Loss Reserving with GLMs: A Case Study

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Summary

This paper provides a case study in the application of generalised linear models ("GLMs") to loss reserving. The study is motivated by approaching the exercise from the viewpoint of an actuary with a predisposition to the application of the chain ladder ("CL").

The data set under study is seen to violate the conditions for application of the CL in a number of ways. The difficulties of adjusting the CL to allow for these features of the data are noted (Sections 3).

Regression, and particularly GLM regression, is introduced as a structured and rigorous form of data analysis. This enables the investigation and modelling of a number of complex features of the data responsible for the violation of the CL conditions. These include superimposed inflation and changes in the rules governing the payment of claims (Sections 4 to 7).

The development of the analysis is traced in some detail, as is the production of a range of diagnostics and tests used to compare candidate models and validate the final one.

The benefits of this approach are discussed in Section 8.

Keywords: chain ladder, generalised linear model, GLM, loss reserving, regression, superimposed inflation.

1. Introduction

Taylor (2000) surveys many of the methods of loss reserving. Although the **chain ladder** ("CL") (Chapter 3) is, in a number of ways, the most elementary, it is also still the most widely used by practitioners.

This method is based, however, on a very restrictive model whose conditions are likely to be breached quite commonly in practice. When this happens the method is liable to material error in the loss reserve it generates.

If such error is to be corrected, the model itself must be subjected to some form of corrective action. This may be difficult on two scores:

- The CL falls within the category of model labelled **phenomenological** by Taylor, McGuire and Greenfield (2003). This means that it reflects little of the underlying mechanism of claim payment, and consequently the required form of correction may not be readily apparent.
- Even if the required form of correction can be identified, perseverance with the CL may be more tedious and less reliable than its abandonment in favour of a fundamentally different approach.

The present paper is concerned with a data set that manifestly fails to meet the conditions under which application of the CL is valid. It then examines the sorts of corrections required, and how they might be implemented most efficiently.

It should be pointed out that there has been no necessity to trawl through numerous data sets to locate one that breaches CL assumptions. The data set used here relates to the Auto Bodily Injury claims of one of the Australian states. The consultancy with which we are associated deals with such claims in four states, and it is fair to say that any one of these could have been used as the example for the present paper.

The viewpoint taken will be that of a reserving actuary with a predisposition to the application of the CL. The validity of its application to the subject data set will be examined (Section 3), as will the materiality of the potential error it introduces. Analysis of the data set will then be directed to the identification of the various breaches of the CL conditions, and their consequences for a loss reserve.

The ultimate purpose of this analysis is not to produce a diatribe against the CL as such, since this may provide a perfectly useful piece of methodology under appropriate conditions. Rather, the purpose is to demonstrate how **Generalised Linear Models ("GLMs")** can provide a structured and rigorous form of data analysis leading to a loss reserving model.

2. The data set

The data set relates to a scheme of Auto Bodily Injury insurance in one state of Australia. This form of insurance is compulsory, and includes no component of property coverage.

The form of coverage, and other conditions under which the scheme operates, are legislated, but it is underwritten by private sector insurers subject to these conditions. Premium rates are partially regulated by the promulgation of acceptable ranges.

Insurers that participate in the underwriting are required to submit their claims data to a centralised data base. The data set used in the present paper is extracted from this data base. It comprises a unit record claim file, containing the following items of information:

- Date of injury;
- Date of notification;
- Histories of:
  - Finalised/unfinalised status (some claims re-open after having been designated finalised), including dates of changes of status
  - Paid losses
  - Case estimates
- Various other claim characteristics (e.g. injury type, injury severity, etc) not used in the present paper.

The scheme of insurance commenced in its present form in September 1994, and the data base contains claims with dates of injury from then. It is current at 30 September 2003.

The purpose of the present paper is to illustrate loss reserving by means of GLMs, rather than to carry out a loss reserving consulting assignment. For this reason, analysis will be limited to **finalised claims**. Some justification for this course will become apparent as the analysis develops, but there will be no attempt to demonstrate beyond doubt that it is the best.

A consequence of this approach is that (for almost all purposes) data are required only in respect of finalised claims. Exceptions are that:

- The ultimate numbers of claims to be notified in each accident quarter have been estimated outside the paper, and will here be taken as given.
- In respect of each accident quarter, the total amount of losses paid to 30 September 2003, whether relating to finalised or unfinalised claims, is used to obtain estimates of outstanding claims in Sections 3.2 and 7.6.

Wherever paid loss amounts are used they have been converted to 30 September 2003 dollar values in accordance with past wage inflation experienced in the state concerned. This is done to eliminate past "normal" inflationary effects on the assumption that wage inflation is the "normal" inflation for this type of claim. Henceforth, any reference to paid losses will carry the tacit implication that they are expressed in these constant dollar values.

Naturally, claims inflation actually experienced differs from wage inflation from time to time, and is the subject of estimation in Sections 7.3.2 and 7.3.3. The excess of claims inflation over wage inflation is referred to as **superimposed inflation** ("SI").

Appendix A.1 provides a triangular summary of the paid loss data in the usual form. In conventional fashion, rows of the triangle represent **accident quarters**, columns **development quarters**, and diagonals **experience quarters** (or quarters of finalisation). Development quarters are labelled 0, 1,..., with development quarter 0 coinciding with the accident quarter.

Let P_{ij} denote claim payments in the (i,j) cell. Let C_{ij} denote their cumulative version:

$$C_{ij} = \sum_{k=0}^{j} P_{ik} \qquad (2.1)$$

Similarly, P^F_{ij} and C^F_{ij} denote the corresponding quantities in respect of just finalised claims. Appendix A.2 provides a triangular summary of these. Each cell of the triangle contains the paid losses, whether paid in that quarter or earlier, **in respect of claims finalised in the cell.**

Let F_{ij} denote number of claims finalised in the (i,j) cell. They are set out in Appendix A.3. Let G_{ij} denote their cumulative version. Define **average sizes** of finalised claims, incremental and cumulative respectively, as follows:

$$S_{ij} = P^F_{ij} / F_{ij} \tag{2.2}$$

$$T_{ij} = C^F_{ij} / G_{ij} . \tag{2.3}$$

Appendices A.4 and A.5 display these average claim sizes.

3. The chain ladder

3.1 Age-to-age factors

Appendix B derives age-to-age factors from the data of Appendix A.

The age-to-age factor linking cells (i,j) and (i,j+1) in the triangle of cumulative paid losses is

$$R^F_{ij} = C^F_{i,j+1} / C^F_{ij}. \tag{3.1}$$

These factors are tabulated in Appendix B.1.

Likewise, the age-to-age factor linking cells (i,j) and (i,j+1) in the triangle of cumulative average claim sizes (Appendix A.4) is

$$Q_{ij} = T_{i,j+1} / T_{ij}. \tag{3.2}$$

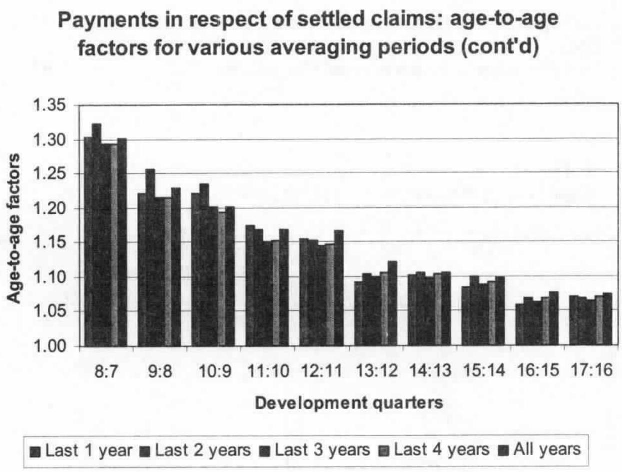
These factors are tabulated in Appendix B.2.

Average age-to-age factors are displayed in Appendices B.1 and B.2. Conventionally, these are taken over various past averaging periods, as some sort of test of stability of the factors over time.

Figures 3.1 and 3.2 chart the average age-to-age factors, showing clear indications of **instability**. In development periods 3 to about 10, the factors show a clear tendency toward higher values for more recent experience years (except the latest year, where they are lower).

Figure 3.1

Figure 3.2



3.2 Sensitivity of loss reserve

While Figures 3.1 and 3.2 demonstrate that different averaging periods lead to different age-to-age factors, and therefore to different loss reserves, the materiality of the differences is not apparent. Table 3.1 sets out the loss reserves calculated according to the various averaging periods.

Inspection of Appendix B.1 reveals that, while the age-to-age factors generally showed increasing trends over recent periods, those recorded in the September 2003 experience quarter (the last diagonal, were particularly low. Table 3.1 includes an examination of the effect of including or excluding this quarter's experience from the averaging.

Omission of the September 2003 experience prevents estimation of a loss reserve for that accident period. Therefore, the loss reserves set out in Table 3.1 relate to all accident quarters except that one.

Table 3.1
Loss reserves according to different averaging periods for age-to-age factors

Averaging period	Loss reserve at 30 September 2003 (excluding September 2003 accident quarter)
	$B
All experience quarters	1.61
Last 8 experience quarters	1.68
All experience quarters except September 2003	1.78
Last 8 experience quarters except September 2003	1.92

Table 3.2
Loss reserve dissected by accident period

Accident quarter	Loss reserve at 30 September 2003 (excluding September 2003 accident quarter)
	$M
Sep 00	176
Sep 01	165
Sep 02	171
Dec 02	124
Mar 03	59
Jun 03	58
Total	1,785

The sensitivity of loss reserve to averaging period is considerable. The largest estimate is 19% larger than the smallest. However, a more detailed examination of the loss reserves quickly reveals that the true sensitivity is much greater than this.

Table 3.2 sets out an accident quarter partial dissection of the "All experience quarters except September 2003" reserve from Table 3.1. It is quite evident that the loss reserve is distorted downward in respect of the latest accident quarters.

This is due to the low cumulative paid losses at the end of this quarter, as evidenced by the low age-to-age factors in this quarter, which serve as the baseline for forecasting future paid losses.

The usefulness of the reserves in Table 3.1 is unclear in the presence of this factor. It is natural to correct for it by adjusting any loss reserve at 30 September 2003 (still excluding the September 2003 accident quarter) by forecasting it on the basis of paid losses to 30 June 2003. Specifically, this consists of:

- calculating a standard chain ladder loss reserve at 30 June 2003; and then
- deducting the forecast September 2003 quarter paid losses included in that reserve.

This makes sense only for reserves based on averaging that excludes the September 2003 experience quarter. Table 3.3 augments Table 3.1 to include such corrections.

Table 3.3
Loss reserves corrected and uncorrected for low September 2003 quarter paid loss experience

Averaging period	Loss reserve at 30 September 2003 (excluding September 2003 accident quarter)	
	Uncorrected	Corrected
	$B	$B
All experience quarters	1.61	
Last 8 experience quarters	1.68	
All experience quarters except September 2003	1.78	1.94
Last 8 experience quarters except September 2003	1.92	2.35

Table 3.4, again dealing with the "All experience quarters except September 2003" case, shows that the corrections introduced into the last two rows of Table 3.3 do at least remove the most obvious implausibility in the trends of those loss reserves over recent accident periods.

This comes, however, at the cost of a considerable widening of the gap between the two versions of the chain ladder that respectively use all experience or just the last 8 experience quarters with the exception of the last. The larger of these two estimates is now 21% larger than the other, compared with 8% previously.

Table 3.4
Loss reserve by accident quarter

Accident quarter	Loss reserve at 30 September 2003 (excluding September 2003 accident quarter) – corrected as in Table 3.3
	$M
Sep 00	96
Sep 01	121
Sep 02	137
Dec 02	119
Mar 03	101
Jun 03	114
Total	1,943

It is submitted that the actuary attempting application of the CL to the example data set is now confronted with a bewildering array of models, corrections to models, and corrections to the corrections.

The principal facts are that:

- There are clear time trends in the data;
- One can attempt to deal with this by limiting the data on which the model relies to those of recent period. Here the example of averaging over the last 8 experience quarters is used, but there is no clear guidance to prefer 8 over say 4, or 6, or some other number.
- In any event, the last experience quarter appears fundamentally different from the preceding 7, and the extremely *ad hoc* procedure of dropping it has been adopted.

While the CL can be applied to any choice of data set, there is no apparent criterion for reliable choice of that data set. Moreover, the CL's phenomenological treatment of the trends is deeply unsatisfying. These trends must have a cause that resides somewhere in the detailed mechanics of loss payment. However, the formulaic nature of the CL renders it incurious as to these details.

3.3 The effect of operational time

It is common for the above type of instability to occur when rates of settlement of claims are changing over time. Berquist and Sherman (1972) suggest adjustment to loss reserving methods to take such movements into account.

They refer to "ultimate claims disposed ratio" to denote the proportion of an accident period's claims settled, and suggest that its outstanding claims should be in some way commensurate with the complement of settlement time. Reid (1978) introduced the term **operational time** to take the same meaning, and this terminology will be used below. This quantity is also referred to sometimes as "settlement time".

Let N_i denote the estimated number of claims incurred in accident quarter i, i.e. the number ultimately to be notified in respect of this accident quarter. Then the operational time associated with (the end of) the (i,j) cell, denoted t_{ij} is

$$t_{ij} = G_{ij} / N_i. \qquad (3.3)$$

Figure 3.3 plots how the operational times associated with various numbers of development years have changed over past accident quarters. It is seen that the operational time attained after 2 development years (i.e. at the end of development year 1) increased from 33% for the September 1994 accident quarter to the 54% for the December 1998 accident quarter, and then declined somewhat for subsequent accident quarters.

Similar trends affected development years 2 and 3, but not lower or higher development periods.

Figure 3.3

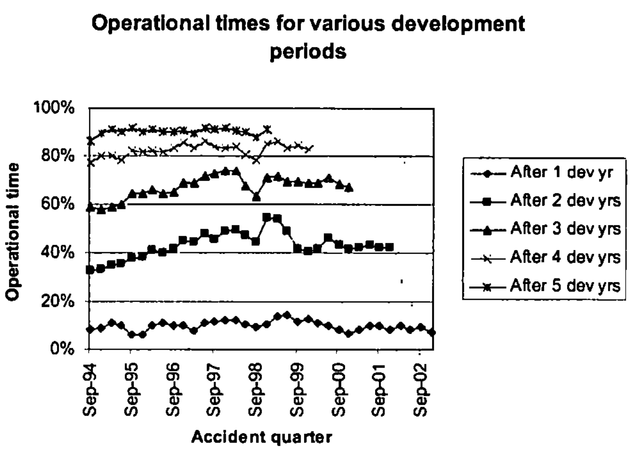


Figure 3.4 superimposes the plot of the quarterly age-to-age factor 3:2 on that of operational time at the end of development quarter 3. Figures 3.5 and 3.6 make the corresponding comparisons for age-to-age factors 7:6 and 11:10 respectively. In the first two of these cases, increases in age-to-age factors appear to coincide with increase in operational time, though the correlation is far from perfect.

Figure 3.4

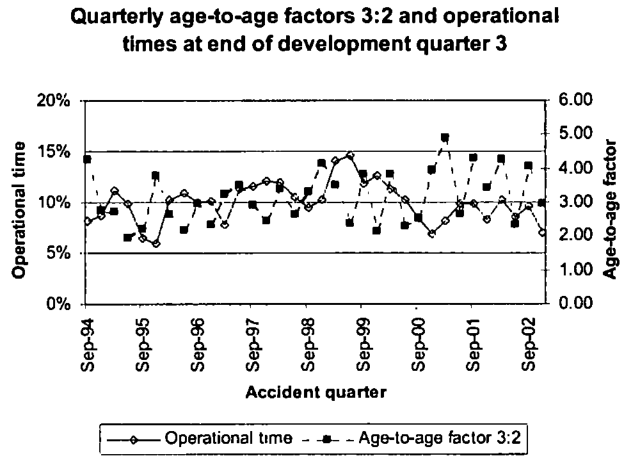


Figure 3.5

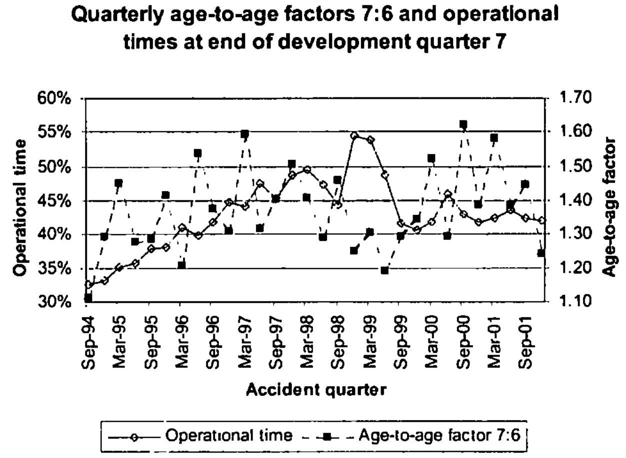
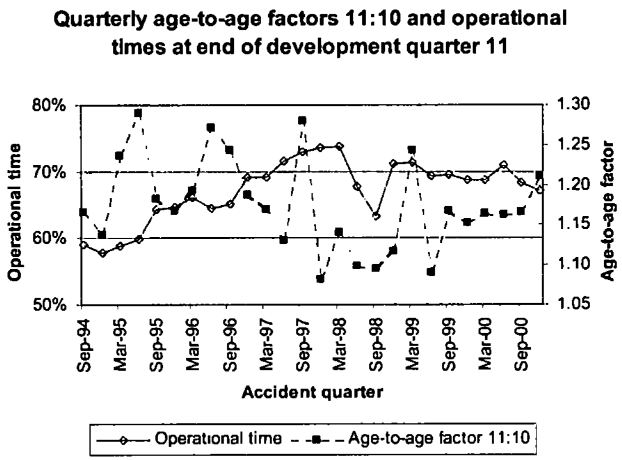


Figure 3.6



An alternative means of controlling for changing operational times is to replace cumulative payments by cumulative average claim sizes in the analysis. The cumulative average claim size (of finalised claims) associated with the (i,j) cell, given by (2.3), may be expressed by means of (3.3) in the alternative form:

$$T_{ij} = [C^F_{ij} / t_{ij}] / N_i. \qquad (3.4)$$

This shows that cumulative average claim size is a multiple of cumulative claim payments per unit of operational time. Such claim sizes might be more stable than payment based age-to-age factors in the presence of changing operational times.

Figure 3.7 plots the cumulative average claim sizes to the end of development quarter 3, for the various accident quarters, against the corresponding operational times. It is found that average claim sizes are not in fact insensitive to variations in operational time, but appear to display a better correlation with operational times than do age-to-age factors.

It will be seen later that this occurs because the claim sizes associated with a particular accident quarter tend to increase with increasing operational time.

A similar improvement in correlation is obtained for development quarter 7, as displayed in Figure 3.8. The corresponding results for development quarter 11 are displayed in Figure 3.9.

Figure 3.7

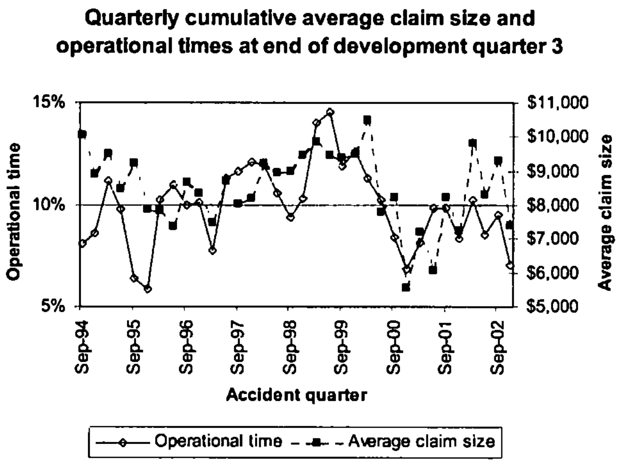


Figure 3.8

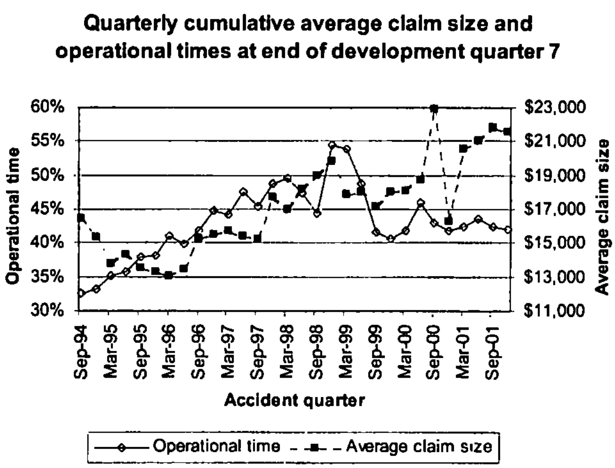
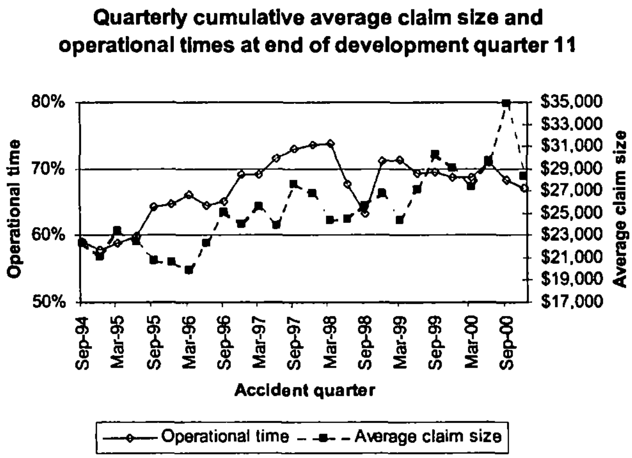


Figure 3.9



4. Exploration of triangular data on average claim size

4.1 Claim development measured by development quarter

The observations made on Figures 3.7 to 3.9 suggest that an average claim size analysis might be preferable to chain ladder analysis. Figures 4.1 to 4.3 therefore explore certain trends in average claim size. Each plots log(average size of finalised claims) against some variable. The triangular form of data is retained.

Figure 4.1 plots log(average size of finalised claims) against development quarter. This could have been carried out as a routine averaging process, but it proved efficient, and in fact more integrated with later sections, to obtain these averages through a modelling process.

Consider the model:

$$\log S_{ij} = \beta_j + \varepsilon_{ij}, \quad (4.1)$$

where

$$\varepsilon_{ij} \sim N(0, \sigma), \quad (4.2)$$

the ε_{ij} are stochastically independent, and the β_j, σ are constants.

Equivalently,

$S_{ij} \sim \log N(\beta_j, \sigma)$ (4.3)

For this model, simple regression estimates of the β_j are equal to the arithmetic means (taken over i) of the observed values of the log S_{ij}. Figure 4.1 could have been derived in this way. EMBLEM software (see also Section 6) has been applied to fit the regression model (4.1) and (4.2) to the data, and the resulting estimates of the β_j plotted against j (see Figure 4.1). The same software is used to produce the remaining plots in this paper.

Figure 4.1
Average claim size by development quarter

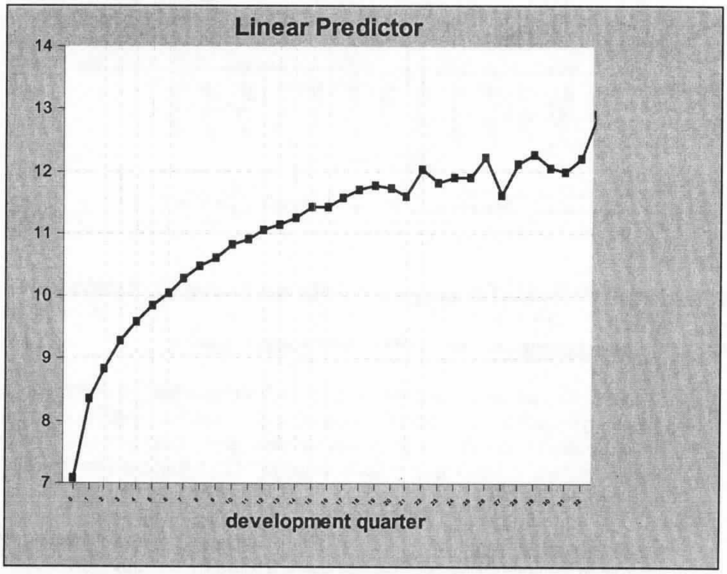


Figure 4.1 shows quite clearly how the average size of finalised claims increases with development quarter, as foreshadowed in Section 3.3.

Figures 3.7 to 3.9 illustrated how (cumulative) average sizes of finalised claims have varied with accident period. Any such effect can be incorporated in the model represented by (4.1) and (4.2) by extending it to the following:

$\log S_{ij} = \beta^d_j + \beta^a_i + \varepsilon_{ij}$, (4.1a)

where the β_j in (4.1) are now denoted β^d_j (the superscript d signifying that these coefficients relate to development quarters), and the accident quarter coefficients β^a_i have also been introduced. The relation (4.2) is retained.

It is worth noting in passing that exponentiation of (4.1a) yields

$$E[S_{ij}] = K \exp \beta^d_j \cdot \exp \beta^a_i, \qquad (4.4)$$

where K is the constant, $E[\exp \varepsilon_{ij}]$.

This is a model with multiplicative row and column effects, and hence is very closely related to the chain ladder. It is the same as the stochastic chain ladder of Hertig (1985) except that Hertig assumed the following in place of (4.2):

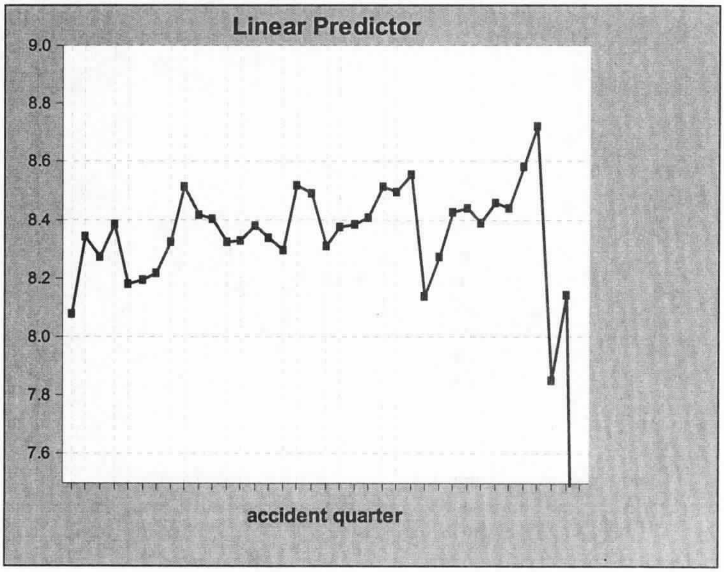
$$\varepsilon_{ij} \sim N(0, \sigma_j). \qquad (4.2a)$$

Though related to the chain ladder of the type discussed in Section 3, models of this type differ from it, as was established by the exchange between Mack (1993, 1994), Mack (2000), Verrall (2000) and England and Verrall (2000).

Stochastic versions of the chain ladder have received extensive treatment in the literature (England and Verrall, 2002; Mack, 1993; Mack and Venter, 2000; Murphy, 1994; Renshaw, 1989; Verrall, 1989, 1990, 1991a, 1991b, 2000).

The coefficients β^d_j and β^a_i are no longer obtainable by simple averaging, but they are obtainable from simple (i.e. unweighted least squares) regression. Figure 4.2 gives the plot of the β^a_i against i.

Figure 4.2
Regression estimate of trend in average claim size by accident quarter



The plotted values become less reliable as one moves from left to right across the figure, because one is considering steadily less developed accident quarters. Hence the downward plunge at the right of the plot can be ignored. The indication is then that, when allowance for a development quarter trend of the type illustrated in Figure 4.1 is made, there remains an increasing trend in claim sizes over time.

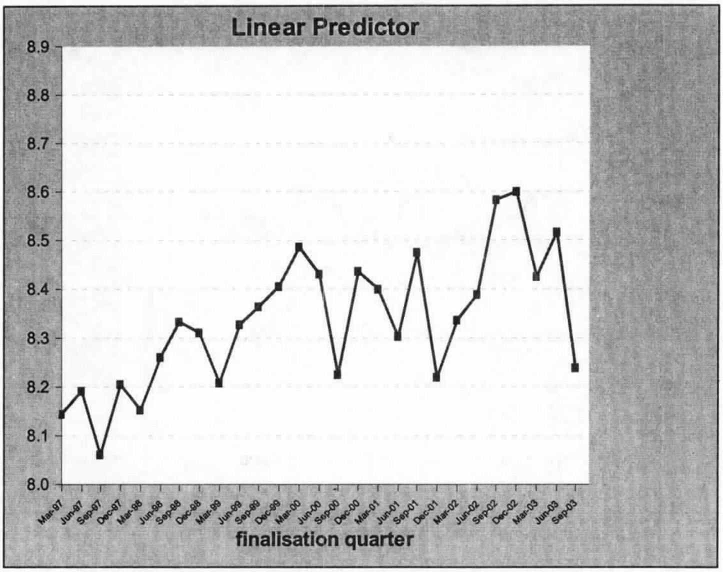
The possibility of a time trend has been incorporated in the model in the form (4.1a), in which the specific time dimension to which it is related is accident quarter, i.e. a row effect. It is possible, however, that the trend occurs over finalisation quarter, i.e. a diagonal effect, represented as follows:

$$\log S_{ij} = \beta^d_j + \beta^f_k + \varepsilon_{ij}, \tag{4.1b}$$

where k = i+j = calendar quarter of finalisation, and (4.2) is still assumed to hold.

Fitting this model to the data yields Figure 4.3 as the plot of the β^f_k against k. This also indicates a time trend. Adjudication on which of (4.1a) and (4.1b) provides the more appropriate representation of the trend may not be easy. This question will be deferred until Section 7 when rather more modelling apparatus is in place.

Figure 4.3
Regression estimate of trend in average claim size by finalisation quarter



4.2 Claim development measured by operational time

The use of operational time as a measure of claim development was introduced in Section 3.3. The models of Section 4.1 may be re-formulated on the basis of it.

The operational time defined in (3.3) related to the end-point of time represented by the (i,j) cell. This was appropriate to the context of average claim sizes that were cumulative to that point. In the context of non-cumulative averages, as currently, the mid-value of operational time for the cell is more appropriate. This is

$$\begin{aligned}\bar{t}_{ij} &= \tfrac{1}{2}\left[t_{ij} + t_{i,j-1}\right] \\ &= \tfrac{1}{2}\left[G_{ij} + G_{i,j-1}\right]/N_i\end{aligned} \qquad (4.5)$$

with the convention in the case j=0 that $t_{i,-1} = G_{i,-1} = 0$.

The quantity $\bar{t}_{ij}$ is a continuous variate in the sense that it may take any value on the continuum [0,1]. It will be convenient, to convert it to a categorical variate by recognising ranges of values in which it might lie.

For the present example, the interval [0,1] has been divided into 50 sub-intervals, [0%,2%), [2%,4%),...,[98%,100%], labelled by the values 1,2,...,50. Then each cell average size S_{ij} may be written in the alternative notation S_{it}, where t is the label corresponding to the mid-quarter operational time $\bar{t}_{ij}$.

Then the re-formulation of model (4.1) in which j is replaced by $\bar{t}_{ij}$ as a measure of development is as follows:

$$\log S_{it} = \beta_t + \varepsilon_{it}, \qquad (4.6)$$

with

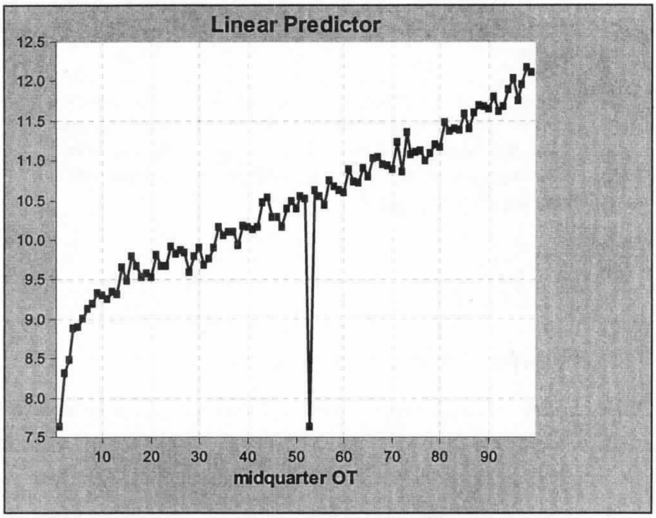
$$\varepsilon_{it} \sim N(0, \sigma). \qquad (4.7)$$

the corresponding re-formulations of (4.1a) and (4.1b) are as follows:

$$\log S_{it} = \beta^d_t + \beta^a_i + \varepsilon_{it} \qquad (4.6a)$$
$$\log S_{it} = \beta^d_t + \beta^f_k + \varepsilon_{it}. \qquad (4.6b)$$

The three models (4.6), (4.6a) and (4.6b) produce the plots in Figures 4.4 to 4.6 in place of 4.1 to 4.3.

Figure 4.4
Regression estimate of trend in average claim size by operational time



Note: The observation at operational time 53 should be ignored as it relates to a point with no data.

Figure 4.5
Regression estimate of trend in average claim size by accident quarter

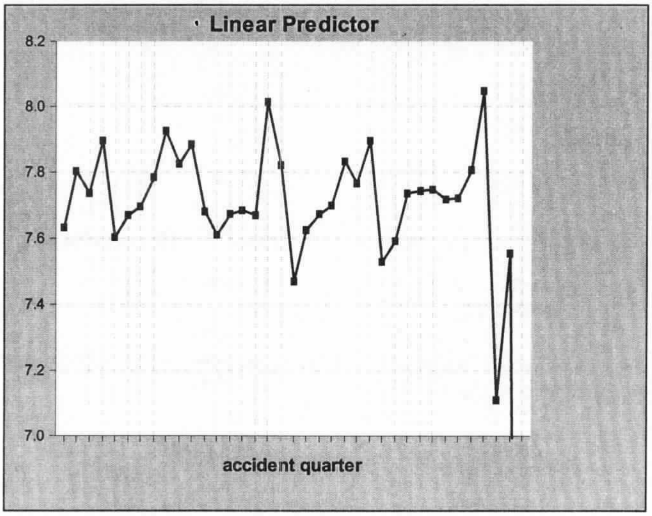
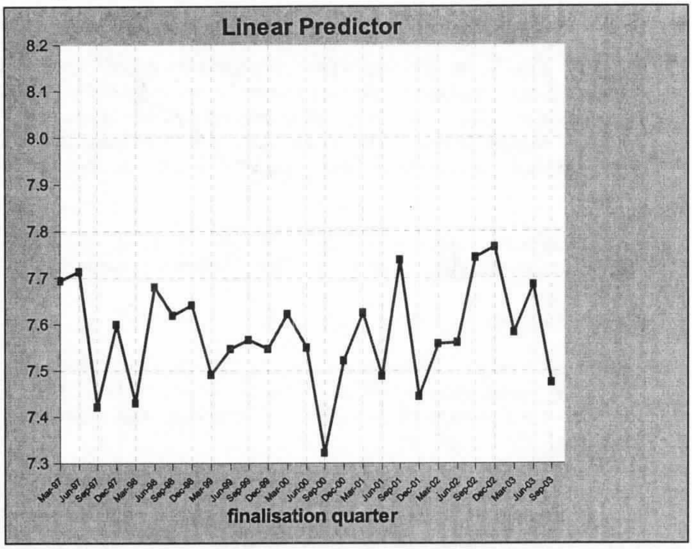


Figure 4.6
Regression estimate of trend in average claim size by finalisation quarter



It is interesting to note, in connection with Figure 4.4, that the use of operational time appears also to have simplified the relation between average claim size and the measure of development of an accident quarter. Indeed, average claim size appears closely approximated by an exponential function of operational time over the interval of roughly [10%,100%].

The actuary responsible for loss reserving against the example data set will by now have reached the following position:

- Any conventional application of a paid loss CL is dubious (Section 3.2).
- It appears that analysis of average claim sizes may be preferable (Section 4.1).
- It may also be desirable to take operational time into account somehow (present sub-section).
- The incorporation of a paid loss development pattern (as a function of operational time) together with the simultaneous identification of a time trend was achieved in Figures 4.4, 4.5 and 4.6 by means of regression.

Further progress by means of modification of a CL model appears difficult in the face of these observations.

5. Modelling individual claim data

5.1 Regression models

If one is impelled toward some form of regression modelling such as in Section 4.2, there is an argument that the regression may as well be carried out by reference to individual claim data as to the triangular summaries used there. The same models as applied in Section 4.2 can be formulated in terms of individual claims, and the use of data summaries then seems unnecessary and artificial.

As a preliminary to this, it will be useful to express (4.6) and its variants in a form more conventional for regression. Thus, (4.6) may be written as:

$$\log S_{it} = X_{it}\,\beta + \varepsilon_{it}, \qquad (5.1)$$

where β is the vector of quantities β_t, viz. $(\beta_1, \beta_2,\ldots,\beta_{50})^T$, with the superscript T denoting matrix transposition, and X_{it} is the row vector $(X_{it1}, X_{it2},\ldots,X_{it50})$ with $X_{itm} = 1$ if operational time label m is associated with S_{it}, and $X_{itm} = 0$ otherwise.

Thus the operational time variate in (4.6) is represented as a 50-vector of binary components. Regression variates of this type are often referred to as **class variates, or factor variates**. The numerical values corresponding to the binary components are called **levels**. Factor variates enable further simplification of the regression equation, with (5.1) being written as:

$$\log S = X\,\beta + \varepsilon, \qquad (5.2)$$

where log S is (with a slight abuse of notation) the column n-vector of all observations $\log S_{it}$, taken in any convenient order, X is the nx50 matrix formed by stacking the n row vectors X_{it}, taken in the same order as the $\log S_{it}$, and ε is the n-vector of the ε_{it}, also taken in the same order.

Let Y_r denote the size of the r-th finalised claim. This claim will have associated values of i, j and k=i+j=calendar quarter of finalisation. It will also have an associated value of t=operational time at finalisation. Let this collection of observations on the r-th claim be denoted i_r, j_r, k_r, t_r.

The quantity t_r may denote operational time specifically, or it may be converted to the categorical form described in Section 4.2. The latter is chosen for the purpose of the present paper.

The model described by (4.6) and (4.7) requires very little modification for application to individual claims. Expressed in the form (5.1), it becomes:

$$\log Y_r = X_r\,\beta + \varepsilon_r, \qquad (5.3)$$

with

$\varepsilon_r \sim N(0, \sigma)$ (5.4)

where X_r is the value of the operational time class variate applicable to the r-th claim and ε_r is the stochastic error term ε_{it} associated with it.

Just as (5.1) was notationally contracted to (5.2), so (5.3) may be abbreviated to:

$$\log Y = X\,\beta + \varepsilon, \quad (5.5)$$

The general idea underlying the models of Section 4.2 is that Y_r takes the form:

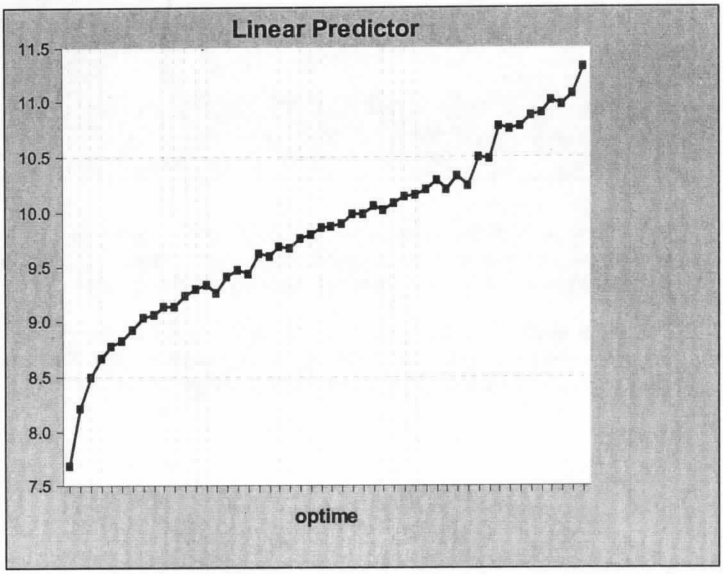
$$\log Y_r = \text{function}(i_r, j_r, k_r, t_r) + \text{stochastic error} \quad (5.6)$$

and that this may be written in the linear form (5.3), and hence (5.5), with X_r denoting a row composed of variates derived from i_r, j_r, k_r, t_r. These may or may not be factor variates.

5.2 Basic trends

Consider the model represented by (5.3) and (5.4), with X_r denoting the operational time factor variate discussed there. Ordinary least squares regression estimation of β yields Figure 5.1, which plots the components β_1, $\beta_2, \ldots, \beta_{50}$ of β against their associated midpoint operational times 1, 3,…,99.

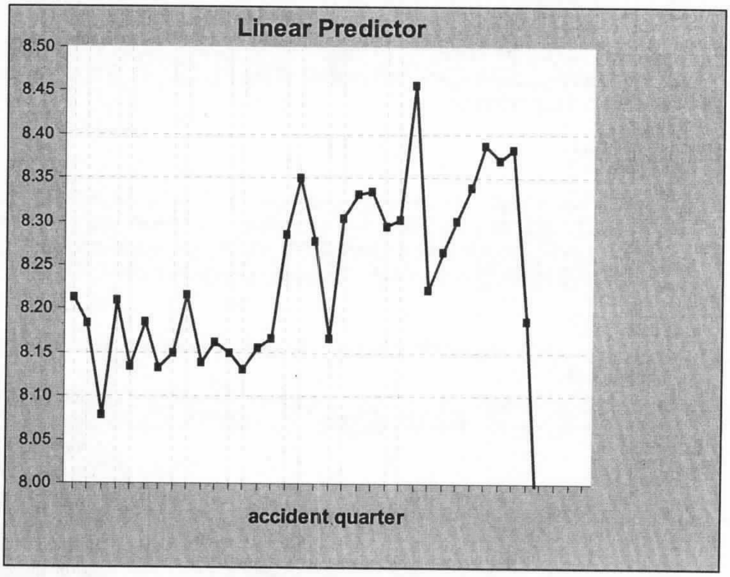
Figure 5.1
Individual claim regression estimate of trend in average claim size by operational time



Not surprisingly, Figure 5.1 closely resembles Figure 4.4, although Figure 5.1 exhibits greater smoothness due to the fact that it is based on about 60,000 observations, compared with ½x38x39=741 in the case of Figure 4.4.

The other models of Section 4.2, namely (4.6a) and (4.6b), may also be adapted to the form (5.3) and (5.4). The adaptation of (4.6a), for example, yields a version of (5.3) in which X_r comprises factor variates for operational time and accident quarter respectively. Figure 5.2 plots the components of the parameter vector β relating to accident quarter.

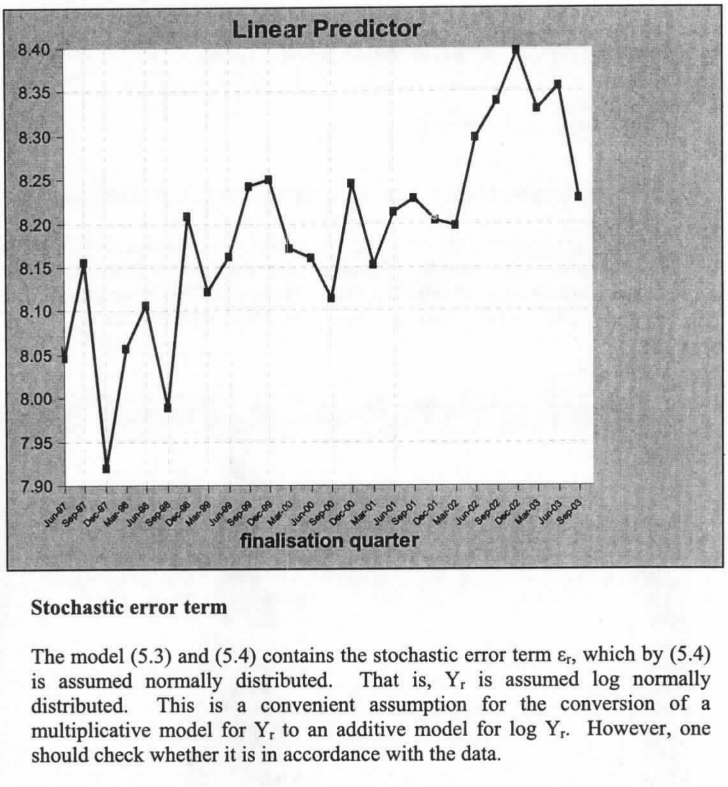
Figure 5.2
Individual claim regression estimate of trend in average claim size by accident quarter



The adaptation of (4.6b) is similar but with X_r comprising factor variates for operational time and finalisation quarter respectively. Figure 5.3 plots the components of the parameter vector β relating to finalisation quarter.

The trends displayed in Figures 5.2 and 5.3 differ somewhat from those in Figures 4.5 and 4.6. Presumably, the additional information included in the regression through the use of individual claims has improved their estimation.

Figure 5.3
Individual claim regression estimate of trend in average claim size by finalisation quarter



5.3 Stochastic error term

The model (5.3) and (5.4) contains the stochastic error term ε_r, which by (5.4) is assumed normally distributed. That is, Y_r is assumed log normally distributed. This is a convenient assumption for the conversion of a multiplicative model for Y_r to an additive model for log Y_r. However, one should check whether it is in accordance with the data.

This question may be investigated by means of residual plots. The residuals naturally adapted to the normal distribution are the **Pearson residuals**, defined as follows.

Consider the general model (5.5) and let $\hat{\beta}, \hat{\sigma}$ denote the regression estimates of β, σ respectively. Define

$$\mu = E[\log Y] = X\beta \quad (5.7)$$

and

$$\hat{\mu} = X\hat{\beta}, \quad (5.8)$$

the estimate of μ, and hence the **fitted value** corresponding to Y.

The Pearson residual associated with observation Y_r is

$$R_r^P = \left(\log Y_r - \hat{\mu}_r\right) / \hat{\phi}^{\frac{1}{2}} \tag{5.9}$$

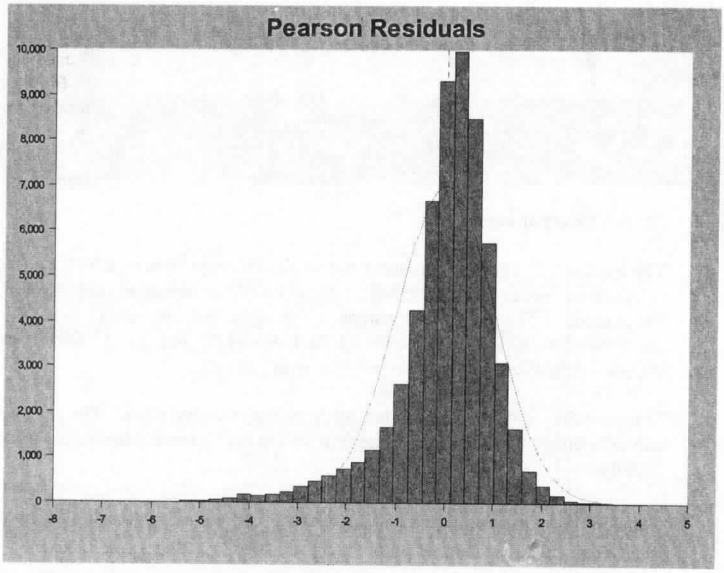
where $\hat{\phi}$ is the following estimator of $V[R^P_r]$:

$$\hat{\phi} = \sum_{r=1}^{n} \left(\log Y_r - \hat{\mu}_r\right)^2 / (n-p) \tag{5.10}$$

with p the dimension of the vector β, i.e. the number of regression parameters.

The Pearson residuals should be approximately unit normal distributed for large samples subject to (5.4). Figure 5.4 plots them for the model underlying Figure 5.3, indicating substantial negative skewness. This is confirmed by the alternative views of the residuals presented in Figures 5.5 and 5.6.

Figure 5.4



This suggests that the logarithmic transformation has over-corrected for the long tail of the Y_r, i.e. these observations, while right skewed, are shorter tailed than log normal. In this event, the choice of working with log transformed data, as in (5.5) is a poor one.

Figure 5.5

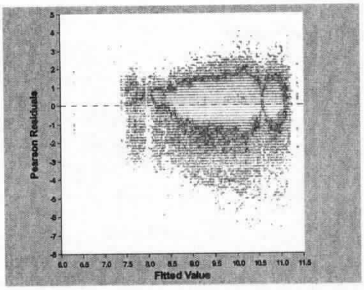
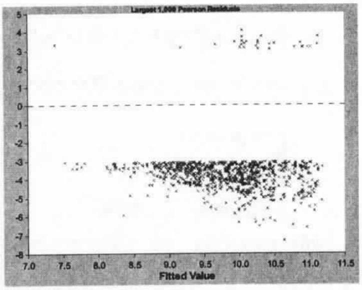


Figure 5.6



6. The exponential dispersion family and generalised linear models

6.1 The exponential dispersion family

One actually requires a distribution of the ε_r that lies between normal and log normal in terms of long-tailedness. The **exponential dispersion family (EDF)** of likelihoods (actually quasi-likelihoods) provides a comprehensive family within which to search for a distribution with suitable tail length.

The EDF comprises the following family of quasi-likelihoods (Nelder and Wedderburn, 1972):

$f(y;\theta,\lambda) = a(\lambda,y) \exp \lambda\, [y\theta - b(\theta)]$
(6.1)

where θ,λ are parameters and a(.) and b(.) are functions characterising the member of the family.

It may be shown that, for this distribution,

$$E[Y|\theta,\lambda] = b'(\theta) \quad (6.1)$$
$$Var[Y|\theta,\lambda] = b''(\theta)/\lambda \quad (6.2)$$

Denote $b'(\theta)$ by $\mu(\theta)$ whence, provided that $\mu(.)$ is one-one,

$$Var[Y|\theta,\lambda] = V(\mu)/\lambda \quad (6.3)$$

for some function V(.) called the **variance function**.

Many applications of the EDF restrict the form of the variance function thus:

$$V(\mu) = \mu^p \quad (6.4)$$

for some constant $p \geq 0$. This likelihood will be referred to as **EDF(p)**.

the quantity $\varphi = 1/\lambda$ is called the **scale parameter**.

Special cases of the EDF are:

p=0: normal
p=1: Poisson
p=2: gamma
p=3: inverse Gaussian.

6.2 Generalised linear models

Now let Y be a random n-vector, as in Section 5. Suppose $Y_1, Y_2, \ldots, Y_n$ to be stochastically independent drawings from the EDF likelihoods

$$f(y_r;\theta_r,\lambda) = a(\lambda,y_r) \exp \lambda \, [y_r\theta_r - b(\theta_r)] \tag{6.5}$$

where the same λ, a(.) and b(.) apply to all r.

Suppose further that $\mu(\theta_r)$ takes the form

$$\mu(\theta_r) = h^{-1}(X_r\beta) \tag{6.6}$$

for some one-one function h(.), called the **link function**, row p-vector X_r and column p-vector β.

With the same slight abuse of notation as occurred in connection with (5.2), the n relations (6.6) may be stacked into the form

$$\mu(\theta) = h^{-1}(X\beta) \tag{6.7}$$

where θ is the column n-vector with r-th component θ_r and X is an nxp **design matrix**. The n-vector $X\beta$ is called the **linear response**.

This specification of the vector Y is called a **Generalised Linear Model (GLM)** (Nelder and Wedderburn, 1972). GLMs are discussed by McCullagh and Nelder (1989). Note that the general linear model arises as the special case of a GLM with normal error term and identity link function.

The parameter vector β may be estimated by maximum likelihood. Generally, closed form solutions are not available, but various software products perform the estimation, e.g. SAS, S-Plus, EMBLEM. This paper uses the last of these, an interactive package produced by EMB Software Ltd of the UK.

Maximisation of the likelihood $L[Y|\theta,\lambda]$ is equivalent to minimisation of the so-called **deviance** $D[Y|\theta,\lambda]$ where

$$D[y \mid \theta,\lambda] = -2\log L[y \mid \theta,\lambda]$$
$$= -2\sum_{r=1}^{n}\{\lambda[y_r\theta_r - b(\theta_r)] + \log a(\lambda, y_r)\} \quad (6.8)$$

6.3 Residuals

In the more general setting of a GLM, the Pearson residual (5.9) becomes

$$R_r^P = (Y_r - \hat{\mu}_r)/[\hat{\phi}V(\hat{\mu})]^{\frac{1}{2}} \quad (6.9)$$

where the observations are now the Y_r instead of the log Y_r, $\hat{\beta}$ is the estimated value of β, $\hat{\mu}=h^{-1}(X\hat{\beta})$ is now the fitted value defined in parallel with (5.8), with $X\hat{\beta}$ now called the **linear predictor**, and

$$\hat{\phi} = D[Y \mid \hat{\phi},\hat{\lambda}]/(n-p). \quad (6.10)$$

Note that, for the identity link and normal error, (5.10) and (6.10) are the same. Then (5.9) and (6.9) are also the same since, for the normal case, $V(\mu)= \mu^0=1$.

Interpretation of Pearson residuals may be difficult for non-normal observations. Since the residual is just a linear transformation of the observation, any feature of non-normality, such as skewness, will be carried directly from one to the other.

An alternative form of residual is often helpful in these circumstances. Note that the deviance (6.8) may be written in the form (argument suppressed for brevity)

$$D = \sum_{r=1}^{n} d_r \quad (6.11)$$

where

$$d_r = -2\log L_r \quad (6.12)$$

with $\log L_r$ the contribution of Y_r to $\log L$.

Now define the **deviance residual**

$$R_r^D = \operatorname{sgn}(Y_r - \hat{\mu}_r)d_r^{\frac{1}{2}} \quad (6.13)$$

The advantage of deviance residuals is that they tend to be closer to normal than Pearson in their distribution. A variant is the **studentised standardised deviance residual**

$$R_r^{SSD} = R_r^D / \left[\hat{\phi}(1 - z_r)\right]^{\frac{1}{2}} \tag{6.14}$$

where z_r is the r-th diagonal element of the nxn matrix $X(X^TX)^{-1}X^T$. These residuals tend to have a distribution close to unit normal.

7. Application of GLM to data set

7.1 Loss reserving with GLMs

Although the use of GLMs in loss reserving is not widespread, it is also not new.

The use of general (as distinct from generalised) linear models can be seen in Taylor and Ashe (1983), Ashe (1986) and Taylor (1988). These two authors were in fact using GLMs for loss reserving consulting assignments during the 1980's.

The general linear model is also inherent in the loss reserving of De Jong and Zehnwirth (1983), based on the Kalman filter, and the related ICRFS software (Zehnwirth, 2003), marketed since the late1980's.

Wright (1990) gave a comprehensive discussion of the application of GLMs to loss reserving. Taylor, McGuire and Greenfield (2003) also made use of them.

All of these models other than in the last reference were applied to summary triangles of claims data, such as used in Section 4, rather than individual claims.

7.2 Choice of error distribution

As suggested at the start of Section 6.1, one requires an error distribution that lies between normal and log normal in terms of long-tailedness. Experimentation might begin with a gamma distribution. This is a more realistic distribution of claim sizes than normal, its density having strictly positive support and positive skewness. It is, however, considerably shorter tailed than log normal.

Consider the gamma (i.e. EDF(2)) GLM corresponding to (5.5). It has the same X and β, but observations are Y_r instead of log Y_r, and the link function is log. For example, the particular form of this model adapted to (4.6b) is as follows:

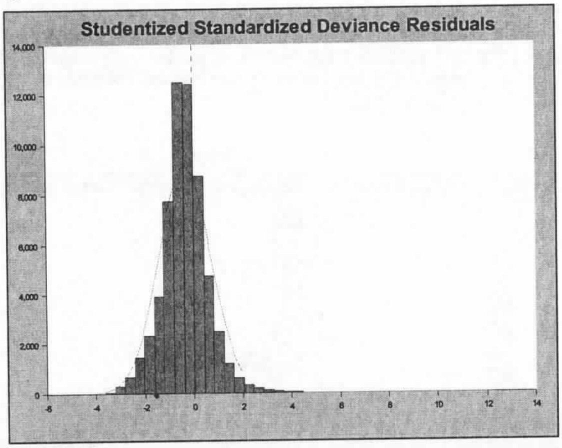
$$Y_r \sim EDF(2) \tag{7.1}$$

$$E[Y] = \exp X\beta = \exp [X^d\beta^d + X^f\beta^f] \tag{7.2}$$

where X^d and X^f are factor variates for operational time and finalisation quarter respectively.

Fitting this model to the data set yields the residual plots set out in Figure 7.1.

Figure 7.1



Comparison of Figure 7.1 with 5.4 reveals that the use of a gamma rather than log normal error has corrected the most obvious left skewness of the residuals. However, Figures 7.2 and 7.3 give more detail of the residuals and indicate that they are not altogether satisfactory.

Figure 7.2

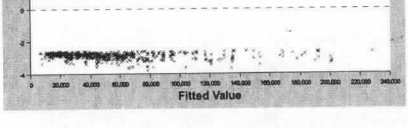
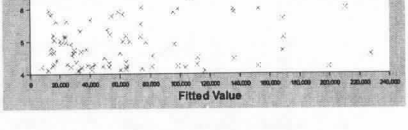


Figure 7.3



The studentised standardised residuals are expected to resemble standardised unit normal residuals. The largest 1,000 of these (from 60,050 observations) would numerically exceed 2.4. Figure 7.2 conforms reasonably well with this requirement, displaying residuals numerically exceeding a threshold value of roughly 2.6.

However, extreme values, up to 12, appear, indicating a much longer tail than normal. This abnormality in the residual plot is emphasised in Figure 7.3, which displays the largest 100 residuals. The unit normal range for these has a threshold value of about 3.1. the observed threshold exceeds 4, and all 100 residuals are positive.

These properties of the residual plots indicate that the distribution of claims sizes is longer tailed than gamma. As indicated by (6.3) and (6.4), a larger EDF exponent p will generate a longer tail. Therefore, one experiments with values of p>2 (gamma). Figures 7.4 to 7.6 are the residual plots for EDF(2.3) corresponding to Figures 7.1 to 7.3.

Figure 7.4

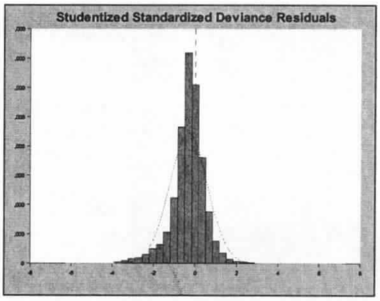


Figure 7.5

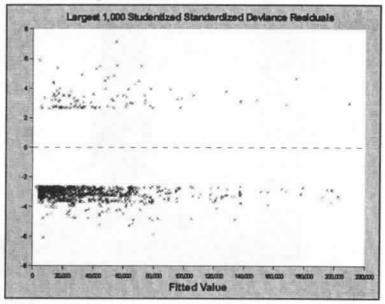


Figure 7.6

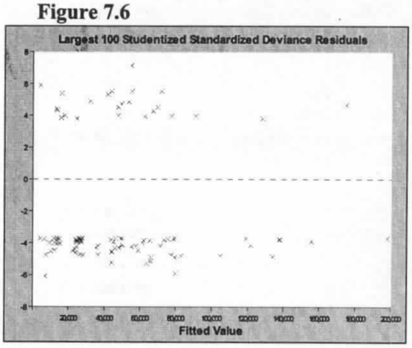


Figure 7.7

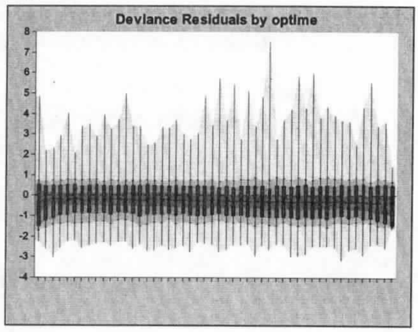


Figure 7.4 shows that the shift to the longer tail of EDF(2.3) has over-compensated somewhat for the right skewness, producing a degree of left skewness. Figure 7.5 shows little change in the threshold value of the largest 1,000 residuals. However, Figure 7.6 shows considerable improvement in the treatment of the extreme tail.

The final choice of claim size distribution needs to balance these observations. Generally, the improved treatment of the tail would be expected to improve robustness of the parameter estimation such that this more than offsets the unwanted skewness near the centre of the distribution. The choice of EDF(2.3) will be retained for the remainder of this paper.

There is a practice, common among actuaries, of separately analysing "small" and "large" claims, however defined, on the ground that the latter group are liable to distort the averaging processes inherent in modelling. It is worth remarking that the explicit incorporation of a (relatively) long tailed error distribution in the model (such as EDF(2.3) as above), and the adoption of a procedure for parameter estimation that is consistent with this distribution, may eliminate the need for this practice.

Figure 7.7 displays a further residual plot in which residuals are plotted in box-whisker form against operational time. The boxes correspond to the range between 10- and 90-percentiles, and the markers on the whiskers are placed at the 5- and 95-percentiles.

Once a tentative choice of claim size distribution has been made, it is necessary to examine plots of this type against each independent variate. These examinations seek two things:

- Trendlessness from left to right (horizontality of the box centres)
- Rough equality of dispersion (boxes all of about the same size).

Violation of the first requirement indicates some dependency of the dependent variable on the independent variate, not already accounted for in the model. The second requirement checks for homoscedasticity, i.e. that (6.3) holds for a value of φ that is constant over the entire range of the independent variate under scrutiny.

7.3 Refinement of the model design

7.3.1 Operational time

The model discussed in Section 7.2 still has the very elementary form set out in (7.1) and (7.2). The factor variate X^d, defined in Section 5.1, has 50 levels, which means that β^d contributes 50 parameters to the model. Inspection of Figure 5.1 indicates, however, these 50 parameters can be closely represented as linearly related to operational time over much of the latter's range.

Write (7.2) in the form:

$$E[Y_r] = \exp X_r\beta = \exp [X^d_r\beta^d + X^f_r\beta^f] \qquad (7.3)$$

where X^d_r and X^f_r are the values of the factor variates X^d and X^f assumed by the r-th observation.

Now replace this by the form:

$$E[Y_r] = \exp X_r\beta = \exp [\beta^d_1 t_r + \beta^d_2 \max(0,10\text{-}t_r) + \beta^d_3 \max(0,t_r-80) + X^f_r\beta^f] \quad (7.4)$$

where t_r is the value of operational time applying to the r-th observation, and β^d_1, β^d_2 and β^d_3 are scalar parameters.

This is equivalent to representing the operational time trend in Figure 5.1 as a piecewise linear trend with breaks in gradient at operational times 10 and 80. The factor variate has been replaced by a set of **continuous variates.**

This enables operational time to be accommodated in the model by means of just 3 parameters, rather than 50. The factor variate representation of finalisation quarter is retained for the time being.

If the model (7.4) is fitted to the data, with error term EDF(2.3), as suggested by Section 7.2, the operational time component of (7.4) is as shown by the piecewise linear plot in Figure 7.8. It is superimposed on the factor variate plot in the figure. The correspondence between the two representations is seen to be quite good, indicating that the 3-parameter representation captures essentially all the information of the 50-parameter one.

7.3.2 Superimposed inflation

Similar economies in the representation of finalisation quarter can be made. Figure 7.9 shows the plot of the parameter vector β^f in the case of a factor variate fitted in the presence of the continuous representation of operational time, as in (7.4).

Figure 7.8
Continuous operational time variate

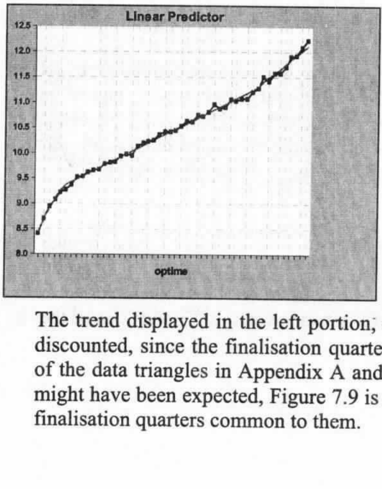
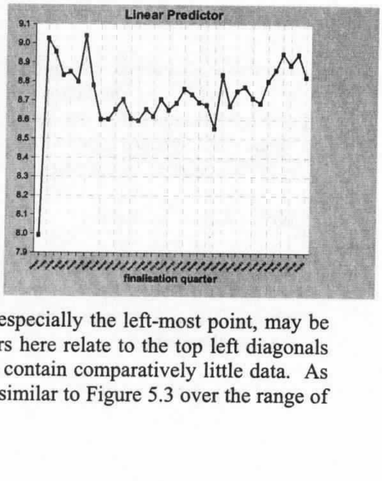


Figure 7.9
Factor variate representation of finalisation quarter



The trend displayed in the left portion, especially the left-most point, may be discounted, since the finalisation quarters here relate to the top left diagonals of the data triangles in Appendix A and contain comparatively little data. As might have been expected, Figure 7.9 is similar to Figure 5.3 over the range of finalisation quarters common to them.

One possibility would be to fit a linear trend from the beginning of 1997. An appropriate choice of model for the earlier finalisation quarters is unclear but, in view of the small quantity of data represented here and its antiquity, the model chosen is unlikely to affect estimation of a loss reserve unduly.

Consequently, Figure 7.10 relates to a model in which the linear trend assumed to apply to finalisation quarters from 1997 onwards is cavalierly assumed to apply to the earlier ones also, though with a step in claim sizes occurring at the start of 1997.

In this case, (7.4) is replaced by:

$$E[Y_r] = \exp [\alpha + \beta^d_1 t_r + \beta^d_2 \max(0,10\text{-}t_r) + \beta^d_3 \max(0,t_r - 80) + \beta^f_1 k_r + \beta^f_2 I(k_r < 97Q1)] \qquad (7.5)$$

where k_r is the number of the finalisation quarter applying to the r-th observation, α, β^f_1 and β^f_2 are scalar parameters, and generally I(.) is the indicator function defined as follows:

$$I(c) = 1 \text{ if condition c holds;} \quad = 0 \text{ if it does not.} \qquad (7.6)$$

The constant α now becomes necessary, having previously been absorbed into β^f.

Figure 7.10
Continuous finalisation quarter variate

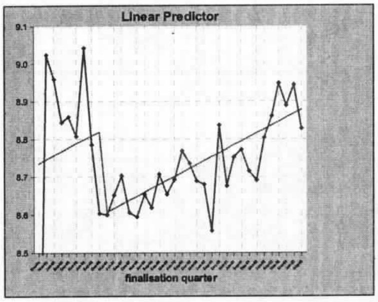
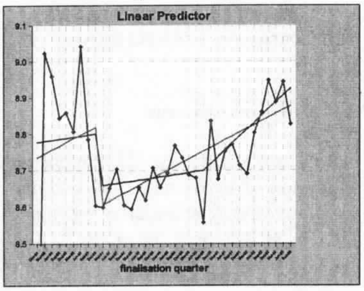


Figure 7.11
Additional break in the finalisation quarter trend



The comparison in Figure 7.10 between the trend of constant gradient over finalisation quarter and the corresponding factor variate hints at an increase in gradient over the more recent finalisation quarters. Figure 7.11 therefore represents an alternative model in which the gradient changes at the end of the September 2000 quarter.

Formally, the model (7.5) is replaced by:

$$E[Y_r] = \exp[\alpha + \beta^d_1 t_r + \beta^d_2 \max(0,10-t_r) + \beta^d_3 \max(0,t_r-80) + \beta^f_1 k_r + \beta^f_2 \max(0,k_r - 2000Q3) + \beta^f_3 I(k_r<97Q1)]. \quad (7.7)$$

One will need to make a choice between models (7.4), (7.5) and (7.7), and possibly others. The choice can be made on the basis of the so-called information criteria, which reward goodness-of-fit but penalise additional parameters. For example, the **Akaike Information Criterion (AIC)** (Akaike, 1969) is defined as:

$$AIC = D + 2p \quad (7.8)$$

where D denotes deviance and p number of parameters. Models with low values of the AIC are to be preferred.

Table 7.1 gives values of the AIC for the three models under consideration, showing that:

- The factor variate model is dramatically inferior to the two involving continuous finalisation quarter variates; and
- Model (7.7), allowing for a change in gradient of the trend is the best of the three.

Table 7.1
AIC for different models of finalisation quarter effect

Model of finalisation quarter effect	AIC
Factor variate (7.4)	-14,517.6
Constant gradient trend (7.5)	-14,566.6
Change in gradient of trend (7.7)	-14,567.1

7.3.3 Interaction terms

The trend over finalisation quarter measures the increase in claim sizes in real terms over calendar time, and may therefore be interpreted as SI. Figure 7.11 indicates that the preferred model estimates the factor of increase as about exp(0.22) over the 3 years from September 2000 to September 2003, or equivalently more than 7% per annum.

While it is quite possible for smaller bodily injury claims to inflate at this rate, it is less usual for the larger and catastrophic claims. A question arises, therefore, as to whether larger and smaller claims might be subject to differing rates of SI.

If operational time is adopted as a proxy for distinguishing between large and small claims, then one might investigate whether different operational times are subject to different rates of SI. This is done by searching for statistically significant interaction effects between operational time and finalisation quarter.

For this purpose, the 0-100 range of operational time is divided into the following 7 bands: 0-6, 6-14, 14-22, 22-40, 40-60, 60-80, 80-100, denoted $b_1,\ldots b_7$ respectively. Let X^{bt} denote the banded operational time factor variate, and let X^{bt}_r be its value for the r-th observation.

The following model is then fitted:

$$E[Y_r] = \exp\,[X^{ct}_r\beta^{ct} + X^{bt\otimes cf}_r\beta^{bt\otimes cf}] \tag{7.9}$$

where X^{ct} represents the set of three continuous operational time variates appearing in (7.7), X^{cf} represents the set of three continuous finalisation quarter variates in the same expression, and $X^{bt\otimes cf}$ denotes the 21-component vector of variates formed as the cartesian product of the 7-component X^{bt} and 3-component X^{cf}. Cartesian products of this type are called **interaction variates** in GLM parlance.

Model (7.9) may be written in the equivalent form:

$$E[Y_r] = \exp\,\{\alpha + \beta^d_1\, t_r + \beta^d_2 \max(0,10\text{-}t_r) + \beta^d_3 \max(0,t_r-80) +$$

$$\sum_{m=1}^{7} I(t_r \otimes b_m)\,[\beta^f_{m1}\, k_r + \beta^f_{m2} \max(0,k_r - 2000Q3) + \beta^f_{m3}\, I(k_r<97Q1)] \tag{7.10}$$

whose square bracketed member retains the same functional dependency on finalisation quarter as in (7.7), but separately for each operational time band. Note that the coefficients β^f_{m1}, β^f_{m2}, β^f_{m3} represent SI in operational time band b_m.

Figure 7.12 provides a display of the interaction term when (7.9) is fitted to the data. Here "opband7(m)" denotes band b_m. For each of these bands, the model's linear predictor, as defined in Section 6.2, is plotted for t_r=0. Features of the plot are as follows:

- The general level of claim size is seen to increase with increasing operational time band (as in Figure 7.8)
- While Figure 7.11 indicated the period since September 2000 to be subject to an increased rate of SI, it is now seen that this is confined to the operational time bands b_2, b_3, and b_4, which cover operational times 6-40. As hinted at the start of the present sub-section, the increased SI does not apply to the larger claims settled at the high operational times.
- The rate of SI over recent periods, which is measured by the gradients of the paths appearing in Figure 7.12, peaks in operational time bands b_3 and b_4, i.e. in the range 14-40.

The last remark suggests that the interaction terms represented by the summation in (7.10) can be simplified by means of continuous variates. An example of such a simplification is the following:

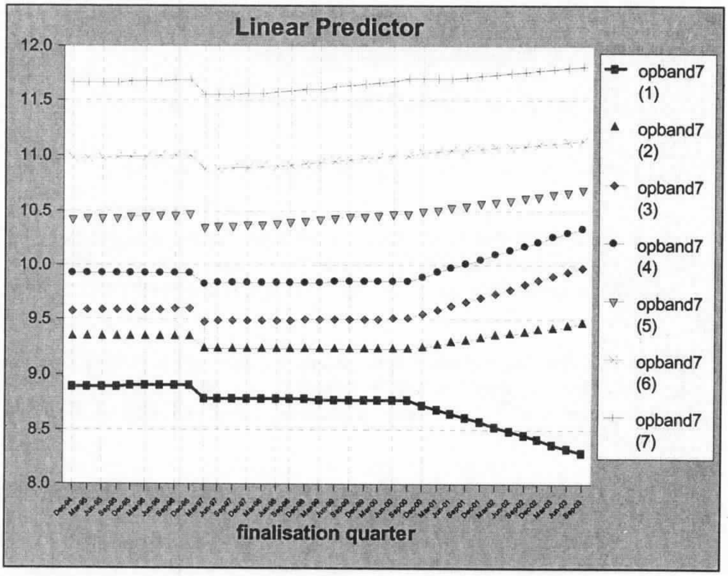
$$E[Y_r] = \exp\{\alpha + \beta^d_1 t_r + \beta^d_2 \max(0,10\text{-}t_r) + \beta^d_3 \max(0,t_r-80) + \beta^f_1 k_r + \beta^f_2 \max(0,k_r - 2000Q3) + \beta^f_3 I(k_r<97Q1) + \gamma(t_r)[\beta^{tf}_1 + \beta^{tf}_2 \max(0,k_r - 2000Q3)]\} \quad (7.11)$$

where

$$\gamma(t) = \min(15,\max(0,t\text{-}10)) - \min(15,\max(0,t\text{-}25)) \quad (7.12)$$

i.e. $\gamma(t)$ describes a function that is zero everywhere on the interval [0,100] except on the sub-interval (10,40), where it describes an isosceles triangle of height 15.

Figure 7.12
Interaction between SI and operational time



It can be seen that (7.11) comprises (7.7) plus a further term representing additional SI in the operational time range 10-40, at a rate that increases steadily from 0 at operational time 10 to a peak at operational time 25, and then declines steadily to 0 at operational time 40.

Fitting this model to the data produces the SI profile illustrated in Figure 7.13. Figure 7.14 provides the same type of display of model (7.11) as appears in Figure 7.12, and facilitates the comparison of model (7.11) with model (7.10). Here "opband7(m)" is as in the earlier figure, and "+opband7(m)" denotes the corresponding plot for the continuous model (7,11), i.e. the plot of the average linear predictor against k for t_r=0 and $t_r \otimes b_m$.

Figure 7.13
Profile of SI allowing for SI x operational time interaction

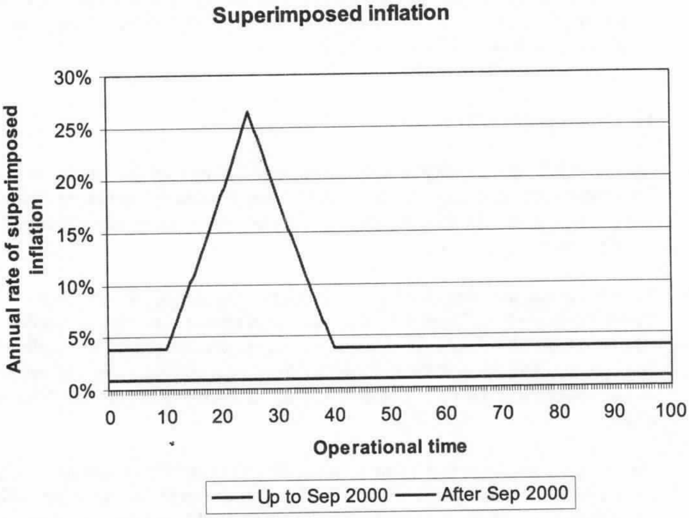
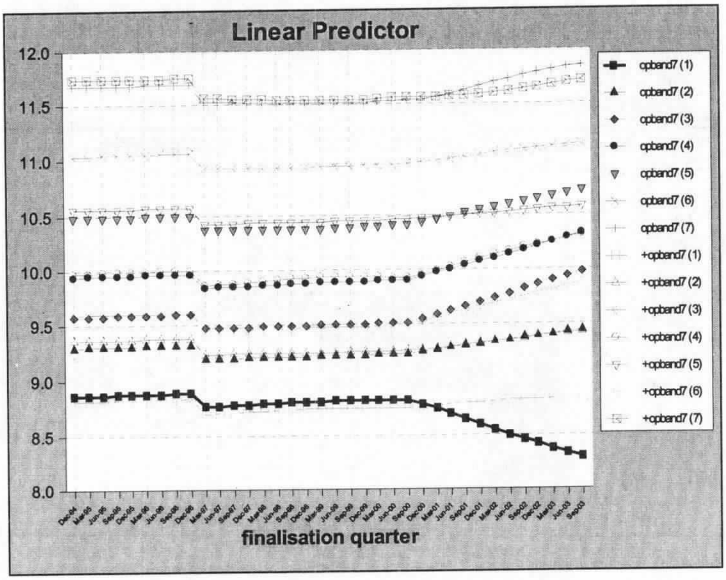


Figure 7.14
Interaction between continuous SI and operational time variates



The simplified model (7.11) is seen to produce a reasonable fit to the more elaborate (7.10). It would not be acceptable as it stands, as there are systematic discrepancies, particularly in relation to opband7(1). However, certain aspects of this model will be superseded in Section 7.3.4, and so detailed improvement of it is not pursued here.

7.3.4 Accident quarter effects

Section 7.3.3 has already noted the change in rate of SI at the end of September 2000, and how the rate changed much more at the low operational times than others. In fact, the legislation governing the scheme changed at precisely this date.

All subsequent accident periods were subject to limitations on payment of plaintiff costs, whose expected effect was to eliminate a certain proportion of smaller claims in the system. Larger claims were expected to be unaffected. The scheme of insurance, as modified by these changed rules, will be referred to as "**the new scheme**". Prior accident quarters make up the "the old scheme".

This strongly suggests that some or all of the SI observed at low operational times after September 2000 might constitute an accident quarter (row) effect rather than finalisation quarter (diagonal) effect. In this connection, it is noted from Figure 3.3 that virtually all of the exceptional operational times (<40) after September 2000 relate to the new scheme.

It is worthwhile returning to the average claim size data in respect of the new scheme. This is done in Table 7.2.

Table 7.2
Average sizes of claim finalisations for old and new schemes

Accident	Average claim sizes (in 30/09/03 values) in development quarter							
quarter	0	1	2	3	4	5	6	7
	$	$	$	$	$	$	$	$
Dec-99	547	6,035	8,934	11,699	18,397	18,062	26,086	32,139
Mar-00	5,050	5,185	6,958	14,904	13,504	20,746	22,489	27,879
Jun-00	2,910	4,177	7,433	10,275	13,895	18,916	26,206	32,897
Sep-00		6,512	7,116	9,917	14,163	24,034	27,392	41,851
Dec-00	221	2,977	4,175	7,571	10,869	17,505	24,393	29,700
Mar-01	792	2,498	4,605	10,000	11,581	20,672	29,574	39,969
Jun-01	1,271	3,342	5,683	7,936	16,207	21,294	34,237	40,814
Sep-01	1,258	3,516	5,127	12,012	21,726	25,997	26,019	38,150
Dec-01	1,355	2,623	5,225	11,374	19,439	22,548	35,709	28,963
Mar-02	1,594	2,658	7,018	14,700	16,768	26,827	26,851	
Jun-02	1,017	3,641	8,669	12,905	17,750	25,063		
Sep-02	3,484	3,303	5,982	14,379	18,852			
Dec-02	8,102	3,118	6,493	10,714				
Mar-03	1,182	2,454	2,931					
Jun-03	2,327	1,568						
Sep-03	103							

The heavy horizontal line in the table marks the passage from old to new scheme. Claim sizes are seen to decline instantaneously and substantially on introduction of the new scheme.

The shaded area marks one in which the reduction in claim size is maintained. Below this shaded area, however, claim sizes increase rapidly, and by the December 2002 finalisation quarter (the fourth last diagonal) are in excess of their old scheme counterparts.

The immediate reduction in claim sizes by the new scheme is certainly a row effect, and needs to be modelled as such. The subsequent increase in claim sizes can be viewed as either:

- a diagonal effect limited to low operational times (as in Section 7.3.3); or
- a row effect limited to low operational times.

In view of its likely origin in the new scheme, it is perhaps better regarded as the latter. This is the view taken in this paper, and reflected in the final model fitted to the data in Section 7.4. Details of the trend identification are similar to the examples dealt with above, and are not given here.

7.4 Final model

The final model fitted to the data set takes into account the issues discussed in Sections 7.1 and 7.2, and also includes a seasonal effect whereby the sizes of claims finalised in the March quarter tend to be slightly lower than in other quarters. It takes the following form:

$$
\begin{aligned}
E[Y_r] = \exp \{\alpha &+ \beta^d_1 t_r + \beta^d_2 \max(0,10-t_r) \\
&+ \beta^d_3 \max(0,t_r-80) + \beta^d_4 I(t_r < 8) && \textit{[Operational time effect]} \\
&+ \beta^s I(k_r = \text{March quarter}) && \textit{[Seasonal effect]} \\
&+ \beta^f_1 k_r + \beta^f_2 \max(0,k_r - 2000Q3) \\
&+ \beta^f_3 I(k_r < 97Q1) && \textit{[Finalisation quarter effect]} \\
&+ k_r [\beta^{tf}_1 t_r + \beta^{tf}_2 \max(0,10-t_r)] && \textit{[Operational time x finalisation quarter interaction]} \\
&+ \max(0,35-t_r) [\beta^{ta}_1 + \beta^{ta}_2 I(i_r > 2000Q3)]\} && \textit{[Operational time x accident quarter interaction]}
\end{aligned}
\quad (7.13)
$$

where i_r is the accident quarter applying to the r-th observation.

The model form (7.13) is set out in a series of components that isolate the different types of effects, labelled in italics on the right.

Comparison of it with (7.11) shows that:

- It retains the concept of an operational time x finalisation quarter interaction, though this now:
  - o has its peak rate of SI shifted from operational time 25 to 10; and
  - o this profile of SI applies to all finalisation periods, not just those that fall within the new scheme.
- There is heightened SI in the new scheme, but affecting all operational times, not just the low range.
- A part of what previously appeared as heightened SI in the new scheme is now accounted for as an accident period effect, with a one-off shift in claim size at introduction of the new scheme, the size of the shift being largest at the low operational times and gradually decreasing with increasing operational time, until petering out at operational time 35.

Table 7.3 compares the AIC for model (7.7) with the final model, showing a considerable improvement achieved by the latter.

Table 7.3
AIC for final model and model (7.7)

Model of finalisation quarter effect	AIC
Model (7.7)	-14,567.1
Final model (7.13)	-14,588.9

7.5 Validation of final model

While (7.13) may appear the best model achievable, it needs to satisfy a number of routine tests before its final acceptance. These are concerned with the properties of residuals, and are illustrated in Figures 7.15 to 7.20.

Figure 7.15

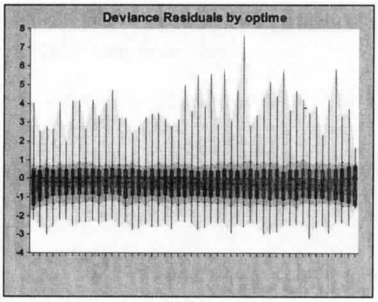
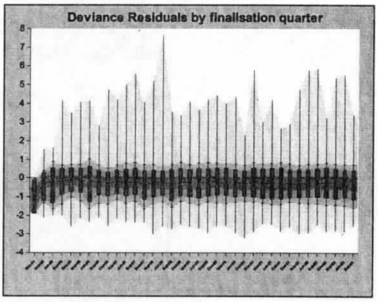


Figure 7.16



Figures 7.15 to 7.17 test for two things:

- Trendlessness, from left to right, with respect to the major variates, checking that no systematic trend in the data remains uncaptured by the model; and
- **Homoscedasticity**, i.e. constant dispersion from left to right.

Both of these tests are concerned just with trends rather than with the magnitude of the residuals. Hence standardisation is unnecessary (though it would do no harm), and just deviance residuals are displayed.

The possible trend at the extreme right of Figure 7.17 is, of course, based on very little data, as it relates to just the last three accident quarters. It has been ignored for the purposes of the present paper.

Figure 7.17

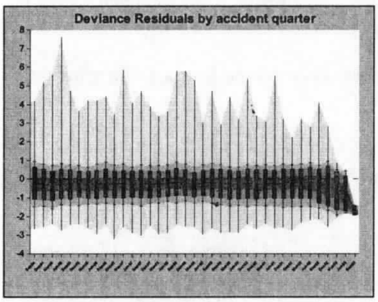


Figure 7.18

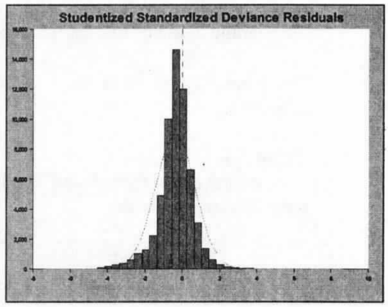


Figure 7.19

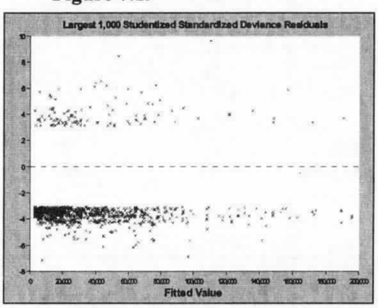
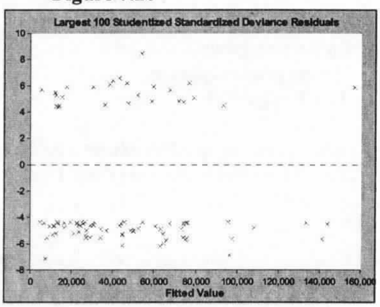


Figure 7.20



Figures 7.18 to 7.20 are concerned with the distribution of the residuals, with the same considerations as discussed in relation to Figures 7.4 to 7.6. Indeed, there is little difference to the naked eye between the two sets of graphs, showing that, once the EDF(2.3) error structure has been chosen, the rather extreme change in model from (7.2) to (7.13) has had little effect on the distribution of residuals.

7.6 Forecast of final model

Table 7.4 repeats Table 3.3, but supplemented by the loss reserve forecast by model (7.13). The following assumptions are made for the purpose of this forecast:

- The experience of finalised claims of an accident period is indicative of its ultimate average claim size.
- Future SI is as experienced to date in the new scheme.
- Future rates of claim finalisation are about the same as experienced over the most recent 8 quarters.

The first of these assumptions is fundamental to the forecasting methodology. It might be violated if, for example, at specific operational times, one observed a trend over time in the ratio of average amount paid to date on open claims to the average paid on finalised claims.

The second assumption has a major influence on the forecast, the third little influence.

Table 7.4
Loss reserves corrected and uncorrected for low September 2003 quarter paid loss experience

Averaging period	Loss reserve at 30 September 2003 (excluding September 2003 accident quarter)	
	Uncorrected	Corrected
	$B	$B
Chain ladder models:		
All experience quarters	1.61	
Last 8 experience quarters	1.68	
All experience quarters except September 2003	1.78	1.94
Last 8 experience quarters except September 2003	1.92	2.35
GLM (7.13)	2.23	

The GLM (7.13) generates a loss reserve near the top of the range of CL results. While there is reasonable agreement with the CL version derived from the experience of the last 8 quarters but one and corrected for the anomalous experience of the last quarter, this is a very detailed choice, and one has no means of determining this model to be superior to many other contenders.

For example, why 8 quarters? Why not 6? Or 10? Why correct for just the last quarter of experience? Why not the last 2? In any event, Table 7.5 shows that, while this version of the CL may produce a total reserve similar to that of the GLM, its composition by accident quarter is very different.

The former produces a reserve for the last accident year that is 19% higher than the GLM. This would lead to much higher estimates of average claim size, and hence to quite different pricing decisions for future underwriting periods.

Table 7.5
GLM and CL loss reserves by accident quarter

Accident quarter	Loss reserve at 30 September 2003 (excluding September 2003 accident quarter)	
	GLM (7.13)	CL based on last 8 experience quarters except the last - corrected
	$M	
Sep 94 – Dec 98	283	200
Mar 99 – Mar 02	1,122	1,174
Jun 02	154	183
Sep 02	159	199
Dec 02	160	201
Mar 03	173	206
Jun 03	179	192
Total	2,229	2,354

The validation devices represented in Figures 7.15 to 7.17 have the common feature that they are all 1-dimensional summaries of residuals. While the residuals may be trendless over the single dimension, finalisation quarter, and may also be trendless over the single dimension, accident quarter, it is possible that there are pockets of cells within the 2-dimensional triangle in which they tend to be systematically of the one sign.

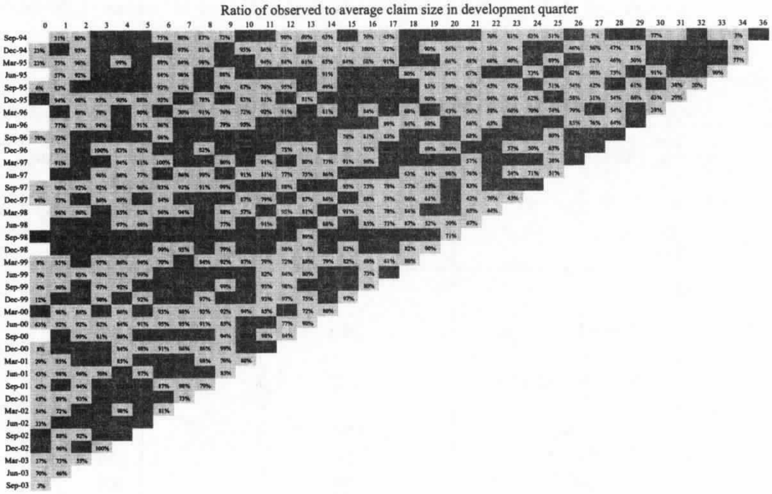
Figure 7.21 provides a simple test of such an eventuality. For each cell of the accident quarter/development quarter triangle, it records the ratio:

Observed average size of claim finalisation / GLM fitted average size.

These ratios are colour coded: red if greater than 100%, blue if less. The fact that the numerical values of the ratios are too small to be legible in the figure as reproduced here does not detract from its value A cursory examination of its colour patterns indicates a generally random scatter of red and blue.

There is no apparent congregation of cells of one or other colour in particular locations within the triangle. This confirms the trendlessness of the residuals over the whole of the 2-dimensional array.

Figure 7.21
Colour coded ratios of observed to fitted average claim sizes



8. Conclusions

The foregoing sections have dealt with a case study involving a loss triangle of obvious complexity. It contains multiple trends.

The triangle has been approached initially from the viewpoint of one with a predisposition to application of the CL. The trends then manifest themselves in the form of non-constancy of age-to-age factors over accident periods.

The complexity of the data set is reflected in the model of claim sizes fitted to it, which includes the following, in addition to the expected variation with operational time:

- a **seasonal effect**;
- **SI** whose rate varies with operational time, and also passes though one change-point;
- recognition of a **new scheme affecting accident periods** after its introduction, but with an effect that varies with operational time.

It is extremely difficult to accommodate such trends within the CL structure and estimate them efficiently. However, the GLM (7.13) adopted here does so **parsimoniously**, using just 13 parameters. This compares with the 73 parameters implicit in a CL applied to a triangle of dimension 37 even before the recognition of any trends.

The GLM is one example of a model with a **fully stochastic specification**, as opposed to the CL which is usually approached in practice as an algorithm (though the stochastic formulations mentioned in Section 4.1 may be noted). The stochastic framework provides a set of diagnostics that may be used to **compare candidate models** in a formal and organised manner, and to **validate the model** finally selected.

The stochastic framework also allows a choice of the **distributional form** from which observations are assumed drawn. This enables an informed treatment of **outliers**.

These properties of the GLM are seen to be more than academic as this model generates a loss reserve that differs vastly from some CL applications. While one CL model is found to produce a somewhat similar reserve (Section 7.6), there is no apparent reliable basis for distinguishing that model as superior to other CL models.

In any event, though the CL model in question appears to produce a total loss reserve that is approximately correct, its dissection by accident period appears quite wrong. Specifically, it over-estimates average claim sizes of recent accident periods by margins approaching 20%. Such estimates, if incorporated in the business process, would be liable to lead to quite **incorrect pricing decisions** for the ensuing underwriting periods.

Finally, but not of least significance, one emerges from the GLM fitting process described in Section 7 with a greatly **enhanced understanding** of one's data. Data exploration forms an integral part of the process, and the GLM provides the framework within which such exploration can be carried out efficiently.

The CL on the other hand provides a sausage machine, a rigid and unenquiring algorithm. This is an advantage in terms of required resources. Only relatively low-skilled resources are required to apply it in its unmodified form. A serious disadvantage to be set against this is that it may produce a totally wrong result, that it may give **precedence to process over substance**.

The CL model may be described as a multiplicative model with categorical accident and development period effects. This is a very simple design, which is highly convenient if justified. It is, however, a design that relies on an assumption of an identical process affecting every accident period.

Beyond this, it is phenomenological in the sense that there is no specification of what that process is. If evidence appears that the CL design is invalid, the lack of process specification leaves one with no indication of how the design should be modified.

One may attempt modification on some empirical basis, such as trending age-to-age factors, but the empiricism itself is a recognition of the lack of understanding of the process. Indeed, because of this, there is in our view a strong case for abandonment of the CL immediately its simple design is found to be violated. One is likely to be better served in this case by an attempt to build understanding of the process and then select the model design accordingly.

These arguments are presented not in the spirit of an anti-CL diatribe, but rather in recognition of the fact that, when the CL (or indeed any other highly standardised model design) turns out to be a poor device in practice, alternatives are required and use of a GLM may well be an effective alternative.

Appendix A
Paid loss data

A.1 Incremental paid losses

accident	development quarter ($000)									
quarter	0	1	2	3	4	5	6	7	8	9
Sep-94	1	61	273	934	1,320	1.017	492	393	1,111	2.096
Dec-94	40	416	1,362	2.348	3 671	2,823	2,207	3.031	5.083	4.987
Mar-95	30	581	1,352	2.452	1,678	1,704	2 603	4.747	3,078	3,868
Jun-95	24	493	1,641	1,504	1,972	3,581	3,318	3,248	4,805	5,714
Sep-95	28	689	876	1,973	2,639	3,823	2.588	4.270	5.290	7.363
Dec-95	59	239	751	1,698	2,526	2.209	3,319	4 812	4.316	4 181
Mar-96	30	268	1,300	2,016	2,732	3,036	3.317	4,058	3 614	3.978
Jun-96	27	488	1,444	1,715	2,492	3.405	3 534	3 471	4,759	8,035
Sep-96	19	459	1,188	2,383	3 485	3.097	3 346	5,426	6,796	6,364
Dec-96	7	315	1,439	2,278	3 213	2 900	5,411	4.532	4,548	5,868
Mar-97	56	381	1,216	2,615	2,290	3.195	5,206	6.497	4,561	7,066
Jun-97	7	486	1,813	2,054	2,970	3,433	5.971	4.222	6.311	4 334
Sep-97	45	557	1,270	2,763	2,714	4.640	3,783	5,336	6.592	10 646
Dec-97	45	447	1,734	2,767	4,107	3,660	5,290	8,830	7,564	6 157
Mar-98	17	385	1,593	3,050	3 344	4,132	5,526	5,433	4,802	5.677
Jun-98	29	746	1,830	3,100	3 599	5,265	7 271	4,743	6,868	4,533
Sep-98	100	678	1,582	3 172	4,391	5.865	5 132	8,321	9,431	7,880
Dec-98	54	533	1,599	4 207	6 823	8.897	10.541	7,628	5,492	5.131
Mar-99	28	721	2,393	4,798	5,052	7,237	6 378	5.879	4,394	6,118
Jun-99	92	725	2,517	3,238	5 455	5,472	7,317	4,549	8.027	6.979
Sep-99	65	649	1.419	3,913	3 531	6.699	5.169	7.277	7,891	16,651
Dec-99	55	740	2,094	2,694	5.952	3.925	6.103	6.790	11,315	7,334
Mar-00	75	666	1,364	3,879	2,758	5.350	6.112	7,328	6.486	7.222
Jun-00	60	571	1.527	2.133	4,521	5,852	8 414	6.501	9,512	6.807
Sep-00	76	810	1,156	2.825	3,602	8,354	7,015	10,612	9.707	9 489
Dec-00	40	476	762	1.576	3 394	3,905	5 806	6.412	8,394	8,060
Mar-01	42	382	950	2.411	3 240	5,281	6 840	10.038	7,674	8,413
Jun-01	71	629	1.203	1,857	4,116	5,433	9.705	7,721	10,723	6,983
Sep-01	63	999	1,180	3,101	4,923	7,240	7,068	8.900	6,862	
Dec-01	59	635	1,209	2,517	5,749	5.112	10.178	7,201		
Mar-02	54	687	1,164	3,445	2,814	7,077	5,729			
Jun-02	134	762	1,513	2,062	4,099	5,285				
Sep-02	67	719	1,316	2,630	3,243					
Dec-02	94	475	978	1 650						
Mar-03	71	473	689							
Jun-03	56	450								
Sep-03	45									

accident	development quarter ($000)									
quarter	10	11	12	13	14	15	16	17	18	19
Sep-94	1 101	1,413	1,839	1,170	1,493	805	2,153	932	1,865	730
Dec-94	4,569	6,094	8,931	4,781	6,972	3,183	6,695	5,344	3,563	1,667
Mar-95	6,165	6,640	2,973	4,302	5,603	5,982	5,248	4,287	3,473	5,550
Jun-95	12,655	5 078	5,780	6,620	7,086	8,035	5,216	3,932	5,322	2,935
Sep-95	4,589	4,753	6,304	6,085	6,043	5,016	10,251	5,847	4,274	2,830
Dec-95	7,169	6,308	8,881	4,183	4,446	5,274	4,247	3,703	4,917	2,656
Mar-96	4,491	5,647	5 015	6,081	5,736	4,635	4,857	4,756	3,793	3,224
Jun-96	5,366	5,246	6,932	7,495	5,589	4,762	9,615	3,532	3,362	2,067
Sep-96	6,984	6,170	5,031	9,244	5,783	4 996	4,842	3,730	2,297	4,424
Dec-96	5,934	6,767	8,576	4,098	7,389	2 687	3,886	1,880	4,534	7,378
Mar-97	5,654	6,678	5,797	4,207	4,167	5,396	3 236	5,807	12,137	3,909
Jun-97	5,225	3,730	7 353	3,374	5,833	2,744	3,950	3,817	2,499	2,694
Sep-97	3,815	10,341	4,479	5,755	3,072	5,046	3,969	2,822	2,666	3,847
Dec-97	6,880	4,670	4,775	4,734	3,146	4,016	5,570	2,002	2,779	2,021
Mar-98	4,215	6 045	3,188	6,368	3,316	3,345	4,198	3,334	2,685	4,675
Jun-98	5,476	5,212	7,386	4,765	7,866	4,308	6,153	3 455	5,819	1,793
Sep-98	4,992	6,735	7,242	7,403	9,829	8,446	7,969	6,711	7,192	2,693
Dec-98	6,237	6,806	10,558	5,085	6,570	4,882	5,377	2,669	4,702	3,006
Mar-99	8,260	6,386	5,277	7,161	4,647	3,459	4,264	4,344	2,455	
Jun-99	8,429	4,465	6,050	7,378	12,514	5,076	5,091	4 303		
Sep-99	8,427	6,730	7,886	9,256	5,401	7,277	5,676			
Dec-99	7,274	7,858	9 303	5,688	5 800	6,527				
Mar-00	7,803	11,137	11,257	5 040	5,261					
Jun-00	9,162	8,265	7,600	5,807						
Sep-00	10,347	8 534	8,310							
Dec-00	8,487	9,557								
Mar-01	6,164									
Jun-01										

accident	development quarter ($000)									
quarter	20	21	22	23	24	25	26	27	28	29
Sep-94	1,708	1,866	314	777	176	281	1,566	124	505	253
Dec-94	2,587	3,694	2,678	3,154	1,827	430	222	1,296	749	542
Mar-95	1,915	1,441	366	1,878	364	1,244	304	594	638	1,745
Jun-95	4,419	2,653	3,034	799	332	597	1,635	611	2,043	3,811
Sep-95	1,780	2,542	1,305	829	1,587	1,317	758	1,366	583	1,473
Dec-95	2,843	764	761	297	1,361	2,814	512	745	1,276	149
Mar-96	896	1,278	1,652	2,242	4,731	682	1,331	1,229	821	1 114
Jun-96	1,882	1,755	7,216	2,366	3,323	861	1,768	712	144	98
Sep-96	3,733	2,530	7,858	2 628	1,218	1,103	3,441	783	694	
Dec-96	972	1 594	2,057	1 644	1,051	1,149	1,858	105		
Mar-97	1,488	4,174	1 330	3 695	410	976	641			
Jun-97	2,406	2,387	2,706	1,725	2,431	785				
Sep-97	2,585	5 581	1,455	1 868	1 740					
Dec-97	3 221	5 013	887	1,711						
Mar-98	2,529	2 058	1,413							
Jun-98	2,426	3,088								
Sep-98	5,601									
Dec-98										

accident	development quarter ($000)						
quarter	30	31	32	33	34	35	36
Sep-94	522	1	-63	108	1	2	92
Dec-94	1,147	145	2,272	400	74	557	
Mar-95	1,892	2,062	88	191	676		
Jun-95	444	3,270	190	20			
Sep-95	1 082	2,675	41				
Dec-95	190	947					
Mar-96	541						
Jun-96							

A.2 Incremental paid losses in respect of finalised claims

accident	development quarter of finalisation ($000)									
quarter	0	1	2	3	4	5	6	7	8	9
Sep-94	0 0	14	145	524	1,254	771	429	351	707	1,852
Dec-94	3 5	277	552	1,474	3.334	2.404	1,125	2,683	4,341	4,203
Mar-95	3 3	211	850	1.834	1,320	1,101	2,158	3.360	2,341	4,804
Jun-95	0 0	197	906	1 032	1,122	2,302	3,466	2,519	4.032	3 352
Sep-95	0 9	293	423	862	2,141	3.461	2,323	2,710	4,087	3,792
Dec-95	54 4	120	212	1,081	2.000	2,055	2,594	3,368	2,878	6,206
Mar-96	0 0	105	794	1,466	2,345	2,280	2,987	2.049	4,942	3,889
Jun-96	0 0	178	869	1.209	1,760	2 353	1,953	4,481	4,497	3,498
Sep-96	5 3	145	743	1,741	1,963	2 497	3,941	4,155	5,150	5,827
Dec-96	0.0	127	910	1,367	1 559	3,490	4,873	3,801	4,398	4.188
Mar-97	0 0	96	447	1,216	2.738	2,725	2,883	6,002	4,586	4,830
Jun-97	0 0	133	762	2,239	2,617	2,446	4,554	4,041	6,119	5,324
Sep-97	0 4	77	895	1,881	2.285	3,567	3,319	4,841	6,014	7,102
Dec-97	10.0	172	1,063	1,785	3,062	3,647	4,147	7,040	8 524	6,175
Mar-98	0 0	134	820	2,298	2,288	4,212	4,079	5,667	5.645	6,282
Jun-98	0 0	201	1,010	1,987	3.540	3,935	7,108	5,173	6,683	3,595
Sep-98	5 8	157	838	2,314	3 376	5,839	4,765	7,974	5,220	5,438
Dec-98	0.0	104	859	3,027	6,470	6,290	8,646	6,389	8,235	3,714
Mar-99	0 4	215	1,327	3,884	4,278	7,361	4,166	6,488	3,916	3,600
Jun-99	0 2	192	1,798	2,708	4,636	5,046	5.928	3,868	5,073	5,491
Sep-99	0 2	231	861	3,100	3,046	4,407	3,779	4,531	7,213	12.158
Dec-99	1 6	368	1,581	2,234	4,581	2,727	4,513	5,496	10,136	8,289
Mar-00	15 1	311	724	2,966	1,877	3,610	4,475	7,277	5,305	8,413
Jun-00	5 8	192	959	1,500	2,626	4,407	8,700	5,428	9,670	6,131
Sep-00	0.0	339	612	1,438	2.294	7,234	5,698	10,923	7,560	7,947
Dec-00	0 4	71	259	977	2,511	3 448	5,806	5,079	6,537	6,609
Mar-01	0 8	62	387	1,750	2,478	5,230	6,033	9,273	7,673	7,299
Jun-01	3 8	217	574	1,317	3,501	4,791	8.593	7,265	9,867	6,866
Sep-01	6 3	176	502	2,258	4,280	6.135	5.126	8,279	5,131	
Dec-01	1 4	121	502	1,524	4,918	4,307	9,820	5,098		
Mar-02	11 2	141	632	2,558	2,280	6,599	4,457			
Jun-02	6 1	189	763	1,265	3,337	3,860				
Sep-02	7.0	175	526	2 171	2.375					
Dec-02	32.4	128	383	1,061						
Mar-03	7.1	96	111							
Jun-03	9 3	39								
Sep-03	0 4									

Note: Paid losses in finalisation quarter x include all amounts paid in quarters up to and including x for claims finalised in x.

accident	development quarter of finalisation ($000)									
quarter	10	11	12	13	14	15	16	17	18	19
Sep-94	1,169	1,192	1,381	1,065	1,149	1,437	1,250	481	926	2,628
Dec-94	3,439	3 270	5,306	10,651	7,187	4,929	4,616	4,471	8,490	3,100
Mar-95	5,531	5,555	5,758	3,769	3,443	5,781	3,887	6,597	5,242	5,367
Jun-95	3,898	6,602	11 973	6,055	4,933	6,079	7,011	7,515	4 888	5,306
Sep-95	5,332	4,648	5,253	8 834	2,824	6,063	8,382	7,525	6,979	2,821
Dec-95	4,295	4,173	7 276	4,211	7,421	5,877	7,486	3,928	5,070	3 583
Mar-96	3,039	4,596	5,485	6,140	3,394	7,740	3 876	8,296	2,885	4,328
Jun-96	4,438	6 842	7,675	5,985	5,869	7,775	6,455	3,315	3,505	897
Sep-96	4,038	7,355	6,985	9,914	7,170	4,608	3,632	3,378	3,166	1,857
Dec-96	6,361	5 805	6,119	4,438	8 435	3,231	2,410	2,775	3,280	3,050
Mar-97	7,444	5,571	6,903	4,754	2,866	3,287	2,015	3,962	5,238	5,091
Jun-97	4,742	4,314	7,397	3,176	3,282	4,055	3,707	2,844	3 510	2,622
Sep-97	6,485	10,205	4 452	6,501	3,640	2,103	2,039	4,868	2,341	5,025
Dec-97	6,292	3,413	7,127	2,846	2,826	4,147	4,940	5,124	4,838	1,528
Mar-98	2,823	4 810	3,227	2,481	5 889	5,258	2,633	3 344	2,715	4,699
Jun-98	5,203	3,783	4 084	5,255	7,258	7,690	6,548	4,481	5,312	2,087
Sep-98	5,117	3,893	7,186	7,966	5,599	11,969	7,303	7,723	7,486	10,009
Dec-98	4,587	5,634	9 425	5,373	8,626	4,608	6,539	4,038	4,868	6,188
Mar-99	4,916	9,749	5,366	8 804	5,391	3,899	3,736	4,402	2,483	
Jun-99	11,923	4,247	7,727	4 678	9,901	6,165	4,279	7,077		
Sep-99	9,317	8,123	8,999	7,495	6,069	8,988	4,428			
Dec-99	9,088	7,461	8,498	4,853	7,232	6,023				
Mar-00	6,589	6,830	11,414	5,911	4,560					
Jun-00	8,683	7,855	7,845	5,033						
Sep-00	10,856	9,088	7,014							
Dec-00	8,466	8,389								
Mar-01	6,256									
Jun-01										

accident	development quarter of finalisation ($000)									
quarter	20	21	22	23	24	25	26	27	28	29
Sep-94	1,164	1,956	1,219	970	672	252	712	9	1,265	1,414
Dec-94	2,619	4,602	2,271	2,610	1,556	2,682	253	625	610	1,318
Mar-95	1 957	1,618	1,326	658	1,033	741	4,524	675	421	472
Jun-95	3,270	1,673	6,170	2,822	850	1,295	1,362	2,958	1,286	1,870
Sep-95	1,469	3,770	426	2,141	1,934	1,547	1,183	631	7,640	816
Dec-95	2,073	2,000	1,702	201	2,263	3,465	1 538	358	311	738
Mar-96	510	1,095	1,137	2,827	1,604	1,265	722	2,736	1,011	4 683
Jun-96	6,680	1,443	3,234	7,912	3,951	1,476	2,503	1,831	500	809
Sep-96	4,437	2,836	3,828	4,531	2,256	1,533	1,817	4,079	1,814	
Dec-96	2,642	6,086	6,398	1,682	1,136	1,169	3,231	2,130		
Mar-97	5,736	2,574	13 854	2,865	2,180	466	2,401			
Jun-97	4,744	1,863	3,693	814	1,772	697				
Sep-97	3,184	2,226	6 450	3 056	1,862					
Dec-97	3,744	1,581	2,566	929						
Mar-98	3,518	2,732	1,189							
Jun-98	1,592	2,262								
Sep-98	3,478									
Dec-98										

accident	development quarter of finalisation ($000)						
quarter	30	31	32	33	34	35	36
Sep-94	140	0	1 009	0	6	0	634
Dec-94	1,147	1 935	1,076	1,827	1,165	0	
Mar-95	2,932	1 329	298	1,787	1,156		
Jun-95	1,398	1,603	914	963			
Sep-95	1,143	327	84				
Dec-95	862	397					
Mar-96	147						
Jun-96							

A.3 Numbers of claim finalisations

accident quarter	development quarter of finalisation									
	0	1	2	3	4	5	6	7	8	9
Sep-94	0	6	26	36	53	37	32	22	35	73
Dec-94	2	37	69	151	200	130	52	131	192	115
Mar-95	2	39	101	163	102	67	141	173	99	125
Jun-95	0	47	110	95	53	147	226	130	150	126
Sep-95	2	51	51	67	189	216	155	171	126	139
Dec-95	6	21	32	127	185	184	173	135	135	176
Mar-96	0	16	113	173	174	185	139	122	184	133
Jun-96	1	37	126	143	148	177	128	191	147	126
Sep-96	1	33	103	167	150	171	222	148	149	136
Dec-96	0	32	115	141	159	246	193	154	157	105
Mar-97	2	22	68	143	246	205	149	187	123	139
Jun-97	0	21	99	240	215	180	176	158	166	116
Sep-97	5	19	140	191	175	217	170	190	181	161
Dec-97	2	46	125	197	242	188	205	178	181	126
Mar-98	0	33	122	198	196	239	171	187	143	146
Jun-98	0	40	130	188	256	220	264	163	166	110
Sep-98	1	27	113	228	227	270	208	257	138	119
Dec-98	0	20	129	272	381	302	306	190	147	98
Mar-99	1	54	160	335	304	338	196	164	109	79
Jun-99	2	44	225	226	307	236	193	108	116	103
Sep-99	2	55	116	273	214	201	148	152	162	279
Dec-99	3	65	180	193	253	155	173	173	282	170
Mar-00	3	69	107	204	140	179	202	268	155	192
Jun-00	3	49	138	150	192	238	333	170	242	134
Sep-00	0	55	89	146	167	307	215	264	168	164
Dec-00	3	29	68	135	240	203	255	182	185	138
Mar-01	2	28	91	184	219	260	208	237	186	184
Jun-01	3	71	102	173	225	232	260	181	198	157
Sep-01	7	53	103	195	202	242	205	221	145	
Dec-01	2	49	101	145	259	204	278	182		
Mar-02	7	58	96	180	148	252	167			
Jun-02	6	55	96	110	192	162				
Sep-02	5	57	94	154	130					
Dec-02	4	44	63	106						
Mar-03	7	40	42							
Jun-03	4	28								
Sep-03	7									

accident	development quarter of finalisation									
quarter	10	11	12	13	14	15	16	17	18	19
Sep-94	30	26	32	26	32	18	19	13	9	10
Dec-94	104	100	142	136	104	68	49	48	61	27
Mar-95	96	135	137	100	75	84	60	63	39	38
Jun-95	95	134	118	77	77	81	72	64	53	48
Sep-95	157	126	98	107	77	68	56	61	39	26
Dec-95	126	104	111	79	79	62	57	37	41	31
Mar-96	101	96	109	83	56	81	49	54	34	20
Jun-96	111	101	126	89	75	64	61	37	35	11
Sep-96	95	121	109	117	88	66	44	36	26	13
Dec-96	136	99	124	69	81	56	24	24	15	33
Mar-97	112	108	98	82	49	40	23	28	31	39
Jun-97	98	95	141	54	42	37	27	20	42	24
Sep-97	117	122	77	57	41	28	27	55	36	42
Dec-97	129	71	80	45	39	41	67	55	36	20
Mar-98	91	76	53	43	46	61	42	35	25	27
Jun-98	111	79	64	69	114	72	80	55	49	30
Sep-98	93	72	101	144	75	89	80	61	53	46
Dec-98	78	89	165	74	74	53	50	33	44	46
Mar-99	106	197	105	116	67	42	45	54	21	
Jun-99	225	89	135	75	78	64	52	50		
Sep-99	138	136	130	90	64	65	47			
Dec-99	162	130	122	87	81	64				
Mar-00	123	124	113	106	61					
Jun-00	132	112	131	65						
Sep-00	116	141	116							
Dec-00	144	114								
Mar-01	127									
Jun-01										

accident	development quarter of finalisation									
quarter	20	21	22	23	24	25	26	27	28	29
Sep-94	9	10	12	8	7	3	1	1	4	5
Dec-94	33	34	25	18	9	8	3	6	7	9
Mar-95	20	23	12	10	6	5	10	8	5	6
Jun-95	27	17	15	8	7	3	13	17	10	5
Sep-95	20	26	6	14	10	17	12	9	6	7
Dec-95	22	22	12	2	22	15	15	6	3	7
Mar-96	9	13	15	29	14	10	5	12	11	6
Jun-96	18	16	33	26	21	7	16	13	4	4
Sep-96	16	29	25	21	10	11	7	12	5	
Dec-96	24	29	15	18	13	10	10	6		
Mar-97	34	30	20	15	12	9	8			
Jun-97	31	15	18	14	15	8				
Sep-97	20	17	10	17	9					
Dec-97	18	24	19	15						
Mar-98	22	28	16							
Jun-98	29	21								
Sep-98	37									
Dec-98										

accident	development quarter of finalisation						
quarter	30	31	32	33	34	35	36
Sep-94	1	0	5	0	1	0	1
Dec-94	6	7	2	3	7	1	
Mar-95	8	5	1	3	7		
Jun-95	8	5	4	5			
Sep-95	4	4	2				
Dec-95	7	7					
Mar-96	2						
Jun-96							

A.4 Incremental average sizes of finalised claims

accident	development quarter of finalisation									
quarter	0	1	2	3	4	5	6	7	8	9
Sep-94		2.382	5,594	14,548	23.662	20,845	13,393	15 952	20.187	25,363
Dec-94	1.735	7.483	8.005	9.761	16.670	18.494	21.625	20,482	22.610	36.551
Mar-95	1.636	5,401	8.415	11,250	12 939	16,427	15.306	19.423	23.650	38.433
Jun-95		4,201	8,235	10.865	21.174	15,658	15,338	19,380	26.883	26.601
Sep-95	433	5,741	8,290	12.863	11.326	16,024	14,984	15,849	32,440	27,282
Dec-95	9.060	5.734	6,634	8,514	10.810	11.168	14.994	24,945	21.316	35,263
Mar-96		6,532	7,028	8,476	13.478	12,324	21,493	16,797	26.858	29,239
Jun-96		4,820	6.896	8,456	11.891	13,291	15,259	23,460	30,592	27 762
Sep-96	5,307	4,384	7,214	10,427	13,090	14.603	17,752	28,077	34,566	42,847
Dec-96		3,967	7,915	9,696	9,805	14,188	25.250	24,684	28.015	39,882
Mar-97		4,351	6,578	8,504	11,132	13,294	19,350	32,097	37.282	34,748
Jun-97		6,340	7,701	9,328	12,174	13,587	25,875	25,577	36,860	45,901
Sep-97	73	4.063	6,393	9,849	13,056	16,439	19.525	25,478	33,226	44,113
Dec-97	5.013	3,749	8.501	9,059	12.652	19,397	20.228	39.553	47.096	49,009
Mar-98		4.069	6.720	11,608	11,671	17,624	23.852	30,306	39,476	43,025
Jun-98		5,032	7,769	10.571	13,827	17,887	26.926	31,734	40,262	32.682
Sep-98	5,828	5,832	7,420	10.149	14.871	21,627	23.007	31,026	37,829	45 701
Dec-98		5,181	6,660	11.127	16.982	20,827	28,255	33,628	56,021	37,895
Mar-99	401	3,986	8,292	11,595	14.073	21,779	21,256	39.558	35,930	45.572
Jun-99	111	4,363	7,990	11.984	15 102	21,380	30.718	35,818	43,731	53,315
Sep-99	97	4,207	7,420	11.354	14,234	21,926	25.532	29.806	44,528	43.578
Dec-99	547	5,663	8,785	11,578	18.106	17.596	26,086	31,767	35,942	48.759
Mar-00	5,050	4,509	6.763	14,539	13,408	20.166	22,155	27.151	34,228	43 820
Jun-00	1 940	3,922	6,948	10,001	13,678	18.518	26,127	31,930	39,958	45,756
Sep-00		6,157	6,876	9,850	13,739	23,564	26.500	41,375	45,000	48,456
Dec-00	147	2,464	3,807	7,235	10.462	16,988	22,767	27,905	35,336	47,889
Mar-01	396	2,231	4,251	9,510	11,317	20.115	29,005	39.125	41,250	39,670
Jun-01	1.271	3,060	5,628	7,615	15,559	20,652	33,051	40.138	49,832	43.731
Sep-01	898	3,317	4,878	11,581	21,188	25,352	25,003	37,460	35,387	
Dec-01	678	2,463	4,966	10,511	18,989	21,111	35,324	28,008		
Mar-02	1,594	2,429	6,579	14,210	15,408	26.188	26,690			
Jun-02	1,017	3,443	7,947	11.497	17,380	23,825				
Sep-02	1,394	3,072	5,600	14,098	18,272					
Dec-02	8.102	2,905	6.081	10,007						
Mar-03	1,013	2,392	2,652							
Jun-03	2,327	1,400								
Sep-03	59									

Note: Each entry is calculated as the quotient of the corresponding entries in Appendices A.2 and A.3.

accident	development quarter of finalisation									
quarter	10	11	12	13	14	15	16	17	18	19
Sep-94	38,981	45,863	43,155	40,948	35,908	79,834	65,785	37,000	102,906	262,773
Dec-94	33,065	32,703	37,365	78,315	69,108	72,480	94,207	93,147	139,178	114,821
Mar-95	57,613	41,147	42,031	37,687	45 913	68,820	64,777	104,722	134,404	141,225
Jun-95	41,032	49.272	101,466	78,637	64 070	75.045	97,381	117 429	92,234	110,545
Sep-95	33.960	36.892	53.606	82.564	36.676	89 167	149,679	123 365	178,941	108,499
Dec-95	34,086	40,123	65,550	53,308	93,931	94.785	131,331	106.164	123,663	115,566
Mar-96	30,093	47,875	50.320	73,979	60.599	95.555	79.094	153,636	84,868	216,397
Jun-96	39,983	67,740	60.913	67,250	78,248	121,485	105,823	89,606	100,151	81,515
Sep-96	42,509	60,787	64.080	84,732	81,474	69,821	82 547	93,824	121 783	142,849
Dec-96	46,771	58.639	49.350	64,319	104,134	57.695	100.423	115,642	218,638	92,438
Mar-97	66.464	51,580	70,439	57,971	58.495	82,164	87.609	141,490	168,964	130,546
Jun-97	48.392	45,413	52.464	58,820	78.146	109.606	137,301	142,201	83,567	109,264
Sep-97	55,427	83,651	57,813	114,059	88.783	75 094	75,506	88,514	65,035	119 652
Dec-97	48,775	48,073	89.093	63,252	72.469	101,136	73,730	93,165	134,379	76,378
Mar-98	31,026	63 288	60.886	57,705	128.024	86 192	62.702	95,530	108.612	174,022
Jun-98	46,875	47.891	63,811	76,160	63.667	106,801	81,854	81,477	108,414	69,581
Sep-98	55,025	54,074	71,146	55,322	74,652	134,479	91,283	126,607	141,252	217,591
Dec-98	58,810	63.307	57.124	72,602	116,566	86.935	130 781	122,363	110,631	134,531
Mar-99	46 377	49,490	51,105	75,895	80.459	92,833	83,014	81,526	118,238	
Jun-99	52,992	47,720	57,234	62,371	126,934	96,335	82 298	141,547		
Sep-99	67,518	59,729	69,221	83,279	94 824	138,272	94,214			
Dec-99	56,099	57,395	69.656	55,780	89.280	94,104				
Mar-00	53,568	55,077	101,005	55,766	74,750					
Jun-00	65,777	70,131	59,887	77,433						
Sep-00	93,585	64,453	60,465							
Dec-00	58.789	73,588								
Mar-01	49,258									
Jun-01										

accident	development quarter of finalisation									
quarter	20	21	22	23	24	25	26	27	28	29
Sep-94	129,349	195,633	101,579	121,257	95.978	84.158	712,264	8,764	316,169	282,728
Dec-94	79,351	135,363	90,855	145,021	172.933	335,296	84,477	104,109	87 098	146 445
Mar-95	97,834	70,355	110,518	65,824	172,169	148,164	452,372	84,431	84,166	78,663
Jun-95	121,116	98,383	411,325	352,690	121,433	431.789	104,799	174,005	128 565	374,070
Sep-95	73,437	144,988	71,009	152,943	193.406	91,006	98,609	70.139	1,273 300	116,584
Dec-95	94,241	90 905	141,854	100.275	102 849	230,998	102,517	59.253	103,639	105,413
Mar-96	56,632	84 235	75,801	97.496	114 575	126,450	144,496	227.986	91,890	780,581
Jun-96	371,135	90,167	98,013	304 316	188 162	210,869	156,466	140.817	124 941	202,313
Sep-96	277,303	97,799	153,110	215,742	225.600	139,336	259,511	339,882	362,793	
Dec-96	110,092	209,851	426,546	93,455	87 416	116,867	323,074	355,008		
Mar-97	168,714	85,802	692,725	190,969	181 670	51,736	300.118			
Jun-97	153,029	124,170	205,154	58,125	118.166	87,164				
Sep-97	159,195	130,970	644,990	179,757	206,863					
Dec-97	207,985	65,860	135,046	61,946						
Mar-98	159,924	97,588	74,291							
Jun-98	54,887	107,701								
Sep-98	94,013									
Dec-98										

accident	development quarter of finalisation						
quarter	30	31	32	33	34	35	36
Sep-94	139,507		201.849		6,200		633,545
Dec-94	191.107	276.459	537,824	608,937	166,449		
Mar-95	366,509	265.796	297,888	595,605	165,077		
Jun-95	174,706	320,567	228,614	192,673			
Sep-95	285,658	81.822	41,975				
Dec-95	123,129	56,756					
Mar-96	73,749						
Jun-96							

A.5 Cumulative average sizes of finalised claims

accident	development quarter of finalisation									
quarter	0	1	2	3	4	5	6	7	8	9
Sep-94		2.382	4,992	10,051	16,013	17,144	16,512	16,454	16,983	18 895
Dec-94	1,735	7,188	7,710	8,906	12,289	13,659	14 305	15,353	16,798	18,904
Mar-95	1,636	5 218	7,492	9,500	10,362	11,219	12,156	13,752	14,856	17,769
Jun-95		4,201	7,027	8 474	10,681	12,300	13,312	14,289	16,261	17,463
Sep-95	433	5,540	6,889	9 229	10,330	12,465	12,999	13,539	15,856	17,217
Dec-95	9,060	6.473	6,560	7,894	9.348	9,951	11,150	13,308	14,391	17.520
Mar-96		6 532	6,967	7,831	9 896	10,575	12,472	13,044	15.342	16.834
Jun-96	0	4,693	6.386	7,350	8 827	10.077	10,950	13,462	15,756	16,992
Sep-96	5,307	4,411	6.518	8 666	10.127	11.352	13,030	15,268	17,781	20,444
Dec-96		3.967	7,056	8,348	8.866	10.755	13,913	15,508	17,148	18.982
Mar-97	0	3,988	5,902	7,485	9,350	10,529	12,103	15,761	18,073	19,878
Jun-97		6,340	7.463	8,706	10.003	10 857	13,696	15.420	18 256	20,595
Sep-97	73	3,232	5 930	8,039	9,695	11,654	13,113	15,236	17,764	20,691
Dec-97	5,013	3,802	7,197	8.189	9.954	12,173	13,816	17,689	21.591	23,909
Mar-98		4.069	6,156	9,214	10.091	12.378	14,422	17,014	19.506	21.899
Jun-98		5,032	7.125	8 934	10.974	12.798	16.195	18,203	20,769	21.622
Sep-98	5.828	5,832	7.105	8,986	11,227	14,470	16,123	19,001	20,769	22,638
Dec-98		5.181	6,462	9.476	13.042	15,172	18.011	19,865	22.908	23.704
Mar-99	401	3,921	7,174	9,867	11,364	14,317	15,297	17,861	19.046	20 251
Jun-99	111	4,178	7,343	9,453	11,610	13,827	16,471	18,029	20,075	22 270
Sep-99	97	4,083	6,314	9,399	10.966	13.525	15,286	17,187	20,535	24 548
Dec-99	547	5,438	7,867	9,491	12 632	13,538	15,662	17,994	21,420	24.242
Mar-00	5,050	4,532	5.866	10,485	11.268	13.537	15,462	18,135	20.015	23,024
Jun-00	1,940	3,807	6,088	7,815	9.931	12 585	16,673	18.711	22,105	24 027
Sep-00		6,157	6.601	8,237	10.247	15,598	17,993	22,959	25,583	27,965
Dec-00	147	2,247	3.308	5,564	8.038	10,718	14,011	16,279	18,991	21,764
Mar-01	396	2,108	3,719	7,213	8,928	12,638	16,070	20,516	23,242	25.132
Jun-01	1,271	2,987	4,517	6,053	9,779	12,909	17,822	21.061	25,003	26,839
Sep-01	898	3,035	4,199	8.220	12,898	16,656	18,355	21,793	23.229	
Dec-01	678	2.393	4,103	7,231	12,708	14.964	20.417	21,549		
Mar-02	1,594	2,339	4,867	9,799	11,496	16,493	18,368			
Jun-02	1,017	3,204	6,104	8.326	12,113	15,168				
Sep-02	1,394	2,936	4,541	9,289	11,943					
Dec-02	8,102	3,338	4,895	7,392						
Mar-03	1,013	2,187	2,406							
Jun-03	2,327	1,516								
Sep-03	59									

Note: Each entry is calculated as the quotient of the corresponding entries in the cumulative versions of Appendices A.2 and A.3.

accident	development quarter of finalisation									
quarter	10	11	12	13	14	15	16	17	18	19
Sep-94	20.617	22.362	23.993	25 009	25,757	27.768	29,204	29.401	30.661	34,999
Dec-94	20,149	21.127	22.745	27,587	30,180	31.840	33,555	35.118	38.475	39,549
Mar-95	21,221	23.385	25.236	26,077	27,034	29.176	30,433	33,089	35,283	37,472
Jun-95	19.362	22,414	28,933	31,471	33,054	35.096	37,676	40,509	41,987	43,716
Sep-95	19,202	20,740	22,820	26,683	27,127	29,471	33 098	35,970	38,821	39,735
Dec-95	19.126	20,681	23,969	25,423	28,658	31,021	34 211	35,667	37,596	38,868
Mar-96	17,833	19,842	21,992	24.643	25,838	29.038	30,390	33.953	34,863	36,752
Jun-96	18,903	22,338	25,450	27,703	29,900	33,174	35.568	36,627	37,783	38.032
Sep-96	21,969	25,109	27,755	31,627	34,050	35,309	36,391	37,448	38,554	39,234
Dec-96	21,610	23,995	25,888	27,421	30,852	31,657	32.530	33,571	35,010	35,975
Mar-97	23,616	25,624	28,365	29,807	30,618	31,781	32,496	34.169	36,422	38,360
Jun-97	22.449	23 844	26,211	27.212	28,400	30,034	31,587	32.761	33,868	34,796
Sep-97	23,287	27,649	28,965	31.627	32,885	33,510	34,101	35,618	36.145	37,855
Dec-97	25,891	26,823	29,637	30.471	31,354	32.864	34,259	35,866	37,593	37.967
Mar-98	22.443	24,381	25.550	26.364	29,045	30.977	31,698	32,885	33.878	35,834
Jun-98	23.323	24,447	25,853	27.720	29,796	32,505	34,362	35,550	37.151	37,582
Sep-98	24,430	25,647	28,126	30,086	31,698	35.929	37.904	40,254	42 526	45,879
Dec-98	25.128	26.817	29,114	30 543	33,281	34.477	36,460	37,612	38,895	40,619
Mar-99	21,751	24.426	25,730	28.300	29,799	30,915	31,885	32,969	33,687	
Jun-99	26.143	27,167	29.188	30,382	33,865	35,661	36,726	38,977		
Sep-99	27.956	30,259	32,784	34,953	36,727	39,693	40,821			
Dec-99	27,095	29,127	31,526	32,508	34,571	36,233				
Mar-00	25.312	27,402	31.828	33,107	34,348					
Jun-00	27,121	29,666	31,622	33,047						
Sep-00	32,466	34.928	36,449							
Dec-00	25.134	28,391								
Mar-01	26,907									
Jun-01										

accident	development quarter of finalisation									
quarter	20	21	22	23	24	25	26	27	28	29
Sep-94	36,560	39,432	40.749	41.871	42,523	42 737	43,882	43.822	45,668	47,660
Dec-94	40 223	41,852	42,462	43.372	43,944	45 084	45,142	45.314	45,456	45,895
Mar-95	38 121	38,515	38,972	39,113	39,531	39.814	41,957	42,133	42,242	42,354
Jun-95	44.799	45,267	48,065	49,302	49,558	50.137	50,493	51,538	51,919	52,715
Sep-95	40.072	41,417	41,504	42,266	43,000	43.393	43,711	43,824	47,334	47,564
Dec-95	39,501	40,083	40,707	40,768	41,457	42 881	43,326	43,374	43,463	43,677
Mar-96	36,844	37,161	37,457	38,333	38,866	39.302	39,562	40,677	40,953	43,122
Jun-96	41.104	41,503	42.435	45,794	47.254	47.811	48,650	49,225	49,370	49,663
Sep-96	41,128	41,934	43 280	45,016	45,878	46.365	47,071	48,724	49.461	
Dec-96	36,870	39,359	42,218	42,668	42.950	43.307	44.651	45,543		
Mar-97	40 659	41,350	47,936	49,013	49,807	49,816	50,806			
Jun-97	36 645	37,303	38 802	38,935	39.517	39,702				
Sep-97	39,027	39,776	42,661	43,762	44,454					
Dec-97	39,431	39,731	40,579	40,729						
Mar-98	37,230	38,082	38,372							
Jun-98	37 801	38,437								
Sep-98	46 609									
Dec-98										

accident	development quarter of finalisation						
quarter	30	31	32	33	34	35	36
Sep-94	47,814	47,814	49,096	49.096	49.025	49.025	49,994
Dec-94	46,315	47,088	47,559	48 366	48,760	48.737	
Mar-95	43,683	44,250	44 380	45 223	45,649		
Jun-95	53.195	53,851	54,193	54 531			
Sep-95	48,014	48,078	48,073				
Dec-95	43,950	43,994					
Mar-96	43,152						
Jun-96							

Appendix B
Age-to-age factors

B.1 Age-to-age factors based on paid losses in respect of finalised claims

accident quarter	development quarter of finalisation 0	1	2	3	4	5	6	7	8	9
Sep-94			11.18	4.28	2.83	1.40	1.16	1.11	1.20	1.44
Dec-94		80.79	2.97	2.77	2.45	1.43	1.14	1.29	1.37	1.26
Mar-95		65.36	4.97	2.72	1.46	1.26	1.41	1.45	1.22	1.36
Jun-95			5.59	1.94	1.53	1.71	1.62	1.28	1.35	1.22
Sep-95		339.23	2.44	2.20	2.36	1.93	1.32	1.29	1.33	1.23
Dec-95		3.22	2.21	3.79	2.36	1.59	1.47	1.41	1.25	1.43
Mar-96			8.60	2.63	1.99	1.48	1.43	1.21	1.41	1.23
Jun-96			5.87	2.15	1.78	1.59	1.31	1.54	1.35	1.20
Sep-96		28.26	5.95	2.95	1.75	1.54	1.56	1.38	1.34	1.29
Dec-96			8.17	2.32	1.65	1.88	1.65	1.31	1.27	1.20
Mar-97			5.67	3.24	2.56	1.61	1.40	1.59	1.28	1.23
Jun-97			6.73	3.50	1.84	1.43	1.56	1.32	1.36	1.23
Sep-97		212.34	12.54	2.93	1.80	1.69	1.38	1.40	1.36	1.31
Dec-97		18.20	6.82	2.43	2.01	1.60	1.43	1.51	1.41	1.21
Mar-98			7.11	3.41	1.70	1.76	1.42	1.41	1.29	1.25
Jun-98			6.02	2.64	2.11	1.58	1.67	1.29	1.29	1.12
Sep-98		28.02	6.13	3.31	2.02	1.87	1.38	1.46	1.21	1.18
Dec-98			9.29	4.14	2.62	1.60	1.52	1.25	1.26	1.09
Mar-99		538.07	7.15	3.52	1.79	1.76	1.24	1.31	1.14	1.11
Jun-99		863.69	10.35	2.36	1.99	1.54	1.41	1.19	1.21	1.19
Sep-99		1,195.03	4.72	3.84	1.73	1.61	1.32	1.29	1.36	1.45
Dec-99		225.16	5.28	2.15	2.09	1.31	1.39	1.34	1.47	1.26
Mar-00		21.54	3.22	3.82	1.47	1.61	1.47	1.52	1.25	1.32
Jun-00		34.02	5.84	2.30	1.99	1.83	1.90	1.30	1.41	1.18
Sep-00			2.81	2.51	1.96	2.54	1.48	1.62	1.26	1.22
Dec-00		162.87	4.60	3.95	2.92	1.90	1.80	1.39	1.36	1.27
Mar-01		79.90	7.12	4.89	2.13	2.12	1.61	1.58	1.30	1.22
Jun-01		57.97	3.60	2.66	2.66	1.85	1.83	1.38	1.38	1.19
Sep-01		28.96	3.76	4.30	2.45	1.85	1.38	1.45	1.19	
Dec-01		90.03	5.11	3.44	3.29	1.61	1.86	1.24		
Mar-02		13.62	5.15	4.26	1.68	2.17	1.36			
Jun-02		32.02	4.90	2.32	2.50	1.69				
Sep-02		26.13	3.89	4.06	1.82					
Dec-02		4.94	3.39	2.95						
Mar-03		14.49	2.08							
Jun-03		5.21								
Sep-03										
last 1 year		8.85	3.78	3.36	2.22	1.82	1.59	1.40	1.30	1.22
last 2 years		14.19	4.03	3.51	2.34	1.94	1.63	1.43	1.32	1.26
last 3 years		17.51	4.12	3.30	2.14	1.79	1.55	1.38	1.29	1.22
last 4 years		24.20	4.59	3.10	2.09	1.76	1.51	1.37	1.29	1.22
all years		32.18	5.16	2.97	2.02	1.69	1.48	1.37	1.30	1.23

accident quarter	development quarter of finalisation									
	10	11	12	13	14	15	16	17	18	19
Sep-94	1.19	1.17	1.16	1.11	1.11	1.12	1.09	1.03	1.06	1.16
Dec-94	1.17	1.14	1.20	1.33	1.17	1.10	1.08	1.07	1.13	1.04
Mar-95	1.31	1.24	1.20	1.11	1.09	1.14	1.08	1.13	1.09	1.08
Jun-95	1.21	1.29	1.41	1.15	1.10	1.12	1.12	1.11	1.07	1.07
Sep-95	1.27	1.18	1.17	1.25	1.06	1.13	1.16	1.12	1.10	1.04
Dec-95	1.21	1.17	1.25	1.12	1.18	1.12	1.14	1.06	1.08	1.05
Mar-96	1.15	1.19	1.19	1.18	1.08	1.18	1.08	1.15	1.05	1.07
Jun-96	1.21	1.27	1.24	1.15	1.13	1.15	1.11	1.05	1.05	1.01
Sep-96	1.15	1.24	1.19	1.22	1.13	1.07	1.05	1.05	1.04	1.02
Dec-96	1.26	1.19	1.17	1.10	1.18	1.06	1.04	1.05	1.05	1.05
Mar-97	1.29	1.17	1.18	1.10	1.06	1.06	1.04	1.07	1.08	1.08
Jun-97	1.17	1.13	1.20	1.07	1.07	1.08	1.07	1.05	1.06	1.04
Sep-97	1.22	1.28	1.10	1.13	1.06	1.03	1.03	1.07	1.03	1.07
Dec-97	1.18	1.08	1.16	1.05	1.05	1.07	1.08	1.08	1.07	1.02
Mar-98	1.09	1.14	1.08	1.06	1.13	1.10	1.05	1.06	1.04	1.07
Jun-98	1.16	1.10	1.10	1.11	1.14	1.13	1.10	1.06	1.07	1.03
Sep-98	1.14	1.09	1.16	1.15	1.09	1.18	1.09	1.09	1.08	1.10
Dec-98	1.10	1.12	1.17	1.08	1.13	1.06	1.08	1.05	1.05	1.06
Mar-99	1.14	1.24	1.11	1.16	1.08	1.06	1.05	1.06	1.03	
Jun-99	1.34	1.09	1.15	1.08	1.16	1.08	1.05	1.08		
Sep-99	1.24	1.17	1.16	1.11	1.08	1.11	1.05			
Dec-99	1.23	1.15	1.15	1.07	1.10	1.08				
Mar-00	1.19	1.16	1.24	1.10	1.07					
Jun-00	1.22	1.16	1.14	1.08						
Sep-00	1.25	1.17	1.11							
Dec-00	1.27	1.21								
Mar-01	1.16									
Jun-01										
last 1 year	1.22	1.17	1.15	1.09	1.10	1.08	1.06	1.07	1.08	1.07
last 2 years	1.23	1.17	1.15	1.10	1.11	1.10	1.07	1.07	1.05	1.06
last 3 years	1.20	1.15	1.14	1.10	1.10	1.09	1.06	1.06	1.06	1.05
last 4 years	1.19	1.15	1.15	1.10	1.10	1.09	1.07	1.07	1.06	1.05
all years	1.20	1.17	1.17	1.12	1.11	1.10	1.08	1.07	1.06	1.05

accident quarter	development quarter of finalisation									
	20	21	22	23	24	25	26	27	28	29
Sep-94	1.06	1.10	1.06	1.04	1.03	1.01	1.03	1.00	1.05	1.05
Dec-94	1.03	1.06	1.03	1.03	1.02	1.03	1.00	1.01	1.01	1.01
Mar-95	1.03	1.02	1.02	1.01	1.01	1.01	1.06	1.01	1.01	1.01
Jun-95	1.04	1.02	1.07	1.03	1.01	1.01	1.01	1.03	1.01	1.02
Sep-95	1.02	1.05	1.01	1.03	1.02	1.02	1.01	1.01	1.08	1.01
Dec-95	1.03	1.03	1.02	1.00	1.03	1.04	1.02	1.00	1.00	1.01
Mar-96	1.01	1.02	1.02	1.04	1.02	1.02	1.01	1.03	1.01	1.06
Jun-96	1.09	1.02	1.04	1.09	1.04	1.02	1.03	1.02	1.00	1.01
Sep-96	1.06	1.03	1.04	1.05	1.02	1.02	1.02	1.04	1.02	
Dec-96	1.04	1.08	1.08	1.02	1.01	1.01	1.04	1.02		
Mar-97	1.08	1.03	1.17	1.03	1.02	1.00	1.02			
Jun-97	1.07	1.03	1.05	1.01	1.02	1.01				
Sep-97	1.04	1.03	1.08	1.03	1.02					
Dec-97	1.05	1.02	1.03	1.01						
Mar-98	1.05	1.04	1.02							
Jun-98	1.02	1.03								
Sep-98	1.03									
Dec-98										
last 1 year	1.04	1.03	1.04	1.02	1.02	1.01	1.03	1.03	1.01	1.02
last 2 years	1.05	1.04	1.06	1.04	1.02	1.02	1.02	1.02	1.02	1.02
last 3 years	1.05	1.03	1.05	1.03	1.02	1.01	1.02	1.02	1.01	
last 4 years	1.04	1.02	1.06	1.01	1.01					
all years	1.04									

accident quarter	development quarter of finalisation						
	30	31	32	33	34	35	36
Sep-94	1.00	1.00	1.04	1.00	1.00	1.00	1.02
Dec-94	1.01	1.02	1.01	1.02	1.01	1.00	
Mar-95	1.04	1.02	1.00	1.02	1.01		
Jun-95	1.01	1.01	1.01	1.01			
Sep-95	1.01	1.00	1.00				
Dec-95	1.01	1.00					
Mar-96	1.00						
Jun-96							
last 1 year	1.01	1.01	1.01	1.01	1.01	0.00	0.00
last 2 years	1.00	1.01	1.01				
last 3 years							
last 4 years							
all years							

B.2 Age-to-age factors based on average sizes of finalised claims

accident quarter	development quarter of finalisation									
	0	1	2	3	4	5	6	7	8	9
Sep-94			2.10	2.01	1.59	1.07	0.96	1.00	1.03	1.11
Dec-94		4.14	1.07	1.16	1.38	1.11	1.05	1.07	1.09	1.13
Mar-95		3.19	1.44	1.27	1.09	1.08	1.08	1.13	1.08	1.20
Jun-95			1.67	1.21	1.26	1.15	1.08	1.07	1.14	1.07
Sep-95		12.80	1.24	1.34	1.12	1.21	1.04	1.04	1.17	1.09
Dec-95		0.71	1.01	1.20	1.18	1.06	1.12	1.19	1.08	1.22
Mar-96			1.07	1.12	1.26	1.07	1.18	1.05	1.18	1.10
Jun-96			1.36	1.15	1.20	1.14	1.09	1.23	1.17	1.08
Sep-96		0.83	1.48	1.33	1.17	1.12	1.15	1.17	1.16	1.15
Dec-96			1.78	1.18	1.06	1.21	1.29	1.11	1.11	1.11
Mar-97			1.48	1.27	1.25	1.13	1.15	1.30	1.15	1.10
Jun-97			1.18	1.17	1.15	1.09	1.26	1.13	1.18	1.13
Sep-97		44.24	1.84	1.36	1.21	1.20	1.13	1.16	1.17	1.16
Dec-97		0.76	1.89	1.14	1.22	1.22	1.13	1.28	1.22	1.11
Mar-98			1.51	1.50	1.10	1.23	1.17	1.18	1.15	1.12
Jun-98			1.42	1.25	1.23	1.17	1.27	1.12	1.14	1.04
Sep-98		1.00	1.22	1.26	1.25	1.29	1.11	1.18	1.09	1.09
Dec-98			1.25	1.47	1.38	1.16	1.19	1.10	1.15	1.03
Mar-99		9.78	1.83	1.38	1.15	1.26	1.07	1.17	1.07	1.06
Jun-99		37.55	1.76	1.29	1.23	1.19	1.19	1.09	1.11	1.11
Sep-99		41.93	1.55	1.49	1.17	1.23	1.13	1.12	1.19	1.20
Dec-99		9.93	1.45	1.21	1.33	1.07	1.16	1.15	1.19	1.13
Mar-00		0.90	1.29	1.79	1.07	1.20	1.14	1.17	1.10	1.15
Jun-00		1.98	1.60	1.28	1.27	1.27	1.32	1.12	1.18	1.09
Sep-00			1.07	1.25	1.24	1.52	1.15	1.28	1.11	1.09
Dec-00		15.27	1.47	1.68	1.44	1.33	1.31	1.16	1.17	1.15
Mar-01		5.33	1.76	1.94	1.24	1.42	1.27	1.28	1.13	1.08
Jun-01		2.35	1.51	1.34	1.62	1.32	1.38	1.18	1.19	1.07
Sep-01		3.38	1.38	1.96	1.57	1.29	1.10	1.19	1.07	
Dec-01		3.53	1.71	1.76	1.76	1.18	1.36	1.06		
Mar-02		1.47	2.08	2.01	1.17	1.43	1.11			
Jun-02		3.15	1.91	1.36	1.45	1.25				
Sep-02		2.11	1.55	2.05	1.29					
Dec-02		0.41	1.47	1.51						
Mar-03		2.16	1.10							
Jun-03		0.65								
Sep-03										
last 1 year		0.78	1.54	1.71	1.39	1.29	1.23	1.17	1.14	1.10
last 2 years		1.23	1.59	1.72	1.42	1.34	1.24	1.18	1.14	1.12
last 3 years		1.50	1.51	1.63	1.35	1.28	1.21	1.16	1.14	1.10
last 4 years		1.74	1.51	1.53	1.32	1.27	1.20	1.16	1.14	1.10
all years		2.65	1.43	1.39	1.27	1.21	1.16	1.15	1.14	1.11

accident quarter	development quarter of finalisation									
	10	11	12	13	14	15	16	17	18	19
Sep-94	1.09	1.08	1.07	1.04	1.03	1.08	1.05	1.01	1.04	1.14
Dec-94	1.07	1.05	1.08	1.21	1.09	1.05	1.05	1.05	1.10	1.03
Mar-95	1.19	1.10	1.08	1.03	1.04	1.08	1.04	1.09	1.07	1.06
Jun-95	1.11	1.16	1.29	1.09	1.05	1.06	1.07	1.08	1.04	1.04
Sep-95	1.12	1.08	1.10	1.17	1.02	1.09	1.12	1.09	1.08	1.02
Dec-95	1.09	1.08	1.16	1.06	1.13	1.08	1.10	1.04	1.05	1.03
Mar-96	1.06	1.11	1.11	1.12	1.05	1.12	1.05	1.12	1.03	1.05
Jun-96	1.11	1.18	1.14	1.09	1.08	1.11	1.07	1.03	1.03	1.01
Sep-96	1.07	1.14	1.11	1.14	1.08	1.04	1.03	1.03	1.03	1.02
Dec-96	1.14	1.11	1.08	1.06	1.13	1.03	1.03	1.03	1.04	1.03
Mar-97	1.19	1.09	1.11	1.05	1.03	1.04	1.02	1.05	1.07	1.05
Jun-97	1.09	1.06	1.10	1.04	1.04	1.06	1.05	1.04	1.03	1.03
Sep-97	1.13	1.19	1.05	1.09	1.04	1.02	1.02	1.04	1.01	1.05
Dec-97	1.08	1.04	1.10	1.03	1.03	1.05	1.04	1.05	1.05	1.01
Mar-98	1.02	1.09	1.05	1.03	1.10	1.07	1.02	1.04	1.03	1.06
Jun-98	1.08	1.05	1.06	1.07	1.07	1.09	1.06	1.03	1.05	1.01
Sep-98	1.08	1.05	1.10	1.07	1.05	1.13	1.05	1.06	1.06	1.08
Dec-98	1.06	1.07	1.09	1.05	1.09	1.04	1.06	1.03	1.03	1.04
Mar-99	1.07	1.12	1.05	1.10	1.05	1.04	1.03	1.03	1.02	
Jun-99	1.17	1.04	1.07	1.04	1.11	1.05	1.03	1.06		
Sep-99	1.14	1.08	1.08	1.07	1.05	1.08	1.03			
Dec-99	1.12	1.07	1.08	1.03	1.06	1.05				
Mar-00	1.10	1.08	1.16	1.04	1.04					
Jun-00	1.13	1.09	1.07	1.05						
Sep-00	1.16	1.08	1.04							
Dec-00	1.15	1.13								
Mar-01	1.07									
Jun-01										
last 1 year	1.13	1.09	1.09	1.05	1.07	1.06	1.04	1.05	1.04	1.05
last 2 years	1.13	1.09	1.08	1.05	1.07	1.07	1.04	1.04	1.04	1.04
last 3 years	1.11	1.08	1.08	1.05	1.06	1.06	1.04	1.04	1.04	1.04
last 4 years	1.10	1.08	1.08	1.06	1.07	1.06	1.04	1.05	1.04	1.04
all years	1.11	1.09	1.09	1.07	1.06	1.06	1.05	1.05	1.05	1.04

accident quarter	development quarter of finalisation									
	20	21	22	23	24	25	26	27	28	29
Sep-94	1.04	1.08	1.03	1.03	1.02	1.01	1.03	1.00	1.04	1.04
Dec-94	1.02	1.04	1.01	1.02	1.01	1.03	1.00	1.00	1.00	1.01
Mar-95	1.02	1.01	1.01	1.00	1.01	1.01	1.05	1.00	1.00	1.00
Jun-95	1.02	1.01	1.06	1.03	1.01	1.01	1.01	1.02	1.01	1.02
Sep-95	1.01	1.03	1.00	1.02	1.02	1.01	1.01	1.00	1.08	1.00
Dec-95	1.02	1.01	1.02	1.00	1.02	1.03	1.01	1.00	1.00	1.00
Mar-96	1.00	1.01	1.01	1.02	1.01	1.01	1.01	1.03	1.01	1.05
Jun-96	1.08	1.01	1.02	1.08	1.03	1.01	1.02	1.01	1.00	1.01
Sep-96	1.05	1.02	1.03	1.04	1.02	1.01	1.02	1.04	1.02	
Dec-96	1.02	1.07	1.07	1.01	1.01	1.01	1.03	1.02		
Mar-97	1.06	1.02	1.16	1.02	1.02	1.00	1.02			
Jun-97	1.05	1.02	1.04	1.00	1.01	1.00				
Sep-97	1.03	1.02	1.07	1.03	1.02					
Dec-97	1.04	1.01	1.02	1.00						
Mar-98	1.04	1.02	1.01							
Jun-98	1.01	1.02								
Sep-98	1.02									
Dec-98										
last 1 year	1.02	1.02	1.04	1.01	1.01	1.01	1.02	1.02	1.01	1.02
last 2 years	1.03	1.02	1.05	1.03	1.02	1.01	1.01	1.02	1.01	1.02
last 3 years	1.03	1.02	1.04	1.02	1.02	1.01	1.02	1.01	1.02	
last 4 years	1.03	1.02	1.04	1.02	1.02					
all years	1.03									

accident	development quarter of finalisation						
quarter	30	31	32	33	34	35	36
Sep-94	1 00	1 00	1 03	1.00	1 00	1 00	1 02
Dec-94	1 01	1 02	1.01	1 02	1 01	1.00	
Mar-95	1.03	1 01	1.00	1 02	1 01		
Jun-95	1 01	1 01	1.01	1.01			
Sep-95	1.01	1 00	1 00				
Dec-95	1.01	1 00					
Mar-96	1.00						
Jun-96							
last 1 year	1 01	1.01	1 00	1 01	1 01	1 00	1 02
last 2 years	1 01	1 01	1 01				
last 3 years							
last 4 years							
all years							

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