A MODEL FOR CALCULATING MINIMUM SURPLUS REQUIREMENTS

BY
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Required capital and surplus has been a much debated problem for many years. Subjective factors, such as the caliber of management, ownership relations, underwriting standards, and future profitability of the company, have dominated such discussions. The quantifiable elements of this problem are numerous and extremely complex for the large insurance company. However, with the advent of captive insurance companies and formalized self-insurance programs in recent years, this age-old problem is at issue again for these new insurance mechanisms. In his paper, Mr. Finger develops a mathematical model and a foundation for analysis of this renewed question.

This reviewer has concentrated his discussions of this paper on particular points worthy of note.

CONFIDENCE INTERVALS FOR LOSS RESERVES

Finger has developed a model relating the amount of surplus needed to maintain solvency to the loss reserves and their probability distribution. This idea is similar to the problem of establishing confidence intervals for loss reserves and Finger's approach is much like that outlined by

Khury in his paper, "Loss Reserves : Performance Standards[1]". The recognition of uncertainty in loss reserve estimates and the measurement of this uncertainty in terms of confidence intervals is an important area for actuarial study. However, an insurance company or self-insured should have the financial capacity to meet the eventual cost of all its open claims. Thus, the uncertainty of loss reserves can be used to establish the surplus needed to protect the solvency of the company.

THE RATIO OF PREMIUM TO SURPLUS

Finger has used ruin theory in his formulation of minimum surplus requirements. The probability distribution that he develops to find the probability of ruin, i.e., insolvency, could also be used to describe a confidence interval for the loss reserves.

The determination of surplus adequacy has traditionally depended on the ratio of premium writings to surplus[2]. This ratio has found almost universal acceptability as a measure

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[1] Khury, C. K., "Loss Reserves : Performance Standards," presented at the November 1977 meeting of the Casualty Actuarial Society.

[2] A discussion of the origins of this ratio rule can be found in Kenney, R., Fundamentals of Fire and Casualty Insurance Strength, The Kenney Insurance Studies (1967), p.97-102.

of adequate surplus. The standard ratios that have been used, 2:1 and 3:1, are basically rules of thumb that have been reinforced by many years of experience. Unfortunately, such a rule cannot be appropriate in all cases and thus can impose undue restrictions or insufficient protection in many individual cases. The model described by Finger establishes surplus requirements based on the specifics of each individual case. Consequently, the use of the Finger model would require an actuarial study in each case. Perhaps this could be done where an exception to the premium-to-surplus rule is believed to be appropriate.

LIMITATIONS OF RUIN THEORY MODEL

Of course, no model is perfect in its description of particular circumstances nor in its predictions. All models require certain assumptions to be made and premises to be accepted. For example, ruin theory establishes acceptance or rejection on the level of probability of ruin but ignores the potential magnitude of ruin. It also fails to consider the variability of losses. However, it is still a valuable theory and for establishing surplus requirements, the effect of its limitations should not be significant.

NEED FOR JUDGMENT

It is important to recognize the need for judgment in any attempt to apply this model to a particular situation.

The selection of distributions, parameters, and other quantitative factors cannot always be based on relevant, credible data. Therefore, some applications will require the use of estimates based on only judgments as to what is reasonable given whatever information is available. A sound actuarial model must make explicit those assumptions and judgment estimates which can significantly affect the answer produced by the model.

THE PROBABILITY OF RUIN

A major factor in the application of ruin theory to establish minimum surplus requirements is the selection of the ruin probability. Finger suggests a 1 per cent figure and he defines the minimum surplus requirement as the difference between the 99th percentile of the aggregate reserve distribution and the aggregate reserve established as liabilities. Is 1 per cent appropriate in all cases? Why not 0.1 per cent or 5 per cent, 10 per cent, 20 per cent? How should the ruin probability be set? Shouldn't there be a range for surplus between an acceptable minimum that is reasonable and prudent, and a maximum that represents an upper limit on necessary insolvency protection?

Surplus in excess of the maximum could be treated as surplus surplus[3].

To test the sensitivity of the selection of the ruin probability to the surplus required, the following examples were computed based on the Case II in the paper.

Ruin Probability (%)	Required Surplus (millions)	Premium/Surplus Ratio
1	$8.4	1.00
2.5	7.0	1.20
5	5.8	1.45
10	4.5	1.88
15	3.6	2.34
20	2.9	2.92

PARAMETER RISK

The author addresses the uncertainty in loss reserves resulting from fluctuations about a known mean and from errors in estimating the mean. The first type of uncertainty, known as "process risk"[4], is a statistical concept based on a probability distribution. In this application, the reserves represent a random variable with frequency

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[3]For a discussion of surplus surplus see Kimball, S. L. and Denenberg, H. S., Eds. Insurance, Government and Social Policy, Irwin (1969), p.64-69.

[4]See Freifelder, L. R., A Decision Theoretic Approach to Insurance Ratemaking, Irwin (1976), p.70-71.

and severity components. Several actuarial papers [5] have been written describing or using this concept. However, the second type of uncertainty has been used infrequently, although the works of Bühlmann and Hewitt do use a formulation quite similar to Finger's. This "parameter risk", as its called by Freifelder[6], should encompass not only the uncertainty in the mean of the aggregate reserve distribution, but also the errors in estimating any of the moments or parameters of the distribution, including even the type of distribution selected.

Finger incorporates the parameter risk by multiplying together two log-normal variables. One of these variables represents the conditional aggregate reserve distribution, but the other is unspecified. A manipulation of formulas in Appendix A.2. indicates that the second log-normal variable has a mean of 1.00 and variance given by the selected coefficient of variation (CV) representing the uncertainty in the mean of the first variable. In other words, the estimated mean severity could be off by a factor which

[5] Hewitt, C.C., "Credibility for Severity." PCAS LVII, 1970, p. 148, and Miccolis, R. S., "On the Theory of Increased Limits and Excess of Loss Pricing." PCAS LXIV, 1977.

[6] Freifelder, op. cit., p.70-71.

ranges from zero to infinity, and is log-normally distributed with mean 1.00. Moreover, it can be shown that this multiplicative factor approach is equivalent to Hewitt's [7] formulation of a composite distribution for loss severity. Hewitt takes a log-normal distribution with parameters μ and σ^2, and keeps a σ^2 fixed but distributes μ as a normal random variable with mean N and variance S^2. The composite distribution he derives is log-normal with parameters N and $\sigma^2 + S^2$. This corresponds to Finger's composite log-normal distribution with N equal to the parameter of the conditional aggregate reserve distribution, σ^2 from the same distribution, and $S^2 = \ln(CV^2 + 1)$ where CV represents the uncertainty in the mean value.

Therefore, while the author has made necessary adjustments in his model for the uncertainty in predicting the mean, his formulation is not complete. Other elements of uncertainty are present which do not fit the multiplicative or the Hewitt formulations. These elements should be studied to determine their significance in establishing surplus requirements.

Parameter risk is, in essence, the risk of making a bad assumption. If each assumption can be identified and

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[7] Hewitt, _op. cit._

assigned a probability distribution, then each assignment becomes another assumption. Obviously, the problem is theoretically boundless, but this does not mean that it is uncontrollable. Through the use of sensitivity analysis and judgment, it should be possible to develop a reasonable framework for analysis.

Finger has developed a useful analytical tool for evaluating surplus adequacy. However, to apply this tool, a ruin probability and an uncertainty measure (CV) must be selected. The sensitivity of ruin probability has previously been illustrated. Similar examples are given below for various values of CV. Case II is used with a 1 per cent ruin probability.

Uncertainty in the Mean (CV)	Required Surplus (millions)	Premium/Surplus Ratio
.00	$ 4.1	2.07
.05	4.5	1.89
.10	5.5	1.54
.15	6.9	1.23
.20	8.4	1.00
.25	10.1	0.84

STEADY STATE RESERVES

Finger addresses the aggregate reserves which include individual case reserves, IBNR, and unearned premium reserves. The unearned premium reserves appear to be missing from the aggregate reserve distribution, presumably because these should have little or no impact on the minimum surplus requirements. Consequently, the model applies primarily to loss reserves. However, such loss reserves will be composed of cl·ims from various accident years at different levels of maturity and different degrees of uncertainty. The author relates these aggregate reserves to the latest year's expected losses using adjustments for inflation and investment income. This part of the model should be expanded to replace the steady state assumptions with a more realistic description of actual reserves.

AGGREGATE EXCESS REINSURANCE

The model presented takes into account various limits of reinsurance coverage, but only on a per claim or per occurrence basis. Excess of aggregate loss reinsurance can be a major element in the operation of a small insurance company, self-insurer or captive. This reinsurance would usually apply on an accident year basis and therefore could not be directly applied to the aggregate reserves. However, for illustration, the effect of this coverage can be readily explained by considering only one accident year.

Suppose it is determined that surplus is required in an amount equal to the difference between the 99th percentile and the expected value of the probability distribution for the aggregate losses from an accident year. Assume further that aggregate excess reinsurance is purchased which will cover aggregate losses in excess of an amount equal to the 80th percentile of the distribution and will continue to pay such losses up to the 99th percentile. In this case, it would be proper to reduce the surplus requirement by the amount of reinsurance coverage provided. Thus, surplus would be needed to complement the 80th percentile rather than the 99th.

FINANCIAL CAPACITY

Traditionally, insurance company financial analysis has primarily been based on the company's statutory financial statements and not on any consolidation with affiliated non-insurance operations. Disclosure of financial inter-relationships with affiliates is required, but this is primarily used to review the potential for impairment from such relationships.

This traditional approach may not be valid when applied to the self-insured or captive. These risk financing mechanisms are established to be self-sustaining and capable of operating within the parent's financial capacity. However, much of the burden of insolvency is born by the parent. While these may be some insolvency protection outside the organization, the parent could very well have to respond at some point since it is the policyholder.

This discussion is not meant to be conclusive, but it should point out that required surplus is not the only criteria for evaluating the viability of an insurance operation.

SUMMARY

Mr. Finger has made a significant contribution to the Society which not only adds to the actuary's analytical tools, but also opens up an avenue of study that heretofore was unexplored . This reviewer has attempted to probe the assumptions and implications of the paper and thus enhance the reader's understanding of this work.

One final point is worth mentioning. Although the author has described his model as computing a minimum surplus requirement, it can also be used, and perhaps more appropriately, to review the adequacy of a company's surplus position in terms of the probability of insolvency.