ESTIMATING PER CAPITA EXPENSES IN MULTIPLE STATE MODELS OF PERMANENT HEALTH INSURANCE

By

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Abstract

The aim of this work is to present a method to compute the expected amount of annual claims in the health insurance. (A more detailed analysis of the method can be found in Kovářová (1998).) We will be especially concerned with the permanent health insurance and compare our numerical results with those computed and published for the permanent health insurance in C.M.I.R. (1991). At the same time we will give a general methodology applicable not only to the permanent health insurance but for other types of health insurance as well.

The methods used today can be divided into three groups: the method of the decrement tables, the Manchester Unity method and the multiple state model. We would like to describe a simple method to compute from the data of the multiple state model the bases for the Manchester United and for the decrement tables methods.

Keywords

Permanent Health Insurance; Multiple State Model; Transition Intensity; Claims; Manchester Unity Model.

1. The Regenerative Model

Let us outline how the decrement tables method is typically used in German insurance companies. For each age x or an age group the average annual amount of claims (called "Kopfschaden") is computed. If the average amount is not evaluated for each single age separately but for age groups, it is assumed that the resulting value corresponds to the central age in this group (e.g., to the 28 years old person in the group of 26-30 years olds). The average annual amounts of claims for all ages are then computed using interpolation (usually quadratic). The bases for the Manchester Unity method are analogous averages.

We will present a method for calculating the expected amount of claims on the basis of a multiple state model. We will use the model for permanent health insurance published in C.M.I.R. (1991), where the observed persons can move between three states: healthy, sick and dead with certain transition intensities (dependent on the age and possibly on the length of the sickness).

Since we are always concerned with just one age group, the dependency on the age is omitted and we use a slightly modified notation for the transition intensities than it has been done in C.M.I.R. (1991): Let μ (denoted μ_x in C.M.I.R. (1991)) be the transition intensity from the state healthy to the state dead, σ (denoted σ_x in C.M.I.R. (1991)) the intensity of becoming sick, $\rho(u)$ (denoted $\rho_{x,u}$ in C.M.I.R. (1991)) the intensity of recovery, $\nu(u)$ (denoted $\nu_{x,u}$ in C.M.I.R. (1991)) the transition intensity from the state sick to the state dead, where u equals the length of the sickness.

As it is often done, we will consider 5 year age groups. Let us suppose that we have a collection of individuals of the same 5 year age group (with known transition intensities) and let us consider the parts of their lives when they belonged to the investigated age group to be the statistical file we are exploring. Our method is based on the idea to compound these parts into a single infinite trajectory. This leads us to the model of observing one hypothetic person for an arbitrary long time period. In order to get a sufficiently large statistical file, we replace the person after his death with "another healthy one". From the mathematical point of view it will be convenient to model a "revival" with transition intensity λ (for the transition from the state dead to the state healthy). The numerical value of the transition intensity λ from the state dead to the state healthy is chosen equal to two for the following reason: Let us suppose that we observe a standard collection of l_x individuals for one year. It can be assumed that those who die will die on average in the middle of the year. Hence, we want the expected length of the time spent in the state dead in our model to be equal to one half. This yields $\lambda = 2$. However, the numerical results showed that the choice of the parameter λ practically does not affect the numerical results (we computed the results also for $\lambda = \infty$).

Now we have a hypothetic person with constant transition intensities from the state healthy to the state sick or dead, with transition intensities dependent on the length of the sickness from the state sick to the state healthy or dead and with a nonzero transition intensity from the state dead to the state healthy.

The transition intensities do not change within the 5 year intervals rapidly. Thus we take a "typical representative" of each group, a person of the age lying in the middle of the corresponding 5 year interval.

The life within the three states of the hypothetical person is a trajectory of a regenerative process since it can be divided into mutually independent cycles. The time points between the cycles are called the regeneration points. As regeneration points we shall use the inceptions of the sickness.

The definition of the regenerative process can be understood more generally – the first cycle can start later, not necessarily in the point of regeneration.

We will further call the model of the health insurance we are interested in the regenerative model of the health insurance, shortly the regenerative model.

Let us denote

τ_i ... the length of the i-th cycle of the regenerative model,

ξ_i ... the cost associated to the i-th cycle of the regenerative model.

This gives us identically distributed variables τ_1, τ_2, ... and associated ξ_1, ξ_2, ... The pairs (τ_i, ξ_i) are mutually independent and identically distributed (except perhaps for $i = 1$).

The meaning of ξ_i is rather general. It could mean some cost during the sickness; it could be the time spent between the n-th and $(n + m)$-th week of sickness, it could be the benefit paid when sick. It could also mean the premium paid by the insured in the time when no benefit from the insurance company is paid.

By using the central limit theorem we get the following results for the asymptotic distribution of the number of claims and of the amount of claims:

Let N_t be the number of cycles of the regenerative model until time t, $E\tau_i = E\tau$ the expected value of the length of the i-th cycle of the regenerative model and $E\xi = E\xi_i$ the cost associated to the i-th cycle of the model. Furthermore, $\tau_i = s^2$ (assumed finite) for all $i \in N$. Then asymptotically (as t tends to infinity)

$$\frac{N_t - \frac{t}{E\tau}}{s\sqrt{\frac{1}{(E\tau)^3}}} \sim N(0,1)\,. \tag{1}$$

Our further interest will be

$$\kappa = \frac{E\xi}{E\tau},$$

which represents "the cost per individual per time unit" or "the annual cost per individual".

Example:

Let

$$\xi \equiv 1\,. \tag{2}$$

Then

$$\kappa = \frac{1}{E\tau} \tag{3}$$

corresponds to the expected number of diseases per time unit.

Let us denote $\sigma^2 = \text{var}(\xi - \kappa\tau)$, X_T the cost until time T (which corresponds to the costs of the number of T individuals observed during a one-year period) and let us suppose our regenerative model as described above. Then

$$\frac{X_T - \kappa T}{\sqrt{T}} \sim N\left(0, \frac{\sigma^2}{E\tau}\right) . \tag{4}$$

With our assumptions, we also get

$$\sigma^2 = \text{var}(\xi - \kappa\tau) = \frac{s^2}{(E\tau)^2} . \tag{5}$$

(See C.M.I.R. (1991).)

2. Calculation of Costs

Let us denote
c_j ... the cost per time unit in the state j,
$k(t)$ ... the state at time t.
Then

$$X_T = \int_0^T c_{k(t)} dt , \tag{6}$$

$$\xi_k = \int_{\tau_{k-1}}^{\tau_k} c_{k(t)} dt . \tag{7}$$

Let us come back to the multiple state model we are dealing with and denote the states by numbers:

1 ... healthy
2 ... sick
3 ... dead.

We suppose c_1, c_3 to be constant, c_2 dependent on the length of the sickness. Then we get for the expected value of costs for the time spent in the three states the following results:

$$\text{state healthy} \ldots \quad \frac{c_1}{\sigma + \mu} \tag{8}$$

$$\text{state dead} \ldots \quad \frac{c_3}{\lambda} \tag{9}$$

$$\text{state sick} \ldots \quad \int_0^\infty c_2(u) e^{-\int_0^u (\rho(s)+\nu(s))ds} du . \tag{10}$$

(In formula (10) we suppose being at the start of the sickness.)

Let us denote $E_j\xi$ the expected value of the cost in one cycle of the regenerative model for the case when we start our observation in the state j. Then we get

$$E_1\xi = \frac{c_1}{\sigma+\mu} + \frac{\sigma}{\sigma+\mu}\cdot 0 + \frac{\mu}{\sigma+\mu}E_3\xi \tag{11}$$

and

$$E_3\xi = \frac{c_3}{\lambda} + E_1\xi\,. \tag{12}$$

From these equations we can compute $E_1\xi$ and $E_3\xi$ as follows:

$$E_1\xi = \frac{c_1}{\sigma} + \frac{c_3\cdot\mu}{\lambda\cdot\sigma}\,, \tag{13}$$

$$E_3\xi = \frac{c_1}{\sigma} + \frac{c_3(\sigma+\mu)}{\lambda\cdot\sigma}\,. \tag{14}$$

For the expected cost of one cycle of the regenerative model (beginning in the moment of becoming sick) we have

$$E_2\xi = \int_0^\infty (\rho(u)(C(u)+E_1\xi) + \nu(u)(C(u)+E_3\xi))\cdot e^{-\int_0^u(\rho(s)+\nu(s))ds}du\,, \tag{15}$$

$$\text{where} \quad C(u) = \int_0^u c_2(s)ds\,. \tag{16}$$

Integrating by parts we get

$$E_2\xi = \int_0^\infty (c_2(u) + \rho(u)E_1\xi + \nu(u)E_3\xi)e^{-\int_0^u(\rho(s)+\nu(s))ds}du\,. \tag{17}$$

For the second moments of the variable ξ, according to the state where we start our observation, we finally get

$$E_1\xi^2 = c_1^2\left(\frac{1}{\sigma+\mu} + \frac{1}{(\sigma+\mu)^2}\right) + \frac{\mu}{\sigma+\mu}\left(2\cdot\frac{c_1}{\sigma+\mu}E_3\xi + E_3\xi^2\right)\,, \tag{18}$$

$$E_3\xi^2 = c_3^2\left(\frac{1}{\lambda} + \frac{1}{\lambda^2}\right) + 2\cdot\frac{c_3}{\lambda}E_1\xi + E_1\xi^2\,, \tag{19}$$

$$E_2\xi^2 = \int_0^\infty \Big(\rho(u)\Big(C(u)^2 + 2C(u)E_1\xi + E_1\xi^2\Big) + \tag{20}$$

$$+ \nu(u)\Big(C(u)^2 + 2C(u)E_3\xi + E_3\xi^2\Big)\Big) \cdot e^{-\int_0^u (\rho+\nu)ds} du \,.$$

So we have found the general form of the expected cost for each stay in a specified state.

To calculate for example the expected cost of claims in one cycle of the regenerative model for a permanent health insurance where benefits are payed between the a-th and $(a + b)$-th week of sickness, we define (to get standardized values)

$$\begin{aligned} c_2(u) &= 1 \quad \text{for } a \text{ weeks } \leq u \leq (a+b) \text{ weeks,} \\ c_2(u) &= 0 \quad \text{otherwise,} \\ c_1 = c_3 &= 0 \,. \end{aligned} \tag{21}$$

These definitions and relations

$$E_1\xi = \frac{c_1}{\sigma+\mu} + \frac{\mu}{\sigma+\mu} E_3\xi \,,$$

$$E_3\xi = \frac{c_3}{\lambda} + E_1\xi$$

yield

$$E_1\xi = E_3\xi = 0 \,. \tag{22}$$

This matches the intuitive idea that in the period starting when the individual is healthy and ending by becoming sick or in the period starting when the individual is dead and ending by becoming sick, no benefit will be paid. We have then

$$E_2\xi = \int_0^\infty c_2(u) \cdot e^{-\int_0^u (\rho(s)+\nu(s))ds} du = \int_{\frac{a}{w}}^{\frac{a+b}{w}} e^{-\int_0^u (\rho(s)+\nu(s))ds} du \,, \tag{23}$$

where $w := 52.18$ is the number of weeks in a year.

To get the expected annual amount of claims ($E_2\xi$ represents the expected claims in one cycle of the regenerative model) we have to compute E_T, i.e.

the expected length of such cycle. For this purpose we define c_1, c_2, c_3 as follows:

$$c_1 = c_2 = c_3 = 1 \tag{24}$$

and we have for $E_1\xi$, $E_3\xi$:

$$E_1\xi = \frac{1}{\sigma+\mu} + \frac{\mu}{\sigma+\mu} E_3\xi \,, \tag{25}$$

$$E_3\xi = \frac{1}{\lambda} + E_1\xi \,. \tag{26}$$

The solution of this system of equations is

$$E_1\xi = \frac{1}{\sigma} + \frac{\mu}{\sigma\cdot\lambda} \,, \tag{27}$$

$$E_3\xi = \frac{1}{\sigma} + \frac{\sigma+\mu}{\sigma\cdot\lambda} \,. \tag{28}$$

For $E_T = E_2\xi$ we get

$$E_T = E_2\xi = \int_0^\infty (\rho(u)E_1\xi + 1 + \nu(u)E_3\xi)\cdot e^{-\int_0^u (\rho(s)+\nu(s))ds} du \tag{29}$$

$$= \int_0^\infty \left(1 + \frac{\lambda+\mu}{\sigma\cdot\lambda}\rho(u) + \frac{\lambda+\sigma+\mu}{\sigma\cdot\lambda}\nu(u)\right)\cdot e^{-\int_0^u (\rho(s)+\nu(s))ds} du \,.$$

The annual expected cost is equal to $\frac{E_2\xi}{E_T}$.

Under the assumption of the regenerative model the standard deviation of the average annual costs per person in a portfolio of T individuals is equal to

$$\frac{\frac{\sigma}{\sqrt{E_T}}}{\sqrt{T}} \,. \tag{30}$$

3. Numerical Results

Using the method we have just described we are able to compute the expected claims for various types of insurance. For computation of integrals the software MATHEMATICA has been used, for transition intensities we use the graduated values published in C.M.I.R. (1991).

The symbol $z_x^{a|b}$ stays for the expected annual amount of money paid to an x-year old person if he has an insurance agreement of the type where one money unit is paid weekly between the a-th and $(a+b)$-th week of sickness.

The following tables compare our results (using the regenerative model) and the results computed in C.M.I.R. (1991) – Tables 23b, 23c and 23d (using selection) where t in $z^{a|b}_{[x-t]+t}$ means that a person now x years old entered the model t years ago as a healthy one.

TABLE 1

x	$z^{1\|3}_{[x]}$ (CMIB)	$z^{1\|3}_{[x-1]+1}$ (CMIB)	$z^{1\|3}_{[x-5]+5}$ (CMIB)	$z^{1\|3}_{x}$ (regen. m.)
30	0.1468	0.1526	0.1525	0.1526
35	0.1617	0.1683	0.1681	0.1671
40	0.1784	0.1856	0.1853	0.1823
45	0.2006	0.2087	0.2079	0.2015
50	0.2342	0.2434	0.2417	0.2284
55	0.2891	0.2996	0.2956	0.2676
60	0.3837	0.3952	0.3842	0.3227
64	0.5123	0.5209	0.4948	0.3773

TABLE 2

x	$z^{4\|9}_{[x]}$ (CMIB)	$z^{4\|9}_{[x-1]+1}$ (CMIB)	$z^{4\|9}_{[x-5]+5}$ (CMIB)	$z^{4\|9}_{x}$ (regen. m.)
30	0.0663	0.0765	0.076	0.0763
35	0.0846	0.0978	0.0977	0.0967
40	0.1078	0.1248	0.1246	0.1218
45	0.1398	0.1619	0.1613	0.1551
50	0.1877	0.2175	0.2159	0.2023
55	0.2661	0.3076	0.3034	0.2721
60	0.4047	0.4653	0.4523	0.3761
64	0.6019	0.6852	0.6495	0.4897

TABLE 3

x	$z^{13\|13}_{[x]}$ (CMIB)	$z^{13\|13}_{[x-1]+1}$ (CMIB)	$z^{13\|13}_{[x-5]+5}$ (CMIB)	$z^{13\|13}_{x}$ (regen. m.)
30	0.016	0.0244	0.0243	0.0242
35	0.0232	0.0353	0.0353	0.0347
40	0.0334	0.0511	0.051	0.0495
45	0.0489	0.0750	0.0747	0.0714
50	0.0742	0.1137	0.1129	0.1005
55	0.1186	0.1814	0.1789	0.1593
60	0.2033	0.3093	0.3006	0.2479
64	0.3325	0.5016	0.4752	0.3545

The influence of the selection disappears after one or two years. Then the values computed in C.M.I.R. (1991) and the values computed by means of the regenerative model correspond well.

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