Fundamentals of Individual Risk Rating, 1992, Part I

Part I

Experience Rating

This study note describes the algebraic development of the 1940 and 1961 Workers Compensation rating plans. The revised plan introduced by the National Council on Compensation Insurance in 1989, generally effective 1991, is included, but covered more thoroughly elsewhere.

A. <u>No Split Experience Rating</u>[1]

Experience rating plans can be classified as split plans or no split plans. In the split plans each loss is divided into primary and excess elements. The primary element reflects loss frequency and is given major weight in the experience rating formula. The excess element reflects loss severity and is given minor weight in the experience rating formula.

Split plans can be further classified as single split and multi-split plans. In single split plans, there is a single split point for dividing losses into primary and excess elements. All amounts of loss below the split point are considered to be primary. All amounts of loss above the split point are excess. In multi-split plans, losses may be thought of as being divided into increments, and a decreasing portion of each successive increment is considered to be primary.

No split plans are the simplest form of experience rating. No attempt is made to divide losses into primary and excess elements. The <u>basic formula</u> for experience rating applies without modification to the no split plans. The formulas for the split plans are derived from the basic formula.

1) The Basic Formula for experience rating is

$$\boxed{M = \frac{ZA + (1 - Z)E}{E}} \quad - - - - - (1),$$

where $M =$ the experience modification.

$A =$ the actual or rated losses for the risk.

$E =$ the expected losses for the risk.

$Z =$ credibility.

2) Analysis and Interpretation

Following a simple transformation, the basic formula easily lends itself to verbal interpretation.

$$M = \frac{ZA + (1 - Z)\,E}{E} = \frac{ZA + E - ZE}{E}$$

$$\boxed{M = 1 + Z\,\frac{(A - E)}{E}} \quad \text{- - - - - (2).}$$

The first term of M is unity, which represents the standard rate. The second term provides a charge if the actual losses A during the experience period exceed the expected E, or a credit if A is less than E. The charge for a single loss is moderated by the factor Z/E.

3) A special adaptation, The Surety Association Plans:

$$M = \frac{ZA + (1 - Z)E}{E}$$

$$= 1 - Z + Z\,\frac{A}{E}$$

$$= 1 - Z + Z\,\frac{A/P}{E/P}$$

where $P =$ the standard premium.

$$\boxed{M = (1 - Z) + \frac{Z}{E/P} \cdot \frac{A}{P}} \text{ - - - - - } (3).$$

If, in formula (3), $(1 - Z)$ is called the premium modifier, $\frac{A}{P}$ the adjusted loss ratio and $\frac{Z}{E/P}$ the adjusted loss multiplier, the formula is then:

Modification = Premium Modifier + Adjusted Loss Multiplier x Adjusted Loss Ratio

which is the peculiar guise of the formula for the Surety Association's experience rating plans for Mercantile Establishments and Financial Institutions.

4) Credibility

The fundamental expression for credibility is

$$Z = \frac{E}{E + K_E}$$

In the fundamental expression, K_E is a function of E. In the simplest case, K_E is a constant K. In olden days, the value of K was determined from the swing desired in the plan: the amount of credit for clear experience, or alternatively, the charge for a single maximum loss, offered to a small risk.

5) Credibility Criteria

In order to determine that the credibility system operates effectively, the credibility function should satisfy the following conditions:

a) Credibility is not less than zero nor greater than unity.

b) Credibility increases (or at least does not decrease) as the size of risk increases.

c) As the size of risk increases, the percentage charge for any loss of a given size decreases.

Mathematically, the conditions are:

$$\boxed{\begin{array}{l} a.\ \ 0 \leq Z \leq 1 \\[1em] b.\ \ \dfrac{d}{dE}(Z) \geq 0 \\[1em] c.\ \ \dfrac{d}{dE}\left[\dfrac{Z}{E}\right] < 0 \end{array}} \quad \text{- - - - - - (4).}$$

As an exercise, the student should show these conditions hold if

$$Z = \frac{E}{E + K}$$

and K is a constant.

6) The K Formula for Experience Rating

The K formula uses the fundamental credibility expression to eliminate the Z term.

$$M = \frac{ZA + (1 - Z)E}{E}$$

$$M = \frac{1}{E}\left[\left(\frac{E}{E + K}\right) A + \left(1 - \left(\frac{E}{E+K}\right)\right) \cdot E\right]$$

$$= \frac{A}{E + K} + \frac{K}{E + K}$$

$$\boxed{M = \frac{A + K}{E + K}} \quad \text{- - - - - - (5).}$$

B. <u>Single Split and Multi-Split Experience Rating</u>

Basic Formula (1) does not work well in Workers Compensation. This is because the loss size distribution is heavy tailed. Venter[2] discusses the problems in application of linear credibility to estimating long tailed distributions. Solutions to this problem include such non-linear transformations of loss as the split of individual claims into primary and excess components. This was a natural choice in compensation, where there are several types of claims, including a group of small, mostly medical-only, claims and another group of larger claims with an indemnity component. Both types of claims can indicate individual risk characteristics, but must be treated differently in the formula.

Historic variants of the split formula are described below in Section E.

1) The Split Plan Formula

The formula for split experience rating plans is an elaboration of formula (1).

$$\boxed{M = \frac{1}{E}\left[Z_p A_p + (1 - Z_p)E_p + Z_e A_e + (1 - Z_e)E_e\right]} \quad \text{- - - (6).}$$

where A_p = Primary actual losses.

E_p = Primary expected losses.

Z_p = Primary credibility.

A_e = Excess actual losses.

E_e = Excess expected losses.

Z_e = Excess credibility.

2) Analysis and Interpretation

$$M = \frac{1}{E}\left[Z_pA_p + (1 - Z_p)E_p + Z_eA_e + (1 - Z_e)E_e\right]$$

$$= \frac{1}{E}\left[Z_pA_p + E_p - Z_pE_p + Z_eA_e + E_e - Z_eE_e\right]$$

$$= \frac{1}{E}\left[(E_p + E_e) + Z_pA_p - Z_pE_p + Z_eA_e - Z_eE_e\right]$$

But $E_p + E_e = E$.

$$\boxed{M = 1 + Z_p\frac{(A_p - E_p)}{E} + Z_e\frac{(A_e - E_e)}{E}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(7).}$$

The verbal interpretation of formula (7) is similar to the interpretation of formula (2).

The analysis of the loss free modification is also extremely useful.

$$\boxed{\bar{M} = 1 - Z_p\frac{E_p}{E} - Z_e\frac{E_e}{E}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(8).}$$

$\bar{M}$ denotes the loss free modification.

If $Z_p = \frac{E}{E+K_E}$ *and* $Z_e = \frac{E}{E+J_E}$, formula (7) an be written in the form originally tested pursuant to the 1991 revisions of the Workers Compensation plan.

$$\boxed{M = 1 + \frac{A_p-E_p}{E+K_E} + \frac{A_e-E_e}{E+J_E}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(9).}$$

C) Perryman's First Formula

A working formula similar to formula (5) was needed for the split plans. Perryman derived such a formula using the fundamental expression for credibility, but without direct evaluation of the credibility constants in the formulas $Z_p = \frac{E}{E+K_E}$ and $Z_e = \frac{E}{E+J_E}$.

1) The Three Cases

He assumed that there existed numbers Q and S such that:

Case I: for $0 < E \leq Q$, $0 < Z_p \leq 1$, $Z_e = 0$;

Case II: for $E \geq S$, $Z_p = 1$, and $Z_e = 1$;

Case III: for $Q < E < S$, $0 < Z_e < Z_p < 1$.

S is the self-rating point which would vary by state. It is the value of E at which a risk's experience was accepted, without modification, as the measure of the correct rate. Q is the value of E below which a risk's excess losses were not used in the rating. Below the Q point, only the primary loss experience of the risk was allowed to modify the rate and only to the extent determined by Z_p. Above the Q point, excess losses were also allowed to modify the rate to the extent determined by Z_e.

Formulas for the credibility constants K_E and J_E were determined by the three cases. It turns out that:

Case I: for $E \leq Q$, $K_E = K$, and $J_E = \infty$;

Case II: for $E \geq S$, $K_E = J_E = 0$;

Case III: for $Q < E < S$ it may be shown that

$$K_E = K(S-E)/(S-Q)$$

$$J_E = -E + \frac{[E\ (S-Q-K)\ + KS]}{(E\ -\ Q)}$$

It is instructive to follow the development of the formulas for the three cases.

Case I: $0 < E \leq Q, 0 < Z_p \leq 1, Z_e = 0$:

$$M = \frac{1}{E} [Z_p A_p + (1 - Z_p) E_p + E_e]$$

$$= \frac{1}{E} [(\frac{E}{E + K}) A_p + (1 - \frac{E}{E + K}) E_p + E_e]$$

$$= \frac{1}{E} [\frac{E}{(E + K)} A_p + \frac{K}{(E + K)} E_p + \frac{E_e (E + K)}{(E + K)}]$$

$$= \frac{1}{E} [\frac{A_p E}{E + K} + \frac{E_p K}{E + K} + \frac{E_e E + E_e K}{E + K}]$$

$$= \frac{1}{E} [\frac{A_p E + (E_p + E_e) K + E_e E}{E + K}]$$

$$= \frac{1}{E} [\frac{A_p E + E_e E + KE}{E + K}]$$

$$\boxed{M = \frac{A_p + E_e + K}{E + K}} \quad \text{- - - - - -} (10)$$

Case II: $E \geq S, Z_p = 1, Z_e = 1$:

$$M = \frac{A}{E}$$

$$= \frac{A_p + E_e + K + A_e - E_e - K}{E + K - K} \quad \text{(anticipating developments below)}$$

Case III: $Q < E < S,\ 0 < Z_e < Z_p < 1.$

Before attempting to develop a formula for Case III, we might note that the numerator of the expanded Case II formula is equal to the numerator of the Case I formula plus some additional elements.

$$A = \boxed{A_p + E_e + K} \boxed{+ A_e - E_e - K}$$

Similarly, the denominator of the expanded Case II formula is equal to the denominator of the Case I formula plus an additional element.

$$E = \boxed{E + K} \boxed{- K}$$

The entire expanded version of the Case II formula might look like this:

$$M = \boxed{\frac{A_p + E_e + K}{E + K}} \boxed{\frac{+ A_e - E_e - K}{- K}}$$

When Case I conditions apply ($E \leq Q$), no additional elements are added to the Case I formula. When Case II conditions apply ($E \geq S$), the additional elements are added to the numerator and denominator of the Case I formula. It seems reasonable to assume that when Case III conditions apply ($Q < E < S$), a portion of the additional elements should be added to the numerator and denominator of the Case I formula. The portion that should be added is determined by a weighting factor, W, which is a linear function of E when $Q < E < S$.

$$\boxed{W = \frac{E-Q}{S-Q}} \quad \text{. . . . .} \quad (11)$$

The Case III formula becomes

$$M = \frac{A_p + E_e + K + W(A_e - E_e - K)}{E + K - WK}$$

$$= \frac{A_p + WA_e + E_e - WE_e + K - WK}{E + K - WK}$$

$$\boxed{M = \frac{A_p + WA_e + (1 - W)E_e + (1 - W)K}{E + (1 - W)K}} \qquad (12)$$

The Case III formula is the working formula in the 1961 Workmen's Compensation Experience Rating Plan. The Case I and Case II formulas are simply special cases of the more general working formula. The manual formula is simplified by the introduction of

$$B = (1 - W)K.$$

$$\boxed{M = \frac{A_p + WA_e + (1 - W)E_e + B}{E + B}} \qquad (13)$$

2) Credibilities for Perryman's First Formula

Formula (12) is Perryman's First Formula. It was not derived directly from credibility principles, but it is useful to back into expressions for Z_p and Z_e in terms of W and E. This can easily be done using the loss free modification as a starting point.

$$\bar{M} = \frac{(1 - W)E_e + (1 - W)K}{E + (1 - W)K} = \frac{(1 - W)(E - E_p) + (1 - W)K}{E + (1 - W)K}$$

$$= \frac{E - WE - E_p + WE_p + (1 - W)K}{E + (1 - W)K}$$

$$= \frac{E + (1 - W)K}{E + (1 - W)K} - \frac{E_p}{E + (1 - W)K} - \frac{W(E - E_p)}{E + (1 - W)K}$$

$$= 1 - \frac{E_p}{E + (1 - W)K} - \frac{WE_e}{E + (1 - W)K}$$

But, by formula (8),

$$\bar{M} = 1 - Z_p\frac{E_p}{E} - Z_e\frac{E_e}{E}$$

By equating terms, $Z_p \frac{E_p}{E} = \frac{E_p}{E + (1 - W)K}$

$$\boxed{Z_p = \frac{E}{E + (1 - W)K}} \quad \text{(14).}$$

Similarly,

$$Z_e\frac{E_e}{E} = \frac{WE_e}{E + (1 - W)K}$$

$$\boxed{Z_e = \frac{WE}{E + (1 - W)K} = WZ_p} \quad \text{(15).}$$

D. <u>The Revised Experience Rating Plan - 1991</u>[3]

The 1991 Workers Compensation Experience Rating Plan is based on a more rigorously correct application of risk theory to individual risk credibility. The expression for credibility may be written:

$$Z = \frac{E+I}{JE+I+K} \quad \text{.............................} \quad (16)$$

This form recognizes both parameter uncertainty and risk heterogeneity in the variance of (conditional) loss ratios, while allowing K, I and J to be constant coefficients.

The simple algebra to transform (16) into the Fundamental Expression for credibility is left to the student. This allows the use of the modification formulas such as (9) or (13) and credibility ballast values K_E, J_E or B to be listed as tabular function of E.

It follows that

$$K_E = E\left[\frac{CE+D}{E+F}\right] \quad \text{.........................} \quad (17)$$

where C, D, and F are constant coefficients. We use K_E for primary credibility and a similar formula for J_E in excess credibility. Principles used to evaluate the coefficients are described in Venter[4] and details of performance testing used to select the final value are described in Gillam[5].

The primary and excess credibilities of the split formula, (6), are indexed to state relative benefit levels, using a scale constant G, in the formulas for D and F. G is 0.001 x (SACC), where SACC is the state average cost per case.

$$K_E = E\left[\frac{0.1E + 2570G}{E + 700G}\right] \text{ subject to a minimum of 7500}$$

$$J_E = E\left[\frac{0.75E + 203825G}{E + 5100G}\right] \text{ subject to a minimum of 150,000}$$

The changes in credibility pursuant to the Revised plan are analyzed in Mahler[3] and noted below.

1) For small risks, primary credibility is larger.

2) For large risks, primary credibility is smaller. The maximum primary credibility is 91%, rather than 100% as under the prior plan; as such, there is no more self-rating.

3) For small risks, excess credibility is a little larger. Even the smallest risks have non-zero excess credibility, where they had none previously.

4) For large risks, excess credibility is much smaller, with a maximum of 57%, rather than the 100% maximum of the prior plan. Because of this change, primary losses are effectively given more weight in the formula.

Using W as the ratio of excess to primary credibility,

$$\boxed{W = \frac{E + K_E}{E + J_E}} \qquad (18)$$

and letting

$$\boxed{B = K_E} \qquad (19)$$

the modification formula (13) is algebraically equivalent to (9). The student should check this. These B and W formulas represent a significant change from those in Perryman's first formula.

E. Primary Value Formulas

1) In a single split plan, losses are split at a single value I, below which losses are all primary.

$$\boxed{\begin{aligned} A_p &= A \text{ if } A < I \\ &= I \text{ if } A \geq I \\ A_e &= \hat{A} - A_p \end{aligned}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

The Revised Experience Rating Plan (1991) uses this split, with I = 5000, and Â is A subject to a maximum ratable value.

2) The 1940 Multi-Split Plan

Using the method applicable to the 1940 plan, each loss was divided into increments of $500. The first increment was considered all primary, 2/3 of the second increment was considered primary, $4/9 = (2/3)^2$ of the third increment was considered primary, etc. A generalized formula can be developed. Let A be the total loss and A_p be the primary loss.

Let N, I, and A be such that

$$NI \leq A < (N+1)I$$

Then

$$\boxed{\begin{aligned} A_p &= I + (1-d)I + (1-d)^2 I + \ldots + (1-d)^{N-1} I + (1-d)^N (A - NI) \text{ and} \\ A_e &= A - A_p \end{aligned}} \quad \ldots \quad (21)$$

where I = width of each increment

d = the rate of discount.

The maximum value of A_p is given by

$$\text{Max } A_p = I \cdot \frac{1}{d}$$

which is also the formula for the present value of a perpetuity due.

Given I = 500 and d = ¼, the reader should verify the entries in the following table:

A	A_p	A_e
400	400	0
500	500	0
600	575	25
1,000	875	125
1,400	1,100	300

3) The 1961 Plan

The 1961 plan included a revision of the method for determining the primary portion of losses. The Workmen's Compensation Experience Rating Plan was then a multi-split plan, and a simpler formula was needed to determine the primary portion of each loss.

The expression for A_p can be subjected to a further generalization,

$$A_p = I + a_1 r_1 + a_2 r_2 + \ldots$$

where a_i is an increment of A and r_i can have any positive value less than one (although, logically, $r_i > r_{i+1}$). If r_i is thought of as the discounting ratio for a_i,

$$A_p = I + (A - I)r$$

Here r is the average discounting ratio,

$$r = \frac{A_p - I}{A - I} = \frac{I + a_1 r_1 + a_2 r_2 + \ldots - I}{A - I}$$

$$r = \frac{a_1 r_1 + a_2 r_2 + \ldots}{A - I}$$

The exact values a_i and r_i are of little concern if a practical expression for A_p can be found. The primary value should approach a maximum as A increases and should equal A when $A = I$. Such an expression is:

$$A_p = \frac{A}{A + C} Max(A_p) \quad , C \text{ is a constant.}$$

Note that $\frac{A}{A + C} Max\ (A_p) \rightarrow Max\ (A_p)\ as\ A \rightarrow \infty$.

If we require $A_p = A$ when $A = I$, then

$$I = \frac{I}{I + C} Max\ (A_p)$$

$$Max(A_p) = \frac{I(I + C)}{I}$$

$$Max\ A_p = I + C$$

$$\boxed{A_p = \frac{A}{A + C} (I + C)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

If \$8,000 is chosen as the value of C and \$2,000 is chosen as the value of I, formula (22) becomes the familiar

$$A_p = \frac{A}{A + 8{,}000} (10{,}000)\ if\ A \geq 2{,}000$$

$$A_p = A\ if\ A < 2{,}000.$$

$A_e = \hat{A} - A_p$, where Â is A subject to a maximum ratable value