

SMOOTHING WEATHER LOSSES: A TWO-SIDED PERCENTILE MODEL

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Abstract

This paper presents a method for smoothing wind losses when calculating rate indications, but it can apply equally well to other weather events such as hail or freezing. It can be used in other situations such as smoothing out the effects of large losses or other large, but infrequent, events. The model is relatively easy to explain to non-actuaries, and it is not difficult to implement. The traditional approach applies a one-sided cap to losses. This paper presents a two-sided model that bounds losses on both the low and high sides.

1. INTRODUCTION

The current state-of-the-art in pricing for major loss-producing events lies in sophisticated computer models that combine both the damageability of risks and the damage causing potential for events such as hurricanes or earthquakes. Much effort and expense are being directed towards applying these models to insurance ratemaking. But, the actuary, like any skilled craftsman, still needs simple, basic tools to handle tasks where more sophisticated methods cannot be readily applied.

2. STABILIZING RATEMAKING LOSS RATIOS

Although premiums and losses over a span of several years provide the basis for calculating a rate change indication, irregular events during that period can produce large swings in the rate indication. One weekend of tornadoes in a state can cause a large increase in a state's property rate indication even though it may be based on five years of data. Conversely, several years of exceptionally good weather in a state may drive the indication in the opposite direction.

Table 1 contains an example of five years of ratemaking data for a line of insurance that includes coverage for wind losses. Partial loss ratios have been computed for wind losses and all other losses. The wind loss ratio ranges from a low of 3.0% in 1994 to a high of 40.0% in 1996.

Table 1				
Accident Year	Earned Premium (\$000)	Wind Loss Ratio	All Other Loss Ratio	Combined Loss Ratio
1992	\$ 714	9.9%	45.0%	54.9%
1993	654	14.0	54.9	68.9
1994	750	3.0	43.4	46.4
1995	870	17.4	49.5	66.9
1996	907	40.0	61.0	101.0
Total	\$3,895	17.9%	51.1%	69.0%

The starting point for applying this smoothing procedure is to collect the wind loss data in the state for a longer time period. Seventeen years¹ of earned premium and wind loss ratios are displayed in Columns 1 and 2 of Table 2. From Column 2, two percentiles are computed and displayed: a 33rd percentile and a 67th percentile.² Normal wind loss ratios as shown in Column 3 are defined to be those loss ratios limited to the range of 5.5% to 14.0%, the 33rd and 67th percentiles. If a wind loss ratio falls below this range, then the 33rd percentile value is substituted. Correspondingly, if a wind loss ratio is above this range then the 67th percentile is used.

¹ Seventeen years of data was available for this line of insurance and state. The procedure does stabilize ratemaking loss ratios using data from this relatively short time period. But, this is too short a record to recognize the potential impact of catastrophic weather losses.

² Arranging the data from smallest to largest, n_1, n_2, \dots, n_m , then n_1 is the 0th percentile and n_m is the 100th percentile. For data points in between, n_k is the $100 \times (k-1) / (m-1)$ percentile. Other percentiles are computed by interpolation.

Table 2

Calculation of Normal Wind Losses And Additional Wind Load

<u>Year</u>	(1) Earned Premium (\$000)	(2) Wind Loss Ratio	(3) Normal Wind Loss Ratio	(4) Difference = (2) - (3)	(5) Load = (1) x (4) (\$000)	(6) Adjusted Wind Loss Ratio = (3) + Wind Load
1980	402	0.0 %	5.5 %	-5.5 %	-22	7.6 %
1981	462	9.6	9.6	0.0	0	11.7
1982	560	19.7	14.0	5.7	32	16.1
1983	601	1.4	5.5	-4.1	-25	7.6
1984	664	13.7	13.7	0.0	0	15.8
1985	691	4.4	5.5	-1.1	-8	7.6
1986	736	4.0	5.5	-1.5	-11	7.6
1987	620	13.9	13.9	0.0	0	16.0
1988	669	0.5	5.5	-5.0	-34	7.6
1989	673	8.4	8.4	0.0	0	10.5
1990	659	21.7	14.0	7.7	51	16.1
1991	710	14.8	14.0	0.8	6	16.1
1992	714	9.9	9.9	0.0	0	12.0
1993	654	14.0	14.0	0.0	0	16.1
1994	750	3.0	5.5	-2.5	-19	7.6
1995	870	17.4	14.0	3.4	30	16.1
1996	<u>907</u>	<u>40.0</u>	14.0	26.0	<u>236</u>	<u>16.1</u>
Total	\$11,342	12.4 %			\$237	12.4 %

Calculation of Normal Range

33rd Percentile	5.5 %
67th Percentile	14.0 %

Calculation of Wind Load

$$\text{Load} = \frac{237}{11,342} = 2.1 \%$$

Normal Wind Loss Ratio

1. If "Wind Loss Ratio" < 33rd Percentile, then "Normal Wind Loss Ratio" = 33rd Percentile
2. If "Wind Loss Ratio" > 67th Percentile, then "Normal Wind Loss Ratio" = 67th Percentile
3. Otherwise, "Normal Wind Loss Ratio" = "Wind Loss Ratio"

The average off-balance of this bounding procedure is computed to determine a wind load. Column 4, the difference between Columns 2 and 3, represents how many loss ratio points to add or subtract to bring the wind loss ratio into the “normal” range. Column 5, the product of Columns 1 and 4, is the dollar impact of this bounding procedure. A load that recognizes the off-balance of the bounding is calculated by dividing the sum of Column 5 by the sum of Column 1.³ An alternative calculation for the load would be to use the average of Column 4 rather than the earned premium weighted average. The last column, the Adjusted Wind Loss Ratio, is the sum of the Normal Wind Loss Ratio and the Wind Load.

Now the results of the calculation can be applied to the ratemaking data in Table 1. There are two steps to adjusting the wind loss ratios: (i) restrict each loss ratio to the normal range [5.5,14.0], the 33rd and 67th percentiles, and (ii) add the balancing wind load factor to each loss ratio. The results are displayed in Table 3.

Accident Year	(1) Unadjusted Wind Loss Ratio	(2) Normal Wind Loss Ratio	(3) Wind Load	(4) Adjusted Wind Loss Ratio	(5) All Other Loss Ratio	(6) Combined Loss Ratio =(4)+(5)
1992	9.9%	9.9%	2.1%	12.0%	45.0%	57.0%
1993	14.0	14.0	2.1	16.1	54.9	71.0
1994	3.0	5.5	2.1	7.6	43.4	51.0
1995	17.4	14.0	2.1	16.1	49.5	65.6
1996	40.0	14.0	2.1	16.1	61.0	77.1
Total	17.9%	11.6%	2.1%	13.7%	51.1%	64.8%

³ This method of calculating a wind load has the effect of allocating the excess wind losses based on earned premium. Sometimes excess losses are allocated using losses, for example by computing the ratio of excess losses to losses and then applying this factor to observed losses, but relating the excess losses to premium usually gives a more stable result.

Compare Columns 1 and 2. Note that three out of five, or 60%, of the Unadjusted Wind Loss Ratios were capped by the bounding procedure. This outcome is consistent with the selection of the 33rd percentile to the 67th percentile as the normal range for the seventeen-year period. On average about 66% of the observed wind loss ratios would fall outside of the normal range. The total Adjusted Wind Loss Ratio (bottom of Column 4) is 13.7% which lies between the unadjusted 17.9 in the five years of ratemaking data and 12.4%, the seventeen-year average wind loss ratio (bottom of Column 2 of Table 2). The procedure blends both current and long-term experience.

3. SELECTION OF NORMAL LOSS RATIO RANGE

The selection of a normal loss ratio range from the 33rd percentile to the 67th percentile was based on judgment. Table 4 shows a sample of ranges.

Table 4				
Percentiles		Normal Loss Ratio Range		Wind Load
Lower	Upper	Lower	Upper	
0%	100%	0.0%	40.0%	0.0%
10	90	1.0	20.5	1.6
20	80	3.2	16.9	1.9
30	70	4.3	14.2	2.4
33	67	5.5	14.0	2.1
40	60	8.9	13.8	1.0
45	55	9.7	12.9	1.1
50	50	9.9	9.9	2.5

The first row in the table results in no wind smoothing, whereas, the last row is equivalent to uniformly spreading the average wind loss ratio to all years. Although the table displays minimum and maximums which are symmetric about the midpoint, a symmetric range is not necessary.

Typically, wind smoothing models only cap “upside” events that push loss ratios above some selected amount. The model presented here also adjusts for years that have very good weather

because these years can drive ratemaking indications towards inadequate rates. For this reason, a two-sided capping model can be more effective at smoothing out loss ratios than a one-sided model. In fact, the one-sided model is just a special case of the two-sided where the lower cap is set at the 0th percentile. Because a two-sided model uses two parameters, it offers the opportunity for a better fit to the data than with a one-sided, one-parameter model.

In selecting a percentile range, the actuary is confronted with the eternal tradeoff in ratemaking: stability versus responsiveness. A narrow percentile range will produce more stable loss ratios. But, a narrow range may not be responsive to longer term changes in weather patterns, the geographic distribution of insureds, construction techniques, underwriting, or other factors that contribute to the level of risk. A wider range allows the ratemaking model to adjust more quickly for the changing level of risk, but at the cost of more year-to-year variability in the loss ratios.

4. FINDING A GOOD FIT TO THE HISTORICAL DATA

The selection of a normal loss ratio interval does not have to be left entirely to judgment. Quantitative measures can help eliminate weaker choices. To demonstrate this, the one-sided capping model mentioned above will be compared to the [33rd percentile, 67th percentile] bounding procedure. The last column of Table 2 shows the Adjusted Wind Loss Ratios after the bounding and load operations. The difference between the highest and lowest loss ratios in Column 6 of Table 2 is 8.5%. The width of the range can be considered a measure of the stability of the loss ratios resulting from application of the procedure with the selected percentiles.

When using one-sided capping for excess wind loss ratios the lower bound is set at 0.0%. This forces the upper bound to be set at the 38th percentile in order to generate a range between the highest and lowest Adjusted Wind Loss Ratios as low as 8.5%! Any higher upper

bound (with the lower bound fixed at the 0th percentile) will result in a wider range between the highest and lowest Adjusted Wind Loss Ratios, reducing stability. Table 5 compares the selected two-sided model in the first box and the one-sided model in the second box with the same level of year-to-year stability.

The difference between the high and low values of the Adjusted Wind Loss Ratios equals the width of the range of the Normal Wind Loss Ratios because the Wind Load is added to both of the endpoints of the normal range.

The range for each set of adjusted loss ratios is displayed in the boxes at the bottom of Table 5. Below these ranges are two measures of how well the Adjusted Wind Loss Ratios fit the raw data in Column 2. The first measure is the sum of the squares of the differences between the adjusted and raw data and the second measure is the sum of the absolute values of the differences. Under both of these measures the [33rd percentile, 67th percentile] bounding beats the one-side capping for a given level of stability. For a selected level of stability represented by the range, the two-sided model produces a better fit to the raw data.

The average Wind Loss Ratio over the seventeen-year period is 12.4%. Year-to-year fluctuation in wind losses could be eliminated by taking all of the wind losses out of the ratemaking data and substituting this long-term average. Note that this 12.4% lies closer to the midpoint of the two-sided range [7.6%, 16.1%] than it does to the midpoint of the one-sided range [6.0%, 14.5%]. The two-sided model produces results which are better balanced about the long-term average for the sample data.

Frequently the actuary must rely on judgment to select ratemaking parameters. With this model the actuary does need to rely on judgment to select the desired degree of stability (i.e. the range in the Adjusted Wind Loss Ratios), but can quantitatively search for minimum and maximum percentiles that fit the historical data well for the chosen level of stability. (It is

**Table 5
Two-Sided Versus One-Sided Capping**

Year	(1)	(2)	Two-Sided Model		One-Sided Model	
	Earned Premium (\$000)	Wind Loss Ratio	(3) Normal Wind Loss Ratio	(4) Adjusted Wind Loss Ratio = (3)+Load	(5) Normal Wind Loss Ratio	(6) Adjusted Wind Loss Ratio = (5)+Load
1980	402	0.0 %	5.5 %	7.6 %	0.0 %	6.0 %
1981	462	9.6	9.6	11.7	8.5	14.5
1982	560	19.7	14.0	16.1	8.5	14.5
1983	601	1.4	5.5	7.6	1.4	7.4
1984	664	13.7	13.7	15.8	8.5	14.5
1985	691	4.4	5.5	7.6	4.4	10.4
1986	736	4.0	5.5	7.6	4.0	10.0
1987	620	13.9	13.9	16.0	8.5	14.5
1988	669	0.5	5.5	7.6	0.5	6.5
1989	673	8.4	8.4	10.5	8.4	14.4
1990	659	21.7	14.0	16.1	8.5	14.5
1991	710	14.8	14.0	16.1	8.5	14.5
1992	714	9.9	9.9	12.0	8.5	14.5
1993	654	14.0	14.0	16.1	8.5	14.5
1994	750	3.0	5.5	7.6	3.0	9.0
1995	870	17.4	14.0	16.1	8.5	14.5
1996	907	40.0	14.0	16.1	8.5	14.5
			<u>Normal Range:</u>		<u>Normal Range:</u>	
			33rd Percentile	5.5 %	0th Percentile	0.0 %
			67th Percentile	14.0 %	38th Percentile	8.5 %
			Wind Load =	2.1 %	Wind Load =	6.0 %
			Range = (Maximum-Minimum)	8.5 %	Range = (Maximum-Minimum)	8.5 %
			Sum of squares of (4) - (2) =	839.0	Sum of squares of (6) - (2) =	1035.5
			Sum of absolute value of (4) - (2) =	80.7	Sum of absolute value of (6) - (2) =	94.3

possible that the sum-of-squares and the sum-of-absolute-values measures may give conflicting signals. The actuary will have to decide which measure is more meaningful for the situation.)

Table 6 shows various ranges for the Adjusted Wind Loss Ratios and corresponding percentiles that produce good fits to the data. The first column shows the stability constraint, how much variability is allowed in the Adjusted Wind Loss Ratios. Then, the next columns display percentiles which satisfy the stability constraint and have low values for the sum-of-squares errors of the fit.⁴ The last column shows the five-year Total Adjusted Wind Loss Ratio after application of the procedure to the ratemaking data in Table 1.

Table 6					
Wind Loss Ratio Range	"Good Fit" Normal Loss Ratio Bounds		Wind Load	Sum-of-squares Error	1992-1996 Total Adjusted Wind Loss Ratio
	Lower	Upper			
0 points	50 th percentile = 9.9%	50 th percentile = 9.9%	2.5%	1608.3	12.4%
5 points	40 th percentile = 8.9	64 th percentile = 13.9	0.9	1078.5	13.1
10 points	37 th percentile = 8.1	83 rd percentile = 18.1	0.1	724.0	13.9
15 points	33 rd percentile = 5.5	90 th percentile = 20.5	0.6	469.1	14.5
20 points	11 th percentile = 1.2	92 nd percentile = 21.2	1.5	342.8	15.1
30 points	11 th percentile = 1.2	97 th percentile = 31.2	0.6	77.0	16.5
40 points	0 th percentile = 0.0	100 th percentile = 40.0	0.0	0.0	17.9

In the first row of the table the Adjusted Wind Loss Ratio is a constant 12.4% = 9.9% + 2.5%; the long-term average wind loss ratio would be substituted for the actual wind loss ratios in the ratemaking experience period. In the last row, there is no smoothing on the data which itself has a 40 point range. Of course, as the stability constraint is loosened, the fit to the data improves.

⁴ The percentiles were computed by solving for the two percentiles which satisfied the stability constraint in the first column and that minimized the sum-of-squares error using the "Solver" routine in a Microsoft Excel spreadsheet. The iteration stopping point of the routine depended on the initial values. So, it was necessary to try a number of initial starting points and compare sum-of-squares errors for the resulting iteration stopping points and then pick the one with the lowest error.

5. CONCLUSION

The two-sided capping model presented here achieves the same end as the traditional “upside” capping model: the stabilization of loss ratios used in ratemaking. But, for the same degree of stabilization, two advantages of the two-sided model with the sample data were noted: (1) it fits the historical data better, and (2) the range of resulting loss ratios is more evenly balanced around the long-term average loss ratio. Also, the two-sided model tempers the impact on the rate indication of unusually good weather during the ratemaking period.

This model offers the actuary considerable flexibility in stabilizing the effects of volatile losses on ratemaking. Choices range from a high degree of stabilization by choosing a [50th percentile, 50th percentile] range to complete responsiveness with a [0th-percentile, 100th percentile] range, or anything in between.

Since percentiles involve ranking and counting, the concept is easier to explain to non-actuaries than a less intuitive concept such as standard deviation. Standard deviation has been used in some actuarial models to define the acceptable range of variability in weather losses, but when dealing with highly skewed distributions, a percentile is more meaningful and easier to understand.

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