Insurance to Value[0]

by Gwendolyn Anderson, ACAS

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Coinsurance: Definitions

Coinsurance is insurance provided jointly with another or others.[i] In primary property insurance, coinsurance is an arrangement by which the insurer and the insured share, in a specific ratio, payment for losses covered by the policy, after the deductible is met.[ii] Under a coinsurance arrangement, the person insured by the primary policy is regarded as a joint insurer and becomes jointly and proportionately responsible for losses. A more specific definition of property coinsurance will be given once a few preliminary concepts are described. Formulas and numerical examples follow the definitions.

While the concept of coinsurance is simple to grasp, the actual term "coinsurance" may be confusing because of the wide range of forms the coinsurance arrangement assumes. The function of coinsurance may differ substantially between primary insurance and reinsurance, and also among various lines of business. Although the term typically applies to an insured, when multiple insurance policies cover a single loss, the insurance companies who share in the liability may also be referred to as "coinsurers." In the study of primary property insurance, it is important that the type of coinsurance arrangement specific to this line and layer not be confused with other forms in existence.

Coinsurance in Health Insurance

Perhaps the most familiar coinsurance arrangement to the public at large exists in the standard health insurance policy. In a health insurance policy, coinsurance is a form of deductible. The coinsurance percentage dictates the fraction of covered medical expenses the insured must pay, after having met a flat deductible.

Example 1: Medical Coinsurance

A health insurance policy has a $500 deductible and a 20% coinsurance clause. If the insured incurs $1,000 of covered medical expenses in the policy year, what amount does the insurer pay?

The insurer pays (100% - 20%) × ($1,000 - $500) = 80% × $500 = $400.

Medical coinsurance is typically complicated by a 'stop loss,' a maximum amount the insured will pay for medical expenses within a policy period.

Example 2: Medical Coinsurance with Stop Loss

A health insurance policy has a $500 deductible, a 20% coinsurance clause, and a $2,000 stop loss. If the insured incurs $15,000 of covered medical expenses in the policy year, what amount does the insurer pay?

The stop loss will come into effect after the insured has paid $2,000 out of pocket, which equals the $500 deductible plus $1,500 coinsurance, or 20% of $7,500. The insurer pays all expenses beyond the stop loss.

The insurer pays		[(100% - 20%) × ($7,500)] + [(100%) × ($15,000 - $7,500 - $500)]
	=	(80%) ($7,500) + (100%) ($7,000)
	=	$13,000
	=	$15,000 minus the $2,000 stop loss.

The purpose of medical coinsurance is to provide a monetary form of disincentive for the insured to incur frivolous medical expenditures. The flat deductible strengthens this effect. The purpose of the stop loss is to prevent catastrophic financial consequences in a major health crisis.

The incentives placed on the insured by coinsurance in property insurance are substantially different than those in health insurance. And beyond its incentive role, property coinsurance is critical to maintaining balance in actuarial rating equations.

Coinsurance in Property Insurance

Property coinsurance is somewhat less familiar to the public than medical coinsurance. First, not everyone has property to insure. Second, not all insurance companies employ coinsurance arrangements. Third, where the coinsurance clause does exist, it will play absolutely no role for policyholders who maintain a level of coverage equal to, or nearly equal to, the value of their property.

In a standard *health* insurance agreement, the policyholder is a joint insurer up to the level of the stop loss. In a standard *property* insurance agreement, the policyholder is regarded as a joint insurer only when insuring property for less than the required portion of its full value; only then does the insured become jointly and proportionately responsible for losses. For policyholders who do not wish to self-insure, the property coinsurance clause will clearly induce them to maintain adequate coverage against insured perils.

Property coinsurance obligates the insured to keep a specific amount of insurance in force on the insured property, or else face penalties in the event of loss. The required level of insurance may be a stated amount or a percentage of the property value. In the event the insured purchases a policy with a face value equal to or greater than the required amount, coinsurance does not play any role in calculating indemnity on insured losses, and a covered loss will be fully insured beyond the deductible. If the insured carries less than the required level of insurance on the property, then the extent coinsurance applies will be proportional to the degree to which the insured falls short of the requirement at the time of loss.

While the coinsurance clause contractually places the burden of maintaining adequate coverage onto the policyholder, the insurance company employing the clause still carries a certain legal responsibility for properly explaining policy terms and advising the insurance consumer on appropriate levels of coverage. Lawsuits in the past have held insurance companies liable for damages beyond the face values of their policies. In instances where insurers are forced to pay greater amounts than defined within the policies, they will undoubtedly not be permitted to collect back premiums on all policies similarly underinsured to compensate. For such reasons, it is important for insurance companies to consistently ensure that their policyholders are maintaining an adequate level of coverage. Maintaining insurance to value defines the goal of maintaining coverage at the level assumed within the actuarial premium rate calculations.

In primary property insurance, <u>coinsurance</u> may be specifically defined as an apportionment of losses between an insurer and its insured, such that the insurer pays a fraction of each insured loss equal to the coinsurance apportionment ratio. The coinsurance apportionment ratio applies after deductibles and other policy restrictions have been met.

The <u>coinsurance apportionment ratio</u> is the ratio of the designated amount of insurance (purchased by the property owner) to either (i) a stated sum, or (ii) a specified percentage of the value of the insured property.

The maximum coinsurance apportionment ratio is *one*. The designated insurance (represented in the numerator of the coinsurance apportionment ratio) may be:

(1) the face amount of the policy requiring insurance,
(2) the total face amounts of the insured's applicable policies, or
(3) under provisional reporting form policies[1], full insurance on the property values last reported before a loss.

A <u>coinsurance clause</u> is any policy provision that establishes a coinsurance arrangement.

The <u>coinsurance requirement</u> is the least amount of insurance for which the coinsurance apportionment ratio will equal *one* in a given coinsurance clause. In keeping with the format of the coinsurance apportionment ratio, the coinsurance requirement may be either (i) a stated sum, or (ii) a specified percentage of the value of the insured property. Under a provisional reporting form policy, the requirement may equal the true aggregate value of all property at the date of the latest report. As stated previously, when the coinsurance requirement is met, the coinsurance clause will not reduce indemnity for any loss.

A <u>coinsurance deficiency</u> is the amount by which a coinsurance requirement exceeds the carried insurance presently applicable to the coinsurance requirement.

A <u>coinsurance penalty</u> is the amount, greater than zero, by which an indemnity payment for a loss is reduced by the operation of a coinsurance clause. If no loss occurs when a coinsurance deficiency exists, no coinsurance penalty arises.

A <u>coinsurer</u> is an insured that fails to meet a coinsurance requirement, and is thus exposed to possible coinsurance penalties. A coinsurer provides a higher level of 'self-insurance' than an insured that meets the coinsurance requirement. A loss need not occur for an insured with a coinsurance deficiency to be a coinsurer; according to the terms of the contract, the insured is providing protection up to the maximum coinsurance penalty, whether or not losses occur. In the instance of a loss where the full face value of the deficient policy is paid, the insured bears a portion of the loss but *not* as a coinsurer. An insured with less than full coverage may be entitled to the full face amount of the policy whether or not a coinsurance clause applies, and in that case, no coinsurance penalties are considered to exist.

<u>Insurance to value</u> exists if property is insured to the exact extent assumed in the premium rate calculation. The rate calculation may assume that the average level of coverage is less than 100% of the value of the property. *Insurance to value* means *insurance to **full** value* only if 100% coverage is assumed in the rate computation. <u>Underinsurance</u> is coverage less than that assumed, and <u>overinsurance</u> is coverage beyond that assumed. As the concept of *insurance to value* is so closely tied to the premium rate calculation, it will become clearer later in the paper once the appropriate equations have been derived.

<u>Coinsurance: Basic Formulas</u>

The following variables will be used to define simple coinsurance relationships:

I	=	the indemnity received by the insured for a loss
L	=	the dollar amount of the loss (after the flat deductible is met)
F	=	the face amount of insurance
V	=	the dollar value of the property
c	=	the coinsurance percentage

[1] Properties whose values are constantly changing, such as inventories, may be insured under periodic reporting forms. These types of policies have no face amounts, but require the insured to report at regular intervals the description and value of the covered property at each location and the amount and terms of specific insurance. Reports are typically made on a monthly basis.

C = the coinsurance requirement
d = the coinsurance deficiency
a = the coinsurance apportionment ratio
e = the coinsurance penalty

For simplicity, the flat deductible will be ignored or it will be assumed that none exists (i.e. the loss amount equals the loss after the flat deductible and similar policy restrictions have been met.) Using the above variables, a formula expression for:

(1) the coinsurance requirement is $C = cV$; **[Equation 1.a]**

(2) the coinsurance deficiency is $d = [\, cV - F \,]$; **[Equation 2]**

(3) the coinsurance apportionment ratio is $a = [\, F / cV \,]$; **[Equation 3.a]**

subject to the constraint: $[\, F / cV \,] \leq 1$; and **[Equation 3.b]**

(4) the coinsurance penalty is:

$e = L - I$ if $L \leq F$, and **[Equation 4.a]**
$e = F - I$ if $F < L \leq cV$ **[Equation 4.b]**
$e = 0$ if $L > cV$ **[Equation 4.c]**

Note that for a coinsurance penalty to arise, the selected policy face value can not exceed the coinsurance requirement. Equation 4.c states that no coinsurance penalty exists when the amount indemnified exceeds the face amount of the policy. In the next section entitled "Coinsurance Penalties," this result will be shown to be true, and it also will be shown that this occurs when the loss amount exceeds the coinsurance requirement. Further, it will be shown that the maximum coinsurance penalty for a given deficiency level occurs for the loss size exactly equal to the policy face.

Example 3: Property Coinsurance

A property is valued at $500,000. The coinsurance requirement for the policy is 80% of the property value. What is the coinsurance apportionment ratio for an insured that chooses (a) a $300,000 face value? (b) a $50,000 face value? What is the coinsurance deficiency in each case? What is the maximum coinsurance penalty the insured faces in each case?

The coinsurance apportionment ratios are:

(a) $300,000 ÷ (80% × $500,000) = 75%
(b) $50,000 ÷ (80% × $500,000) = 12.5%

The coinsurance deficiencies are:

(a) [(80% × $500,000) - $300,000] = $400,000 - $300,000 = $100,000
(b) [(80% × $500,000) - $50,000] = $400,000 - $50,000 = $350,000

The maximum coinsurance penalties the insured faces are:

(a) $300,000 × { 100% - [$300,000 ÷ (80% × $500,000)] } = $75,000
(b) $50,000 × { 100% - [$50,000 ÷ (80% × $500,000)] } = $43,750

Note that the maximum coinsurance penalty is smaller in the second instance, even though the coinsurance apportionment ratio is smaller, because the insured has purchased less coverage in total.

The Coinsurance Clause

The standard coinsurance mechanism may be represented by a simple formula. The coinsurance clause establishing the most general coinsurance arrangement provides that:

$$I = L\,[\,F / cV\,]. \qquad \textbf{[Equation 5.a]}$$

subject to two constraints:

$$I \leq L \qquad \textbf{[Equation 5.b]}$$
$$I \leq F \qquad \textbf{[Equation 5.c]}$$

Equation 5.a dictates that losses are apportioned between the insurer and its insured according to the standard definition of coinsurance, such that the insurer pays a fraction of the insured loss equal to the coinsurance apportionment ratio. The first constraint is that the indemnity payments be limited to the loss amount. This constraint is due to the principle of indemnity, the concept that no insured should profit from any loss. It also follows mathematically from Equation 4 and the limitation of *one* placed on the coinsurance apportionment ratio, $[\,F / cV\,]$: if the ratio were allowed to exceed one, the amount indemnified could exceed the loss amount; but the indemnity payment must never exceed the loss amount, even if the insured selects a high face value, choosing to insure beyond the coinsurance requirement. The second constraint limits the indemnity payment to the face value of the policy, which sets the overall limit on the amount of insurance payable on a single occurrence.

Coinsurance Penalties

By definition, a policyholder is deemed a *coinsurer* simply by holding a policy with a *coinsurance deficiency*; however, three conditions are necessary for an actual coinsurance penalty to arise:

(1) a covered loss greater than zero must occur which exceeds any flat deductible, such that an amount greater than zero is payable beyond the flat deductible and any other restrictions on the policy:

$$L > 0\ ; \qquad \textbf{[Equation 6.a]}$$

(2) a coinsurance deficiency must exist on the policy insuring the loss, i.e. - the insured must fail to meet the coinsurance requirement, such that:

$$F < cV\ ;\ \text{and} \qquad \textbf{[Equation 6.b]}$$

(3) as no coinsurance penalty is said to exist if the indemnity is capped by the policy face (as in Equation 5.b), it follows that for a coinsurance penalty to exist, the indemnity payable must fall strictly beneath the face value of the policy:

$$I = L\,[\,F / cV\,] < F\ ;\ \text{or} \qquad \textbf{[Equation 6.c]}$$
$$L < cV \qquad \textbf{[Equation 6.d]}$$

By definition, the coinsurance penalty ceases to exist at the point where the indemnity payable equals the policy face. The loss amount corresponding to this indemnity payment may be fully derived as follows:

$$I = F$$
$$a\,L = F$$
$$[\,F / cV\,]\,L = F$$
$$[\,1 / cV\,]\,L = 1$$
$$L = cV$$
$$L = C$$

A similar situation may be presented in which a coinsurance deficiency exists but *no coinsurance penalty arises* from a covered loss. This case requires only a slight modification to the three conditions given above under which a coinsurance penalty would arise. Equations 7.a and 7.b will not be stated as they are identical to Equations 6.a and 6.b; the third condition is reversed such that the indemnity is capped by the policy face, which occurs when the loss amount exceeds the coinsurance requirement:

$$I = L [F / cV] > F \text{ ; or} \quad \textbf{[Equation 7.c]}$$
$$L \geq cV \quad \textbf{[Equation 7.d]}$$

Recall that the coinsurance penalty is given by:

$$e = L - I \quad \text{if } L \leq F\text{, and} \quad \textbf{[Equation 4.a]}$$
$$e = F - I \quad \text{if } F < L \leq cV \quad \textbf{[Equation 4.b]}$$
$$e = 0 \quad \text{if } L > cV. \quad \textbf{[Equation 4.c]}$$

For a coinsurance penalty to arise, the face value of the policy can not exceed the coinsurance requirement. The coinsurance penalty is equal to the difference between loss capped at the policy face and the indemnity payable. This is true by definition of the coinsurance penalty, which does not apply to losses beyond the amount the policy is intended to pay according to its face value. Since the difference between the loss and the indemnity payment is an increasing function of loss, and the difference between the face amount and the loss is a decreasing function of loss, it follows that the maximum coinsurance penalty, for a *given coinsurance deficiency*, will occur at the endpoint maximums where the loss amount for the two differences are equal:

$$L - I = F - I \text{ ; or} \quad \textbf{[Equation 8.a]}$$
$$L = F \quad \textbf{[Equation 8.b]}$$

The graph on the following page shows the coinsurance penalty as a function of the loss amount, and illustrates where the maximum occurs.

Example 4: The Standard Coinsurance Clause

Immediately before a fire breaks out, a property is valued at \$500,000. The coinsurance requirement for the policy is 80% of the property value. The building suffers a covered loss of \$40,000. What is the indemnity payment on the loss, and what is the coinsurance penalty, if the insured carries a policy with (1) a \$300,000 face value? (2) a \$500,000 face value?

(1)
$$I = (\$40{,}000) \times [\ \$300{,}000 \div (80\%) \times (\$500{,}000)\ ]$$
$$= (\$40{,}000) \times (0.75)$$
$$I = \$30{,}000 \quad \text{= the indemnity payment}$$
$$L - I = \$40{,}000 - \$30{,}000 = \$10{,}000 \quad \text{= the coinsurance penalty}$$

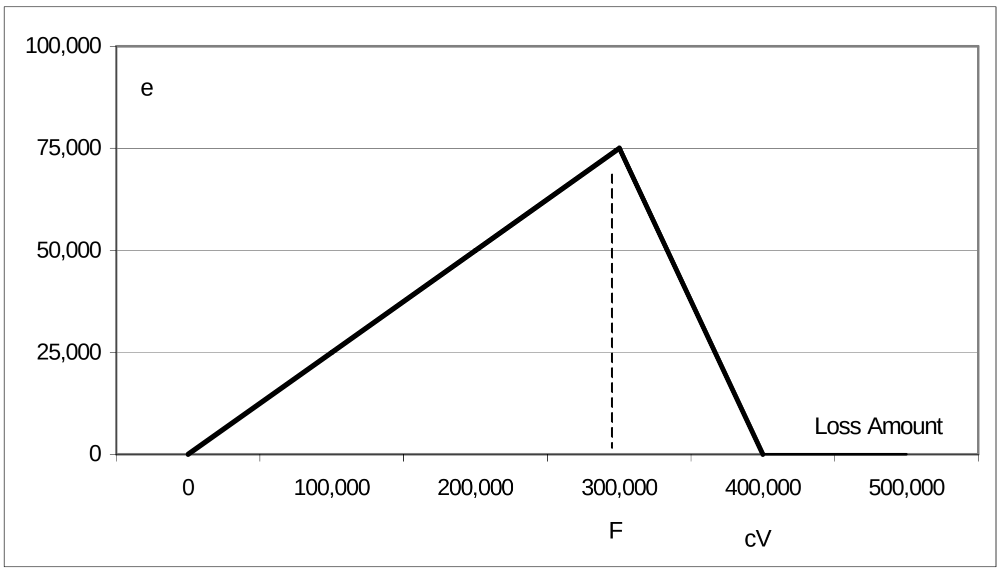
(2)
$$I = (\$40{,}000) \times [\ \$500{,}000 \div (80\%) \times (\$500{,}000)\ ]$$
$$= (\$40{,}000) \times (1.25)$$
I = \$50,000, but the coinsurance apportionment ratio can not exceed *one*, and
I ≤ \$40,000, the loss amount.
$$I = \$40{,}000 \quad \text{= the indemnity payment}$$
$$L - I = \$40{,}000 - \$40{,}000 = \$0 \quad \text{=> no coinsurance penalty exists}$$

Example 5: The Standard Coinsurance Clause

Immediately before a fire breaks out, a property is valued at $500,000. The coinsurance requirement for the policy is 80% of the property value. The building suffers a covered loss of $450,000. What is the indemnity payment on the loss, and what is the coinsurance penalty, if the insured carries a policy with (1) a $300,000 face value? (2) a $500,000 face value?

(1)	*I*	=	($450,000) × [$300,000 ÷ (80%) × ($500,000)]	
		=	($450,000) × (0.75)	
		=	$337,500 but	
	I	≤	$300,000, the policy face amount.	
	I	=	$300,000	= the indemnity payment
	F - I	=	$0	=> no coinsurance penalty exists

(2)	*I*	=	($450,000) × [$500,000 ÷ (80%) × ($500,000)]	
		=	($450,000) × (1.25)	
	I	=	$562,500, but the coinsurance apportionment ratio can not exceed *one*, and	
	I	≤	$450,000, the loss amount.	
	I	=	$450,000	= the indemnity payment
	L - I	=	$450,000 - $450,000 = $0	=> no coinsurance penalty exists



Graph 1. *The Coinsurance Penalty* The coinsurance penalty for a given coinsurance requirement is an increasing function of the loss amount up to the policy face. For loss amounts greater than the policy face, the penalty is a decreasing function up to the coinsurance requirement. For losses equal to or exceeding the coinsurance requirement, the full face value is paid and no coinsurance penalty exists. Graph 1 is based on the values in Example 4.

The Agreed Amount Endorsement

Another type of coinsurance arrangement is the agreed amount endorsement (also known as the guaranteed amount endorsement or stated amount coinsurance clause), which specifies a set dollar value for the item of property to be insured. These types of endorsements typically apply to objects of art or antiques like classic automobiles which are not readily replaceable and for which a "retail" or market dollar value can not be accurately quoted. The insured object may be subject to appraisal, but the insurer and the insured must come to an agreement on a value. The agreed amount generally dictates the level of coverage. It is rare that the insured would be granted the option to purchase a policy with a face less than the agreed amount. The coinsurance clause under a stated amount coinsurance clause may still be described mathematically. Define the following variable:

A = the agreed amount of insurance under an agreed amount endorsement

Under a policy with an agreed amount endorsement, the endorsement would provide that:

$$I = L\,[\,F / A\,]\ ; \qquad \textbf{[Equation 5.d]}$$

subject to the same constraints as given under the standard coinsurance agreement:

$$I \leq L\ ; \text{ and} \qquad \textbf{[Equation 5.b]}$$
$$I \leq F\ ; \qquad \textbf{[Equation 5.c]}$$

where the coinsurance apportionment ratio is given by:

$$a = [\,F / \mathrm{A}\,]\ ; \qquad \textbf{[Equation 3.c]}$$

subject to the constraint: $$[\,F / \mathrm{A}\,] \leq 1\ . \qquad \textbf{[Equation 3.a]}$$

The coinsurance requirement will be a fixed dollar amount rather than a percentage value, such that:

$$C = A. \qquad \textbf{[Equation 1.b]}$$

Insurers may be reluctant to offer this type of a policy. If the property value rises during the policy period, the insurer assumes the risk of inadequate premium. If the property is insured to the agreed amount, for example, the insurer will be liable to pay any loss up to that amount in full. The value of the property at the time of the loss is not relevant to the contract. While the agreed amount endorsement is usually offered simply because valuation difficulties exist, in some cases, the insurer may accept the terms of an agreed amount endorsement because competitive pressures in the insurance market give the insured greater bargaining power.

For other types of coinsurance arrangements and their formulas, refer to *Insurance to Value*, by George L. Head, CPCU, CLU, published by the S. S. Huebner Foundation for Insurance Education © 1971.

Loss Severity Distributions

Loss-severity distributions are conditional probability distributions of losses, the condition being that a loss of some size occurs. These distributions may reflect losses either by absolute dollar size or as a percentage of the property value.

Conditional and Unconditional Probability Distributions

A probability distribution is a function that includes all possible outcomes and for each outcome specifies a probability, the total value of which sum to *one*, that is, to one hundred percent. A conditional probability distribution is a probability distribution for which the set of outcomes have been restricted by a condition. An unconditional probability distribution has no restrictions on the set of outcomes.

Conditional probability may be defined mathematically. Suppose that A and B are two sets of outcomes. The probability of B under the condition that A has happened, denoted by P(B | A) is given by:

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} \qquad \textbf{[Equation 9]}$$

Generalized Loss-Severity Functions

The following variables were previously defined:

L	=	the dollar amount of the loss (after the flat deductible is met)
F	=	the face amount of insurance
V	=	the dollar value of the property

The unit exposed to loss is an individual property. By assumption, each property is insured under only one policy, and no policy insures more than one property.

The unconditional probability of a loss of size L, p(L), is a joint probability equal to the product of the probability of a nonzero loss and the conditional probability of a loss of that given size, s(L). If N risks are insured, and, of these, M suffer a loss greater than zero during a policy period, M/N is the probability f of some nonzero loss during that period. The unconditional probability of a loss of L, denoted as p(L), is given by:

$$p(L) = (M/N)\, s(L) = f\, s(L). \qquad \textbf{[Equation 10]}$$

The variable f represents the frequency of loss, and the function s(L) represents the loss-severity distribution. Since few risks suffer a loss during a policy period, the integral of the *unconditional* probabilities from *more than zero* to V is much less than *one*, but over the range *zero* to V, including "zero loss":

$$\int_0^V p(L)\,dL = 1\,, \qquad \textbf{[Equation 11]}$$

and it equals *one* because the distribution accounts for all possible outcomes. The generalized loss-severity function s(L) represents the *conditional* probability of the size of loss given a nonzero loss has occurred, and excludes the outcome of 'zero' losses, that is, that no loss occurs.

Let s(L) represent the percentage of all losses from a given peril that are of size L, where L may be represented either by the dollar amount of loss or its fraction of the property value. Then s(L) is the conditional probability of a loss of L, under the condition that a nonzero loss has occurred. If no loss can be greater than the property value V, then all possible outcomes are encompassed by loss sizes from zero to V. Then:

$$\int_0^V s(L)dL = 1 \,, \qquad \textbf{[Equation 12]}$$

and it equals *one* because the distribution accounts for all possible loss sizes. This is the equation of a generalized loss-severity function. It differs from the unconditional loss probability distribution, which would include the outcome of no loss.

Both the conditional probabilities of losses greater than zero and the unconditional probabilities of all losses including "zero" losses, form complete probability distributions.

Example 6: Probability Distributions

Assume that a company writes 1,000 policies, that each policyholder purchases only one policy, and that each policy has a face value equal to the value of the insured property ($F = V$). Assume only one loss per policy per period is possible. Exactly 12 insureds will suffer a loss of some size during any one policy period. If a loss occurs, in half of the cases the amount of damage will amount to only 10% of the property value. In one in four cases, the damage will amount to 50% of the property value. In one case in twenty, the damage will amount to 90% of the property value. One loss in five will be total. Write expressions for:

(1) f, the probability of any loss greater than zero faced by each insured property per policy period;
(2) s(L), the conditional probability of a loss exactly equal to L, given some loss greater than zero; and
(3) p(L), the unconditional probability of a loss exactly equal to L (for all possible values of L).

(1) f = M / N = 12 / 1,000 = 1.2%

(2)

$$s(L) = \begin{cases} 50\% & L = .10V \\ 25\% & L = .50V \\ 5\% & L = .90V \\ 20\% & L = V \end{cases}$$

(3)

$$p(L) = f\,s(L) = \begin{cases} 98.80\% & L = 0 \\ 0.60\% & L = .10V \\ 0.30\% & L = .50V \\ 0.06\% & L = .90V \\ 0.24\% & L = V \end{cases}$$

p(0) = 1 - P(x≠0) = 100% - 1.2%
p($.10V$) = fs($.10V$) = (1.2%)(50%)

Coinsurance Illustrations

This section demonstrates the function of coinsurance arrangement, as well as its short-comings, in solving or preventing problems of underinsurance. The two illustrations which follow examine the inequities that could result if insureds were to purchase less insurance coverage than the amount assumed in the calculation of the pure premium rate. In the absence of coinsurance, policyholders might find they benefit by carrying low levels of insurance, and an insurer with a number of underinsured risks might not collect sufficient premium to meet its liabilities. The illustrations show how the coinsurance mechanism may be implemented to help the insurer collect adequate premiums, how coinsurance provides incentive for policyholders to maintain full coverage on insured property, and how coinsurance serves to promote equity among insureds. Following the illustrations, the case study presents a real-life example of the extreme troubles that can strike insurers and insureds alike when underinsurance is severe, far beyond remedy by the coinsurance clause.

Illustration 1: Premium

Consider two people who own similar homes valued at $500,000 each. Anna insures her home for full value, as expected. However, Blanca chooses a policy face of half the property value, $250,000, in order to pay lower monthly premiums. Blanca knowingly retains the upper layer of risk for losses beyond $250,000. Neither policy contains any form of deductible.

In the absence of a coinsurance clause, each insured would be indemnified equivalently for any fire loss under $250,000. Blanca has paid a lower premium, but has received the same level of protection on any partial loss below her selected policy face. Is this arrangement "fair?"

For simplicity, assume that the severity distribution is uniform, that all sizes of loss are equally likely. A loss above $250,000 is equally likely as a loss below $250,000. Assuming the insurer believes that all of its insureds are purchasing full coverage, it will charge Blanca exactly half the pure premium it charges Anna. If a fire occurs, in half the instances, Anna and Blanca would receive the same indemnity payment equal to the loss amount; this case applies to losses of up to $250,000. In the other half of the instances, when losses exceed $250,000, Anna would receive the full amount of the loss while Blanca would receive her policy face of $250,000. Anna's expected indemnity payment is equal to her expected loss of $250,000, while Blanca's expected indemnity payment is $187,500[2]. Note that this amount is 50% higher than *half* of Anna's. While Blanca pays only half of the full-coverage premium, she can expect to be indemnified by more than half of the loss amount for any loss size short of a total loss. Blanca has selected less coverage but is clearly getting a better deal for the price. If these were the only two insureds, Anna's premiums would subsidize some of Blanca's exposure to loss. Anna would have clear incentive to lower her coverage.

Since the insurer has assumed in its premium rate calculation that each insured will maintain coverage equal to 100% of their property's value, the insurer could remedy the situation by adding a coinsurance clause to the policy establishing a 100% coinsurance requirement. If such a requirement were added to the policy, Blanca's coinsurance apportionment ratio would be 50%. She would coinsure half of every partial loss below the policy face, and a portion of every partial loss beyond the face up to the coinsurance requirement which in this case is the total property value. On a fire loss of $200,000, Blanca would be indemnified $100,000, suffering a coinsurance penalty of $100,000. Such indemnity payments would balance with the premium payments. Similar results would follow even if all loss amounts were not equally likely.

[2] In half of the instances, Blanca receives the policy face of $250,000. In the other half of the instances, the average payment is uniformly distributed from $0 to $250,000. Blanca's average indemnity payment is therefore [(0.50) × ($250,000) + (0.50) × ($125,000)] = $187,500.

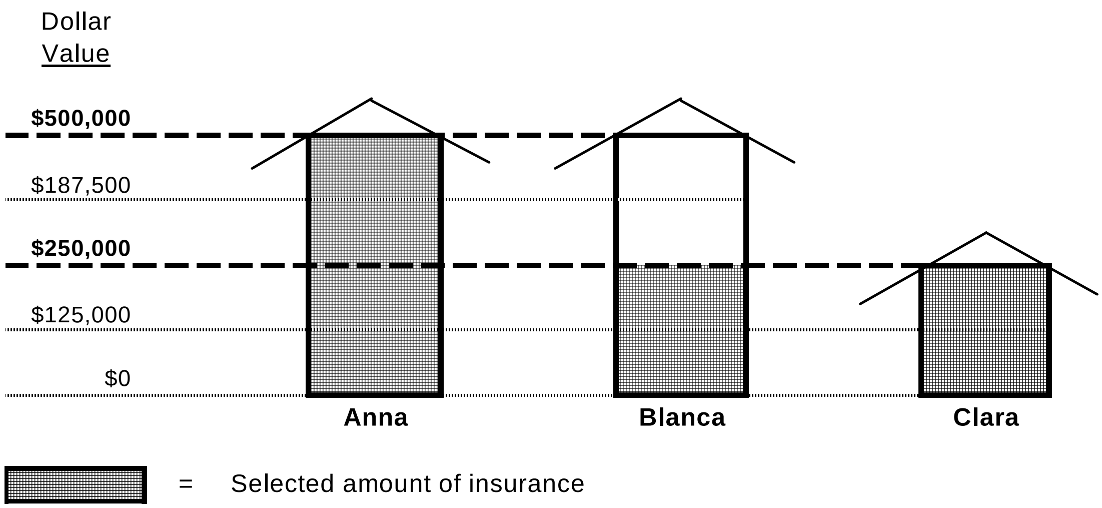
Illustration 2: Exposure

Consider two people who own similar homes of different sizes. Blanca's home is valued at $500,000 but she chooses to insure it for $250,000 as in the previous illustration. Clara's home is valued at $250,000 and she chooses to insure it for its full value. Again, Blanca knowingly maintains the upper layer of risk for losses beyond $250,000, and neither policy contains any form of deductible. The insurer has mistakenly assumed that all policyholders purchase insurance coverage equivalent to the value of their property.

In the absence of a coinsurance clause, each insured would be indemnified equivalently for any fire loss under $250,000. Both insureds are charged the same premium for the same face amount of coverage. Is this arrangement "fair?"

For simplicity, again assume that the loss severity distribution is uniform, that all sizes of loss are equally likely. For Blanca, a loss above $250,000 is equally likely as a loss below $250,000. If a fire occurs, in half the instances, Blanca would receive her policy face of $250,000; this case applies for losses over $250,000.   For Clara, the loss amounts for the most severe half of her loss exposure, in the range from $125,000 to $250,000, average only $187,500; the probability of a total loss for which she would be paid the full $250,000 face value, is quite small. Clara's expected indemnity payment is equal to her expected loss of $125,000, while Blanca's expected indemnity payment is again $187,500. Note that this amount is 50% higher than Clara's. Again, with the same premium but different loss exposure, Blanca is getting a better deal for the price. If these were the only two insureds, Clara's premiums would subsidize some of Blanca's exposure to loss. Clara would have clear incentive to lower her coverage.

If a 100% coinsurance requirement were added to the policy, Blanca's coinsurance apportionment ratio would be 50%. She would coinsure half of every partial loss above or below the policy face. On a fire loss of $200,000, Blanca would be indemnified $100,000, suffering a coinsurance penalty of $100,000 as in the first illustration. Such indemnity payments would balance with the loss exposure. Similar results would follow if all loss sizes were not equally likely.



Graph 2. ***Coinsurance Illustrations***. Anna and Blanca own similar properties of equal value. Anna is fully insured to a face amount equal to her property's value , but Blanca carries only half of the full coverage. In this case, there is a difference between Anna and Blanca in the total premium dollars paid but no difference in their exposure to losses. Blanca and Clara carry insurance policies of the same face value, but Clara is fully insured for her property that is worth half the value of Blanca's. In this case, there is no difference in the total premium dollars paid between Blanca and Clara but there is a difference in their exposure to losses.

Case Study: The Oakland Fires[iii,iv]

On October 20, 1991, a firestorm swept the hills of Oakland and Berkeley, California, to the east of the San Francisco Bay. In the fires, 25 people were killed and 150 injured, and a total of 3,354 dwellings were destroyed at once with damages estimated at over $1.5 billion. In the aftermath, many homeowners were shocked to find their properties grossly underinsured.

The Oakland fires were characterized by underinsurance so severe as to arouse panic surrounding the efficacy of the insurance industry. Claimants who had paid insurance premiums for years feared most of their losses - their homes, cars, and all of their possessions - would be irrecoverable. The policyholders of one insurance group estimated the cost of rebuilding their homes at $150 to $300 per square foot, but found their insurance would pay only $66 to $93 per square foot. Payments made on behalf of the forty insureds of another company fell short by an average of $130,000 per insured structure. In a survey of 27 insurance companies concerning 2,465 of the covered structures, 49% felt that the structures included in the survey had been underinsured by an average of $102,000. In a separate survey of 665 homeowners, 83% felt they were underinsured by an average of $194,000.

Insurance companies said that under the law, it is the homeowner's responsibility to make sure they have enough coverage. Many of the policyholders said that they had not understood the promise of "guaranteed replacement coverage" in their contracts. Others had requested higher levels of coverage prior to the fires which had been denied. Appraisal methods used by some insurers to establish replacement cost, such as a method of appraising the structures room by room, were found to be terribly inaccurate. Political pressure was placed on the insurance industry to provide full coverage, and lawsuits were numerous. In the end, upgrades were granted to 500 insureds costing insurance companies $80 million.

Two insurance groups received commendations from the California Department of Insurance. One of these, the California Casualty Group, made a decision early in the process to cover all policyholders to the full extent of their losses, regardless of the varying extents to which they were underinsured. The decision was based largely on the group's claims philosophy, described by Ed McKeon, Vice President of Corporate Relations: "Find a way to pay the claim, not a way to deny it." McKeon said that because the California Casualty Group specializes in educator and public safety associations and does not insure "every third home on the street," it felt an extra-contractual obligation to its insureds. At the time, California Casualty would not have expected its policyholders to be carrying enough insurance to recover from such an extreme situation. "Insurance appraisers aren't expecting foundations to crumble and driveways to slide away. Usually a lot of the structure remains [following a fire]."

There were drawbacks to the decision. McKeon felt that the California Casualty Group set a standard that the Department of Insurance used as a basis for measuring the performance of other companies not specializing in associations and not sharing the same claims philosophy, creating some tension within the local industry. Similar friction arose within the group, as some underinsured claimants in single family fires following the Oakland disaster expected similar treatment from the group. Clearly, it would not be feasible to provide full coverage to underinsured policyholders under all circumstances.

Since the Oakland fires, many improvements have been made both in coverage terms and in the tools and techniques of appraisal. Some policies contain a clause which, in the event of a total or near-total loss, allow the policyholder to be reimbursed by a proportional amount in excess of the face amount if they have maintained the required coverage level. Appraisals have become far more accurate so the face amount is not left entirely up to the insured to determine as might have been the case in the past. "Today, most underinsureds are people who have made improvements and upgrades or put on new additions, and forgot to tell us," said McKeon.

Large settlements emphasize the importance for insurance companies to monitor the level of insurance selected by their policyholders. The need exists to train employees, appraisers and agents to properly advise insureds, inform them of contract terms, take accurate measurements and encourage insurance to value. In most instances, the coinsurance clause places the burden of maintaining adequate coverage on the insured, but attempts to rely exclusively on it may not ultimately benefit the insurance company.

Loss Severity Curves

The Fire Loss Severity Curve

In the illustrations of the preceding section, fire losses were assumed to follow uniform size of loss distributions for simplicity. It is clear that in actuality fires do not produce uniform loss sizes. The fire size of loss severity distribution generally follows a downward-sloping curve, where the x-axis represents the size of loss either in dollars or as a percentage of the property value, and the y-axis represents the probability of the loss size. The actual shape of the curve will depend on the type of property that is included in the distribution. Care must be taken to separate losses into an adequately homogeneous populations in order to achieve an estimation of any accuracy. There are many factors which can introduce distortions into the data collected for approximating a size of loss curve.

The use of the actual fire loss-severity distribution would not substantially alter the conclusions of the illustrations from the preceding section; similar results concerning the equity among insureds would hold whether the actual distribution were a decreasing, flat, or increasing function of the loss size.

Dollar Size or Percentage of Value

A loss-severity distribution defines the conditional probabilities of loss sizes greater than zero. In graphical form, the size of loss would generally be shown on the x-axis and the corresponding probability of each size of loss on the y-axis. The size of loss may either be stated in absolute dollars or as a fraction of full property value which that loss represents.

Many diverse factors impact the size of loss distribution. The shape of the distribution might depend, for example, on the total value of the property, its physical size, the immediate surroundings and the building materials used. Compiling data on which to base a loss severity distribution can be problematic, whether expressed in absolute dollars or as fractions of the corresponding property values. Distortions may be introduced through the combination of loss data from dissimilar properties, or through estimation errors such as the use of outdated appraisals of property values.

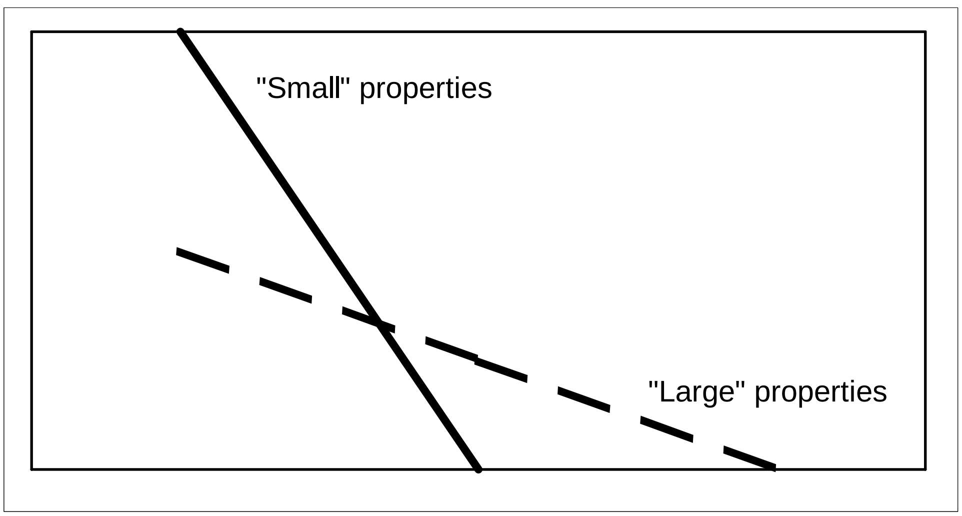
When representing loss values in absolute dollars, the unweighted grouping of all losses incurred from properties of varying worth could easily result in distorted size of loss distributions. The data collected may be totally inapplicable to arriving at equitable premium rates to charge. For instance, properties worth over a million dollars may be relatively infrequent compared to properties worth a quarter of that amount. If all observed losses are recorded as they occur with equal weight, then the chance of losses exceeding a quarter of a million dollars, computed specifically for a high-valued property, would be underrepresented in the compiled distribution, simply because properties exposed to losses of such a magnitude are scarce. If such a data compilation were used to develop a schedule of premium rates, those rates would be sorely understated for properties of high value.

In <u>*Loss Distributions*</u> by Hogg and Klugman, the authors suggest the estimation problem known as ***mixture of models*** can be solved by combining each distribution with the probability with which it occurs. This type of aggregated distribution is most applicable to predicting losses for a population as a whole. It would not serve the rate making process well in pricing policies applicable to individual properties. It may be possible to weight the tail of a combined distribution to account for the relative infrequency of the higher valued properties, but the combination of partial losses might still have the effect of distorting the severity distribution for both the higher and the lower valued properties. One solution is to develop several loss distributions, grouping observed losses according to reasonably narrow ranges of property values.

The problems associated with variations in property values can also be eliminated somewhat by representing losses as fractions of full property value. A combination of such a form would assume that the loss distributions of all the included properties were similarly shaped, and would provide an accurate basis for developing a premium rate schedule if this were the case. This assumption will not always be reasonable, and other types of distortions may be introduced into such a model.

The shape of the size of loss distribution can be expected to vary according to numerous attributes. One variation straightforward to imagine is the physical size. Properties which are small in size are more prone to total and near total losses than large structures. For instance, imagine a small historic home in an upscale urban area which is appraised at an equivalent dollar value as a modern ranch house five times its size in a remote, low-cost rural area. While the two structures may be insured by policies with equivalent face values, both their loss frequencies and their size of loss distributions differ. The territory relativities applied in rating these policies will most likely account for the difference in frequencies; this may be due to the greater fire hazard in historic areas caused by faulty old wiring or the greater speed by which fires spread in densely populated regions. Should both buildings incur fire damage, it is more likely that the small home will be damaged to a greater extent or completely destroyed than the large ranch house, simply because it is comprised of far less materials taking less time to burn. Territory relativities applied as straight factors will generally not adjust for differences in the shape of loss distributions. It is not a common practice to separate small- versus large-sized properties in developing premium rates, but greater accuracy in rate making could be achieved by doing so.

A common practice is to separate properties into distributions according to reasonably narrow ranges of property values. This practice could eliminate much of the distortion in combined distributions, whether losses are reflected in dollar amounts or as percentages of the property value. Such a division may even serve as a reasonable proxy to size, depending on how well territories delineate variations in cost levels and how territorial differences are addressed.



Graph 2. ***Small versus large properties***. Assume for simplicity the loss-severity distributions of all properties are similarly shaped, linear and downward sloping. It is apparent that combining the observed dollar losses from properties of different values can distort the true loss severity distribution, leading to a model that is inappropriate for pricing properties of any size. Scaling the losses, by representing each loss as a percentage of its property value, can eliminate this type of distortion if the loss distributions for the individual properties are similarly shaped. The shape of the size of loss distribution should be expected to vary among properties which are dissimilar in various ways including in their physical size, immediate surroundings, or construction materials used.

Other distortions may arise as a matter of policy. For instance, mortgage companies may require insureds to carry insurance on the dwelling structure equal to the full value of the mortgage at a minimum. In regions where land values have appreciated rapidly, the difference in mortgage values would be more likely to reflect variations in year of purchase than variations in costs of rebuilding a structure, or variations in the percentage of down payment. The size of loss distribution could be severely distorted if losses were reflected as a percentage of mortgage amounts rather than the property value. Arriving at an accurate size of loss distribution on which to base an equitable premium rate structure would be least viable where land values fluctuate widely over a small area, particularly over an area which must be contained within a single territory.

The representation of losses as percentages of full property value poses added difficulty when property values are not known with accuracy or when they are rapidly changing. If an accurate property value is not known or can not be determined, the insured value is commonly substituted for the true property value when such loss observations are made.

Distortions in the loss amounts may arise in a catastrophic event, such as in the Oakland fires. A large number of simultaneous losses may lead to a demand surge, the increase in the costs of goods and services, such as construction materials and labor, due to the large number of people in need at one time.

General Pure Premium Rate Equation

The pure premium rate the insurer will charge for a property insurance policy depends on the expected value of the indemnity payments and the face value of the policy. Expected indemnity payments are a function of frequency, severity, and the face value or the coinsurance requirement. The model depends on three parameters: f, the frequency of loss; $s(L)$, the distribution of losses by size; and $C = cV$, the coinsurance requirement.

Assumptions

There are several assumptions that must be made before an equation for the premium rate can be developed. The assumptions underlying the model are that:

- premium rates are equitable and just adequate
- all insured properties are homogeneous
- the rate determinants - frequency and severity - are known and stable
- losses are precisely adjusted
- each insured carries no other property insurance
- there is no interest on the pure premiums collected
- each policy has no more than one loss per period

It is not necessary that each of the theoretical assumptions be met to the letter in practical applications. The assumptions are, however, completely necessary to produce an approximately correct and reasonably simple mathematical model of the insurance pure premium rate. In real applications, most of the assumptions will not hold precisely, but closely enough to ensure a reasonable approximation of the appropriate premium rate. Additional accuracy in the assumptions would add complexity that might not be warranted by the degree of precision gained.

The pure premium rate equation is founded on the notion that the pure premium charged match the expected indemnity payments in the policy period, and that it not exceed that expectation. This condition is particularly important to rate regulation, which in most states requires rates to be “adequate, reasonable, and not unfairly discriminatory.”

The homogeneity of insured properties dictates that the properties of a given class have the same relative frequency of loss, f, and the same loss size distributions, s(L). In reality, these rate determinants for a given property are unknown. For example, a home with a fire hydrant directly in front of it may follow a different severity function than homes two and three blocks from the hydrant, yet all of the homes in this neighborhood may be deemed similar enough to group in the same class for rating purposes. To develop the rating model, it is assumed that the frequency and severity variables for the given property class are known and are not subject to changes due to dynamic influences (e.g. technological advances, changes in economic conditions, etc.).

For the coinsurance mechanism to function accurately, it is necessary for the value of each insured property to be determinable and for coinsurance penalties to be accurately assessed in the event of loss. Valuation and loss adjustment are assumed to be precise in order to arrive at presumably accurate pure premium rates.

The complication of pro-ration among insurers can be eliminated from the model through a simple assumption. Assuming that the property is covered by one policy only, the estimation of the expected value of indemnity payments is greatly simplified. The model also ignores interest on pure premiums collected for the sake of simplicity. Pure premium is usually collected at the beginning of the policy period, or at quarterly or monthly intervals, while the loss payment would occur at the middle of the period on average. A greater degree of precision could be achieved by adding interest to the model, but this rating determinant does not benefit the study of coinsurance and therefore does not warrant the increased complexity.

Although it will be assumed otherwise, it is definitely possible for a single property to incur more than one loss in a policy period. The model accounts for this by incorporating multiple losses to one property within the frequency variable. The loss frequency is estimated by studying the total number of losses to many properties over many policy periods. The relative frequency selected to represent the loss potential assumed for a single property is actually based on data that reflects multiple losses to a single property during a single policy period. The exclusion of multiple losses simplifies rate computation, avoiding the problem of double weight being given to multiple losses, an error to which the formula is prone, and ultimately avoiding an inflated frequency which would result in redundant aggregate premiums.

Notation

The following variables and notations were previously defined:

	L	=	the dollar amount of the loss
	F	=	the face amount of insurance
	V	=	the dollar value of the property
	f	=	the probability of any loss of whatever size greater than zero, to each insured property per policy period.
	s(L)	=	the percentage of losses exactly equaling L, or the conditional probability of a loss of L, given some loss greater than zero
p(L) =	f s(L)	=	the unconditional probability of a loss exactly equal to L to each insured property per policy period

The following variables and notations are now defined:

P	=	the pure premium charged each insured per policy period
R	=	the pure premium rate per dollar of face amount per policy period ($R = P/F$)

$E(I)$ = the expected value of the insurer's indemnity payments for insured losses of whatever size greater than zero to each per policy period

$E(I \mid x < L \leq y)$ = the expected value of the insurer's indemnity payments to each insured per policy period for losses greater than x but not greater than y.

Derivation of the Equation

The pure premium rate per dollar of face amount per policy period, by definition is given by:

$$R = P / F. \qquad \textbf{[Equation 13]}$$

If pure premium equals expected indemnity, then:

$$P = FR = E(I) \qquad \textbf{[Equation 14]}$$

or:

$$R = E(I) / F \qquad \textbf{[Equation 15]}$$

The foundation of the pure premium rate equation is represented mathematically by Equation 14: the pure premium charged equals the expected indemnity payments in the policy period. This equation is intended to set rates to be "adequate, reasonable, and not unfairly discriminatory." Equation 15 restates Equation 14 in terms of a rate per dollar of coverage as opposed to a total pure premium. The equitable, adequate, and reasonable price per $100 of property insurance should vary with the amount of insurance, the policy face.

Only the amount of the loss and the policy face are assumed to limit the insurer's liability, as given in Equations 5.b and 5.c. For the expected indemnity payments, both the discrete case and the continuous case will be examined. The discrete case assumes whole dollar losses only for simplicity. An insurer's expected indemnity payments can be divided into two parts by the face amount of the policy. First, for losses not exceeding the policy face, the expected value of these losses is the sum of the products of the amount of each loss times the unconditional probability of each loss, scaled by the probability of not exceeding the policy face in the denominator. This expected value can be expressed as:

$$E\,(I \mid 0 < L \leq F) = \frac{\sum_{L=1}^{L=F} Lf\,\mathrm{s}(L)}{\sum_{L=1}^{L=F} \mathrm{s}(L)} \qquad \textbf{[Equation 16.a]}$$

if L can take on only discrete integer values, or:

$$E\,(I \mid 0 < L \le F) \quad = \quad \frac{\int_0^F L f\, \mathrm{s}(L)\, dL}{\int_0^F \mathrm{s}(L)\, dL} \qquad \textbf{[Equation 16.b]}$$

if the values of L are continuous. The second element of an insurer's expected indemnity payments, the portion for losses exceeding the policy face, is equal to the policy face limit times the probability of losses exceeding the policy face, scaled by the probability the loss exceeds the policy face. The probability of losses exceeding the policy face is the unconditional probability of any loss of whatever size greater than the policy face, that is, f multiplied by the difference between one and the percentage of losses not exceeding the policy face.

This expected value can be expressed as:

$$E\,(I \mid F < L < \infty) \quad = \quad \frac{Ff \left[1 - \sum_{L=1}^{L=F} \mathrm{s}(L)\right]}{\left[1 - \sum_{L=1}^{L=F} \mathrm{s}(L)\right]} \qquad \textbf{[Equation 17.a]}$$

if L can take only discrete integer values, or:

$$E\,(I \mid F < L < \infty) \quad = \quad \frac{Ff \left[1 - \int_0^F \mathrm{s}(L) dL\right]}{\left[1 - \int_0^F \mathrm{s}(L)\, dL\right]} \qquad \textbf{[Equation 17.b]}$$

if L is a continuous variable. Note that Equation 17 simplifies to $E\,(I \mid F < L < \infty) = F f$.

Combining the above expressions, the total of an insurer's expected indemnity payments under one policy during one policy period can be expressed as:

$$E(I) = \quad f\left(\sum_{L=1}^{L=F} L\, \mathrm{s}(L) \; + \; F\,[1 - \sum_{L=1}^{L=F} \mathrm{s}(L)]\right) \qquad \textbf{[Equation 18.a]}$$

if the values of *L* are discrete integers, or:

$$E(I) = f\left(\int_0^F L\, s(L)\, dL + F\left[1 - \int_0^F s(L)\, dL \right] \right) \quad \textbf{[Equation 18.b]}$$

if the values are continuous.

Recalling Equation 15:

$$R = E(I) / F, \quad \textbf{[Equation 15]}$$

the general expression for the pure premium rate results from substituting Equation 18 into Equation 15. The pure premium rate for each unit of face amount F is:

$$R = \frac{f\left\{ \sum_{L=1}^{L=F} L\, s(L)\, dL + \left(F\left[1 - \sum_{L=1}^{L=F} s(L)\, dL \right] \right) \right\}}{F} \quad \textbf{[Equation 19.a]}$$

if *L* can take only discrete integer values,

or:

$$R = \frac{f\left\{ \int_0^F L\, s(L)\, dL + \left(F\left[1 - \int_0^F s(L)\, dL \right] \right) \right\}}{F} \quad \textbf{[Equation 19.b]}$$

if the values are continuous. Equation 19 applies to four broad types of rate calculations, some of which require variations of the general equations as stated above in Equations 19.a and 19.b.

First, Equation 19 applies as stated above whenever the loss to the insured is unlimited, but the policy face limits the insurer's liability. General liability insurance is an example.

Second, if both the insured's loss and the insurer's aggregate liability are unlimited, the integral in the numerator of the right-hand side of Equation 19 is revised as the expected value of losses from zero to infinity. Workers' Compensation insurance is an example. Policies of this type lack a face amount, so the equation represents a *total* premium ($P = FR$) rather than a premium *rate* per unit of coverage:

$$P = f\left(\sum_{L=1}^{\infty} L\, s(L)\, dL \right) \quad \text{or} \quad P = f\left(\int_0^{\infty} L\, s(L)\, dL \right)$$

[Equation 20.a] **[Equation 20.b]**

Third, if the insured's loss and the insurer's liability are limited only by the property value, the numerator on the right-hand side of Equation 19 is the expected value of losses from zero to full value. The second, bracketed, element in the numerator of Equation 18 is not needed for losses above the policy face. Automobile physical damage insurance is a common example.

$$R = \frac{f\left(\sum_{L=1}^{L=V} L\, s(L)\, dL \right)}{F} \quad \text{or} \quad R = \frac{f\left(\int_{0}^{V} L\, s(L)\, dL \right)}{F}$$

[Equation 21.a] **[Equation 21.b]**

A variation of this case is a policy with an agreed amount endorsement. Since, under an agreed amount clause, the policy face generally equals the full property value determined by an insurance agent and an insured, losses exceeding the policy face can be presumed impossible. Therefore, the rate equation for each $100 of an agreed amount of insurance, *A*, is:

$$R = \frac{f\left(\sum_{L=0}^{L=F} L\, s(L)\, dL \right)}{A/100} \quad \text{or} \quad R = \frac{f\left(\int_{0}^{F} L\, s(L)\, dL \right)}{A/100}$$

[Equation 22.a] **[Equation 22.b]**

Fourth, if the insured's loss is limited by the property value and the insurer's liability cannot exceed the policy face, which may be less than the property value, Equation 19 requires no change. This is the case, where property values and amounts of insurance are definite and separable, that entails the topic of insurance to value. A special problem arises when each insured has the choice of policy face, yet a premium rate can be appropriate only if each insured chooses the policy face assumed in the premium rate calculation. Coinsurance is one solution to the problem of insureds selecting policy faces different from that assumed in the premium rate computation.

As stated earlier, insurance to value exists if property is insured to the exact extent, either dollar amount or percentage of value, as assumed in the premium rate calculation. The rate calculation may assume that the average level of coverage is less than 100% of the value of the property. *Insurance to value* means *insurance to **full** value* only if 100% coverage is assumed in the rate computation.

The term "value" refers to the value of the property, on the same basis used in indemnifying losses; that basis is usually *actual cash value* or *replacement cost*. The replacement value of property is equal to the amount it would cost to fully repair or replace the property if it must be reconstructed or purchased new. The actual cash value (ACV) is equal to the "…replacement cost [adjusted] by subtracting an amount that reflects depreciation. …The actual cash value of an item can be depressingly small after only a brief period of ownership."[ii] If the premium rate is based on 80 percent insurance to replacement cost, neither coverage to 80 percent of actual cash value nor coverage to 90 percent of replacement cost constitutes "*insurance to value*." In the first case, the property is underinsured if actual cash value is less than replacement cost; in the second instance, the property is overinsured relative to the insurance to value relationship assumed in the premium rate.

Example 7: Premium Rate Calculation (Discrete Case)

Assume losses are distributed as in example 6, all properties are valued at $500,000, and no appreciation occurs in property values. A company writes 1,000 policies. Each policyholder purchases only one policy and each policy has a face values equal to the value of the insured property ($F = V$). Assume only one loss per policy per period is possible. Exactly 12 insureds will suffer a loss of some size during any one policy period. If a loss occurs, in half of the cases the amount of damage will amount to only 10% of the property value. In one in four cases, the damage will amount to 50% of the property value. In one case in twenty, the damage will amount to 90% of the property value. One loss in five will be total. Find the premium rate per $100 of insurance for a policy face equaling (1) $50,000; (2) $100,000; (3) $250,000; (4) $475,000; (5) $500,000.

Solution:

$$s(L) = \begin{cases} 50\% & L = \$50{,}000 \\ 25\% & L = \$250{,}000 \\ 5\% & L = \$450{,}000 \\ 20\% & L = \$500{,}000 \end{cases}$$

(1) $$R = \frac{1.2\% \times \{ (\$50{,}000)(.50) + (\$50{,}000)[(.25) + (.05) + (.20)] \}}{\$50{,}000 / \$100} = \$1.20$$

(2) $$R = \frac{1.2\% \times \{ (\$50{,}000)(.50) + (\$100{,}000)[(.25) + (.05) + (.20)] \}}{\$100{,}000 / \$100} = \$0.90$$

(3) $$R = \frac{1.2\% \times \{ (\$50{,}000)(.50) + (\$250{,}000)(.25) + (\$250{,}000)[(.05) + (.20)] \}}{\$250{,}000 / \$100} = \$0.72$$

(4) $$R = \frac{1.2\% \times \{ (\$50{,}000)(.50) + (\$250{,}000)(.25) + (\$450{,}000)(.05) + (\$475{,}000)(.20) \}}{\$475{,}000 / \$100} \approx \$0.52$$

(5) $$R = \frac{1.2\% \times \{ (\$50{,}000)(.50) + (\$250{,}000)(.25) + (\$450{,}000)(.05) + (\$500{,}000)(.20) \}}{\$500{,}000 / \$100} \approx \$0.50$$

An important property of the premium rate calculation emerges in Example 7. Notice that the premium rate per $100 of insurance declines as the policy face increases. This is even true even when larger losses are more likely, as when coverage increases from $475,000 to $500,000; the premium rate per $100 of insurance declines even though total losses are four times as likely as near-total losses.

It is always true that the premium rate per $100 of coverage will decrease as the policy face increases, for any size of loss distribution, even if large losses predominate. One way to explain this phenomenon is to consider the consecutive layers of insurance. A loss of $500,000 encompasses the layer of loss up to $100,000, but the opposite is not true. Even if a $500,000 loss is more likely than a $100,000 loss, the layer up to $100,000 is reached in both instances, while for the latter all of the layers beyond $100,000 are not. The rate must decline as the policy face increases because the higher the loss layer, the fewer losses will be a size large enough to reach it. The only theoretical instance in which this would not hold would be when all losses were of the same size.

In the coinsurance illustrations in an earlier section of this paper, the insurer had assumed in its pure premium rate calculation that its policyholders would purchase insurance equal to the full value of their insured properties. From Example 7, it can be seen that the total pure premium that should be charged an insured choosing a face value at only half the value of her property will be an amount quite a bit greater than half of the full coverage pure premium. Although the illustrations assumed a uniform loss size distribution, the equitable premium for the significantly underinsured will still be a greater proportion of the full-coverage premium than the proportion of the property value insured.

Premium gradation may be viewed as an alternative to coinsurance; however, such a method requires the insurer to be able to accurately assess the true full value of the covered property and determine when and by how much it is changing.

Discrete versus Continuous notation

The pure premium rate was derived using both discrete and continuous notations to represent loss amounts. Since every currency has a smallest monetary unit, such as the *cent* in the United States, the continuous case is not technically valid. Since fractional cents are not possible loss amounts, the continuity of the loss severity distribution is broken at the regular small increments of the cent. However, this technicality does not render the continuous case inappropriate for illustration since the smallest monetary unit is generally very small in relation to typical loss sizes. In light of penny increments, the shape of the size of loss distribution can be expected to at least approach the continuous case. A greater hindrance to achieving an accurate continuous representation of the loss size distribution is the tendency for loss amounts to cluster. Insurers and insureds alike tend to settle for and to prefer round values such as $10,000 instead of $9,765.24 or $10,014.12; or $50,000 as opposed to $49,983.87 or $50,374.28.

There are certain aspects of the loss variable which are difficult to describe in the discrete notation. One example is the mathematical derivation of the change of rate with face. The assumption that *L* is a continuous variable will be assumed in this instance where the relationships would be awkward to derive in discrete notation.

Change of Rate with Face

Whenever losses less than the policy face are possible, the pure premium rate should fall as the policy face increases, either absolutely or relative to value. This property holds regardless of the shape of the size of loss distribution, and even if large losses predominate. In the words of the *1911 Merritt Committee Report*:

> *The principle that the rate falls as the ratio of insurance to value increases holds whatever the figures that are used, even though large losses were relatively more frequent than small ones.*

Proof of this principle is that the first derivative of the premium rate with respect to the policy face in Equation 19.b is negative if losses less than the face are possible. Recalling Equation 19.b:

$$R = \frac{f\left\{ \int_0^F L\, s(L)\, dL + \left(F\left[1 - \int_0^F s(L)\, dL\right] \right) \right\}}{F}, \qquad \textbf{[Equation 19.b]}$$

the derivative is:

$$dR/dF = \left(1/F^2\right) F f \left[F\, s(F) + \left(1 - \int_0^F s(L) dL \right) + F\left(- s(F)\right) \right]$$
$$- \left(1/F^2\right) f \left[\int_0^F L\, s(L)\, dL + F \left(1 - \int_0^F s(L) dL \right) \right]$$

$$= \left(1/F^2\right) F f \left[F\, s(F) - F\, s(F)\right] + \left(1/F^2\right) F f \left(1 - \int_0^F s(L)dL\right)$$

$$- \left(1/F^2\right) f \left[\int_0^F L\, s(L)\, dL\right] - \left(1/F^2\right) F f \left(1 - \int_0^F s(L)dL\right)$$

$$= \frac{-f \int_0^F L\, s(L)\, dL}{F^2} \qquad \textbf{[Equation 23]}$$

Because the variables *f, L, s(L),* and *F* can only assume positive values, and the average loss size represented by the integer expression of Equation 23 must be positive, *dR/dF* is always negative; therefore, if losses less than the policy face are possible, the pure premium rate must decline as the policy face increases. As the policy face may represent the coinsurance requirement, the pure premium rate must also decline as the coinsurance requirement increases.

Example 8: Premium Rate Calculation and Change of Rate with Face (Continuous Case)

Assume losses are distributed according to the linear function:

$$s(L) = (-5.0 \times 10^{-11})\, L + 10^{-5},$$

and that all properties are valued at $200,000. For simplicity, assume no appreciation in property values occurs. A company writes 1,000 policies. Exactly 180 insureds will suffer a loss of some size during any one policy period. Each policyholder purchases only one policy and can choose any positive face value up to the value of the insured property ($0 < F \le V$). Assume only one loss per policy per period is possible.

(1) Show that the s(*L*) function as defined in this example forms a complete probability distribution.
(2) Find the expected average loss per policy.
(3) Find the equation for the pure premium rate per $100 of insurance, *R*, as a function of the face value of insurance, *F*, based on the given loss size function, s(*L*).
(4) Compare the pure premium rate for $100,000 and $105,000 of coverage, using the equation from (3).
(5) Calculate the pure premium *rate of change* at $100,000 and compare it to the answer found in (4).

(1) $$\int_0^V s(L)dL = \int_0^{200,000} \left[(-5.0 \times 10^{-11})\, L + 10^{-5}\right] dL$$

$$= \left. \frac{(-5.0 \times 10^{-11})\, L^2}{2} + 10^{-5} L \right|_0^{200,000}$$

$$= (-2.5 \times 10^{-11})(200,000)^2 + (10^{-5})(200,000)$$

$$= -1 + 2$$

$$= 1$$

(continued)

(continued)

(2)
$$f \int_0^V L\ s(L)dL = \frac{180}{1{,}000} \int_0^{200{,}000} [(-5.0 \times 10^{-11}) L^2 + 10^{-5} L]\ dL$$

$$= 0.18 \left\{ \frac{(-5.0 \times 10^{-11}) L^3}{3} + \frac{10^{-5} L^2}{2} \right\} \Bigg|_0^{200{,}000}$$

$$= (-3.0 \times 10^{-12})(200{,}000)^3 + (9.0 \times 10^{-7})(200{,}000)^2$$
$$= -24{,}000 + 36{,}000$$
$$= \$12{,}000$$

(3)
$$R = \frac{f\left(\int_0^F L\ s(L)\ dL + \{ F [1 - \int_0^F s(L)\ dL] \}\right)}{(F / 100)}$$

$$R = \frac{(0.18)\left(\int_0^F [(-5.0 \times 10^{-11}) L^2 + 10^{-5} L]\ dL + \{ F [1 - \int_0^F (-5.0 \times 10^{-11}) L + (10^{-5}) dL] \}\right)}{(F / 100)}$$

$$R = \frac{(0.18) ([(-5.0 \times 10^{-11}) F^3 / 3 + (10^{-5}) F^2 / 2] + \{F [1 - (-5.0 \times 10^{-11}) F^2 / 2 - (10^{-5}) F] \})}{(F / 100)}$$

$$R = (18) ([(-5.0 \times 10^{-11}) F^2 / 3 + (10^{-5}) F / 2] + [1 - [(-5.0 \times 10^{-11}) F^2 / 2 + (10^{-5}) F])$$

$$R = [(-3.0 \times 10^{-10}) F^2 + (9.0 \times 10^{-5}) F] + [(4.5 \times 10^{-10}) F^2 - (1.8 \times 10^{-4}) F + 18]$$

$$R = (1.5 \times 10^{-10}) F^2 - (9.0 \times 10^{-5}) F + (1.8 \times 10^{1})$$

(4) $R_{100,000} = (1.5 \times 10^{-10})(100{,}000)^2 - (9.0 \times 10^{-5})(100{,}000) + 18 = \$10.50$ per \$100 of insurance

$R_{105,000} = (1.5 \times 10^{-10})(105{,}000)^2 - (9.0 \times 10^{-5})(105{,}000) + 18 = \$10.20$ per \$100 of insurance

(5) $R = (1.5 \times 10^{-10}) F^2 - (9.0 \times 10^{-5}) F + (18)$

$dR/dF = (3.0 \times 10^{-10}) F - (9.0 \times 10^{-5})$

$dR/d(100{,}000) = (3.0 \times 10^{-10})(100{,}000) - (9.0 \times 10^{-5}) = 6.0 \times 10^{-5}$

$(5{,}000)(6.0 \times 10^{-5}) = -\$0.30$

$\$10.50 - \$10.20 = -\$0.30$

(continued)

(continued)

Note that Equation 23 may be used, slightly modified to reflect $100 units of coverage, to find the derivative:

$$dR/dF = \frac{-f \int_0^F L\, s(L)\, dL}{F^2 / (100)}$$

$$= \frac{(-0.18)\,([(-5.0 \times 10^{-11})\, F^3 / 3 + (10^{-5})\, F^2 / 2]}{F^2 / (100)}$$

$$= (3.0 \times 10^{-10})\, F - (9.0 \times 10^{-5})$$

Rate of Premium Rate Change

As shown in Equation 20, the pure premium rate should fall as the policy face increases, whether the actual distribution is a decreasing, flat, or increasing function of the loss size, so long as losses less than the policy face are possible. Aside from losses being all of one size, no other shape of the size of loss distribution will void this property; however, the shape will impact the *rate* at which the premium rate change decreases.

The *rate of decrease* in the pure premium rate depends on whether small or large losses predominate. These relationships stem from the second derivative of the pure premium rate with respect to the policy face, i.e. the second derivative of Equation 19.b. The relationships are shown below in Table I.

- ***T A B L E I*** -

Shape of the size of loss distribution:	**Second derivative of the pure premium rate:**	**Rates decrease at:**
(a) small losses outnumber large ones	(a) positive	(a) a decreasing rate
(b) losses of all sizes are equally numerous	(b) zero	(b) a constant rate
(c) large losses outnumber small ones	(c) negative	(c) an increasing rate

The second derivative is an indicator of concavity. A curve with a positive second derivative is concave up, and one with a negative second derivative is concave down. Given that losses less than the policy face are possible, the pure premium rate will be decreasing. On a decreasing curve, concave up suggests flattening such that the rate of change is declining, while concave down suggests steepening such that the rate of change is increasing.

Example 9: Small Losses Predominating versus Large Losses Predominating (Continuous Case)

In Example 8, it is assumed that losses are distributed according to a linear function and all properties are valued at $200,000. The company writes 1,000 policies, with exactly 180 insureds suffering a loss of some size during a policy period. Each policyholder purchases only one policy and can choose any positive face value up to the value of the insured property ($0 < F \le V$), where only one loss per policy per period is possible. Under the same assumptions, compare the two linear functions:

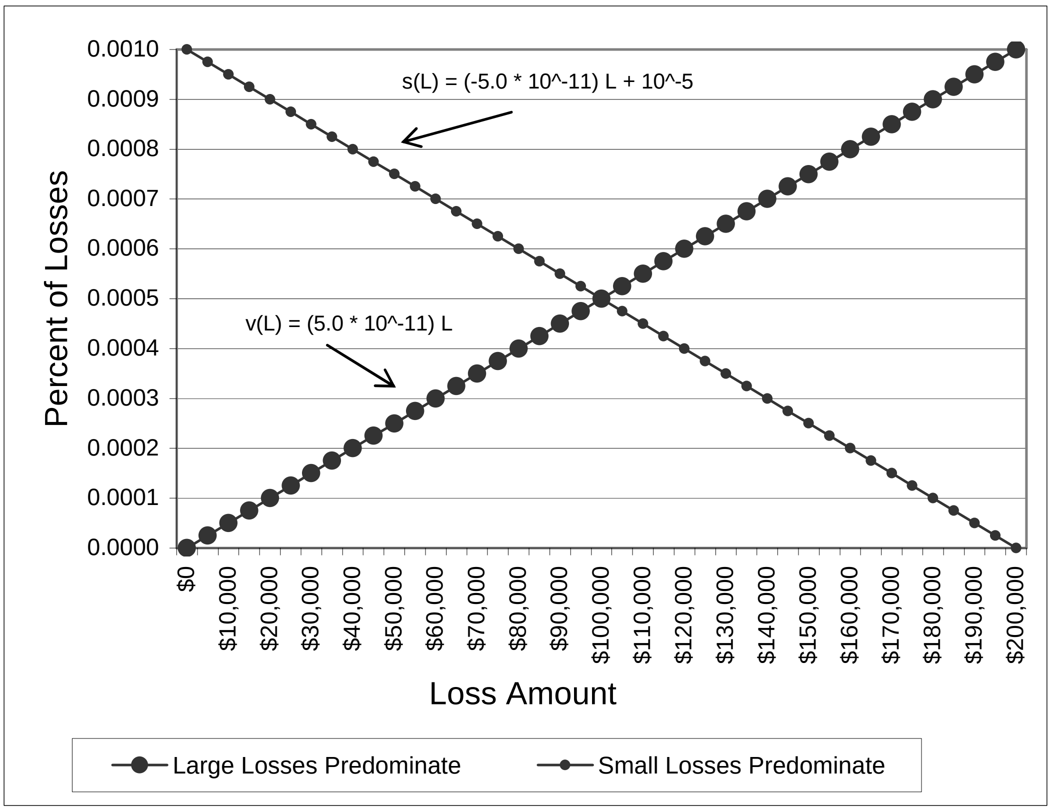
$$s(L) = (-5.0 \times 10^{-11})\, L + 10^{-5},$$
$$v(L) = (5.0 \times 10^{-11})\, L$$

(continued)

(continued)

(1) Graph the two functions on the interval $0 \leq L \leq 200{,}000$, to confirm graphically that small losses predominate for s(L) while large losses predominate for v(L).
(2) Show mathematically that s(L) is a decreasing function of L and v(L) is an increasing function of L.
(3) Find the equation for the pure premium rate per $100 of insurance, *R*, as a function of the face value of insurance, *F*, based on the loss size function, v(*L*).
(4) Graph the pure premium rates for s(L) and v(L) using the equations found in Part (3) of Example 8 and Part (3) of this example.
(5) Find the second derivatives of the pure premium rates for s(L) and v(L), and explain what the second derivatives imply about the shape of the pure premium rate curves.

(1)



(2) Find the first derivatives of the two linear functions:

$s'(L) = -5.0 \times 10^{-11} < 0$ ➔ s(L) is a decreasing function of L
$v'(L) = 5.0 \times 10^{-11} > 0$ ➔ v(L) is an increasing function of L

(3)

$$R = \frac{f\left(\int_0^F L\, v(L)\, dL + \left\{ F \left[1 - \int_0^F v(L)\, dL\right] \right\} \right)}{(F / 100)}$$

(continued)

(continued)

$$R = \frac{(0.18)\left(\int_0^F (5.0 \times 10^{-11})\, L^2 \, dL + \left\{ F\left[1 - \int_0^F (5.0 \times 10^{-11})\, L \, dL\right]\right\}\right)}{(F / 100)}$$

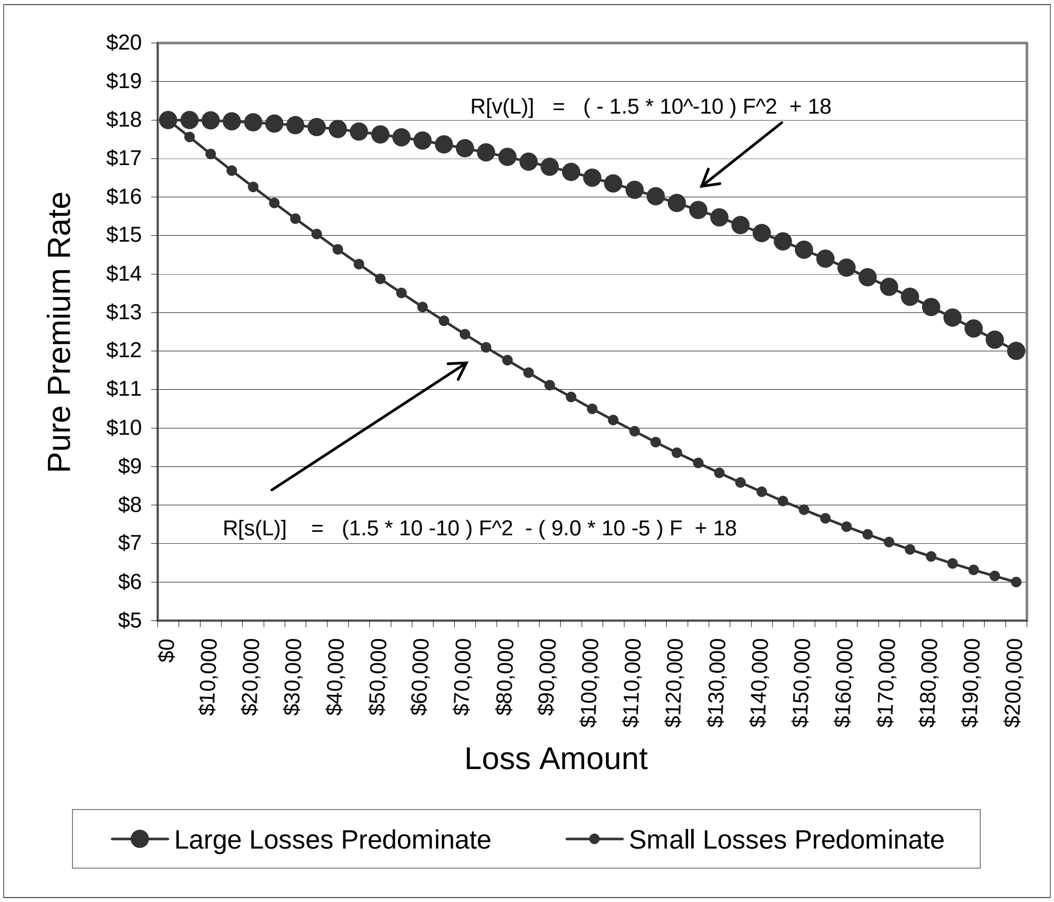
$$R = \frac{(0.18)\left([(5.0 \times 10^{-11})\, F^3 / 3] + \{F[1 - (5.0 \times 10^{-11})\, F^2/2]\}\right)}{(F / 100)}$$

$$R = (18)\left(\left[(5.0 \times 10^{-11})\, F^2 / 3\right] + \left[1 - (5.0 \times 10^{-11})\, F^2 / 2\right]\right)$$

$$R = (3.0 \times 10^{-10})\, F^2 + (-4.5 \times 10^{-10})\, F^2 + 18$$

$$R = (-1.5 \times 10^{-10})\, F^2 + 18$$

(4)



(5) R[s(L)] was found in Part (3) of Example 8:

$$R[s(L)] = (1.5 \times 10^{-10})\, F^2 - (9.0 \times 10^{-5})\, F + (1.8 \times 10^{1})$$
$$dR[s(L)]/dF = (3.0 \times 10^{-10})\, F - (9.0 \times 10^{-5})$$
$$d^2R[s(L)]/dF^2 = (3.0 \times 10^{-10}) > 0$$

➔ R[s(L)] is concave up.

(continued)

(continued)

R[v(L)] was found in Part (3) of this example:

$$R[v(L)] = (-1.5 \times 10^{-10})\,F^2 + 18$$
$$dR[v(L)]/dF = (-3.0 \times 10^{-10})\,F$$
$$d^2R[v(L)]/dF^2 = (-3.0 \times 10^{-10}) < 0 \quad \Rightarrow \quad R[v(L)] \text{ is concave down.}$$

Since $d^2R[s(L)]/dF^2$ is positive, R[s(L)] is concave up and rates are decreasing at a decreasing rate.
Since $d^2R[v(L)]/dF^2$ is negative, R[v(L)] is concave down and rates are decreasing at an increasing rate.

Pure Premium Coinsurance Rates

As stated earlier, a particular pricing problem arises when each insured has the choice of policy face, yet premium rates can be appropriate only if each insured chooses the policy face assumed in the rate calculation. *Pure premium coinsurance rates* are computed on the assumption of a policy face equal to the coinsurance requirement. If the policy face is less than or equal to that assumed, the coinsurance mechanism will balance pure premiums and expected indemnity payments.

While the coinsurance mechanism can ensure balance when the policy face falls *below* the level assumed in the rates, such a balance is not achieved when the policy face *exceeds* that which is assumed. Since proper pure premium rates vary inversely with coinsurance requirements, the insured who selects coverage exceeding that requirement will be paying a rate too high. Because the policy will never indemnify beyond the full property value, it is not in the insured's best interest to select a face value exceeding the full property value. A coinsurance rate for full coverage must be made available to ensure equity to all insureds.
Note that the insured may collect total indemnity payments greater than the full property value when "additional coverages," such as debris removal, apply. Such coverages may be included in a homeowner's policy in addition to the face value applicable to the dwelling structure. It is unnecessary to purchase a level of insurance coverage exceeding the value of the dwelling to receive the additional coverages.

Percentage Coinsurance Rates

Under a percentage of value coinsurance clause, the insured is offered a choice of coinsurance percentage, *c*, from those percentages made available. Recall that the coinsurance requirement, *C*, is given by:

$$C = cV; \qquad \textbf{[Equation 1]}$$

Since the pure premium coinsurance rate relativities are determined by the distribution of losses as percentages of value, any property value, *V* can be used with the distribution. Because the assumed policy face equals *C*, the premium rate equations, from Equations 19.a and 19.b, become:

$$R = \frac{f\left(\sum_{L=1}^{L=C} L\, s(L)\, dL + \left\{ F\left[1 - \sum_{L=1}^{L=C} s(L)\, dL\right] \right\}\right)}{F} \qquad \textbf{[Equation 24.a]}$$

if *L* is discrete,

or:

$$R = \frac{f\left(\int_0^C L\, s(L)\, dL + \left\{ F\left[1 - \int_0^C s(L)\, dL\right] \right\} \right)}{F} \qquad \textbf{[Equation 24.b]}$$

if L is continuous.

Example 10: Rate of Premium Rate Change - Coinsurance Rates

Assume that losses from three different populations are distributed according to three distinct loss size distributions $s_1(L)$, $s_2(L)$, and $s_3(L)$, as given below, that and all distributions have the same loss frequency, $f = .04$, and that all properties are valued at $500,000. Note that large losses predominate in $s_1(L)$, loss sizes are uniform in $s_2(L)$, and small losses predominate in $s_3(L)$. For simplicity, assume that the arithmetic mean loss in each loss size interval is the midpoint of the interval, except for the first and last intervals of $s_1(L)$ and $s_3(L)$, which are slightly skew. For each distribution, find the coinsurance premium rate per $100 of coverage on each interval, and calculate the decrease in the premium rate between intervals.

Coinsurance percentage	Loss Size L (in $000s) $x_1 < L < x_2$ x_1	x_2	Percentage of Losses in interval $s_1(L)$	$s_2(L)$	$s_3(L)$	Arithmetic Mean Loss in interval(in $000s) $s_1(L)$	$s_2(L)$	$s_3(L)$
20%	$0	$100	.05	.20	.50	$25	$50	$75
40%	$100	$200	.10	.20	.20	$150	$150	$150
60%	$200	$300	.15	.20	.15	$250	$250	$250
80%	$300	$400	.20	.20	.10	$350	$350	$350
100%	$400	$500	.50	.20	.05	$475	$450	$425

Coinsurance percentage	Coinsurance Premium Rate per $100 $s_1(L)$	$s_2(L)$	$s_3(L)$	Rate Change from prior interval $s_1(L)$	$s_2(L)$	$s_3(L)$
20%	3.85	3.60	3.50			
40%	3.73	3.20	2.55	-0.12	-0.40	-0.95
60%	3.52	2.80	2.00	-0.21	-0.40	-0.55
80%	3.24	2.40	1.60	-0.28	-0.40	-0.40
100%	2.89	2.00	1.29	-0.35	-0.40	-0.31

Example 10 illustrates that premium rates decrease at a increasing rate when large losses predominate, at a constant rate when loss sizes are uniform, and at a decreasing rate when small losses predominate.

Adjustments for Expense Allowances

Up to this point, this paper has dealt only with *pure* premium rates. The equation for the pure premium rate is designed to cover the insurer's expected indemnity payments only. The *gross* premium rate equals the pure premium rate plus a loading for operating expenses, contingency reserves, and profit. The gross premium rate is the rate that the insurer will actually charge. Both the gross and the pure premium rates are generally stated as rates per $100 of property insurance. It has been shown that the pure premium rate will vary with the amount of insurance, the policy face. The gross premium rate should also vary with the policy face. The question is whether or not gross premium rates decline in the same manner as pure premium rates, as outlined in Table I?

Rating formulas will often represent loadings as a constant percentage of pure premium. Alternatively, expense loadings may also commonly be fixed in amount. If all loadings are a constant percentage of the pure premium rate, gross rates will be a constant multiple of pure rates; the gross rates will have the same relativities of the pure rates, and Table I will be unaltered. Loadings that are a fixed amount represent a decreasing percentage of the premium rate as the policy face increases, and its percentage decreases at a decreasing rate. The average of this fixed cost falls at a decreasing rate as the policy face increases. When small losses outnumber large ones, as in case (a) of Table I, fixed loadings will cause gross rates to still decrease at a decreasing rate, although faster than the underlying pure rates. When large losses outnumber small ones, as in case (c), whether the rate of decrease in gross rates is increasing or decreasing will depend on both the skewness of the size of loss distribution and the amount of the fixed loadings. Fixed loadings will more certainly impact case (b), where loss sizes are uniform, causing the gross rates to decrease at a decreasing rather than a constant rate.

In most size of loss distributions, small losses - losses that are small percentages of value - outnumber large ones. This property will lead to both pure and gross coinsurance rates which decrease by decreasing amounts as the coinsurance requirement increases over equally-sized increments. This will be true whether loadings are variable or fixed.

Premium Reversals

A premium reversal occurs when the difference in incremental premium rates is so great that the cost of more insurance is less than the cost of less insurance. If the percentage decrease in the gross premium rate is greater than the percentage increase in the policy face, a premium reversal will result on that interval. A premium reversal will lead to a negative marginal revenue for the insurer, suggesting that the marginal cost of that additional insurance is also negative. As this can not be the case, most premium reversals are actuarially incorrect and should be avoided.

There are a few normal circumstances under which minor premium reversals may occur. In the instance where coinsurance rates are grouped in rating bands, for instance, reversals will be inevitable at the boundaries of the bands. For example, if coinsurance percentages are grouped in bands at 10 percent intervals, a premium reversal could exist between 89 and 90 percent of coverage, such that the policy would cost less at 90 percent coinsurance than at 89 percent coinsurance. Even so, no reversals should be found within each band, such as within the range from 80 to 89 percent coinsurance.

Range of Coinsurance Requirements and Credits

As noted earlier, the coinsurance mechanism balances premium rates with expected indemnity payments so that rates will be equitable and just adequate. While the coinsurance mechanism allows for rate adequacy and equity when the policy face falls *below* the level assumed in the rates, such a result is not achieved when the selected policy face *exceeds* that which is assumed.

For coinsurance to function as intended, the insurer must provide a full range of coinsurance requirements and credits. Each coinsurance requirement should specify a separate rate which dictates the appropriate credit. The coinsurance requirements should be offered at small intervals to the greatest extent practical. A coinsurance clause should be attached to even the smallest of policies to create a balance of expected indemnity and pure premium.

Summary and Conclusion

Insurance to value exists if property is insured to the exact extent assumed in the premium rate calculation. Maintaining insurance to value defines the goal of maintaining coverage among insureds at a level equal to that assumed within the actuarial premium rate calculations.

A coinsurance clause will automatically render the policyholder a joint insurer anytime a coinsurance deficiency exists, that is, when the policy face falls short of the required level of coverage according to the coinsurance requirement as defined by the insurer. The coinsurance clause is a tool for maintaining balance in actuarial premium rate calculations in those instances in which the policyholder is underinsured. The clause will not have the effect of creating balance when the policyholder is overinsured.

A coinsurer will be subject to a coinsurance penalty in the event of any loss beneath the coinsurance requirement. If the loss exceeds the coinsurance requirement, the indemnity is limited by the face value of the policy and no coinsurance penalty exists. Assuming the face value of the policy is less than the coinsurance requirement, losses exceeding the policy face but below the coinsurance requirement are still subject to a coinsurance penalty.

It is rare for a coinsurance penalty to exist under an agreed amount endorsement, where the value of the property may be established by appraisal and must be agreed upon between the insurer and the insured. Under this type of endorsement, the insurer is subject to a greater risk of underinsurance than under the percentage coinsurance requirement. Under an agreed amount endorsement, the insurer must pay the full loss amount for any loss up to the policy face, even if the property appreciates during the policy period.

The fire size of loss severity distribution generally follows a downward-sloping curve. The actual shape of the curve will depend on the type of property that is included in the distribution. There are many factors which can introduce distortions into the data collected for the purpose of approximating a size of loss curve.

The pure premium rate is the expected dollar amount of indemnity per unit of face value. The general pure premium rate equation is a mathematical expression simplified due to several assumptions. The assumptions are that premium rates are equitable and just adequate, that all insured properties are homogeneous, that the rate determinants (frequency and severity) are known and stable, that losses are precisely adjusted, that each insured carries no other property insurance, that there is no interest on the pure premiums collected, and that each policy has no more than one loss per period. All assumptions are not necessarily expected to hold in application. The impact that the divergence of these assumptions from reality has on the accuracy of premium rates derived from the equation should be negligible.

The pure premium rate declines as the policy face increases, according to the first derivative of the general pure premium rate equation which will always assume a negative value. According to the second derivative, the pure premium rate will decrease (over equal-sized incremental increases in the face value) at an increasing rate if large losses predominate, at a constant rate if all loss sizes are equal, and at a decreasing rate if small losses predominate. The latter case is typical for homeowner pure premiums.

There are three major implications of the model for gross premium coinsurance rates: a full range of coinsurance requirements and credits should be available at separate premium rates; no premium "reversals" should exist, except at borders between bands of premium rates; and both the gross coinsurance rates and the underlying pure rates should decrease at a declining rate with added coverage, given that small losses predominate. These three results are implied by both agreed amount clauses and percentage coinsurance requirements.

The coinsurance clause solves some of the problems that arise when insurance to value is not achieved, but care must be taken to avoid severe and widespread underinsurance. Tools and techniques of appraisal and policy provisions exist which help to maintain insurance to value. Homeowners policies typically include a clause which ties the face value to a construction price index, leading to automatic increases in both the coverage level and the total premium with inflation. The problem of underinsurance may also be alleviated by pricing policies to cover a proportion of losses greater than the coinsurance requirement in instances that the coverage level requirement is met. As demonstrated by the Oakland firestorms of 1991, it is in the best interest of the insurer to strive to maintain insurance to value.

Endnotes:

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[0] The source of the core definitions and equations used in this paper is *Insurance to Value*, by George L. Head, CPCU, CLU, published by the S. S. Huebner Foundation for Insurance Education © 1971.

[i] Source: The Random House Dictionary of the English Language, Second Edition, Unabridged, ©1987 by Random House, Inc.

[ii] Source: www.insure.com

[iii] Source: The Oakland Tribune, April 1, 1992, page A-3, and April 2, 1992, page A-1, and June 11, 1992, page A-3.

[iv] Source: Interview with Ed McKeon, Vice President of Corporate Relations, California Casualty Group, May 14, 2003.