Going from a Pure Premium to a Rate

By

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The CAS Statement of Principles Regarding Property and Casualty Insurance Ratemaking, Principle 3 states:

A rate provides for the costs associated with an individual transfer of risk.

The purpose of this principle, as stated in the SOP, is to maintain equity among insureds. Implicit in this principle is the fact that costs vary with each individual transfer of risk. Different types of cost have different correlations with the overall cost of the transfer of risk. For instance, the commission expense is more highly correlated with the cost of the transfer of risk than are overhead expenses such as the home office salary expense and the home office electric bill. Each is a cost of providing the insurance product and must be allocated in some way to the individual policyholder. In practice, expenses are placed into one of two categories, those expenses that vary with total cost of the transfer of risk and those that do not.

This Study Note provides a simple model for addressing the issue of allocating these different types of expenses. Two specific variations of the model are presented to the actuarial student. These variations are known as 1) the Expense Fee Method and 2) the Workers' Comp Method. Used in the proper context, these techniques can help satisfy the requirements set forth in Principle 3.

There are three assumptions made in this Study Note. They are:

1) that all quantities are accurately measured

2) that all rates are at an adequate level, and

3) that the rating plans are multiplicative.

It will be helpful if the student understands clearly the difference between a rate and a premium. According to The American Heritage Dictionary, a rate is "the cost per unit of a commodity or service". Exposures are the units used in insurance rates. Premium is total cost of the insurance policy, calculated simply as:

Premium = Rate x Exposures

If Premium is measured in units such as "dollars", Exposures in units such as "Car Years" then the Rate would be measured in "dollars per Car Year". Fixed expenses can be allocated based on the rate being charged the policyholder, the premium being paid, or a combination of the two. The methods and examples used in this Study Note will indicate which of these types of allocation is being used.

In this paper, costs (or expenses) are categorized several different ways. First, they are categorized by their Insurance Expense Exhibit (IEE) expense class. The IEE contains data on a company's premium and expenses by line of insurance. It contains detailed information about written and earned premiums, losses, allocated and unallocated loss adjustment expenses as well as administrative expenses. It is the administrative expenses, those expenses of running the business not directly related to the settlement of claims, that are of interest in this Study Note; those expenses, as categorized in the IEE, are:

1) Commission and Brokerage

2) Taxes, Licenses and Fees

3) Other Acquisition

4) General Expense

5) Pretax Profit.

Each of these classes is then separated into the two broad categories of fixed and variable expense. The fixed and variable expense categories are used to recognize that some expenses are independent of the rate or premium being paid (e.g., company overhead) while other expenses vary with the rate or premium (e.g., commissions).

The general model used in this Study Note provides a framework for the actuarially fair allocation of expenses to insureds. The total rate is modeled as the sum of its component parts, the pure premium, fixed expenses, variable expenses and a provision for profit and contingencies. It is assumed that each of these components has been estimated and measured accurately. The notation used throughout the Study Note is intended to be consistent with that used in "Ratemaking" by McClenahan in Foundations of Casualty Actuarial Science. The McClenahan Pure Premium Method is actually a special case of the general model used in this Study Note, with the general model's functions f(E) and v(R,E) taking on a specific functional form. That special case is a topic of discussion later in this Study Note. The general model is given by the equation:

$$R = P + f(E) + v(R,E) + Q \cdot R \qquad (1)$$

where R = rate per unit of exposure

P = pure premium (expected loss cost) per unit exposure

E = number of written exposures on the policy

f(E) = fixed expenses as a function of total written exposures

v(R,E) = variable expenses as a function of the rate and written exposures

Q = profit and contingencies factor (as a proportion of the rate)

It should be noted that R is a function of exposures (E) through f(E) and v(R,E).

The actuary's first concern is to decide whether fixed expenses should be allocated to a policy or to an exposure. Since the rate is determined on an *exposure basis*, fixed expenses calculated on a *policy basis* need to be converted to an *exposure basis*. For example, the actuary determines that for Private Passenger Auto there are $50 of fixed expenses per policy with an average of 2 exposures per policy. That $50 is spread across the number of exposures on the policy to get the fixed expenses per exposure, f(E). Table 1 shows f(E) for various values of E.

E	f(E)	Ef(E)
1	$50.00	$50.00
2	$25.00	$50.00
3	$16.67	$50.00
4	$12.50	$50.00

Table 1. **Fixed expenses calculated on a *policy basis*.**
When fixed expenses are constant for a policy, fixed expenses vary by exposure.

Here f(E) is a function of the number of exposures on the policy, namely f(E) = $50/E. The amount Ef(E), is used to cover the fixed expenses the actuary has determined are associated with the policy. When fixed expenses are determined on a policy basis, the amount Ef(E) will be a constant, and f(E) varies by exposure.

If the fixed expenses are calculated on an *exposure basis*, no conversion is necessary. The premium for each exposure will need to cover a specified fixed expense dollar amount. In our example, the average dollars of fixed expense per exposure is $25. Since f(E) = $25 for all values of E, f(E) is independent of the number of exposures on the policy. Table 2 shows the relationship between E and f(E) when fixed expenses are calculated on an exposure basis. Again, the amount Ef(E) is used to cover the fixed expenses associated with the policy.

E	f(E)	Ef(E)
1	$25.00	$25.00
2	25.00	50.00
3	25.00	75.00
4	25.00	100.00

Table 2. **Fixed expenses calculated on an *exposure basis*.**
When fixed expenses are constant per exposure, fixed expenses vary by policy.

In deciding whether to calculate fixed expenses on an exposure or a policy basis, consideration should be given to the range of exposures on a policy, the correlation of overhead expenses with policy exposures and current company practices with regard to allocation of expenses. Once that determination has been made, the actuary can choose an appropriate mathematical form of f(E).

Treatment of fixed and variable expenses can be done on a per exposure basis (e.g., Expense Fee Method) or on a per policy basis (e.g., Workers' Comp Method). The per

exposure basis is used when the range of exposures written on a policy is small. It is often used in Private Passenger Auto where exposures (earned car years) typically range from 0.5 to 4.0. The per policy basis is used when a wider range of exposures on a policy can exist, such as in Workers' Comp.

The actuary can use the information in the Insurance Expense Exhibit, along with any supplementary expense information (e.g., internal company reports or audits, external industry data, economic data) to estimate the percentage of premium attributable to each of the expense categories (Commissions, Other Acquisition, General, etc.). These same sources may also give the actuary information about the proportion of each expense category that can be considered "fixed" or "variable" expense. Some jurisdictions, such as Florida, have requirements about partitioning expenses into fixed and variable components; statutory restrictions need to be considered when making selections.

Expense Fee Method

An expense fee is an amount paid to cover the cost of fixed expenses associated with writing and servicing a policy. If fixed and variable expense treatment is done on a per exposure basis, then f(E) and v(R,E) will be independent of the number of exposures on the policy (E). In the Expense Fee Method, f(E) and v(R,E) take on the following values:

$$f(E) = F \qquad (2)$$
$$= H \cdot R$$

$$v(R,E) = V \cdot R \qquad (3)$$

Where:

F = fixed expenses per exposure

H = fixed expense ratio (fixed expenses as a proportion of the rate)

V = variable expense ratio (variable expenses as a proportion of the rate)

Equation (1) now reduces to

$$R = P + F + V \cdot R + Q \cdot R \tag{4}$$

Solving for R gives us the formula for the Pure Premium Method used by McClenahan:

$$R = \frac{P + F}{1 - V - Q}$$

or

$$R = \frac{P}{1 - V - Q} + \frac{F}{1 - V - Q} \tag{5}$$

$\frac{P}{1 - V - Q} \equiv$ variable rate

$\frac{F}{1 - V - Q} \equiv$ expense fee

Here, R is independent of the number of exposures written with the policy (because the functional forms of f(E) and v(R,E) in Equations (2) and (3) are independent of E) and each exposure pays the same dollars of fixed expense. As seen in Table 2, a policy written with multiple exposures (i.e., a multicar policy) would pay proportionately more dollars of fixed expense than a policy written with one exposure. In most cases, the pure premium, P, is calculated as some base rate times all applicable rating plan factors

Exhibit 1 shows an example of the Expense Fee Method as used in Private Passenger Auto - Bodily Injury coverage. The technique shown can be applied to other lines of business. The example shows how expense fees are calculated the first time they are

included in the rates. In addition, it shows how territorial base rates are offset so that there is no overall rate level effect of implementing expense fees.

Implementing Expense Fees

Exhibits 1-4 show how Expense Fees can be introduced into the rates. The procedures shown on these Exhibits are mathematically equivalent to those in the ISO paper "Methodologies for Expense Fee Program Implementation" written in the late 1970's. It is assumed that the current rates are adequate at the time of the review. The actuary must first determine the fixed and variable proportion of the rate. Exhibit 1 shows a breakdown of fixed and variable expenses by IEE category for Private Passenger Auto Bodily Injury Liability coverage. The "Total Expenses" are partitioned into "Fixed" and "Variable" expenses using the percentages in the "% Fixed" column. The ISO Expense Fee Program, implemented in 1978, selected 75% of General Expense and Miscellaneous Taxes, Licenses and Fees as fixed expenses. It is not clear if the 75% factor was supported by an analysis of expense data, or if it was judgmentally selected. Since 1978, significant changes have taken place in the use of technology and the distribution of computer resources within insurance companies. This may have an effect on the allocation by department of computer related expenses and decrease the fixed proportion of General Expense. The actuary should use any additional information (internal audits, procedures for the allocation of corporate overhead to individual departments, etc.) and his judgment in selecting appropriate values for the "% Fixed" for a particular expense type.

In the example, the total expense has been partitioned into fixed and variable portions, given by

$$H = 0.0683 \quad \text{(fixed)}$$

$$V + Q = 0.2817 \quad \text{(variable)}$$

The Expense Fee Ratio is then given by

$$\text{Expense Fee Ratio} = \frac{H}{1-V-Q} = \frac{0.0683}{1-0.2817} = 0.0951$$

The Expense Fee Ratio is the proportion of the total rate that is needed to cover the fixed expenses (i.e., the fixed expense ratio loaded for variable expenses). The Expected Loss Ratio (T or Target Loss Ratio according to McClenahan) is simply

$$\text{Expected Loss Ratio} = 1 - V - Q - H = 0.65$$

The actual calculation of the expense fee requires the Statewide Average Rate. This Statewide Average Rate can be calculated a number of different ways:

$$\text{Statewide Average Rate} = \frac{\text{Earned Premium @ Current Rate Level}}{\text{Earned Exposures}}$$

$$= \text{Statewide Average Base Rate} \cdot \text{Average Rating Plan Factor}$$

$$= \frac{\text{Statewide Average Loss Cost}}{\text{Expected Loss Ratio}}$$

The last two equations are known as the Premium and Lost Cost methods, respectively. Calculation of the Average Rating Plan Factor is shown on Exhibit 2. The Average Rating Plan Factor is just the average effect of all rating plans (for instance, territory, classification, increased limits, deductibles) on the final rate per exposure. Of course, the Base Rates and the Rating Plan factors represent the current rates. The expense fee is just

the Statewide Average Rate times the Expense Fee Ratio. Exhibit 3 shows the calculation of the Expense Fee using the Premium method and the Loss Cost method. Once the Statewide Average Rate has been determined, the Premium and the Loss Cost Methods are identical.

When incorporating expense fees into the rates, it is necessary to offset the otherwise applicable rates. Several methods of offsetting are available. Each of the following equations should produce the appropriate Variable Base Rates:

$$\text{Variable Base Rate} = \frac{(\text{Statewide Average Rate} - \text{Expense Fee}) \cdot \text{Territory Relativity}}{\text{Average Rating Plan Factor}}$$

$$= \frac{\text{Base Loss Cost}}{1 - V - Q}$$

$$= \text{Current Territorial Base Rate} \cdot (1 - \text{Expense Fee Ratio})$$

Each of these offset methods assumes that all rating plans are multiplicative.

Subsequent indicated rate changes need to take into account the expense fees embedded in the rates. If the method in Exhibits 1-4 is to be used to implement subsequent expense fees, a proposed Statewide Average Rate is required. The proposed average rate is found by multiplying the current average rate by the indicated change. The calculation of the indicated change should use premium at current rates and expense fees, since the Statewide Average Rate includes expense fees.

The major difference between implementing and revising expense fees is in the calculation of the Variable Base Rate. When implementing expense fees and variable base rates, we were given a set of territorial base rates. When revising the expense fees, we do not have the same type of territorial base rates, we have variable base rates. The same procedure can be followed with the exception that the Variable Base Rates should not be calculated as the Current Territorial Base Rates times (1 - Expense Fee Ratio), since the Current Territorial Base Rates are already Variable Base Rates.

Workers' Comp Method

Under the Workers' Comp Method, the fixed and variable expense treatment is done on a per policy basis. As was seen in Table 1, the rate per exposure of covering the fixed expenses is a function of the number of exposures on the policy (E). Every policyholder pays a fixed dollar amount to cover fixed expenses (known as an "expense constant" in Workers' Comp) associated with his policy. In general, the expense constant currently ranges from $140 to $200. The function f(E) has the following form:

$$f(E) = \frac{\text{Policy Expense Constant}}{E}$$

Expenses not covered by the expense constant are either variable (a constant proportion of premium) or some non-constant function of policy exposures or premium. In the Expense Fee Method, the variable expense percentages were a constant; they did not vary with the level of premium being charged. For instance, the commission ratio was 20.0% whether the policy was written for $500 or $5,000. In the Workers' Comp method, the variable expense ratio is not constant for all expense categories. In particular, the Commission and General Expense ratios decrease as premium increases. In effect, this

gives a discount to policyholders with large policy premiums. The determination of the "Premium Discount" is a goal of the Workers' Comp Method. The idea behind the premium discount is that not all expenses are directly proportional to the premium or rate.

Exhibit 5 shows an example of how the premium discount is determined. In this example, it is assumed that a particular policyholder has $7,000,000 in limited payroll, with an exposure base of $1,000 of payroll giving 7,000 exposures. There is also a $200 expense fee charged to every policy. The policy's Standard Premium is $350,000. This $350,000 is partitioned into "gradations"; the first $5,000 of premium; the next $95,000 of premium, and so on. Each gradation of premium has a set of expense percentages associated with it. In this example, the expenses are Production, General, Taxes and Profit & Contingencies. The expenses shown on Exhibit 5 are illustrative only, and to not reflect any particular company or rating organization's expense structure. Some rating organizations file several different premium discounts because of the varying expense structures of their member companies. Expense structures can differ form many reasons, including type of entity (stock or non-stock), type of production (policy distribution) system, etc. For instance, direct writers, captive agency companies and companies that use independent agents will all have different production costs.

In addition to the variable expenses discussed earlier, Feldblum, in his Study Note article "Workers' Compensation Ratemaking" indicates that other variable expenses may be included; he specifies assessments and the residual market burden. The Production and General Expenses percentages vary with the premium gradation and represent

percentages of Standard Premium. All other expenses are shown as percentages the Discounted Premium. The percent reduction in expenses is simply the difference between the expenses in a particular Premium Range and those expenses in the Premium Range of $1 - $5,000. The Discount Percent is calculated as:

$$\text{Discount Percent} = \frac{\text{Expense Reduction}}{1 - \text{expenses that are percentages of discounted premium}}$$

$$= \frac{\text{Expense Reduction}}{1 - \text{Taxes} - \text{Profit \& Contingencies}}$$

$$\text{Total Discount} = \sum_{\text{Premium Range}} (\text{Discount Percent}) \cdot (\text{Premium in Range})$$
$$= 0.00\% \cdot 5{,}000 + 7.61\% \cdot 95{,}000 + 12.5\% \cdot 250{,}000 + 17.39\% \cdot 0$$
$$= 0 + 7{,}228 + 31{,}250 + 0$$
$$= 38{,}478$$

This results in a Discounted Premium of $311,522 and a total discount of 10.99%. After adding the expense fee of $200, we get the net premium of $311,722. The rate here can then be calculated as the premium per exposure or $44.53 per $1,000 of limited payroll. As was mentioned earlier, this technique does not lend itself to the explicit formulation of v(R,E); however, the variable expenses are a function of the rate and exposure level through the Standard Premium level and the associated total discount.

Conclusion

Proper allocation of expenses is an important aspect of producing an actuarially fair rate. This Study Note has provided the student with a general model showing the components of the total rate, including the fixed and variable expense portions of the rate. The techniques provided in this Study Note, the Expense Fee Method and the Workers' Comp

Method, provide the student with a framework for fairly allocating expenses to policies of differing premium and rate levels.

Exhibit 1

Private Passenger Auto Liability
Expense Summary

Expense Type	Total Expense	% Fixed	Fixed	Variable
Commission	0.2000	0.0%	0.0000	0.2000
Other Acquisition	0.0300	75.0%	0.0225	0.0075
General	0.0400	80.0%	0.0320	0.0080
Premium Tax	0.0150	0.0%	0.0000	0.0150
Misc. Licenses & Fees	0.0050	75.0%	0.0038	0.0012
Profit & Contingencies	0.0500	0.0%	0.0000	0.0500
Other	0.0100	100.0%	0.0100	0.0000
Total	0.3500		0.0683	0.2817

H	=	0.0683
V + Q	=	0.2817
1 - V - Q	=	0.7183

Expense Fee Ratio	=	0.0951 = 0.0683/(1-0.2817) = H / (1 - V - Q)
Expected Loss Ratio (T)	=	0.6500 = 1 - 0.35 = 1 - V - Q - H

Exhibit 2

Exposure Distribution and Rating Plan Factors

	Exposure Distribution Increased Limits Factor			
Territory	20/40	50/100	100/300	Total
A	80	202	118	400
B	48	52	100	200
C	21	31	48	100
D	150	50	100	300
Total	299	335	366	1000

Territory	Exposures	Territory Relativity
A	400	1.000
B	200	1.222
C	100	0.711
D	300	0.578
Total	1000	0.8889

Limits	Exposures	Increased Limits Factor
20/40	299	1.000
50/100	335	1.200
100/300	366	1.500
Total	1000	1.2500

Average Rating Plan Factor = Avg. Territory Relativity x Avg. Increased Limits Factor

= 0.8889 x 1.2500
= 1.1111

Exhibit 3

Implementing Expense Fees
Premium & Loss Cost Methods

Premium:

Territory A (Base) Base Rate	225.00			
Average Rating Plan Factor	1.1111		See Exhibit 2	
Statewide Average Rate	250.00	=	225 x 1.1111	
H	0.0683		See Exhibit 1	
1 - V - Q	0.7183		See Exhibit 1	
Expense Fee Ratio	0.0951		See Exhibit 1	
Fixed Expense Per Exposure (F)	17.08	=	250 x 0.0683	Statewide Average Rate x H
Expense Fee	23.78	=	250 x 0.0951	Statewide Average Rate x Expense Fee Ratio
		=	17.08 / 0.7183	F x H / (1 - V - Q)

Loss Cost:

Statewide Average Loss Cost Per Exposure	162.50			
Expected Loss Ratio (T)	0.65			
Statewide Average Rate	250.00	=	162.50 / 0.65	Statewide Avg. Loss Cost / ELR
H	0.0683		See Exhibit 1	
1 - V - Q	0.7183		See Exhibit 1	
Expense Fee Ratio	0.0951		See Exhibit 1	
Fixed Expense Per Exposure (F)	17.08	=	250 x 0.0683	Statewide Average Rate x H
Expense Fee	23.78	=	250 x 0.0951	Statewide Average Rate x Expense Fee Ratio
		=	17.08 / 0.7183	F x H / (1 - V - Q)

Exhibit 4

Variable Base Rates

Territory	Earned Car Years	Current Territorial Base Rate	Territorial Base Rate Relativity	Base Loss Cost	Variable Base Rate
A	400	225	1.000	146.25	203.61
B	200	275	1.222	178.75	248.85
C	100	160	0.711	104.00	144.79
D	300	130	0.578	84.50	117.64
Total	1000	200	0.889	130.00	180.99

Base Loss Cost = Base Rate x ELR

$$\text{Variable Base Rate} = \frac{\text{Base Loss Cost}}{1 - V - Q}$$

= Current Territorial Base Rate x (1 - Expense Fee Ratio)

$$= \frac{\text{(Statewide Average Rate - Expense Fee) x Territory Relativity}}{\text{Average Rating Plan Factor}}$$

Exhibit 5

Workers' Compensation Method

Premium Range	Gradation of Premium	Premium in Range	Production	General	Taxes	Profit & Cont.	Production and General Expense Reduction	Discount Percent	Discount Dollars
1-5,000	5,000	5,000	15.00%	10.00%	3.00%	5.0%	0.00%	0.00%	0
5,001-100,000	95,000	95,000	10.00%	8.00%	3.00%	5.0%	7.00%	7.61%	7,228
100,001-500,000	400,000	250,000	7.50%	6.00%	3.00%	5.0%	11.50%	12.50%	31,250
500,001+	500,000+	0	5.00%	4.00%	3.00%	5.0%	16.00%	17.39%	0
Total		350,000							38,478

Example:
Assume Premium = $350,000
Calculations below are for the 100,001 - 500,000 Premium Range

Premium in Range
= Min(400,000, Max(350,000 - 100,000, 0))
= Min(400,000, Max(250,000, 0))
= Min(400,000, 250,000)
= 250,000

Production and General Expense Reduction
= (15.00% + 10.00%) - (7.50% + 6.00%)
= 11.50%

Discount Percent
= 11.50% / (1.00 - 0.03 - 0.05)
= 12.50%

Discount Dollar
= 250,000 x 12.50%
= 31,250