THE RELATIONSHIP BETWEEN UNDERWRITING PROFIT AND THE SURPLUS RATIO: A MODEL

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ABSTRACT

This paper presents an approach to measuring the trade-off between two contrasting goals of an insurance company: surplus growth based on profitability vs. competitiveness based on a reduced price.

The traditional surplus-to-written premium ratio is used to measure financial strength, while competitiveness is measured by the percentage of profit (or loss) present in the rates. Formulas are developed in the paper to relate these two quantities, and predict a company's future financial position under a wide variety of assumptions regarding inflation, exposure growth, and investment return.

The material presented here can serve as a framework to which could be added the detail appropriate for an individual company situation. Using the approach discussed in this paper, a company can test whether or not its profitability goals and its competitive goals are compatible. In this paper we present such an example. A future target surplus ratio can be selected, as well as growth, inflation, and investment return parameters. The model can be used to compute the underwriting profit or loss margin needed to reach the target.

When the profit loadings are compared for different surplus ratio targets under identical growth; inflation and investment assumptions, the result can be taken to be a measure of the competitive cost of the higher surplus ratio target.

I. INTRODUCTION

When planning a financial strategy for an insurance company, managers are presented with the task of balancing two seemingly contradictory goals. On one hand they must seek to build financial strength through profitability of investments and underwriting, and on the other they must maintain a strong competitive position, which requires lower prices and, therefore, lower profits to attract and hold a desirable book of business.

In this paper we will, for a specific situation, quantify the relationship between the two opposing goals of increasing surplus growth and retaining a strong competitive position. We also develop formulas which can be expanded and applied to more general situations. Using the kind of analysis represented here, a company can be certain that its profitability and competitive goals are compatible.

The specific situation we will examine is that confronted by a company currently writing at a surplus to written premium ratio of .50 to 1. The company has the option of continuing to write at this ratio, or it may opt to improve its financial position by moving to a 1.00 to 1 ratio over a six year period. We will compare the underwriting profit ratios needed for each option, and measure the cost in competitive position necessary to achieve the 1.00 surplus ratio goal. These will be measured under a variety of assumptions concerning expected rates of inflation, interest, and growth. In addition, in Section V, we will measure the cost of maintaining a 1.00 surplus ratio as compared to .50. Only internal financing will be considered, although there are other ways to finance surplus growth.

An insurance company's premium growth can be separated into two components, real growth and inflation. For this paper we define growth to be measured in some appropriate non-inflationary exposure units. However, any exposure units could be used in the model as long as exposure growth and all other premium growth are properly identified and accounted for.

We measure the competitiveness of the company by the underwriting profit margin earned. There are other ways to attract business than by reducing the price by lowering the profit margin, but these will not be considered here.

We have developed for our purpose a simplified model of the financial transactions of an insurance company having our desired characteristics. Utilizing this model, we project operating results and surplus ratios under our selected assumptions.

We will not, however, consider the question of what the proper surplus ratio should be for such a company. This is a very important but very difficult question and is beyond the scope of this paper.

II. DEVELOPMENT OF MODEL

Our method is an iterative one. We begin with the basic equation for underwriting profit margin and solve for Written Premium in year n+1 based on year n financial results and growth, inflation, and interest rate assumptions. The basic equation for underwriting profit margin P in year n+1 is:

$$(1)\ P(n+1) = \frac{EP\ (n+1) - L(n+1) - E(n+1) - D(n+1)}{EP\ (n+1)}$$

where EP(x) is earned premium in year x.

L(x) is incurred loss and loss adjustment expense in year x.

E(x) is underwriting expense in year x.

D(x) is incurred dividends in year x.

If we assume all policies are written for a term of one year, and are written uniformly throughout the year, this can be written as

$$(2)\ P(n+1) = [.5\ WP(n+1) + .5\ WP(n) - L_p(n+1) - L_r(n+1) + L_r(n) - E_1(n+1) - E_2(n+1) - E_3(n+1) - d\ EP(n+1)] \div EP(n+1)$$

where WP(x) is written premium in year x, and D(x) = dEP(x).

where $L_p(x)$ is paid loss and loss adjustment expense in year x.

$L_r(x)$ is outstanding loss and loss adjustment expense in year x.

$E_1(x)$ is the portion of expense in year x which does not vary with premium, but increases yearly in proportion to policy growth and an inflation rate I_1.

$E_2(x)$ is the portion of expense in year x which varies with earned premium. Let $E_2(x)=mEP(x)$

$E_3(x)$ is the portion of expense in year x which varies with written premium. Let $E_3(x) = t\ WP(x)$

then

$$
\begin{aligned}
(3)\quad P(n+1) = [&.5\ WP(n+1) + .5\ WP(n) - (1+g)(1+I)\ L_p(n) \\
&-(1+g)(1+I)L_r(n) + L_r(n) \\
&-(1+g)(1+I_1)E_1(n) - .5m\ WP(n+1) - .5m\ WP\ (n) \\
&-t\ WP(n+1) - .5d\ WP(n) - .5d\ WP(n+1)] \\
&\div [.5\ WP(n) + .5\ WP(n+1)]
\end{aligned}
$$

where g is the annual rate of growth in exposures

I is the rate of change in losses per exposure, reflecting not only inflation but also changes in policy distribution which effect losses such as increased amounts of insurance.*

Assuming the profit goal is the same every year, i.e. $P = P(n) = P(n+1) = \ldots$, we have

*One could reflect changes in reserve adequacy, by year by using different inflation rates for paid and outstanding losses. This is not considered in this paper.

$$(4)\ P\,[.5\,WP(n+1) + .5\,WP(n)] = WP(n+1)\,[.5 - .5m - .5d - t] + .5\,WP(n)$$
$$- (1+g)(1+I)L_p(n) - L_r(n)\,[(1+g)(1+I)-1]$$
$$- (1+g)(1+I_1)E_1(n) - .5(m+d)\,WP(n)$$

and

$$(5)\ WP(n+1) = .5\,(1 - P - m - d)\,WP(n) - (1+g)(1+I)L_p(n)$$
$$-[(1+g)(1+I)-1]L_r(n) - (1+g)(1+I_1)E_1(n)$$
$$\div\ .5\,(P + m + d - 1) + t$$

We will assume that the company can hit its profit margin target exactly every year. In practice, regulatory and other constraints as well as the effects of randomness on underwriting results may prevent this. We now proceed to the computation of the surplus at the end of the year n+1, labelled S(n+1), based on the following formula:

(6) S(n+1) = (assets at the end of year n+1)
- (liabilities at the end of year n+1)

= (accumulated value of assets at the end of year n) + (accumulated value of income received during year n+1) - (accumulated value of payments during year n+1) - (liabilities at the end of year n+1)

= (accumulated value of liabilities at the end of year n) + (accumulated value of surplus at end of year n) + (accumulated value of income received during year n+1) - (accumulated value of payments during year n+1) - (liabilities at the end of year n+1)

Since it is impossible to segregate these quantities into separate investment categories, it is assumed an average interest rate i is appropriate to compute all the accumulated values. The investment income generated by i is considered to be net of investment expenses.

To simplify the model, we have assumed the existence of only three different liabilities, unpaid dividends, the unearned premium reserve, and a combined loss and loss adjustment expense reserve. These constitute the bulk of an insurer's liabilities, and are usually the ones which can be projected into the future with some degree of accuracy.

The quantities and entry dates of our income stream are listed in the table below.

Investment Income Calculation

Quantity	Date	Accumulated Value to 12/31/n+1
Beginning Outstanding Loss & LAE Reserve	1/1/n+1	$L_r(n)(1+i)$
Beginning Unearned Premium Reserve	1/1/n+1	$.5\ WP(n)(1+i)$
Beginning Unpaid Dividends	1/1/n+1	$.5d\ WP(n)(1+i)$
Beginning Surplus	1/1/n+1	$S(n)\ (1+i)$
Written Premium	7/1/n+1	$WP(n+1)(1+i)^{.5} \div (1+i)^a$
Loss and LAE Paid	7/1/n+1	$L_p(n+1)(1+i)^{.5}$
Underwriting Expenses Paid	7/1/n+1	$E(n+1)\ (1+i)^{.5}$

Dividends Paid	7/1/n+1	$d\ WP(n)\ (1+i)^{.5}$
Federal Income Tax	9/1/n+1	$T \cdot P \cdot EP(n+1)(1+i)^{.33}$ (if P>0)
Ending Loss Reserves Outstanding	12/31/n+1	$L_r\ (n+1)$
Ending Unearned Premium Reserve	12/31/n+1	$.5\ WP(n+1)$
Ending Unpaid Dividends	12/31/n+1	$.5d\ WP(n+1)$
Ending Surplus	12/31/n+1	$S(n+1)$

Written premiums have been discounted a years for delayed remission. $(0 \leq a < 1)$

Federal Income Tax is computed as a percentage of underwriting gain T $(0 \leq T < 1)$ if underwriting gain is positive. We assume no tax liability on investment income, since many insurers invest in tax-free instruments, and also can reduce their taxes through astute timing of the realization of capital gains and losses. Also as a simplification we have allowed no tax carry-overs.

Substituting the accumulated values into equation (6) we have

$$
\begin{aligned}
(7)\quad S(n+1) = {} & L_r(n)\ (1+i) + .5\ WP(n)(1+i) + .5dWP(n)(1+i) \\
& + S(n)\ (1+i) + WP(n+1)(1+i)^{.3} \\
& - L_p(n+1)(1+i)^{.5} - E(n+1)(1+i)^{.5} \\
& - dWP(n)(1+i)^{.5} - \max\ \ .46P \cdot EP(n+1)(1+i)^{.33}\ ,\ 0 \\
& - L_r(n+1) - .5\ WP(n+1) - .5\ d\ WP(n+1)
\end{aligned}
$$

Since the only sources of income the model considers are underwriting income and investment income, we can compute the investment income earned in year n+1 from

$$\text{Investment Income} = S(n+1) - S(n) - P \cdot EP(n+1)$$

Once we have completed the calculation of all necessary quantities for year n+1, we can proceed iteratively to years n+2, n+3, etc.

In order to handle the large number of calculations, we programmed the needed formulas using APL.

III. INITIAL VALUES

We display here a simplified balance sheet and income statement for our hypothetical company.

12-31-82 Balance Sheet
(thousands)

Assets		Liabilities	
Total Assets	193,850	Loss and LAE Outstanding	80,000
		Unearned Premium	55,000
		Dividends Declared but unpaid	3,850
		Surplus	
		Surplus	55,000

1982 Income Statement
(thousands)

<u>Underwriting Income</u>	
Written Premium	$110,000
Earned Premium	100,000
Incurred Loss and LAE	60,000
Paid Loss and LAE	55,000
Underwriting Expense Incurred	30,400
Net Underwriting Gain/Loss	9,600
<u>Investment Income</u>	
Investment Income	10,000
<u>Other Income</u>	
Dividends to Policyholders	-7,000
Federal Income Tax Incurred	-1,196
Net Income	11,404

Values of Other Parameters
(dollars in thousands)

Fixed Expenses	E_1(1982)	20,000
Dividend Ratio to Earned Premium	d	.07
Ratio of Expenses Varying with E.P.	m	.06
Ratio of Expenses Varying with WP*	t	.04
Effective Income Tax rate on Underwriting Income	t	.46
Number of years to discount WP for delayed remission	a	.20

*We assume here that the company is a direct writer. Agency companies would have substantially more expense varying with Written Premiums due to Broker's Commissions. Other expense categories would be correspondingly smaller.

AN EXAMPLE

To show how the formulas work, we will use the data presented in Section III to complete the 1983 expected results based on assumptions of 5% growth, 10% inflation, and investment return of 10%, with an underwriting profit target of 4%.

1. Written Premium

From Formula (5), Section II

$$WP(1983) = .5(1 - .04 - .06 - .07)(10{,}000)$$
$$- (1.05)(1.10)(55{,}000)$$
$$- [(1.05)(1.10) - 1](80{,}000)$$
$$- (1.05)(1.10)(20{,}000)$$
$$+ \quad .5\ (.04 + .06 + .07 - 1) + .04$$
$$= 142{,}333$$

2. Earned Premium

$$EP(1983) = .5\ WP(1982) + .5\ WP(1983)$$
$$= 126{,}167$$

3. Incurred Loss and LAE

$$L_r(1983) - L_r(1982) + L_p\ (1983)$$
$$= (1.05)\ (1.10)\ L_r(1982) - L_r\ (1982) + (1.05)(1.10)L_p(1982)$$
$$= (1.05)\ (1.10)\ (80{,}000) - 80{,}000 + (1.05)(1.10)(55{,}000)$$
$$= 75{,}925$$

4. Underwriting Expense

$(1.05)(1.10)E_1(1982) + .06\ EP(1983) + .04\ WP(1983)$

$= (1.05)(1.10)(20{,}000) + .06(126{,}167) + .04\ (142{,}333)$

$= 36{,}363$

5. Incurred Dividends

$.07\ EP(1983)$

$= .07\ (126{,}167)$

$= 8{,}832$

6. Underwriting Gain Before Tax

$126{,}167 - 75{,}925 - 36{,}363 - 8832$

$= 5047$

$= .04 \times EP(1983)$

7. Surplus

From Formula (7), Section II

$S(1983) = (80{,}000)(1.10) + .5(110{,}000)(1.10)$

$+ .5\ (.07)(110{,}000)(1.10) + 55{,}000\ (1.10)$

$+ (142{,}333)(1.10)^{.3} - 63{,}525(1.10)^{.5}$

$- 36{,}363\ (1.10)^{.5} - .07\ (110{,}000)(1.10)^{.5}$

$- (1.05)(1.10)(80{,}000) - .46\ (5047)(1.10)^{.33}$

$- .5\ (142{,}333) - .5\ (.07)(142{,}333)$

$= 75{,}913$

8. Surplus Ratio

(75,913) ÷ (142,333)

= .533

Applying the formulas repeatedly, we obtain the following surplus ratios by year:

Year	Written Premium	Surplus	Ratio
1982	110,000	55,000	.500
1983	142,333	75,913	.533
1984	147,481	101,600	.689
1985	189,059	132,336	.700
1986	197,649	169,693	.859
1987	251,208	214,129	.852
1988	264,776	267,705	1.011

Note the pattern of alternating jumps in the growth of the Written Premium and the Surplus Ratio. We have worked through many examples using this model and this pattern appears regularly. It seems to be inherent in the use of the Surplus Ratio as a measurement, and stems from the fact that the Surplus Ratio can be increased by either increasing surplus or by decreasing Written Premium.

In practical applications of the model we can mitigate this problem by adding additional constraints on the year-to-year change in Written Premium per Exposure.

V. RESULTS AND CONCLUSION

Summarized below are the results for the various sets of assumptions. The per cent underwriting profit shown is the profit goal which gives the ending surplus ratio closest to the target (.50 or 1.00).

We have chosen these for illustrative purposes, and are not attempting here to predict the future. Therefore some are very extreme cases. This is desirable to show the sensitivity of the model.

Table 1

Growth Assumption (% per year)	Inflation Assumption (% per year)	Investment Return (% per year)	Underwriting Profit Margin Needed (%) .50 Target	Underwriting Profit Margin Needed (%) 1.00 Target
5%	5%	5%	-3%	11%
5	5	10	-11	-1
5	10	5	0	16
5	10	10	- 8	4
5	15	10	- 5	10
5	15	15	-13	-2
10	5	5	0	16
10	5	10	- 8	4
10	10	5	4	22
10	10	10	- 5	10
10	15	10	- 2	17
10	15	15	-10	4

We note that the model is symmetric with respect to the growth and inflation parameters. For example, g = 5% and I = 10% give the same result as g = 10% and I = 5%, all else being equal. Thus it makes no difference to the outcome of the model what the exposure units are.

Examining the results shown in Table 1, we see that if the underwriting profit is 0 or negative (where there is no Federal Tax), a decrease of 5% in the inflation rate or growth rate will decrease the necessary profit margin by about 3%. The amount of this decrease is not much affected by either the investment return or the surplus ratio target. If the profit margin is positive, the same decrease in inflation or growth decreases the profit margin needed by 5-7%. This is due to the Federal Tax provision, which results in a smaller proportion of the change in the profit being retained by the company.

On the other hand, an increase of 5% in the investment return assumption will decrease the needed profit margin by about 8% when unaffected by the tax provision, and about 12% otherwise.

Apparently, profit margins needed for a particular surplus ratio target are much more sensitive to interest rates than inflation, at least under our chosen assumptions.

Table 1 also confirms our expectation that a company which opts to increase its surplus ratio will lose ground to a similarly situated competitor. In our example the difference is very large because we have deliberately selected an extreme case. But this should not be viewed as an either-or situation. The company can select a more modest surplus ratio goal and still remain reasonably competitive. The point is that by using this type of analysis, the company can be certain that its profitability and financial goals are compatible with its competitive goals, and quantify the trade-off between them.

In Table 2 below we turn to the measurement of the competitive advantage of maintaining a .50 ratio as compared to maintaining a 1.00 ratio. We do this by altering the 1982 balance sheet in Section III by giving the company an additional $55,000,000 in assets, bringing the total surplus to $110,000,000 and the initial surplus ratio to 1.00. Other quantities are unchanged. The table below shows that there is definitely a competitive advantage in maintaining a lower surplus ratio. The amount of difference varies, but depends mainly on the size of the investment return relative to the growth and inflation.

Table 2

Growth Assumption (% per year)	Inflation Assumption (% per year)	Investment Return (% per year)	Underwriting Profit Margin Needed (%)	
			Maintain .50	Maintain 1.00
5%	5%	5%	-3%	0
5	5	10	-11	-11
5	10	5	0	8
5	10	10	- 8	- 6
5	15	10	- 5	0
5	15	15	-13	-11
10	5	5	0	8
10	5	10	- 8	- 6
10	10	5	4	16
10	10	10	- 5	0
10	15	10	- 2	10
10	15	15	-10	- 4

Anyone who wishes to perform this type of analysis will need to modify the formulas to fit individual company situations. Enhancements which could be introduced include Dividends to Stockholders, differing Income Tax Strategies, varying Loss Reserve adequacy, the inclusion of Expense Reserve computations, and adjustments to account for policy terms longer or shorter than one year. We will not give details on how this might be done since the variations could be virtually limitless. It should be emphasized these can have a substantial effect on the results of the analysis.