# Risk-Adjusted Performance Measurement for P&C Insurers

Richard Goldfarb, FCAS, CFA, FRM

Original Draft: December 2006 (Revised: October 2010)

Abstract

This paper was prepared as an introduction to risk-adjusted performance measurement for P&C insurance companies. A simplified numerical example is used to demonstrate how measures such as risk-adjusted return on capital (RAROC) can be used to guide certain strategic decisions. While the discussion is simplified throughout, the numerical examples are used to highlight the important challenges associated with this methodology and clarify some of its limitations.

Acknowledgments

Several individuals read earlier drafts of this paper and provided feedback that greatly improved the content and helped to streamline the presentation. I would like to particularly thank Walt Stewart, Emily Gilde, Ralph Blanchard, Don Mango, Bill Panning, Edward Chiang, Tom Conway, Jenny Lai, Parr Schoolman and Jessica Somerfeld for their contributions.

Note Regarding October 2010 Revision

The October 2010 revision merely reflects a change to the title of the study note resulting from revisions to the numbering convention used for the CAS exam for which this study note was originally produced. All of the content remains the same as the December 2006 version.

**Table of Contents**

## 1. Introduction

This paper was prepared as an introduction to risk-adjusted performance measurement for a P&C insurance company.

The paper is organized as follows. Section 2 provides an overview of risk-adjusted performance measurement, with an emphasis on one particular implementation of risk-adjusted return on capital (RAROC). The emphasis on RAROC is used solely to focus the discussion, as many of the issues presented in subsequent sections are relevant to alternative methodologies that also attempt to risk-adjust performance measures.

Section 3 discusses the techniques used to characterize the risk distributions for different risk sources and the issues associated with developing a firm's aggregate risk profile. Section 4 then presents a simplified numerical example and uses it to demonstrate various techniques used to calculate the firm's aggregate risk capital and then allocate, or attribute, this risk capital to individual business units.

In Section 5, various applications that make use of the allocated risk capital are discussed in the context of the numerical example presented in Section 4. While not intended to be exhaustive, the discussion of these applications will help to emphasize the strengths, weaknesses and limitations of the specific RAROC application presented.

Finally, Section 6 summarizes some of the refinements that might be needed for certain applications, some of which can be useful for overcoming the limitations discussed in Section 5.

## 2. Overview of Risk-Adjusted Performance Measurement

Risk-adjusted performance measures are intended to improve upon the metrics used to make capital planning, risk management and corporate strategy decisions by explicitly reflecting the risks inherent in different businesses.

In a simple one-period case[1] in which a business requires an investment of a specific amount of capital and earns (or is expected to earn) a given dollar amount of income (profit) during the period, the return on capital is simply calculated as:

$$\text{Return on Capital} = \frac{\text{Income}}{\text{Capital}}$$

This is, of course, a very general form of a "return" calculation and in practice there are a wide variety of approaches that can be used to determine the amounts used for both the numerator and denominator. In many instances, adjustments made to either the numerator or denominator will have the effect of transforming the resulting measure into less of a "rate of return" than is commonly acknowledged. The resulting metrics are more accurately described as profitability indices or, more generally, financial ratios. This distinction between a rate of return and a financial ratio will be explored further when the challenges associated with developing an objective benchmark for the metric is discussed. For now, the ratio of income to capital will be referred to as a return on capital measure in the usual manner.

A variety of standard return on capital measures such as return on equity (ROE), return on assets (ROA) or total shareholder return (TSR) are often reviewed to assess *ex post* or *ex ante* performance of different business units within a firm or to assess the firm's overall performance relative to peers. However, because these measures often do not explicitly distinguish between activities with varying degrees of risk or uncertainty, they can sometimes result in misleading indications of relative performance and value creation.

Insurance companies commonly attempt to overcome this weakness associated with conventional ROE measures by allocating, or more accurately attributing, their capital or surplus to different

[1] A one-period model is rarely adequate for insurance businesses, since the capital required to support these businesses is committed and the income is earned over many periods. This issue will be explored further in Section 5.5.

business units using either premium to surplus ratios or reserve to surplus ratios that vary by line of business. This can serve to "risk-adjust" the return on capital measure by attributing more capital or surplus to business segments with more perceived risk, though often the premium to surplus and reserve to surplus ratios used are selected judgmentally or without the use of quantitative models.

An alternative approach is to make the risk-adjustment more explicit. Many banks and insurance companies have adopted risk-adjusted return on capital measures in which either the "return" is risk-adjusted, the "capital" is risk-adjusted, or in some cases both are risk-adjusted. Often all three instances are generically referred to as RAROC (**R**isk **A**djusted **R**eturn **O**n **C**apital), a convention that will be used here for convenience. But for clarity, throughout this discussion the emphasis will be on a measure based on income that is not risk-adjusted and capital that is risk-adjusted[2]:

$$\text{Risk - Adjusted Return on Capital} = RAROC = \frac{\text{Income}}{\text{Risk - Adjusted Capital}}$$

### 2.1 Income Measures

A wide variety of income measures exist, all of which are intended to reflect the profit, in dollars, during a specific measurement period. Four relevant choices include:

- GAAP Net Income – This measures the income earned according to GAAP accounting conventions. Use of this measure is convenient when RAROC is intended to be used to guide management decision-making, since the measurement basis is already in use within the firm.
- Statutory Net Income – In countries where separate statutory (regulatory) accounting frameworks are used, the income component may also be measured using these statutory accounting conventions.
- IASB Fair Value Basis Net Income – Although not yet formally adopted, efforts are underway by the International Accounting Standards Board (IASB) to develop "fair value" accounting standards. These standards are intended to remove many existing biases in various accounting conventions used throughout the world. For insurance companies, this measure of profit differs from GAAP net income primarily due to the discounting of loss reserves to reflect their present value and the inclusion of a risk margin on loss reserves to approximate a risk charge that would typically be included in an arms-length transaction designed to transfer the risk to a third party.
- Economic Profit – A more general method for measuring profit that further eliminates many accounting biases is often referred to as *economic profit*. Unfortunately, this term is often used to refer to many different types of adjustments to the GAAP income measures. Generally it refers to the total change in the "economic value" of the assets and liabilities of the firm, where asset values reflect their market value and the liabilities are discounted to reflect their present value. Whether this discounting of the liabilities includes a risk margin, as in the IASB definition of fair value, often varies.

  Some believe that estimates of the change in the "economic value" of assets and liabilities represent a more meaningful measure of the gain or loss in a given period. But there are limitations associated with this measure:

  - To accurately reflect the change in value for a firm, changes in the value of its *future* profits must also be taken into account. This *franchise value* can be a significant source of value for firms (well in excess of the value of the assets and liabilities on its balance sheet) and changes in this value will clearly impact total shareholder returns.

---

[2] To clarify, the methodology used throughout this paper is referred to as RAROC. However, because it is calculated as **R**eturn **O**ver **R**isk-**A**djusted **C**apital, it is often referred to as RORAC to indicate that it is the capital amount that is risk-adjusted. The RAROC terminology is often reserved for measures of **R**isk-**A**djusted **R**eturn **O**ver **C**apital, where the return measure is risk-adjusted. However, in both cases, Return on Capital is being measured and in both cases it is Risk-Adjusted, so in another sense *both* can legitimately be referred to as **R**isk-**A**djusted **R**eturn **O**n **C**apital, RAROC.

- The use of economic profit as an income measure also complicates reconciliation to GAAP income or other more familiar measures of profitability. This reconciliation issue is often important in practice because management may have more difficulty interpreting income measures that deviate significantly from commonly used measures.
- If the economic profit measures are not disclosed to external parties such as investors, regulators or rating agencies, management may have more difficulty communicating the basis for their decisions. These external parties may only have access to GAAP and statutory financial statements and they may be unable to reproduce internally generated economic profit estimates.

In the discussion that follows, a specific measure of "economic profit" will be used, merely for convenience. A variety of adjustments often made to the selected income measure will be ignored in Sections 2 through 5, but will be addressed briefly in Section 6.

### 2.2 Capital Measures

There are numerous ways to measure the capital required for a given firm or for specific business units within the firm. Some of these capital measures are risk-adjusted and some are not.

Two measures that are not risk-adjusted include:

- Actual Committed Capital – This is the actual cash capital provided to the company by its shareholders and used to generate income for the firm and its respective business units. This is typically an accounting book value equal to contributed capital plus retained earnings and can be based on GAAP, Statutory or IASB accounting conventions.
- Market Value of Equity – As discussed in Section 2.1, the committed capital measure described above could be adjusted to reflect market values of the assets and liabilities, though this will still reflect only the value of the net assets on the balance sheet. An alternative is to actually use the market value of the firm's equity, which will generally be larger than the committed capital because of the inclusion of the franchise value of the firm.

Four measures that explicitly reflect risk-adjustments, to varying degrees, include:

- Regulatory Required Capital – This is the capital required to satisfy minimum regulatory requirements. This is typically determined by explicit application of the appropriate regulatory capital requirement model.
- Rating Agency Required Capital – This is the capital required to achieve a stated credit rating from one or more credit rating agencies (S&P, A.M. Best, Moody's or Fitch). This is usually determined by explicit application of the respective credit rating agencies' capital models and by reference to the standards each rating agency has established for capital levels required to achieve specific ratings[3].
- Economic Capital – This term is commonly used but often defined differently, which leads to unnecessary confusion. In its most general sense, economic capital could be defined as *the capital required to ensure a specified probability (level of confidence) that the firm can achieve a specified objective over a given time horizon*. The objective that the risk capital is intended to achieve can vary based on the circumstances and can vary depending upon whether the focus is on the policyholder, debtholder or shareholder perspectives.

---

[3] It is important to note that the capital models used by the rating agencies represent just one of many factors that are used to assign a credit rating to any particular firm. Other factors include the strength of the management team, historical experience, access to capital and other related considerations. Nonetheless, the rating agencies typically provide indications of the rating levels associated with different levels of capital adequacy that result from the application of their capital models. These indications are used by firms to determine the "required capital" for a given rating, independent of all of these other rating factors.

- Solvency Objective – The most common approach used by rating agencies and regulators could be referred to as a *solvency objective*. A solvency objective focuses on holding sufficient capital today to ensure that the firm can meet its existing obligations to policyholders (and perhaps debtholders as well). This approach clearly reflects a policyholder or debtholder perspective.
- Capital Adequacy Objective – An alternative approach is to use what could be referred to as a *capital adequacy objective.* This objective focuses on holding sufficient capital to ensure that the firm can continue to pay dividends, support premium growth in line with long-term business plans or maintain a certain degree of financial strength over an extended horizon so as to maximize the franchise value of the firm.

These two approaches can lead to substantially different indications of the capital required for the firm or any individual business. The "solvency" perspective is currently quite commonly used, so for convenience this perspective will be adopted throughout this paper[4]. When using this definition of economic capital, the focus is typically on ensuring that there are sufficient financial resources (in cash and marketable securities) to satisfy policyholder (and debtholder) obligations. However, there will necessarily be a somewhat arbitrary separation of the total financial resources into a portion that represents a "liability" and a portion that represents "capital". This separation will usually follow applicable accounting conventions, but can lead to meaningful differences in practice.

For instance, some practitioners define economic capital as the difference between the total financial resources needed *less* the undiscounted value of the (expected) liability. This is consistent with how the firm's resources would be classified under U.S. statutory accounting. Others prefer to define the economic capital as the amount that the total financial resources needed exceeds the discounted value of the (expected) liability. Still others might choose to incorporate a risk margin in the liability and treat the economic capital as the amount by which the total financial resources needed exceeds the fair value of the liability.

Any of these approaches could be used, so long as they are used consistently across different risks.

- Risk Capital – The range of different interpretations of the term *economic capital* is worrisome and can lead to a variety of inconsistent adjustments in practice. For instance, the choices described above all define economic capital as the portion in excess of the discounted expected liability, the undiscounted expected liability or the fair value of the liability under the assumption that funding for these amounts are already accounted for in the firm's financial statements. This is not the case for all risks – some could not be reflected at all on the balance sheet, in which case the economic capital has to account for all of the potential liabilities, while others could be funded by an amount well in excess of the discounted value, undiscounted value or fair value of the expected liability.

  To avoid confusion in this paper, a closely related measure referred to here as *risk capital* will be used instead of any of the definitions of economic capital. Risk capital is defined as the amount of capital that must be contributed by the **shareholders** of the firm in order to absorb the risk that liabilities will exceed the funds already provided for in either the loss reserves or in the policyholder premiums. Under this definition, any conservatism in the loss reserves or any risk margins included in the premiums will reduce the amount of risk capital that must be provided by shareholders.

  Notice that in the absence of a risk margin included in the premiums or the reserves, the risk capital and the economic capital may be identical. As a result, for many of the applications discussed later in this paper either amount could be used. However, for some of the main

[4] Panning's *Managing the Invisible* contains a thorough discussion of the alternative perspective and its importance for managing a firm.

applications that involve evaluating specific business unit results or pricing for new business, the use of risk capital will more fairly account for the risk from the shareholder's perspective.

As a result, the term *risk capital* will be used here, even in instances where it is equivalent to the common definition of economic capital.

***Are Risk-Based Capital Measures Superior?***

Before proceeding, it is worthwhile to consider whether the risk-based measures of capital are necessarily more insightful or meaningful for various strategic decisions than the measures of actual committed capital or market value capital. These risk-based measures of "required" capital are quite often substantially lower than either the book value of the firm or the market value of the firm's equity. As a result, attempts to reflect the "cost" of the capital allocated to specific business units will potentially understate the true costs by ignoring a substantial amount of unallocated capital.

Some practitioners attempt to compensate for this weakness through the use of so-called *stranded capital* charges that further adjust the return measure to reflect a cost associated with the actual capital held in excess of the risk-based capital.

Issues associated with this adjustment will be discussed in Section 6. At this point it is sufficient to emphasize that there is an alternative approach to ensuring that all of the capital held by the firm is taken into account that preserves the risk-based nature of the allocation of capital. In this method, the firm's actual capital is used, but the *allocation method* is risk-based. In other words, measures of risk capital for each business unit serve as the basis for the allocation, but the total amount of capital allocated is simply the firm's actual book value or its market value.

## *3. Measuring Risk Capital*

A critical component of the RAROC measure described in the previous section is the calculation of the risk capital for the firm and, more importantly, the risk capital allocated to various business units. For clarity, the allocation methods will be discussed in Section 4 in the context of a simplified numerical example. In this section, a variety of risk measures and the methods used to measure the firm's overall risk capital will be described.

### 3.1 Risk Measures

Four common risk measures will be described in this section and then used in Section 4 in the context of a specific numerical example.

#### *Probability of Ruin*

The Probability of Ruin is the (estimated) probability that a "ruin" scenario will occur. This is often defined specifically to refer to "default", where the assets are insufficient to fully settle all liabilities, but other definitions of ruin could easily be substituted. For instance, risk capital might be determined based upon an objective of maintaining a particular credit rating over some specified time horizon. In this context, ruin could be defined as a decline in the credit rating below some specified threshold.

#### *Percentile Risk Measure (Value at Risk)*

In practice, calculating the firm's actual probability of ruin is often of less interest than a closely related measure – the dollar amount of capital required to achieve a specific probability of ruin target.

Suppose the full distribution indicating the amount, in dollars, that could be lost over a given time horizon was known. Because each dollar of loss will destroy one dollar of "capital", each percentile of this distribution indicates the amount of starting capital required so that the losses do not exceed the capital, resulting in "ruin", with a given probability. For example, the $99^{th}$ percentile of this distribution determines the amount of capital required to limit the probability of ruin to 1%. Similarly, the $95^{th}$ percentile determines the amount of capital required to limit the probability of ruin to 5%. This is best demonstrated with a numerical example.

The following table represents 1,000 simulated values from an insurance liability claim distribution[5], with most of the values not shown for convenience and the values sorted in descending order. Here, the expected value of the claims equals $5,000 and the premium charged is $6,000.

**Table 1: Simulated Underwriting Loss**

| Scenario | Liability | Premium | Loss |
|---:|---:|---:|---:|
| 1,000 | 7,356 | 6,000 | 1,356 |
| 999 | 7,354 | 6,000 | 1,354 |
| 998 | 7,269 | 6,000 | 1,269 |
| 997 | 7,199 | 6,000 | 1,199 |
| 996 | 7,178 | 6,000 | 1,178 |
| 995 | 7,039 | 6,000 | 1,039 |
| 994 | 7,021 | 6,000 | 1,021 |
| 993 | 6,949 | 6,000 | 949 |
| 992 | 6,946 | 6,000 | 946 |
| 991 | 6,908 | 6,000 | 908 |
| **990** | **6,908** | **6,000** | **908** |
| 989 | 6,811 | 6,000 | 811 |
| 988 | 6,797 | 6,000 | 797 |
| 987 | 6,792 | 6,000 | 792 |
| 986 | 6,787 | 6,000 | 787 |
| 985 | 6,767 | 6,000 | 767 |
| : | : | : | : |
| 5 | 3,323 | 6,000 | -2,677 |
| 4 | 3,261 | 6,000 | -2,739 |
| 3 | 3,248 | 6,000 | -2,752 |
| 2 | 3,243 | 6,000 | -2,757 |
| 1 | 2,735 | 6,000 | -3,265 |

[5] Simulated liability distributions are used in this section to avoid mathematical details and provide an intuitive discussion of the differences in allocation methodologies. Imprecision introduced through the use of too few simulated values should be ignored.

The last column reflects the "loss" in the profit and loss sense (e.g. as in the calculation of an underwriting loss), with losses depicted as positive amounts and profits as negative amounts. This reversal of the signs is done to facilitate the discussion of both liability claim distributions and asset distributions later in this paper. Note though that care must be taken to distinguish between losses in this profit/loss sense and claim amount distributions, which actuaries commonly refer to as "loss distributions". Note as well that the losses (again, in the profit/loss sense) are shown here net of the premiums charged and other expenses incurred.

The 99$^{th}$ percentile risk measure is the loss amount that is exceeded only 1% of the time. In this specific example, this is equal to $908. If the firm had an additional $908 of risk capital, then it would have sufficient funds ($6,908 in total when the premiums are taken into account) to pay all claims 99% of the time and would suffer partial "default" in only 1% of the scenarios.

This percentile risk measure is essentially identical to the risk measure known as Value at Risk (VaR). There are two minor distinctions that are worth noting:

- Value vs. Nominal Loss Amount – When VaR is calculated for marketable securities such as equities, bonds or derivative instruments, the quantity of interest is the change in value of the instrument over a specific time horizon. In some applications, including the one discussed here, the quantity modeled may not necessarily be the *value* of the cash flows, which would include the effects of discounting for the time value of money and a risk margin. Instead, often the quantity being modeled is simply the total amount of the cash flows or simply the discounted value of these cash flows without consideration of a risk margin. As a result, it may be more accurate to refer to risk capital as a percentile risk measure, rather than a "value at risk".
- Relative vs. Absolute VaR – In some textbooks VaR is defined as the amount by which the percentile deviates from the mean of the profit/loss distribution rather than the amount by which it falls below zero. In the context of the previous numerical example, since the expected liability amount is $5,000 and the premium is $6,000, the expected "loss" is -$1,000 (technically, an expected profit of $1,000). The 99$^{th}$ percentile loss amount is $908, so in a relative sense this is $1,908 worse than the expected loss.

  However, in the application discussed here the goal is to understand how much risk capital is needed. Therefore, the absolute measure of $908 is more relevant than the deviation from the mean, which can be viewed as a relative measure.

Despite these two minor distinctions, the percentile risk measure and the VaR terminology are commonly used interchangeably. This will be the case in various sections of this paper, where the VaR terminology is used to remain consistent with common practice.

***Conditional Tail Expectation***

The conditional tail expectation (CTE), which is also known as the Tail VaR (TVaR) or the Tail Conditional Expectation (TCE), is similar to the percentile risk measure (VaR) in some respects. The difference is that rather than reflect the value at a single percentile of the distribution, the CTE represents the average loss for those losses that exceed the chosen percentile. Once again, note that the use of the term "loss distribution" refers to the profit/loss sense of the word. When dealing with insurance liabilities, the premiums or the carried reserves should be subtracted from the "claim" amount when calculating the CTE.

Continuing with the previous example, the CTE can be calculated as the average of the 10 scenarios that exceed the 99$^{th}$ percentile value. These scenarios have an average loss of $1,122 and are shown as the boxed values in the following table.

**Table 2: Calculation of the CTE**

| Scenario | Liability | Premium | Loss |
|---:|---:|---:|---:|
| 1,000 | 7,356 | 6,000 | **1,356** |
| 999 | 7,354 | 6,000 | **1,354** |
| 998 | 7,269 | 6,000 | **1,269** |
| 997 | 7,199 | 6,000 | **1,199** |
| 996 | 7,178 | 6,000 | **1,178** |
| 995 | 7,039 | 6,000 | **1,039** |
| 994 | 7,021 | 6,000 | **1,021** |
| 993 | 6,949 | 6,000 | **949** |
| 992 | 6,946 | 6,000 | **946** |
| **991** | **6,908** | **6,000** | **908** |
| 990 | 6,908 | 6,000 | 908 |
| 989 | 6,811 | 6,000 | 811 |
| 988 | 6,797 | 6,000 | 797 |
| 987 | 6,792 | 6,000 | 792 |
| 986 | 6,787 | 6,000 | 787 |
| 985 | 6,767 | 6,000 | 767 |
| : | : | : | : |
| 5 | 3,323 | 6,000 | -2,677 |
| 4 | 3,261 | 6,000 | -2,739 |
| 3 | 3,248 | 6,000 | -2,752 |
| 2 | 3,243 | 6,000 | -2,757 |
| 1 | 2,735 | 6,000 | -3,265 |

Due to certain desirable mathematical properties[6], the CTE has become an increasingly common risk measure used in practice. Interestingly, using this risk measure results in a more ambiguous relationship between the *risk measure* and the *capital* needed to satisfy a specific objective. In the case of the percentile risk measure (VaR), it is easy to see that when the firm's capital is equal to the $X^{th}$ percentile (the *X*% VaR) then the default probability is 1-*X*%. But when capital is equal to the *X*% CTE, the default probability is some amount less than 1-*X*%.

The precise default probability is dependent upon the particular shape of the loss distribution, though some practitioners commonly assume that it is roughly equal to (1-*X*%)/2. In the example shown here, capital equal to the 99% CTE = $1,122 would result in defaults in 5 of the scenarios, or .5% of the time. The reliability of this approximation depends heavily on the shape of the aggregate loss distribution.

### *Expected Policyholder Deficit Ratio*

The Expected Policyholder Deficit (EPD) is closely related to the CTE risk measure. However, the CTE is conditional on the losses exceeding an arbitrarily selected percentile while the EPD is somewhat less arbitrary. The EPD is driven by the average value of the *shortfall* between the assets and liabilities. All liability scenarios are included in this calculation, in contrast to the CTE risk measure that uses only those scenarios for which the liabilities exceed a selected percentile. But in the EPD calculation, scenarios for which there is no "shortfall" are assigned a value of zero.

Again using the same example and assuming that the premiums collected represent the only assets the firm carries, the average shortfall is calculated using all of the highlighted values in the following table.

[6] See Artzner, et al.

**Table 3: Calculation of the EPD**

| Scenario | Liability | Premium | Shortfall |
|---|---|---|---|
| 1,000 | 7,356 | 6,000 | **1,356** |
| 999 | 7,354 | 6,000 | **1,354** |
| 998 | 7,269 | 6,000 | **1,269** |
| 997 | 7,199 | 6,000 | **1,199** |
| 996 | 7,178 | 6,000 | **1,178** |
| 995 | 7,039 | 6,000 | **1,039** |
| 994 | 7,021 | 6,000 | **1,021** |
| 993 | 6,949 | 6,000 | **949** |
| 992 | 6,946 | 6,000 | **946** |
| 991 | 6,908 | 6,000 | **908** |
| : | : | : | **:** |
| 908 | 6,032 | 6,000 | **32** |
| 907 | 6,024 | 6,000 | **24** |
| 906 | 6,022 | 6,000 | **22** |
| 905 | 6,019 | 6,000 | **19** |
| 904 | 6,015 | 6,000 | **15** |
| 903 | 6,012 | 6,000 | **12** |
| 902 | 6,008 | 6,000 | **8** |
| 901 | 6,006 | 6,000 | **6** |
| 900 | 6,006 | 6,000 | **6** |
| 899 | 6,003 | 6,000 | **3** |
| : | : | : | **0** |
| 5 | 3,323 | 6,000 | **0** |
| 4 | 3,261 | 6,000 | **0** |
| 3 | 3,248 | 6,000 | **0** |
| 2 | 3,243 | 6,000 | **0** |
| 1 | 2,735 | 6,000 | **0** |

The EPD in this case is equal to $38.72. It is closely related to the *value* of shortfall protection, though it does not take into consideration discounting for the time value of money or the inclusion of a risk margin.

To use the EPD as the basis for risk capital, a target ratio of the EPD to the expected liabilities, referred to as the EPD Ratio, is assumed. For instance, if 0.5% is used as the EPD ratio target, then the risk capital would be determined such that the EPD is equal to 0.5% of the expected liability amount, or $25. In the case of fixed assets and lognormally distributed liabilities as shown here, Butsic's formulas[7] can be used to derive risk capital equal to $253.86. In a more general case or when using simulation as the basis for the liability values, an iterative process will be needed because the EPD calculation itself depends on the total assets, which equal the policyholder provided funds as well as the risk capital.

### 3.2 Risk Measurement Threshold

For each of the risk measures described above, a critical input is the threshold at which the risk is measured. For instance, in the case of default probability, a specific target probability of default must be selected. In the case of the percentile risk measure or the CTE, a specific percentile must be selected. In the case of the EPD Ratio, a specific target ratio must be used.

There are a variety of methods that could be used:

- Bond Default Probabilities at Selected Credit Rating Level – Practitioners commonly rely on bond default statistics to determine a risk measurement threshold. It is often argued that once the firm's managers decide that they desire a "AA" rating they merely need to select a level of risk capital such that their probability of default is consistent with that of an "AA-rated" bond.

[7] See Butsic 1994.

An obvious weakness of this approach is that it does not address the more fundamental question of which rating to target. For the present discussion, this decision is assumed to be based upon knowledge of the firm's business strategy and target customer base.

The more important issue with this approach that is often overlooked is the need to distinguish between i) a probability of default assuming the firm is immediately (or at the end of some chosen time horizon) placed into run-off and ii) a probability of being downgraded over a specific time horizon. To manage a firm and maximize shareholder value, what should matter most to a firm that targets an "AA-rating" is their ability to **retain** that rating with a high probability. However, commonly used risk capital models do not attempt to measure this probability. Instead, they assume a run-off scenario (either immediately or after a specified time period) and assess whether the current capital base is sufficient to withstand a "tail event".

When this run-off approach is used along with a risk measurement threshold tied to default probabilities, a critical question to address is what bond default probabilities to use. One set of statistics which are often quoted are those that appeared in a paper discussing Bank of America's implementation of RAROC[8]. In that paper, the following bond default data was used:

**Table 4: Estimated Default Probabilities by Rating**

| S&P Rating | Moody's Equivalent | 1-Year Default Probability | Percentile |
|---|---|---|---|
| AAA | Aaa | 0.01% | 99.99% |
| AA | Aa3/A1 | 0.03% | 99.97% |
| A | A2/A3 | 0.11% | 99.89% |
| BBB | Baa2 | 0.30% | 99.70% |
| BB | Ba1/Ba2 | 0.81% | 99.19% |
| B | Ba3/B1 | 2.21% | 97.79% |
| CCC | B2/B3 | 6.00% | 94.00% |
| CC | B3/Caa | 11.68% | 88.32% |
| C | Caa/Ca | 16.29% | 83.71% |

Based on this table, many firms have adopted the 0.03% probability of default and, by extension, the 99.97% threshold as an appropriate percentile on the distribution to measure risk. Aside from the obvious danger of placing too much reliance on risk measurements this far out in the tail, there are several subtleties that should be considered:

- Historical vs. Current Estimates – A choice between historical default rates and current market estimates of default rates must be made. The former will be somewhat more stable, but the latter will more accurately reflect current market conditions.
- Source of Historical Default Statistics – The table above contains average default rates that are not consistent with more recent estimates of long term default rates by rating. For example, the following tables show statistics based on both S&P and Moody's analysis of historical data from roughly equivalent time periods (note that some years are not shown).

---

[8] Source: James, "RAROC Based Capital Budgeting and Performance Evaluation: A Case Study of Bank Capital Allocation", 1996, Wharton Working Paper 96-40. Author cited Bank of America as his source.

**Table 5: Alternative Estimates of Historical Default Rates by Rating**

| | | Default % - Data 1981 through | | | |
|---|---|---|---|---|---|
| | | 1997 | 2000 | 2002 | 2003 |
| S&P | AAA | 0.00 | 0.00 | 0.00 | 0.00 |
| | AA | 0.00 | 0.01 | 0.01 | 0.01 |
| | A | 0.05 | 0.04 | 0.05 | 0.05 |

| | | Default % - Data 1970 through | | | | |
|---|---|---|---|---|---|---|
| | | 1996 | 1999 | 2001 | 2002 | 2003 |
| Moody's | Aaa | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | Aa | 0.03 | 0.03 | 0.02 | 0.02 | 0.02 |
| | A | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 |

The default statistics from the S&P data differ noticeably from the figures quoted in the Bank of America data. The Moody's data also exhibits an unusual relationship between the AA- and A-rated categories and has significantly lower default rates for the A-rated bonds than the S&P data indicates.

- Time Horizon – All of the default rates shown above reflect *annual* default probabilities. In cases where the risk is being measured over a single annual period, these data may be applicable. In many instances though, "default" in risk capital models is often assessed over the lifetime of the liabilities, which have varying time horizons based on the nature of the risk. Some practitioners modify the threshold to account for these varying horizons, arguing that over longer horizons there is a larger probability of a bond defaulting and therefore over longer horizons it is acceptable for the insurer to have a higher probability of default.

- Management's Risk Preferences – Some practitioners argue that the risk measurement threshold that is most relevant is the one that matches the risk preferences of the firm's management. For instance, if the firm's management prefers to limit its probability of default to a particular value, then perhaps that amount should be used to measure the risk?

  Getting the firm's management to articulate and agree upon a particular threshold can be challenging. Attitudes towards risk are often inconsistent and context-specific[9]. In addition, the risk preferences of management, the risk preferences of the board of directors and the risk preferences of the firm's shareholders will often differ, which further complicates this exercise in practice.

  More importantly, effective risk preference statements go beyond articulating a "probability of default". To begin, effective risk preference statements should reflect both the *risk* and the potential *reward* for taking risk. Secondly, shareholder value for an insurer is ultimately driven by events that may cause a ratings downgrade, a weakened financial position or any other event that diminishes the firm's ability to remain a going concern and continue to write profitable insurance business in perpetuity. Risk preferences intended to capture the shareholders' perspective are unlikely to focus on the probability of default.

- Arbitrary Default Probability, Percentile or EPD Ratio – While it may not be scientific, one could choose an arbitrary threshold such that the risk measure can be reliably estimated and reflect the appropriate relative views of "risk". As will be shown in Section 5, in many applications it is the relative measures of risk associated with a firm's different activities that matter the most. In addition, even under the most ideal circumstances it may be very difficult to reliably and accurately measurer *any* loss distribution's 99.97$^{th}$ percentile. This is especially true when dealing with insurance liability risk models for which significant model and parameter uncertainty exists.

---

[9] See Bazerman for a detailed discussion of the many behavioral biases that complicate this process.

For the sake of brevity, these issues will not be fully resolved here. When various applications of risk capital and RAROC measures are used in Section 5, the sensitivity of the results to the choice of risk measurement thresholds will be explored.

## 3.3 Risk Sources

### *3.3.1 Overview*

While practices vary, the conventional approach to measuring a firm's aggregate risk profile segregates the risks into five main categories following the framework adopted by the NAIC and several rating agencies:

- **Market Risk –** This measures the potential loss in value, over the selected risk exposure horizon, that results from the impact that changes in equity indices, interest rates, foreign exchange rates and other similar "market" variables have on the firm's *current* investments in equities, fixed income securities or derivative securities.

  Standard practice is to estimate the distribution of portfolio profits or losses over the selected horizon and use risk measures such as VaR or CTE. A critical issue though is to identify the appropriate time horizon over which to measure the profit/loss distribution and the resulting risk measure. Calculations of VaR for these classes of investments are typically performed over a horizon on the order of 10 or fewer days, which roughly coincides with estimates of the time required to divest risky positions. Calculating the VaR or CTE over longer horizons can be quite challenging, given limitations in historical data used to calibrate the models, the need to account for potential non-stationary models, the need to reflect mean reversion and autocorrelation across periods and the need to account for changes in portfolio composition over longer horizons[10]. For risk-adjusted performance measurement within an insurance company though, the risk exposure horizon for analyzing the insurance liabilities is necessarily much longer because their underwriting and reserve risk exposures generally must be held to maturity. Aggregating market risk with the other risks is therefore inherently problematic due to differences in these time horizons.

  For the moment, and at the risk of confusing matters, this potential disparity in the time horizons will be ignored and a one-year horizon will be selected for measuring market risk. This simplified approach is consistent with current insurance industry practice and allows the discussion to focus on other aspects of this methodology. Discussion of the challenges associated with the time horizon inconsistency will be deferred until Section 6.

  The specific methods used to calculate VaR for various asset classes are covered extensively in various readings on the current CAS Syllabus and will not be discussed in detail here.

- **Credit Risk –** This measures the potential loss in value due to *credit events*, such as counterparty default, changes in counterparty credit rating or changes in credit-rating specific yield spreads[11]. These credit-related risk exposures can impact the firm in a variety of ways, but the three that are the most important include:

  - Marketable Securities, Derivative and Swap Positions – A firm's marketable securities, derivative positions and swap positions may be subject to specific exposure to the various credit events noted above. It is somewhat arbitrary to categorize these exposures within the credit risk category, as opposed to the market risk category, but for practical purposes the methods and models used for the various credit risks are likely to overlap and so it is natural to include these along with the other sources of credit risk.

---

[10] See Rebonato & Pimbley for an insightful discussion of this topic.

[11] Some practitioners classify certain components of credit risk, such as changes in credit spreads unrelated to changes in the underlying counterparty's rating, along with the market risks discussed earlier. Depending on the methods used though, it may be difficult to separate these components cleanly. For presentation purposes, this discussion assumes that all credit-related risks are included as a single risk source.

- Insured's Contingent Premiums and Deductibles – These reflect policyholder obligations in the form of loss-sensitive premium adjustments, deductibles, etc. that, in some instances, cannot be readily offset against claim payments and therefore create a counterparty credit exposure.
- Reinsurance Recoveries – This category represents the most challenging source of credit risk to an insurance company. While the same methods used for the other sources of credit risk are generally applicable here, there are three unique aspects to this category:
  - Definition of Default – For reinsurers, the definition of "default" may need to be adjusted to properly account for the fact that a credit downgrade below the equivalent to an investment-grade rating could, and often does, create a "death spiral" for the firm. Their ability to write future business will be substantially impacted and many existing policyholders will rush to commute or otherwise settle outstanding and potential recoveries. This could create a severe liquidity crisis and result in settlement amounts far less than 100% of potential recoveries for the reinsureds. As a result, a broader definition of default may be necessary.

    In addition, disputes between insurers and their reinsurers are common and often result in settlements of less than 100% of potential recoveries. To the extent that the risk from such disputes can be quantified, they may be treated as the equivalent of a partial default.
  - Substantial Contingent Exposure – Potential exposure to reinsurers' credit risk can far exceed the reinsurance recoverable balances currently on the balance sheet. The balance sheet entries reflect only the receivables relating to paid claims and the expected recoveries against current estimates of *gross* loss reserves. They do not include the potential recoveries from reinsurers in the event of adverse loss development or in the event that losses on new written and earned premiums exceed their expected values. In practice, these contingent exposures need to also be reflected[12].
  - Correlation with Other Insurance Risks – It should be obvious from the previous point that reinsurance credit risk is likely to be highly correlated with the underlying insurance risks. As a result, it is harder to rely on external credit-risk only models for this category of credit risk exposure than it is for investment portfolio or other assets with credit exposure.

The specific methods used to measure credit risk are covered in other readings on the current CAS Syllabus and will not be discussed in detail here.

- **Insurance Underwriting Risk –** This category includes the three primary categories of insurance risk:
  - Loss Reserves on Prior Policy Years – Potential adverse development from existing estimates.
  - Underwriting Risk for Current Period Policy Year – Potential losses (and expenses) in excess of premiums charged for the "current" policy period. In some cases, the definition may include only unearned premiums, but in general it is assumed that one year of new business will be written and so the underwriting risk will also include the potential losses associated with those premiums as well.

[12] Moody's P&C capital model documentation discusses this issue and presents one method for doing this.

- Property Catastrophe Risk – Due to the unique modeling needs of catastrophe risk associated with earthquakes, hurricanes and other weather-related events, these risks are often segregated.

  Each of these three categories of insurance underwriting risk will be described in more detail in the subsequent portions of Section 3.

- **Other Risk Sources –** The above list is far from exhaustive. There are a variety of additional "risks" that could impact a firm, including a wide variety of operational risks associated with the failure of people, systems or processes, as well as a wide variety of strategic risks related to competitors. While these are important risks for a firm to understand, anticipate and manage, they are generally less quantifiable and therefore do not serve a critical role in the current discussion. For convenience, they will be ignored in the discussion that follows.

Given this overview of the typical risk categories used, the rest of this section will explore the insurance risk category in more detail.

### *3.3.2 Loss Reserve Risk*

For most P&C insurers, the magnitude of carried loss and expense reserves, as well as the uncertainty associated with the estimation of these reserves, makes the risk inherent in loss reserves the dominant risk to the firm.

To fully appreciate what is being measured with respect to loss reserve risk, it is useful to make a distinction between three components of the total risk:

- Process Risk – This is the risk that actual results will deviate from their expected value due to the random variation inherent in the underlying claim development process.
- Parameter Risk – This is the risk that the *actual*, but unknown, expected value of the liability deviates from the *estimate* of that expected value due to inaccurate parameter estimates in the models.
- Model Risk – This is the risk that the *actual*, but unknown, expected value of the liability deviates from the *estimate* of that expected value due to the use of the wrong models.

A variety of actuarial methods exist to establish loss reserves. Some of these lend themselves to a statistical analysis of two closely related concepts:

- Reserve Estimation Error – This represents the range of uncertainty associated with a given reserve estimate, rather than the uncertainty with regard to the ultimate "outcome". It is a measure of how statistically reliable a given *estimate* is relative to the true, but currently unknown, value. This uncertainty is usually depicted as a *confidence interval* for a given estimate.
- Reserve Distribution – This represents the full distribution of the unpaid loss amount and is intended to estimate the likelihood that the ultimate outcome deviates from the current estimate. It is a depiction of the full range of possible values for the unpaid loss, along with their associated probabilities, from which the realized value will be drawn. This distribution is often expressed in terms of the *percentile* values (e.g. the $98^{th}$ percentile is the value that is larger than 98% of all other possible values).

The primary goal for this paper is to obtain the full distribution of unpaid losses (perhaps at some particular valuation date) and not merely a confidence interval for the estimate.

### *Alternative Methods for Measuring Loss Reserve Risk*

With the above distinction in mind, some common methods used for determining the loss reserve distribution can be summarized as follows:

- Mack Methods (1993, 1999) – These are analytical methods for estimating the standard error of the reserves based on the traditional chain ladder model for estimating ultimate losses. Their analytical tractability makes them ideal for the current purposes, where frequent stresstesting of assumptions and methods is required, despite some inherent weaknesses of the methods[13].

- Hodes, Feldblum, Blumsohn – This is a simulation method that is also based on the Chain Ladder model for estimating ultimate losses. The approach involves simulating age-to-age loss development factors for each development period, rather than relying on various averages. This approach is intuitively appealing, and quite flexible, though the use of simulation could impact run-time and the reliability of the results.

- Bootstrapping Method – A variety of "bootstrapping" methods exists. One method discussed by England and Verrall uses the distribution of incremental paid or incurred loss amounts to simulate a new, hypothetical loss triangle, from which loss development factors can be derived and new ultimate loss amounts estimated. The result of a large number of similar simulations produces a distribution of ultimate losses and reserves.

- Zehnwirth Methods – While the methods above attempt to adapt existing actuarial methods to produce estimates of the full distribution, Zehnwirth has proposed a different modeling framework that relies on a ground-up probabilistic model of the loss development process. His model works with the (log) incremental paid losses and identifies common trends impacting accident years, calendar years and development periods simultaneously. Using these more elaborate probabilistic models, estimates of the full distribution of ultimate outcomes follow more naturally.

- Panning Econometric Approach – In a recent paper[14], Panning addressed three common weaknesses of some of the previous methods. First, they tend to be derived from chain ladder loss development estimation methods which are *ad hoc* and do not rely on objective criteria for measuring and maximizing the goodness of fit to the observed data. Second, they often rely on cumulative loss data, which introduces serial correlation. And third, they often incorrectly assume constant variance across development periods, even though the development periods should be expected to exhibit heteroskedasticity.

  Panning's method corrects for these three characteristics by relying on linear regression techniques that minimize the squared errors, uses incremental rather than cumulative data and models each development period separately to account for the non-constant variance in the error terms for each development period.

- Collective Risk Model – As will be discussed in the next section with regard to underwriting risk, it is conceptually possible to use claim frequency and severity distribution assumptions, so long as they both represent the distributions of *outstanding* frequency and *outstanding* severity. However, because the severity distributions used at inception for <u>all</u> claims will include the smaller, simpler and more quickly reported and paid claims as well as the larger, more complex and slower reported and paid claims, it is critical to use severity distributions conditional on the age of the outstanding claims. Few entities are likely to have sufficient data to accomplish this parameterization reliably, though research by the Insurance Services Office (ISO) has produced interesting results[15].

- Relationship to Underwriting Risk – In the absence of robust loss reserve data, the coefficient of variation for the ultimate loss distributions could be based on the coefficient of variation for the underwriting risk distributions for similar classes of business. To use this information, the underwriting model parameters would have to be adjusted to reflect the declining coefficient of variation relative to the ultimate losses as a given accident year ages.

---

[13] See Hayne and the CAS Working Party on Quantifying Variability in Reserve Estimates

[14] See Panning, 2005.

[15] See Meyers, Klinker and Lalonde

Given the variety of methods available, this paper will not attempt to address the many technical differences that may result from each of them. Readers interested in a more thorough treatment of these various methods, and in particular their strengths and weaknesses, are encouraged to review the report issued by the CAS Working Party on Quantifying Variability in Reserve Estimates.

On a conceptual level all of these methods attempt to quantify the distribution of outstanding claims as of a given date. In the discussion that follows the Mack Method will be used. This particular method was chosen solely for convenience, though its analytical tractability is particularly appealing.

While the details of the calculations will not be shown here, the following numerical example uses industry data[16] for Commercial Auto Liability and the formulas from Mack's 1993 paper to demonstrate the method.

**Table 6: Sample Paid Loss Data for Mack Method Example**

**Sample Insurer**
**Commercial Auto Liability**

| Accident | Valuation Month | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Year | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 |
| 1994 | 357,848 | 1,124,788 | 1,735,330 | 2,218,270 | 2,745,596 | 3,319,994 | 3,466,336 | 3,606,286 | 3,833,515 | 3,901,463 |
| 1995 | 352,118 | 1,236,139 | 2,170,033 | 3,353,322 | 3,799,067 | 4,120,063 | 4,647,867 | 4,914,039 | 5,339,085 | |
| 1996 | 290,507 | 1,292,306 | 2,218,525 | 3,235,179 | 3,985,995 | 4,132,918 | 4,628,910 | 4,909,315 | | |
| 1997 | 310,608 | 1,418,858 | 2,195,047 | 3,757,447 | 4,029,929 | 4,381,982 | 4,588,268 | | | |
| 1998 | 443,160 | 1,136,350 | 2,128,333 | 2,897,821 | 3,402,672 | 3,873,311 | | | | |
| 1999 | 396,132 | 1,333,217 | 2,180,715 | 2,985,752 | 3,691,712 | | | | | |
| 2000 | 440,832 | 1,288,463 | 2,419,861 | 3,483,130 | | | | | | |
| 2001 | 359,480 | 1,421,128 | 2,864,498 | | | | | | | |
| 2002 | 376,686 | 1,363,294 | | | | | | | | |
| 2003 | 344,014 | | | | | | | | | |

| Accident | Age to Age Factors | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Year | 12:24 | 24:36 | 36:48 | 48:60 | 60:72 | 72:84 | 84:96 | 96:108 | 108:120 | 120:ULT |
| 1994 | 3.143 | 1.543 | 1.278 | 1.238 | 1.209 | 1.044 | 1.040 | 1.063 | 1.018 | |
| 1995 | 3.511 | 1.755 | 1.545 | 1.133 | 1.084 | 1.128 | 1.057 | 1.086 | | |
| 1996 | 4.448 | 1.717 | 1.458 | 1.232 | 1.037 | 1.120 | 1.061 | | | |
| 1997 | 4.568 | 1.547 | 1.712 | 1.073 | 1.087 | 1.047 | | | | |
| 1998 | 2.564 | 1.873 | 1.362 | 1.174 | 1.138 | | | | | |
| 1999 | 3.366 | 1.636 | 1.369 | 1.236 | | | | | | |
| 2000 | 2.923 | 1.878 | 1.439 | | | | | | | |
| 2001 | 3.953 | 2.016 | | | | | | | | |
| 2002 | 3.619 | | | | | | | | | |
| Wtd | 3.491 | 1.747 | 1.457 | 1.174 | 1.104 | 1.086 | 1.054 | 1.077 | 1.018 | |
| Simple | 3.566 | 1.746 | 1.452 | 1.181 | 1.111 | 1.085 | 1.053 | 1.075 | 1.018 | |
| Select | 3.491 | 1.747 | 1.457 | 1.174 | 1.104 | 1.086 | 1.054 | 1.077 | 1.018 | 1.018 |
| To Ultimate | 14.703 | 4.212 | 2.411 | 1.654 | 1.409 | 1.277 | 1.175 | 1.115 | 1.036 | 1.018 |

The following table summarizes the estimated reserve on a nominal basis and the Mack Method standard errors both by accident year and in the aggregate.

[16] Source: AM Best

**Table 7: Mack Method Example**

**Sample Insurer**
**Commercial Auto Liability**

| Accident Year | (1) Paid Loss | (2) LDF | (3) Ultimate Loss | (4) Selected Ultimate | (5) Reserve | (6) Mack Method Std Error |
|---|---|---|---|---|---|---|
| 1994 | 3,901,463 | 1.018 | 3,970,615 | 3,970,615 | 69,152 | 0 |
| 1995 | 5,339,085 | 1.036 | 5,530,030 | 5,530,030 | 190,945 | 76,874 |
| 1996 | 4,909,315 | 1.115 | 5,474,165 | 5,474,165 | 564,850 | 123,856 |
| 1997 | 4,588,268 | 1.175 | 5,391,810 | 5,391,810 | 803,542 | 135,916 |
| 1998 | 3,873,311 | 1.277 | 4,944,310 | 4,944,310 | 1,070,999 | 266,040 |
| 1999 | 3,691,712 | 1.409 | 5,201,766 | 5,201,766 | 1,510,054 | 418,295 |
| 2000 | 3,483,130 | 1.654 | 5,761,106 | 5,761,106 | 2,277,976 | 568,213 |
| 2001 | 2,864,498 | 2.411 | 6,905,058 | 6,905,058 | 4,040,560 | 890,842 |
| 2002 | 1,363,294 | 4.212 | 5,742,274 | 5,742,274 | 4,378,980 | 988,473 |
| 2003 | 344,014 | 14.703 | 5,057,913 | 5,057,913 | 4,713,899 | 1,387,316 |
| | 34,358,090 | | 53,979,046 | 53,979,046 | 19,620,956 | 2,490,469 |
| | | | | | Coefficient of Variation | 0.127 |

The key result for the present purposes is the estimated coefficient of variation for the aggregate unpaid liabilities. From this, and the mean of the reserve risk distribution, a lognormal distribution is assumed for the outstanding losses and the parameters estimated. The lognormal assumption was chosen arbitrarily; in practice it may be important to confirm whether this is a reasonable assumption and to consider other distributions as well.

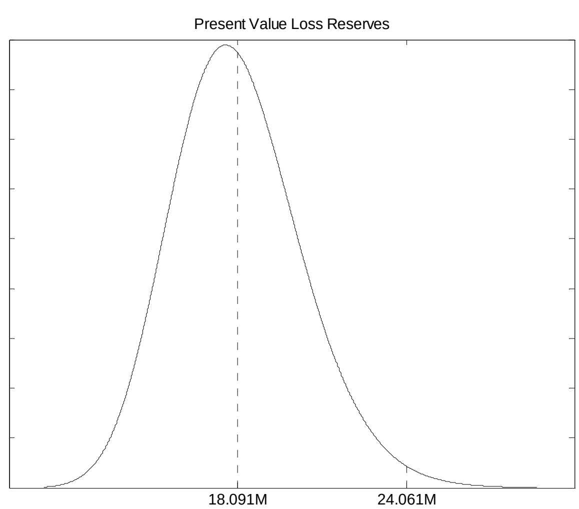
Note also that the loss reserve distribution parameters should be adjusted to reflect their *discounted* values, where the discount rate is based on a risk-free rate (4.0% in this case) and the discounting is done to the *end* of the one-year period consistent with an assumption that all payments are made at the end of the year. This is one approach to normalizing the models to account for the different time horizons over which the claim payments will be made.

The parameters of the lognormal distribution are calculated as follows using the method of moments:

**Table 8: Reserve Risk – Lognormal Parameters**

| | Undiscounted | Discounted |
|---|---|---|
| Mu | 16.784 | 16.703 |
| Sigma | 0.126 | 0.126 |

The resulting distribution of outstanding losses, on a present value basis, can be shown graphically as follows with the mean ($18.091M) and the 99$^{th}$ percentile ($24.061M) values highlighted:



***Ultimate Liability vs. Loss Development During Horizon***

Some practitioners advocate measuring the reserve risk over a finite horizon, such as one year, and reflecting only the degree to which the ultimate liability may need to be restated as of the end of this horizon. This is in contrast to the measure described above, which reflects the uncertainty in the loss reserves that comes from, for instance, unknown rates of loss severity trend over the lifetime of the liability. The one-year measurement reflects only the degree to which the *best estimate* could change over this time horizon, making it more compatible conceptually with the market VaR and credit VaR calculations discussed previously and more consistent with calendar year measures of income that are often used.

For many lines of insurance business, the differences between a lifetime of liability horizon and a one-year horizon is likely to be small and perhaps insignificant. For lines such as high-layer, excess of loss general liability where there is little new information that emerges over a short horizon, the differences can be significant[17].

For the present purposes, these potential differences will be ignored and lifetime of liability insurance risk distributions will be used. However, the distributions will be adjusted to reflect present value loss amounts as of the end of the one-year horizon, as if all claims are known and paid at the end of this period. Appendix A of this paper discusses the issues associated with this choice of *risk exposure horizon* in more detail, including the potential inconsistency with the market and credit risk measures.

***Ceded Reinsurance Recoveries***

The analysis of ceded reinsurance can be modeled directly using similar models as is done for the gross loss reserves, adjusted of course to take into consideration the specific nature of the reinsurance agreements, or indirectly by modeling the *net* reserve risk. Typically, modeling the net reserves will be the easiest approach. However, the need to take into consideration the credit risk on reinsurance recoveries may favor a more direct estimation of ceded reserves.

***3.3.3 Underwriting Risk***

The term *underwriting risk* is used to reflect the risk that total claim and expense costs on new business written and/or earned during a specified *risk assumption horizon*[18] exceed the premiums collected during the same period. This new business written reflects both renewals as well as policies for "new" customers – it is all business written during the risk assumption horizon but that is not currently reflected on the firm's balance sheet. In some rating agency and regulatory capital models, this is often referred to as *premium* risk or *new business* risk.

A variety of risk factors affect the distribution of potential claims and expenses on new business. To simplify the discussion of the quantification of this risk the following methods will be discussed:

- Loss Ratio Distribution Models
- Frequency & Severity Models
- Inference from Reserve Risk Models

***Loss Ratio Distribution Models***

One easy approach to implement relies on an assumed distribution of loss ratios. Combined with an estimate of written premium during the risk assumption horizon, either deterministically or

---

[17] Notice that by ignoring the risk of subsequent deviations beyond the selected horizon there is an implicit assumption that the liabilities could, if necessary, be transferred to a third party subsequent to the restatement, since there will still be risk of further adverse deviations but the firm will have no capital to support this risk. For this assumption to be reasonable, the firm will *also* need to have sufficient additional resources to pay a fair market risk margin to the assuming party.

[18] The term *risk assumption horizon* is being used here to refer to the period over which <u>new</u> exposures to risk are being added to the firm. This is in contrast to the *risk exposure horizon*, discussed in Appendix A, which refers to the period over which the risks are assumed to affect the firm. In an insurance context, the risk assumption horizon reflects how much new business is assumed to be written.

stochastically generated, the full distribution of potential liabilities and expenses net of premium can be determined.

Among the most important considerations in applying this approach are the following:

- Source of Model Parameters – Loss ratio distribution parameters may be based on either historical loss ratio experience for the company or on industry data if the company's own data lacks sufficient credibility.

  When using the company's historical loss ratio data, it is important to adequately reflect changes in claim cost trends, premium adequacy and the relative volume over the data analysis period.

  Industry loss ratio data from external sources (e.g. ISO or NCCI in the U.S.) can either supplement or replace company data in certain circumstances.

- Choice of Distribution Models – The foundation of the risk capital framework discussed here is the explicit recognition of uncertainty. This makes the choice of distribution models, their applicability to the particular risk and their fit with the historical data critically important.

  While common models for loss ratios will often be limited to Normal, Lognormal and Gamma distributions, others are certainly feasible. Special attention however should be paid to the model fit and often-encountered inconsistencies should be avoided.

  For instance, lognormal distributions are commonly used to model loss ratios due to their desirable quality of a heavy right tail that seems to reflect reality. However, when applied to loss ratios, this model tends to exhibit a left tail that is too heavy and a right tail that is too light. In other words, a lognormal model could result in a high probability of the loss ratios being well below the mean and too little probability of loss ratios well above the mean.

  One way to correct for this is to use a mixture of two lognormal models, one with a very small coefficient of variation and one with a very large coefficient of variation. Using "small" losses to calibrate the first model and "large" losses to calibrate the second, the two models can be combined to produce a more reasonable overall loss ratio distribution[19].

As an example of this approach, consider the following hypothetical data for a commercial auto insurer. The estimated loss ratios for the past ten accident years are assumed to be representative of the prospective years' results.

[19] See Mildenhall, 1997

**Table 9: Hypothetical Historical Loss Ratios**

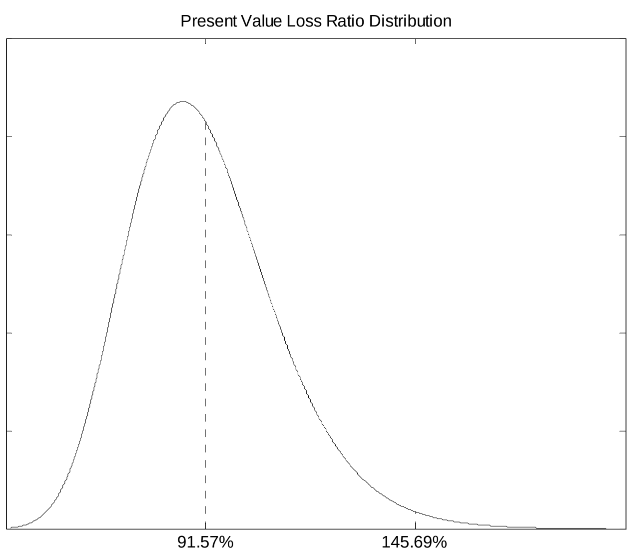
| Year | Earned Premium | Ultimate Loss | Loss Ratio |
|---|---|---|---|
| 1994 | 5,272,000 | 3,970,615 | 75.3% |
| 1995 | 5,188,000 | 5,530,030 | 106.6% |
| 1996 | 4,212,000 | 5,474,165 | 130.0% |
| 1997 | 3,656,000 | 5,391,810 | 147.5% |
| 1998 | 4,528,000 | 4,944,310 | 109.2% |
| 1999 | 5,012,000 | 5,201,766 | 103.8% |
| 2000 | 6,174,000 | 5,761,106 | 93.3% |
| 2001 | 6,202,000 | 6,905,058 | 111.3% |
| 2002 | 6,528,000 | 5,742,274 | 88.0% |
| 2003 | 6,276,000 | 5,057,913 | 80.6% |
| | | Mean | 104.6% |
| | | Std Deviation | 22.1% |
| | | CV | 0.2113 |

Assuming a standard lognormal distribution but adjusting the parameters to reflect the discounted loss ratio as of the end of the year (i.e. assuming all claims are paid at the end of the year), the following parameters can be estimated using the method of moments:

**Table 10: Discounted Distribution Parameters**

| | |
|---|---|
| Discounted Mean | 91.6% |
| CV | 0.2113 |
| Mu | -0.1099 |
| Sigma | 0.2090 |

The resulting lognormal distribution is as shown in the following graph:




### *Frequency & Severity Models*

While the loss ratio distribution model can be based on rather limited historical data, more robust models can be developed which rely on separate frequency and severity models to determine the aggregate loss distribution. When sufficiently detailed and relevant claim data is available, this approach can have a number of key advantages. Klugman, Panjer and Wilmot provide the following list of advantages that this approach has over the previous method[20]:

1. Growth in volume of business can be more easily accounted for;
2. Inflation can be more accurately reflected, particularly when there are deductibles and policy limits;
3. Changes in limit and deductible profiles can be directly reflected;
4. Impact of deductibles on claim frequency can be reflected;
5. Estimates of the split of losses between insured, insurer and reinsurer can be mutually consistent.

Using these models, separate probability distributions for claim frequency can be developed based upon Poisson, Negative Binomial or Normal distributions and separate claim severity models can be developed using any number of distributions such as the Lognormal, Gamma, Exponential or Beta distributions. The aggregate loss distribution can then be determined via a variety of methods:

- Analytical Solution – For certain choices of frequency and severity models, it may be possible to determine a closed form solution for the aggregate loss distribution (sometimes referred to as the *collective risk model*) based on the frequency and severity parameters.
- Numerical Methods – Numerical approximations based upon the Fast Fourier Transform can be used to determine the aggregate loss distribution based on the frequency and severity parameters.
- Approximations – Using the mean, variance and possibly higher moments of the collective risk model, an aggregate loss distribution can be approximated with parameters that provide a best fit to these moments.

  If $N$ is random number of claims and $S_i$ is the random severity for each claim, the collective risk model (without parameter uncertainty) suggests the following mean and variance of the aggregate distribution:

  Aggregate Loss Mean = E(N)E(S)

  Aggregate Loss Variance = $E(N)Var(S) + Var(N)E(S)^2$

  The previous formulas can be trivially adjusted to reflect parameter uncertainty by introducing a "shock" or random deviation to both of the claim counts, $N$, and the severity $S_i$. Heckman and Meyers[21] introduce *contagion* parameters $c$ for the frequency shock and $b$ for the severity shock and derive the following modified formulas for the mean and variance of the aggregate distribution:

  Aggregate Loss Mean = E(N)E(S)

  Aggregate Loss Variance = $E(N)E(S^2)(1 + b) + Var(N)E(S)^2(b + c + bc)$

  To demonstrate the flexibility of this method, consider the following example taken from the IAA Report on Insurer Solvency. Assume that all (ground-up, unlimited) commercial auto liability claims follow a lognormal distribution with a mean value of $6,000 and a coefficient of variation of 7.0. The claim frequency is assumed to be Poisson. Further assume that the contagion parameters are $c = 0.02$ and $b = 0.003$. If an insurer has $600 million of written premium and an expected loss ratio of 105% on an undiscounted basis, this implies the following expected claim counts and aggregate claim costs:

[20] See Klugman, Panjer and Wilmot, Page 292.

[21] See Heckman and Meyers

**Table 11: Aggregate Loss Distribution**

| | |
|---|---|
| Expected Claim Count | 105,000 |
| Expected Aggregate Loss | 630,000,000 |
| Std Dev of Aggregate Loss | 13,771,498 |

These parameters could then be used to fit an appropriate distribution for the aggregate claim costs for the line of business. The model can also be readily adjusted to reflect different premium volumes, different expected loss ratios or different attachment point and limit profiles[22]. This flexibility is particularly appealing because it can be achieved in a consistent fashion across different lines of business.

In addition, aggregating the moments (mean and standard deviation) across different lines of business is also convenient using this model. If the frequency and severity distributions across different lines of business are assumed to be impacted by *common* shocks, though with different sensitivity to these common shocks, this will induce *dependency* across different lines of business.

These two features, consistency across lines of business and the ability to infer dependency across lines of business, are particularly beneficial in the application discussed here.

- Simulation – Aggregate losses can also be estimated via simulation and the simulated results can either be used directly (via the empirical distribution) or fit to a particular distribution model. While this approach has the advantage of allowing for complex policy structures to be modeled with minimal mathematical complexity, the results can be unstable without a large number of iterations. This, as well as the processing time required to run a large number of iterations, can make it difficult to test the sensitivity to the assumptions.

***Inference from Reserve Risk Models***

An alternative to the direct analysis of the insurance pricing risk for the new business is to infer the magnitude of the risk on new business from the reserve risk models described earlier. The reserve risk models estimate the potential variability in unpaid losses as of some date after the policies are written and certain previously unknown parameters have been determined, such as the premium volume, the number of catastrophe events, some portion of the total claim counts, etc.

Recognizing that the reserve risk models reflect the risk *conditional* on this particular information, *unconditional* models can be inferred from these models and applied to the current risk assumption horizon at inception. Alternatively, a rough approximation that assumes the coefficient of variation for the most recent accident year can be used for the prospective accident year can be used.

### *3.3.4 Property-Catastrophe Risk*

Ever since the large insured hurricane and earthquake losses of the early 1990s, natural catastrophe risk modeling has become substantially more sophisticated.

Prior to the development of these models, insurers often relied upon historical loss experience to assess their potential losses from these natural catastrophes. But historical catastrophe loss experience can be a misleading indicator of potential losses for a variety of reasons, including the fact that the events are rare, the exposure changes over time, severities change over time based on building materials and designs, repair and contents replacement costs are poor indicators of current costs, etc.

In contrast, the leading catastrophe risk models rely on meteorological, seismological and engineering data to produce a probability distribution of potential catastrophe losses. Through simulation of various events, an assessment of damage to affected property is determined together with an assessment of the impact of specific insurance and reinsurance coverages. From this, probability

[22] To adjust the results to reflect different attachment points and limits, the mean and the standard deviation of the ground up and unlimited claim severity distribution would be adjusted to reflect the mean and the standard deviation of the appropriate layer.

distributions of the insurers' potential gross and net losses from earthquakes, hurricanes, severe storms and related events can be derived.

These models typically have various modules, such as:

- Stochastic Module/Hazard Module – These modules jointly determine the possible events such as earthquakes and hurricanes that can occur, as well as their location, intensity, etc.
- Damage Module (Vulnerability Module) – This uses the exposure information for the insurer to determine the damage that would occur for any given event.
- Financial Analysis Module – This applies the insurance and reinsurance policy terms to the losses to determine the financial impact to the insurer.

The specific capabilities and model features vary from vendor to vendor and will not be addressed in this paper.

## 3.4 Risk Aggregation

Given the risk distributions for market, credit, loss reserves, underwriting and property-catastrophe risk, the next step is to determine an aggregate risk distribution for all risk sources combined[23]. As noted earlier, in instances where the risk distributions for each category were not derived using comparable risk exposure horizons, this may be far more challenging than it appears.

Ignoring that complexity for the moment, the critical issue to address is the degree of correlation or dependency across the various risk categories.

### *3.4.1 Correlation vs. Dependency*

These two terms are often used interchangeably, though technically there is an important difference between correlation and dependency. Mathematically, the term *correlation* refers to a specific measure of <u>linear</u> dependency, while *dependency* reflects a more general measure of the degree to which different random variables depend on each other.

The best way to see the distinction is through a very simple example. Assume that $X$ represents a Standard Normal random variable and that $Y = X^3$. For any given values of $X$ and $Y$, it is clear that $Y_i$ is entirely dependent upon the value of $X_i$. However, if the values were simulated and the correlation were measured, the correlation would be estimated to be only approximately 0.78. In this case, perfect dependency does not imply perfect correlation.

### *3.4.2 Measures of Dependency*

Recognizing this distinction between correlation and dependency, how would one derive measures of dependency across risk categories, or within risk categories and across different asset classes or lines of business?

Despite the importance of this question, there are currently no entirely satisfying answers. Instead, there are three common methods used, each of which suffer from various practical limitations.

1. Empirical Analysis of Historical Data – Despite the obvious appeal of this approach, the reality is that often the data required for this analysis does not exist. Worse, even when data does exist the measurement errors often produce estimates of correlations that are unreliable, inconsistent and counter-intuitive. Finally, by definition historical data contains very little insight into how much correlation or dependency exists when "tail events" occur. Seemingly independent events under normal conditions may turn out to be more highly dependent under extreme conditions. For example, when Russia defaulted on its debt in the Summer of 1998, a flight to quality and demand for liquidity caused simultaneous disruptions in a variety of sectors and led to the collapse of LTCM, a multi-billion dollar hedge fund.

---

[23] Some practitioners do not derive an aggregate risk distribution and instead use the stand-alone risk distributions to derive risk measures for each risk and then simply aggregate these risk measures into an overall aggregate risk measure. This simplified approach is discussed below in Section 3.4.3.

2. Subjective Estimates – Subjective estimates can be made that reflect dependency during tail events and that reflect the user's intuition, so at times these are preferred. However, this approach suffers from the fact that as the number of unique risk categories or lines of business increases, the number of pairwise correlation/dependency assumptions that must be made grows exponentially.

   Aside from the obvious burden this places on the user, the task of ensuring internal consistency becomes onerous. For instance, with three lines of business, subjective estimates of the correlation between Line A and Line B and subjective estimates of the correlation between Line A and Line C necessarily restrict the range of internally consistent subjective estimates of the correlation between Line B and Line C[24].

   It may be possible to enforce a bit more structure on this process by adopting certain explicit conventions with regard to what can or cannot cause correlation, which will help to avoid these potential inconsistencies. The results will still be subjective, but may be slightly less onerous to produce.

3. Explicit Factor Models – In many advanced applications of Value at Risk for asset portfolios, explicit factor models are used to link the variability of different assets or asset classes to common *factors*. The consequence of this is that correlations across assets or asset classes can be derived based on each asset's respective sensitivity to these common factors.

   An insightful application of this approach was alluded to earlier in the discussion of the collective risk model with common "shocks", which is described in great detail by Meyers, Klinker and Lalonde. Given the assumption of common shocks to the frequency and severity parameters, correlations across lines of business can be derived based upon the contagion parameters.

   This particular approach is intended to reflect the dependency across lines of business and across the reserve and underwriting risk categories. Separate assumptions would generally have to be made to reflect dependency across other risk categories (e.g. market, credit and property catastrophe).

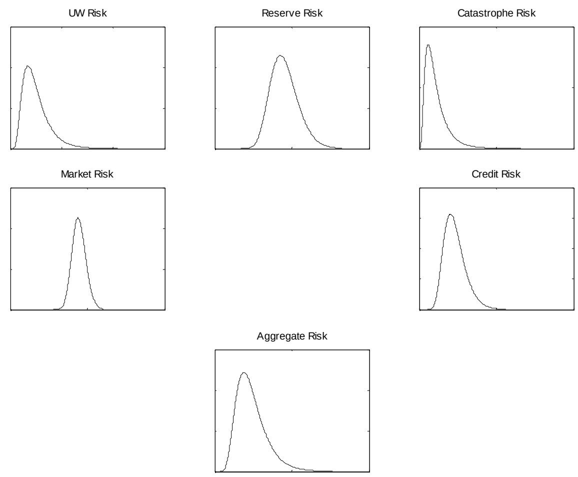
#### *3.4.3 Aggregate Risk Distribution*

Given the selected correlation or dependency measures, the next step is to create an aggregate risk distribution using each of the component risk distributions.

Ignoring, for now, the potential inconsistency of the measurement basis for the different risk types, the goal is to combine the market, credit, loss reserve, underwriting and property catastrophe distributions into a single aggregate distribution. The following diagram depicts each of the stand-alone risk distributions and the resulting aggregate risk distribution that can be obtained using any of the methods described in this section.

[24] The technical requirement is that the correlation matrix must be positive, semi-definite. See Rebonato, 1999, for a discussion of a technique that can be used to "tweak" an invalid correlation matrix so that this requirement is met.

**Table 12: Aggregate Risk Distribution**




To derive this aggregate distribution, a variety of mathematical techniques can be used:

- Closed Form Solutions – In highly stylized and simplified cases, it may be possible to derive closed form, analytical formulas for the aggregate risk distribution. However, in practice the wide variety of stand-alone risk distributions may make this impractical.
- Approximation Methods – Several approximation methods are available to overcome the complexity of deriving analytical formulas, including assuming that all risk distributions are Normal or Lognormal and deriving the model parameters from either the actual moments of the respective distribution or specific percentiles of the actual distributions.
- Simulation Methods – Simulation methods are often favored by practitioners because of their intuitive interpretation and the "brute force" way in which the results can be derived. However, run-time and stability concerns can make these approaches impractical and hamper the users' ability to thoroughly test the implications of the model and assumptions.

  When simulation is used, it is important to reflect the dependency across the simulated variables and the uniqueness of the stand-alone, marginal distributions. Two general approaches are common:

  1. Iman-Conover Method – This method, which is used in the commercial software package @Risk, uses a *rank correlation* measure that effectively simulates each random variable separately based on its marginal distribution and then "reshuffles" the stand-alone results in such a way as to preserve the specific rank correlations[25].
  2. Copula Method – Copula methods are similar conceptually to the Iman-Conover method and attempt to ensure, for instance, that when two variables are highly and positively dependent upon each other, a value for one variable in the far right tail of its distribution will *generally* result in a value for the second variable that is also in the far right tail of its distribution. Alternatively, a highly negative dependency will result in the second variable *generally* being in the far left tail of its distribution.

     To achieve the goal stated above, one could simulate correlated/dependent percentiles, which will reflect values between 0 and 1 and then use these correlated/dependent percentiles to obtain values from the respective distributions. Copulas represent multivariate distributions with values that range from 0 to 1 and therefore they can naturally be used to represent these dependent percentiles.

[25] See Mildenhall's Chapter in the CAS Working Party on Correlation Report for a thorough discussion of this method.

Different copulas will result in different degrees of dependency, particularly in the tail.

A *normal copula* uses a multivariate standard normal distribution to generate correlated standard normal values and then maps these into their corresponding percentiles by inverting the standard normal distribution function. The resulting percentiles will themselves be dependent, in the sense that when the correlations are assumed to be high, the values of the various simulated percentiles will be generally similar.

However, normal copulas tend to induce rather low degrees of dependency in the tails of the distributions. Therefore, other copulas with more tail dependency are often used. For instance, if correlated values from a *Student's t* distribution are used in place of the multivariate normal distribution, more tail dependency results, depending on the number of degrees of freedom assumed for the *t* distributions.

Other much more complex copulas can also be used, which are conceptually the same but significantly more difficult to use.

### *3.4.4 Alternative Approach – Aggregate Risk Measures*

The description in the last section assumed that the intent is to model the aggregate risk distribution, from which an aggregate risk measure can be calculated. It is quite common for practitioners to avoid this step of first deriving the aggregate distribution and instead aggregate the stand-alone risk measures directly using a simple formula.

This approach was adopted by the NAIC for use in their RBC formula and was referred to as the *square root rule*[26]. Under the square root rule,

$$C = \sqrt{\sum C_i^{\,2} + \sum_i \sum_{j \neq i} \rho_{ij} C_i C_j}$$

where $C$ represents the risk measure for the aggregate risk distribution, $C_i$ represents the risk measure for risk source $i$ and $\rho$ is the assumed correlation between the risk sources.

This approximation is exact when the risk measure is proportional to the standard deviation (as it is for the relative VaR risk measure) and all of the risk distributions are normal. This is because with normally distributed risks, the aggregate risk distribution's standard deviation is calculated using the same "square root rule". The proportionality constant can therefore be factored out and the resulting aggregate risk measure is simply the proportionality constant times the aggregate standard deviation.

In symbols, if $\alpha$ is the proportionality constant, then $C_i = \alpha\sigma_i$ for each risk type and for the aggregate risk. In this case, the square root rule can be shown to be exact:

$$\begin{aligned} C &= \sqrt{\sum C_i^{\,2} + \sum_i \sum_{j \neq i} \rho_{ij} C_i C_j} \\ &= \sqrt{\sum (\alpha\sigma_i)^2 + \sum_i \sum_{j \neq i} \rho_{ij} (\alpha\sigma_i)(\alpha\sigma_j)} \\ &= \alpha\sqrt{\sum (\sigma_i)^2 + \sum_i \sum_{j \neq i} \rho_{ij} (\sigma_i)(\sigma_j)} \\ &= \alpha\sigma \end{aligned}$$

This method of aggregating risk measures has become widely used, despite the fact that it represents a crude approximation in more general cases when the distributions are not normal and the risk measures are not a constant multiple of the stand-alone standard deviations[27].

---

[26] See Butsic, 1993.

[27] In his 1993 paper for the NAIC RBC Task Force, Butsic argued that when using the EPD Ratio as the standard to determine stand-alone and aggregate capital requirements, this square root rule leads to a conservative estimate of the required capital.

## *4. Aggregate Risk Capital and Allocation to Business Units*

In this section, the calculation of a firm's aggregate risk capital and the allocation of this to various business units or risk sources will be demonstrated using a highly simplified numerical example. To ensure that the focus is properly placed on the methods and their implications, the numerical examples will intentionally ignore some risk types that would have to be reflected in a more realistic application.

### 4.1 Assumptions

A hypothetical insurance company, Sample Insurance Company (SIC), will be used for the numerical example based on the following key assumptions:

- Invested Assets – SIC is assumed to have $19.6 million in invested assets, equal to the undiscounted value of their existing loss reserves. New premiums collected, net of operating expenses, will be invested in identical assets. The rate of return (change in value) of the invested asset portfolio is assumed to be normally distributed and uncorrelated with all other risk categories. The expected return is 5% per annum with a standard deviation of 3.75% per annum.
- Loss Reserves – SIC is assumed to have $19.6 million in *undiscounted* loss reserves associated with the premiums written and earned in the past. For convenience, the reserve risk distribution parameters will be assumed to be identical to those used in the loss reserve risk example shown in Section 3. Recalling the details from that section, the reserve risk distribution has the following lognormal parameters: $\mu = 16.703$, $\sigma = 0.126$.
- Written Premium – Over the prospective year, SIC will be assumed to write a total of $6.4 million of new premium in each of two lines of business – Line A and Line B. The premium will be assumed to be written on the first day of the year and, after paying up-front expenses, invested entirely in marketable securities identical to those in which the existing assets are invested. The premium is assumed to be fully earned during the year and the loss payment patterns for both lines of business will be assumed to be the same.
    - Line A – This line is expected to have a *present value*[28] loss ratio of 91.6%, with discounting to the end of the first year, and lognormal distribution parameters as shown in the underwriting risk example in Section 3 ($\mu = -0.1099$, $\sigma = 0.2090$). These losses will be assumed to have a correlation coefficient of 0.50 with the reserve risk and a correlation coefficient of 0.25 with the underwriting results for Line B (described below).
    - Line B – This line is also expected to have a present value loss ratio of 91.6%, with discounting to the end of the first year, and a coefficient of variation that is 50% higher than the example shown in Section 3 ($\mu = -0.1359$, $\sigma = 0.3094$). The higher coefficient of variation is assumed so that differences in risk between the lines of business can be emphasized in the examples that follow. The losses for this line are assumed to have a correlation coefficient of 0.25 with the reserve risk and a correlation coefficient of 0.25 with the Line A underwriting risk.
- Expense Ratio – Expenses are assumed to equal 5.0% of the written and earned premium, and paid entirely at the beginning of the year.

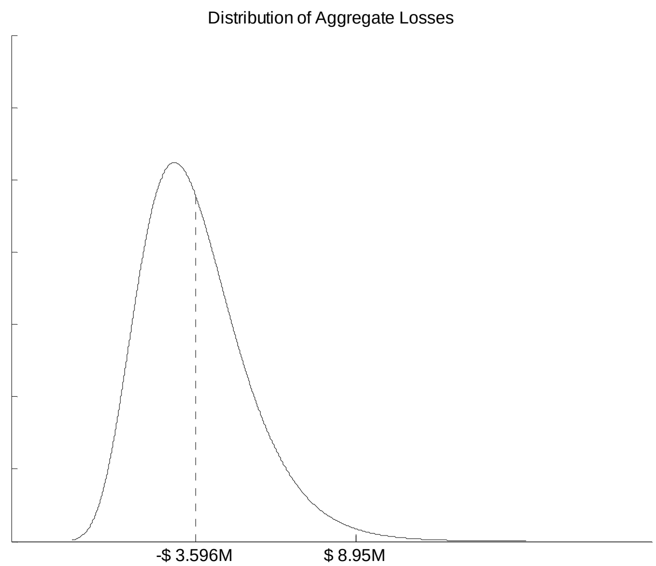
---

[28] The loss payments are discounted to the *end* of the first year, to mirror a simplifying assumption that all outstanding losses are paid, on a discounted basis, at the end of the year. This assumption is used to simplify the aggregation of the insurance risks with the market risk and to define "loss" consistently throughout the model.

- Key Simplifications – For convenience, the example ignores the following risk sources that would normally be included:
  - Credit Risk
  - Property-Catastrophe Risk
  - Operational and Strategic Risks

### 4.2 Aggregate Risk and Risk Capital

Given the assumptions described above, the aggregate risk distribution for SIC can be readily obtained by simulating from each of the stand-alone risk distributions and using a normal copula to account for the desired correlation/dependency among the risk types. For presentation purposes, a lognormal distribution was fit to the empirical aggregate distribution[29], using the method of moments. The resulting distribution is shown below:




For the purposes of this distribution, the amounts shown represent the potential "loss", again with profits depicted as negative amounts and losses as positive amounts. Because both assets and liabilities are included in this calculation, the aggregate loss includes the losses (or profits) in the investment portfolio and the insurance claim costs and expenses in excess of the premiums.

Recall that the company was assumed to have $19.6 million of invested assets initially, collected $12.8 million of premium for the two lines of business and paid 5% of that premium in expenses. These amounts are available to pay claims but are assumed to be contributed by the policyholders, to distinguish them from the *risk capital* that must be contributed by shareholders[30].

---

[29] Because of the desire to assume a lognormal distribution, which cannot take on negative values, the distribution was shifted by the amount of the minimum loss (maximum profit) in the simulation to determine the parameters and then shifted back for the purposes of labeling the graph shown.

[30] In an actual application the company is likely to have committed capital in excess of the capital funded by policyholders, though for the purposes of the calculations that follow it is easiest to ignore the existence of this committed capital. The goal of these calculations is to assess the amount of risk capital needed, so including some of what will account for this risk capital only confuses matters. Whether the assets supporting the reserves are determined on a discounted or undiscounted basis will, of course, impact the dollar amount of risk capital determined and may need to be taken into account for different uses of the results.

From this aggregate risk distribution, the 99th percentile risk measure is \$8.95 million. If this amount of risk capital were contributed to the firm the probability of having insufficient assets to pay all of the claims fully would be 1%[31].

Because of the use of the percentile risk measure and the connection this has to the amount of capital needed to reduce the risk of default to the stated probability, the \$8.95 million figure can be interpreted as an amount of risk capital. It is then natural to allocate this risk capital to the various risks or business units that create the need for this risk capital. As stated earlier, it may also be appropriate to simply allocate the firm's actual capital, a point that will be discussed in Section 6.

### 4.3 Allocation of Risk Capital

In this section, several capital allocation approaches will be discussed and demonstrated using the SIC numerical example described above. In each case, the \$8.95 million of risk capital will be allocated to some or all of the major risk sources. The methods discussed include the following:

- Proportional Allocation Based on a Risk Measure – This method simply calculates stand-alone risk measures for each risk source (market risk, reserve risk, Line A underwriting risk, Line B underwriting risk) and then allocates the total risk capital in proportion to the separate risk measures.
- Incremental Allocation – This method determines the impact that each risk source has on the aggregate risk measure and allocates the total risk capital in proportion to these incremental amounts.
- Marginal Allocation (Myers-Read Method) – This method determines the impact of a small change in the risk exposure for each risk source (e.g. amount of assets, amount of reserves, premium volume) and allocates the total risk capital in proportion to these marginal amounts. One particular method that will be demonstrated is the Myers-Read method.
- Co-Measures Approach (Kreps, Ruhm-Mango) – This method determines the contribution each risk source has to the aggregate risk measure. The method that was independently developed by Kreps and by Ruhm and Mango will be demonstrated.

***Proportional Allocation Based on a Risk Measure***

Using any selected risk measure, such as a percentile risk measure (VaR) or the CTE, each unit's proportional risk measure to the sum of all the risk measures is applied to the total capital. For example, if the stand-alone 99th percentile risk measure, which will be referred to here as the 99% VaR, is used for each risk source the following allocation percentages would be calculated:

**Table 13: Capital Allocation – Proportional to 99% VaR**

| | 99.00% VaR | % of Total | Allocated Capital |
|---|---|---|---|
| Market Risk | 1,183,461 | 8% | 742,665 |
| Reserve Risk | 4,440,453 | 31% | 2,786,545 |
| Line A UW Risk | 3,243,793 | 23% | 2,035,598 |
| Line B UW Risk | 5,394,016 | 38% | 3,384,941 |
| Total | 14,261,723 | | 8,949,750 |

Because the 99% VaR risk measure was used to determine the aggregate capital, it seems reasonable to use the same risk measure to perform the allocation. However, some practitioners choose to use a

[31] Note that this is not entirely accurate due to the fact that the risk in the marketable securities has only been measured over a 1 year horizon, whereas the claims will be paid over a longer horizon. The current simplified model assumes that the present value of the outstanding claims will all be paid at the end of the year, or equivalently that after the end of the year the invested assets will no longer generate risk. More importantly, the model assumes a run-off scenario after one year of premiums are written, which is rarely a realistic assumption. As a result, the term *default probability* should be very carefully interpreted.

different risk measure as the basis for allocating risk than is used to measure the aggregate risk capital.

For instance, if the 99.97% VaR is used to allocate risk capital but the same total amount of risk capital from the previous example is allocated, the following would be obtained:

**Table 14: Capital Allocation – Proportional to 99.97% VaR**

| | 99.97% VaR | % of Total | Allocated Capital |
|---|---|---|---|
| Market Risk | 2,500,702 | 10% | 851,813 |
| Reserve Risk | 8,035,878 | 31% | 2,737,259 |
| Line A UW Risk | 5,666,239 | 22% | 1,930,089 |
| Line B UW Risk | 10,071,313 | 38% | 3,430,588 |
| Total | 26,274,131 | | 8,949,750 |

Similarly, if the 99% CTE is used as the risk measure[32], the following would be obtained:

**Table 15: Capital Allocation – Proportional to 99% CTE**

| | 99.00% CTE | % of Total | Allocated Capital |
|---|---|---|---|
| Market Risk | 1,593,170 | 9% | 799,365 |
| Reserve Risk | 5,441,265 | 31% | 2,730,126 |
| Line A UW Risk | 3,922,399 | 22% | 1,968,043 |
| Line B UW Risk | 6,880,426 | 39% | 3,452,217 |
| Total | 17,837,260 | | 8,949,750 |

Notice that in all three cases here, the allocations are quite similar. In other applications, particularly those that include highly skewed risks such as property-catastrophe risk, this will not always be the case. In addition, as will be discussed in Section 6, there are many instances where it may be appropriate to use risk measures that are not "tail" based. In these instances, the differences that result could be more significant.

For example, the following two tables show the allocations that would result using the 80% VaR and the 80% CTE risk measures:

**Table 16: Capital Allocation – Proportional to 80% VaR**

| | 80.00% VaR | % of Total | Allocated Capital |
|---|---|---|---|
| Market Risk | -586,016 | -35% | -3,132,546 |
| Reserve Risk | 335,121 | 20% | 1,791,389 |
| Line A UW Risk | 756,744 | 45% | 4,045,176 |
| Line B UW Risk | 1,168,409 | 70% | 6,245,731 |
| Total | 1,674,258 | | 8,949,750 |

**Table 17: Capital Allocation – Proportional to 80% CTE**

| | 80.00% CTE | % of Total | Allocated Capital |
|---|---|---|---|
| Market Risk | 80,957 | 1% | 117,902 |
| Reserve Risk | 1,809,817 | 29% | 2,635,718 |
| Line A UW Risk | 1,622,380 | 26% | 2,362,745 |
| Line B UW Risk | 2,632,196 | 43% | 3,833,385 |
| Total | 6,145,350 | | 8,949,750 |

---

32 Given the simplicity of the model, the CTE could be calculated analytically. In this case, the CTE has been calculated using the simulated results instead. See Hardy for details of the analytical formulas.

Notice that in the 80% VaR allocation, the market risk allocation is negative. This reflects the fact that at the 80th percentile of the market risk distribution, the market returns are positive and *reduce* the aggregate capital needs. The impact of this is offset significantly in the 80% CTE allocation because the entire tail of the market risk distribution is taken into account and therefore the scenarios in which the market returns are negative impact the overall capital allocation to market risk.

***Incremental Allocation***

Under this approach, an aggregate risk measure is selected and calculated for the aggregate risk distribution. Then, the same risk measure is recalculated after removing one of the business units. The difference in the capital requirement with and without the selected business unit then represents the *incremental*[33] capital requirement for the business unit.

Using the incremental capital requirements for each business unit, the firm's capital can then be allocated to each unit in proportion to its respective incremental capital requirements. This is demonstrated in the following table.

**Table 18: Capital Allocation – Incremental Based on 99% VaR**

| | Total 99.00% VaR | All-Other 99.00% VaR | Incremental 99.00% VaR | % of Total | Allocated Capital |
|---|---|---|---|---|---|
| Market Risk | 8,949,750 | 8,661,043 | 288,707 | 3% | 241,168 |
| Reserve Risk | 8,949,750 | 5,510,089 | 3,439,661 | 32% | 2,873,285 |
| Line A UW Risk | 8,949,750 | 5,869,650 | 3,080,099 | 29% | 2,572,929 |
| Line B UW Risk | 8,949,750 | 5,044,312 | 3,905,437 | 36% | 3,262,367 |
| Total | | | 10,713,904 | | 8,949,750 |

An important characteristic of this allocation method is that the incremental amounts do not add up to the total capital, even though the same risk measure was used. This is a characteristic that some practitioners find troublesome and there is disagreement over whether the "excess" amount should be allocated[34].

***Marginal Allocation***

The incremental allocation eliminates an entire business unit to determine its capital requirements. Instead, one could eliminate one dollar of revenue or one dollar of expected loss from each unit sequentially and use the change in the firm's total capital requirement as an estimate of the marginal capital requirement for the unit.

Applying this marginal requirement to the total revenue or total expected losses for the business unit provides an alternative measure of the capital needed for the unit. This can then be allocated in the same manner as described above for the incremental allocation method.

This approach typically results in a more appropriate result, however it may be impractical in certain circumstances where not all risk sources can be represented relative to revenue or expected loss or their marginal impacts easily determined.

[33] Different authors have adopted different terminology for incremental and marginal methods. Throughout this document, the term *incremental method* is used to refer to calculations with and without entire business units or risk sources and the term *marginal method* is used to refer to calculations before and after a small change in the risk exposure.

[34] In an influential paper by Merton & Perold that used a different risk measure, they argued against allocation of the excess. In that paper the risk capital was defined as the cost of purchasing protection against default, which is similar to an EPD risk measure. Mango has extended the Merton & Perold approach to insurance applications and argues persuasively that capital allocation is only an intermediate step towards the real goal, which is to allocate the costs of risk capital and not the capital itself. As a result, allocation of all of the capital is not necessary.

### ***Myers-Read Method***[35]

This is a specific type of marginal allocation method, but its basis is somewhat different than those described above. Because an insurance company's total potential losses almost always exceed its assets, its owners have an option to default on the firm's obligations and *put* the claims (or some portion thereof) back to the policyholders. The value of this put option will decline as the amount of capital held increases for the same exposures. The Myers-Read method allocates capital so as to equalize the marginal impact that each business unit has on the value of this put option.

To apply this method, the value of the default option is calculated based on the firm's current capital and its current exposures. The exposure for a given business unit is then increased and the capital needed to maintain the same value of the firm's aggregate default option is determined. This capital then represents a marginal requirement per unit of expected loss for each unit that can be applied to the unit's expected losses.

The results of this method are demonstrated in the table below (see Appendix B for the technical details). For this example, the target EPD Ratio has been set arbitrarily to 0.186% so that the resulting aggregate risk capital is identical to the 99% VaR risk measure used in the other allocation method examples. In addition, the methodology takes into account the market risk in the invested assets, though it does not allocate capital to the market risk component. All capital is allocated to the lines of business for which there are liabilities, since it is only the need to pay liabilities that gives rise to the need to hold capital[36].

**Table 19: Capital Allocation – Myers-Read Method (0.186% EPD Ratio)**

| | Capital to Loss Ratio | Expected Claims | Capital | % of Total | Allocated Capital |
|---|---|---|---|---|---|
| Reserve Risk | 21.78% | 18,091,233 | 3,939,466 | 44% | 3,939,466 |
| Line A UW Risk | 33.92% | 5,860,732 | 1,988,079 | 22% | 1,988,079 |
| Line B UW Risk | 51.57% | 5,860,732 | 3,022,205 | 34% | 3,022,205 |
| Total | | | 8,949,750 | | 8,949,750 |

This particular method has become popular because it produces additive capital requirements that sum to the total capital requirement for the firm when the same risk measure is used. Three points are worth noting with respect to this method:

- The method was not developed as a means for determining risk-adjusted capital requirements; it was developed as a means to allocate the frictional costs of capital to various businesses. While it may be used for the former purpose, it is not necessarily more appropriate for this purpose than the other methods discussed.
- Because this method requires the valuation of the default option, its application may require substantially more quantitative resources compared to other methods, except in certain limited circumstances.
- Significant mathematical challenges have been raised that suggest that the Myers-Read method is not appropriate for most insurance applications. The method assumes that risk exposure in a business unit can be increased or decreased without impacting the shape of the loss distribution, a property referred to as homogeneity[37]. Except when risk can be increased

[35] The original Myers-Read paper presents the derivation of their approach. Butsic (1999) and Venter (2003) each present insightful discussions of this method and present simplified formulas that can be used to implement Myers-Read.

[36] This is a subtle point that is often overlooked because while it is true that a risk measure such as VaR can be used for an asset portfolio, in the absence of liabilities *risk capital* is not needed in the sense discussed here. There is clearly risk associated with marketable securities, though it would be odd to invest capital to protect against this risk. This is another example that highlights the need to understand the distinction between a risk measure and risk capital as it has been defined here as the amount needed to ensure that liabilities can be satisfied.

[37] See Mildenhall's discussion of the Myers-Read method for details.

or decreased through changes in *quota share* percentages, insurance loss distributions will not exhibit homogeneity when adding or removing policies from the firm's mix of business.

***Co-Measures Approach***[38]

This approach establishes the firmwide capital requirement using a particular conditional risk measure, such as VaR or CTE, and then calculates the Co-Measure for each business unit by calculating the comparable risk measure for the unit subject to the *condition* applied to the entire firm.

For example, consider the case where the risk measure selected is the CTE. The firmwide CTE is the average value of the losses *given that* the losses for the firm exceed the Value at Risk at a chosen percentile. To determine the Co-CTE for a given business unit, simply calculate the average losses *for each business* subject to the same firmwide condition that the total losses for the firm exceeds the chosen percentile.

This is very easy to implement in a simulation context. For example, the four key risk components for the SIC example were simulated using a normal copula method and the aggregate "loss" was determined for each of 50,000 simulation scenarios. The results were sorted in descending order based on the total loss and the worst 1% of the scenarios (the top 500 scenarios) were identified, as shown below:

**Table 20: Co-CTE Calculations**

| Sorted Scenario | Market | Reserves | Line A | Line B | Total |
|---:|---:|---:|---:|---:|---:|
| 1 | 779,323 | 12,180,298 | 3,188,429 | 4,994,583 | 21,142,632 |
| 2 | 494,425 | 8,169,822 | 3,734,913 | 8,695,665 | 21,094,825 |
| 3 | -3,407,081 | 13,140,377 | 7,607,985 | 788,471 | 18,129,751 |
| 4 | -779,922 | 2,587,705 | 5,675,660 | 10,386,216 | 17,869,658 |
| 5 | -1,311,004 | -1,203,142 | 3,238,333 | 16,924,158 | 17,648,345 |
| 6 | -1,392,828 | 5,488,457 | 6,646,703 | 6,799,820 | 17,542,152 |
| 7 | -255,475 | 4,812,487 | 4,018,249 | 7,904,885 | 16,480,145 |
| 8 | -10,210 | 6,710,721 | 2,273,968 | 7,472,474 | 16,446,953 |
| 9 | -1,896,169 | 4,433,724 | 1,652,542 | 12,169,231 | 16,359,328 |
| 10 | 758,494 | 3,132,459 | 2,330,630 | 10,003,805 | 16,225,388 |
| 11 | -1,291,494 | 8,133,807 | 5,475,393 | 3,899,206 | 16,216,912 |
| 12 | 1,523,399 | 8,164,027 | 1,320,562 | 4,996,263 | 16,004,250 |
| 13 | -1,507,026 | 8,701,922 | 4,941,913 | 3,358,494 | 15,495,303 |
| 14 | -418,192 | -390,473 | 1,172,596 | 15,112,222 | 15,476,153 |
| 15 | 348,569 | 4,904,846 | 4,173,982 | 6,001,026 | 15,428,423 |
| : | : | : | : | : | : |
| 490 | -470,761 | 3,622,090 | -148,615 | 4,519,262 | 7,521,976 |
| 491 | -980,559 | 3,630,412 | 1,980,834 | 2,889,533 | 7,520,220 |
| 492 | -2,921,510 | 2,906,628 | -200,015 | 7,730,833 | 7,515,936 |
| 493 | -1,179,044 | 3,552,559 | 2,343,631 | 2,794,807 | 7,511,953 |
| 494 | -2,744,202 | 2,173,409 | 4,717,356 | 3,364,141 | 7,510,703 |
| 495 | 127,947 | 1,318,389 | 4,749,312 | 1,308,659 | 7,504,307 |
| 496 | 42,016 | 1,663,231 | 1,653,643 | 4,143,005 | 7,501,894 |
| 497 | -1,062,298 | 2,170,695 | 6,366,285 | 27,183 | 7,501,865 |
| 498 | -901,735 | 4,579,393 | -124,816 | 3,947,145 | 7,499,986 |
| 499 | -2,782,565 | 972,163 | 1,896,786 | 7,411,779 | 7,498,163 |
| 500 | -2,959,845 | 6,146,281 | 863,894 | 3,441,193 | 7,491,523 |
| **Co-CTE** | **-908,399** | **3,715,533** | **2,279,319** | **4,549,138** | **9,635,591** |

The overall 99% CTE is simply the average total loss for the 500 worst scenarios, or $9.635 million. For each of these specific scenarios, the four main risk components make a different contribution to the total loss. For example, in Scenario 1, 58% of the total loss came from the reserve risk, 24% came

[38] See Kreps or Ruhm and Mango for a complete discussion of this approach.

from Line B's underwriting risk, 15% came from Line A's underwriting risk and 3% came from the market risk. Note though that, on average over these 500 scenarios, the market risk component actually *reduced* the total loss (due to profits in the investment portfolio rather than losses). Taking an average for each of these risk components, not across each of their own respective worst 1% of outcomes but rather across these specific 500 scenarios that represent the worst 1% of the total outcomes, the Co-CTE's are calculated as shown in the bottom row of the table. These reflect the average contribution each makes to the total losses.

**Table 21: Capital Allocation – Proportional to 99% Co-CTE**

| | 99.00% Co-CTE | % of Total | Allocated Capital |
|---|---|---|---|
| Market Risk | -908,399 | -9% | -843,742 |
| Reserve Risk | 3,715,533 | 39% | 3,451,069 |
| Line A UW Risk | 2,279,319 | 24% | 2,117,082 |
| Line B UW Risk | 4,549,138 | 47% | 4,225,340 |
| Total | 9,635,591 | | 8,949,750 |

As shown in this table, on average the reserve risk contributes 39% of the total losses, Line A's underwriting risk contributes 24% of the total losses and Line B's underwriting risk contributes 47% of the total losses.

In addition, the Co-CTE's "add-up" to the total CTE as shown in the bottom row of the scenario summary. But to remain consistent with the other allocation examples and to highlight the ability to separate the allocation method from the amount allocated, the final allocation in the last column uses the Co-CTE allocation percentages applied to the 99$^{th}$ percentile risk measure (99% VaR) total risk capital figure used earlier.

## *5. Guiding Strategic Decisions*

In this section, five specific applications of risk-adjusted performance metrics and the methods discussed in the previous sections are presented:

- Assessing Capital Adequacy
- Setting Risk Management Priorities
- Evaluating Alternative Risk Management Strategies
- Risk-Adjusted Performance Measurement
- Insurance Policy Pricing

### 5.1 Assessing Capital Adequacy

Insurers sell a promise to pay claims that, under certain conditions, could far exceed the premiums collected. As a result, in addition to carrying reserves for expected claims, they must also hold capital to provide their policyholders with reasonable assurances that their claims will be paid.

Regulators require certain minimum capital levels and various rating agencies have their own methods of assessing the adequacy of an insurer's capital base and assigning a financial strength or claims paying ability rating. Key questions that these rating agencies seek to have answered include:

- Is the firm sufficiently capitalized to meet current policyholder obligations?
- Does management understand the source of risk in the business?
- Does management actively measure and manage its exposure to risk?

The aggregate risk profile and the aggregate risk measures used to determine the firm's risk capital are useful in addressing these questions. They require the firm to develop risk models for each type of risk, select an aggregation method and choose an appropriate risk measure.

Firms capable of performing these calculations should be in a better position to demonstrate their claims paying ability and should have the tools they need to understand what drives the risk in their business.

### 5.2 Setting Risk Management Priorities

To assess firmwide capital adequacy, the capital allocation methods presented in Section 4 are not needed. By incorporating these allocation methods, firms can identify those business units or those activities that generate the greatest need for risk capital. Those business units may offer the greatest opportunity to reduce capital needs through effective risk management actions aimed at mitigating or transferring risk.

### 5.3 Evaluating Alternative Risk Management Strategies

Going further, measures of expected profitability can be incorporated and risk-adjusted return on capital (RAROC) measures can be calculated. This provides a means to test the impact of alternative strategies aimed at reducing risk, by comparing the costs and benefits of risk reduction. For example, a firm's overall RAROC or the RAROC for a particular business unit could be compared before and after a specific risk mitigation strategy to determine whether the transaction increases or decreases the return per unit of risk. Such an analysis is commonly performed to evaluate alternative reinsurance programs, for example.

### 5.4 Risk-Adjusted Performance Measurement

It is often desirable to evaluate actual, *ex post*, performance of different business units. Traditional measures of financial performance for insurers, such as historical loss ratios, can provide misleading indications of relative results for two business units with different levels of risk. For instance, if a business unit with a high degree of risk were to have a lower loss ratio than a business unit with a low amount of risk, the loss ratios alone may not properly identify which of the two business units performed "better". The use of a risk-adjusted performance metric such as RAROC may allow these business units to be more fairly compared. The explicit risk-adjustment may also be an improvement over judgmental premium to surplus ratios.

As an example of this process, consider the Sample Insurance Company presented in Section 4. Rather than rely upon the expected loss ratios, hypothetical values for the actual loss ratios realized over the year will be used. For this example, the actual loss ratios will be assumed to equal 92% for Line A and 86% for Line B.

Based solely on the loss ratios, it is natural to assume that Line B performed better. Calculation of an "economic profit" could also be used to show that Line B had a larger present value profit. For example, assuming that the actual market returns were 5%, then each line of business would have had economic profit at the end of the year equal to the following[39]:

**Table 22: Calculation of Actual Economic Profit**

| | Line A | Line B | Calculations |
|---|---|---|---|
| (1) Premium | 6,400,000 | 6,400,000 | Actual |
| (2) Expense Ratio | 5.00% | 5.00% | Actual |
| (3) Expenses | 320,000 | 320,000 | (3) = (1) * (2) |
| (4) Investment Return | 5.00% | 5.00% | Actual |
| (5) Investment Income | 304,000 | 304,000 | (5) = (4) * [(1) - (3)] |
| (6) Discounted Loss Ratio | 92.00% | 86.00% | Actual |
| (7) Discounted Claim Costs | 5,888,000 | 5,504,000 | (7) = (6) * (1) |
| (8) **Economic Profit** | **496,000** | **880,000** | (8) = (1) - (3) + (5) - (7) |

As shown in Section 4, Line B exposed the firm to substantially more risk than Line A and its profit per dollar of risk capital was actually lower. For instance, if the 99% Co-CTE allocation method were used, the following table shows the RAROC for these two business units:

**Table 23: Comparison of RAROC – Using Co-CTE Allocation**

| | Economic Profit | Co-CTE (99%) Allocated Capital | RAROC |
|---|---|---|---|
| Line A | 496,000 | 2,117,082 | 23.4% |
| Line B | 880,000 | 4,225,340 | 20.8% |

By rescaling the profit by the allocated capital for the underwriting risk, the risk-adjusted profitability measure shows that despite the lower loss ratio and higher economic profit, Line B required far more capital to support its operations and as a result did *not* outperform Line A.

This use of RAROC to better inform the assessment of performance shows that it is possible to take risk into consideration in a relatively simple manner. However, Section 4 showed that there were a variety of allocation methods that could be used. For instance, if the proportional allocation based on the 99$^{th}$ percentile (99% VaR) risk measure were used, the following alternative results would be obtained:

[39] Recall that the present value, or discounted, loss ratio reflects the value of the losses at the *end* of the year.

**Table 24: Comparison of RAROC – Using Proportional 99% VaR Allocation**

| | Economic Profit | 99% VaR Allocated Capital | RAROC |
|---|---|---|---|
| Line A | 496,000 | 2,035,598 | 24.4% |
| Line B | 880,000 | 3,384,941 | 26.0% |

This comparison shows that RAROC, despite its appeal as a means to risk-adjust performance metrics, does not necessarily produce unambiguously superior performance measures. Depending upon the method used for the allocation, the RAROC for Line B could be either lower than or higher than the RAROC for Line A. These results are highly sensitive to a variety of implicit and explicit assumptions that can materially impact the allocation of capital to specific business units.

## 5.5 Insurance Policy Pricing

A natural extension of the RAROC analysis just demonstrated, which focused on a relative comparison of two business units, is to use RAROC directly in insurance policy pricing. The rationale would be to set the price such that the expected RAROC is above a specified target rate.

Suppose, for instance, that an acceptable RAROC target of 15% is assumed. The premium that should be charged such that Line B's expected RAROC was equal to at least 15% would then be easy to determine. One approach, albeit overly simplified and somewhat naïve, is to simply choose one of the many capital allocation methods and then solve for the additional risk margin, which will be denoted by $\pi$ here, such that the RAROC equals the target rate of 15%.

For the sake of a numerical example, consider the allocation of risk capital to Line B using the Co-CTE allocation method. Based on the existing assumptions regarding Line B's *expected* loss ratio rather than the actual loss ratio used in the previous example, this produces the following expected economic profit and expected RAROC for Line B:

**Table 25: Expected Economic Profit – Line B**

| | Line B | Calculations |
|---|---|---|
| (1) Premium | 6,400,000 | Expected |
| (2) Expense Ratio | 5.00% | Expected |
| (3) Expenses | 320,000 | (3) = (1) * (2) |
| (4) Investment Return | 5.00% | Expected |
| (5) Investment Income | 304,000 | (5) = (4) * [(1) - (3)] |
| (6) Discounted Loss Ratio | 91.60% | Expected |
| (7) Discounted Claim Costs | 5,862,400 | (7) = (6) * (1) |
| (8) **Expected Economic Profit** | **521,600** | (8) = (1) - (3) + (5) - (7) |

**Table 26: Expected RAROC – Using Co-CTE Allocation**

| | Expected Economic Profit | Co-CTE (99%) Allocated Capital | RAROC |
|---|---|---|---|
| Line B | 521,600 | 4,225,340 | 12.3% |

With no additional risk margin, the RAROC is below the target rate. The following equation can be used to solve for the additional risk margin, π, that produces the target rate of 15%[40]:

$$RAROC = \frac{[\text{Original Premium} + \text{Additional Risk Margin - Expenses}] * (1 + \text{Expected Investment Income}) \text{ - PV of Expected Claims}}{\text{Allocated Risk Capital}}$$

$$= \frac{[6{,}400{,}000 + \pi - 320{,}000] * (1 + 5\%) - 5{,}862{,}400}{4{,}225{,}340} = 15\%$$

$$\pi = 106{,}858$$

This solution can also be derived using what is often referred to as an Economic Value Added or EVA™ approach[41]. If the $4,255,340 is treated as the "required capital" to write Line A and the 15% RAROC target is the "per unit cost of capital", then the total dollar cost of the capital is 15% * $4,255,340 = $633,801. This is the amount of expected economic profit that would have to be incorporated into the premium. Since the original premium already accounted for $521,600 of this expected profit, only $106,858 of additional risk margin would have to be incorporated to meet the RAROC target rate.

**Table 27: Calculation of Additional Risk Margin Required**

| | Amount | Calculations |
|---|---|---|
| (1) Allocated Risk Capital | 4,225,340 | Based on Co-CTE Allocation |
| (2) Target RAROC | 15.0% | Assumed |
| (3) Required Economic Profit | 633,801 | (3) = (1) * (2) |
| (4) Current Economic Profit | 521,600 | Based on Assumptions |
| (5) Shortfall | 112,201 | (5) = (3) - (4) |
| (6) Expected Investment Income | 5.00% | Based on Assumptions |
| (7) Additional Risk Margin Required | 106,858 | (7) = (5)/[1 + (6)] |

Notice that in this calculation the additional risk margin is assumed to earn the same expected rate of investment income as the net premiums. An argument could be made that the additional risk margin should be assumed to be invested in risk-free assets only, to avoid the need to calculate the additional risk capital that investing these funds in risky assets might produce. But the impact of this is likely to be small and can usually be ignored.

***Additional Considerations***

Using RAROC for pricing, as in this example, is appealing because the steps are logical and easy to explain. However, some subtle complications can arise in practice that are not as obvious in this example due to some of the simplifications made. In this section, the consequences of three specific simplifications of importance to pricing applications will be discussed (additional complications relevant to all applications will be discussed in Section 6):

---

[40] For simplicity, the additional risk margin in this section will be assumed to not affect expenses such as commissions or premium taxes that are commonly proportional to total premium. In practice, the formulas shown here would have to be adjusted for such expenses.

[41] EVA™ is the trademarked terminology used by Stern Stewart & Co. This approach is compatible with RAROC though it is expressed in dollars instead of as a ratio. See Brealey & Myers for a discussion of the advantages of using profitability measures denominated in dollars rather than profitability ratios.

- Investment Income on Allocated Capital
- Multi-Period Capital Commitment
- Cost of Capital (Target RAROC Rate)

Investment Income on Allocated Capital

In the simplified example shown above, it was assumed that the target return on the allocated risk capital was 15%. How this target return is calculated depends on how the economic profit is defined.

The definition of economic profit used in the example above did not include the investment income that can be expected to be earned on the allocated risk capital itself. As a result, the 15% target return also excludes the investment rate assumed to be earned on the allocated risk capital. The target return is technically an *excess return* over the investment rate in this case.

Alternatively, the investment income expected to be earned on the allocated risk capital could be included in the calculation of economic profit. In this case, the target return should be inclusive of the investment rate assumed to be earned on the allocated risk capital. In a single period context, the two approaches lead to the same risk margin. However, when risk capital is required over multiple periods, the approach used in the examples above is easier to apply.

Multi-Period Capital Commitment

Up until now, the allocated risk capital was assumed to be exposed to risk for only a single period. This allowed the discussion of RAROC to be somewhat simplified and did not impact any of the conclusions drawn from the previous examples, in part because both Line A and Line B had the same claim payment patterns and the comparisons were made *relative* to each other rather than on an absolute basis.

But in the context of policy pricing, it is important to recognize that the *initial* capital required to write the policy does not fully reflect the total capital costs. The risk will not be fully resolved in a single period and so some risk capital will be needed in subsequent periods as well, perhaps until the final claims are paid. It is easy to see how one might account for this in practice. One common approach is to assume an *average* pattern for the release of the risk capital and then use that pattern either to adjust the RAROC ratio or to modify the target rate.

To see how these adjustments could be made, consider an assumed claim payment pattern for Line A as follows (chosen arbitrarily for simplicity):

**Table 28: Claim Payment Pattern - Line A**

| Year | % Paid |
|---|---|
| 1 | 50% |
| 2 | 30% |
| 3 | 15% |
| 4 | 5% |

Further, assume that the risk capital will be released, on average, at the same rate as the claims are paid. In reality, under some scenarios more capital will be released, perhaps faster or slower than this pattern, and under some scenarios more capital may even be committed to support this line of business. But given the assumed release pattern for the allocated risk capital, the cost per unit of risk capital (15% for the sake of this example) can be applied to the outstanding risk capital each period and the aggregate cost of risk capital over the life of the exposures determined as shown below:

**Table 29: Aggregate Cost of Risk Capital – Multi-Period Release of Risk Capital**

| Year | % Paid | Beginning Risk Capital | Cost of Risk Capital | PV Cost of Risk Capital | Risk Capital Released | Ending Risk Capital |
|---|---|---|---|---|---|---|
| 1 | 50% | 4,225,340 | 633,801 | 603,620 | 2,112,670 | 2,112,670 |
| 2 | 30% | 2,112,670 | 316,901 | 287,438 | 1,267,602 | 845,068 |
| 3 | 15% | 845,068 | 126,760 | 109,500 | 633,801 | 211,267 |
| 4 | 5% | 211,267 | 31,690 | 26,071 | 211,267 | 0 |
| | | | 1,109,152 | 1,026,630 | | |

Using the EVA™ approach demonstrated above for the single period case, where the premium is adjusted to ensure that the total dollar cost of risk capital is recovered through the premium charges, it is easy to see how this total cost of risk capital might be reflected in the policy pricing.

To stay within the RAROC framework, it is sometimes helpful to convert this solution that takes into account the multi-period nature of the risk capital commitment into either an adjusted RAROC metric or an adjusted target rate.

To see the adjusted target rate first, note that the EVA™ approach makes it clear that if $C_i$ reflects the beginning risk capital each period and $R$ is the constant cost of risk capital each period, and $r$ is the investment income rate expected to be earned on the risk margin[42] (assumed to be 5% in the previous examples), then the policy pricing should reflect the following expected economic profit:

$$\text{Economic Profit} = \sum C_i R(1+r)^{-i}$$

From this, both sides can be divided by the initial risk capital, $C_1$, to determine the adjustment factor (shown in brackets below) to use to modify the target rate, $R$:

$$\frac{\text{Economic Profit}}{\text{Initial Risk Capital}} = \frac{\sum C_i R(1+r)^{-i}}{C_1}$$

$$\text{RAROC} = R\left[\frac{\sum C_i (1+r)^{-i}}{C_1}\right]$$

Using the example shown above, the RAROC target rate would be the original rate $R$ = 15% adjusted by the factor given in the brackets, or 1.62. Given this average pattern for the release of capital, the RAROC target rate would have to be adjusted to 15%*1.62 = 24.3%. Then, the target economic profit needed to achieve this target RAROC would simply be $4,225,340 * 24.3% = $1,026,630.

Alternatively, some practitioners[43] suggest altering the calculation of RAROC by including the present value of the risk capital commitments in the denominator instead of simply using the *initial* capital (denoted $C_1$ to reflect the capital at the beginning of the first period). This is algebraically the same as the formula above:

$$\frac{\text{Economic Profit}}{C_1} = R\left[\frac{\sum C_i(1+r)^{-i}}{C_1}\right]$$

$$\frac{\text{Economic Profit}}{C_1} * \left[\frac{C_1}{\sum C_i(1+r)^{-i}}\right] = R$$

$$\frac{\text{Economic Profit}}{\sum C_i(1+r)^{-i}} = R$$

*Steady State Assumption*

A common simplification assumes a "steady state" and incorporates the reserve risk capital into the calculations of the initial required capital for each line of business. Then, instead of reflecting the present value of all future capital commitments from the underwriting risk alone (the $\sum C_i (1+r)^{-i}$ term in the formulas above), the initial capital requirements for *both* the underwriting and reserve risk are used.

---

[42] Earlier it was noted that the examples here use the same investment income assumptions used to derive the risk capital. The question of whether this rate should really just be the risk-free rate will not be addressed here. In addition, corporate income taxes are ignored. With corporate income taxes, and assuming that the risk margin is treated as taxable income, these numbers may need to be grossed up to reflect the after-tax funds contributed by policyholders.

[43] See Nakada, et al

In the example here, the Sample Insurance Company was assumed to have existing loss reserves, but the line of business was not specified. While the existing reserves also required risk capital, it was ignored in all of the numerical calculations that were aimed at assessing the pricing for the *new* business only. But under certain limited circumstances, it may be possible to include the risk capital associated with the reserves along with the underwriting risk capital as an approximation for the denominator shown above. For example, if the reserves were all for Line A and the riskiness of this line of business has not changed, then combining the total reserve and underwriting risk capital may serve as an approximation for the denominator in the previous equation. Because of differences in how the diversification impacts the different formulas, this approximation may be relatively poor in some cases.

Of course, in other cases, such an approximation will clearly not be appropriate. For instance, if Line B were an entirely new line of business, there would be no way to approximate the denominator by including any portion of the reserve risk capital into the calculations. The inability to use this simplification across all lines of business would further complicate comparisons across different lines of business.

Cost of Risk Capital

In the above discussion, a constant 15% cost of risk capital was assumed, without explanation or justification. It is worth exploring, particularly in the context of insurance policy pricing, how this cost of risk capital should be determined in practice.

Although the RAROC measure is intuitively appealing, it is more *ad hoc* than many practitioners often recognize. Because it is referred to as a return on capital, it is quite common for practitioners to assume that standard "rate of return" benchmarks, such as those derived from models such as Capital Asset Pricing Model (CAPM) or the Fama-French 3-Factor Model, are applicable. In reality, the rate used for the cost of risk capital must take into account the specific way in which the RAROC metric is defined.

The most significant issues include the following:

- Numerous textbook discussions of RAROC suggest using risk-adjusted return models such as CAPM to establish the cost of risk capital and to assess whether or not the RAROC exceeds this value. Despite the fact that both RAROC and CAPM produce "risk-adjusted returns", the risk adjustment in RAROC reflects a different definition of "risk" than is used in these theoretical models.

  Models such as CAPM measure the systematic risk associated with an investment, which accounts for the *marginal* contribution the investment adds to an existing portfolio of diversified investments. RAROC, even for the total firm, incorporates an entirely different measure of risk based on the relationship between a cash flow's expected value and certain values in the tail of its probability distribution[44].

- RAROC is artificially "leveraged". The denominator reflects neither the total market value of the "invested" capital (as is assumed in the theoretical return models) nor the firm's actual capital that could be exposed to loss (the committed capital).

  If the firm's shareholders desire a given target rate of return on their investment, the dollar value of their target "income" will depend on the total market value of the firm's equity. This will almost always exceed the value of the firm's book equity (the difference being attributed to their franchise value), though under certain assumptions regarding the stability of the

[44] It should be noted that the distinction being made here between systematic risk in CAPM, on the one hand, and tail measures of risk in RAROC, on the other hand, may not necessarily be as stark as implied here. Many academic researchers have begun to question the focus on systematic risk in models of return expectations and have suggested a variety of methods to also incorporate measures of non-systematic risk (see Froot & Stein and Shimko). Others have instead suggested that RAROC itself could be adapted to also incorporate measures of systematic risk (see Crouhy, Turnbull & Wakeman or Wilson).

firm's market to book value multiple, the rate of return on market value and the rate of return on book value may be equal.

Nevertheless, earning this rate of return solely on the firm's *risk capital* will not necessarily be sufficient to satisfy the income expectations of the shareholders. Using this lower base in the denominator of RAROC artificially inflates the rate of return on "capital", with only a modest offset due to the fact that the numerator also ignores a component of "income" based on changes in the franchise value of the firm.

When RAROC is measured for distinct business units within the firm, the capital allocated to those business units will depend upon the degree to which diversification effects are reflected in the amount allocated, the risk measure used and other somewhat arbitrary decisions. The business unit losses are not literally limited to the amount of risk capital allocated to it, so this leverage effect on the RAROC is even more artificial.

Taking these considerations into account is a bigger challenge than is often recognized and entirely satisfactory methods for calibrating the cost of risk capital do not exist.

One acceptable compromise is to recognize that models such as CAPM or the Fama-French 3-Factor Model are reasonable means to quantify shareholders' target return on the firm's total capital (e.g. GAAP book value). Under a conservative assumption that only the total risk capital is "at risk", the CAPM return can be adjusted upwards by the ratio of the firm's total capital to the firm's risk capital. Alternatively, rather than using the (arbitrary) risk capital in the RAROC calculation, the firm's total capital could be used along with the allocation methods discussed here. In either case, this allows the pricing model to reflect the aggregate compensation required by the shareholders for assuming systematic risk and then allocates this total amount to different business units (or policies) in a manner that reflects the relative "risk" of each.

This approach does not account for the differential degrees of leverage in each business unit. This is because after taking into account diversification benefits, it is quite difficult to quantify how much additional leverage has been introduced into the calculation.

This approach also does not address the potential for different business units to have different degrees of systematic risk. Theoretically this should be easy to deal with, though in practice adjustments to reflect differing degrees of systematic risk across segments of the total firm are quite difficult to make because of the limited ability to reliably quantify these differences[45].

Many alternative methods for quantifying the cost of risk capital have been proposed. For example, Feldblum suggests incorporating the frictional costs of holding capital, such as those that result from the double taxation of investment income. Venter points out though that this is incomplete because it doesn't address the compensation required for assuming the *risk* that would reasonably need to be included even in the absence of corporate taxes (e.g. for Bermuda-based reinsurance firms that are not subject to corporate income taxes).

Yet another approach has been suggested by Mango. Mango notes that while it is common to refer to the allocation of "capital", it is really just an allocation of underwriting *capacity* and therefore the policy or business unit must earn adequate profits to pay for this capacity. In addition, each policy or business unit also is given the ability to call upon the capital not explicitly allocated to it, if needed to pay claims, and therefore must also earn adequate profits to compensate the firm for the value of this capital call. These costs could be combined and reflected as a cost per unit of allocated risk capital.

---

[45] See Cummins and Phillips for an example of how one might be able to use the *full information beta* approach to distinguish between measures of systematic risk for different lines of business.

## 6. Practical Considerations

In an effort to streamline the discussion and introduce the RAROC concept as fully as possible, many real-world considerations that can complicate this type of analysis have been intentionally ignored. In this section, some of these issues are highlighted and briefly described.

### 6.1 Time Horizons

In Section 3, it was briefly noted that inconsistent time horizons are often used to measure a firm's aggregate risk profile. For instance, market risk is typically measured based on the potential change in the value of the assets over a one-year period, while the insurance risks are measured based on the potential ultimate liability. In many instances, such as property-catastrophe or other very short-tailed insurance risks, this distinction is trivial. But in some instances, particularly those for which the ultimate liability is highly dependent upon the realization of unknown claim severity trends, this distinction could be material.

Some practitioners argue that to resolve this issue, the market and credit risks could be measured over the entire lifetime of the insurance liabilities. This significantly complicates the mechanics of the models (requiring complex DFA models) and introduces new challenges to estimate the parameters for the models. It is far more difficult to quantify equity market, interest rate and foreign exchange rate parameters over long horizons due, for instance, to limited availability of long-horizon historical data and more significant serial correlations. More importantly, over longer horizons it is far less reasonable to assume a fixed portfolio and to ignore important strategic decisions that may be made in response to market movements.

An alternative approach being explored by European insurance regulators in conjunction with Solvency II is to focus on the change in *value* of the insurance liabilities over a one-year period. Although conceptually more consistent with the methods used to measure market risks, there are serious questions that have been raised about this approach for certain classes of insurance. In many cases, information relevant to the revaluation of the liabilities is not available over a short horizon and so this approach will result in limited potential change in the value, even in cases where there is substantial risk over a longer horizon.

An example of this can be found in high layer excess general liability insurance policies. Over a short horizon, such as one year, the premium required to transfer the risk to a third party (a standard measure of the *value* of an insurance risk) is unlikely to differ materially from the premium initially charged to write the policy. But as the underlying claims are reported and settled and claim severity trends accumulate over a long horizon, there very well could be material risk associated with these policies.

There does not appear to be consensus on how to express the time horizons over which the risk is measured when market risk, credit risk and long-term insurance risks are combined. Regulatory discussions in Europe suggest a trend towards measuring insurance risks over shorter horizons[46] rather than over the lifetime of the liabilities. But practices vary and many implementation details remain unresolved.

### 6.2 Alternative Return Measures

The discussion to this point has assumed that the return measure reflects economic profit, though this is just one of many variations of the RAROC approach that can be used.

<u>Benefits of Accounting Measures of Income</u>

Some firms have found it challenging to introduce RAROC considerations into their organizations using a return measure that is substantially different from the GAAP or Statutory calendar year income measures with which senior management is familiar. For this reason, they prefer to use these accounting metrics in the RAROC calculation. When this is done, the result becomes something akin

---

[46] See Conway and McCluskey, 2006.

to a *calendar year* RAROC calculation, though inconsistencies with the denominator of the RAROC calculation become inevitable.

Taxes

One clearly important variation of the calculations shown in this paper is to include the effect of corporate income taxes on any measures of return. Depending upon the tax jurisdiction and the specific tax position of the firm, this could prove to be a fairly complex issue. Nonetheless, corporate income taxes are a real expense that should be reflected, if applicable.

Stranded Capital

Some practitioners, in an attempt to account for the leverage effect noted in Section 5.5, reduce the return measures used in RAROC for what they describe the cost of *stranded capital*. The stranded capital is defined as the amount of capital held in excess of the (allocated) risk capital. In some cases, this amount is limited to the amount of regulatory or rating agency measures of "required" capital and in other cases it reflects the difference between all of the firm's capital and the (allocated) risk capital. Reducing the rate of return by this cost is conceptually identical to the adjustment noted in Section 5.5, where all of the firm's capital is allocated rather than just the firm's risk capital. However, depending on the rate of return used, some small differences between the two approaches may result.

Investment Income

In the numerical examples that dealt primarily with single-period capital commitments, investment income was included in the definition of economic profit because all of the calculations were assumed to occur at the end of the period. When multiple periods are reflected, it is easier to work with *present value* amounts. When this is done, including investment income becomes more complicated and the RAROC ratio is not a true "rate of return".

### 6.3 Risk-Based Allocation

In Section 4, all of the methods discussed to allocate risk capital relied upon "tail" measures of risk using either a percentile (VaR), CTE, EPD Ratio or Co-Measure methodology. While this produces a "risk-based" allocation, it does so using a rather limited view of what drives the risk to the firm and tends to allocate capital primarily to those businesses with the most extreme levels of skewness, such as businesses exposed to property-catastrophe risks. This may make sense in the regulatory or rating agency applications where many of these risk capital models were first developed, but this is less appropriate when these models are used to manage the interests of the firm's shareholders.

An alternative approach alluded to in Section 3 starts with the observation that the firm's shareholders could be severely impacted by less extreme events that, while not in the "tail", would materially affect the firm's credit rating, financial strength or ability to operate as a going concern. Then, the business units that most impact these measures of "risk" would be allocated more of the firm's capital and its associated cost.

Similar risk measures may still be appropriate, though the percentiles at which they are measured are less likely to be values such as 99% or 99.97%. Instead, thresholds based on a target percentage loss of surplus are more likely to be used.

### 6.4 Diversification Adjustments

In Section 3, numerous challenges associated with estimating correlation or dependency parameters across business units and risk sources were mentioned. What wasn't emphasized is that these difficult assumptions often drive the determination of the firm's aggregate risk profile, the allocation of risk capital to business units and the resulting RAROC calculations. In light of this, it is worth questioning the role that RAROC measures should play in important managerial decisions. They can be informative and insightful, but they should not serve as the sole metric that drives such decisions.

## *7. Conclusion*

This paper has presented an introductory overview of risk-adjusted performance measurement. Using a simplified version of a commonly used performance metric, Risk-Adjusted Return on Capital (RAROC), new insights into common managerial decisions may be possible.

The method shown here began with the development of an aggregate, firmwide risk profile and then used various risk measures to quantify how much of the firm's capital was "at risk". Aside from highlighting the level and sources of risk in the firm, this measure of risk capital was allocated to various business units or activities and then used to compare relative performance or to guide pricing decisions.

Through the examples shown, a variety of critical challenges likely to be encountered when using this framework were presented to highlight the strengths as well as the limitations of the methodology.

## Appendix A: Risk Exposure Horizon

The distinction between the lifetime of liability uncertainty and the one-year horizon uncertainty relates to the *risk exposure horizon*, or the time period over which the risk can materialize.

As alluded to in the discussion of market risk, consistency of the risk exposure horizon is particularly challenging in risk capital models. There are several schools of thought regarding how to handle this issue among practitioners:

- Use One-Year Risk Exposure Horizon for All Risk Types
- Use Multi-Period "DFA" Models
- Ignore the Inconsistencies

Each of these approaches will be discussed briefly below.

### One-Year Risk Exposure Horizon for All Risk Types

Some practitioners want to ensure that the risk measures for market, credit and insurance risks can be aggregated easily and therefore they measure risk from *all* sources by estimating their potential change in value over a common time horizon, typically one year to coincide with typical accounting-based profitability measures.

While this approach appears more mathematically pure, in practice this is very difficult to achieve, for several reasons.

First, there are no reliable methods that can be used to estimate the timing of the recognition of adverse loss development for loss reserves. Limited historical data does exist, but because practices among firms vary considerably with regard to how they report their losses by line and whether they test reserve adequacy at any particular point in time on a by-line basis or in the aggregate, this data is unlikely to prove reliable.

Second, this "change in value" perspective is not entirely consistent with a market VaR measure if it only reflects the change in the *best estimate* of the reserve over the time horizon. Market VaR calculations reflect the potential change in the *market value* and therefore for the reserve risk to truly reflect the change in *value*, it would also have to reflect a risk margin. Only when a risk margin is included will the amount reflect the price at which the risk could realistically be transferred to a third party. In the absence of such adjustments, the figures cannot fairly be represented as a change in "value" in the same sense that the market VaR reflects the change in value for the invested assets.

And third, the vast majority of the "risk" inherent in loss reserves will <u>not</u> be resolved over the course of a single one-year period. As a result, methods that focus on only a single period will necessarily ignore a significant amount of the total risk for an insurer.

### Multi-Period "DFA" Models

Dynamic Financial Analysis (DFA) models make it theoretically possible to account for market and credit risks throughout the lifetime of the liabilities, thereby ensuring a consistent risk exposure horizon across all risk categories. However, this advantage does not come without some associated complexity. In particular, modeling long term market and credit risk exposure is far more complex than simply extending short-term market and credit risk measurement metrics, since over long horizons the issues associated with model parameters, serial correlation and management's strategy become significant drivers of the results.

### Ignore the Inconsistencies

A common approach inherent in virtually all regulatory and rating agency capital models is to measure some risks over a one-year horizon and other risks, notably the insurance risks, over a lifetime of liability horizon. This renders the aggregate risk models difficult to interpret, but it does greatly simplify the modeling effort.

Interestingly, this approach may not actually be problematic for some applications. Many applications assume the existence of an *aggregate* risk distribution and in those instances, mixing risk exposure horizons clearly leads to resulting measures that are difficult to interpret. However, one could take the view that only the insurance risks need to be aggregated and used to determine the "economic capital" needed today to satisfy current obligations (see Section 2.2 for a discussion of the distinction between economic capital and risk capital, as used here).

The market and credit risk measures do not have to be explicitly aggregated with this longer-term risk measure. Instead, they can be used to reflect a "haircut" to the current asset balances to account for the potential loss in value of the assets that could occur over the course of one year. This recognizes that the firm always has some flexibility to alter its allocation from risky assets to lower risk, or even risk-free, assets at the end of the year (or some other chosen horizon).

In many respects, this is the approach reflected in the current S&P Capital Adequacy Ratio (CAR), which does not attempt to aggregate market, credit and insurance risk. Instead, the S&P CAR merely attempts to compare the economic capital required for the insurance risks to the "adjusted" assets actually held (net of the haircut based on a one-year market VaR and a 10-year measure of expected credit losses).

## Appendix B: Myers-Read Capital Allocation

Introduction

In this Appendix, the calculations used to produce the Myers-Read allocation will be shown in detail. The reader is encouraged to review the Myers-Read paper for the theoretical basis for these calculations and to the papers by Butsic and Venter for insightful discussions of the method as well as simplified calculations.

The Myers-Read methodology is also described in some detail by Cummins in a paper that appears on the CAS examination syllabus. For convenience, the calculations that follow are based on the methodology presented in the Cummins paper.

Default Option

A critical element of the Myers-Read method is the evaluation of the firm's so-called default option, which reflects the value of their right to default (in whole or in part) on their obligations to policyholders.

With fixed liabilities and risky assets, this can be evaluated as simply a put option on the assets of the firm with a strike price equal to the (fixed) value of the liabilities. When the assets are fixed and the liabilities are risky, the default option is more accurately described as a call option on the liabilities with a strike price equal to the assets. And in the most general case, when both assets and liabilities are risky, the default option is technically an option to exchange the assets for the liabilities. This option to exchange the assets for the liabilities is more complicated than a standard put or call option and cannot be easily quantified using the Black-Scholes option pricing formula.

Cummins simplifies this otherwise complex option by characterizing it as a put option on the *asset to liability ratio* rather than an option on either the assets or the liabilities. Specifically, the default option is a standard put option on the ratio of the assets to the liabilities with a strike price equal to 1.0. If the ratio of assets to liabilities is less than 1.0, then the firm is insolvent and the deficit (as a percent of the liabilities) is the amount by which the ratio is below 1.0.

The volatility parameter used in the Black-Scholes put formula therefore represents the volatility of the asset-to-liability ratio. It reflects not just the separate asset and liability standard deviations but also their correlations. In the simplifying case where the assets and liabilities are independent, the volatility of the asset to liability ratio, $\sigma_{V/L}$, is related to the asset volatility, $\sigma_V$, and the liability volatility, $\sigma_L$, by the following equation[47]:

$$\sigma_{V/L}^2 = \sigma_V^2 + \sigma_L^2$$

Myers-Read Allocation

The Myers-Read method estimates the marginal capital for a particular line of business by determining the effect on the default option of a small increase in the size of the line (based on expected loss amount). They begin with the formula for the default option as a function of the expected loss amounts for each line and calculate the partial derivatives with respect to each line. The capital needed for each line is then determined such that each line has the same marginal impact on the firm's overall default option value (as a percentage of the expected losses).

This method is designed to allocate a set amount of capital and so it does not necessarily specify what that total capital amount should be. In the Cummins example, he assumes a 5% EPD Ratio target for the firm overall, which is equivalent to assuming that the default option should be worth 5% of expected aggregate losses.

---

[47] This relationship follows from Ito's Lemma when both the assets and liabilities are assumed to follow geometric Brownian motion.

The resulting formula for the capital to liability ratio, $s_i$, for each line is given as:

$$s_i = s - \left(\frac{\partial p}{\partial s}\right)^{-1}\left(\frac{\partial p}{\partial \sigma}\right)\left[(\sigma_{iL} - \sigma_L^2) - (\sigma_{iV} - \sigma_{LV})\right]\frac{1}{\sigma}$$

In this formula, the $\sigma$ terms with the $V$ subscripts reflect the covariance of the line $i$ losses with the assets and the covariance of the total losses with the assets, respectively. Since the assets are assumed to be independent of the liabilities, both of those terms are zero in the case discussed in this paper. In addition, the σ parameter with no subscripts reflects the overall volatility of the assets to liability ratio and uses the formula shown above. Finally, the term $\sigma_{iL}$ reflects the covariance of line $i$ with the total losses for all lines. Using the expression for the total variance, the formula for this covariance term is follows:

$$\begin{aligned}\sigma_L^2 &= \Sigma\, w_i^2\, \sigma_i^2 + \Sigma\Sigma_{i\gamma j}\, w_i w_j \sigma_{ij} \\ &= \Sigma\Sigma w_i w_j \sigma_i \sigma_j \rho_{ij} \\ &= \Sigma w_i[\Sigma w_j \sigma_i \sigma_j \rho_{ij}] \\ &= \Sigma w_i[\text{Covariance of Line } i \text{ with Total Losses}]\end{aligned}$$

For the two partial derivative terms, these can be derived simply by writing the formula for the default option value as a put option on the asset to liability ratio A/L = (1+$s$) with a strike price of 1.0. The notation is simplified by assuming $r = 0$ and $T = 1$.

$$p = N(-d_2) - (1+s)N(-d_1)$$

where $d_1 = \frac{\ln(1+s)}{\sigma} + \sigma/2$ and $d_2 = d_1 - \sigma$.

From this it is relatively easy to calculate the two partial derivatives needed for the formula for the surplus to liability ratios. The first is found by taking the derivative of $p$ with respect to $s$ but paying attention to the fact that there is an $s$ term in the $d_1$ and $d_2$ terms that makes the derivative a bit more complicated than it first appears. When this is done, the option delta is $\partial p/\partial s = -N(-d_1)$.

Similarly, the option vega is $\partial p/\partial \sigma = N'(-d_2)$.

Surplus to Liability Ratios - Numerical Example

In the paper, the example involved three lines of business (reserve risk, Line A underwriting risk and Line B underwriting risk). The following are the calculations for each of the components of the surplus to liability ratios for these three lines of business.

*Summary of Key Liability Assumptions*

Expected Loss and Volatility Assumptions

| | Expected Liability | CV | Volatility |
|---|---|---|---|
| Reserve | 18,091,233 | 12.7% | 12.6% |
| Line A UW | 5,860,732 | 21.1% | 20.9% |
| Line B UW | 5,860,732 | 31.7% | 30.9% |
| Total | 29,812,697 | 13.6% | 13.5% |

Liability Correlation Assumptions

| | Reserve | Line A UW | Line B UW |
|---|---|---|---|
| Reserve | 1.00 | 0.50 | 0.25 |
| Line A UW | 0.50 | 1.00 | 0.25 |
| Line B UW | 0.25 | 0.25 | 1.00 |

*Covariance of Each Liability Line with the Total Liability Distribution*

Covariance of Each Liability Line with Total Liability

| Line | Covariance with Total |
|---|---|
| Reserve | 0.0141 |
| Line A UW | 0.0198 |
| Line B UW | 0.0279 |
| | |
| Total Liability Volatility Squared | 0.0179 |
| Total Liability Volatility | 0.1340 |

*Volatility of the Asset to Liability Ratio*

Asset to Liability Ratio Volatility

| | |
|---|---|
| Liability Volatility | 0.1340 |
| Asset Volatility | 0.0400 |
| | |
| Asset to Liability Ratio Volatility | 0.1398 |

*Overall Capital to Liability Ratio*

| | |
|---|---|
| Total Capital | 8,949,750 |
| Total Liability | 29,812,697 |
| Capital to Liability Ratio | 0.3002 |

*Calculation of Current Insolvency Put Value and Partial Derivatives*

Default Option Value

| | |
|---|---|
| Spot Price | 1.30 |
| Strike Price | 1.00 |
| Volatility | 0.1398 |
| | |
| d1 | 1.9477 |
| d2 | 1.8079 |
| | |
| N(-d1) | 0.0257 |
| N(-d2) | 0.0353 |
| | |
| Put Value | 0.186% |
| | |
| Delta | -0.0257 |
| Vega | 0.0778 |

*Calculate Capital to Liability Ratios for Each Line*

This calculation uses the formula: $s_i = s - \left(\frac{\partial p}{\partial s}\right)^{-1}\left(\frac{\partial p}{\partial \sigma}\right)\left[(\sigma_{iL} - \sigma_L^2) - (\sigma_{iV} - \sigma_{LV})\right]\frac{1}{\sigma}$

Capital to Liability Ratio

| | |
|---|---|
| Reserve | 21.78% |
| Line A UW | 33.92% |
| Line B UW | 51.57% |

*Determine Capital Allocation by Line*

Capital Allocation - Myers-Read Method (0.186% EPD Ratio)

| | Capital to Loss Ratio | Expected Claims | Capital |
|---|---|---|---|
| Reserve Risk | 21.78% | 18,091,233 | 3,939,466 |
| Line A UW Risk | 33.92% | 5,860,732 | 1,988,079 |
| Line B UW Risk | 51.57% | 5,860,732 | 3,022,205 |
| Total | | | 8,949,750 |

## Bibliography

Artzner, P., Delbaen, F., Eber, J.-M. & Heath, D., 1999, "Coherent Risk Measures", Mathematical Finance 9, 207–225.

Bazerman, M., *Judgment in Managerial Decision Making*

Brealey and Myers, 2006, *Corporate Finance*

Butsic, R. P, 1994, "Solvency Measurement for Property-Liability Risk Based Capital Applications", Journal of Risk and Insurance, Vol 61, 4, 656-690.

Butsic, R., 1993, "Report On Covariance Method For Property-Casualty Risk-Based Capital"

Butsic, Robert, 1999, "Capital Allocation for Property-Liability Insurers: A Catastrophe Reinsurance Application". CAS Forum, Vol: Spring, Page(s): 1-70.

CAS Correlation Working Party, 2004

CAS Fair Value Steering Committee, 2004 "Fair Value of P&C Liabilities: Practical Implications", Casualty Actuarial Society.

CAS Working Party on Quantifying Variability in Reserve Estimates, 2005, "The Analysis and Estimation of Loss & ALAE Variability: A Summary Report"

Conway, Thomas P. and McCluskey, 2006, "Property/Casualty Insurer Economic Capital Using a VaR Model", *2006 SOA/CAS ERM Symposium Call for Papers*.

Crouhy, Turnbull and Wakeman, 1999, "Measuring Risk-Adjusted Performance", Journal of Risk 2(1), 5-35.

Cummins, J. David, 2000, "Allocation of Capital in the Insurance Industry", Risk Management and Insurance Review, Spring, Vol 3, 1, 7-28.

Cummins, J. David and Richard D. Phillips, "Estimating the Cost of Equity Capital for Property-Liability Insurers", *The Journal of Risk and Insurance*, 2005, Vol. 72, No. 3, 441-478.

England, P. D. and R. J. Verrall, 1999, "Analytic and Bootstrap Estimates of Prediction Errors in Claims", *Insurance: Mathematics and Economics* 25, 281-293.

Feldblum, S., 2006, "Fair Value Accounting For Property-Casualty Insurance Liabilities"

Froot, K., and J. Stein, 1998, "Risk Management, Capital Budgeting and Capital Structure Policy for Financial Institutions: An Integrated Approach," Journal of Financial Economics, 47, January 1998: 55-82.

Hardy, M., 2006, "An Introduction to Risk Measures for Actuarial Applications", SOA Syllabus Study Note.

Hayne, R. M., 2003, "Measurement of Reserve Variability", *Casualty Actuarial Society Forum*, 141-171.

Heckman, Philip and Glenn G. Meyers, 1983, "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions" Proceedings of the Casualty Actuarial Society, Vol. LXXX pp 22–61.

Hodes, Feldblum and Blumsohn, "Workers Compensation Reserve Uncertainty", Proceedings of the Casualty Actuarial Society, Volume LXXXVI (p. 263-392).

James, 1996, "RAROC Based Capital Budgeting and Performance Evaluation: A Case Study of Bank Capital Allocation", Wharton Working Paper 96-40.

Klugman, Stuart A., Harry H. Panjer and Gordon E. Willmot. 1998. *Loss Models: From Data to Decisions*, J. Wiley and Sons.

Kreps, Rodney, "Riskiness Leverage Models", Instrat Working Paper.

Mack, T., 1993, "Distribution-Free Calculation of the Standard Error of Chain Ladder Reserve Estimates", ASTIN Bulletin 23, 213-225.

Mack, T., 1999, "The Standard Error of Chain Ladder Reserve Estimates: Recursive Calculation and Inclusion of a Tail Factor", ASTIN Bulletin 29, 361-366.

Mango, D. F. (2003) "Capital Consumption: An Alternative Methodology for Pricing Reinsurance," Casualty Actuarial Society Forum, Winter pp. 351-378.

Mango, D.F., 2005, "Insurance Capital as a Shared Asset", Accepted to *ASTIN Bulletin*.

Merton, R. C. and A. F. Perold, 1993, "Theory of Risk Capital in Financial Firms", Journal of Applied Corporate Finance, 6, 16-32.

Meyers, Klinker and Lalonde, 2003, "The Aggregation and Correlation of Insurance Exposure", CAS Forum, Vol: Summer, Pages: 15-82.

Mildenhall, S., 1997, "Modeling Loss Ratios for Aggregate Features", Unpublished Working Paper

Mildenhall, S., 2004, "A Note on the Myers and Read Capital Allocation Formula", North American Actuarial Journal, Vol 8, No 2, 32-44.

Moody's Investors Services, 2004, "Moody's Risk Adjusted Capital Model For Property & Casualty Insurers"

Myers, S. and J. Read, 2001, "Capital Allocation for Insurance Companies," Journal of Risk and Insurance, 68 (4), pp. 545-580.

Nakada,P., Shah, H., Koyluoglu, H.U. and Collingnon O., "P&C RAROC: A Catalyst for Improved Capital Management in the Property and Casualty Insurance Industry", The Journal of Risk and Finance, Fall 1999.

Panning, William H., 2005, "Measuring Loss Reserve Uncertainty", Willis Re Working Paper.

Panning, William H., 2006, "Managing the Invisible: Measuring Risk, Managing Capital, Maximizing Value", SOA/CAS 2006 ERM Symposium Call for Papers.

Rebonato, R. and J. Pimbley, 2005, "Modelling Economic Capital: Clever Strategy or Dead Alley?", Global Association of Risk Professionals, March/April Issue 23.

Rebonato, R., 1999, "The Most General Methodology To Create A Valid Correlation Matrix For Risk Management And Option Pricing Purposes", Working Paper.

Ruhm, David and Donald Mango, 2003, "A Risk Charge Calculation Based on Conditional Probability", 2003 Bowles Symposium, Atlanta, GA.

Ruhm, David, Donald Mango and Rodney Kreps, 2004, "Applications of the Ruhm-Mango-Kreps Conditional Risk Charge Algorithm", Presented at the 2004 Applied Actuarial Research Conference, University of Central Florida, Orlando, FL.

Shimko, D., 2002, "Project Valuation When Risk Capital is Limited", Unpublished Working Paper.

Stulz, R., 1999, "What's Wrong with Modern Capital Budgeting?" Address Delivered at the Eastern Finance Association Meeting, Miami Beach, FL.

Venter, G., 2003. Capital allocation: An opinionated survey. *Casualty Actuarial Society Forum*, Summer: 279-306.

Venter, G., 2003, "Discussion of 'Capital Allocation for Insurance Companies' by Myers and Read", Vol: Fall Pages: 459-478.

Venter, G., 2006, "Adapting Banking Models to Insurance ERM", SOA/CAS ERM Symposium Call for Papers.

Wilson, T., 1997, "RAROC Remodelled", *RISK*