OPTIMIZATION OF EXCESS PORTFOLIOS

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Mr. Heckman and Mr. Norton have written a paper which addresses an important topic: balancing risk reduction versus cost in insurance transactions. This should be of interest to people involved in four areas:

(1) the risk manager of a medium to large insurance purchaser,

(2) the person in the primary insurance company in charge of providing insurance to the medium to large insurance purchasers,

(3) the person in charge of ceded reinsurance in a primary insurer or reinsurer,

(4) the person in charge of assuming reinsurance from primary insurers or reinsurers.

My review is divided into two parts. First, I will describe briefly the approach used by the authors. I will conclude with a discussion of practical applications of this approach.

Description

The authors use an example of a risk manager buying specific excess and aggregate excess over a self-retention on a workers' compensation portfolio. The result of the authors' approach is a curve of the efficient frontier of retained risk vs cost of coverage (Exhibit II). In economic terms, this is the insurer's supply curve. The commodity being purchased is reduction in the buyer's risk.

The approach is to first apply the collective risk model (CRM) to the buyer's insurance portfolio to determine the portfolio's mean and variance. The frequency is assumed to be a negative binomial distribution with the expected (mean) frequency input. The severity is estimated from a table of probabilities for any claim exceeding a certain amount. This table is input and can be based on observations or judgment. The inputs for the authors' example are shown in Exhibit I. This model is necessary to calculate the mean and variance of the losses retained by the buyer and the mean and variance of the losses assumed by the insurer. The approach here is very flexible and realistic, although the input values may be difficult to estimate.

The next step is to develop the insurer's pricing procedure. This is done in the excess pricing model where the insurer's price is the sum of a flat charge for expenses plus a factor times the expected value of the ceded losses plus a risk charge. This risk

charge is proportional to the standard deviation of the insurer's losses from this risk. The price for the specific excess is added to the price for the aggregate excess to get the total price.

The authors assume that the specific excess and aggregate excess are independent. This is correct if they are written by different insurers, which is unlikely. The two coverages are positively correlated, so the approach here is somewhat inaccurate when both coverages are written by the same insurer. This is probably not a serious problem in practice.

The final step is to apply a constrained optimization to the selection of the retentions and limits of the specific excess and aggregate excess. This equalizes the ratio of marginal risk reduction of the buyer to cost for both specific excess and aggregate excess. This gives an infinite number of answers which are graphed on Exhibit II. In practice, a reasonable number of points are calculated and plotted and the graph connects the points. Exhibit III shows the eight solutions the authors calculated.

The result of these procedures is the insurer's supply curve. The insurer is indifferent to which point on this supply curve the buyer chooses. This curve gives the point with the most risk reduction for the buyer for any given cost available from the insurer. The buyer thus has a great deal of knowledge and must

only decide how much he is willing to pay for risk reduction and hence which point on the curve to choose.

In practice buyers will have a maximum risk they can keep, regardless of cost, to reasonably assure their continued solvency. I would call this the solvency constraint and this would show in Exhibit II as a horizontal line. Also, buyers will have a maximum cost they can afford before eliminating the exposure becomes more desirable. This I would call the cost constraint which would show in Exhibit II as a vertical line. Thus, only the part of the curve between the solvency constraint and the cost constraint is available to the buyer.

I feel that for most buyers, cost net of expected ceded losses should be used instead of gross cost, although some consideration of investment income and cash flow would be desirable.

Applications

The authors give an example of a risk manager purchasing insurance over a self-insured retention. The same approach could be used for a retrospective rating plan where the maximum is equivalent to the aggregate excess retention, the loss limitation is the specific excess retention, and there are no limits. This could also be used by insurance companies as a tool in serving their clients.

This approach is usable in reinsurance transactions as well. It could be used where there is specific excess and aggregate excess as well as for retrospective rating. It would be difficult, if not impossible, to apply where there is surplus share pro rata and aggregate excess reinsurance. This would be useful though because a combination of pro rata and aggregate excess is the norm in crop-hail insurance and is fairly common in property insurance.

It seems to me that the process could be applied in reverse to give demand curves. The price the buyer is willing to pay would be equal to an expense constant plus the expected ceded losses plus a risk charge. In this case, the risk charge would be proportional to the risk reduction to the buyer. This would relate to the ratio of the standard deviation of the whole portfolio to the standard deviation of the retained business. The result would be a demand curve where the buyer was indifferent between any two points on the curve. The intersection between the demand and supply curves would give the retentions and limits at which the contract should be written.

There are three major difficulties to be overcome by the actuary in applying this approach in practice. The first is the difficulty of getting the mathematics on his computer and using these computer programs. Mr. Heckman should be able to help the interested actuary with this problem, since he has computer programs which do the calculations.

The second difficulty is to determine the input for the collective risk model. Sometimes reasonable data is not available at all. Often even good data are not credible for determining the severity. More work in overcoming this difficulty is necessary.

The third difficulty is to determine the formula for the excess pricing model. It is difficult to determine how much risk an insurer will assume for a given price. An analysis of several rates determined by the insurer's underwriters might be adequate. More work is necessary in overcoming this difficulty.

Conclusion

Mr. Heckman and Mr. Norton have written a fine paper that clearly lays out an approach to solving the problem of selection of an insurance or reinsurance portfolio that most reduces risk for the least cost. The approach is very reasonable and flexible and should have practical uses. I hope to see more development in the future in applying this approach to practical problems.