PROFIT/CONTINGENCY LOADINGS AND SURPLUS: RUIN AND RETURN IMPLICATIONS

A Review by Lee R. Steeneck

The line between failure and success is so fine that we scarcely know when we pass it - so fine that we often are on the line and do not know it.

Ralph Waldo Emerson

We have come a long way in our thinking about surplus requirements and the amount of premium that can be written comfortably - confident of our company's immortality. As a student some years back I vaguely remember reading: if an insurer had a mean composite ratio of 100% with a standard deviation of 10% and if ruin were restricted to a probability of .001 of occurring, then using a normal approximation for the distribution of composite results, the maximum composite ratio which could be withstood would be 131% (100 + 3.1 x 10). Hence, one shouldn't write at more than a 100/31 or approximately 3:1 written premium to surplus ratio.

Are we on the line at this point or are our assumptions too simplistic to evaluate our position? Certainly we casualty actuaries have been devoting insufficient resources to researching our own potential ruin. The author has constructed an excellent paper which suggests that not only do profit/contingency loadings lead to a return, and surplus guards against ruin, but there are other inter-relationships.

We now find that the normal approximation is not necessarily a very good one and that the profit/contingency load in the rates which should bring the expected composite ratio under 100% also should affect solvency. Financial leverage, solvency, and profit/security loads are all interrelated.

Given (a) a portfolio loss distribution rather than a composite ratio distribution, (b) a desired return goal, and (c) a method for accurately determining expected losses for each exposure unit, the author goes on to show what profit/contingency loading should be charged to the entire loss base if the ruin probability is set at some arbitrarily small value. The author then demonstrates how the total loading can be divided among the exposures.

The logic follows the "composite example" cited to a certain extent but builds upon it a great deal. The additions are logical and straightforward. Hence, in the subsequent paragraphs I will attempt to amplify and lend additional precision to this very compact paper. To make the review flôw along with the original paper, I will comment on various items in the order of their appearance.

Purposes of Profit/Contingency

Expanding on the theme that profit/contingency loadings in the insurance rate serves two purposes, it can be noted that "the fluctuation in loss experience" really includes both reserve inadequacies and possible prospective rate deficiencies. In part they also protect against adverse fluctuations in investment results. And, of course, they do provide for a return which will be partially paid out in dividends but mostly retained to allow for increased capacity.

The Portfolio Loss Distribution

One must be precise in defining a portfolio loss distribution. If we mean to speak of all losses coming from a group of homogeneous risks with similar loss distributions for frequency and severity, then later on in the paper when we allocate loading to contract based on expected losses a simple proportionality assumption will hold. Since it is more likely that the portfolio loss distribution is meant to cover all losses from groups of nonhomogeneous exposures, the per contract loading procedure, later to be described, should not theoretically be divided proportionally among expected losses. Rather the riskiness associated with the particular contracts' loss distribution should be used. We will see that this implies different values for T_{ϵ} and σ_L . All the contracts' separate loadings would then be added together and balanced if necessary to the total.

Volume Considerations

It is to be noted that the loading will be divided among expected losses so it will be necessary to make volume predictions when setting rates. Since Insurance Commissioners and others are quite sensitive to premium to surplus ratios the equation $R = (W - L)/S$ is limited practically either in the ratio of W:S or L:S.

Scope/Exclusions from Consideration

The author wishes to highlight the interplay between surplus and the profit/contingency loading so he excludes investment income and overhead considerations. Overhead or expense considerations can readily be overlooked since they contribute little to (from) solvency usually.

Investment income is important, however, since our industry thrives on it. Although it is not included in this paper it must be remembered that it often contributes much more to surplus than gains due to favorable underwriting results. This income would tend to reduce the probability of ruin if it weren't for the fact that yearly more risk is usually assumed, so the portfolio loss distribution increases as well. With an assumption of constant premium to surplus writing annually (investment income going toward increased capacity) I believe it is possible to assume away any investment considerations here. Variations in investment results can be minimized by investing in fixed income instruments.

It should also probably have been stated that federal income tax considerations are also excluded. They will affect your desired rate of return since underwriting profits are typically taxed at a 46% rate. Federal income taxes will also affect your ruin point set at T_α standard deviations above expected losses since there is a possibility of securing loss carrybacks. If you had been operating profitably prior to the year in which actual losses far exceeded expected losses, then you could recover part or all of your taxes paid in the prior several years. And this would aid solvency.

Return on Surplus

Let us now return momentarily to the value of S. Surplus doesn't necessarily mean that figure from the Annual Statement on the Liabilities, Surplus and Other Funds page called Surplus as Regards

Policyholders. Masterson[1] suggests that we can measure return on a net worth basis using policyholders surplus + unauthorized reinsurance liabilities + other surplus type liabilities + nonadmitted assets + the equity in the unearned premium reserve net of any taxes - the tax liability on unrealized capital gains.

Calculation of Ruin Point

The value for T_ϵ is not strictly a standard deviation figure. For a given value of ϵ it depends on more than the first 2 moments of the portfolio loss distribution. In a personal conversation with the author he suggests that a Cornish-Fisher type expansion would be an appropriate means of calculating T_ϵ . This expansion starts with a normal variate and makes adjustments or uses correction terms for skewedness, peakedness, etc. Presumably one could also use an Edgeworth expansion, the normal power expansion, or the Esscher method[2]. The simplest, yet most accurate, of the three is the normal power expansion. If we limit it to two terms, then

$$\frac{x - P}{\sigma\sqrt{n}} = y_\epsilon + \frac{1}{6}\gamma_1(y_\epsilon^2 - 1)$$

where y_ϵ is found in the normal function tables and γ_1 is given by

$$\int_0^\infty z^3\,dL(z) \Big/ \left[\int_0^\infty z\,dL(z)\right]^3 \sqrt{n}$$

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1 NAIC, Measurement of Profitability and Treatment of Investment Income in Property and Liability Insurance. June 1970

2 R. E. Beard, T. Pentikainen, E. Pesonen, Risk Theory, Metheun and Company, London.

Expected Rate of Return, σ_R

If management seeks to have an expected rate of return greater than or equal to a + bσ_R, then to be more precise b should be greater than 0, otherwise as the standard deviation of R increases the expected rate of return decreases. If "a" were set sufficiently low, then "b" would naturally be positive.

The author correctly states that $\sigma_R = \sigma_L/S$ is implied from R = (W - L)/S. This follows if one expands each side given

$$\sigma_x^2 = E(x)^2 - [E(x)]^2$$

Expected Rate of Return, σ_R^2

As a second example of the frontier given a solvency equation and a return equation the author chooses to set the return as a constant plus a percentage of the variance of R. This example was chosen to show that the most competitive price doesn't necessarily place one on the frontier of the solvency equation. Unfortunately, Exhibit 2 doesn't graphically emphasize that S = 3 is the minimum point for the return curve SE(R) = .04S + .36 σ_L^2/S. But the word "minimum" is given. What this is saying is that even with a minimal loading the real probability of ruin is less than the value assumed.

It would be interesting to see what Sharpe would say about the fit if a variance loading form of expected return were used. If the correlation was sufficiently high, then this would lend more importance to the illustration in Exhibit 2.

Loading Allocations

The section on allocation to contracts discusses the time-worn problem of whether 100 individual exposures taken separately should equal a single 100 exposure contract. To the insurance company it probably matters little which 100 are preferable (except for per contract volume and economy of scale considerations). But as far as the insured is concerned and the way he views his risk it is proper, I too believe, to maintain separate higher profit/contingency charges for the smaller insureds.

I would be remiss as an actuary in the reinsurance field if I didn't mention that reinsurance is frequently purchased because of risk considerations. As an insurer determines that because of writing risky business (meaning lines of insurance or exposures where the variance in losses is relatively high) he may find that he needs or would like to reduce risk and the probability of insolvency (or a "heavy hit" on surplus). Risk reduction can be accomplished through the purchase of excess of loss reinsurance if the program is properly established. Wilhelmsen[3] and others have noted equations associated with the Collective Theory of Risk demonstrating risk reduction via reinsurance. The theory builds on a foundation that underwriting results (net of expenses) come from a distribution of results of a group or collective of policies. Equations can be

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3 International Congress of Actuaries. 1954

solved for M, the excess of loss reinsurance retention, or λ the security loading or profit/contingency loading required for prolonged solvency, (where λ is related to the loss and loss expense items, E(L), only). Given a judicious choice for M and an efficient reinsurer it is possible that primary company gross competitive rates will give more solvency protection than otherwise indicated before reinsurance. One would need to relate (W - E(L))/E(L) to λ . The following formulas can be used to determine the security loading on a net of reinsurance loss distribution.

The ruin probability $\epsilon = e^{-Ru}$ where $u = S/\bar{x}$. This unitizes surplus on the basis of an average claim. $\bar{x}$ is given by $\int_0^M zL(z)dz + M\int_M^\infty L(z)dz$. This will set R (not to be confused with the rate of return R used elsewhere in the paper). If we further let $R = K\bar{x}$, then we can solve the equation $\int_0^M e^{Kz}L(z)dz + e^{KM}\int_M^\infty L(z)dz = 1 + (1+\lambda)K\bar{x}$, for λ.

It is personally and professionally gratifying to me to see actuaries mathematically tackling this intricate topic. I commend Mr. Venter for his efforts.

ERRATA

1. On page 4 "This point is to be determined by simultaneously solving the 2 equations . . . for S and SE(R). Doing this yields S = an expression and SE(R) = an expression."
2. In appendix 2 the numerical references to Seal and Hastings/Peacock are reversed. Seal should be (4) etc.
3. The value for $T_\epsilon \sigma_L$ for the portfolio (103,200) does not follow from the separate values of T_ϵ and σ_L . I believe σ_L should be 22,360 rather than 23,360. Then the portfolio loading dollars under the stated (a,b) assumptions works out to be $14,280 not $14,920.
4. In appendix 2 the "total stand alone loading" amounts to $76,640 not $74,640. I arrive at my sum by adding $10,210 to 100 x $664.30.
5. Lastly, in appendix 2 I believe the various σ's should be subscripted with an L.