Title: EVALUATION OF THE FINANCIAL CONDITION OF INSURANCE COMPANIES -- A THEORETICAL APPROACH

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Abstract:

This paper is concerned with the methodology for evaluating the financial condition of an insurer. To date, the methodology has focused on empirical approaches. This paper presents a theoretical framework for this evaluation. An insurer's financial condition at a given point in time is defined as a point (within a corresponding confidence interval) on a solvency continuum with insolvency and solidity as end points. The paper proposes that a particular insurer's position on the solvency continuum, at a point in time, is a mathematical function of underlying financial measures (such as investment portfolio composition and reserve accuracy) observed at that same point in time. The proposed concept is illustrated for selected variables. The proposed system provides an absolute evaluation of financial condition as well as a basis for evaluation of the contribution of individual variables to overall financial condition.

I. INTRODUCTION AND DEFINITIONS

This paper presents a theoretical framework for the evaluation of the financial condition of an insurer. For use in this presentation, three terms associated with the financial condition of an insurer -- insolvency, solvency, and solidity are defined as follows:

A. Insolvency

Insolvency means that an insurer cannot, at a particular point in time, meet its current and long-term obligations to its customers, owners, and/or creditors.[1] That is, the company, having sold its product, is unable to pay for the cost of production of the product (which, for insurance, lags sale and delivery of the product). The cost of production of the insurance product includes the payment of all claims, expenses of operation, debts, and return on investment to owners.

More specifically, insolvency may be defined to occur at that point in time at which surplus becomes either:

- less than the minimum required surplus (as established by law or theoretically as mentioned in Section VII.A),
- equal to 0, or
- less than 0, i.e., assets are less than or equal to liabilities stated at long-term values.

Note that a company may be insolvent for some period of time prior to the time that it cannot meet its current obligations out of cash flow.

The insolvency condition can be caused by many factors but basically can occur in one of two ways (or some combination thereof):

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[1] Allen L. Mayerson, "Ensuring the Solvency of Property and Liability Insurance Companies," in Insurance Government and Social Policy, ed. Spencer L. Kimball and Herbert S. Denenberg (Homewood, Illinois: Richard D. Irwin, Inc., 1969), p. 147.

. the value of the company's asset account unexpectedly declines;

. the value of the company's liability account unexpectedly increases.

B. Solvency

Solvency means that a company is not insolvent, as defined above. That is, the company is able to continue operations. The definition of solvency implies an ability to meet those obligations that can now be predicted, using the best available judgment, but provides no indication of the ability to continue to meet those obligations.[2]

C. Solidity

Solidity, for the purpose of this discussion, means that an insurer is not insolvent, as defined above, and has sufficient capacity to achieve at least the following over a long period of time:[3]

. meet its obligation to pay for all claims and expenses associated with contingencies, and

. consistently pay a fair rate of return to its owners, and

. retain sufficient earnings to grow in accordance with its company philosophy, and

. maintain its operations at a consistent level in accordance with its company philosophy.

Similarly, according to the Special Committee on Insurance Holding Companies report to the New York Insurance Department:

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[2] Ibid., p. 147.

[3] Ibid., p. 147.

"The central objective of insurance regulation is to ensure the 'solidity' of every insurance company. What is sought is more than 'solvency' in the traditional senses, of (a) excess of assets over liabilities, or (b) ability to pay debts as they mature. What is sought is a more stringent test of soundness that will provide assurance of solvency lasting long enough into the future for any dangerous development to be detected and the surplus drain resulting from it stopped."[4]

Thus, solidity is a much more stringent condition than solvency.

II. PURPOSE AND SCOPE

This paper defines insurer financial condition at a given point in time as a point (within a corresponding confidence interval) on a solvency continuum with insolvency and solidity as end points. The purpose of the paper is to propose that a particular insurer's position on the solvency continuum, at a given point in time, is a mathematical function of underlying measures (such as investment portfolio composition and reserve accuracy) observed at that same point in time. This concept is developed and illustrations of its application are provided.

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[4] "The Special Committee on Insurance Holding Companies Report to the New York Insurance Superintendent on February 15, 1968" excerpt in Insurance Government and Social Policy, ed. Spencer L. Kimball and Herbert S. Denenberg (Homewood, Illinois: Richard D. Irwin, Inc., 1969), pp. 64-69.

III. BACKGROUND -- CURRENT METHODOLOGY FOR EVALUATION OF INSURANCE COMPANY FINANCIAL CONDITION

Evaluation of the financial condition of insurance companies is a primary goal of regulators, investors, and insurer management. Each group is interested in a different aspect of financial condition. Regulators must ascertain a company's ability to operate and meet its obligations while investors and insurer management are interested in a company's long term growth and profit potential. The evaluation methodology currently in use by each of these groups is discussed below.

A. Regulators

To date, the most systematic methods of evaluating insurance company financial condition have been developed and applied by regulators. The basic goals of these methods are to categorize insurers into two groups: (1) those now insolvent or potentially insolvent within a short period of time and (2) all others.

The methods and their historical development are briefly outlined below.

1. The NAIC Early Warning System (EWS).

The EWS was developed in 1971 based on research by the California, Illinois and Michigan Insurance Departments.[5] The purposes of the EWS are "to help State Insurance Department personnel quickly identify companies requiring close surveillance and determine the form that surveillance should take."[6]

Beginning with over 24 financial ratios, the EWS later became

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[5]Cormick L. Breslin and Terrie E. Trozel, Property-Liability Insurance Accounting and Finance (Malvern, Pennsylvania: American Institute for Property and Liability Underwriters, 1978), pp. 287-290.

[6]"Using the Early Warning System," NAIC Tests for Property and Liability Insurers, 1977.

the Insurance Regulatory Information System[7] (IRIS) based on the computation and review of eleven financial ratios in four categories as follows:

Category	NAIC IRIS Test
. Overall	Premium to Surplus Change in Writings Surplus Aid to Surplus
. Profitability	Two-Year Adjusted Underwriting Ratio Investment Yield Change in Surplus
. Liquidity	Liabilities to Liquid Assets Agents' Balances to Surplus
. Reserves	One-year Reserve Development to Surplus Two-year Reserve Development to Surplus Current Estimated Reserve Deficiency to Surplus

2. Illinois Solidity Tests.

The Illinois Insurance Department developed its own set of financial tests. The purpose of these tests, similar to the EWS but in reverse order, is to determine areas where corrective action appears most necessary and to identify those companies threatened with insolvency.[8] The Illinois tests are similar to IRIS, consisting of ten tests in total. Three of the Illinois tests are not used by the IRIS: (1) Projected equity in the unearned premium, (2) Surplus divided by liabilities, and (3) Projected underwriting gain divided by surplus. The four IRIS tests not used by Illinois are (1) Two-year adjusted underwriting ratio, (2) Investment yield, (3) Surplus aid, and (4) Estimated current reserve deficiency. Although the

[7]Steven Brostoff, "NAIC's IRIS Program Strives to Nip Insolvencies Before They Bud," The National Underwriter.

[8]Illinois Insurance Guaranty Fund, Illinois Department of Insurance, Milliman & Robertson, Inc., "Property and Liability Solidity Testing Programs: An Analysis,", May, 1979.

intent of the Illinois System is nearly the same as for the IRIS System, the scoring method is somewhat unique. The results of each test are compared to the average of all results for that test, then values are assigned to a company's variation from the average and weighted to reflect the relevant value of each test. The result is a measure which attempts to extend the basic purpose of the system by quantifying a company's solidity rather than merely categorizing it as troubled/not troubled.[9]

3. American Insurance Alliance (AIA).

In 1978 the AIA Discriminant Analysis Program was completed by the Aetna Life and Casualty Company. After consideration of more than 150 variables, the AIA approach provides for the calculation of five key financial ratios [(1) Two-year Operating Ratio, (2) Liabilities to Liquid Assets, (3) Change in Surplus, (4) Net Written Premium to Loss and Loss Adjustment Expense Reserves, (5) Change in Liability Mix] and combines them using multivariate discriminant analysis to develop a "score". The score (0 to 10), or index of financial strength was, as for the IRIS and Illinois tests, intended to categorize companies into two groups - troubled (0-5) and not troubled (6-10). The score provides an indication of the degree to which the test results match those of troubled or not troubled companies using historical data: "the closer the score to the end point, the greater the certainty of classification."[10] For example, a company with a score of 0 is more likely to be a truly troubled

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[9] Ibid.

[10] Aetna Life & Casualty, American Insurance Association Property/Liability Early Warning System Proposal," July, 1978, p. 24.

company than a company with a score of 5. Similarly, a score of 10 indicates a company that is not troubled with greater certainty than would a score of 6.

In response to a request from the Illinois Insurance Department, Milliman and Robertson,[11] reviewed and evaluated the three systems described above and found the AIA approach, with proposed enhancements, to be the best predictor for correctly categorizing companies as troubled/not troubled.

4. Trieschmann and Pinches.

Although not among the above, multivariate discriminant analysis was also studied by Trieschmann and Pinches[12] and the results published in their 1972 paper "Multivariate Model for Predicting Financially Distressed P-L Insurers." The model presented in that paper was developed with the purpose of discriminating statistically between distressed and non-distressed firms. After review of more than 70 possible predictor variables, the authors used the following six: (1) Agents Balances/Total Assets, (2) Stocks at Cost/Stocks at Market, (3) Bonds at Cost/Bonds at Market, (4) (Loss Adjustment Expenses Paid + Underwriting Expenses Paid)/Net Premiums Written, (5) Combined Ratio, and (6) Premiums Written Direct/Surplus. The authors note that a key factor governing the selection of variables

[11] Illinois Insurance Guaranty Fund, Illinois Department of Insurance, Milliman & Robertson, Inc., "Property and Liability Solidity Testing Programs: An Analysis," May, 1979.

[12] James S. Trieschmann and George E. Pinches, "A Multivariate Model for Predicting Financially Distressed P-L Insurers," The Journal of Risk and Insurance, pp. 327-338.

is that the methodology is based on the assumption that the predictor variables are independent of one another.[13] Therefore, variables such as loss ratio and policyholder's surplus/total assets were omitted. Using the six selected variables, a Z-score was produced. A Z-score cutoff to classify firms as distressed or non-distressed was selected based on known Z-score results for distressed and non-distressed firms in the historical data base.

These techniques as currently used or proposed for use by regulators basically share the common purpose of categorizing companies into one of two groups -- troubled/not troubled.

B. Investors

Investors are interested in more than the troubled/not troubled dichotomy of firms. Investors must evaluate potential return to owners in the form of capital appreciation and/or dividends. Specifically, one security analyst cites the objectives of security analysts, on behalf of investors, as follows:

"1. To determine earnings on a consistent basis both from year to year and from company to company.

2. To determine the factors which cause changes in earning power within a company, and which cause differences in profitability between companies.

[13] Ibid., p. 331.

3. To be able to compare earnings of an insurance company with those of alternative investment mediums.
4. To be able to adjust book values on a consistent basis from company to company."[14]

Although less systematic than the methods described above for regulators, the investor also relies on financial or annual statement type data to discriminate the investment value of firms. The data compiled regularly by Merrill Lynch include items such as: net written premium, net earned premium, losses paid, increase in loss reserves, underwriting expenses, combined ratio, after-tax investment income, operating income, cash flow, premium to surplus ratio, loss reserves to surplus ratio, earnings per share, return on equity, and a measure of exposure of GAAP equity to stock market risk.[15] The data are compiled for all stock companies and overall averages are developed. The analysis is completed by observation of current values of each variable with particular emphasis on trends in the values over a period of time for both individual companies and the overall average.

C. Insurer Management

Company executives are interested in monitoring their progress toward company growth and profit goals. Included in this monitoring is recognition of the company's evaluation by its other observers -- regulators, auditors, and investors.[16] Therefore, company

[14]Thomas K. Meakin, "Analysis of Financial Conditions - Security Analyst Viewpoint," Proceedings of IMA (1967), pp. 700-701.

[15]"The Insurance Quarterly," Merrill Lynch, June, 1983.

[16]Theodore A. Davidson, "Analysis of Financial Conditions - Company Point of View," Proceedings of IMA (1967), pp. 695-696.

managements have the two-fold task of not only preserving solvency but also striving to grow and earn profits to thereby make its stock an attractive investment.[17] Although documentation of specific systematic procedures is not available, company monitoring of its financial condition is generally accomplished using a combination of financial and management information. These data include, at a minimum, annual statement data in addition to underlying statistics such as claim frequency, claim severity, and pure premium.

IV. DEFINITION - SOLVENCY AS A CONTINUUM

It is proposed that insurance company solvency may be represented by a continuum beginning at insolvency and continuing through solidity.[18] This concept may be illustrated using the analogy of the relationship between molecular activity and the temperature scale.

At absolute zero (-273°C) on the temperature scale there is no molecular activity. Similarly, at insolvency on the solvency continuum, there is no company activity, i.e., the company can no longer operate. In contrast, at the boiling point (100°C) and beyond there is rapid molecular motion. Similarly, at solidity on the solvency continuum, there is the activity of the company achieving the

[17] Allen L. Mayerson, "Ensuring the Solvency of Property and Liability Insurance Companies," in Insurance, Government and Social Policy, ed. Spencer L. Kimball and Herbert S. Denenberg (Homewood, Illinois: Richard D. Irwin, Inc., 1969), pp. 149-150.

[18] The end points of insolvency and solidity were selected for purposes of the discussion of solvency as a continuum. However, the solvency continuum actually ranges from $-\infty$ to $+\infty$. That is, insolvency is a condition of differing degrees--for example, one insolvent company may pay 80¢ on $1 to its creditors while another insolvent company pays 10¢ on $1 to its creditors. On the opposite end of the solvency spectrum, solidity, although less easily quantified, is also a matter of degree with the possibility of companies becoming infinitely sound.

goals characteristic of solid companies as identified in Section I.C (growth, profits, etc.). *Between absolute zero and the boiling point there are distinct* measurable differences in molecular activity. Increases in molecular activity correspond to increases in temperature. Similarly, it is proposed here that there are distinct measurable differences in a specific company's achievement of the goals characteristic of solid companies. Just as temperature provides a measure of the degree of molecular activity, position on the solvency continuum provides a measure of the degree of company solvency. Increases in company achievement of goals correspond to measurable movement toward the point of solidity on the solvency continuum.

Finally, the definition of solvency as a continuum implies that a particular company's position on the solvency continuum differs over time and that, at an instant in time, position on the solvency continuum differs for different companies.

For use in this paper, the solvency continuum is converted to an index from 0 to 100 where 0 represents the point of insolvency and 100 represents the point of solidity where the upper bound or "solidity" is actually an asymptote since the condition of solvency has no theoretical absolute limit. (Note that if the entire continuum were referenced, the index would range from -100 to +100.)

Mathematically, the conversion to an index, beginning at 0, may be represented as follows:

$$I_S = f(S)$$

where I_S = the solvency continuum index

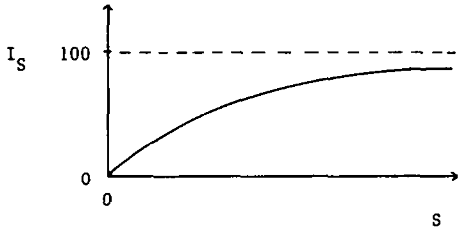
$0 \leq I_S \leq 100$

S = the solvency continuum

$0 \leq S < \infty$

f = the index conversion function.

The conversion of solvency to an index may be represented graphically as follows (note that the graph corresponds to the mathematical representation above and displays the conversion beginning at 0):



V. PROPOSAL - DETERMINATION OF POSITION ON SOLVENCY CONTINUUM

Given the above definition, the problem is to determine company position and its associated confidence interval on the solvency continuum at a given point in time and, further, to determine the direction of a company's movement on the continuum over time. As evidenced by the system of regulatory tests described in Section III. A. above, it is generally recognized by regulators that there exist variables such as the premium to surplus ratio, combined ratio, etc., that provide an indication of company financial condition. These variables are, in turn, a function of various factors such as premium, expenses, etc.

The relationship between the variables affecting company financial condition and company position on the solvency continuum may be illustrated by continuing the analogy of temperature. Company position on the solvency continuum is represented by the temperature. As the company operates, variables affecting its financial condition each contribute a distinct measurable amount of change (within a confidence interval) in the company temperature. Contributions of individual variables may differ over time. The combined contributions of the change in temperature associated with each variable produces the overall company temperature, or the company position on the solvency continuum.

Using the above analogy, the principles advanced here are for the single variable and multi-variable cases, respectively:

1. In the single variable case, for the ith variable, V_i, affecting company solvency, there exists an identifiable mathematical function, h_i, between values of that variable (within a confidence interval) and company position on the solvency continuum (within a confidence interval). Mathematically this may be represented as follows:

$$V_i = g_i(F_{i1}, F_{i2}, \ldots, F_{ik})$$
$$S = h_i(V_i)$$
$$I_S = f(S)$$

where,

F_{ij} = ijth factor underlying V_i, which in turn affects company position on the solvency continuum

g_i = ith mathematical function relating the F_{ij} factors to the ith variable

V_i = ith variable affecting company position on the solvency continuum

h_i = the mathematical function relating the ith variable to company position on the solvency continuum

S = the solvency continuum

f = the mathematical function relating the solvency continuum to the index of solvency

I_S = the index of solvency

2. The contributions of each variable blend pairwise or in any combination such that there exists an identifiable mathematical function between the values of each combination of variables (within a

confidence interval) and company position on the solvency continuum (within a confidence interval). Mathematically, this concept is represented for two variables as follows:

$$V_i = g_i(F_{i1}, F_{i2}, \ldots, F_{ik})$$
$$V_j = g_j(F_{j1}, F_{j2}, \ldots, F_{jk})$$
$$S = h_{ij}(V_i, V_j)$$
$$I_S = f(S)$$

where,

F_{ij}, g_i, V_i, S, f, and I_S are defined as above and

h_{ij} = the mathematical function relating the ith and jth variable to company position on the solvency continuum

And, mathematically, the concept is represented for n variables, as follows:

$$V_1 = g_1(F_{11}, F_{12}, \ldots, F_{1k})$$
$$\vdots$$
$$V_i = g_i(F_{i1}, F_{i2}, \ldots, F_{ik})$$
$$\vdots$$
$$V_n = g_n(F_{n1}, F_{n2}, \ldots, F_{nk})$$
$$S = h_{1,2,\ldots,n}(V_1, V_2, \ldots, V_n)$$
$$I_S = f(S)$$

where,

F_{ij}, g_i, V_i, S, f, and I_S are defined as above and

$h_{1,2,\ldots,n}$ = the mathematical function relating the n variables to company position on the solvency continuum.

VI. PROPOSAL - INTERPRETATION OF COMPANY POSITION ON THE SOLVENCY CONTINUUM

Two types of interpretation can be made using the results of the proposal -- comparative and absolute. These are discussed separately below.

A. Comparative Interpretation

The above proposal may be used to determine the following for an individual company:

1. The impact on company position on the solvency continuum of any one variable acting alone.
2. The relative impact on company position on the solvency continuum of any combination of two or more variables.
3. The combined impact on company position on the solvency continuum of all variables acting together.
4. The impact over time on company position on the solvency continuum of any one variable acting alone.
5. The relative impact over time on company position on the solvency continuum of any combination of two or more variables.
6. The combined impact over time on company position on the solvency continuum of all variables acting together.

Application 1 above provides the basis for sensitivity testing of the relative importance of individual variables to company position on the solvency continuum. Application 2 combined with Application 1 provides the basis for sensitivity testing of the relative contribution to company position on the solvency continuum of individual or combinations of variables when acting in combination with other variables. These applications are of fundamental importance in assessing the reason for a company's position on the solvency continuum, as determined from Application 3 above.

Applications 4 through 6 enhance the results of Applications 1 through 3 by extending the analysis to include results over time. For example, the relative importance of a single variable may differ with different observations of data. Several observations are required to ascertain the consistent significance of a particular variable. Again, Applications 4 and 5 are of fundamental importance in assessing the reason for a company's position on the solvency continuum at a point in time or the trend in the movement of its position on the continuum over time (Application 6). For example, assuming that company positions on the solvency continuum had been 10, 40, 30, 50, 40 and 60, respectively, for each of the last six years, it could be concluded that the company was moving gradually toward increasing solidity.

The results of Applications 1 through 6 for an individual company may also be used as a basis for comparison between or among companies.

F. Absolute Interpretation

The resulting I_S values, as proposed, will range from 0 to 100 (insolvency to solidity) within a confidence interval. It is proposed that there exists a theoretical basis for the absolute interpretation of a specific result. Development of this theory, however, is beyond the scope of this paper and has been identified as an area for further study. In the meantime, absolute interpretation of specific results may be achieved in one of the following ways:

- Judgmental assignment of evaluative ranks to each score.
- Empirical and judgmental assignment of evaluative ranks to each score.

For example, the continuum may be segregated into bands -- with the end points of each band surrounded by a confidence interval. A possible set of bands is displayed in the figure below.

0 10 20 40 60 80 90 100

Solvency Continuum Bands

One possible interpretation of these bands is:

Position on Solvency Continuum	Evaluation of Company Financial Condition
0	Insolvent
1 - 10	Seriously Impaired
11- 20	Poor
21- 40	Fair
41- 60	Average
61- 80	Sound
81- 90	Excellent
91-100	Solid

VII. ILLUSTRATION OF PROPOSAL USING SELECTED VARIABLES UNDERLYING THE CONDITION OF SOLVENCY

As exemplified by the research underlying the AIA[19] and the Trieschmann and Pinches[20] multivariate discriminant analysis work in which over 70 and 150 variables, respectively, potentially affecting solvency were studied, there are many variables affecting the condition of company solvency. Four key variables resulting from these studies as well as from various other sources are briefly described below in order to illustrate the proposal described in the preceding

[19]Aetna Life and Casualty, American Insurance Association Property/Liability Early Warning System Proposal," July, 1978, p. 24.

[20]James S. Trieschmann and George E. Pinches, "Multivariate Model for Predicting Financially Distressed P-L Insurers," The Journal of Risk and Insurance, pp. 327-338.

sections for the single variable case. Two of these variables are then combined to illustrate the proposal for the two variable case.

A. Premium to Surplus Ratio.

Premium is a measure of risk (or exposure) assumed by an insurer. The risk arises because the premium realized through the sale of the insurance product represents only an estimate of the ultimate cost of production (claims and expenses) of the product -- the ultimate cost to be known only after all possible claims are closed. The more exposure an insurer assumes, the greater is the exposure to environmental risks such as inflation, changes in technology, and changes in legal climate and, hence, the greater is the variance in the estimate of the ultimate cost of production of the product.[21] The greater the variance in the estimate of the ultimate cost of production of the product, the greater is the insurer's need for surplus to be available to cover all possible contingencies.[22] Therefore, there exists a relationship between premiums written and surplus and, hence, between the variable, premium to surplus ratio, and company position on the solvency continuum. In fact, because of the need for surplus to cover contingencies arising due to the variance in the actual cost of the product from the cost anticipated in the premium, the greater the premium to surplus ratio, the closer is the company's position on the solvency continuum to the insolvency point.

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[21] Cormick L. Breslin and Terrie E. Troxel, Property-Liability Insurance Acounting and Finance (Malvern, Pennsylvania: American Institute for Property and Liability Underwriters, 1978), p. 290.

[22] Determination of the required amount of surplus is beyond the scope of this paper. There are several references on this subject. One of these is "The Special Committee on Insurance Holding Companies Report to the New York Insurance Superintendent on February 15, 1968" excerpt in Insurance, Government and Social Policy, ed. Spencer L. Kimball and Herbert S. Denenberg (Homewood, Illinois: Richard D. Irwin, Inc., 1969).

The measurement of the premium to surplus ratio may be obtained using annual statement data. Collected premium is a known amount. Surplus is, however, an estimate because it is based on estimates of asset values and liability values which reflect the uncertainty in the cost of the product through the estimated claim reserves.[23] These values are displayed as point estimates, without display of associated surrounding confidence interval, on the annual statement. Therefore, the premium to surplus ratio based on annual statement data is a point estimate within a confidence interval.

The relationship between the premium to surplus ratio and company position on the solvency continuum may be expressed mathematically as follows:

$$F_{1,1} = P$$

$$F_{1,2} = PS$$

$$V_1 = g_1(F_{1,1}, F_{1,2}) = g_1(P, PS) = P/PS$$

$$S = h_1(V_1)$$

$$I_S = f(S)$$

where,

P = Premium

PS = Policyholder's Surplus.

and, $F_{1,1}$, $F_{1,2}$, g_1, V_1, h_1, S, f, and I_S are as defined in Section V.

[23] C. K. Khury, "Loss Reserves: Performance Standards," PCAS LXVII, 1980, pp. 2-9.

Using hypothetical data, let P = \$1,000,000, S = \$250,000 ± \$25,000, $S = h_1(V_1) = 1/V_1$, and $f(S) = 100\ [S/(1+S)]$. Then,

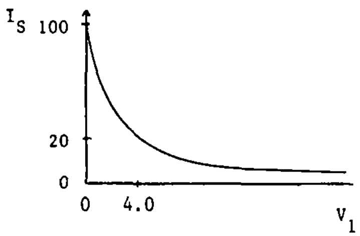
$$F_{1,1} = P = \$1,000,000$$

$$F_{1,2} = S = \$250,000 \pm \$25,000$$

$$V_1 = g_1(F_{1,1},F_{1,2}) = g_1(P,PS) = \$1,000,000/(\$250,000 \pm \$25,000) = 4.0 \pm .4$$

$$I_S = f(S) = f[h_1(V_1)] = 100\ [1/(1+V_1)] = 20 \pm 1.5$$

Graphically,



B. Investment Portfolio.

The composition of the investment portfolio is a large subject which may be viewed from different perspectives to produce many different variables which affect company solvency. Examples of these different perspectives include:

Liquidity -- Does the portfolio consist of short-term or long-term investments? What are the penalties associated with early relinquishment of long-term holdings?

. Yield -- Does the portfolio consist of investments which anticipate capital gains or dividends? Does the portfolio anticipate high or low yields? Is the income taxable or tax free?

Examples of solvency-related variables which result from these perspectives include:

. Liabilities to Liquid Assets (AIA[24] and IRIS[25])

. Investment Yield (IRIS[26] and Illinois[27])

. Stocks at Cost/Stocks at Market (Trieschmann & Pinches[28])

. Bonds at Cost/Bonds at Market (Trieschmann & Pinches[29])

Each of the above variables provides a way to measure the degree of risk of a decline in surplus due to an unexpected decline in income from or value of the investment portfolio. Each of the portfolio related variables is also related to company position on the solvency continuum. For example, the liabilities to liquid assets ratio measures a company's ability to respond to the need to meet its liabilities as if they were all due immediately. If liquid assets are not sufficient to meet immediate liability needs, then surplus must be used to fill the void. Thus,

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[24]Aetna Life & Casualty, American Insurance Association Property/Liability Early Wrning System Proposal," July, 1978, p. 24.

[25]<u>Using the NAIC Insurance Regulatory Information System</u>: <u>Property AND Liability Insurers</u>, 1978.

[26]<u>Ibid</u>.

[27]Illinois Insurance Guaranty Fund, Illinois Department of Insurance, Milliman & Robertson, Inc.," Property and Liability Solidity Testing Programs: An Analysis," May, 1979.

[28]James S. Trieschmann and George E. Pinches, "A Multivariate Model for Predicting Financially Distressed P-L Insurers," <u>The Journal of Risk and Insurance</u>, pp. 327-338.

[29]<u>Ibid</u>.

the higher the ratio of liabilities to liquid assets, the greater the risk that surplus must be used, and possibly exhausted, and, hence, the closer the company's position on the solvency continuum is to the insolvency point.

As for the premium to surplus ratio, the ratio of liabilities to liquid assets can be measured using annual statement data. Annual statement liabilities, as mentioned above, are an estimate, within a confidence interval,[30] of the ultimate cost of the insurance product due to claims. Similarly, annual statement liquid assets are an estimate within a confidence interval, of the true value of liquid assets at a given point in time. Thus, the liabilities to liquid asset ratio based on annual statement data is a point estimate within a confidence interval.

The relationship between the liabilities to liquid assets ratio and company position on the solvency continuum may be expressed mathematically as follows:

$$F_{2,1} = L$$
$$F_{2,2} = LA$$
$$V_2 = g_2(F_{2,1},F_{2,2}) = g_2(L,LA) = L/LA$$
$$S = h_2(V_2)$$
$$I_S = f(S)$$

where,

L = Liabilities

LA = Liquid Assets

and, $F_{2,1}$, $F_{2,2}$, V_2, g_2, h_2, S, f and I_S are as defined in Section V.

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[30] C. K. Khury, "Loss Reserves: Performance Standards," PCAS LXVII, 1980, pp. 2-9.

Using hypothetical data, let $L = \$20{,}000{,}000 \pm \$2{,}000{,}000$, $LA = \$1{,}000{,}000 \pm \$10{,}000$, $S = h_2(V_2) = 1/V_2$, and $f(S) = 100\,[S/(1+S)]$ then,

$$F_{2,1} = L = \$20{,}000{,}000 \pm \$2{,}000{,}000$$

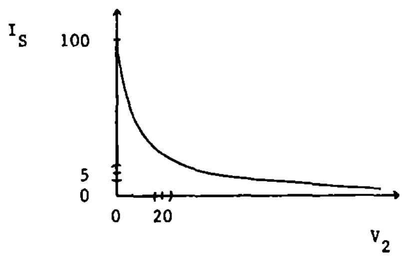
$$F_{2,2} = LA = \$1{,}000{,}000 \pm \$10{,}000$$

$$V_2 = g_2(F_{2,1},F_{2,2}) = g_2(L,LA) = (\$20{,}000{,}000 \pm \$2{,}000{,}000)/(\$1{,}000{,}000 \pm \$10{,}000)$$

$$= 20 \pm 2.2$$

$$I_S = f(S) = f[h_2(V_2)] = 100\,[1/(1+V_2)] = 5 \pm .5$$

Graphically,



C. Reinsurance. Reinsurance and its relationship to solvency may be *established first by examining the quantitative purposes of reinsurance:*[31]

- Stability -- Avoid violent swings in underwriting results. Avoid incidents of high frequency or severity which might threaten surplus.

[31]Allen L. Mayerson, "Ensuring the Solvency of Property and Liability Insurance Companies," in Insurance, Government and Social Policy, ed. Spencer L. Kimball and Herbert S. Denenberg (Homewood, Illinois: Richard D. Irwin, Inc., 1969), p. 185.

- Capacity -- Insurer can write large risks but share the premium and cost of the product with the reinsurer.
- Financing -- Insurer can recover the equity in the unearned premium reserve.

These purposes are accomplished using either pro rata, excess of loss or some combination of the two classes of reinsurance treaty. These classes in turn are comprised of five types of reinsurance treaty:[32]

- Quota Share (Pro Rata) -- The primary insurer cedes to the reinsurer a fixed percentage of every risk it writes.
- Surplus Share (Pro Rata) -- A surplus share treaty provides for ceding to the reinsurer any amount written by the primary insurer over its retention.
- Each Risk/Each Occurrence (Excess of Loss) -- The reinsurer pays the first $X of each claim arising out of any one accident.
- Each Occurrence (Excess of Loss) -- The reinsurer pays the amount in excess of $X for all claims in the aggregate arising out of any one occurrence.
- Stop-Loss (Excess of Loss) -- The reinsurer pays the amount in excess of $X for all claims in the aggregate arising during a calendar year.

Reinsurance as it affects solvency is reflected in the regulatory tests as surplus aid tests to ascertain whether reinsurance is used as a loan from the reinsurer to the ceding company in order to overstate ceding company surplus. The test of surplus aid to surplus is included by IRIS

[32] Ibid., pp. 185-186.

and Illinois. Reinsurance affects company position on the solvency continuum also in a more general or pervasive way as discussed below.

Companies, of course, generally own several reinsurance contracts. For simplicity in the illustration of the relationship between reinsurance and company position on the solvency continuum, ownership of a stop-loss contract alone is assumed. Two key aspects of the stop-loss contract are the retention (or anticipated maximum ceding company loss ratio) and the cost of the reinsurance contract. The lower the retention the lower is the ceding company's maximum loss ratio and the smaller is the range of possible loss ratios, i.e., the smaller is the variance in the ceding company loss ratio and the less likely that ceding company surplus will suffer a decline due to unexpected claims. Conversely, the higher the retention, the larger is the variance in the ceding company loss ratio and, for company's of the same premium volume and mix of business, the closer to the insolvency point is the company's position on the solvency continuum.

Therefore, under the stop-loss contract, the risk to ceding company surplus is limited to the amount of claims between breakeven and the stop-loss combined ratio produced by the selected retention. Thus, the ratio of the potential claims under the stop-loss contract to surplus is the variable relating the stop-loss reinsurance retention to company position on the solvency continuum.

As before, annual statement data are the basis for the surplus value. Hence, surplus as used in this variable is an estimate within a confidence interval and the ratio of potential claims under the stop-loss contract to surplus is a point estimate within a confidence interval.

The relationship between the reinsurance variable and the solvency continuum may be expressed mathematically as follows:

$$F_{3,1} = P$$

$$F_{3,2} = PS$$

$$F_{3,3} = SCR$$

$$V_3 = g_3(F_{3,1},F_{3,2},F_{3,3}) = g_3(P,PS,SCR) = P[(SCR-100)/100]/PS$$

$$S = h_3(V_3)$$

$$I_S = f(S)$$

where,

P = Premium

PS = Surplus

SCR = *the stop-loss combined ratio*

and $F_{3,1}$, $F_{3,2}$, $F_{3,3}$, g_3, V_3, h_3, S, f and I_S are as defined in Section V.

Using hypothetical data, let P = \$15,000,000, PS = \$5,000,000 ± \$1,000,000, SCR = 150, $S = h_3(V_3) = P[(SCR-100)/100]/PS$, and $f(S) = 100[S/(1+S)]$, then,

$$F_{3,1} = \$15,000,000$$

$$F_{3,2} = \$5,000,000 \pm \$1,000,000$$

$$F_{3,3} = 150$$

$$V_3 = g_3(F_{3,1},F_{3,2},F_{3,3}) = g_3(P,PS,SCR) = P[(SCR-100)/100]/PS$$

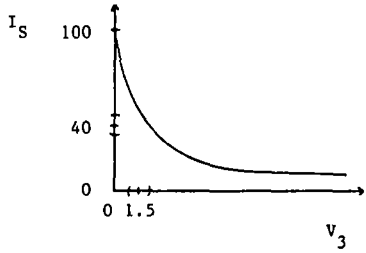
$$= \frac{\$15,000,000 \times .50}{\$5,000,000 \pm \$1,000,000}$$

$$= 1.5 \pm .3$$

$$I_S = f(S) = f[h_3(V_3)]$$

$$= 100[1/(1+V_3)] = 40 \pm 5$$

Graphically,



D. Reserve Accuracy.

The question of reserve accuracy is, in itself, a large subject covered in many other sources. Claim reserves generally account for a large proportion of an insurer's liabilities[33] and are affected by many factors. These factors include inflation, claim settlement philosophy, claim department expertise, type of business, mix of business, regulatory climate, reserve methods, expertise or ability of reserve estimators.[34] These factors all contribute to the variance in the annual statement reserve estimate. Thus, it is universally recognized that claim reserve inadequacy is a key element leading to insolvency. All of the regulatory tests described in Section III include measures of reserve adequacy. These were:

. One-Year Reserve Development to Surplus (IRIS)[35]

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[33] Timothy M. Peterson, Loss Reserving Property/Casualty Insurance (USA: Ernst & Whinney, 1981).

[34] Ibid.

[35] Using the NAIC Insurance Regulatory Information System: Property and Liability Insurers, 1978.

. Two-Year Reserve Development to Surplus (IRIS)[36]

. Estimated Current Reserve Deficiency to Surplus (IRIS[37] and Illinois[38])

. Net Written Premium to Loss and Loss Adjustment Expense Reserves (AIA)[39]

These tests measure not only reserve adequacy, but also reserve accuracy. For example, in the Reserve Deficiency to Surplus test, reserve deficiency is calculated by comparing required reserves to stated reserves. The extent of reserve error (deficiency or redundancy) is then compared to surplus to ascertain the proportion of surplus which may be required to pay claims (if the error is a deficiency). The greater the estimated reserve deficiency the closer to the insolvency point is the company's position on the solvency continuum.

As for the other variables discussed, the factors in the reserve deficiency ratio may be calculated based on annual statement data. In the computation of reserve error, required reserves must be estimated and then compared to the reserve amount displayed in the annual statement. Both the required reserves and the annual statement reserves are point estimates within a confidence interval. The size of the reserve estimate confidence interval is directly determinable from the size of the

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[36] Ibid.

[37] Ibid.

[38] Illinois Insurance Guaranty Fund, Illinois Department of Insurance, Milliman & Robertson, Inc.," Property and Liability Solidity Testing Programs: An Analysis," May, 1979.

[39] Aetna Life & Casualty, American Insurance Association Property/Liability Early Warning System Proposal," July, 1978, p. 24.

confidence interval around the estimated product cost per exposure[40] (or premium). The wider the confidence interval associated with the statement reserves, the greater the variability and the wider the confidence interval around the estimated reserve error. The greater the variance in the estimated reserve error, the wider the confidence interval around the company's position on the solvency continuum.

The relationship between reserve accuracy and company position on the solvency continuum may be represented mathematically as follows:

$$F_{4,1} = PS$$

$$F_{4,2} = RRE$$

$$F_{4,3} = SRE$$

$$V_4 = g_4(F_{4,1}, F_{4,2}, F_{4,3}) = g_4(PS, RRE, SRE) = |(RRE-SRE)|/PS$$

$$S = h_4(V_4)$$

$$I_S = f(S)$$

where,

PS = surplus

RRE = required reserve estimate

SRE = statement reserve estimate

and, $F_{4,1}$, $F_{4,2}$, $F_{4,3}$, g_4, V_4, h_4, and I_S are as defined in Section V.

Using hypothetical data, let PS = \$25,000,000, RRE = \$15,000,000, SRE = \$5,000,000, $h_4(V_4) = V_4$ and $f(S) = 100[S/(1+S)]$, then,

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[40] C. K. Khury, "Loss Reserves: Performance Standards," PCAS LXVII, 1980, p. 1.

$$F_{4,1} = \$25,000,000 \pm \$5,000,000$$

$$F_{4,2} = \$15,000,000 \pm \$3,000,000$$

$$F_{4,3} = \$5,000,000 \pm \$1,000,000$$

$$V_4 = g_4(F_{4,1},F_{4,2},F_{4,3}) = g_4(PS,RRE,SRE) = [|(RRE-SRE)|/PS]$$

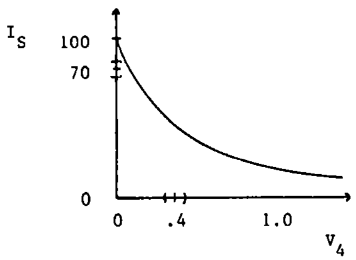
$$= \frac{|(\$15,000,000\pm\$3,000,000) - (\$5,000,000\pm\$1,000,000)|}{\$25,000,000\pm\$5,000,000}$$

$$= .4 \pm .2$$

$$I_S = f(S) = f[h_4(V_4)]$$

$$= 100[V_4/(1+V_4)] = 71 \pm 10$$

Graphically,



E. <u>More than One Variable</u>

The preceding illustrations were all for the one variable case. Using the variables, premium to surplus ratio and liabilities to liquid assets ratio, and the hypothetical data referred to above, the two variable case may be illustrated as follows. First, algebraically:

$$V_1 = g_1(F_{1,1},F_{1,2})$$

$$V_2 = g_2(F_{2,1},F_{1,2})$$

$$S = h_{1,2}(V_1,V_2)$$

$$I_S = f(S)$$

and, numerically, let,

$$F_{1,1} = \$1,000,000$$

$$F_{1,2} = \$250,000 \pm \$25,000$$

$$F_{2,1} = \$20,000,000 \pm \$2,000,000$$

$$F_{2,2} = \$1,000,000 \pm \$10,000$$

$$V_1 = g_1(F_{1,1},F_{1,2}) = g_1(P,PS) = P/PS = 4 \pm .4$$

$$V_2 = g_2(F_{2,1},F_{2,2}) = g_2(L,LA) = L/LA = 20 \pm 2.2$$

$$h_{1,2}(V_{1,2}) = 1/(V_1+V_2)$$

$$f(S) = 100[1/(1+S)]$$

$$I_S = f(S) = f[h_{1,2}(V_1,V_2)]$$

$$= 100[1/(1+V_1+V_2)]$$

$$= 4 \pm .5$$

Similarly, given known functional relationships, g, h, and f, any combination of n variables may be combined, as proposed in Section V, to determine company position on the solvency continuum.

VIII. APPLICATIONS OF THE PROPOSAL

Current methodology to evaluate the financial condition of insurance companies was discussed in Section III. for three groups -- regulators, investors and insurer management. This section illustrates how the proposed method meets the needs of all three groups for a systematic method to evaluate insurance company financial condition.

A. Regulators

Implementation of the proposed method to evaluate insurance company financial condition would meet the specific needs of regulators by providing a system to:

- specifically evaluate an individual company's financial condition as a point within a confidence interval on the solvency continuum at a given point in time so that companies may be classified into the traditional categories as troubled or not troubled, plus provide an additional measure of the condition of all companies.
- evaluate the trend in a company's financial condition over time so that not troubled companies with a deteriorating trend may be identified long before reaching the troubled class.
- quantify the contributions of each area of company operations to overall company financial condition at a given point in time and over time so that poorly functioning areas can be identified and corrected prior to irreversible financial impairment.

Thus, the proposed system would provide a more complete measure of company financial condition than the current IRIS or Illinois systems and fill the major gap[41] of multivariate discriminant analysis (the AIA system) by providing a diagnostic tool to evaluate the causes of company financial condition.

B. Investors

The proposed system would meet the needs of investors by providing a system to:

- specifically evaluate an individual company's financial condition as a point within a confidence interval on the solvency continuum

[41] Aetna Life & Casualty, American Insurance Association Property/Liability Early Warning System Proposal," July, 1978, p. 24.

at a given point in time so that the potential investor can determine the degree of a company's financial strength and potential earning capacity.

- evaluate the trend in a company's financial condition over time so that changes in a company's financial strength and potential earning capacity can be anticipated.
- compare on an absolute scale the financial condition of different companies -- at specific points in time and the trend over time -- so that potential earnings of companies can be differentiated.

Thus, the proposed system would add a systematic process to the investor's system for the evaluation of the financial condition of insurance companies.

C. Company

The proposed system would meet the needs of the company to evaluate its financial condition by providing a system to:

- specifically evaluate its financial condition as a point within a confidence interval on the solvency continuum at a given point in time and the trend in its position on the solvency continuum over time so that it can evaluate its progress versus its goals and be aware of its impact on its owners and regulators.
- compare its financial strength and implied earning potential with that of other companies with which it must compete for invested funds.

IX. CONCLUSION

This paper has presented the following three basic principles regarding the evaluation of the financial condition of insurance companies:

- The financial condition of insurance companies may be defined as a continuum with insolvency and solidity as end points (labeled the solvency continuum).
- Various factors (such as premiums, expenses, etc.) may be combined into various variables (such as premium to surplus ratio) which affect insurance company financial condition.
- There exists a specific mathematical function relating individual variables or any combination of two or more variables to company position, within a confidence interval, on the solvency continuum.

The application of these principles results in a theoretical system to evaluate the financial condition of insurance companies. The proposed system provides for:

- Specific mathematical measurement of an absolute value of an individual insurance company's financial condition as a point within a confidence interval on the solvency continuum.
- Specific mathematical measurement of the trend in an individual insurance company's financial condition as the change in the company's position within a confidence interval on the solvency continuum over time.
- Specific mathematical measurement of the relative contribution of individual variables to a company's financial condition.
- Comparability of measurements between companies.

The proposed system differs from the multivariate discriminant analysis approach of the AIA[42] or Trieschmann and Pinches[43] in that the multivariate discriminant analysis approach is an empirical method which relies on historical data to be used to determine current financial condition. In addition, application of the multivariate discriminant analysis approach requires independence of the predictor variables so that many variables must be eliminated and some amount of covariance tolerated. The proposed method, however, relies solely on a theoretical relationship among the various variables to determine company financial condition in terms of a position on the solvency continuum.

The value of the proposed theoretical approach versus current methodology for the evaluation of insurance company financial condition may be likened to the value of the knowledge of the theoretical size of loss distribution to determine claim amounts over a selected size versus the use of empirical data to determine that amount.

X. AREAS FOR FURTHER STUDY

Areas in need of further study in order to implement the proposed system include:

- Identify and define the variables affecting insurance company position on the solvency continuum.

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[42] Ibid.

[43] James S. Trieschmann and George E. Pinches, "A Multivariate Model for Predicting Financially Distressed P-L Insurers," The Journal of Risk and Insurance, pp. 327-338.

- Development of a procedure to determine the functions relating the variables to position on the solvency continuum.
- Development of a procedure to combine the functions relating individual variables into the function relating combinations of two or more variables to position on the solvency continuum.
- Development of a procedure to evaluate the absolute position on the solvency continuum, perhaps by using bands.