TITLE: REINSURING THE CAPTIVE/SPECIALTY COMPANY

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REINSURING THE CAPTIVE/SPECIALTY COMPANY

INTRODUCTION

It is coincidental that the 1982 call paper program on the pricing, underwriting, and managing of large risks is being held here in Florida. Only several months ago, I was approached as a professional reinsurance actuary to give input to a reinsurance program for a Florida physicians group. Recent legislation in the state (presumably to alleviate some of the rate pressure) had restricted annual loss payments on claims to $100,000. Of course, juries were entitled to hand down verdicts of their choice. Deferred claim payments are subject to an interest penalty. In certain cases, this leads to a $100,000 annuity in perpetuity (!) passed down from generation to generation. Although our approach has not been finalized, this exemplifies the peculiar needs today of captive or specialty companies.

Much has been written about reinsurance lately. The topic has scored highly in items of current interest to our membership. It occurred to me that a paper illustrating the complete pricing and underwriting process from the reinsurer's point of view would be appreciated.

The paper could easily be 100% dialogue. I will discuss reinsurance in general terms detailing approaches and advantages/disadvantages. Primarily, however, I would like to concentrate on the actuarial problem of selecting an excess of loss retention level. From my U.S. literature search, including the Proceedings, the mathematical treatment of retention setting has gone undocumented. I intend to introduce risk theory but concentrate on utility theory concepts. It is my desire to target everyone in this actuarial audience piquing your interest. I visualize utility concepts coming into common usage. To help those of you interested in being on the American vanguard of this revolution, I have enriched my footnotes into a complete bibliography of important readings.

Traditionally, my company has been organized into separate Treaty and Facultative divisions. Treaties deal with classes of insureds or lines of business. Facultative deals with reinsurance of a single insurance contract. Several years ago as the Captive and Self-Insurer movement escalated, a subunit of the Facultative department was established.

To our company, a Florida doctor program, under one management, falls into the definition of large risk. Premiums for a single policy, can exceed millions of dollars per annum. Although, there are many insureds within the program, characteristic of a treaty, the nature of a specialty or captive company has led us to service these accounts facultatively.

Most of these companies employ actuarial consultants to establish primary rate levels and supplementary reserves. Some have hired actuaries full time. In any event, we find whether our insured is a specialty company or treaty client, casualty/property actuaries are playing an important role in the negotiation of the reinsurance program. This includes the form the program takes on, the retention level, the pricing, and the premium/claim reporting systems.

Charles Bragg, a contemporary artist, unknowingly illustrated the actuary to perfection. Actuaries walk a thin line, making life and death (ruin) decisions. They rely on averages which seldom (in reinsurance) materialize, and through it all, are expected to remain in good humor.

This section will be expository, narrating thoughts expressed by many authors on forms and functions of reinsurance including advantages/disadvantages of each.

Virtually every insurance company must concern itself with the various forms of reinsurance that are available and the functions they perform. The establishment of a good reinsurance program is essential to (a) contain to a manageable level claim variance and (b) to reduce adverse effects on company growth and solvency caused by claim fluctuations.

> It has been said that the object of reinsurance is, in the first place, to protect the direct writing company, the cedent, against payments of such claims as would threaten his solvency, and, secondly, to secure the cedent a result of his risk business as even as possible.[1]

To purchase reinsurance economically means to select a form suitable to the needs of the company with a retention high enough to control costs yet low enough to minimize loss experience fluctuations over the years.

There are basically two types of Treaty reinsurance (a) pro rata and (b) excess of loss.

Pro rata or proportional reinsurance calls for the equal sharing of premiums and losses. The shares are determined at policy inception. In quota share reinsurance, the percentage reinsured is constant across policy limits. For X% of premium (less a commission), X% of losses incurred on the policy are ceded. Surplus share behaves similarly but with one large exception. The predetermined share X% will be determined on the basis of the insured's policy limit and the company's line guide regarding maximum retention. There is still a constant share of premiums and losses, but the company sets the share per policy according to a guide.

Pro rata reinsurance emphasizes two of the four functions that reinsurance provides. Because of the ceding commission paid by the reinsurer and the sharing of all including the larger losses, pro rata reinsurance is particularly effective in providing financial aid and capacity. Stabilization is generally inoperative since results continue to be as variable just on a smaller scale. There is some minor catastrophe protection. In an era of high investment earnings, the large premium outlays associated with pro rata reinsurance cannot be easily justified unless financial aid is neccessary (surplus relief).

Most reinsurance sold today is on an excess of loss form. Coverage can apply (a) per occurrence to an individual insured or (b) per event to a group of insureds. Event reinsurance is termed catastrophe reinsurance. Excess of loss can also be time dependent as opposed to occurrence dependent. Aggregate or stop loss reinsurance is used to restrict total claims incurred for typically an annual period on either (a) a per risk basis or (b) for a collection of risks. Sometimes insurance companies seeking to protect composite ratio results reinsure on an accident year or calendar basis the ratio of losses incurred to premiums earned. This is a stop loss cover.

Aggregate or stop loss reinsurance can be quite advantageous from the insurer's prospective. Whether there be an unexpected increase in the frequency of claims, or a few more than expected larger losses or a combination of the two, if total losses exceed the reinsured retention, then coverage attaches. Unfortunately, this type of reinsurance tends to be gimmicky.(2) Protecting calendar year results is illusory as no standard, such as accident year losses can be used to track payments over time.

Even on an accident year basis, the cash flow support is postponed indefinitely. In effect, the paid accident year loss ratio must exceed the established trigger. Lastly, losses to this type of coverage are volatile. Reinsurers in the past have typically limited their risk to a few loss ratio points above high attachment ratios. This coverage is viewed as high risk/high reward and priced accordingly.

What remains is the popular excess of loss per risk or per occurrence. Coverage will usually be divided into several layers. (Since everyone is familiar with it, I'll move along and discuss structure.) According to Reinarz(3) layers are either exposed or unexposed. An exposed or working layer is expected to have reasonably predictable frequency/severity characteristics. If a moderate sized hospital company issues $1 million policy limits and the appropriate retention is $250,000 (more on this later), the layer $250,000 xs 250,000 will be a working layer. This narrow layer with substantial premium per annum should be self-funding over a 3-5 year time horizon. Being somewhat stable, it can also be subject to commission, experience, or retrospective rating plans. A layer of $500,000 xs 500,000 would also be exposed since any single loss could attach, but the layer would not work as often. Presumably, there would not be enough premium in the second layer to sustain full layer losses; (an unbalanced condition) hence, giving the reinsurer highly variable accident year results. This layer would be expected to be self funding over a much longer time horizon. Since chronological stabilization is more valuable here (and riskier to the reinsurer), rates for this layer would include a higher profit and risk charge than for the layer $250,000 xs 250,000 and be guaranteed cost.

Of course, two doctors with separate policies may have attended the claimant. Although, in my example, an individual policy would not pay beyond $1 million, the claimant might sue for $1.5 million and win. Both policies would be expected to contribute. Reinsurance can protect against these multiple claims, through a "clash cover".

Excess reinsurance provides substantial amounts of capacity and catastrophe protection. Stability is enhanced as losses are truncated as far as the insurer is concerned at a cost of modest premium outlay. Modest premium outlay is again important in these days of high investment returns on funds withheld. Many excess of loss contracts also call for payment of ceding commissions. Insurers are then able to capture (recognize) prepaid acquistion costs. The unearned premium reserve is reduced by the amount of the cession but the reinsurer is paid an amount equal to unearned premium less commission.

It is my experience that actuarial consultants are advising clients to begin life with an adequate capital base. Captives and specialty companies are soliciting reinsurance bids directly; having a real stake in the outcome of the risk business. They are typically concerned with capacity and stability so they seek excess reinsurance. Reinsurance markets are quite competitive. The environment is—knowledgeable buyer dealing with knowledgeable seller so transactions are free of rate and form regulation.

The reader is referred to footnotes (4)-(6) for additional reading.

Reinsurance actuaries hold to the common doctrine that contracts exhibiting low risk should be priced at low expected reward, and conversely; high risk reinsurance should be priced with a high expected reward. When we divide the variance in a loss portfolio between insurer and reinsurer, we have a 2-person game. More determined attempts to minimize variance on the retained portfolio concomitantly bring about more costly reinsurance. European actuarial literature discusses this.

Lambert[(7)] notes that the reinsurance loading generally increases with the following forms of reinsurance — (a) pro rata, (b) excess of loss, and (c) stop loss or aggregate excess. Vajda[(8)] demonstrates that for a given level of premium, the reinsurer's variance is minimized if the form is pro rata—quota share. Borch[(9)] notes that stop loss reinsurance minimizes the variance to the retained portfolio.

It is no wonder that most reinsurance sold for capacity and catastrophe protection today is of the excess of loss form. In an era where investment income on retained funds is extremely important, excess of loss reinsurance makes sense. Pro rata requires large premium outlay. Stop loss is heavily loaded for profit and contingency. Furthermore, it does not return cash quickly. Excess of loss is by far our best selling product.

GAME AND UTILITY THEORY

Very little has been written in the U.S. about the quantitative study of the relative costs of various reinsurance forms and methods of establishing retentions. One text, however, by Reinarz[10] illustrates several pragmatic approaches which can be taken. If the excess of loss form is chosen, a cost effective retention can be viewed in light of (a) the reinsurer's loss loading, (b) minimizing the variation in retained loss ratio, (c) reinsuring where claims frequency drops off, and others. These are judgmental approaches calling for the actuary or reinsurance purchaser to guess at relative effectiveness. Can the consequences of the decision be objectly measured in advance?

In European literature, beginning in the 1960's, we see Risk Theory being applied in the insurance context. Using Collective Risk Theory, a "Safety First" concept is applied in loss loadings. Retention and reinsurance programs are selected to minimize the probability of ruin sometimes defined as a fraction of surplus placed at risk.

Safety first is not the subject of this study. For those interested, I suggest reading Gerber[11], Seal[12], Buhlmann[13], Philipson[14], Wilhelmsen[15], Bjerreskov[16], Pentikainen[17], Wooddy[18], and Beard et. al.[19]. The methods they note are generally complicated in theory-boiled down for practice-and may not be as safety oriented as stated.

Game theory can be viewed in the insurance context[20]. Various players, employing competing strategies, obtain payoffs which they seek to maximize by some measure. Payoffs depend on each player's strategy but all strategies are interactive. The simplest is the 2-person zero sum game where "my gain is your loss".

Traditional economic theory at first glance does not apply to insurance. Businessmen seek to maximize profits. The purchase of insurance at a cost greater than expected losses is, therefore, an irrational business decision. The resulting reduction in profits is contrary to the businessman's primary motive. But Bernoulli[21] states that a rational man does not seek to maximize gain but instead maximize the expected utility of gain. This brings insurance within the theory of economic activity.

Wald[22] relates a "game against Nature". Begin with 3 players. Two insurers cooperate against Nature with acts irrationally but not completely erratic in behavior. The single player cannot protect himself against the vagaries of Nature. By introducing the third player, eg. the reinsurer, the primary insurer transfers some of Nature's erratic behavior at a price. The origins of game theory go back to a classic book by VonNeumann and Morgenstern[23].

Utility theory and gamesmanship are normative. We begin with axioms of behavior and derive rules for decision making. Instead of observing behavior and noting its reasonability, we use the direct (as opposed to indirect) method of study and predict behavior then observe.

While studing for part 4 of the Casualty Actuarial exams, many of us studied Raiffa[24] for the economics portion of the test. A similar decision analysis text has been written by Schlaifer[25]. These books are introductory. Raiffa first deals with the risk neutral individual, this person only pays the expected loss price. Most everyone, however, is either a risk taker or a risk avoider. Utility theory explains decisions made by individuals who Raiffa calls non-EMV's (expected monetary value). Later, I intend on showing that risk averseness can lead to quantitative cost-benefit analysis.

Briefly, the value individuals place on an amount of money varies depending on wealth. Each of us views $1, $10, and $1000 differently. $1,000 to the beggar is worth infinitely more than $1,000 to the millionaire. To the beggar, it represents food, shelter, and warmth. To the millionaire, it may not cover repaires to his Rolls-Royce or Ferrari. The graph given below illustrates one utility curve. The 45-degree line suggests that each dollar is worth no less and no more than the previous one. This ideal of a dollar equals a dollar equals a dollar is seldom a realistic assumption.

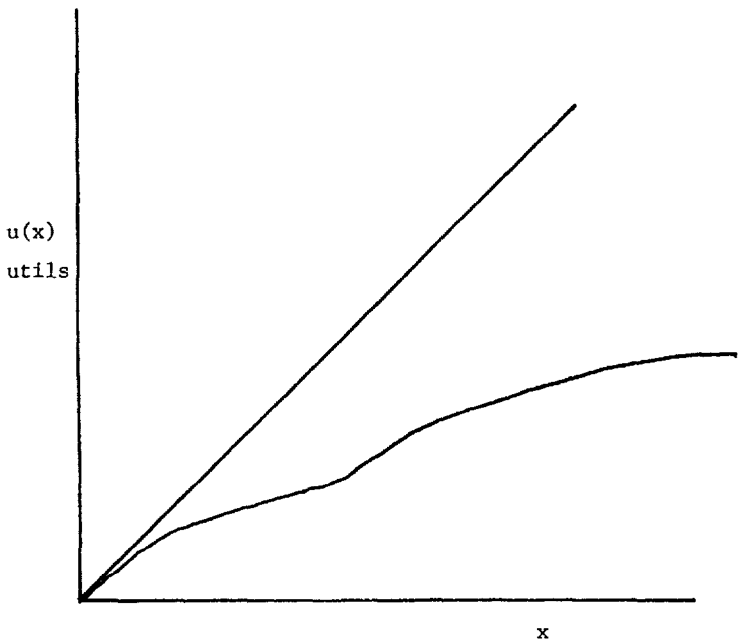
Instead most likely, we see a convex down curve. The value in "utils" of additional dollars decreases generally over the length of the curve. There may be risk taking sections of the curve, however, where we play unfair lotteries because of our aspirations. Siegel[(26)] writes about levels of aspiration.

We often see charities offering $10 tickets on a chance to win a new car. Although an unfair game if ticket sales are brisk, we might aspire to own that new car so we take a chance. The point is the ticket price has lower utility than our aspiration to own the car. The striking price for indifference is what utility theory measures. How much premium is one willing to pay (the certain result) so as to escape an uncertain loss process. This is the mirror image of the car lottery example. In the car problem, you pay to gain (utility) and for insurance, you pay not to lose (disutility).

Savage[(27)] gives an interesting history of utility and the papers written about it. Arrow [(28)] and Pratt[(29)] give accurate and meaningful interpretation to the concepts of risk aversion and risk preference. DeGroot[(30)] has demonstrated axiomatically that utility theory exists.

Finally, it may appear that insurers are nearly risk indifferent. Only recently, has the ISO varied profit and contingency loadings from the traditional 5% generally used. Certain insureds are desirable as evidenced in Bailey's paper on "Skimming the

Cream"[31]. Just as this demonstrates risk preference, we see FAIR plans with loss free insureds. Insurers obviously prefer not to insure these policyholders at the voluntary market price. Utility theory isn't abstract, incapable of practical use. Insurers can specify preferences. It is a wonder utility theory hasn't been explored in our Proceedings.



The graphs on the following page illustrate 4-families of utility functions.

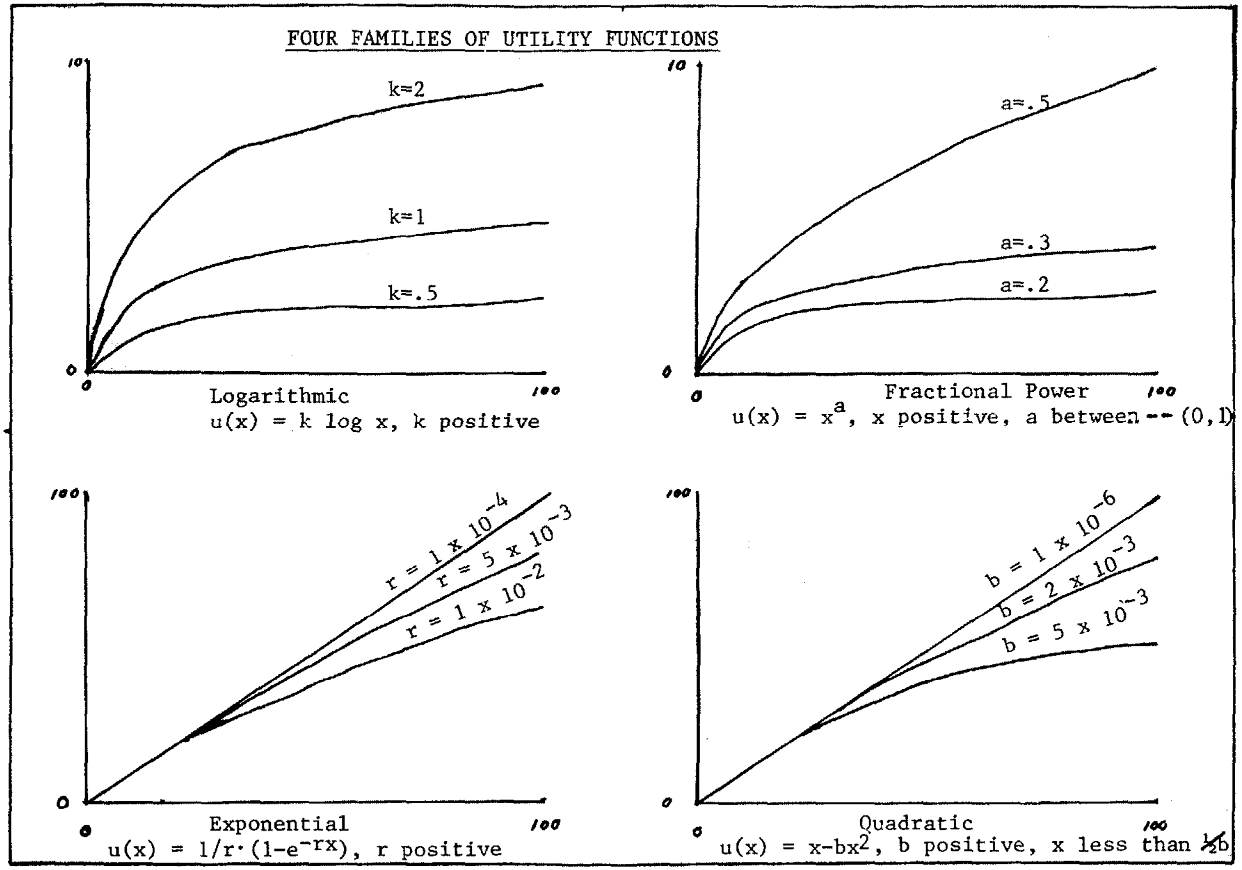
Logarithmic utility was first suggested by Bernoulli[32]. It implies decreasing risk aversion (more on this later). The family is particularly useful for insurers because in the long run, insurers become more risk prone or daring as they develop more wealth.

Quadratic utility may also be useful for insurance companies. Markowitz[33] shows that if a decision maker maximizes expected utility and always prices on a best mean-minimum variance principle, he will develop a Pareto optimal portfolio. This occurs only if his utility function is quadratic. Borch[34] demonstrates that stop loss reinsurance should be preferred for insurers exhibiting quadratic utility toward risk.

But quadratic utility curves have 2 draw-backs. First, the curves only increase up to 1/2b. Secondly, it can be demonstrated that as wealth increases, there is an increasing aversion to risk. So the larger the insurer gets, the more likely he will raise prices and reinsure more of his business. It is questionable whether insurers should behave and do behave in this manner.

Recently, it has become popular to extol exponential utility. Gould[35] shows that consumers choose deductibles consistent with those that exponential utility dictates. Shpilberg and DeNeufville[36] note that results are not particularly sensitive to the family of utility functions used; so since the exponential is easiest to work with, use it. Cozzolino[37] and Freifelder[38] have developed rate making models relying exclusively on exponential utility.

Today, insurance is largely priced on expected value (after expenses). The typical 5% underwriting margin in rates is fixed but only occasionally attained. For the utility function $u(x) = x$, our 45 degree angle line is implied. Underwriters,



FOUR FAMILIES OF UTILITY FUNCTIONS

however, do not accept this risk neutrality or indifference. This is a special case of the exponential utility family where as r approaches zero, the quantity $(1/r)(1 - e^{-rx})$ approaches x.

Exponential utility is reasonable. It implies a constant aversion to risk regardless of wealth level. Utility functions are generally concave down in the first quadrant. "More is better" so the curve is increasing, but the rate of climb slows since added dollars are worth slightly less than prior dollars. Mathematically, the first derivative $u'(x)$ is greater than zero, but the second derivative $du'(x)/dx$ is negitive. If we calculate $-u''(x)/u'(x)$ as an index of risk preference then for the exponential family, everything cancels, and we are left with r— constant risk aversion. This is the only distribution where wealth is immaterial.

This makes pricing multiple prospects over time easier. Decisions can be made independent of order or time. Other families rely on wealth for pricing purposes, and all decisions must be made in light of others. The exponential is both clean and aesthetically appealing.

Although slightly flawed, we can assume that for the long run r can vary; call it a different r each year. Then, exponential utility can displace logarithmic utility's prime appeal.

One final and most important item. Using exponential utility and given particular reinsurance terms for a book of business, you the insurer can determine an indifference price such that you do not care whether you are ceding the business or accepting it. This will be useful when we are selecting retention levels. This seller of reinsurance (the primary company) must be willing to make a fair offer of reinsurance to the reinsurer. We have often heard the words "put yourself in my place". The striking price for reinsurance can be determined using exponential utility theory as a guide. The retention can then be the most cost effective one of a group tested.

I speak in mathematical and statistical terms and do not try to bring in management expertise, knowledgeable claims staff, or quality of underwriting staff. These affect the price and retention terms offered by a reinsurer as well.

Suppose an insured with wealth "a" is given a choice of self-insuring completely a loss process "X" or paying a gross premium "G" for full coverage. Assume the insured has a linear utility attitude so that $u(x) = bx + d$.

To determine G, we solve the general equation $u(a - G) = E(u(a - X))$. The utility of net wealth after insurance must equate to the expectation of the utility of wealth without insurance. From our expression $bx + d$, we substitute $a - G$ and $a - X$ respectively for x, and get:

$$\begin{aligned} b(a - G) + d &= E(b(a - X) + d) \\ &= b(a - E(X)) + d \\ &= b(a - m) + d, \text{ where } E(X) = m, \text{ the mean expect losses} \\ G &= m \end{aligned}$$

Recall I said linear utility implied risk indifference.

Now suppose the insured's utility function is exponential so $u(x) = 1/r(1 - \exp(-rx))$. Let us modify this somewhat. Let us make the process X negative so the function relates to losses. Let us also negate the entire expression and speak of the disutility of losses. (See Cozzolino). In this case, G is given by:

$$\begin{aligned} Du\ (a - G) &= E(u(a - X)) \\ -1/r\ (1 - \exp(r(a - G))) &= E(-1/r\ (1 - \exp(r(a - X)))) \\ &= -1/r\ (1 - E\ (\exp(r(a - X)))) \\ \exp(r(a - G)) &= E(\exp(r(a - X))) \\ \exp\ (-rG) &= E(\exp(-rX)) \\ G &= -1/r \ln E(\exp(-rX)) \\ G &= 1/r \ln E(\exp(rX)) \qquad \text{translated back} \end{aligned}$$

To make this arithmetically workable, we can take the claim size distribution and discretize it into n partitions, if neccessary each with probability p. Then if we assume a uniform distribution over the interval (x_i,x_{i+1}), the risk adjusted severity is given by the following formula:

$$G = \frac{1}{r} \sum_{i=1}^{n} \frac{p_i}{x_{i+1} - x_i} \left(\frac{\exp(r^{x_{i+1}}) - \exp(r^{x_i})}{r} \right)$$

It is now only necessary to bring in the frequency distribution. Let k represent the number of claims. Then the frequency and severity adjusted risk premium equals:

$$G' = \frac{1}{r} \ln \left(\sum_{k=0}^{\infty} p(k) \exp(krG) \right)$$

In the case where frequency is Poisson distributed with parameter k, we have G' = (k/r)(exp(rG)- 1). If frequency is distributed according to the negative binomial with parameters p and b, (mean b(1 -p)/p, variance b(1 - p)/p^2) then $G' = \frac{b}{r} \ln (p/(1 - (1 -p)\exp(rG)))$

At this point, perhaps an illustration is in order. Suppose a property owner has a utility function $u(x) = \exp(-.005x)$. Further suppose, there is a 1 in 10 chance of a property loss whose distribution is $f(x) = .10\ (.01 \exp(-.01x))$. Then expected loss is given by:

$$E(X) = (.90)(0) + .10 \int_0^{\infty} x(.01 \exp(-.01x)\, dx = 10$$

Risk adjusted premium, G' is given by:

$$u(a - G') = .90\ u(a) + \int_0^{\infty} u(a - x)\ f(x)dx$$

$$-\exp(-.005(a - G')) = -.90 \exp(-.005a) - .10 \int_0^{\infty} \exp(-.005(a - x))\ (.01 \exp(-.01x))dx$$

$$\exp .005G' = .90 + (.10)\ (2)$$

$$G' = 200 \ln(1.10)$$

$$G' = 19.06$$

The insured is willing to pay almost double expected losses because of the danger in the frequency/severity distributions coupled with his risk averseness.

A TEST CASE

Assume a hospital company only writes policy limits of $5 million. According to recent ISO increased limit studies, losses can be modeled by a Shifted Pareto distribution. The following chart indicates average severities per Insurance Services Office.

Policy Limit	Average Loss	Average Allocated Loss Expense	Sum
$ 250,000	54,402	15,000	$ 69,402
$5,000,000	112,227	15,000	$127,227

Further assume a claim frequency of .006 against 16,667 occupied beds giving 100 expected claims. If acquisition, general expenses, taxes-licenses-fees, and profit amount to 25%, premium volume at total limits should be:

$$100\ (127{,}227)/.75 = \$16{,}963{,}600$$

Expected losses in the $4,750,000 xs $250,000 layer (excluding pro rata allocated loss adjustment expenses) equal:

$$100\ (127{,}227 - 69{,}402) = \$5{,}782{,}500 \text{ and}$$

divided over the 10 claims implied by the Shifted Pareto, yields an average loss of $578,250 each.

Now let us view the reinsurer's losss distribution. If we move the y-axis of the gross loss distribution over to the right to $250,000, we have a decreasing reinsurance loss function defined on the interval (0; $4,750,000). Let us assume it is nearly exponential.

A characterisitic of the exponential is that the mean $578,250 here, is the reciprocal of the value "r", so $r = 1.729\ 10^{-6}$. The loss function is then given by:

$$f(x) = .10\ (.000001729 \exp(-.000001729x)),\ x \text{ positive.}$$

Mean losses are given by:

$$E(X) = 100((.90)(0) + .10 \int_{0}^{4,750,000} .000001729\ x\ \exp(-.000001729\ x)dx$$

$$= 100\ (0 + (.10)\ (578,250))$$

$$= \$5,782,500$$

Suppose the insurer has a utility function given by:

$$u(x) = -\exp\ (-.00000025x)$$

Then,

$$-\exp\ \left(-.00000025\ \left(a-\frac{G'}{100}\right)\right) = .90\ u(a)+$$

$$\int_{0}^{4,750,000} \exp(-.00000025(a-x))\ (.10)\ (.000001729\ \exp(-.000001729))dx$$

Dividing through by u(a) gives

$$\exp\ \left(.00000025\ \frac{G'}{100}\right) = .90 + .10\ (.000001729) \int_{0}^{4,750,000} \exp((.00000025 - .000001729)x)dx$$

$$= .90 + .10(.000001729)\ (675,000)$$

$$= 1.0167$$

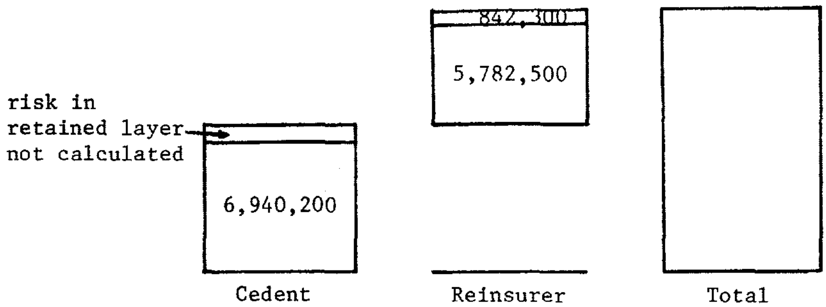
Finally, we have

$$\frac{G'}{100} = \frac{\ln\ (1.0167)}{.00000025} = 66,248$$

or $$G' = \$6,624,800*$$

In this example, the reinsured should be willing to cede \$4,750,000 xs 250,000 for \$6,624,800 - 5,782,500 or \$842,300 more than expected losses. If the reinsurer has the same utility function or is less risk averse, a deal can be struck. The reinsurer might express this as \$16,963,600 (57,825/127,227) = \$7,710,000 less a 14% ceding commission or \$6,630,600. The \$842,300 loading would have to cover all reinsurer operating expenses, including service, and a profit/risk charge.

This retention pricing example probably is not optimal. Other retention levels should be studied. Diagramatically, this first attempt shows loss costs to be:



An approach would be to minimize the sum of total subject to a restriction on the risk proneness of the ceding company and a reasonably risk averse function for the reinsurer.

*Note: The appendix gives the framework of a more complete mathematical/statistical analysis.

"r" VALUES

The question always arises, "How can management specifiy their risk aversion function?" Kalcek and McIntyre (39) begin to explore this. They suggest risk capital can be detrmined as: (a) 1 to 5% of working capital, (b) 1 to 3% of total assets, (c) 3 to 5% of earnings, or (d) .1 to .5% of sales. Suppose we set a value on risk capital of "x".

From our exponential disutility (Du) function, we can take the first derivative. The slope constantly increases. Suppose we set our pain threshold at 10:1. One dollar is worth the risk of 10. Also suppose for a small company "x" must be $1 million. We can then calculate the "r" value.

$$Du(x) = -1/r\ (1-\exp(-rx))$$

$$\frac{d}{dx} Du(x) = -\exp(-rx)$$

$$10 = -\exp\ (-1{,}000{,}000\ r)$$

$$\ln 10/1{,}000{,}000 = r$$

$$.000002 = r$$

We can also use a polling technique. By interviewing management, we can determine risk propensity. Ask what premium they would charge for several loss/no-loss situations, then graph expected payoffs (abscissa) against premium (ordinate). For example, how much would you pay for a lottery ticket with a .001 chance of winning $1 million. Although, the expected value is $1,000, the risk avoider might only pay $500. If the question were asked, however, in the disutility context where it is a .001 chance of losing $1 million, he might say $1,500. By getting premiums for a wide variety of expected payoffs, utility or disutility curves can be constructed.

RISK ASSESSMENT

Once an appropriate excess of loss retention is determined, underwriters and actuaries meet to discuss pricing techniques. Proposed treaty rates must be assessed both analytically and judgmentally. The pricing method previously described is completely analytic once utility is specified. This price indication can be compared with both empirical and exposure methods. Empirically, the company would have a history of observed losses per exposure unit by layer (admittedly after trend and development/IBNR). The gross price for insurance can also be layered by exposure. National Council ELPF's, ISO increased limit factors and property distributions such as published by Salzmann[40] are useful.

If no credible past data exists, we use our collective experience and judgment to rate the account. It is of great benefit to reinsure or quote on many state doctor and hospital companies. Each lacks complete credibility, but our collective experience makes up the difference. When a totally new risk presents itself, such as in 1973 New Jersey No-Fault excess of loss coverage, we price by analogy. This No-Fault should be similar to a combination of first party long-tail Workers' Compensation and Automobile Liability/Medical Payments.

The degree to which we become aggressive in pricing depends on our assessment of the company. Reinsurers, in fact, underwrite underwriters.

> The treaty reinsurer, on the other hand, knows nothing about the individual risk and relies almost entirely on assessment of the primary company's management and judgment of its underwriting program... Almost every insurance company buys treaty reinsurance... (41)

We perform both claim and underwriting reviews to satisfy ourselves that the reinsured is effectively managing his business.

In the claim review process, we seek to get historical losses for our empirical rate making. But the scope extends much further. We develop what I call "caring indicies". The five components are: (a) promptness of claimant contact, (b) promptness and completeness of investigation, (c) adequacy of initial reserves, (d) realistic evaluations, and (e) early reasonable offers. I cannot over emphasize the quality of life issues. Claimants want fair immediate attention.

There are four other general areas where claim reviews seek to "paint a claim picture". A well run home office claims department has suit and claim counts under control. Supervisors act and work professionally. This includes skill in dealing with excess of policy limits exposure. Finally, a general assessment is made of staff, methods, and procedures.

Being in an excess position is dangerous! We often hear the expression "the tail wagging the dog". When you are a reinsurer holding on to that tail, the ride can be both exhilarating and numbing.

We perform underwriting reviews to determine the company's market position and underwriting and pricing techniques. We begin by interviewing executive and underwriting management about their philosophy. Is their market thrust substandard, standard, or superstandard? We review the company underwriting guide. We also speak with line underwriters to be sure the executive philosophy filters all the way down. This can be verified by auditing several files. Well documented files will include loss control reports, MVR's, etc. There will be follow up remarks on recommendations. It is easy to determine whether undesirable classes or individuals within a class slip through the guide lines and are written anyway.

All these impressions we get by underwriting company staff are used in risk assessment. Results from our claim and underwriting reviews affect our judgment on rates directly. Only after actuarial-claims-underwriting have evlauated the company's experience, do we quote a concensus rate.

THE POST MORTEM - EXPERIENCE

Our company monitors experience by client, line of reinsurance, contract and layer of coverage. Quarterly, we print calendar/accident year results before retrocessions (our reinsurance). Calendar year premiums are both on a cedent and reinsurer month of account. Gross premiums earned are reduced by commissions earned, paid and outstanding case basis losses, the loss development and IBNR provision, and overhead expenses. The resulting profit margin is related back to gross premium earned. This is done for each calendar/accident year in the contract, line by line and layer by layer.

Corporate indications of accident year loss development and IBNR factors vary by line of reinsurance and department. Account sensitive development/IBNR is included in the analysis to the extent credible. Credibility is measured using a premium volume-expected frequency concept mixed with a consistency measure in the account's loss development triangle. Account indicated factors move corporate factors off center depending on this composite credibility factor.

As mentioned earlier, heavily worked layers are expected to be slightly profitable, even under short time horizons. Lesser worked layers are expected to generate moderate underwriting profits over a moderate time horizon. Catastrophe layers need not be profitable account by account. Using the insurance principle, combined catastrophe margins should be very profitable. We seek overall balance in this book. Eventual payback by any customer is not expected.

Account experience variability by line of reinsurance is tremendous. We, therefore, typically review account rating in total. Occasionally, as needed, there is a general rate realignment negotiated. In the past this occurred infrequently. Savage competition has forced us to place more emphasis on rate equitibility by line of reinsurance and account.

CONCLUSION

It is not surprising that reinsurance has received little attention until lately in the Proceedings. Until recently, there have been but a handful of actuarial practitioners in the field. Mathematical and statistical tools, such as Risk Theory, Game Theory, and Utility Theory were not studied in the U.S. for application to reinsurance. Utility theory, I believe, is a key to understanding what reinsurance forms make sense when and what retentions are desirable. I hope other actuaries extend ideas expressed here.

Throughout history, reinsurance has operated along traditional lines. Excess of loss reinsurance is most popular today. The burning question is "what retention is appropriate for me?" This essay primarily attempts to seek an analytical solution to an otherwise judgmental decision.

To the extent, it helps you to understand how one reinsurer handles the pricing, underwriting, and managing of large risks this essay is doubly rewarding. My thanks go to William Weimer for extending my example. He eliminated the constraint that reinsurance frequency of loss be constant and I am grateful to him for the mathematics expressed in the appendix.

APPENDIX

We can formalize the mathematical structure of the hospital example stated earlier. Specifically, we can eliminate the assumption of a constant number of excess \$250,000 claims. Our choice of a Poisson frequency distribution will provide an elegant path to follow. In a Collective Risk Theory framework, this will be a derivation using a particular frequency distribution and a particular severity distribution. We hope that after reviewing this example, the reader will gain more insight into the general formulas stated in the mathematical section and be able to apply them with distributions of his or her choice.

Assumptions:

Total losses (each is the excess of \$250,000 portion)

$$X = X_1 + X_2 + \dots + X_N$$

Frequency of Claims distribution: Poisson (h) with h = 10.

$$P(N = n) = \exp(-h)\, h^n/n! \quad n = 0,1,2,\dots$$

Severity of Claims distribution: Exponential with mean = \$5,782,500

$$f(x) = s \exp(-sx) \quad (x>0) \text{ with } s = .000\ 001\ 729$$

Utility function:

$$u(x) = -\exp(-rx) \text{ with } r = .000\ 000\ 25$$

Initial Net Worth = a

With these assumptions, the hospital should be willing to pay an amount G' for a \$250,000 excess of loss cover, where G' satisfies the equation:

$$u(a - G') = E(u(a - X)).$$

The "no memory" property of u(x) leaves us with:

$$\begin{aligned}
\exp(rG') &= E(\exp(rX)) \\
&= E(E(\exp(rX)/N)) \\
&= \sum_n P(N = n)\, E\,(\exp(r(X_1 + X_x + \ldots + X_n))) \\
&= \sum_n P(N = n)\, (s/(s - r))^n \\
&= \sum_n (\exp(-h)h^n/n!)\, (s/(s - r))^n \\
&= \exp\,(-h) \sum_n (hs/(s - r))^n/n!
\end{aligned}$$

Solving for G' gives $G' = (h/r)\,((s/(s - r)) - 1)$

Replacing h, s, and r with their selected values leaves G' = $6,761,325.

We see that by letting the frequency vary, we are introducing more uncertainty into our problem, and the premium G' has gone up from $6,624,800. (Actually, some of the increase, $41,578, is due to the severity distribution no longer being truncated.)

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