ASTIN COLLOQUIUM AT RANDERS

Subject A

Report

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Our discussions today will concern Subject A: Risk Theory, in particular the overall risk involved in operating an insurance concern. 7 papers have been handed in to the Colloquium on this subject.

To be quite candid, the referee would like to say that only one of these papers really treats the whole of the topic foreseen, and this is the paper by Mr. Colin M. Stewart (U.K.): The Assessment of Solvency. The other papers treat different special practical sides of the problem as well as general risk theoretical questions. One of the papers, that of Mr. Brichler, could have been attributed to Subject B as well as to Subject A, but it has been decided to take it up for treatment today.

Mr. Stewart delivers a lot of interesting points, which, I may say, take into account the solvency aspect as it looks to the supervisory authorities much more than has been done within the ASTIN discussions earlier. This, of course, is very related to the fact that regional as well as Common Market discussions have resulted in or will result in legislative rules for judging the solvency of insurance concerns, which rules will have great practical importance in the future. Mr. Stewart stresses the point that in case of non-life concerns it is the business on the books and its future contracted periods, short time business to be signed before judgment can be practically undertaken, claims reported, claims occurred but not yet reported, as compared with free reserves available that really decide the matter of sovency. This is by no means the same as a certain percentage of premiums and, least of all, the same percentage in all branches. It is necessary to account for how a concerns business is built up in different branches and with respect to nearby risks as motor insurance as compared with other risks as marine, aviation and transit ones. On all these points, Mr. Stewart argues, a close cooperation between underwriters, actuaries and management must govern the business as a whole. Mr. Stewart also takes up a somewhat unusual side of the matter as he also discusses the influence of variations in the values of the assets. On this point, however, law rules in different countries, especially rules of taxation, have so much influence that a direct comparison must account for that side of the matter first Mr. Stewart, as is natural, gives his interesting points of view out from conditions in the U.K.

The paper by Mr. Brichler: Étude sur la survenance des sinistres en assurance automobile consists essentially of two quite different parts, the first part giving some theory and interesting practical experience results concerning the number of vehicles subject to 0, 1, 2, and so on accidents in one year, the other part discussing in theory and practice the relation between number of accidents first year and the number for each of the subsequent years.

In the first part Mr. Brichler describes the ordinary Polya process with the Delaporte extension, i.e. the basic probabilities according to Ammeter are distributed according to a Gamma function beginning at point s_0. Naturally, s_0 cannot be negative as it should mean the risk intensity of some of the Poisson processes involved, and if s_0 is positive, this means the cutting away of the possibility of too little risk per year. This may, of course, apply to certain branches, but, as Mr. Brichler shows, it does not apply to motor insurance according to the material by Depoid that he utilizes. It would be of great interest to the referee if Mr. Brichler would give the difference between what he calls the "formules de Brichler" and the ordinary Polya methods (or negative binomial distribution) introduced 25 years ago by Ove Lundberg and Ammeter and since then used by lots of authors. Mr. Brichler in the second part of his paper gives very interesting figures showing that the original Delaporte attempt with the simplifications referred to above gives, in wide fields, very good practical results. It is only natural that the number of accidents incurred during the first insurance year shows a gradually weakening tendency to govern the results during subsequent years, resulting in fairly unrealistic results during the 4th and later years of insurance

In his paper Correlations between excess of loss reinsurance covers and reinsurance of the n largest claims dr. Baruch Berliner of Switzerland has taken up a problem earlier treated by E. Franckx and Hans Ammeter. By developing Ammeter's methods and under the condition that the number of claims is Poisson distributed whilst the size of the claims is Pareto distributed, he gives the expected value of the product of the n^{th} and the m^{th} largest claims. This result is extended to the calculation of the expected value and the variance of the sum of the n largest claims by further elaborate methods. Finally dr. Berliner can calculate the correlation between the sum of the *n largest claims and the total loss amount.* The paper ends up by some very interesting tables showing the dominating influence of the two or three largest claims under different circumstances as to time and Pareto constants. The tables show at what rate the influence of the two or three largest claims gradually fades away as time goes on—although this fading away is certainly slower than most insurers would believe—and how it fades away with growing Pareto constant—which is also a rather slow rate.

Mr. E. Straub of Switzerland in his paper: Application of Reliability theory to insurance brings in new general methods from other fields to calculate the probability that during a period of a certain length the result of the business shall be a technical loss is smaller than a given percentage. The classical method for treating this problem, the Esscher method, is considered to be not always easy to apply—if at all possible to apply as in the case when individual claims are Pareto distributed—, and the methods worked out in the general theory of reliability might give some help to solve this problem. Mr. Straub assumes the number of claims to be Poisson distributed, other assumptions being more difficult to handle in practice. He only uses the mean and standard deviation of the claims distribution on top of the Poisson constant (including the factor arising from the time period under consideration), and gives results for two different classes of claims distributions. These are the case when the density function divided by its right hand integral is steadily decreasing or steadily increasing. Unfortunately, also other distributions of claims exist, which are not steadily increasing or decreasing. Such is f.i. the lognormal distribution. For such cases the methods presented do not apply. For exponentially distributed claims distributions it holds that the quotient mentioned is constant, i.e both non-decreasing and non-increasing steadily. The methods used might seem to be laborious to the reader, but so is hardly the case at a closer inspection. Incidentally, two "alphas" have disappeared on page 6 of the paper, one in the exponent of the 9th line from below and one before the log $(1 + \frac{x}{n})$ in the formula line 6 from below. The results reached are generalized to other functions than those used in the beginning. The author also takes from reliability theory the notion of ordering functions as to their convexity resp. concavity with respect to other functions. In two Appendices Mr. Straub uses his results to give upper bounds for the probability of loss for different Poisson constants (including time), standard deviations of the claims distribution and quantities of premiums (net) earned during the corresponding periods. He also gives upper as well as lower bounds in the case when the individual claims distributions is the Pareto distribution with different parameters. The precision of the upper bounds cannot be judged, and the difference between the upper and lower bounds calculated naturally gets more and more wide so as to become at the end practically useless in special limit cases of the parameters studied. For other cases the difference is not large and practically possible to use.

Mr. Carl Philipson (Sweden) has presented a paper on The ruin function for positive risk sums and for unlimited time by using the Thyrion and the Ammeter transforms of extended Hofmann processes with a specified dependence ot time for the structure function.

This paper essentially consists of two different parts. In the first part the author gives an elaborate analysis and survey of the different ways of arriving at the Polya process used by Ammeter, Thyrion and Segerdahl. There has been a lot of discussion about the real meaning of the limit assumptions used, and a comprehensive simple survey of these questions seems to lack in almost every paper or text-book on the matter. Such an analysis requires quite a good deal of thinking, and I am afraid most readers are not quite aware of the differences existing. Mr. Philipson also gives the different series expressions arrived at for the probability of ruin in the different cases—both the total series and approximate expressions arrived at—and shows the internal correspondence between these results. He also arrives at some other, more general processes, which are related to the ones mentioned above. After this eloquent presentation, which is by no means altogether easily understood by the ordinary reader, I am afraid, there remains for an easy understanding a good and simpler presentation of the results reached—if such a presentation is possible to make.

As is well known, the Poisson and Polya processes are special cases of the Hofmann processes, and these, in their turn, can be further generalized. For this case, which includes the ordinary Hofmann processes, the author arrives at an approximate expression for the probability of ruin, which is not very complicated but still needs an estimate of how much it differs from the exact values.

Mr. Olof Thorin (Sweden) in his paper *Analytical steps towards a numerical calculation of the ruin probability for a finite period when the risk process is of the Poisson type or of the more general type studied by Sparre Andersen* gives an advance release of the theoretical parts of a Swedish committee set up to continue the calculations of the so-called convolution committee whose results were presented by Bohman-Esscher in 1963-64 in the Skandinavisk Aktuarietidskrift. The problem is now to calculate with high precision the probability of ruin within a limited time period and to compare this with the results earlier reached by Cramer-Arfwedson and Segerdahl. Also a generalization to the interesting proposal of widening the use of the Poisson process put forward by Sparre Andersen in New York 1957 is undertaken, and further generalizations are under consideration. The work involves a numerical double inversion of characteristic functions and is by no means easy. Maybe simulation will prove to be the method possible to use in practice—for reasons of cost—the author says. Mr. Thorin's paper is very thoroughly worked out and interesting to study. He also shows that if we limit ourselves to study claims distributions of exponential polynomial type considerable simplifications can be reached. This corresponds very well to conditions at least in motor insurance as Almer and Philipson have showed. The methods under consideration are also applicable when the Sparre Andersen approach is made, and for one special case here, the solution can be shown to consist of an expression of Bessel functions, which are rather handy for practical use.

Although the paper seems to be highly mathematical and theoretical, it certainly aims at results of a numerical and practical character, and it will be very interesting to follow the results reached by Mr. Thorin and his coworkers.

Mr. Jan Grandell (Sweden) has taken up a question that has been under consideration in many private talks in his paper. On risk processes with stochastic intensity function.

As the intensity function is allowed to change at different time points in for instance the Ammeter approach to reach the Polya expressions—which are of a type necessary to apply in order to arrive at the rate of growth in time of the dispersion of processes intended to suit practical materials—it could be asked what happens if such changes are continuously made. And, if so, why should the changes not be allowed to be stochastically changing. Mr. Grandell gives a very complete treatment of his subject, showing under which circumstances the normal function is the limit type or some other function, which he can also indicate and which often has a normal component. He also gives methods of how to estimate the intensity function of the process under different assumptions.

As was mentioned in the beginning all the papers presented to this subject are very unsimilar to one another. They are, nevertheless, all very interesting. Their contents range from a survey of solvency criterions in practice and how to make such criterions simple and suitable and possible for supervisory authorities to apply, over the eternal problem of how to handle the tails of skew distributions and how to get models suiting a dispersion that grows very fast in time and how to evaluate the ruin probabilities involved, both within a limited and an unlimited time span. I think we ought to thank the authors for all work they have performed and, personally, I beg to thank them and the Board of the Colloquium for the pleasure and honour it has been to me to make this report.