TITLE: SELECTION OF THE OPTIMUM ASSET PORTFOLIO TO SATISFY CASH NEEDS

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Introduction

One goal of an insurance company in the management of assets is to have enough cash on hand (or invested assets that can be sold for cash) to pay claims and expenses when they are due. The recent economic environment of inflation and the attendant high level of interest rates has had two adverse effects on insurance companies' ability to attain this goal. One adverse impact is the increase in cash needs to pay claims and expenses. This is due to the general increase in costs. The second adverse impact of this environment is a reduction in the market value of bonds, which typically comprise a large portion of an insurance company's invested assets. The reduction in market value of bonds has been caused, in large part, by the inflation-induced increase in the market rate of interest to record levels. There are two key reasons why the inflationary environment causes an increase in the market rate of interest. First, in an inflation, lenders will only loan money at a higher rate of interest in order to make up for the loss in purchasing power between the time money is loaned out in the present and repaid in the future.[1] A second cause of the increase in interest rates during an inflation is the Federal Government policy to control the inflation. This policy often

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Footnotes appear at the end of the paper.

includes restrictions on credit and on the money supply. These restrictions are intended to raise the market rate of interest to discourage consumption and investment spending. However, the effect of the inflation-induced increase in interest rates on investment portfolios is to reduce the market value of bonds by increasing the discount the market applies to the future income stream of the bonds. The potential result of the simultaneous increased need for cash and the reduction in the market value of invested assets, which are brought about by inflation, is that an insurance company may realize a capital loss if some of the bonds in its investment portfolio have to be sold at reduced market value in order to pay claims and expenses.

The magnitude of the effect of the reduction in bond values on property/casualty insurance company portfolios at any point in time can be seen by comparing the amortized value of bonds (at which they are usually carried in the statutory annual statement) to the market value of the bonds. This writer made this comparison for a group of eight major insurers at 12/79 based on figures supplied in the insurers' annual statements to shareholders. At 12/79, for this group of insurers, the market value of bonds had declined to 88% of the amortized value. To put it more dramatically, this decline in market value caused the market value of policy-hoders' surplus to decline to 70% of statutory value at 12/79. According to an article in the financial press, this decline in market value of statutory surplus has continued into 1980.[2] The reason that the decline in market value of bonds has such a dramatic impact on the market value of surplus is that bonds comprise a large portion of property/casualty insurance companies' assets. A published study of the composition of the asset portfolios of the leading 100 property/casualty insurers at 12/79 shows that, overall, bonds are 64% of total admitted assets and 270% of statutory policyholders' surplus.[3] Therefore, a 10% decline in the market value of bonds translates into approximately a 27% decline in the market value of the combined policy-holders' surplus for these companies.

At the same time that this reduction in the market value of assets occurred, the inflationary environment may have also made it necessary to meet unexpectedly large claim and expense payments. Thus, the potential exists for some insurers to have to sell their bonds at depressed market prices in order to meet their cash obligations. This situation has been noted by industry organizations as well as regulators. For example, in September, 1980 an official of the American

Institute for Property and Liability Underwriters indicated that if insurers had to sell a significant portion of their bond holdings, they would have realized losses "substantially greater than their surplus positions."[4] In addition, a California regulatory official is quoted as suggesting that if market values were substitued for amortized values in the balance sheet of the statutory annual statement then some California insurance companies would appear insolvent.[5] The official also surmised that on a national level there would be "scores" of apparent insolvencies.[6]

In view of this potential loss, this writer sought to determine what insurers could have done in the past (or could do in the future) so as to minimize the impact of inflation-induced fluctuations in the market value of assets on the insurers' ability to satisfy cash needs for claim and expense payments. It quickly became apparent that a criterion is needed to select a portfolio of investments that would balance the goal of maximum gain from investment activity with the goal of being reasonably certain that cash needs will be met. The balance is needed because these two goals may conflict. They may conflict because investment markets are risk averse. Due to risk aversity, in order to hold a portfolio of investments with greater expected value, the insurance company has to tolerate a greater variation in the market value of the portfolio. On the other hand, the need to be reasonably certain of market value of assets (to meet cash needs) may require a portfolio that has relatively little variation in market value, and therefore a relatively lower expected value.

It is the purpose of this paper to provide a general criterion for selecting an investment portfolio that will enable the insurer to maximize investment income subject to the constraint of being relatively sure of satisfying cash needs. The criterion depends on the following parameters: the expected value and variation of the market value of invested assets; the expected value and variation of the cash need; and a measure of the potential cost of having less than a selected cash level at some point in time. The parameters can be affected by inflation as well as other economic influences. However, as described above, recent experience has shown that inflation can be a major cause of fluctuations in the market value of an insurance company's invested assets, and a major determinant of the level of cash needs. Therefore, a criterion such as the one presented in this paper is necessary for an insurer to determine an investment stategy in an inflationary environment.

Financial Environment

In order to develop a criterion for selecting an investment portfolio, it is first necessary to describe the financial environment in which the insurance company operates. One key assumption we will make is that the insurer seeks to maximize its net worth. For our purposes, net worth is the difference between the insurer's assets (valued at market value) and the insurer's liabilities. One characteristic of the insurance operation that enables the insurer to increase net worth is that typically the insurer receives dollars as premiums in advance of paying dollars to insureds as losses. This enables the insurer to increase net worth by creating a "cash flow" situation in which the insurer can invest the premium dollars before they are paid out as losses. We will make further simplifying assumptions about the timing of the receipt of premium dollars and the payment of loss dollars below.

First, however, consider the types of assets the insurer owns. For simplification, we classify the assets into two general categories. We assume that all the insurer's assets fall into one of these categories. The first category of assets is invested assets, and the second category of assets is fixed assets. Invested assets are items such as stocks and bonds, which are expected to earn dividends, interest, or capital gains. It is expected that invested assets will either mature or be sold for cash at some point. Invested assets are those that are used to pay claims and expenses of the insurer. Therefore, it is the selection of the invested asset portfolio with which this paper is concerned. The second set of assets consists of items such as EDP equipment and home office real estate. We will call these fixed assets. Fixed assets do not earn income in the form of dividends, interest, or capital gains. Rather they are used to write policies, process claims, maintain data files, and otherwise service the underwriting, investment, and other activities of the insurer. The fixed assets determine the scale of the insurance operation. For example, a certain size computer can handle a maximum number of policy transactions. Or, a given size office building can hold only a certain number of employees. In order to simplify the model presented in this paper, we assume that over the time horizon for which invested asset portfolio decisions are to be made, the level of fixed assets does not change. This is realistic because decisions on large expenditures such as EDP equipment and on the scale of operations of the insurer are generally made less frequently than the decision on how to invest the flow of cash the insurer receives. Also, once a certain scale of operations has been decided upon, it may be very costly to make changes to that decision. For example, selling the home office building and moving to different size quarters involves quite a bit of disruption and expense. Over a longer time horizon, the scale of operations could be changed. However, we assume that this does not happen over the short run time horizon for which the portfolio of invested assets is selected.

For purposes of presentation, we will now make simplifying assumptions as to the cash flow process. It is the purpose of these assumptions to establish a framework in which we can easily show the relationship between cash needs and the investment portfolio selected. The key point is that the principles developed would apply to a more complex model of insurance company operations. The simplifying assumptions are:

1. At point in time 0, the insurer writes a number of policies and receives a certain number of dollars as a result of having written the policies.

2. Prior to time 0, the insurer has no cash or invested assets. The insurer does, however, have a given level of fixed assets.

3. Upon receipt of premiums at time 0, the premiums are used to purchase a portfolio of invested assets. There is no transaction cost such as stockbroker commissions incurred in the purchase of the invested assets.

4. At point 0, it is known that all losses will be paid at

a later point in time, point 1, but the amount to be paid is not known.

5. At point 1 the policies expire, and at that time the insurer converts its invested assets into cash (again with no transaction cost) in order to pay losses. Any dividends or interest payments to the asset holder are received at point 1 just before the asset is sold.

6. All losses incurred by the insurer under policies written at point 0 are paid at point 1. There are no reserves.

7. The company has no other obligations than losses on policies it writes. Therefore, the only obligations the insurer has at point 1 are losses on policies written at point 0.

8. The insurer writes no other policies between points 0 and 1.

9. The insurer receives no cash between points 0 and 1.

10. The insurer makes no payments until point 1.

In short, these 10 assumptions mean that the process is one of converting cash at point 0 into invested assets whose future value (at point 1) is uncertain. At point 1, these invested assets will be sold to convert them back to cash to meet cash needs. The level of cash need at point 1 is also unknown at point 0. Net worth will increase if the cash value of the invested assets at point 1 is greater than the cash need. And net worth will decrease if the cash need is greater than the cash value of invested assets. The constraint on the choice of invested asset portfolio at point 0 is, therefore, that the insurer wants to be reasonably sure of meeting cash needs at point 1.

There is uncertainty at point 0 as to whether cash needs will be met at point 1 because of two reasons. First, there is uncertainty as to what the level of cash needs will be. This stems from the fact that the insurer cannot know at point 0 what the liability under the insurance contract will be at point 1. We do assume that the insurer can estimate the statistical distribution of the cash need at point 1. This may be based on subjective judgment or on objective statistical analysis or both. The second reason for uncertainty as to whether cash needs will be met at point 1 is that there is uncertainty at point 0 as to the cash value of assets at point 1. This is due to the uncertainty as to financial conditions at point 1. We assume that, as for the cash need, the insurer can estimate the statistical distribution of the cash value of invested assets at point 1.

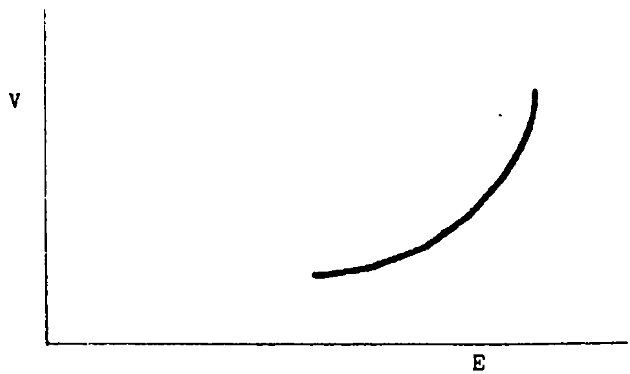
Portfolios Available

We have described the financial environment in which the insurer operates. In order to further develop the criterion for selection of an invested asset portfolio, we now discuss the choices available to the insurer. The various portfolios to be selected at point 0 can be characterized by the expected rate of return and the variation around the rate of return. As used in this paper, the rate of return is the change in the cash value of invested assets between points 0 and 1 as a ratio to the value of the invested assets when purchased at point 0.[7] For example, a common stock purchased for $2 at point 0 that provides a dividend of $0.05 and is sold for $2.50 at point 1 has a rate of return of 27.5% (= 100 X (2.50+.05-2.00/2.00)).

A portfolio of invested assets is made up of individual invested assets. In the market place there are many types of assets the insurer can invest in. At the extremely safe end of the spectrum, the insurer can purchase Treasury Bills at point 0 that mature at point 1. This is the most riskless investment since the government can easily pay its obligations by printing more money, if need be. At the risky end are speculative investments such as common stocks in gold mining ventures in foreign countries. In general, it is seen in the market place that invested assets whose return is more uncertain have a greater expected return. That is to say, in general, investors have to be offered a higher expected rate of return in order to induce them to accept a greater variability in the return. Actually, it is not individual assets, rather portfolios of assets, that exhibit this tradeoff between expected return and variability of return. For individual assets, the return is related to that portion of the variability in return which cannot be eliminated by combination of the asset with other assets in portfolio.[8] The phenomenon of the investor having to be offered a higher expected return in order to hold a portfolio with greater variation in return is referred to as risk aversion. Risk aversion can be explained by the utility analysis of choices involving risk. Very briefly, risk aversion is explained by what is called the diminishing marginal utility of wealth.[9]

For the given number of dollars that the insurer has received at point 0, the insurer can purchase portfolios of assets that will exhibit different rates of return and variations in rates of return. The portfolio rate of return and variation in rate of return depend on the number of dollars invested in each individual asset; the expected return of each asset; the variation in return for each asset; and the covariation among the returns of the different assets.[10] As indicated above, we expect that the investor will consider portfolios with greater variation in return only if they provide a greater expected return. Markowitz refers to the set of portfolios where the rate of return is maximized for any given variation in return as "efficient" portfolios.[11] For a given number of dollars to invest and the various individual assets that can be included in a portfolio, the efficient portfolios can be enumerated using the technique of quadratic programming, as described by Markowitz.[12]

It is not our purpose here to discuss the techniques used to enumerate the efficient portfolios. Suffice it to say that once the efficient portfolios are determined, they will show that an increasing return is accompanied by an increasing variation in return. This is shown graphically as follows:[13]



In the above graph, V is the statistical variance of the return, and E is the statistical expected value of the return. The question is, which efficient portfolio (i.e. which combination of expected return and variation) do we choose. We proceed to the answer.

The Criterion

To reiterate the development so far, the insurer wishes to maximize net worth by taking advantage of the cash flow situation through investment activity. When the insurer goes to the market place to select a portfolio of investments, the insurer will select that portfolio that maximizes expected return for any degree of risk (i.e. the insurer will select an efficient portfolio). Due to risk aversity in the market place, the insurer will find that in order to increase the expected return on the portfolio the insurer purchases, the insurer will have to tolerate a greater variation in the return. Which portfolio (i.e. which combination of expected return and variation around the expected return) should the insurer select? We will present a selection criterion based on a balance between potential financial gains and losses to net worth of each portfolio. Further in the paper, we will discuss modifications to the criterion based on the insurer's attitude toward risk.

Consider the gains and losses to net worth in the cash flow process. There will be a gain to net worth if the cash value of assets at point 1 is greater than the cash need. There is a loss to net worth if cash value of assets is less than cash need at point 1. The loss is composed of two elements. First, net worth declines by the amount that cash need exceeds cash value of assets. Second, net worth declines by the cost of raising cash to make up the difference between the cash need and the cash value of invested assets. One possible way to raise cash to make up the difference is a bank loan using fixed assets as collateral. There are other ways, and these are discussed below. For now, consider the implications of a bank loan. There may be sound reasons why a bank would be willing to loan the insurer money to make up the difference between cash need and cash value. The insurer may have a good organization, an established distribution system, a set of loyal customers, and a name that is recognized by the public. Therefore, the future prospects of the insurer may be bright enough so that in the future the business will once again be profitable. Now consider the interest charge the insurer will incur. The insurer is using its fixed assets as collateral for this loan. We can assume that for a given amount of collateral, the interest rate charged to the insurer will rise as the amount borrowed increases.[14]

There are other ways in which the insurer could raise cash at point 1. For example, the insurer may be owned by a holding company that would be willing to temporarily supply cash to the insurer. In this case, the cost of supplying the cash is the use to which the holding company would have put the cash if it did not give the cash to the insurer. This alternative use is referred to as the "opportunity" cost.[15] Another potential way for the insurer to raise cash is to sell its fixed assets and then rent them from the buyer. In this case, the cost is the extra expense of renting the fixed assets rather than owning them. The point is that whatever way the insurer can raise cash at point 1 to make up the difference between the cash need and the cash value of invested assets, we assume that the cost behaves as the cost on a bank loan. That is, the interest rate increases as the amount of cash to be borrowed increases.

Now, how does the portfolio selected influence the potential gains and losses to net worth described above? The potential for gains and losses to net worth depends on the expected return and the variation around the expected return of the portfolio. In general, the higher the expected return of the portfolio, the greater is the potential for gain to net worth. However, the greater the variation around the expected return, the greater is the potential that the cash value will be less than the cash need, with the resulting loss to net worth.

The following example illustrates these ideas. (Note that the data presented in this example and in the other examples in this section are illustrative of the underlying principles involved, but they do not represent any company's actual situation.)

Assume the following is the financial data for an insurer:

Net Worth (just prior to point 0) = 50, composed entirely of fixed assets

Premiums received at point 0 = 100

Cash need at point 1 = 105

Assume there are two efficient portfolios that the insurer can choose from as follows:

Portfolio	Cash Value at Point 1	Probability
I	115	1
II	100 116 132	.25 .50 .25

The expected cash value at point 1 for portfolio I is 115 (=(115)(1)), and the expected rate of return is 15%.

The expected cash value for portfolio II is 116 (=(.25)(100)+(.50)(116)+(.25)(132)), and the expected rate of return is 16%. Portfolio II offers a higher expected return than portfolio I, but clearly portfolio II exhibits greater variation (which is what we would expect per the discussion of efficient portfolios). The greater variation in portfolio II is not a bad thing in itself. For example, the variation in portfolio II means that there is a possibility of gain to net worth of as much as 27. This would occur if the cash value is 132. Since the cash need is 105, the gain would be 132-105=27. However, the potential disadvantage of portfolio II as compared to portfolio I is that there is a possibility of loss to net worth if the cash value is 100. The loss is 5 (=105-100) plus the interest charge to borrow the 5 to make up the difference.

As a first approach to selecting between these two portfolios, use the criterion that the insurer selects the portfolio that maximizes the expected increase to net worth. After the implications of this criterion are presented, we will discuss later in the paper other criteria for selecting the investment portfolio. In order to use the expected value criterion, we must specify the interest cost of borrowing against fixed assets. As discussed the interest rate will increase as the amount borrowed increases. Continuing with the numerical illustration, assume that the interest cost function is as follows:

$$Z = .03B^2 + .2B$$

where Z is the interest charge in dollars

B is the amount borrowed.

This function has the characteristic that the interest rate increases as the amount borrowed increases.

Now we have all the elements we need to use the criterion of maximizing the expected increase to net worth. The expected net gain to net worth is the expected cash value of invested assets minus the expected cash need minus the expected interest cost of borrowing. First analyze portfolio I. The expected cash value is 115. The expected cash need is 105. There is no possibility of interest cost since there is no possibility that the cash value of invested assets will be below the cash need. Therefore, the expected interest cost is 0. The expected net gain to net worth is therefore 10 (= 115 - 105 - 0). For portfolio II the expected cash value is 116. The expected cash need is 105. The potential interest cost will occur if the cash value is less than the cash need. The cash value will be less than the cash need only if the cash value is 100. This occurs with probability of .25. If this occurs the amount to be borrowed is 5 (= 105 - 100). And the interest cost of borrowing 5 is $(.03)(5^2)+(.2)(5) = 1.75$. Therefore, the expected interest cost of borrowing is .44 (= (1.75)(.25)). Therefore for portfolio II the expected net gain is 10.56 (= 116 - 105 - .44). Since the expected net gain for portfolio II is greater than that of portfolio I, the insurer would select portfolio II.

Expressing the criterion for selecting the portfolio in symbols, let

G = expected net gain to net worth

L = expected cash value of the portfolio at point 1

C = expected cash need at point 1

Z^* = expected interest cost at point 1

Then by definition,

$$G = L - C - Z^* .$$

Having specified L, C, and Z* as we just did for portfolios I and II, above, we could merely choose the portfolio that results in the largest G. In order to further analyze the selection of the optimum portfolio, however, we wish to separate the effects of the expected cash values and expected interest costs as between portfolios. To do this, assume that the portfolios are considered one after another in order of increasing expected rate of return. Then as we hypothetically move from one portfolio to the next, the change in net gain to net worth is the change in the expected cash value minus the change in the expected cash need minus the change in the expected interest cost. In symbols,

$\Delta G = \Delta L - \Delta C - \Delta Z^*$, where Δ means change in a variable as we move from one portfolio to the next. In our example so far, C can only take the value of 105. Therefore, ΔC is 0. In general, we would not expect the cash need to vary with the portfolio we select.

Then we're left with

$\Delta G = \Delta L - \Delta Z^*$.

Then G, the net gain to net worth will increase (i.e. $\Delta G > 0$) if the increase in cash value (ΔL) is greater than the increase in expected interest cost (ΔZ^*). Again, the reason to state the criterion in terms of changes to cash value and interest cost is to explicitly show the effects of increasing both the expected return and variation as we move from one portfolio to another.

We will apply this criterion to an extended example in which there are seven portfolios in the efficient set. In the extended example, the cash need has the following probability distribution:

Cash Need	Probability
100	.50
105	.50

For the extended example we also assume that the level of cash need will not be determined by the portfolio we select. Therefore in moving from one portfolio to the next, $\Delta C = 0$. The seven portfolios are shown in Table 1. The elements ΔL and ΔZ^* are developed in Tables 1 through 4. Table 2 shows the potential amounts to be borrowed for each possible cash need and cash value. Table 3 shows the interest cost of each potential amount borrowed shown in Table 2. Table 3 also shows the expected interest cost, Z*, for each portfolio ("Weighted Interest Cost"). Note that, in order to simplify the example, it is assumed that the level of cash need is statistically independent of the cash value of any portfolio. Finally, Table 4 shows the change in expected cash values and interest costs as alternative portfolios are considered. It is seen from Table 4 that moving from portfolios I to II, etc. that the cash value increases by 1 ($\Delta I=1$). However, the expected interest cost, Z*, increases by increasing amounts. Up to portfolio V, the cash value increases more than the interest cost. Moving from V to VI, or VI to VII, the interest cost increases more than the cash value. Therefore, portfolio V is the one that maximizes expected net gain to net worth, and therefore, is the optimum portfolio.

Table 1

PORTFOLIOS AVAILABLE

AT POINT 0

Portfolio	Cash Value at Point 1	Probability	Expected Cash Value at Point 1
I	115	1	115
II	100	.25	
	116	.50	
	132	.25	116
III	96	.25	
	117	.50	
	138	.25	117
IV	92	.25	
	118	.50	
	144	.25	118
V	88	.25	
	119	.50	
	150	.25	119
VI	84	.25	
	120	.50	
	156	.25	120
VII	80	.25	
	121	.50	
	162	.25	121[1]

Explanatory Notes:

1. 121 = .25(80) + .50(121) + .25(162)

POTENTIAL AMOUNTS BORROWED AT POINT 1

Table 2

Portfolio	Cash Value at Point 1	Probability	Amount Borrowed, Cash Need 100	Amount Borrowed, Cash Need 105
I	115	1	0	0
II	100	.25	0	5
	116	.50	0	0
	132	.25	0	0
III	96	.25	4	9
	117	.50	0	0
	138	.25	0	0
IV	92	.25	8	13
	118	.50	0	0
	144	.25	0	0
V	88	.25	12	17
	119	.50	0	0
	150	.25	0	0
VI	84	.25	16	21
	120	.50	0	0
	156	.25	0	0
VII	80	.25	20	25[1]
	121	.50	0	0
	162	.25	0	0

Explanatory Notes:

1. Cash Need = 105
   Cash Value = 80
   Difference = 25

Table 3

INTEREST COSTS FOR POTENTIAL AMOUNTS BORROWED AT POINT 1

Portfolio	Cash Value at Point 1	Probability	Cash Need 100; Probability of Cash Need .50; Interest Cost (=Z)	Cash Need 105; Probability of Cash Need .50; Interest Cost (=Z)	Weighted Interest Cost (=Z*)
I	115	1	0	0	0
II	100	.25	0	1.75	
	116	.50	0	0	
	132	.25	0	0	0.22
III	96	.25	1.28	4.23	
	117	.50	0	0	
	138	.25	0	0	0.69
IV	92	.25	3.52	7.67	
	118	.50	0	0	
	144	.25	0	0	1.40
V	88	.25	6.72	12.07	
	119	.50	0	0	
	150	.25	0	0	2.35
VI	84	.25	10.88	17.43	
	120	.50	0	0	
	156	.25	0	0	3.54
VII	80	.25	16.00	23.75[1]	
	121	.50	0	0	
	162	.25	0	0	4.97[2]

Explanatory Notes (See following page)

Explanatory Notes to Table 3:

1. Potential amount borrowed = 25 (See Table 2)

Interest Cost Function:

$$Z = (.03)B^2 + (.20)B \quad \text{(See text)}$$

where Z = interest cost
B = amount borrowed

For B = 25, Z=23.75

2. The weighted interest cost for a portfolio is each interest cost weighted by the joint probability of cash need and cash value.

Here, 4.97 = (.50)((.25)(16.00)+(.50)(0)+(.25)(0))+
(.50)((.25)(23.75)+(.50)(0)+(.25)(0))

EXPECTED NET GAIN TO NET WORTH AT POINT 1

Table 4

Portfolio	L	ΔL	C	ΔC	Z*	ΔZ*	Net Gain L-C-Z*
I	115[1]		102.5[2]		0		12.50
II	116	1[3]	102.5	0[4]	0.22	0.22	13.28
III	117	1	102.5	0	0.69[5]	0.47[6]	13.81[7]
IV	118	1	102.5	0	1.40	0.71	14.10
V	119	1	102.5	0	2.35	0.95	14.15
VI	120	1	102.5	0	3.54	1.19	13.96
VII	121	1	102.5	0	4.97	1.43	13.53

Explanatory Notes:

1. L is the Expected Cash Value at Point 1 from Table 1.

2. C is the Expected Cash Need at Point 1. That is, 102.5 = .50(100) + .50(105)

3. ΔL is the difference in Expected Cash Value at Point 1 as we hypothetically consider alternative portfolios in the order of increasing Expected Cash Value at Point 1. Here, 1 = 116 - 115.

4. ΔC is the difference in Expected Cash Need at Point 1 as we hypothetically consider alternative portfolios in the order of increasing Expected Cash Value at Point 1. Since we are assuming cash need is not determined by portfolio selected, ΔC = 0 throughout.

5. Z* is the Weighted Interest Cost from Table 3.

6. ΔZ* is the difference in Z* as we go from one portfolio to another. Here, 0.47 = 0.69 - 0.22

7. 13.81 = 117 - 102.5 - 0.69

Conclusion

The criterion presented above uses a balancing of financial gains and costs to determine the optimum portfolio. The actual portfolio selected will also depend on the attitude toward risk of the person within the insurance organization that makes the decision. For example, assume the insurer selects portfolio V per the criterion presented. According to our extended example, the cash need can be 105 at the same time the cash value of invested assets is 88, if portfolio V is selected. This would mean a loss to net worth of 17 (see Table 2) plus the interest cost of 12.07 (see Table 3) for a total loss to net worth of 29.07. The decision maker may decide that he or she does not wish to expose the insurer to this magnitude of loss under any circumstances. The point is that the selection of the portfolio depends finally on the risk preferences of the decision maker. Nevertheless, the potential financial gains and losses to net worth between portfolios as developed in this paper constitute the data on which the decision will be based.

As a practical matter, in the real, complicated world, selection of an invested asset portfolio would involve the expertise of many different specialists. For example, there would be income tax implications in the selection of any portfolio. As another example, determination of the expected return to the various securities that could be included in a portfolio would require the expertise of financial analysts. There is at least one key area in the selection process that the services of the casualty actuary would be required. This is in the estimation of the level of and the timing of the cash need for claim and related expense payments. It is felt by this writer that this is a potentially fruitful area for casualty actuaries to develop further methodology to contribute to the financial planning process.

FOOTNOTES

1. Michael Quint, "What Inflation Has Done to Bonds," New York Times, November 9, 1980,Section 3, p.8.

2. Ben Weberman, "Here We Go Again?", Forbes, April 28, 1980, p. 34.

3. "Allocation of Assets," Best's Insurance Management Reports, October 6, 1980, Release Number 25.

4. David C. Jones, "Worrisome Bond Picture Painted," The National Underwriter, September 19,1980, p. 1.

5. Ibid., p. 66.

6. Ibid.

7. Franco Modigliani and Gerald Pogue, "An Introduction to Risk and Return," Financial Analysts Journal, March-April, 1974, p. 68.

8. Ibid., p. 74.

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