774	0.0	78.2	*	80	8,238	95.1
TOTAL	99,358	60,428	1,062.3	834.1	*	TOTAL	1,119,252	627,266	12,118.7	8,710.7	*	TOTAL	1,538,836	12,512.6

EXPECTED NET $ & # IBNR INCREASE (i.e. increase - decrease)
CAL MONTH = 38,929 228 YEAR TO DATE = 491,986 3,408

EXHIBIT 15B

EXHIBIT 15C

EXAMPLE REINSURANCE COMPANY
Projected IBNR Review as of Dec 31, 1984
Run at Sept 30, 1984
Based on Reserving Parameters as of Dec 31, 1984 (see Exhibit 14)
by Group, by Accident Year

CALENDAR MONTH IBNR SUMMARY FOR 12/84

UNDERWRITING AREA	COST CENTER	@MONTHLY NET EP	@MONTHLY INCREASE TO IBNR	>@YEAR-TO-DATE INCREASE TO IBNR	*CUMULATIVE IBNR	CURR
FACULTATIVE	ALL	83,333	38,929	491,986	1,538,836	US$

* EXACT, OFFICIAL FIGURE @ NOT EXACT DUE TO ROUNDING
> NOTE: DOES NOT EQUAL ACTUAL YTD TOTAL, UNLESS THE PARAMETERS CURRENTLY IN USE WERE EMPLOYED THROUGHOUT THIS YEAR
THIS FORMULA EXCLUDES IBNR FOR: CASE RESERVE DEVELOPMENT

EXHIBIT 15C

EXHIBIT 16

EXAMPLE REINSURANCE COMPANY
Reserving Parameters as of Dec 31, 1985 *
Based on Reserve Study as of June 30, 1984
by Group

GROUP: PROPERTY

```
EP  = (12ρ50175),(3ρ55750),54858,52182,53520,54000,53000,52000,51000,50000,49000,48000
      ,47000,46000,45000,44000,43000,42000,41000,40000,39000,38000,37000,36000,35000,3
      4000,33000,32000,31000,30000,29000,28000,27000,26000,25000,24000,23000,22000,210
      00,20000,19000,18000,17000,16000,15000,14000,13000,12000,11000,10000,9000,8000,7
      000,6000,5000,4000,3000,2000,1000
MAF = (12ρ0.792),(12ρ0.66),(12ρ0.73),(12ρ0.81),(12ρ0.9),(12ρ1)
XLR = (36ρ0.6),(24ρ0.55),(12ρ0.5)
SEV = (72ρ50)
LAG = 2 2.24 0.86
```

GROUP: CASUALTY

```
EP  = (12ρ24825),(3ρ27583),27142,25818,26480,(19ρ24000),23000,22000,21000,20000,19000,
      18000,17000,16000,15000,14000,13000,12000,11000,10000,9000,8000,7000,6000,5000,4
      000,3000,2000,1000,(12ρ0)
MAF = (12ρ0.612),(12ρ0.51),(12ρ0.64),(12ρ0.8),(24ρ1)
XLR = (60ρ0.9),(12ρ0)
SEV = (60ρ1000),(12ρ1)
LAG = 1 0.02 0
```

GROUP: MARINE

```
EP  = (36ρ0),1000,2000,3000,4000,5000,6000,7000,8000,9000,10000,11000,(14ρ12000),11000
      ,10000,9000,8000,7000,6000,5000,4000,3000,2000,1000
MAF = (12ρ1.2),(60ρ1)
XLR = (72ρ1.2)
SEV = (3[illegible]),(36ρ100)
LAG = 1 0 [illegible]5 0
```

1) 'ρ' [illegible]n be defined as follows: '5ρb' means b,b,b,b,b.
2) In each vector of monthly parameters, the first component is 12/85 and the final component is 1/80.

* Extension rule: Parameters for months after 6/84 are set at the 6/84 value (i.e. MAF, XLR, SEV). To reflect market rate increases, the MAF is multiplied by 1.20 beginning with 1/85.
Earned premiums for July, Aug, Sept, 1984 are actual figures from the Comptroller, and for Oct 1984 through Dec 1985 are based on Underwriter projections.

EXHIBIT 16

EXAMPLE REINSURANCE COMPANY
Projected IBNR Review as of Dec 31, 1985
Run at Sept 30, 1984
Based on Reserving Parameters as of Dec 31, 1985 (see Exhibit 16)
by Group, by Accident Year

GROUP: PROPERTY

EXPECTED IBNR INCREMENT

FOR CALENDAR MONTH: 12/85					YEAR TO DATE: 1/85 TO 12/85					CUMULATIVE IBNR AS OF: 12/85		
ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	ACC YR	CUM $ IBNR	CUM # IBNR
85	38,011	22,288	760.2	445.8	85	456,136	136,329	9,122.7	2,726.6	85	319,808	6,396.2
84	0	12,324	0.0	246.5	84	0	262,512	0.0	5,250.2	84	147,775	2,955.5
83	0	3,054	0.0	61.1	83	0	61,999	0.0	1,240.0	83	46,218	924.4
82	0	756	0.0	15.1	82	0	13,828	0.0	276.6	82	13,758	275.2
81	0	199	0.0	4.0	81	0	3,405	0.0	68.1	81	4,203	84.1
80	0	32	0.0	0.6	80	0	519	0.0	10.4	80	753	15.1
TOTAL	38,011	38,653	760.2	773.1	TOTAL	456,136	478,592	9,122.7	9,571.8	TOTAL	532,514	10,650.3

EXPECTED NET $ & # IBNR INCREASE (i.e. increase - decrease)
CAL MONTH = -641 -13 YEAR TO DATE = -22,455 -449

GROUP: CASUALTY

EXPECTED IBNR INCREMENT

FOR CALENDAR MONTH: 12/85					YEAR TO DATE: 1/85 TO 12/85					CUMULATIVE IBNR AS OF: 12/85		
ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.	ACC YR	CUM $ IBNR	CUM # IBNR
85	36,507	7,501	36.5	7.5	85	438,088	48,612	438.1	48.6	85	389,476	389.5
84	0	7,664	0.0	7.7	84	0	102,904	0.0	102.9	84	379,369	379.4
83	0	4,501	0.0	4.5	83	0	60,434	0.0	60.4	83	222,799	222.8
82	0	2,211	0.0	2.2	82	0	29,693	0.0	29.7	82	109,468	109.5
81	0	500	0.0	0.5	81	0	6,719	0.0	6.7	81	24,772	24.8
80	0	0	0.0	0.0	80	0	0	0.0	0.0	80	0	0.0
TOTAL	36,507	22,377	36.5	22.4	TOTAL	438,088	248,362	438.1	248.4	TOTAL	1,125,884	1,125.9

EXPECTED NET $ & # IBNR INCREASE (i.e. increase - decrease)
CAL MONTH = 14,130 14 YEAR TO DATE = 189,726 190

EXHIBIT 17A

EXAMPLE REINSURANCE COMPANY
Projected IBNR Review as of Dec 31, 1985
Run at Sept 30, 1984
Based on Reserving Parameters as of Dec 31, 1985 (see Exhibit 16)
by Group, by Accident Year

GROUP: MARINE

EXPECTED IBNR INCREMENT

FOR CALENDAR MONTH: 12/85

ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.
85	0	0	0.0	0.0
84	0	0	0.0	0.0
83	0	0	0.0	0.0
82	0	542	0.0	5.4
81	0	604	0.0	6.0
80	0	196	0.0	2.0
TOTAL	0	1,342	0.0	13.4

YEAR TO DATE: 1/85 TO 12/85

ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.
85	0	0	0.0	0.0
84	0	0	0.0	0.0
83	0	0	0.0	0.0
82	0	8,693	0.0	86.9
81	0	9,690	0.0	96.9
80	0	3,143	0.0	31.4
TOTAL	0	21,526	0.0	215.3

CUMULATIVE IBNR

AS OF:12/85

ACC YR	CUM $ IBNR	CUM # IBNR
85	0	0.0
84	0	0.0
83	0	0.0
82	10,573	105.7
81	11,787	117.9
80	3,823	38.2
TOTAL	26,183	261.8

EXPECTED NET $ & # IBNR INCREASE (i.e. increase - decrease)
CAL MONTH = -1,342 -13 YEAR TO DATE = -21,526 -215

GROUP: ALL

EXPECTED IBNR INCREMENT

FOR CALENDAR MONTH: 9/84

ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.
84	99,358	37,456	1,062.3	573.3
83	0	14,773	0.0	186.8
82	0	5,625	0.0	49.2
81	0	2,156	0.0	20.0
80	0	418	0.0	4.8
TOTAL	99,358	60,428	1,062.3	834.1

YEAR TO DATE: 1/84 TO 9/84

ACC YR	$ IBNR INCREASE	$ IBNR DECREASE	# IBNR INCR.	# IBNR DECR.
84	1,119,252	226,691	12,118.7	3,430.7
83	0	269,227	0.0	3,924.8
82	0	90,693	0.0	938.3
81	0	33,881	0.0	338.8
80	0	6,774	0.0	78.2
TOTAL	1,119,252	627,266	12,118.7	8,710.7

CUMULATIVE IBNR

AS OF:9/84

ACC YR	CUM $ IBNR	CUM # IBNR
84	892,561	8,688.0
83	391,450	2,447.6
82	186,012	883.5
81	60,576	398.4
80	8,238	95.1
TOTAL	1,538,836	12,512.6

EXPECTED NET $ & # IBNR INCREASE (i.e. increase - decrease)
CAL MONTH = 38,929 228 YEAR TO DATE = 491,986 3,408

EXHIBIT 17B

EXHIBIT 17C

EXAMPLE REINSURANCE COMPANY
Projected IBNR Review as of Dec 31, 1985
Run at Sept 30, 1984
Based on Reserving Parameters as of Dec 31, 1985 (see Exhibit 16)
by Group, by Accident Year

CALENDAR MONTH IBNR SUMMARY FOR 12/85

UNDERWRITING AREA	COST CENTER	@MONTHLY NET EP	@MONTHLY INCREASE TO IBNR	>@YEAR-TO-DATE INCREASE TO IBNR	*CUMULATIVE IBNR	CURR
FACULTATIVE	ALL	75,000	12,146	145,745	1,684,582	US$

* EXACT, OFFICIAL FIGURE @ NOT EXACT DUE TO ROUNDING
> NOTE: DOES NOT EQUAL ACTUAL YTD TOTAL, UNLESS THE PARAMETERS CURRENTLY IN USE WERE EMPLOYED THROUGHOUT THIS YEAR
THIS FORMULA EXCLUDES IBNR FOR: CASE RESERVE DEVELOPMENT

EXHIBIT 17C