

DISCUSSION OF PAPER PRESENTED AT THE
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"A CONSTRAINED PROFIT MAXIMIZATION MODEL
FOR A MULTI-LINE PROPERTY/LIABILITY COMPANY"

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DISCUSSION BY MICHAEL L TOOTHMAN

At last, a paper applying microeconomic theory to the insurance industry! Mr Brubaker's paper is extremely basic, and I wish he had carried his thought process further. The result is a model which needs a great deal of additional work and refinement before it can even begin to approach reality. Still, this is the first paper I recall in our Proceedings which applies economic theory to the insurance industry. For that we owe Mr Brubaker a great deal of gratitude, and we can only hope that many others will take the cue and expand upon this work.

Mr Brubaker has used the model of the firm with one input, in this case capital, and several outputs. He uses Earned Premiums for each product line to measure the outputs. I believe Written Premiums would be preferable in this regard as Written Premiums can be more directly controlled by management. This change does no damage to the balance of the paper and really has no adverse effect on Mr Brubaker's approach.

The model is technically sound. The problem reduces to a constrained maximization problem which is solved through the use of LaGrange multipliers.

As the resulting set of simultaneous equations is generally difficult to solve, an iterative technique is introduced to determine a solution satisfying the profit - maximizing condition that the ratios of marginal profit per unit of output to the marginal input requirement per unit of output are equalized for all outputs. The examples solved in the paper are for a maximum of three outputs. These examples demonstrate well the relative sensitivities of the optimal solution to changes in the underlying assumptions in means, variances, or covariances of the involved product lines. The calculations involved even for the simple examples are tedious, however. I developed a more realistic example involving seven product lines and found the calculations to be entirely unrealistic without having the technique and an appropriate algorithm programmed. The programming of this technique would not seem to present any great disadvantage to use of the technique.

The model of the firm with one input and two outputs is typically presented in most economics texts in a graphical fashion. The limitation on input defines a product transformation curve, the locus of all output combinations that can be produced from the given level of input. The locus of all output combinations earning a specific revenue defines an isorevenue line. Graphically, the optimal solution is obtained as that combination of outputs that is located on the highest isorevenue line, which is one tangential to the product transformation curve. For simplicity, Mr. Brubaker has limited the set of possible solutions to that quadrant where Earned Premiums are non-negative. In reality, there may be constraints which further restrict the set of possible solutions, and these should be considered as the model is refined. For example, some major companies recently discovered that the desire to continue writing commercial lines in some states requires that they also continue to write personal lines in that state.

It should be realized that conceptually a separate product line needs to be defined for each separate geographical area. If management might consider reducing volume for a given line in one location while increasing in another, the two locations should be viewed as separate product lines for the purposes of the model. Also, writing a product line on an admitted basis might be treated separately from writing the same product on a non-admitted basis. The explosion in the number of product lines should cause no problem once an appropriate algorithm is developed for computer application.

Finally, the results for each product line are assumed to conform to a multivariate normal distribution. This distribution is easy to handle and allows one to easily demonstrate the sensitivities of the optimal solution to changes in the means, variances, or covariances of the various product lines. This is the area that offers the greatest potential for significant additional refinements, however. Most importantly, the effects of alternative reinsurance arrangements need to be factored into the model. Reinsurance can have such a significant effect on the underlying distributions that it must be considered before the model could ever be used. Quota-share reinsurance would be expected to leave the mean unchanged. The variance would also remain unchanged except to the extent that a given capital level can support net premiums which are made up of more risks, which may reduce the variance. 1) Excess of loss reinsurance, on the other hand, has the effect of reducing the variance on the upside, thereby resulting in a skewed distribution. The amount of premium which can safely be written would then increase. The mean

1) Note that Mr. Brubaker assumes that the variance for each product line is independent of Earned Premium. In this case quota share reinsurance has no effect on the result.

may also be affected, although the direction of the change is not obvious. The cost of the reinsurance would lower the expected profit, but the reduction in large losses would raise expected profit. Even simpler in terms of analysis is the effect of stop-loss reinsurance. Here the assumed normal distribution becomes a truncated normal, and the analysis proceeds relatively smoothly.

Obviously all this makes the model much more complicated. All modeling involves the use of simplifying assumptions, and Mr. Brubaker has presented a basic model which is conceptually useful. Some refinements must be made, however, to translate the conceptually useful model to one which is practically useful as well.

The field of economics contains much wealth. The theory is reasonably well-developed, yet the theory has rarely been used to either explain the activities of the insurance industry or to develop models which can be used by management in the decision-making process. So much of economic theory can be expressed so simply in mathematical form that this would seem to be a natural and fertile field for actuaries. In addition to expanding upon the theory of the firm as basically outlined in this paper and developing a model which recognizes the real constraints to which management is subject and the opportunities available to management to alter the underlying distribution for a given line of insurance, actuarial research might address many other areas. For instance, what is the demand curve for insurance? Does the curve vary significantly for different lines of business, in particular commercial lines versus personal lines? Is our

product price elastic or price inelastic? 2) Also, to what degree does product differentiation exist within the industry? What effect does advertising have? To what extent do barriers to entry exist and to what extent do they change the economic model applicable to our industry from a purely competitive to an oligopolistic model? Many additional questions can quickly come to mind. The point is that the answers to these and similar questions are not just academically interesting, but also a great potential worth to the decision-making process in the real world. Mr. Brubaker's paper, basic as it is, is a start in this direction. I hope that others will follow his lead, pick up the gauntlet, and pursue additional research in this field.

- 2) One would expect that the industry demand curve is relatively inelastic with regard to price, but that the demand curve for an individual company exhibits a relatively high degree of price elasticity. Witness the ease with which company managements, particularly marketing and field management, will agree to follow a bureau rate increase but the extreme reluctance to take the same or a smaller rate increase ahead of the rest of the industry. Also, to the extent that expected profits remain positive, perhaps total profit can be increased by lowering rates, contrary to the thinking of many actuaries. We really can "make it up in volume," if (1) expected profit remains positive and (2) the demand curve is sufficiently price elastic.

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