VERIFICATION OF OUTSTANDING CLAIM PROVISIONS— SEPARATION TECHNIQUE

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In reference [1] Dr. G. C. Taylor has described a useful advance in the techniques available for verification of outstanding claims estimates when the data provided is the cohort development of numbers and amounts of claims. In this note it is assumed that the numbers relate to settled claims and that the amounts relate to claim payments, so there is an implicit assumption that the pattern of partial payments is constant. If the amounts of settled claims were to be used, there would be a one/one relationship between the numbers and amounts, but the effect of the exogeneous factor would be blurred because the settlements in a year other than the first include partial payments made some time previously, and, by hypothesis, based on different factors. If information relating to partial payments is available the data can be examined for any major fluctuation in the pattern and allowance made accordingly.

2. In paragraph (2) of reference [1] a brief description is given of a standard routine calculation in which the average distribution function of claim payments in time is estimated from the triangle of payments by a chain ladder technique. This distribution function is then used to estimate the expected development of the incomplete cohorts, the implicit assumption being made that the function was stable in time. With a constant rate of inflation the results obtained by this technique were found to be satisfactory but with a rapid increase in the rate of inflation the distribution function changed so that projection led to underestimates of the future claims payments. Various methods of adjusting the projections to allow for the change in the rate of inflation have been investigated, but they all involve an important element of subjective judgment and so far no generally suitable basis for "automatic" verification by this particular technique has been discovered. See however reference [2].

3. Dr. Taylor's separation technique provides an alternative approach and has been found of value in a number of practical applications in that it has been possible to identify deviations from the underlying hypothetical model with administrative changes within companies. This feature of the technique is a useful addition to the analytical tools available to controllers or auditors. It also provides an "objective" method of allowing for irregular changes in the rate of inflation.

4. As set out, the separation method uses an appropriate index of numbers of claims as a standardisation measure. On occasions a suitable figure for the numbers of claims is not available or the figures available may be suspect for various reasons. Other quantities, such as premiums, may be used as a proxy for the numbers of claims-but if this is done some care is needed because other variations may be introduced into the model. For example if premiums are used, the results will reflect changes in the relationship between premiums and claims.

5. If the number of claims is not available it would be useful to have a separation technique based solely on the amounts of claims. Dr. Taylor's comments in para 7 of (1) are relevant. Accordingly when two sets of claims development data covering 7 and 12 years respectively became available recently, consideration was given to devising a separation technique. This proved effective in these cases and although for reasons of confidentiality the figures cannot be quoted, it is considered of value to record the method used.

6. The data are assumed to be provided in the following form:

	Development Year				
Year of Origin	0	1	2	....	k
0	P_{00}	P_{01}	P_{02}		P_{0k}
1	P_{10}	P_{11}			
2	$\vdots$				
$\vdots$	$\vdots$				
k	P_{k0}				

where P_{ij} is the amount of the claims paid in development year j in respect of year of origin i.

We assume that this is to be represented by the form:

Year of Origin	Development Year 0	1	2	....	k
0	$n_0r_0\lambda_0$	$n_0r_1\lambda_1$	$\vdots$		$n_0r_k\lambda_k$
1	$n_1r_0\lambda_1$				
2	$\vdots$				
$\vdots$					
k	$n_kr_0\lambda_k$				

Where n_s is the (unknown) total number of claims for year s, r_i is the proportion of the total number settled in year i (assumed to be solely dependent on i) and λ_j is the index of exogeneous influences applicable to year of payment j. λ_0 is an index of average claims cost in the first settlement year of year 0.

7. We first eliminate the n_s by forming the "development" ratios along each cohort. (It should be noted that these are based on payments in each year and not cumulative figures as used in the "basic" chain ladder technique for finding the distribution function.) If we denote the ratios r_{s+1}/r_s by R_s and λ_{s+1}/λ_s by L_s the triangle then takes the form:

Year of Origin	Development Year 0	1	2	....	$k-1$
0	R_0L_0	R_1L_1	R_2L_2		$R_{k-1}L_{k-1}$
1	R_0L_1	R_1L_2			
$\vdots$					
$k-1$	R_0L_{k-1}				

The separation technique can now be applied to this array but since the R's are the ratios of the proportions in successive durations we assume that $\sum_0^{k-1} R_s = z$ say and obtain a general solution:

$$\hat{R}_s = \hat{k}_s z,\ \hat{L}_s = \hat{l}_s / z.$$

8. Now z cannot be obtained from the triangle and is discussed later. If we put $z = 1$, $\hat{k}_s = \hat{R}'_s$ and $\hat{l}_s = \hat{L}'_s$ we can complete the rectangle by extrapolating on $\hat{L}'_s$ since $\hat{R}'_s\hat{L}'_s = \hat{R}_s\hat{L}_s$. The products of the successive terms along each cohort can then be calculated

and grossing up factors to apply to the cumulative claim payments follow. Two difficulties have been glossed over. The first is one of bias and arises from the calculation of the successive development ratios. If for some reason the claim payments in year s are low because of delay in some payments to year $s + 1$ then the ratio $R_{s-1} L_{s-1}$ will be relatively low and the ratio $R_s L_s$ relatively high—the effect of a shift of a given amount of claims on the two ratios will differ. Thus the effect on the vertical and diagonal sums will differ and the resulting bias can distort the sequence of values of R and L. This must not be overlooked in making projections or in examining the sequence for evidence of abnormal features.

9. The second difficulty is concerned with the extrapolation of L'_s. Now $L'_s = z\lambda_{s+1} / \lambda_s$ and $\lambda_{s+1} / \lambda_s$ gives the relation between the exogeneous influences in years $s + 1$ and s. If for example only monetary inflation were involved then $\lambda_{s+1} / \lambda_s$ gives the relative increase from inflation between the two successive years. If we form the ratios L'_{s+1} / L'_s we eliminate the z factor and obtain an index of the change in the rate of inflation. Thus, in extrapolating on L'_s we have to bring in the expected or assumed future changes in the rate of inflation.

10. It may be observed at this point that an alternative model is to base the calculations on the logarithms of payments. This then becomes an additive model and admits of a straightforward algebraic solution, but the bias referred to in para 8 will not, of course, be eliminated by this device.

11. The estimate of total claims is derived as follows:

We first form the products along each cohort

Year of Origin	Development year 0	1	2		k	Sum	Est. tail
0	1	$\hat{R}'_0\hat{L}'_0$	$\hat{R}'_0\hat{L}'_0\hat{R}'_1\hat{L}'_1$	....	$\hat{R}'_0\hat{R}'_{k-1}\hat{L}'_0..\hat{L}'_{k-1}$	$= S_0$	t_0
1	1	$\hat{R}'_0\hat{L}'_1$	...			$= S_1$	t_1
2							
⋮							
k	1	$\hat{R}'_0\hat{L}'_k$	....			$= S_k$	t_k

Where the values of $\hat{L}'_s$, $s = k, k + 1, \ldots\ldots..$ are projected from the series $\hat{L}^0_1$, $\hat{L}'_1 \ldots . \hat{L}'_{k-1}$ bearing in mind the comments in para 9. If the last term in the first cohort is not very small, as will occur for some classes of business when k is small, an estimate is made of the remaining tail values. The total of the terms in cohort s is then $S_s + t_s$ and if the sum of the "observed" terms is denoted by S_s^{k-s} then the grossing up factor is S_s / S_s^{k-s}. These factors are then applied to the cumulative payments to give an estimate of the ultimate total claims (O_s) for each cohort.

12. The foregoing provides a verification (or projection) technique for the total expected claims from which the outstanding claims are derived by deduction of the cumulative payments. It is however of interest to consider the possibility of estimating z so that the values of r and λ can be found. If we replace $\hat{R}_s$ by $\hat{r}_{s+1} \mid \hat{r}_s$ and $\hat{L}_s$ by $\hat{\lambda}_{s+1} / \hat{\lambda}_s$ we find:

$$(\hat{r}_0\hat{\lambda}_s + \hat{r}_1\hat{\lambda}_{s+1} + \ldots + \hat{r}_k\hat{\lambda}_{s+k}) = r_0\hat{\lambda}_s S_s = \hat{u}_s/n_s = \bar{c}_s \text{ say}$$

and

$$\frac{\hat{\lambda}_s}{\hat{\lambda}_{s+1}} = \frac{\hat{S}_{s+1}}{S_s} \cdot \frac{\hat{u}_s}{\hat{u}_{s+1}} \cdot \frac{n_{s+1}}{n_s}.$$

But

$$\frac{\hat{\lambda}_s}{\hat{\lambda}_{s+1}} = \frac{z}{\hat{L}'_s}$$

so

$$z = \frac{S_{s+1}}{S_s} \cdot \frac{\hat{u}_s}{\hat{u}_{s+1}} \cdot \frac{n_{s+1}}{n_s} \cdot \hat{L}'_s = \frac{S_{s+1}}{S_s} \cdot \frac{\bar{c}_s}{\bar{c}_{s+1}} \cdot \hat{L}'_s.$$

13. Provided the claim settlement distribution was steady and the exogeneous factors were steady or subject only to smooth changes this relationship shows that z is related to $\hat{L}'_s$ by the change in the numbers of claims. If the numbers are unknown, the situation when calculations are based solely on the total payments, then the exogeneous factors derived will be greater than their true values by the increase in the numbers of claims. This is as would be expected since any increase associated with the year of origin will become incorporated in the relationship of the λ'_s. Thus, if some idea of the rate of growth of the numbers of claims is available, it would be feasible to adjust the values of $\hat{L}'_s$ to correct for

the growth factor. If the actual numbers are available then, of course, the solution is equivalent to that derived by Dr. Taylor, (but the bias referred to earlier may lead to minor differences).

14. Now the claim numbers settlement pattern in $r_0 + r_1 + \ldots .$ can be written

$$r_0 \left(1 + \frac{r_1}{r_0} + \frac{r_1}{r_0}\frac{r_2}{r_1} + \ldots .\right)$$

$$= r_0 \ (1 + R_0 + R_0R_1 + \ldots .)$$

$$= r_0 \ (1 + \hat{R}_0^1 z + \hat{R}_0^1\hat{R}_1^1 z^2 + \ldots .).$$

If we select a suitable value of $\hat{L}_s'$, judged from the trend of the values of S_s and $\hat{u}_s$ and the relationships in para 12, and use this as an approximation to z, we can calculate a value for r_0 (and hence the settlement distribution). Using this same value of z we can also calculate values of $(\lambda_{s+1}/\lambda_s) = (L_s'/z)$ so that the relationships between the successive exogeneous influences can be found. The earliest cohort gives the relation $r_0\lambda_0S_0n_0 = u_0$ or

$$\lambda_0 n_0 = \frac{u_0}{r_0S_0}.$$

Since the numbers are not known, we can find values of λ_0n_0, λ_1n_1, ... etc. to complete the solution. If some information about growth is available, it is then possible to modify the values of λ to, say, λ_0n_0, $(\lambda_1 (n_0/n_1))n_1$, ... etc. and thus eliminate the growth element.

15. It will be obvious from the foregoing that to use claims amounts as a basis for projection when conditions are changing rapidly or discontinuously involves some nice judgment decisions but these can be considerably eased when claim numbers are available. This facility is available from the current statutory returns in the UK, which call for both numbers and amounts. It has been found that the claims settlement pattern estimated by the basic chain ladder method on total claims is closely similar to the pattern from the separation method, but the advantages of the latter in providing values for the exogeneous factors which are essentially discontinuous in form, can be significant. In practice

it is advantageous to use both techniques when the data is available as the differences between the results may provide useful information regarding the claim settlement structure.

References

[1] TAYLOR, Dr. G. C., Separation of Inflation and other effects from the distribution of non-life insurance claim delays. XIIth ASTIN Colloquium, Portimao (Algarve) Portugal, October 1975.

[2] VERBEEK, H. G., (1972). An approach to the analysis of claims-experience in motor liability excess of loss reinsurance. ASTIN Bulletin, Vol. VI, 195-202.