ADJUSTING SIZE OF LOSS DISTRIBUTIONS FOR TREND

by Sheldon Rosenberg

and Aaron Halpert

Review by Charles F. Cook

The stated aim of this lucid and straightforward paper is to forecast the <u>shape</u> (rather than the mean) of a size- of- loss distribution in some future time period, and this aim is accomplished in a clear presentation which I can still read even ten years away from any hard mathematics. I can see the practical uses more clearly if I think of the subject as stratification of Trend by layer of coverage, which immediately brings to mind applications to Excess- of- Loss reinsurance pricing, Increased Limits Tables, and Loss Elimination Ratios. That image is a bit unfair to the authors, however, because trend by layer is but one specific and deductive result from their more general development. Applications to such arcane areas as Risk Theory, which might flow from projected future changes in the variance, skewness, et cetera, are beyond my review, but I suspect such applications exist.

The authors begin from first principles and develop the concept quite directly through to an example of an application. The mathematics is handled well, using several appendices and guiding the reader through the development in detail. I suggest that the reader not look at a formula or two and back off. It is easier than it first appears. The idea is not new-- we have all believed that trend varies by size of claim-- but the rigourous development is satisfying. In Section 2 of the paper, the authors provide two clever proofs that (at least for the sub-lines and time periods they tested) trend in claims cost is in fact higher for larger claims.

In Section 3, they consider various models and propose a specific two- constant model:

$$\text{trend } (x_t) = a\ x_t^{\,b}$$

which depends explicitly and directly on the size of claim x_t being projected. This model is shown to represent a reasonable improvement over the traditional constant model:

$$\text{trend } (x_t) = a \text{ (for all } x_t)$$

Its weaknesses as well as strengths are evaluated, and its fit is tested with some real data. The fit is good for large claim layers, but unfortunately as $x_t \longrightarrow 0$, trend $(x_t) \longrightarrow 0$. The authors suggest (and later use) a fixed lower bound (a minimum trend) rather than a modification of the function such as:

$$\text{trend } (x_t) = a\ (x_t + c)^b$$

which would generate a minimum value internally. This alternative may merit further evaluation than they gave it.

Very conveniently, the proposed model can directly yield the parameter of the claim distribution at time t + k from those measured at time t , provided that the distribution of claims by size is Lognormal, Weibull, or Transformed Pareto. The projected distributions will be of the same type with modified shape.

Finally they give illustrations using the constant model, the proposed model, and the hybrid model (with constant lower bound), to show their effects on Increased Limits pricing as a specific application. The results are as intuitively satisfying as they are mathematically sound.

CAS papers are often either good theory or good practice. Rosenberg and Halpert have managed to produce a paper which is both. As mathematically oriented papers go, it is extremely clear and practical. The concepts are adequately developed to permit serious immediate application, and I most heartily recommend such applications. A substantial data base and adequate computing power are required, but given that the authors have been working with the ISO Increased Limits Subcommittee, I expect such support to be made available for at least this application. Further developments in Reinsurance pricing and Retro rating are to be hoped for, and I am quite confident that such application will reap the same reward - demonstrably better results by sound mathematics rather than intuitive judgment.

It is a good paper, opening up a new area of Actuarial application where it is needed, promising good use of data bases and computing power which have been so painfully put in place.