

MAGNITUDE CONTROL OF TECHNICAL RESERVES IN FINLAND

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This paper deals with some requirements put forward by the Finnish Supervisory Service on technical reserves mainly for solvency reasons. Consequently only one but nevertheless an important aspect of solvency problem is touched upon. The solvency question in its entirety is discussed in a paper of Dr. Pentikäinen at this same colloquium.

According to the instructions of the Supervisory Service, given by virtue of the Insurance Companies Act, insurance companies are obliged to calculate a so-called equalisation reserve in a specific way. Simplifying slightly, this equalisation reserve is that part of the technical reserves which exceeds the premium reserve and the claims reserve calculated according to classical actuarial principles; that is to say, the present value of the expectation value of the net insurance liabilities. The Insurance Companies Act calls for this reserve because conventional technical reserves do not take into account the random feature of claims amount.

A nonlife company has to verify that its equalisation reserve (E , say) is between a minimum amount (E_{\min}) and a maximum amount (E_{\max}) both defined in instructions given by the Supervisory Service. The former bound is linked to the solvency requirements implying that in case of $E \geq E_{\min}$ the company is allowed to continue its activity at least one more year, whereas the latter amount, E_{\max} , is associated with taxation. This upper bound is considered indispensable owing to the deductability of transfers to the technical reserves.

Let X be the total amount of claims after deducting cessions and retrocessions to reinsurers; let P be the annual earned premiums after deducting reinsurers' share and administration costs. Further let C be the capital and free reserves. Then the

minimum amount E_{\min} is defined in principle by the equation

$$P \{1.05(E_{\min} + C) + \sqrt{1.05} (P - X) \geq 0\} = .99, \quad (1)$$

where the factor 1.05 refers to the rate of interest. There is, however, a further restriction,

$$E_{\min} \geq \text{Max} \{0, M - C\},$$

where M is the greatest realistically possible size of a single claim (after deduction of cessions and retrocessions). There are some specific instructions on how to estimate M if the company underwrites Stop Loss reinsurance or any direct insurance in classes with unlimited liability.

Consequently the distribution function $F(x)$ of the random variable X is needed for the calculation of E_{\min} . In order to eliminate speculations in estimation of the function $F(x)$ certain schematic instructions are given which contain some simplifications of reality.

The portfolio is partitioned in a given way into separate classes (k) of insurance so that $X = \sum X_k$, where claims amounts X_k are supposed to be mutually independent. If the mean number of claims in class k is equal to n_k and if $S_k(x)$ is the distribution function of the size of one claim in class k (after deduction of cessions and retrocessions), then the distribution function $F_k(x)$ of the random variable X_k is defined in the instructions by means of equation

$$F_k(x) = e^{-(1+q_k)n_k} \sum_0^{\infty} \frac{[1 + q_k]n_k^r}{r!} S_k^*(x),$$

where q_k is a fluctuation constant which depends on the class k . The purpose of this constant is to pay attention to the well-known fact that in estimating $F_k(x)$ the ordinary generalised Poisson function as such does not correspond to the real situation. In most classes q_k is between .2 and .4, but e.g. in forest insurance it is as high as 6.0. The introduction of this kind of technique instead of a compound Poisson distribution is not induced merely from a practical point of view. It is also motivated in that in general, an exceptionally high number of claims appears in many companies at the same time.

The Supervisory Service does not consider a system desirable if it renders possible many cases of ruin during the same year.

If for simplicity's sake the special case of Stop Loss reinsurance is excluded, it then follows that

$$F(x) = \Pi * F_k(x) = \sum_0^{\infty} e^{-n} \frac{n^r}{r!} S^{r*}(x),$$

$$\text{where } n = \sum (1 + q_k) n_k \text{ and } S = \frac{1}{n} \sum (1 + q_k) n_k S_k.$$

Consequently if a company has to calculate the function $F(x)$ it must estimate the numbers n_k and the distribution functions S_k . In order to facilitate estimation a large set of common claims statistics has been collected, particularly useful for estimation of the tails of the functions S_k . Specific instructions are needed for reinsurance accepted and for cessions.

The direct calculation of E_{\min} can however usually be avoided. This is done by use of the following simple test. If the actual amount of equalisation reserve fulfills the condition

$$E \geq \text{Max} \left\{ 0, \frac{1}{\sqrt{1.05}} (M y(\tau) - P) - C \right\} \quad (2)$$

where $\tau = E\{X\}/M$ and where $y(\tau)$ is the smallest integer ≥ 2 satisfying

$$e^{-\tau} \sum_0^{\tau-1} \frac{\tau^r}{r!} \geq .99,$$

then $E \geq E_{\min}$. With all the more reason this is true if $\tau = \frac{1}{M} \sum (1 + q_k) P_k$ ($\sum P_k = P$) since in practice it may always be assumed that $E\{X\} \leq \sum (1 + q_k) P_k$. This simple test is based on a proposition¹⁾ that for values of the argument greater than the mean value the ordinary Poisson function with parameter $E\{X\}/M$

¹⁾ Since the proof of this proposition is known only in a special case the term hypothesis is more appropriate. More details are found in the paper "On the Calculation of the Generalised Poisson Function" by the present author.

and with the size M of one claim is more dangerous than $F(x)$ if the disturbance caused by steps in the Poisson function is removed. Since the function $y(\tau)$ is given as a table in the instructions, the test calculation becomes elementary.

The maximum amount E_{\max} is based on the condition that the equalisation reserve may remain at most with a probability of 99 per cent positive during the period of the next five years assuming that each year $n = \Sigma (1 + q_k)n_k$. Consequently E_{\max} is determined by means of a five fold integral which is easily calculated as soon as $F(x)$ is known (this kind of integral and also more complicated five year assumptions can be treated conveniently by a Monte Carlo method). In practice, however, the calculation of E_{\max} also becomes necessary only exceptionally, because in this case too it is possible to construct elementary tests which normally verify directly that $E \leq E_{\max}$.

In the rare event that a company cannot avoid the calculation of $F(x)$ the main task is the estimation of functions S_k . As soon as these are known there is a standard computer program which calculates $F(x)$ fairly quickly and consequently at a low price. By means of this standard program companies are released from the necessity of having experts familiar with mathematical problems in connection with the calculation of generalised Poisson functions.