PARAMETER UNCERTAINTY IN THE COLLECTIVE RISK MODEL

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INTRODUCTION

As the two major industry rating bureaus currently grapple with the problems of appropriate Table M charges, this paper must be considered particularly timely. The concept of parameter risk as an integral part of the total risk inherent in any insurance contract has long been recognized and discussed by actuaries. Although not always an equal partner with process risk in terms of explaining total loss ratio variance, parameter uncertainty becomes more and more important as the premium size of the insured increases. This paper goes further than any previous effort to explain, analyze and quantify parameter uncertainty as it relates to insurance pricing.

Although this paper certainly stands on its own merits, for a greater appreciation of its value and purpose, it should be read in conjunction with another paper recently co-authored by Mr. Meyers. That paper, "The Calculation of Aggregate Loss Distributions From Claim Severity and Claim Count Distributions" by Heckman and Meyers, was recently submitted to the Casualty Actuarial Society for publication.

THE ACTUARY'S DILEMMA

The authors clearly state the advantages and disadvantages of modelling versus the use of empirical data. Their arguments are familiar to most actuaries. They stress the importance of testing a
model against actual data to ascertain whether it is a fair representation of the real world. The greatest value of all models, including the one discussed, is their ability to predict results in situations where insufficient historical data exists for an empirical approach.

In the particular case of the National Council on Compensation Insurance's (NCCI) efforts to revise Table M, a combination of empirical and modelling approaches was used. For this study a large amount of data was available on a diverse group of small and medium size risks. The disadvantage of combining such heterogeneous risks, however, was mitigated by the fact that a typical retrospectively rated risk is multi-state in nature and usually involves several classification codes. But, to the extent that one is using Table M to rate a small single-state, single-class risks, this disadvantage becomes pronounced. More on these particular problems will follow.

THE COLLECTIVE RISK MODEL

The paper describes both the frequency and severity distributions as the combination of an assumed gamma distribution, representing parameter uncertainty, with another assumed distribution, representing process risk only. Although this separation of total variance into its component parts is fundamental to the paper, it must be realized that the split is, in a sense, arbitrary. The paper later measures parameter variance by considering the difference between total variance and the assumed process variance. The measure of process variance, however, is itself an estimate.
There may be times when the frequency distribution can be estimated directly as a negative binomial, rather than through the two step process of estimating a value for parameter variance and then combining a gamma and poisson distribution. The same comments apply to severity. In some cases, however, depending on what data is available and what premium size is being modelled, the approach described in the paper may be the most expedient. Obviously, the greatest value of separate measurement of parameter risk is its use in estimating the limiting value of loss ratio variance as premium size increases.

ESTIMATING THE PARAMETERS

The paper discusses the estimation of the contagion parameter (c) and the mixing parameter (b) in detail. These estimates are derived from empirical data using one of several techniques, depending on the data available. This approach would seem to measure only that portion of parameter uncertainty associated with sampling error and data heterogeneity. Other sources of parameter uncertainty which are not measured include adjustment for future inflation, changes in the insured population and future development on open claims. Thus, although the proposed approach to modelling does improve the predictability of the model, some sources of uncertainty remain unmeasured.

Turning now to the specific parameters calculated using the NCCI data, a few comments are in order. The paper describes the circumstances under which a negative value for b or c may be calculated and suggests setting the value equal to zero in those instances. Unless, however, there is strong evidence of random error, a negative value for b or c may indicate an underlying problem in some elements of the model and suggests further research is needed. In the case of
the two negative values of $c$ estimated from the NCCI data, the
standard errors of $A$ were among the highest, suggesting random
error was the cause.

The authors discuss the evidence (estimates of $B$) which indicates
that parameter uncertainty declines as premium size increases. Given
the existence of experience rating plans, this result is to be
expected. In two of the three cases studied, however, rather than
decrease, the estimate of $A$, and thus the estimate of $b$, increased
as premium increased. Although admittedly three trials does not
constitute overwhelming evidence, this apparent contradiction suggests
further refinements may be necessary.

When considering the limiting value of the loss ratio variance, the
estimate of $B$, the values seem surprisingly high. Averaging 0.37 for
the large premium size groups, the estimate of $B$ is much higher than
indicated by the last three points on exhibit II b or any previous
estimate.

The authors seem unconcerned about the rejection of the model by the
$X^2$ test. Although the model excess pure premium ratios match very
closely those produced by empirical data (exhibit V), the model con-
sistently underestimates the charge at an entry ratio of 3.00. These
two failures could be serious flaws.

PARAMETER UNCERTAINTY IN LARGE RISKS
After testing the feasibility of estimating parameter uncertainty from
the past experience of a single risk, the authors are forced to
conclude that such an approach appears unrealistic. They demonstrate, however, the importance and impact of parameter uncertainty on excess pure premium ratios. After offering areas for further research, the paper suggests that at this point the model should be used for sensitivity testing.

The authors seem to feel that large risks are unique and consequently should not be combined for analysis of loss ratio variance. In fact, most large workers' compensation risks are quite similar, involving many job classifications, wage levels and geographic locations. In pricing their insurance, moreover, companies encounter similar problems, such as sampling error and the determination of trend and loss development factors. Finally, these risks are in general, subject to the same economic, social and political forces which affect pricing. For these reasons, the combination of large risks of similar premium size for the purpose of estimating b and c may be more appropriate than the paper suggests. This is not meant to suggest that underwriting judgement is unnecessary in large risk pricing, for it certainly plays a major role. If for nothing else, however, underwriters need actuaries to give them a benchmark from which to begin pricing. We owe them at least that much.

To obtain a measure of the limiting value of loss ratio variance the NCCI Subcommittee studied the loss ratios of the states as if each were a single large risk. This approach deserves mention and further study.
CONCLUSION

The primary value of the Collective Risk Model in pricing will be in estimating the loss ratio distribution for small to medium sized insureds with specific characteristics were insufficient historical data exists, e.g. by state, by hazard group. In those instances where sufficient data is available, the model can be fit to the data, but will not provide additional information. Some aspects of parameter uncertainty inherent in the data will be recognized but other sources of uncertainty will remain unmeasured.

The paper currently cannot provide measures of the mixing or contagion parameters for large risks with any degree of accuracy or confidence. The insurer must continue to rely on underwriting judgement and insurance contracts which are tailored to individual risks.

In conclusion, despite the strides made by this paper in analyzing the nature and measurement of parameter uncertainty, a great deal of work remains. With this paper as a point of departure it is hoped that further progress can be made. At this point in its development, however, there is still much uncertainty in measuring parameter uncertainty.

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