INTRODUCTORY REPORT

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Introduction

As reports on work done on subjects discussed at earlier colloquia and on new lines of investigation 6 papers have been laid down on my table (To be exact, 5 on my table and the last one on my chair at the beginning of this colloquium on Wednesday morning).

Four of them deal with problems concerning the estimation of reserves. These are the papers by Beard, Molinaro, Benedikt and Hovinen.

The other two papers, each of which has two authors, namely Kauppi and Ojantakanen and van Klinken and Groenenberg deal with mathematical questions of more general interest.

I can repeat what my colleagues have said this week:

Although the number of the papers is rather small, the importance and the quality of them seem to me to be very great.

The paper of R. E. Beard concerns the studies he has made on the data provided by the 6th Conference of European Insurance Supervisory Services. Suitable statistics of motor insurance on which to experiment have been made available relating to 28 European insurance companies. Beard has made certain preliminary studies of these statistics. His paper has been written against the background of these studies. Although only the broadest indications of numerical values are given, since the detailed statistics are confidential, nevertheless the comments made are based on the studies of the figures. Therefore these comments have a considerable practical utility.

As a basic principle on which to build calculations of technical reserves it is assumed that the claim frequency rate will be constant and that the distribution of claims by amount will be stable, both apart from random fluctuations. Although it is hardly possible to prove the validity of these assumptions, there could be no basis for calculation if these principles were not true.

The technical reserves are classified in 4 groups:

a) Reserves for unexpired risks.
b) Reserves for incurred but unreported claims.
c) Reserves for outstanding claims.
d) Fluctuation reserves.

The reserves for unexpired risks and for incurred but unreported claims are calculated by finding the expected number of claims and multiplying these numbers by an appropriate average claim. The reserve for outstanding claims is found by multiplying the number of incurred claims in each year by an appropriate average and deducting therefrom the actual settlements to the close of the account concerned. A statement is to be made with the columns: year, number of claims, expected average settlement, expected settlements, payments to the end of the year of account and in the last column the outstanding. The total of the last column is the reserve for outstanding claims. The number of claims and the payment are factual figures derived from the company records. The expected average settlement is a judgement figure to be derived. The number of years, i.e. the rows of the statement depends on the class of business. For own damage claims in motor insurance two years will be enough, for motor third party claims at least 5 years will be needed.

A practical difficulty arises from the fact that the number of claims is defined in different ways with the different companies. Also the treatment of zero claims varies between companies. The incidence of zero claims can be influenced by the administration of the company. The proportion of zero claims varies from a minimum of a few percent to over 50 %. Therefore the separate recording of zero claims seems the better treatment.

The late reportings are of the order of a few percent of the notified claims, the maximum being 13½ %. With the rapid growth of motor insurance a bias exists on the frequency rate if the figures are not corrected according to the late reportings. Moreover a warning is made that a rapidly growing company should not underprovide the outstandings by closing cases as zeros, only to reopen them at a future date, thus throwing forward a part of its claims.

A statement is suggested relating to the numbers of claims. Several calculations can be made to determine the internal consistency of the figures.

After this treatment of the number of claims several sections of the paper concern the amounts of claims. The basic hypothesis is that the average claim is a reasonably stable quantity in a homogeneous portfolio. Nevertheless a number of factors obscure this stability. The average settled claim is the total payments divided by the number of claims in respect of which the payments are made. Thus are calculated the average 1st year settlement, the average 1st and 2nd year settlement and so on. Generally the first year settlements are of course the smaller claims. The average settled first year claim is about one half of the ultimate average. The problem of estimating the trend from the 5th year is not easy. Some countries show about 5 % of outstanding claims by amount, others about 15 %. The increase in the average claim is associated with the change in the value of money. From studies of the 28 companies there appears to have been a tendency for early duration claims to have been overestimated whilst later duration claims have been underestimated. For increasing portfolios this is of course a dangerous situation.

Another point which should be noted is the so-called "mix" and what happens if the "mix" should change.

Beard concludes that there are so many possible variations that it is difficult to lay down any hard and fast rule but that the study suggested that extensive subdivision was not essential. If there are no claims bonuses the frequency will fall with higher bonus, the average claim however will show a much smaller variation. Separation into bonus classes will be hardly necessary.

Since the premium for reinsurance is usually linked closely to the excess claims there is an arguable case for ignoring the reinsurance in making the basic calculations.

If the systematic variations are eliminated after appropriate calculations of the expected values of claims of unexpired risks and on emerged claims then the fluctuation reserve can be determined from statistical theory.

If the number of expected claims on unexpired risks is reasonably large, i.e. of the order of a few hundreds calculation from the data provided shows that a simple method will be sufficiently accurate. If this number is 1.000 the distribution of total amount of claims will be fairly closely approximated by the normal curve for deviations not too remote from the mean. For smaller values of the number of expected claims an adjustment can easily be made.

Thus the problem is reduced to estimating the standard deviation of the total claim distribution. The coefficient of variation, σ divided by m_1 has been calculated for a number of claim distributions. For own damage claims this coefficient is 1.8, for liability claims and for mixed liability and damage claims values lie between 2.5 and 5.0, with a tendency to cluster around 3.0. For mixed portfolios the figure was about 3.5. Each country showed distinctive values. As the claim process is not a simple Poisson process an adjustment should be made. From other data an increase of 30 % seemed to be reasonable. Thus the standard deviation of the mean total claim m would be taken as $3.5 \times 1.3 \times m$ or $5\ m$. If the expected number of claims is n then the total expected claim is $m\ n$ and the approximate standard deviation of the expected claims $5\ m\sqrt{n}$. The total of the outstanding claims is $n_0\, m_0$, the standard deviation $5\ m_0 \sqrt{n_0}$. For the total of expected claims on the unexpired risks and on the outstanding claims the standard deviation is $5\ \bar{m} \sqrt{n_t}$ where $n_t = n + n_0$.

The fluctuation margin is 3σ, i.e. $15\bar{m} \sqrt{n_t}$ at the 1 per mille level. In other words the probability that the total claims will exceed $\bar{m} \times n_t + 15\ \bar{m} \sqrt{n_t}$ is 1 in 1.000. It is clear that the fluctuation reserve increases in proportion to the square root of the total number of claims. Since this is roughly proportional to the premium income, the theoretical reserve is roughly proportional to the square root of the premium income and not to the premium income itself. This is not dissimilar to the risk reserve derived from risk theoretical studies. Pentikainen has shown that the risk reserve is of the form $k \sqrt{P} - \lambda P$. λ is the profit margin in the premiums. If this is ignored then the risk reserve and the fluctuation reserve derived by Beard are identical.

Beard suggests that the calculation of a fluctuation reserve does not involve any very elaborate computations and that there is little justification for a precise calculation of the distribution of total claims; at the end of the paper some remarks are made about the influence of the inflation. 5 % inflation a year would mean a 10 % increase in the cost of outstanding liabilities.

The conclusion of the paper is that provided statistics are properly and consistently compiled subdivision into claim frequency and average claim does provide a practical method of reserve requirements.

In the paper of Luigi Molinaro "Sur la détermination de la reserve pour sinistres en suspens dans l'assurance automobile" a method is developed for calculating the reserve for outstanding claims. The reserve for outstanding claims is equal to the total amount for all claims diminished by the amount for the settled claims. It is suggested that the law of extinction of a group of claims must be found. This law of extinction must be found for the amount of claims as well as for the number of claims. Then the average amount of outstanding claims can be calculated from a coefficient which depends on ratios which are derived from the paid amounts and from the number of settled claims.

In his paper Molinaro gives some figures of the development of claims in a number of insurance companies in Italy. In the first year 66 % of the number of claims are settled. They represent 38 % of the ultimate total amount. After the third year 2, 5 % of the claims are outstanding, representing 19 % of the ultimate total amount.

If the before mentioned coefficient is supposed to be stable, the reserve for outstanding claims can be calculated. For the validity of the method it is necessary that the number of claims is not too small, that there is a certain homogenity, and that the zero claims and the partial payments are excluded.

At the end of the paper some remarks are made concerning two factors which disturb the stability, i.e. the influence of the type of accidents such as damage claims and personal injuries and the influence of the inflation. Molinaro suggests damage claims and personal injuries to be treated separately. Damage claims are settled in Italy within 3 years, whereas personal injuries can take 10 years. So far as the inflation is concerned, Molinaro suggests to make allowance by increasing the reserves with 10 %. This is the same figure which was mentioned by Beard.

In the paper of V. Benedikt "Estimating incurred claims" the writer puts that methods of considerable precision have been proposed for estimating the final amount in Motor Insurance. He suggests a less exact but simple method. His method uses the so-called chain-relative. The chain-relative is the ratio of the amount of losses paid in a certain year and the amount of losses paid in the year before. Both amounts relate to accidents which took place in the same year. These "chain-relatives" are calculated for several years of accidents. The arithmetic mean is calculated from these several chain-relatives. For the first, second, third and further years after the year of accident thus the mean value of the chain-relative is calculated. The chain relatives for future years are supposed to be the mean-values which are calculated. Thus the estimate of the final amount can be calculated as a product of the amount for the last year and of several values of chain-relatives.

The generalized Poisson function gives in risk theory the probability that the total amount of claims does not exceed a certain limit. In the paper of Lauri Kauppi and Perti Ojantakanen "Approximations of the generalized Poisson function" it is shown that the well known expansion for the generalized Poisson function is too inconvenient for numerical computations.

An often used approximation is to replace the generalized Poisson function by the normal distribution function having the same mean and standard deviation. For large values of n, the expected number of claims, this approximation gives good results. For smaller values of the expected number of claims this approximation does not always give satisfactory accuracy.

In the paper three approximation formulae are compared. Computations have been made based on the material concerning fire insurance and third party motor insurance in Finland. In each of the three formula's appears the skewness γ_1. Calculations have been made for the ruin limits $\varepsilon = 0{,}05$, $0{,}01$ and $0{,}001$. x_ε denotes the standardized variable and y_ε the corresponding normal variable.

The first formula is

$$x_\varepsilon = y_\varepsilon\,(1 + C_\varepsilon \cdot \gamma_1^{0{,}98})$$

and is trying to approximate a formula given by Esscher.

The second formula is $x_\varepsilon = y_\varepsilon + B_\varepsilon \cdot \gamma_1$ and approximates the results computed by the Monte Carlo method.

The third formula is

$x_\varepsilon = y_\varepsilon + \frac{\gamma_1}{6}(y_\varepsilon^2 - 1) + o\left(\frac{1}{n}\right)$ and was derived theoretically.

It is shown that the last formula is fit for use in the same area as the Esscher formula and almost equal in accuracy. Which is the area of fitness for the formula's is an open question. It is acceptable for $\gamma_1 < 2.5$.

This last formula has been applied in the paper of Esa Hovinen "Procedures and basic statistics to be used in magnitude control of equalization reserves in Finland." In this paper minimum and maximum amounts for equalization reserves are derived.

In the paper of J. van Klinken and C. J. Groenenberg, "Estimation of the coefficient of variation of the excess of loss by means of Jensen's inequality for convex functions. The connection with the theory of life times" there are determined upper and lower bounds for the coefficient of variation of the total claim cost in a year in excess of a certain limit value. In studying the problems of "Statistics of large claims" the coefficient of variation, defined as the ratio of the standard deviation and mean is a useful tool. In the paper a differential equation of the square of the coefficient of variation of the excess of loss is derived. Formal integration of this differential equation gives the square of the coefficient of variation as a solution for this differential equation.

By making use of some alternative formulae for the first and second moment of the distribution of $x - X$ (X being the non-random limit value) some expressions for the square of the coefficient of variation are given. If Jensen's inequality for convex functions is applied in relation with these expressions then the bounds for the square of the coefficient of variation can be derived. A different approach is developed by Groenenberg. By applying the mean value theorem to the expression emerging from the differential equation some other bounds are found. The last mentioned bounds have a simpler analytical form than those derived from Jensen's inequality. If the excess of loss $x-X$ is interpreted as the time still to live after reaching age X, and the first moment of $x - X$ consequently as the expectation of life some of the bounds are also of interest for the theory of life times.

At the end of this summary I want to thank the authors for their interesting papers and you, ladies and gentlemen, for your attention.