OPTIMUM STRUCTURE OF TRENDS OF ACTIVITIES TO PREVENT CHANCE DAMAGES

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In the socialist countries, where insurance — like the other primary sectors of the national economy — has been nationalized and is conducted by a single state-owned insurance company, a whole number of actuarial problems have arisen. These are problems which either did not interest the private insurance companies or else were marginal questions for them, of no great practical importance. The actuarial problems which have cropped up or have acquired particular importance in the conditions of socialist economy, include questions connected with evaluation of the efficiency of outlays for the insurance company's preventive activities intended to restrict the number and scope of chance damages [1]. Henceforth we shall refer to expenditures of this sort as *preventive outlays /expenditures/*.

One of the initial problems pertaining to the preventive activities of the state insurance monopoly is one which could be called the optimum structure of the trends of activities to prevent chance damages.

We present the problem in the most simple form, formulating it so that it can be solved by generally-known calculus methods employed in linear programming (e.g. the simplex method given by G. B. Danzig in 1951).

Let us assume that in a given period, e.g. in a year, the insurance

[1] A characteristic feature of insurance of the socialist type, it seems, is the attachment of special importance to activities to prevent chance damages, connection of these activities with the insurance operations and the assignment of these activities within the given range to the state insurance company. An instance which confirms this thesis is the new law of Dec. 2, 1958, on property and personal insurance in Poland; this law attaches no less importance to the activities of Polish National Insurance to prevent chance damages than it does to the insurance operations and it allocates for preventive activities part of the insurance rates and the bulk of the profits from insurance operations.

company allocates a certain sum I for preventive purposes. Realization of these outlays is expected to reduce damage payments made by the insurance company during that period; let us call that decrease Q, the „savings" obtained by the insurance company.

It is possible to spend the total preventive outlays I on n different types (trends) of preventive activities e.g. expenditures for fire damages prevention (with possible further division into expenditures to increase the fire resistance of the insured buildings, for the installation of lightning arresters, for improvements in the water supply or for better equipment for the fire fighters, etc.), flood damages, prevention to damages due to motor traffic, etc. Denoting by $I_1, I_2, \ldots, I_n$ the preventive outlays in the individual fields of preventive activities, we have

$$I = \sum_{i=1}^{n} I_i.$$

Further, we denote by $\lambda_i = \frac{I_i}{I}$ $(i = 1, 2, \ldots, n)$

the *sector indices of structure of preventive outlays*; of course

$$0 \leqslant \lambda_i \leqslant 1 \text{ and } \sum_{i=1}^{n} \lambda_i = 1.$$

The problem is to find such a structure of preventive outlays, i.e. the coefficients $\lambda_1, \lambda_2, \ldots, \lambda_n$, that their economic effect, that is „the savings" on damage payments, is the maximum ($Q = \max.$).

In turn, let us assume that we know the *sector coefficients of the efficiency of the preventive outlays* β_i in each sector of preventive activity; we define the coefficient as: $\beta_i = Q_i/I_i$ $(i = 1, 2, \ldots, n)$, where Q_i stands for the saving effected by the preventive outlay realized in the i-th sector of preventive activity. Therefore, the efficiency coefficient β_i denotes the saving resulting in the i-th sector of preventive activity from the realization in that sector of a unit preventive outlay (e.g. 1000 zlotys).

We have: $Q = \sum_{i=1}^{n} Q_i = \sum_{i=1}^{n} \beta_i I_i = I \sum_{i=1}^{n} \beta_i \lambda_i.$

Of course, the condition $Q = \max$ can be replaced by the condition $Z = \sum_{i=1}^{n} \beta_i \lambda_i = \max.$, since $I = \text{const.}$

The function $Z = \sum_{i=1}^{n} \beta_i \lambda_i = \frac{\sum_{i=1}^{n} \beta_i \lambda_i}{\sum_{i=1}^{n} \lambda_i}$, for which we intend to find the maximum value, can be defined as the *overall index of the given structure of preventive outlays.* It is the weighted average of the sector coefficients of efficiency for the preventive outlays, the weights being the sector indices of the structure of preventive outlays.

The problem of finding the maximum value of the function of purpose $Q = \sum_{i=1}^{n} \beta_i I_i$ or the function $Z = \sum_{i=1}^{n} \beta_i \lambda_i$ would be a trivial problem if there were generally speaking unlimited possibilities for the realization of the preventive outlays in every sector of preventive activity and if the realization of these outlays in one sector were not connected in some way with the realization of such outlays in another sector. Naturally, then for $Q = \max.$ the entire sum assigned for prevention of chance damages should be spent in the sector in which the sector coefficient of efficiency of preventive outlays is the largest.

If there were some restrictions on the preventive outlays in the particular sectors and if $\beta_1 > \beta_2 > \ldots > \beta_n$, then an appropriate proportion of the total investment outlays should be used above in sector 1 to an extent corresponding to the size of the limit on outlays in that sector, then in sector 2 also to the limit on outlays in sector 2, and so on until the entire fund I was exhausted.

It appears in practice, however, that the realization of the investment outlays in one sector generally reduces the possibility of the realization of the preventive outlays in other sectors. For instance, for the implementation of its intentions to restrict the chance damages the insurance company has a given number of specialists; increased employment of these specialists at fire prevention, for instance, reduces the possibilities for the realization of outlays intended to reduce damages in communication, etc. It is a similar story with the material and financial resources for the execution of prevention plans. There is a limit to the means of transport connected with propaganda and realization of prevention goals, there is a limit to the foreign currency expenditures assigned for the purchase abroad of certain technical equipment designed to improve fire safety in the insured objects and so forth. These limited means for the realization of prevention plans are in principle utilized to a different extent in the various sectors of preventive activity. The realization of preventive outlays of one million zlotys, for instance, in the fire prevention sector, requires the employment of one specialist and the realization of the same outlays in personal prevention calls for two specialists etc.

Let us assume that there are m limited means of realizing the programme of chance damage prevention and by a_{ri} we denote the coefficients characterizing the utilization of the r-th means in the realization of preventive outlays in the i-th sector of preventive activity. Therefore, by definition

$$a_{ri} = \frac{s_{ri}}{I_i} = \frac{s_{ri}}{I\lambda_i}, \ (i = 1, 2, \ldots, n; r = 1, 2, \ldots, m),$$

where s_{ri} stands for the quantity or value of the r-th means, expressed in natural or monetary units, utilized in the realization of the preventive outlays to the size of Q_i in the i-th sector of preventive activity.

Assuming that the coefficients a_{ri} are known (it is not difficult to find them statistically or by calculation) we obtain

$$s_{ri} = I\lambda_i a_{ri} \ (i = 1, 2, \ldots, n; r = 1, 2, \ldots, m).$$

In turn let us assume that $b_1, b_2, \ldots, b_m$ denote the limits of the individual means available that are indispensable for the realization of the given programme of preventive activity; then, the „balance-sheet relationship" limiting the size of the sector indices of the structure of preventive outlays, can be represented in the form

$$\sum_{i=1}^{n} I \lambda_i a_{ri} = I \sum_{i=1}^{n} a_{ri} \lambda_i \leqslant b_r \; (r = 1, 2, \ldots, m)$$

$$\text{or} \sum_{i=1}^{n} a_{ri} \lambda_i \leqslant c_r \; (r = 1, 2, \ldots, m), \text{ where } c_r = \frac{b_r}{I}.$$

In this way, the problem of finding the optimum structure of trends of activities by the insurance company to prevent chance damages, assuming that a sum of money I is available for the purpose and that there are limited means for the realization of the preventive programme, can be represented mathematically as follows:

The point is to determine the values of the variable $\lambda_1, \lambda_2, \ldots \lambda_n$, (sector indices of the structure of preventive outlays), for which the function of purpose Z (overall index the given structure of preventive outlays) and at the same time ,,savings on damage payments'' Q reaches the maximum:

$$(1) \; Z = \sum_{i=1}^{n} \beta_i \lambda_i = \max.$$

with the satisfaction of the following side conditions

$$(2) \; \sum_{i=1}^{n} a_{ri} \lambda_i = c_r \; (r = 1, 2, \ldots, m)$$

and the additional ,,boundary conditions''

$$(3) \; \lambda_i \geqslant 0 \; (i = 1, 2, \ldots, n)$$

$$\text{and} \sum_{i=1}^{n} \lambda_i = 1.$$

The coefficients appearing in formulae (1), (2) and (3), that is, β_i, a_{ri} and c_r $(i = 1, 2, \ldots, n; r = 1, 2, \ldots, m)$ are known by assumption.

Note that the balance-sheet conditions (2) given in the form of an equality can be replaced by the inequality:

$$\sum_{i=1}^{n} a_{ri}\lambda_i \leqslant c_r \ (r = 1, 2, \ldots, m).$$

This last-mentioned condition means that the staff and material or financial resources available for the realization of outlays in the particular sectors of preventive activity do not have to be used up completely. It is not difficult to show that replacement of the balance-sheet equations by balance-sheet inequalities does not change the methods for the solution of our problem.

The additional condition $\sum_{i=1}^{n} \lambda_i = 1$ denotes that in the problem there are in fact $n - 1$ unknowns. An unknown of the type λ_n, for instance, can be determined from the other $n - 1$ unknowns: $\lambda_1 \lambda_2, \ldots, \lambda_{n-1}$.

From the above mathematical formulation of the problem of finding the optimum program of preventive activities, it appears that it is a typical problem of linear programming, which is not difficult to solve in calculations, especially since the number n of possible directions of this activity and the number m of limiting conditions $(m < n)$ is small. When, for instance, $n = 3$ and $m = 2$, the solution can be obtained graphically without difficulty.

On the other hand, there are some practical difficulties involved in determining the parameters β_i and a_{ri} .Note, first of all, that the sector coefficients of efficiency of the preventive outlays λ_i denoting the ratio of savings on payment damages Q_i to the total preventive outlays I_i can be found by the statistical method with a certain definite probability. We have, therefore, a model of programming in conditions of stochastic uncertainty. The simplest way out of this situation is, it seems, to take coefficients λ_i at their expected values. The size of the savings on payment damages can be found in relatively simple manner if it turns out that the realized preventive outlays I_i will cause the given group of insured objects to pass from one class of hazard to another; it is assumed at the outset that the statistically determined rate of chance damage (ratio of damage payments to value of objects) in both classes of hazard is known.

A further difficulty involved in the determination of the coefficients, generally speaking, may depend on the size of the outlays realized in the given sector of preventive activity. For instance, the economic effect of preventive outlays of the first one million zlotys may be less than the economic effect obtained from the second million zlotys of preventive outlays.

Rejection of what is in actual fact a realistic assumption that the savings on damage payments Q_i are proportional to the preventive outlays I_i (and likewise, that s_{ri} are proportional to I_i) would alter the presented model of linear programming into one of non-linear programming which could be solved by methods of marginal calculus.

Evaluation of Q_i also requires that account be taken of the fact that the economic effects from preventive outlays realized in a given year, in principle, last more than a year. Moreover, apart from the evaluation of the direct economic effects from preventive activity account should be taken of the indirect economic effects of this activity and the resulting general social benefits.

In this short presentation of the problem of programming of the optimum structure of directions of activity to prevent chance damage, consideration of all the complications mentioned and others, was left out.