THE INFLUENCE OF THE FRANCHISE ON THE NUMBER OF CLAIMS IN MOTOR INSURANCE

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1. If you introduce a franchise or raise its amount, the number of claims will decrease. One reason why you will operate a franchise in car damage insurance is to avoid the troublesome and expensive handling of the large number of very small claims.

Therefore, the management could use a mathematical model describing the number of claims as related to the size of the franchise. By varying this figure one may find the franchise that provides for the optimal business conditions.

2. The mathematical model to be described in this paper was found by studying the distribution of the claims in two portfolios. The general conditions of the two groups were as equal as possible, there was no-claim bonus in neither group, but group B had a franchise of 250 DKr. (about US $ 35) while group A covered the smaller claims also. Each policy had been unaltered for at least one year. The number of claims during 12 successive months was registered as follows:

	Number of policies Group A no franchise	Group B franchise 250 DKr.
No claim	52.147	28.907
1 claim	21.770	7 007
2 claims	9.105	1.461
3 —	3.437	258
4 —	1.212	49
5 —	341	13
6 —	137	1
7 —	33	1
8 —	7	0
9 —	3	0
Total	88.192	33.697

The distribution could be described by a negative binomial distribution

$$n(X) = \binom{-\alpha}{X} \left(\frac{\gamma}{\gamma + 1}\right)^{\alpha} \left(-\frac{1}{\gamma + 1}\right)^{X}$$

where the parameters α and γ were estimated by

$$\alpha \backsim \frac{\overline{X}^2}{S^2 - \overline{X}} \qquad \gamma \backsim \frac{\overline{X}}{S^2 - \overline{X}}$$

These estimates are not maximum likelihood estimates but it was not regarded necessary to improve the estimation as the χ^2 values of the incomplete χ^2-test were found as

$$\chi^2_{\mathrm{I}} = 100.7 \qquad \chi^2_{\mathrm{II}} = 3.98$$

The parameters were estimated as follows:

	$\overline{X}$	S^2	α	γ
Group A	0,657	0,977	1,353	2,057
Group B	0,291	0,349	1,457	5,003

3. Special interest is attached to the variations of the γ-parameter. It is well known, that if $\gamma \to \infty$ and $\alpha \to \infty$ under the condition $\frac{\alpha}{\gamma + 1} \to \lambda$ then the negative binomial distribution will converge towards a Poisson distribution with the parameter λ.

The variance V of the negative binomial distribution is

$$V = \alpha \left(\frac{1}{\gamma^2} + \frac{1}{\gamma}\right)$$

so that an increase of the γ parameter corresponds to a decrease of the variance.

The figures indicated that the introduction of a franchise will reduce the variance considerably.

This consideration led to the questions whether a relation exists between the γ-parameter and the size of the franchise and whether the level of the α-parameter of a portfolio might be regarded as uncorrelated to the franchise.

4. In order to study these questions 5 homogenous groups of policies were observed over 2 years. All policies were unaltered during the observation period. The groups 1, 2 and 3 had no bonus clause while in the groups 4 and 5 a no claim bonus deduction of the premium was granted after 2 years of claim-free driving. The franchises of the 5 groups together with the estimates of the parameters are given below.

Group	franchise	bonus conditions	$\overline{X}$	s^2	α	γ
1	250 DKr.	No bonus clause	1,0467	1,2961	4,3929	4,1969
2	500 DKr.	—	0,7912	0,9268	4,6165	5,8348
3	1000 DKr.	—	0,5618	0,6363	4,2365	7,5409
4	0	No claim bonus	1,5087	0,4023	2,5472	1,6883
5	250 DKr.	after 2 years	0,5073	0,6041	2,6586	5,2407

The α-parameter of the first 3 groups are very close to each other while the γ-parameter shows a considerable variation with the franchise.

5. If we consider the negative binomial distribution as an example of a compound Poisson distribution then its structure function will be a gamma distribution

$$P(\lambda) = \frac{\gamma^\alpha}{(\alpha - 1)!} e^{-\gamma\lambda} \lambda^{\alpha-1}$$

It will be seen that when α is constant, the effect of structure function will decrease with increasing γ. A constant increase of γ will have an exponential influence on $P(\lambda)$. This indicates, that the relation between the franchise f and X has the form

$$\gamma = F(\log f)$$

A closer study of the figures showed that the function

$$\gamma = a + b \log f$$

gave a very close approximation for the groups 1, 2 and 3.

This formula, however, gives no reasonable value for $f = 0$ γ assumes the value 0 for $f = 45$ DKr. which might indicate the existence of a certain lower limit for f, with the effect that a franchise smaller than that limit will not influence the number of claims.

For the groups 4 and 5, the α-parameter shows a similar stability but on a lower level, depending on another bonus situation. The γ-parameter shows a corresponding variation with the franchise.

The α-parameter could be regarded as a numerical measure of the intensity of the bonus hunger.

6. Next, a simulation technique was introduced using a computer to find the correlation between the franchise and the total number of claims. For each risk group, the α-parameter was kept constant in order to forecast the number of claims for a given franchise.

This model has been worked out and it has in practice shown its value as a practical tool for the management to decide the strategy in the Danish motor insurance market. Details can yet be published for competitive reasons.

The results in this limited field have given impulses to more general numerical investigations for studying the connection between the α-parameter and the characteristics of the bonus system.