"Fundamentals of Individual Risk Rating", 1992, Part III

FUNDAMENTALS OF INDIVIDUAL RISK RATING

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The purpose of this study note is to consolidate the basic actuarial concepts of individual risk rating into a single source and, in so doing, to provide standard notation for the formulation and solution of problems. It is intended that the elementary ideas will be identified and explained in a straightforward manner, with sufficient detail so the student can easily follow all steps in the development.

FUNDAMENTALS OF INDIVIDUAL RISK RATING AND RELATED TOPICS

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PART III

Related Topics

A. Deductible and Excess Insurance[11]

Deductible and excess insurance, as the terms are used with respect to liability coverage, differ in several important aspects. Deductible insurance is usually written for relatively small retentions in connection with risks with rather high claim frequency. The insurer is required to settle all losses and to indemnify the insured for losses in excess of the retention. Excess insurance usually refers to coverage written at high retentions for risks who desire to self-insure all except the more costly claims. The insurer is expected to investigate and settle only those losses which might exceed the retention. Thus, a portion of loss adjustment expense is eliminated along with the losses.

Recognizing the essential differences between deductible and excess coverage, a more or less uniform approach can be followed in the determination of the discount. This is accomplished in two steps:

a) Determine the loss elimination ratios for retentions under consideration. Empirical methods were commonly used in the past, but loss distributions are frequently fitted today.

b) Making certain assumptions concerning expenses, determine the discount corresponding to the loss elimination ratio.

1) The Loss Elimination Ratio

The loss elimination ratio is usually determined from a study of a representative group of claims. The most common form of deductible coverage is the straight deductible. When the straight deductible is employed, the amount of the retention is deducted from all losses.

Denote the loss elimination ratio by k and let

N = The total number of claims in the study

n = The number of claims that do not exceed the retention

L = The total losses in the study

L_r = Losses due to claims which do not exceed the retention

r = The retention

$$k = \frac{L_r + (N - n)r}{L} \qquad \text{- - - - - - (1).}$$

The disappearing deductible is an interesting variation of the deductible concept employed in homeowners insurance and other property lines. If an individual claim, λ, is equal to or less than the retention, r, there is no indemnification. However, there is a number $R > r$ such that, for $\lambda \geq R$, indemnification is in full. When $r < \lambda < R$, the amount of indemnification is equal to $(\lambda - r)\left[\frac{R}{R-r}\right]$. The aggregate payment for losses between r and R is $(L_R - rN_R)\left[\frac{R}{R-r}\right]$

where L_R = the total losses in the study between r and R, and

N_R = the number of claims in the study in excess of r but less than R.

The amount saved on losses between r and R is given by

$$L_R - (L_R - rN_R)\left[\frac{R}{R-r}\right]$$

and the loss elimination ratio is given by

$$k = \frac{L_r + L_R - (L_R - rN_R)\left(\frac{R}{R-r}\right)}{L} \qquad \text{- - - - - - (2).}$$

The calculated value cannot be used without modification. The loss elimination ratio cannot be expected to be fully realized. Therefore, a safety factor, denoted by f, is required. The value of the safety factor is set by judgment. The product of the loss elimination ratio and the safety factor, fk, is sometimes known as the tempered loss elimination ratio.

The procedure for determining loss elimination ratios for excess coverage is identical with the procedure employed for deductible coverage.

2) Discount Formula for Deductible Coverage

After k is obtained, certain assumptions regarding expenses must be made before the discount can be determined. The usual assumptions made with respect to deductible coverage for liability insurance are:

a) The provisions for acquisition, taxes and profit vary with premium.

b) The provision for all other expenses are fixed portions of the full coverage premium.

The full coverage premium, P, is given by

$$\boxed{P = EP + nP + (A + T + p)\,P} \quad \text{- - - - - (3)}$$

where

E = Expected loss ratio including allocated claim adjustment expense,

n = The provision for expenses other than acquisition, taxes, profit and allocated adjustment expense,

A = The provision for acquisition,

T = The provision for taxes, and

p = The provision for profit.

Define a as the provision for allocated adjustment expenses and e as the provision for expenses other than acquisition, taxes and profit. Then $e = n + a$, and formula (3) can be rewritten as

$$P = EP + nP - aP + aP + (A+T+p)P$$

$$= (E - a)P + (n + a)P + (A + T + p)P$$

$$\boxed{P = (E - a)P + eP + (A + T + p)P} \quad \text{- - - - - (4).}$$

Then $$\boxed{P = \frac{(E - a)P + eP}{1 - A - T - p}} \quad \text{- - - - - (5).}$$

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Let P' be the premium for the deductible coverage. Clearly, the discount, D, is determined by

$$D = 1 - \frac{P'}{P}$$

and

$$\boxed{P' = (1 - fk)(E - a)P + eP + (A + T + p)P'} \quad \text{- - - - - - (6).}$$

$$\boxed{P' = \frac{(1 - fk)(E - a)P + eP}{1 - A - T - p}} \quad \text{- - - - - - (7).}$$

Using Formula (5),

$$\frac{P'}{P} = \frac{(1 - fk)(E - a)P + eP}{(E - a)P + eP}$$

$$D = 1 - \frac{P'}{P} = 1 - \frac{(1 - fk)(E - a)P + eP}{(E - a)P + eP}$$

$$= \frac{(E - a)P + eP - (E - a)P + fk(E - a)P - eP}{(E - a)P + eP}$$

$$= \frac{fk\,(E - a)P}{(E - a)P + eP}$$

From formula (4),

$$eP = P\,(1 - A - T - p) - (E - a)P.$$

Therefore,

$$D = \frac{fk\,(E - a)P}{(E-a)\,P + P(1-A-T-p) - (E-a)\,P} = \frac{fk\,(E - a)\,P}{P(1-A-T-p)}$$

$$\boxed{D = \frac{fk\,(E - a)}{1 - A - T - p}} \quad \text{- - - - - -(8).}$$

3) Discount Formulas for Excess Coverage

Slightly different expense assumptions usually apply in the case of excess coverage.

a) Case I

1. A portion of allocated claim adjustment expense is eliminated which is proportional to the losses eliminated.

2. Acquisition, taxes, profit, inspection expense, unallocated claim adjustment expense and a portion of home office administration expense vary with premiums.

3. The provision for all other expenses, *e*, is a fixed portion of the full coverage premium. (Notice that this *e* has a slightly different meaning from that in the previous section on Deductible Coverage.)

4. *E* retains its meaning as the provision for losses and ALAE.

$$\boxed{P = EP + eP + (A + T + p + i + u + gh)\,P} \quad \text{- - - - - (9)}$$

$$\boxed{P = \frac{EP + eP}{1 - A - T - p - i - u - gh}} \quad \text{- - - - - - (10)}$$

where i = the provision for inspection expense,

u = the provision for unallocated claim expense,

h = the provision for home office administration expense, and

g = the portion of h that varies with premium.

$$P' = (1 - fk)\, EP + eP + (A + T + p + i + u + gh)P' \quad \text{---(11)}$$

$$P' = \frac{(1 - fk)\, EP + eP}{1 - A - T - p - i - u - gh} \quad \text{- - - - -(12)}$$

$$\frac{P'}{P} = \frac{(1 - fk)\, EP + eP}{EP + eP}, \text{ using (10) and (12)}$$

$$D = 1 - \frac{P'}{P} = 1 - \frac{(1 - fk)\, EP + eP}{EP + eP}$$

$$= \frac{\cancel{EP} + \cancel{eP} - \cancel{EP} + fk\, EP - \cancel{eP}}{EP + eP} = \frac{fk\, EP}{EP + eP}$$

From formula (9),

$$eP = P\,(1 - A - T - p - i - u - gh) - EP.$$

Therefore,

$$D = \frac{fk\, EP}{\cancel{EP} + P\,(1 - A - T - p - i - u - gh) - \cancel{EP}} = \frac{fk\, E\cancel{P}}{\cancel{P}\,(1 - A - T - p - i - u - gh)}$$

$$D = \frac{fk\, E}{1 - A - T - p - i - u - gh} \quad \text{- - - - - -(13).}$$

b) <u>Case II</u>

1. A portion of allocated adjustment expense is eliminated which is proportional to the losses eliminated.
2. Acquisition, taxes and profit vary with the premium.
3. Inspection, unallocated claim expense and a portion of home office administration expense vary with excess coverage losses and allocated adjustment expense instead of premium.

4. The provision for all other expenses, e, is a fixed portion of the full coverage premium.

Let i_E represent the provision for inspection expense as a percentage of losses and ALAE. Similarly, let u_E and h_E represent the provisions for ULAE and home office administrative expense, respectively, as percentages of losses and ALAE. Let g be the portion of home office administrative expense which varies with losses and ALAE. The symbols E, A, T and p retain their meanings from Case I.

Once again, we write the appropriate formulas for P and P′:

$$\boxed{P = EP + (A + T + p)P + (i_E + u_E + gh_E)EP + eP} \quad \ldots\ldots\ldots \quad (14)$$

$$P' = (1 - fk)EP + (A + T + p)P' + (i_E + u_E + gh_E)(1 - fk)EP + eP$$

Then

$$\frac{P'}{P} = (1 - fk)E + (A + T + p)\frac{P'}{P} + (i_E + u_E + gh_E)(1 - fk)E + e$$

and so,

$$\frac{P'}{P} = \frac{(1 - fk)E + (i_E + u_E + gh_E)(1 - fk)E + e}{1 - A - T - p}$$

$$= \frac{(1 - fk)\ E\ (1 + i_E + u_E + gh_E) + e}{1 - A - T - p}$$

$$D = 1 - \frac{P'}{P} = 1 - \frac{(1 - fk)(1 + i_E + u_E + gh_E)E + e}{1 - A - T - p}$$

$$= \frac{(1 - A - T - p - (1 + i_E + u_E + gh_E)E - e) + fk(1 + i_E + u_E + gh_E)E}{1 - A - T - p}$$

From (14), $(A + T + p)P + (1+i_E+u_E+gh_E)EP + eP = P$, so

$$\boxed{D = \frac{fkE(1 + i_E + u_E + gh_E)}{1 - A - T - p}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

4) Higher Optional Deductibles

In the 1980s and '90s, interest in higher deductible coverage has grown considerably. Among the reasons for this are the following:

a) Trend toward self-insurance, with its promise of savings to the insured, who hopes to keep profits and expenses that would otherwise go to the insurer.

b) Tax savings which may inure to the employer who can deduct a liability for the insurance deductible on an unpaid insured claim, but may be unable to deduct a loss reserve on a retained claim.

c) Positive cash flow to the insurer, who may not have to pay excess claims for years after the policy is written.

d) In lines with residual market pools, reduction of assessments to the carrier, whose net written premium will be that for the excess only.

Loss elimination ratios for such deductibles have been calculated using data underlying the calculation of ELFs.

B. Workers Compensation Ex-Medical Coverage[12]

The procedure used to obtain the ex-medical discount (the terminology of the manual is ex-medical ratio) is strikingly similar to the one followed for deductible and excess insurance. The only exceptions are that the loss elimination ratio (which usually has a high value) applies only to the medical pure premium and that profit is considered to be a fixed portion of the full coverage premium.

1) Basic Assumptions:

a. The ex-medical pure premium is equal to the total pure premium less a portion of the medical pure premium.

b. Expenses other than acquisition and taxes are not reduced. Acquisition and taxes are a function of premium.

Initially, it might be thought that the entire medical pure premium should be eliminated. However, the following considerations indicate that a small part of it should be retained.

a. Selection of ex-medical coverage will likely be adverse to the insurer.

b. Payment of certain medical costs may be required even though the policy may be ex-medical.

c. The insurer retains an obligation to pay medical loss in the event the employer is unable to pay.

2) The Ex-Medical Discount

These considerations lead to the derivation of the ex-medical discount. Let P denote the full coverage premium.

$$\boxed{P = E + eP + (A + T)P} \qquad (16)$$

$$\boxed{P = \frac{E + eP}{1 - A - T}} \quad \text{(17)}$$

where E = expected losses or total pure premium, not including ALAE.

e = provision for expenses other than acquisition and taxes.

A = provision for acquisition.

T = provision for taxes.

Let P' denote the ex-medical premium:

$$\boxed{P' = E - kE_M + eP + (A + T)P'} \quad \text{(18)}$$

$$\boxed{P' = \frac{E - kE_M + eP}{1 - A - T}} \quad \text{(19)}$$

where E_M = the medical pure premium.

k = the portion of medical pure premium eliminated, which is determined by judgment.

$$\frac{P'}{P} = \frac{E - kE_M + eP}{E + eP}$$

Denote the discount by D:

$$D = 1 - \frac{P'}{P} = 1 - \frac{E - kE_M + eP}{E + eP}$$

$$= \frac{E + eP - E + kE_M - eP}{E + eP} = \frac{kE_M}{E + eP}$$

From formula (15),

$$eP = P\,(1 - A - T) - E.$$

Therefore,

$$D = \frac{kE_M}{E + P(1 - A - T) - E} = \frac{kE_M}{P(1 - A - T)}$$

Also from formula (15),

$$P = \frac{E}{1 - A - T - e}$$

Therefore, $D = \dfrac{kE_M}{\dfrac{E}{1-A-T-e}(1-A-T)}$

$$\boxed{D = \frac{1 - A - T - e}{1 - A - T} \cdot \frac{kE_M}{E}} \quad \text{- - - - - (19).}$$

3) Adjustments Under Retrospective Rating

When ex-medical coverage is rated retrospectively, certain adjustments are indicated (although seldom made). For example, the loss conversion factor might also require adjustment if the dollars of expense provided by the ex-medical loss conversion factor are to equal the dollars of expense provided by the statutory loss conversion factor. Let $J' = c' - 1$ and $J = c - 1$, where c is the usual loss conversion factor.

Then,

$$J' = J \cdot \frac{E}{E - kE_M} ;$$

From formula (19),

$$D = \frac{1 - A - T - e}{1 - A - T} \cdot \frac{kE_M}{E}$$

$$kE_M = D \cdot E \cdot \frac{1 - A - T}{1 - A - T - e}$$

Substituting,

$$J' = J \cdot \frac{E}{E - DE \frac{1-A-T}{1-A-T-e}}$$

$$= \frac{J}{1 - D \cdot \frac{1-A-T}{1-A-T-e}}$$

$$= \frac{J(1 - A - T - e)}{1 - A - T - e - D(1 - A - T)}$$

The adjusted loss conversion factor for the ex-medical retro plan is $c' = 1 + J'$, where

$$\boxed{J' = \frac{J(1 - A - T - e)}{(1 - D)(1 - A - T) - e}} \quad \text{- - - - - - (20)}$$

ACKNOWLEDGMENT

This study note is a living document. It was given birth by Richard Snader, who still deserves the most credit for making sense out of a disparate group of source materials.

Mr. Snader, in turn, gives credit to "giants of our profession," particularly the authors of the footnoted documents, as well as John R. Bevan, who contributed new material on Retrospective Rating.

I have attempted to enhance the vitality of this study note by making improvements and updates as indicated. This could not have been done without help, and I wish to acknowledge that of several people, notably word processor par-excellence Dariene Browning, editor Robert A. Bear, critic Eugene McGovern, reviewer Paul Martin, PC specialist David Michael and educator Gary Venter.

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