TITLE: A CAPACITY MANAGEMENT MODEL BASED ON UTILITY THEORY

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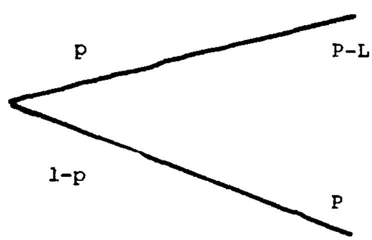
The purpose of this paper is to examine the process by which underwriting decisions are generally made in insurance companies today and to propose an alternative methodology for making such decisions that will encompass concepts of capacity, survival, and stability.

In insurance companies, a decision to underwrite a given risk is frequently made within a given line of business. Little or no attention is given to other lines in a multiple line insurer. In addition, it tends to be the case that the limits for what will be insured are set by top management and could be considered to fall in one of three categories: insuring only the best risks; insuring "the cream of the crap" and best risks; and insuring less desirable risks for a price.

James Stone in his articles on capacity and catastrophe risks develops a structure in which one can look at underwriting and capacity problems in the context of a portfolio which extends across lines of business. He turns the focus of capacity away from issues of capitalization towards those of operational stability. If capacity is viewed as a positively priced commodity, Stone maintains that a portfolio of risks can be formed that may not individually satisfy constraints on survival, stability, and profit, but as an aggregate will. Working under these assumptions one can see that it certainly would be possible to have a more efficient allocation of risks particularly if personal lines insurers are willing to enter the catastrophe risk market. This in turn does show that selfinsurance aggravates some capacity coverages and while it remains justifiable for many, there should in some cases be a trend away from it.

Many aspects of Stone's argument are appealing. Rather than viewing capacity as a positively priced commodity, however, one may prefer to look to the generally accepted view of capacity as the amount of capital which an underwriter can commit to insure a portfolio of risks. It is certainly a much simpler concept, and is intuitively more accurate. Using this as a starting point, one can now apply concepts of exponential utility theory and risk analysis to elements of Stone's model that will create a new, and hopefully more powerful, model from which one can make underwriting decisions.

This method of risk analysis can be used with any probability distribution. For simplicity's sake, we will look at a case of insurance in which there is either no loss or total loss and the amount of loss is fixed. Thus one has a lottery as follows:




P = premium

L = loss

p = probability of loss

In terms of exponential utility, one can compute a risk adjusted value for the lottery:

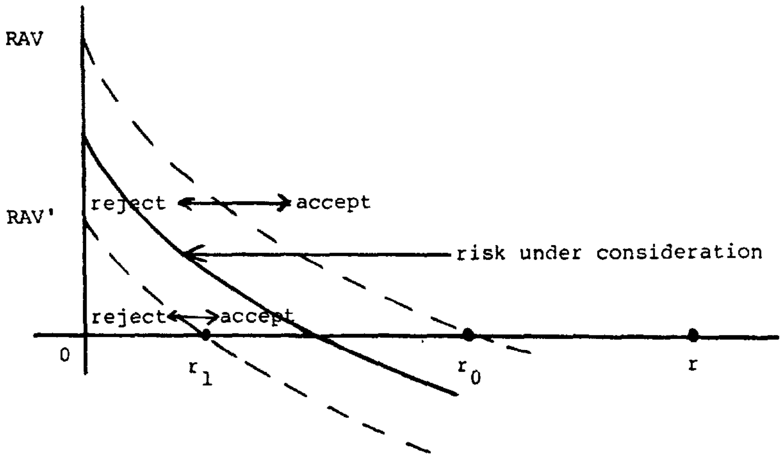
$$RAV = P - \frac{1}{r} \ln \left\{ p\, e^{rL} + (1-p) \right\}$$

where:

RAV = risk adjusted value

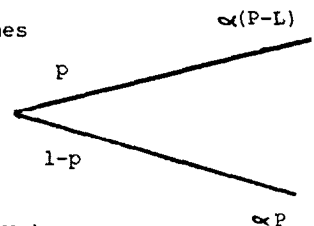
r = measure of risk aversion

One of the first dilemmas encountered is determination of an insurance company's r-value. At this point, one can look to the exponential utility theory literature on capital budgeting in which an acceptable r-value is considered to be the inverse of the budget - the maximum amount that can be spent. In insurance, there is similarly an amount of losses and expenses in the portfolio which one is willing to have a probability of less than p (extremely low) of exceeding in a given period. This amount is often considered to be the sum of capital funds and new premium income. It is this amount that is the capacity of the company. Using the inverse of the company's capacity as a measure of risk aversion makes sense. When a company is new, and capital and premium income small, it will be highly risk averse. As the company and its capital grow, it will be willing to consider riskier projects and will have a correspondingly lower risk aversion level.




Looking at the graph one can see that as r becomes smaller, risks that were previously unacceptable will be underwritten. In addition, although we do not consider price to be a decision variable, it will become possible to price more competitively (a lower premium implies a lower RAV, all other things equal).

There are large risks, however, which cannot be safely underwritten by any one company, no matter how large. In this case, one should consider the possibility of sharing the risk (through co-insurance) rather than retaining it. The lottery then becomes




$\alpha$ = share of risk

and the RAV is

$$RAV - P\alpha - \frac{1}{r}\ \ln\left\{pe^{rL\alpha} + (1-p)\right\}$$

To find the best $\alpha$, take the derivative with respect to $\alpha$ and set it equal to 0.

$$\frac{d\ RAV}{d\,\alpha} = P + \left(-\frac{1}{r}\right) \frac{rLpe^{rL\alpha}}{pe^{rL\alpha} + (1-p)} = 0$$

$$P \left\{ pe^{rL\alpha} + (1-p) \right\} = Lpe^{rL\alpha}$$

$$(P-L) \quad pe^{rL\alpha} = -(1-p)P$$

$$(L-P)pe^{rL\alpha} = (1-p)P$$

$$\alpha^* = \frac{1}{r} \quad \frac{1}{L} \quad \ln \quad \frac{P(1-p)}{p(L-P)}$$

Note that the best share, $\alpha^*$, increases as the premium, P, increases making a more profitable risk more desirable; decreases as the probability of loss, p, increases making a larger loss less desirable. Note also that the function is monotonic in all three parameters. Thus this formula for best share appears to satisfy intuitive ideas on how it should function.

An insurance company is also concerned with potential profitability of the projects it underwrites. Thus it must decide what measure to use. Clearly one wants to measure the expected profit (here expected value = P-pL) in relation to the resource that is constrained. Capacity is constrained here, and it is the loss amount, L, which is using up that capacity. Therefore, an appropriate measure of profitability is $\frac{\text{Expected Value}}{\text{Loss}} = \frac{P-pL}{L}$.

Now, the best share formula should be modified to include the constraint on loss amount. If one uses Lagrangian multipliers, the following computations are required.

$\alpha L \leq B \Rightarrow \left[-\lambda(\alpha L-B)\right]$ is added to the objective function.

$$\text{Max } \alpha P-\frac{1}{r} \ln \left\{pe^{r\alpha L}+(1-p)\right\} -\lambda\alpha L+\underbrace{\lambda B}_{\text{constant}}$$

$$\text{Max}\left\{\alpha(P-\lambda L)-\frac{1}{r} \ln \left\{pe^{r\alpha L}+(1-p)\right\}\right\}$$

$$\frac{d}{d\alpha}=(P-\lambda L)-\frac{1}{r}\frac{prLe^{r\alpha L}}{pe^{r\alpha L}+(1-p)}=0$$

$$\left[pe^{r\alpha L}+(1-p)\right](P-\lambda L)=pLe^{r\alpha L}$$

$$-\left[p(P-\lambda L)-pL\right]e^{r\alpha L}=(1-p)(P-\lambda L)$$

$$p\left[L(1+\lambda)-P\right]e^{r\alpha L}=(1-p)(P-\lambda L)$$

$$e^{r\alpha L}=\frac{(1-p)(P-\lambda L)}{P\left[L(1+\lambda)-P\right]}$$

$$\alpha^{*}=\frac{1}{r}\frac{1}{L}\ln\left\{\frac{(1-p)(P-\lambda L)}{P\left[L(1+\lambda)-P\right]}\right\}$$

Note that the $\lambda$ that will force $\alpha^*$ to zero can be derived as follows:

$$\frac{(1-p)\ (P-\lambda L)}{P\ \left[L(1+\lambda)\ -\ P\right]} = 1$$

$$(1-p)\ (P-\lambda L) = P\ \left[L(1+\lambda)\ -\ P\right]$$

$$(1-p)P\ -\ (1-p)\ \lambda L = pL + p\lambda L - pP$$

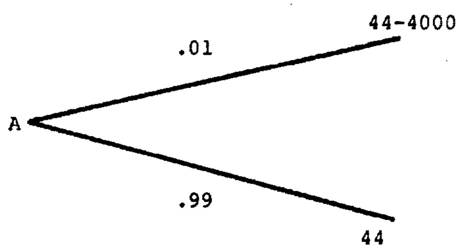
$$pL\ \lambda = (1-p)\ \lambda L = (1-p)P - pL + pP$$

$$L\ \lambda = -pL + P$$

$$\lambda = \frac{P-pL}{L}$$

which equals our measure of profitability.

Let us look at two examples to illustrate the above proofs.




$p = .01$

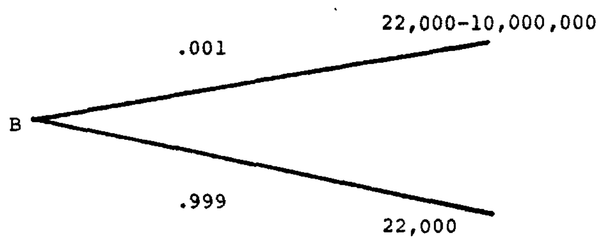
$P = 44$

$L = 4000$

$EV = 4$

$\frac{P-pL}{L} = .001$

This could represent insurance on a truck.




$p = .001$

$P = 22,000$

$L = 10,000,000$

$EV = 12,000$

$\frac{P-pL}{L} = .0012$

This could represent insurance on a bridge.

Now calculate best share: $r = .125 \times 10^{-6}$

$$\alpha^* = \frac{1}{r}\,\frac{1}{L}\,\ln\,\frac{(1-p)P}{P(L-P)}$$

$$\alpha^*(A) = \frac{1}{.125 \times 10^{-6}} \times \frac{1}{4 \times 10^{3}} \quad \ln\left\{\frac{.99(44)}{.01\ (3956)}\right\} = 192.64^{1}$$

$$\alpha^*(B) = \frac{1}{.125 \times 10^{-6}} \times \frac{1}{10 \times 10^{6}} \quad \ln\left\{\frac{.999\ (22,000)}{.001(9,978,000)}\right\} = .63$$

This means that one would certainly insure all of as many risks of type A as possible. It would only be desirable to insure 63% of any type B risk that arose, and if there are two independent risks, take 63% of each.

[1]Any number greater than 1 implies take as many of that risk as are available.

Next look at the case where there is a minimum acceptable return on losses:

$$\alpha^*(A) = \frac{1}{.125 \times 10^{-6}} \times \frac{1}{4 \times 10^{3}} \ln \left\{ \frac{.99\ (44-\lambda(4000))}{.01(4000(1+\lambda) - 44)} \right\}$$

$$\alpha^*(B) = \frac{1}{.125 \times 10^{-6}} \times \frac{1}{10 \times 10^{6}} \ln \left\{ \frac{.999\ (22{,}000 - \lambda(10{,}000{,}000))}{.001(10{,}000{,}000(1+\lambda)-22{,}000)} \right\}$$

| $\lambda$ | %A | %B |
|---|---|---|
| .0005 | 9859.00 | 42.50 |
| .00099 | 201.90 | 15.27 |
| .000999 | 20.20 | 14.67 |
| .0009999 | 2.02 | 14.61 |
| .001 | 0.00 | 14.60 |
| .0012 | 0.00 | 0.00 |

Note, however, that as $\lambda$ increases, the truck insurance falls out as an insurable risk before the bridge even though the truck was initially more desirable. Thus a decision can be made between two alternatives as far as which to insure.

A model has been built that includes not just Stone's constraints, but provides the capability to do much more. The survival and stability criteria he mentions are included in the determination of a risk aversion level and corresponding risk adjusted value. The profit objective is met in two ways: in computing risk adjusted value (and possibly adjusting premium upward), and in adding lambda. The concept of capacity has been simplified. In as much as Stone recognizes the potential for the sharing of risks, a process for determining best share has been delineated eliminating guesswork on what share to take.

This has been a first attempt at looking at underwriting decisions in a utility theory context. While the simplified approach has yielded a wealth of material, it will certainly be desirable to explore the topic even further to consider more realistic losses and investment income. This first step, however, has been a big one in helping change the methodology used in underwriting insurance risks.

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