

BONUS-MALUS SYSTEMS:  
“LACK OF TRANSPARENCY” AND ADEQUACY MEASURE

BY

PAOLA VERICO

ABSTRACT

Based on a recent contribution by Baione and others in this Journal, some consequences of the decrease of the mean merit coefficient for portfolios of Bonus-Malus policies, and some alternative ways to measure the “quality” of a Bonus-Malus system are discussed.

1. THE “TRANSPARENCY” OF B-M SYSTEMS

It has long been known that, in most existing Bonus-Malus systems, the policyholders tend to concentrate on the high discount classes, without this tendency being counterbalanced by an adequate scale of merit factors; so that — as a final effect — the yearly “mean merit coefficient” progressively decreases. Yet, it is my opinion that the practical consequences thereof have been largely underestimated.

In a recent paper of Baione and others (henceforth: BLM 2002), in which the concept of “lack of transparency” (Verico 2000) is resumed, the question is briefly discussed under the point of view of commercial correctness. Indeed the insurers, in order to grant to the large numbers of good drivers the reductions they have promised them, find themselves forced to increase the base premium: in this way, as BLM say, “most of the bonus evaporates”.

But much more can be said. The decrease of the mean factor actually determines a transfer of financial weight among the generations of policyholders (as already observed — f.i. — in Lemaire 1995), which is really hard to justify; and causes a yearly automatic increase of the premium that, in its turn, is very hard to accept by the insured.

To make the preceding observations quantitative, we start by considering one of the examples in (BLM 2002): namely, the “kenyan” B-M system. We remember that the merit coefficients are:

$$c_1 = 0.4; c_2 = 0.5; c_3 = 0.6; c_4 = 0.7; c_5 = 0.8; c_6 = 0.9; c_7 = 1.$$

Note that it is a “pure bonus” system: the merit factors go from 1 — which is the coefficient for the entry class — to 0.4: it is quite evident that no potential

customer may expect such a system to be transparent, where none apparently pays for the reductions granted to good drivers.

BLM consider a closed portfolio, in the “steady state” situation. We will assume the portfolio to be open, with a yearly renewal rate, say, of 3% (every year, 3% of each class policyholders leave, and are substituted by an equal number of newly insured: who are, of course, inserted into the entry class). Assuming all others of BLM hypotheses, the sequence of the portfolio mean factor coefficients starting from “year 1” (the year when the system is proposed on the market) is:

$$C(1) = 1; C(2) = 0.915; C(3) = 0.843; C(4) = 0.780; C(5) = 0.726; \\ C(6) = 0.679; C(7) = 0.638 = C(\infty)$$

(the system is so simple, that 7 years are enough for it to reach the steady state).

Suppose that the base premium is constantly equal to 1; then the premium due by a policyholder who, during the  $t$ -th year of life of the system, belongs to the  $j$ -th merit class, is given by  $c_j/C(t)$ . It is trivial to calculate that, for a driver who enters the system in its year of birth, and reports no claim during 10 years, the overall amount to be paid is 7.90 ( $= c_7/C(1) + c_6/C(2) + \dots + 4c_1/C(7)$ ); whereas, under the same conditions, a driver who enters 7 years afterwards will pay an amount of 9.56. So, the second policyholder, who has in principle the right to be treated in exactly the same way as the first, is called upon to pay 21% more.

As a second example, we consider the italian standard system. The merit classes here are 18, the entry one being the number 14, and the merit factors are:

$$c_1 = 0.5; c_2 = 0.53; c_3 = 0.56; c_4 = 0.59; c_5 = 0.62; c_6 = 0.66; c_7 = 0.7; \\ c_8 = 0.74; c_9 = 0.78; c_{10} = 0.82; c_{11} = 0.88; c_{12} = 0.94; c_{13} = 1; c_{14} = 1.15; \\ c_{15} = 1.3; c_{16} = 1.5; c_{17} = 1.75; c_{18} = 2$$

For the same portfolio as above, the mean merit factors  $C(t)$  are:

$$C(1) = 1.15; C(2) = 1.072; C(3) = 1.053; C(4) = 1.022; C(5) = 0.976; \\ C(6) = 0.957; C(7) = 0.932; C(8) = 0.902; C(9) = 0.883; C(10) = 0.862; \dots \\ C(20) = 0.750; C(30) = 0.729; C(40) = 0.723; \dots; C(\infty) = 0.721$$

The overall expense for a driver who reports no claim during ten years depends on the year of his/her entering into this insurance, and goes from a minimum of 8.379 (if he/she buys the first policy the very year the system appears) up to a maximum of 11.498 (if he/she enters when the system has reached the steady state). For people who enter the system in its 10-th year of life, the overall expense amounts to 10.337: this means that the policyholders of this generation pay, during ten years of good conduct, more than what they should in a system where everyone paid the average premium (exactly 10). And things get worse and worse as the maturity of the system grows.

This redistribution of the financial weight, for which more recent policyholders pay for the advantages of the older ones, is not only surreptitious, but also — in our opinion — totally unjustified.

A second annoying consequence of the "lack of transparency" is the fact that, after a claimless year, a policyholder may be asked to pay, at his/her renewal, a higher premium than the one paid the year before: and this not (or not only) because of inflation, but just because of the diminution of the average merit factor. Although he/she is being promoted from class  $j$  up to class  $j-1$ , it may indeed happen that  $c_j/C(t) < c_{j-1}/C(t+1)$ . Note that this is always the case for people belonging to the best merit class: at least, until the steady state has not been reached (50 years are not enough for it, in Italy!).

Now, people may be not aware of the fact that, because of the mean merit factor effect, classes presented as "bonus" ones are, in reality, "malus"; but many are able to compare one year's premium with that of the preceding one. Maybe the scarce popularity of the automobile insurance in many countries can be better understood on this basis.

## 2. HOW TO MEASURE ADEQUACY

There is a second stimulating argument dealt with in (BLM 2002), and that is the way one should better measure the quality, or "adequacy" of a B-M system (its capability to bring every policyholder up to the point where he pays a premium fair for him). For this problem, many solutions have been suggested by several authors. BLM propose this partially new formula:

$$\sum_{i=1}^s \int_0^{\infty} a_i(\lambda) [c_i \lambda_0 - \lambda]^2 u_i(\lambda) d\lambda \tag{1}$$

in which  $\lambda_0$  stands for the portfolio mean claim frequency,  $a_i(\lambda)$  is the probability of belonging to merit class  $i$  for a policyholder with claim frequency  $\lambda$ , and  $u_i(\lambda)$  is the density of claim frequency for individuals belonging to merit class  $i$  (both  $a_i(\lambda)$  and  $u_i(\lambda)$  refer to the "steady state" situation). The quantity (1) should give the error a system makes when stability has been reached, in letting each policyholder pay a premium different from the one fair for him; and the authors look for a vector ( $c_i$ ) which minimizes it, under the constraints — among others — that the resulting system is fully transparent (otherwise stated: the corresponding mean factor coefficient is equal to 1. The same idea is in (Verico 2000)), where a different measure of "adequacy" is employed).

In (1), the conditional density  $u_i(\lambda)$  (conditioning event: "the policyholder is in class  $i$ ") appears combined with the conditional probability  $a_i(\lambda)$  (conditioning event: "the policyholder has claim frequency  $\lambda$ "); their simultaneous use is questionable. Neither is the fact, explicitly stated by BLM, that the densities  $u_i(\lambda)$  correspond to gamma distributions, at all clear; the hypothesis being only that this is true for the density  $u(\lambda)$  of the whole portfolio (an explicit expression for  $u_i(\lambda)$  should be:

$$u_i(\lambda) = \frac{a_i(\lambda)u(\lambda)}{\int_0^{+\infty} a_i(\lambda)u(\lambda)d\lambda}$$

A last observation: if we want to compare the fair premium for an individual of claim frequency  $\lambda$  with the steady state actual premium for an  $i$  merit class policyholder, then the difference to be considered is not  $c_i\lambda_0 - \lambda$  (the one that appears in (1)), but rather  $c_i\lambda_0/C(\infty) - \lambda$ . Taking both remarks into account, (1) should be substituted by:

$$\sum_{i=1}^s \int_0^{\infty} a_i(\lambda) \left[ \frac{c_i\lambda_0}{C(\infty)} - \lambda \right]^2 u(\lambda) d\lambda. \quad (2)$$

Formula (2) yields the mean of the square of the error the system makes. My personal opinion is that a still better measure of the adequacy of a B-M mechanism is given by the square of the mean of the same error, that is:

$$\int_0^{\infty} \left[ \sum_{i=1}^s a_i(\lambda) \frac{c_i\lambda_0}{C(\infty)} - \lambda \right]^2 u(\lambda) d\lambda. \quad (3)$$

The significant difference between (3) and (2) is that using (3) means making, for every individual frequency  $\lambda$ , a compensation among the errors made in letting people pay more or less than their due, according to the class in which they are inserted. With (2), no such compensation is made, and every error is accounted for.

Both choices may be regarded as logical, but I think that (3) is to be preferred for the following reason. The probabilities  $a_i(\lambda)$  can be interpreted as the time percentage a  $\lambda$  claim frequency policyholder spends in class  $i$ , when the system is in the steady state. Now, the fact that the system is in such a state does not mean that every policyholder has once and for all reached his/her definitive merit class; but only that his/her probabilities of belonging to the different classes no longer vary over the years. Each policyholder will, then, continue wandering up and down the class scale. In this situation, for some of the years he/she will have to pay more, and for some others less than what is fair for him/her: some of the errors the tariff makes are to his/her advantage, some are to his/her disadvantage; so that compensating them, as (3) does, is in my opinion not only fair, but almost — in some sense — compulsory.

#### REFERENCES

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Paola VERICO  
 Dipartimento di Scienze Attuariali e Finanziarie  
 via Nomentana, 41  
 00161 – Roma, Italia  
 E-mail: [paola.verico@unirom.it](mailto:paola.verico@unirom.it)