ANALYTICAL STEPS TOWARDS A NUMERICAL CALCULATION OF THE RUIN PROBABILITY FOR A FINITE PERIOD WHEN THE RISKPROCESS IS OF THE POISSON TYPE OR OF THE MORE GENERAL TYPE STUDIED BY SPARRE ANDERSEN *

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1. *Introduction*

As is well-known, in the early 60's a Swedish committee set to work at the numerical calculation of the distribution function of the total amount of claims and of the related stop loss premiums in the Poisson and Polya cases (Bohman and Esscher [6]). Since the characteristic function for the said distribution function was easily available in terms of the characteristic function for the distribution function of an individual claim, the committee chose to base the numerical calculations on the C-method by H. Bohman (Bohman [5]). The calculation of the ruin probability for a finite or infinite period was not considered by the committee.

The last-mentioned problem has now been taken up by a new committee formed by the Swedish Council for Actuarial Science and Insurance Statistics. The committee—consisting of H. Bohman, J. Jung, N. Wikstad and the present author—has to consider several aspects of the practical applicability of the collective risk theory. However, without possibilities of calculating—at least approximately—the ruin probability for a finite period the applicability of the existing ruin theories seems to be rather limited, so the committee has looked around for such possibilities. At the present stage the committee is considering the classical Poisson theory and Sparre Andersen's generalization of this theory [2]. It is the hope of the committee that, at a later stage, also the Polya theory and the theory recently presented by Segerdahl [11] combining the Sparre Andersen theory and the Polya theory may be treated.

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As one of its first steps the committee has entrusted to me to bring together the analytical results which seem feasible for the above purpose *in the case with only positive risk sums* (claims). Since at the present time some of the analytical results in view are still unpublished it is supposed that it may be of interest to give a short survey including also these results.

In section 2 the results will be reviewed which concern the case with a general distribution of the individual claims. In section 3 simplifications are pointed out in the case when the characteristic function of the said distribution is a rational function (including the case when the tail of the claim distribution is an exponential polynomial).

2. *The formulas for the ruin probability in case the risk sums are positive*

The following formulas are essentially taken from Cramér's book [8] as far as the classical Poisson type process is concerned [1]). However, some results obtained by the present author [12] are also used. The formulas pertaining to the Sparre Andersen generalization are taken from the author's report to the ASTIN Colloquium at Sopot [13] completed with two unpublished papers ([14], [15]).

Let $\psi(u, T)$ denote the probability of ruin within the time interval $(0, T]$ when the initial risk reserve is $u \geq 0$. It is assumed that the distribution function of the independent individual claims, $P(y)$, is such that $P(0) = P(0+) = 0$. The times between successive claims are supposed to be independent and identically distributed with the distribution function $K(t)$, $t \geq 0$, $K(0) = K(0+) = 0$. In the Poisson theory we have $K(t) = 1 - e^{-t}$. In the Sparre Andersen generalization $K(t)$ is arbitrary. However, in the following treatment we assume that $k(s) = \int_0^\infty e^{st}\, dK(t)$ is a rational function of s, or in other words that $K(t)$ is a general Erlangian distribution. That $k(s)$ is a rational function means that $k(s)$ may be written as the quotient of two polynomials. As a consequence of the condition $K(0) = K(0+) = 0$ the degree of the numerator must be lower than that of the denominator. Note that in the Sparre Andersen

1) In Cramér's book there are also complete historical references up to 1954 More recently Beekman has given an alternative approach in [4]. The analytical connections between Cramér's and Beekman's approaches have been investigated in [12].

theory we consider the process as beginning immediately after a claim. In the Poisson theory this assumption is not needed.

In order to illustrate our assumption concerning $K(t)$ we point out some simple well-known distributions for which $k(s)$ is a rational function.

a. $K(t) = 1 - e^{-\beta t}$ i.e. the Poisson case,

$$k(s) = \frac{1}{1 - s/\beta}.$$

b. $K(t) = 1 - b_1 e^{-\beta_1 t} - b_2 e^{-\beta_2 t}$, $0 < \beta_1 < \beta_2$, $b_1 + b_2 = 1$,
$b_1 > 0$, $b_1\beta_1 + b_2\beta_2 \geq 0$,

$$k(s) = \frac{b_1}{1 - s/\beta_1} + \frac{b_2}{1 - s/\beta_2}.$$

This distribution was considered by Sparre Andersen himself when $b_1 > 0$, $b_2 > 0$. When $b_1 = \dfrac{\beta_2}{\beta_2 - \beta_1}$, $b_2 = \dfrac{-\beta_1}{\beta_2 - \beta_1}$ we get the convolution of $1 - e^{-\beta_1 t}$ with $1 - e^{-\beta_2 t}$.

c. $K(t) = 1 - (1 + \beta t)\, e^{-\beta t}$ i.e. the convolution of $1 - e^{-\beta t}$ with itself,

$$k(s) = \frac{1}{(1 - s/\beta)^2}.$$

This is a limiting case of b.

d. $K(t) = 1 - b_1 e^{-\beta_1 t} - b_2 e^{-\beta_2 t} \cos[\gamma(t + t_0)]$,
$0 < \beta_1 < \beta_2$, $b_1 + b_2 \cos \gamma t_0 = 1$, $b_1 > 0$ and
$b_1\beta_1 + \inf[b_2 e^{-(\beta_2 - \beta_1)t} \{\gamma \sin[\gamma(t + t_0)] + \beta_2 \cos[\gamma(t + t_0)]\}] \geq 0$

$$k(s) = \frac{b_1}{1 - s/\beta_1} + b_2 \frac{(\beta_2^2 + \gamma^2)\cos \gamma t_0 - s(\gamma \sin \gamma t_0 + \beta_2 \cos \gamma t_0)}{\beta_2^2 + \gamma^2 - 2\beta_2 s + s^2}$$

{E.g.: $b_1 = 4/5$, $b_2 = \sqrt{2}/5$, $\beta_1 = 1$, $\beta_2 = 2$, $\gamma = 2$, $t_0 = \pi/8$.}

e. $K(t) = 1 - \sum_{v=1}^{n} b_v e^{-\beta_v t}$,

$0 < \beta_1 < \beta_2 < \ldots\ldots < \beta_n$, $b_1 + b_2 + \ldots + b_n = 1$,

$$\sum_{v=1}^{\mu} b_v\beta_v \geq 0, \quad \mu = 1, 2, \ldots, n,$$

$$k(s) = \sum_{v=1}^{n} \frac{b_v}{1 - s/\beta_v}.$$

This is a simple generalization of b.

f. $K(t) = 1 - \sum_{v=1}^{2n+1} b_v e^{-\beta_v t}$,

$$0 < \beta_1 < Re(\beta_2) = Re(\beta_3) < \ldots < Re(\beta_{2n}) = Re(\beta_{2n+1}),$$
$$Im(\beta_2) = - Im(\beta_3), \ \ldots, \ Im(\beta_{2n}) = - Im(\beta_{2n+1}),$$
$$\sum_{v=1}^{2n+1} b_v = 1, \ b_1 > 0, \ b_{2\mu} = b_{2\mu+1}, \ \mu = 1, 2, \ldots, n,$$
and $\sum_{v=1}^{2n+1} b_v \beta_v e^{-\beta_v t} \geq 0$ for all $t \geq 0$,
$$k(s) = \sum_{v=1}^{2n+1} \frac{b_v}{1 - s/\beta_v}.$$

Apart from limiting cases with multiplicities in the β's (generalizations of c.) f. is the most general form of a general Erlangian distribution. (Cf. Feller [9] p. 438.)

In the usual sense of weak convergence of probability laws the class c. is dense in the class of all probability laws concentrated on the positive half-axis (cf. Cox-Miller [7] pp. 257-258). A fortiori, the same is true of the class f. However, such statements are of less practical value. More interesting would it be if experience were in favour of the conjecture that most practical distributions of interoccurrence times can be well approximated by distributions of the type c. or f. with only a very limited number of terms. As to the corresponding problem concerning the function $P(y)$ it seems that such experience is available (see Almer [1]).

After this digression about the function $K(t)$ we return to the formulas giving the probability of ruin within a finite period $(0, T]$, i.e. $\psi(u, T)$. We denote the probability of ruin at any time in the future by $\psi(u) = \psi(u, \infty)$. Then, for fixed $u \geq 0$, $\frac{\psi(u, T)}{\psi(u)}$ with $\psi(u, 0) = 0$ is a distribution function in T (giving the probability that—if ruin occurs—the epoch of ruin is $\leq T$). Therefore, it is natural to consider its characteristic function. More generally (Cramér [8] p. 73) consider $\bar{\psi}(u, z) = \int_0^\infty e^{zT} d_T \psi(u, T)$, $Re(z) \leq 0$. The characteristic function of $\frac{\psi(u, T)}{\psi(u)}$ is then $\frac{\bar{\psi}(u, i\eta)}{\psi(u)}$. If we can deduce a tractable formula for $\psi(u, z)$ we may thus get $\psi(u, T)$ by the Lévy inversion formula. Numerically this might be done e.g. by Bohman's C-method.

However, in order to obtain a formula for $\psi(u, z)$ it is, generally speaking, necessary to consider a "characteristic function" according to u of $\bar{\psi}(u, z)$. In fact, we consider $\bar{\varphi}(s, z) = 1 - \int_{0-}^{\infty} e^{su}\, d_u\, \bar{\psi}(u, z)$ where we have defined $\psi(u, z) = 0$ for $u < 0$. Here $Re(s) < R$ where R is the least positive root, assumed to exist, of the equation $k(-cs)\, p(s) = 1$. The function $k(s)$ has been defined above. By analogy $p(s) = \int_{-\infty}^{\infty} e^{sy}\, dP(y) = \int_{0}^{\infty} e^{sy}\, dP(y)$ since we have assumed $P(0) = P(0+) = 0$. Furthermore we denote by ct the gross risk premium for a period of length t. Of course, we assume that $c > \frac{p_1}{k_1}$ where p_1 and k_1 denote the mean of $P(y)$ and the mean of $K(t)$ respectively i.e. we assume what we call a positive safety loading. (Note that $\psi(u, T) \leq e^{-Ru}$.)

Then we have [13]

$$\bar{\varphi}(s, z) = \frac{A(s, z)}{A(0, z)}$$

where $A(s, z)$ in a simple way depends on the function

$$H(s, z) = -\log\,(1 - k(z - cs)\, p(s)).$$

In fact, it may be shown that there exist a function $M(x, z)$ such that for $0 < Re(s) < R$

$$H(s, z) = \int_{-\infty}^{\infty} e^{sx}\, d_x M(x, z) = \log A(s, z) - \log B(s, z)$$

where

$$\log A(s, z) = \int_{0}^{\infty} e^{sx}\, d_x M(x, z),\ Re(s) < R,$$

$$\log B(s, z) = -\int_{-\infty}^{0} e^{sx}\, d_x M(x, z),\ Re(s) > 0$$

Thus

$$1 - k(z - cs)\, p(s) = \frac{B(s, z)}{A(s, z)},\quad 0 < Re(s) < R.$$

In the paper [12] the present author has pointed out that in the Poisson case, $K(t) = 1 - e^{-t}$, there is another formula connecting A, B and H namely

$$\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{H(s^1, z)}{s^1 - s}\,ds^1 = \begin{Bmatrix} \log A(s, z),\ Re(s) < 0 \\ \log B(s, z),\ Re(s) > 0 \end{Bmatrix},\quad Re(z) < 0,$$

a formula, which the author i.a. used to deduce simple expressions for A and B when $P(0) = P(0+) = 0$. However, this formula is also valid in the situation when $K(t)$ has a more general form (see [15]). If we assume that $k(s)$ is a rational function the proof can be repeated almost word for word. The expressions for A and B become, however, a little more complicated.

In order to avoid trivial complications we give the formulas for A and B when $K(t) = 1 - \sum_{v=1}^{n} b_v\, e^{-\beta_v t}$, $\sum_{v=1}^{n} b_v = 1$ where β_v are distinct and such that $Re(\beta_v) > 0$ and together with b_v are so chosen that $K(t)$ is a distribution function. (The formulas are derived by use of the Cauchy theorem applied to a contour in the left halfplane enclosing the logarithmic singularities and, in the case of A, a simple pole.) We get

$$A(s, z) = $$

$$= \frac{\prod_{j=1}^{n}(cs - c\, s_{1j}(z))}{\prod_{v=1}^{n}(\beta_v + cs - z) - p(s)\sum_{v=1}^{n} b_v\beta_v \prod_{\substack{j=1 \\ j\neq v}}^{n}(\beta_j + cs - z)},\quad Re(s) < R$$

$$B(s, z) = \prod_{j=1}^{n}\frac{cs - cs_{1j}(z)}{\beta_j + cs - z},\ Re(s) > 0. \tag{2.1}$$

Here $s_{1j}(z), j = 1, 2, \ldots, n$ denote the n roots in the left halfplane $Re(s) \leq 0$ of the equation

$$k(z - cs)\, p(s) = 1,\quad Re(z) < 0.$$

(By the Rouché theorem it is found that the number of the $s_{1j}(z)$ must be the same as the number of poles of $k(z - cs)$ in the same halfplane. These poles are in the points $\frac{z-\beta_v}{c}$, $v = 1, 2, \ldots, n$.)

By continuity the formulas are true not only for $Re(z) < 0$ but also for $Re(z) \leq 0$.

We thus have obtained tractable formulas for

$$\bar{\varphi}(s, z) = \frac{A\,(s, z)}{A\,(0, z)} \tag{2.2}$$

as far as we can find the roots $s_{1j}(z), j = 1, 2, \ldots, n$.

In order to obtain $\psi(u, T)$ we must first invert the formula

$$\int_{0-}^{\infty} e^{su}\, d_u\, \psi(u, z) = 1 - \bar{\varphi}(s, z) = 1 - \frac{A\,(s, z)}{A\,(0, z)}$$

obtaining $\bar{\psi}(u, z)$ and second we have to invert the formula

$$\psi(u, z) = \int_0^{\infty} e^{zT}\, d_T\psi(u, T).$$

If we only had in mind to get $\psi(u)$ the first inversion with $z = 0$ would be sufficient since $\psi(u, 0) = \psi(u)$. However, with the aim to compute $\psi(u, T)$ for a set of finite values of T we are facing the task of two successive numerical inversions. In principle this could be done by use of a numerical inversion method, say Bohman's C-method. However, the first inversion must be done for a large number of z-values in order to get a sufficient basis for the second inversion. Furthermore, a high precision in this first inversion seems to be required. Of course, this might be done but perhaps this way would be too expensive, at least as a standard method. Therefore, in the next section we try to get around the first inversion. Needless to say, this cannot be done without paying a certain price.

3. *Simplifications in the case when the risk sums obey a law expressible by a finite number of exponential terms*

It is natural to try to specialize $P(y)$, $P(0) = P(0+) = 0$, in such a way that $p(s)$ by analytic continuation can be extended to a rational function of s expressible as a quotient of two polynomials without common factors, the degree of the numerator being lower than that of the denominator. In order to avoid trivial complications we assume that the zeros of the denominator are simple. Let us denote them by $\alpha_1, \alpha_2, \ldots, \alpha_m$ implying that $P(y) = 1 - \sum_{v=1}^{m} a_v e^{-\alpha_v y}$, $\sum_{v=1}^{m} a_v = 1$, $Re(\alpha_v) > 0$. ($\{a_v\}$ and $\{\alpha_v\}$ cannot be chosen completely arbitrarily since $P(y)$ must be a distribution function.) It is easy to see that the poles of $p(s)$ must be located in the halfplane $Re(s) > R$. As a consequence of this and of our assumption that $k(s)$ also is a rational function we see that the crucial function $1 - k(z - cs)\, p(s)$ for fixed z also is rational in s. At least if $Re(z) \leq 0$ there are no poles in $0 \leq Re(s) \leq R$. The poles for $Re(s) > R$ are determined by $p(s)$ and are located in α_v, $v = 1, 2, \ldots, m$. However, still more interesting than the poles are the zeros of $1 - k(z - cs)\, p(s)$ for $Re(s) \geq R$ since these zeros are the poles of $A(s, z)$ as is seen from the formula (2.1) in the previous section and from $A(s, z)$ we can derive $\bar{\psi}(u, z)$ by inverting (2.2). The number of these zeros is m, i.e. equal to the number of poles of $p(s)$, as follows from the same formula. Let us denote these zeros by $s_{2v}(z)$, $v = 1, 2, \ldots, m$. Since $A(s, z)$ according to (2.1) after insertion of the expression for $p(s) = \sum_{v=1}^{m} \dfrac{a_v}{1 - s/\alpha_v}$ becomes the quotient of two polynomials of the degree $n + m$, which however have the common factor $\prod_{j=1}^{n} (cs - cs_{1j}(z))$ we get $A(s, z)$ as a quotient of two polynomials of the degree m. In fact we find

$$A(s, z) = \frac{\prod_{v=1}^{m} (s - \alpha_v)}{\prod_{v=1}^{m} (s - s_{2v}(z))}.$$

For $s = 0$ we get

$$A(0, z) = \frac{\prod_{v=1}^{m} \alpha_v}{\prod_{v=1}^{m} s_{2v}(z)}.$$

Thus

$$\frac{A(s, z)}{A(0, z)} = \frac{\prod_{v=1}^{m} (1 - s/\alpha_v)}{\prod_{v=1}^{m} (1 - s/s_{2v}(z))}. \tag{3.1}$$

Let us now assume that there are no multiplicities among the $s_{2v}(z)$, $v = 1, 2, \ldots, m$.

Then, developing in partial fractions we may write

$$\frac{A(s, z)}{A(0, z)} = g_0(z) + \sum_{v=1}^{m} g_v(z) \frac{1}{1 - s/s_{2v}(z)}.$$

Comparing with (3.1) we find after multiplying by $1 - s/s_{2j}(z)$ and letting $s \to s_{2j}(z)$ that

$$g_j(z) = \frac{\prod\limits_{v=1}^{m} \left(1 - \frac{s_{2j}(z)}{\alpha_v}\right)}{\prod\limits_{\substack{v=1 \\ v \neq j}}^{m} \left(1 - \frac{s_{2j}(z)}{s_{2v}(z)}\right)}, \quad j = 1, 2, \ldots, m. \tag{3.2}$$

Taking $s = 0$ we further get

$$1 = g_0(z) + \sum_{v=1}^{m} g_v(z). \tag{3.3}$$

We now have

$$\int_{0-}^{\infty} e^{su}\, d_u\, \bar{\psi}(u, z) = 1 - g_0(z) - \sum_{v=1}^{m} g_v(z) \frac{1}{1 - s/s_{2v}(z)} \quad Re(s) < R.$$

Since

$$\frac{1}{1 - s/s_{2v}(z)} = \int_{0}^{\infty} e^{su}\, d_u(1 - e^{-us_{2v}(z)}), \quad Re(s) < R,$$

$$1 = \int_{0}^{\infty} e^{su}\, d\varepsilon(u), \text{ we get for } u \geq 0$$

$$\bar{\psi}(u, z) = 1 - g_0(z) - \sum_{v=1}^{m} g_v(z)\,(1 - e^{-us_{2v}(z)})$$

Observing (3.3) we can write this

$$\boxed{\psi(u, z) = \sum_{v=1}^{m} g_v(z)\, e^{-us_{2v}(z)}, \quad u \geq 0} \tag{3.4}$$

where $g_v(z)$, $v = 1, 2, \ldots, m$ are given by (3.2).

If our assumption that the roots $s_{2v}(z)$ are simple is not satisfied we instead get limiting forms of (3.4).

Note that the formula (3.4) generalizes a formula given by Cramér ([8] p. 82) for $\psi(u)$ when $P'(y)$ is an exponential polynomial with positive coefficients and $K(t) = 1 - e^{-t}$. Note also that for the truth of formula (3.4) it is not fundamental that $k(s)$ is just rational. However, the assumption about the rationality of $p(s)$ seems essential if m is to be finite. If m is permitted to be infinite the formula (3.4) is open for generalizations to wider classes of distribution functions $P(y)$.

By the formula (3.4) we have got $\bar{\psi}(u, z)$ without numerical inversion of formula (2 2). However, we must pay for this advantage in two ways.

First, we must restrict ourselves to use only a number of exponential terms when we represent in analytical form our experience of the distribution of individual claims. This restriction is perhaps not too serious. The experience presented by Almer [1] seems to justify the use of three or four exponential terms in most practical cases. Also Philipson [10] seems to accept such a view.

Second, we must be able to compute $s_{2v}(z)$, $v = 1, 2, \ldots, m$, the zeros of $1 - k(z - cs)\,p(s)$ for $Re(s) > 0$ with great precision for a large number of z-values (whereas the roots $s_{1j}(z)$, $j = 1, 2, \ldots, n$ do not enter the formula). Let us see what this means.

Take first the simple Poisson process where $K(t) = 1 - e^{-t}$, Then the equation $k(z - cs)\,p(s) = 1$ takes the well-known form

$$p(s) = 1 + cs - z.$$

If $P(y)$ can be represented by at most 4 exponential terms then $p(s)$ becomes a rational function where the denominator polynomial is at most of the 4th degree i.e. $m \leq 4$. Our equation then becomes an equation of the 5th degree at most i.e. $m + 1 \leq 5$. At most we have to compute 4 roots since the root in the left halfplane does not enter the formula (3.4).

Let us now consider the Sparre Andersen generalization with $K(t)$ expressed by n exponential terms. The equation $k(z - cs)\cdot p(s) = 1$ then becomes an equation of degree $m + n$. At the first stage it seems reasonable to let $n = 2$ corresponding to two exponential terms in $K(t)$. If $m < 4$ we thus have $m + n \leq 6$ i.e.

we must compute the 4 roots in the right halfplane $Re(s) > 0$ of an equation of the 6th degree since the two roots in the left halfplane does not enter (3.4).

The equations we have to solve have complex coefficients as a consequence of the appearance of z. Computer programs do exist which are claimed to give all zeros of a complex polynomial in a rapid way. It is the intention of the committee to try to use this method to get $\bar{\psi}(u, z)$ and to compute $\psi(u, T)$ by a numerical inversion e.g. according to the Bohman C-method. The committee will also make attempts to determine $\bar{\psi}(u, z)$ in some cases by numerical inversion in order to compare the precision obtained.

In the simple case when $m = 1$ formula (3.4) takes the form

$$\psi(u, z) = \left(1 - \frac{s_2(z)}{\alpha}\right) e^{-us_2(z)} \tag{3.5}$$

(where we have dropped the index 1). If also $n = 1$ then $s_2(z)$ is one of the roots of a second degree equation and it is known (Arfwedson [3] p. 21) that $\psi(u, T)$ can be expressed by Bessel functions. In this case it is thus possible to avoid even the numerical inversion of (3.4). If, in one way or another, this may be generalized to $n > 1$ or/and $m > 1$ is unknown to the author.

4. *Closing remarks*

As pointed out above the simplifications presented in section 3 have been possible only by paying a certain price. Obviously there are cases where this price becomes too high. Let us consider e.g. the risk situation characteristic for the portfolio retained by the cedant in an Excess of Loss treaty. In such a case it seems natural to consider a distribution function $P(y)$ with $P(M-) < 1$, $P(M) = P(M+) = 1$, for a finite M. Then $p(s)$ becomes an entire function (non-rational) and section 3 gives no help. Thus one has to use the double numerical inversion of section 2 or, if this turns out to be too expensive, one can—as Mr. Bohman has proposed—try to use simulation techniques. In order to get a certain idea about the precision and the cost of such techniques it is the intention of the committee to use them also in some cases where the method of section 3 turns out to succeed. A third way to tackle the indicated problem could be an attempt to generalize the method of section 3 to include the risk situation mentioned above.

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* Added in proof: After that the present paper was read at the ASTIN-Colloquium at Randers 1970, the paper [14] has been published in *Skandinavisk Aktuarietidskrift* 1970, Nos 1-2, 29-50. With the latter paper in hand it is not necessary to consult the paper [13]