REVIEW OF "RISK AND RETURN FOR PROPERTY-CASUALTY INSURERS

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Some of the important problems of actuarial science lie also within the bounds of financial theory. The individual's and corporation's decision on the proper amount of insurance to buy can be viewed a sub-problem of the overall issue of how economic risk, insurable or not, should be handled. The emergence of the discipline of corporate risk management shows that it is anachronistic to view the insurance problem separately from other financial decisions. As actuaries we must carefully examine the underlying assumptions, the empirical validity and the resulting implications of applying financial theory to insurance problems.

Mr. Bustic's paper is a welcome contribution to this conference for several reasons. It is a good example of two of the potential areas of application of financial theory to insurance: 1. applying portfolio theory to an insurance company's portfolio of policies, and 2. analyzing the insurance company as one of many publicly traded companies whose stock should obey the same rules as any others in the capital markets. Secondly, the paper covers an overview of three of financial theory's main tools: 1. mean/variance portfolio theory, 2. the capital asset pricing model (CAPM), and 3. the concept of efficient markets. Finally, the paper sets the stage for a discussion of the underlying assumptions and implications of this methodology.

My review will not repeat results from the paper, but will discuss the following extensions:

1. The basic form of the CAPM implies that insurance companies should not exist.

2. The risk (additional variance) an insurance company faces because premiums are set based on an estimate of expected losses rather than the true (but unknowable) value of expected losses is to a great extent not diversifiable (from the company's point of view).

3. It is not clear whether the assumptions underlying mean/variance portfolio theory and the efficient set theorem hold for an insurance company.

And a technical note on Mr. Bustic's model

4. The use of a formula containing the term v, reserves to premium ratio, for pricing decisions is not valid, except in a no growth situation.

1. THE CAPM

The CAPM states that the equilibrium expected rate of return for asset i is

$$R_i = R_f + b_i\,(R_M - R_f)$$

where $b_i = \mathrm{Cov}\,(\tilde{R}_i, \tilde{R}_M)/\mathrm{Var}\,(\tilde{R}_M)$ = "beta of i"

R_M = expected return on the market

R_f = riskless interest rate

The CAPM implies that in equilibrium the market will not reward you (i.e., allow a higher expected return) for bearing diversifiable risk (the portion of variance that is independent of the market's movements). If the risks that an insurer covers are independent of the movement of the stock market and, therefore, represent diversifiable risk, then the insurer should not be allowed any return for bearing those risks - all he should earn is the investment income on his surplus, which would be his even if he did not issue any policies.[1] Mr. Bustic makes this point following formula (33), however, he does not express any concern with the result, in fact, he suggests formula (32) be used by management as a pricing guideline. I argue that for a regulator or company manager to use this formula for calculating allowable underwriting return is unsound because it allows for no incentive for an insurer to bear non-systematic risk, which is its business. It might as well close up shop. To turn this conclusion around, I think that the existence of a competitive reinsurance market indicates that a market, if left to its own devices, will allow additional return for bearing diversifiable risk. According to the CAPM, capital should continue to be attracted to this industry until these additional returns were erased. Although I am speculating on an empirical matter without any data, I do not believe that this has been the case.

I think it is important to make the distinction between 1. constructing a model that describes what will occur in long run equilibrium if companies are free to pursue their own profit maximizing courses and 2. using that model as a procedure as to how an individual company should act. As you see in this case, a company acting or forced to act according to this model will have no reason to pursue its business. An analogy can be drawn with the general long run pure competition solution in economics: in long run equilibrium there will be no economic profits -- all the factors of production including capital and entrepreneurial ability will be rewarded according to their contribution, but there will be no "profit" in addition to this. If there was a profit, the system would not be an equilibrium, more firms would be drawn to provide the service until this profit had been competed away. However, the individual motivation of each firm is to earn a profit. If the long run no profit equilibrium is forced on the firm's pricing decisions, the firm would have no motivation to produce.

Another way of looking at this conclusion from the CAPM is from a corporate insurance buyer's point of view. That corporation gets no reward in the capital market for bearing diversifiable risk. However, it also gets no reward for relieving itself of diversifiable risk. This implies that a corporation should not be willing to pay more than the pure premium (discounted for investment income) for insurance; otherwise, it is harming its stockholders. The one exception is that if having insurance would let a firm get a particular (profitable) factory running again faster than it could without insurance, then the system of insurance would have a real economic gain for the firm. However, in this case, it is not the risk transfer that is critical, but the availability of funds to cover the loss. If a company had a line of credit available equal to the amount of insurance, or could raise funds in the capital market quickly then the insurance would be of no benefit in this situation. The only issue would be whether the insurance company paid the loss or the corporation had to pay back the debt over a long run.

This analysis, however, does not apply to individuals, who presumably have risk averse utility functions and can rationally want to purchase insurance at more than the pure premium. The difference between these two situations is that risk that the individual is averse to is variance of returns to himself. The risk that a corporation is averse to is only the beta component of its variance because its owners can diversify away the non beta risk. Even though individuals will demand insurance at rates higher than actuarily fair ones the CAPM implies that stock insurance companies will compete away any profit over this. Capital will continue to be drawn to the industry until the bearing of diversifiable risk is not rewarded.

In the world of the CAPM, the only assets that should be insured are those that are not owned by pubicly traded companies. The CAPM says a rational investor will hold some tiny fraction of every asset available. If a certain factory burns, our investor only suffers a tiny loss. It would not help him if the factory was insured, because he would have a tiny share of the insurer too. However, assuming he has not issued shares in his home (or his future earning potential) he should insure these.

Of course, all the preceding assumes that insurance underwriting risk is not correlated with the stock market, which seems to me to be a reasonable assumption, although it can be argued that worker's compensation, disability insurance, surety bonds and fire insurance results are partially determined by business conditions. And as Mr. Bustic points out, inflation affects both insurance results and the stock market.

2. DIVERSIFICATION OF RISK WITHIN THE COMPANY

Mr. Bustic's formula (19) shows that, as the number of insureds goes to infinity, the only contribution that underwriting variance gives to the variance of return on surplus comes from co-variance terms. Remember that this equation is derived assuming that the written premium to surplus ratio (k) and the reserves to written premium ratio (v) are fixed. This means that as N goes to infinity, either surplus must go to infinity or the average policy size must go to zero. In other words, if we are dealing with a fixed surplus "N goes to infinity" means that we are taking a smaller and smaller share of a larger and larger number of policies while keeping our total premium fixed. Simply adding independent policies to a fixed surplus increases the variance of return on surplus.

$$R_s = \frac{1}{St} \quad (I(t) + U(t))$$

$$Var(R_s) = \frac{1}{S_t^2} \quad (Var\ (I(t)) + Var\ (U(t)) + 2\ Cov\ (I(t)\ ,\ U(t)))$$

$$U(t) = \$\ \text{underwriting result} = \sum_{i=1}^{N} \tilde{u}_i p_i \quad (\text{pi is premium on } i^{th} \text{ risk})$$

$$Var\ (U(t)) \to \infty \quad \text{As } N \to \infty$$

Also note that as a single company adds policies beyond some point it will be taking poorer risks or will have to compete on price, which will decrease its expected return.

An interesting issue is what portion of underwriting risk is non-diversifiable from the company's point of view (not from the capital market's point of view).[2] If we remember that, the premium charged is an estimator of the expected losses (loaded for profit and expenses) and as an estimator it is itself a random variable, the variance of this estimator represents systematic risk; that is, it can not be driven to zero by insuring smaller and smaller pieces of more and more policies. This can be demonstrated as follows:

Let μ_ℓ = true, but unknowable, expected losses per risk in a given class

$\hat{\mu}_\ell$ = our estimator of μ_ℓ

The underlying ratemaking statistics are a random sample drawn from a loss distribution whose parameters we wish to estimate. $\hat{\mu}_\ell$ is therefore a random variable.

PLR = permissible loss ratio used in ratemaking (expenses are considered part of losses)

$\tilde{\ell}_i + \mu_\ell$ = i^{th} risk's losses

Assume that $E[\tilde{\ell}_i]=0$; $Var(\tilde{\ell}_i)=\sigma_\ell^2$; $\tilde{\ell}_i, \tilde{\ell}_j$ are independent

$\tilde{u}_i$ = i^{th} risk's underwriting results as a percent of premium

$\tilde{U}$ = total company underwriting results as a percent of premium

$\tilde{p}$ = premium per risk

Now $$\tilde{p} = \frac{\hat{\mu}_\ell}{PLR}$$

$$\tilde{u}_i = \frac{\tilde{p}-\mu_\ell-\tilde{\ell}_i}{\tilde{p}} = 1 - \frac{(\mu_\ell+\tilde{\ell}_i)PLR}{\hat{\mu}_\ell}$$

$$\tilde{U} = \frac{1}{N}\sum_{i=1}^{N}\tilde{u}_i = 1 - PLR\,\frac{\mu_\ell}{\hat{\mu}_\ell} - PLR\,\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\ell}_i}{\hat{\mu}_\ell}$$

Assume that $\tilde{\ell}_i$ and $\hat{\mu}_\ell$ are independent (the sample of losses on which $\hat{\mu}_\ell$ is based does not include $\tilde{\ell}_i$, which represents next year's losses).

$$Var(\tilde{U}) = Var\left(PLR\,\frac{\mu_\ell}{\hat{\mu}_\ell} + PLR\,\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\tilde{\ell}_i}{\hat{\mu}_\ell}\right)\right)$$

$$= PLR^2\left[Var\left(\frac{\mu_\ell}{\hat{\mu}_\ell}\right) + \frac{1}{N^2}Var\left(\sum_{i=1}^{N}\frac{\tilde{\ell}_i}{\hat{\mu}_\ell}\right) + 2\,Cov\left(\frac{\mu_\ell}{\hat{\mu}_\ell}, \frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\ell}_i}{\hat{\mu}_\ell}\right)\right]$$

Now $$Cov\left(\frac{\mu_\ell}{\hat{\mu}_\ell}, \frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\ell}_i}{\hat{\mu}_\ell}\right) = E\left[\frac{\mu_\ell}{\hat{\mu}_\ell}\cdot\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\ell}_i}{\hat{\mu}_\ell}\right] - E\left[\frac{\mu_\ell}{\hat{\mu}_\ell}\right]E\left[\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\ell}_i}{\hat{\mu}_\ell}\right]$$

$$= E\left[\frac{\mu_\ell}{\hat{\mu}_\ell^2}\cdot\frac{1}{N}\right]\left(\sum_{i=1}^{N}E[\tilde{\ell}_i]\right) - E\left[\frac{\mu_\ell}{\hat{\mu}_\ell}\right]E\left[\frac{1}{N\hat{\mu}_\ell}\right]\left(\sum_{i=1}^{N}E[\tilde{\ell}_i]\right) = 0$$

Because $E[\tilde{\ell}_i]=0$

So $$Var(\tilde{U}) = PLR^2\left[Var\left(\frac{\mu_\ell}{\hat{\mu}_\ell}\right) + \frac{1}{N}Var\left(\frac{\tilde{\ell}_i}{\hat{\mu}_\ell}\right)\right]$$

So $$Var(\tilde{U}) \to PLR^2\,Var\left(\frac{\mu_\ell}{\hat{\mu}_\ell}\right) \text{ as } N\to\infty$$

Two things in this analysis might be objected to. First, in primary ratemaking, the sample is usually so large that the variance of the estimator should be very small and second, that after the ratemaking statistics are collected, the premium is no longer a random variable. If you take a Bayesian point of view and allow $\tilde{\mu}_{\ell}$, the true expected losses for each risk in the class, to be a random variable because it is unknown, then $V(U) \to \frac{1}{P^2} V(\tilde{\mu}_{\ell})$ as $N \to \infty$ where $V(\tilde{\mu}_{\ell})$ is the variance of the posterior distribution, which will include all the uncertainty involved in forcasting trend conditions over the settlement period of next year's claims. P is now fixed total premium.

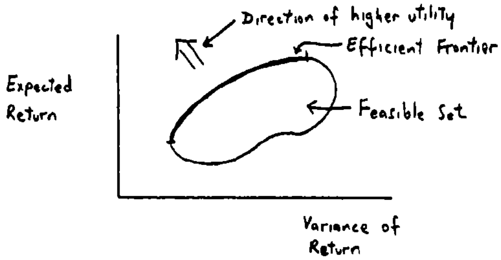
The point is that the risk the insurer faces has two components: first even if the parameters of the loss distribution are known with certainty there is the variance inherant in the random loss process. If the frequency and severity of each risk are independent from other risks then this variance can be diversified away. However, the second component of the total variance facing the company is due to the fact that the parameters of the loss distribution are not known but are only estimated. This component of risk is clearly dependent risk by risk for risks using the same class rate.

This is an oversimplification in order to make the point of the systematic nature of this "ratemaking" risk. In concrete terms this is the risk that your loss development procedure is giving you biased answers, or that a regulator disallows a valid indicated rate change. Very complicated models would have to be constructed to analyze the systematic and non-systematic components of an actual rate making system involving loss development, trend, and credibility weighted class relativities. However, from a reinsurers point of view I feel this component of risk is very significant and I imagine it would be for a primary company also. This component of risk can be diversified to some extent by writing different lines of business and in different states. However, there are practical marketing and organizational limitations on how much this can be done.

3. THE EFFICIENT SET THEOREM

Figure 1. in Mr. Bustic's paper illustrates the common technique in portfolio theory of plotting the expected return and variance (or standard deviation) of all possible investment strategies (referred to as the feasible set), then identifying part of the outer shell as being the efficient frontier, that is, investment strategies that must dominate all others. The standard approach is then to look for the highest utility curve an investor can reach. Using this type of analysis it is very tempting to define the feasible set as all possible underwriting and investment strategies an insurance company might follow and then looking for the tangent between the efficient set and the insurance company's utility function, as Mr. Bustic does in his figure 2.

However, we must analyze whether the restrictive return distribution and utility function assumptions underlying this technique hold in the case of insurance. We should also consider whether it is meaningful to say that an insurance company (rather than an individual investor) has a utility function.



The efficient set theorem is what allows you to reject all portfolios but the ones lying on the efficient frontier. It can be shown that regardless of the utility function, if returns are distributed normally, the efficient set theorem holds.

However, the underwriting returns to an insurer are clearly non normal as is evidenced by the literature on the N-P approximation to aggregate loss distribution. It is possible under non normal return distributions for preferred portfolios to lie in the interior of the feasible set.

This is because such preferred portfolios might have desirable properties in their higher moments which more than counterbalance the disutility of their increased variance. Of course, if it is possible for preferred portfolios to lie in the interior then we can not be sure that we are maximizing utility by moving farther to the upper left of the diagram.

The second sufficient condition for the efficient set theorem is that the utility function is quadratic (regardless of the distribution of returns). A quadratic utility function is one of the form,

$$\text{utility of return } r = a + br + cr^2,\ c < o$$

However, quadratic utility functions have the undesirable property of increasing absolute risk aversion. This means that as a person's wealth increases the dollar amount he invests in risky assets decreases. It is sometimes argued that the quadratic can be used as an approximation for most reasonable utility functions.

Finally, if a corporation is only risk averse to beta, not total variance of returns, then the usual utility functions of $\tilde{R}_i$ (or of $\tilde{\omega}_i$, end of period wealth, which for a given initial wealth, W_0, can be shown to be equivalent to functions of $\tilde{R}_i$ and W_0) will be insufficient to represent a corporation's decision making process. If it is possible to represent its decisions as maximizing expected utility, then this utility function must be a function of $\tilde{R}_M$ in addition to $\tilde{R}_i$.

The summaries of the issues that I see are - 1. Can an insurer have a utility function based on variance or must it also consider beta? - 2. If variance is an appropriate component of the risk measure, is it the only component - can higher moments be ignored? Only if the answers to the above are positive can we use standard portfolio theory.

4. THE RESERVES TO PREMIUM RATIO, v

Mr. Bustic developed a very simplified model of a property and casualty insurer so that the basic financial theoretic ideas he wanted to convey would not be obscured by the details of a more complicated model. One property of his model that should be noted is that his basic pricing equation (32)

$$U = -v\, R_f + b_u\, (R_m - R_f)$$

should not be used to determine an underwriting profit provision (even if you wish to practice "CAPM Pricing") unless v is constant thru time. The reason for this can be most easily seen in the case $b_u = 0$ which we have already argued means that you wish to price so that expected return from underwriting and investment income from insurance operations (not from surplus) is zero. For $b_u = 0$ the above equaltion becomes $U + v R_f = 0$. Remember that v, the ratio of reserves to this year's premium, heavily depends on the pattern of prior year's premium writings. For example, if we are starting a new company then $v = 0$. What we really want $v R_f$ to measure is the present value of expected investment income from future premiums (minus their corresponding paid losses), that is, we want to discount our premium for expected investment income due to that premium. If we do not do this, but use the $v R_f$ term instead, we can manipulate our ultimate profit by changing our premium writings and, therefore, changing our future v.

However, note that using a present value term instead will give anomolous results from an accounting point of view because expected investment income is not earned the same year the premium is earned, but the expected underwriting loss affects earnings as the premium earns.

CONCLUSION

I feel that a financial theoretic view of insurance such as Mr. Bustic took has substantial validity -- an insurer's stocks do not occupy some exalted position outside the influences affecting the stock prices of other corporations -- but this approach also has many unresolved problems. There must be some price (greater than zero) for bearing non beta risk, otherwise the insurance market cannot operate. However, I also believe that that price will be less than the one for bearing beta risk, otherwise there would be great unexploited profit opportunities for forming large pools of non beta risk. If my conjecture of a "two price system" is true, then we must not talk about insurer's utility functions of $\tilde{R}_i$ (insurer's return) only, but must consider them as preference rankings of joint distributions of $\tilde{R}_i$ and $\tilde{R}_m$ – if we talk about them at all.

Notes

1. The basic CAPM implies this for any type of corporation, not just an insurance company. This is because the CAPM implies that a corporation should select any investment project with a return greater than $R_f + b\ (R_m - R_f)$ where b is the beta for that project. Note that this selection criterion ignores the variance of returns on the project.

2. If we believe in CAPM pricing then the following analysis is of academic interest only, because even though parameter variance puts a limit on how much the insurance company can decrease its variance of returns, the company would only be allowed to charge for this to the extent its b_u was affected.