

RECURSIVE EVALUATION OF SOME BIVARIATE COMPOUND DISTRIBUTIONS

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ABSTRACT

In this paper we consider compound distributions where the counting distribution is a bivariate distribution with the probability function $(p_{n_1, n_2})_{n_1, n_2 \geq 0}$ that satisfies a recursion in the form

$$p_{n_1, n_2} = \left(a_0 + \frac{a_1}{n_1} + \frac{a_2}{n_2} + \frac{a_{12}}{n_1 n_2} \right) p_{n_1-1, n_2-1} + \left(b_0 + \frac{b_1}{n_1} \right) p_{n_1-1, n_2} \\ + \left(c_0 + \frac{c_2}{n_2} \right) p_{n_1, n_2-1}, \quad n_1, n_2 = 1, 2, \dots$$

We present an algorithm for recursive evaluation of the corresponding compound distributions and some examples of distributions in this class.

KEYWORDS

Recursions; compound distributions; bivariate distributions: Trinomial distribution, bivariate Negative Binomial, bivariate Poisson.

1. INTRODUCTION

Let N denote the number of claims occurring in an insurance portfolio within a given period, and Y_i the amount of the i th of these claims. We assume that these claim amounts are positive, integer-valued, mutually independent and identically distributed with the common probability function (p.f.) f , and independent of N . If p denotes the p.f. of N , then the distribution of the aggregate claims $X = \sum_{i=1}^N Y_i$ is a compound distribution with p.f.

$$g = \sum_{n=0}^{\infty} p_n f^{*n}.$$

We make the convention that $\sum_{i=a}^b = 0$ when $b < a$.

Panjer (1981) showed that when p satisfies a recursion in the form

$$p_n = \left(a + \frac{b}{n}\right)p_{n-1}, \quad n = 1, 2, \dots,$$

than g satisfies the recursion

$$g(x) = \sum_{y=1}^x \left(a + \frac{by}{x}\right)f(y)g(x-y), \quad x = 1, 2, \dots$$

Let us now consider that the policies of an insurance portfolio are submitted to claims of two kinds and N_i is the random variable (r.v.) yearly frequency of type i claims, $i = 1, 2$. The distribution of the pair (N_1, N_2) is provided by the joint probabilities

$$p_{n_1, n_2} = P(N_1 = n_1, N_2 = n_2), \quad n_1, n_2 = 0, 1, \dots$$

For the compound model, the aggregate amount of claims of the portfolio is $X = X_1 + X_2$ where, for $i = 1, 2$, $X_i = \sum_{j=1}^{N_i} Y_{ij}$ is the aggregate amount of type i claims. In addition to N_i , this expression needs the sequence $(Y_{ij})_{j \geq 1}$ of the amounts of type i claims.

We assume that the following hypotheses are fulfilled (Partrat, 1993):

H1: For $i = 1, 2$ the r.v. N_i and $(Y_{ij})_{j \geq 1}$ are independent, and the r.v. N_i and $(Y_{3-i,j})_{j \geq 1}$ are independent.

H2: The r.v. $(Y_{1j})_{j \geq 1}$ and $(Y_{2j})_{j \geq 1}$ are independent.

H3: For $i = 1, 2$ the r.v. $Y_{ij}, j \geq 1$, are independent and identically distributed with the common p.f. $f_i(x) = P(Y_{ij} = x), x = 0, 1, \dots$ We assume that $P(Y_{ij} > 0) = 1$.

Then the aggregate amounts of claims (X_1, X_2) have a bivariate compound distribution with the p.f.

$$g(x_1, x_2) = P(X_1 = x_1, X_2 = x_2) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} p_{n_1, n_2} f_1^{*n_1}(x_1) f_2^{*n_2}(x_2), \quad (1.1)$$

where $f_i^{*n_i}$ denotes the n_i -th convolution of $f_i, i = 1, 2$.

Since Panjer (1981), many recursive algorithms have been derived from compound distributions with univariate counting distribution (see Willmot, 1986; Willmot and Panjer, 1987; Panjer and Willmot, 1992; Sundt, 1992; Willmot, 1993; Hesselager, 1994; and others) and also for bi- and multivariate distributions (see Hesselager, 1996; Ambagaspitiya, 1998; Sundt, 1998a, b, 1999a, b).

Hesselager (1996) presented some bivariate extensions of Panjer’s recursion, using bivariate generalisations of the counting distribution. He deduced recursions for g in some cases when p can be interpreted as a bivariate extension of Panjer’s counting distributions. Sundt (1999a) studied a multivariate generalisation of Panjer’s recursion in another direction than Hesselager, assuming that the claim number is one-dimensional, but each claim is an m -dimensional random vector. He also discussed Hesselager’s recursion in Model A, giving some modifications.

In this paper (section 2), we derive a recursive scheme for the compound distribution (1.1) when the counting distribution satisfies the recursive relation

$$\begin{aligned}
 p_{n_1, n_2} = & \left(a_0 + \frac{a_1}{n_1} + \frac{a_2}{n_2} + \frac{a_{12}}{n_1 n_2} \right) p_{n_1-1, n_2-1} + \left(b_0 + \frac{b_1}{n_1} \right) p_{n_1-1, n_2} \\
 & + \left(c_0 + \frac{c_2}{n_2} \right) p_{n_1, n_2-1}, \quad n_1, n_2 \geq 1,
 \end{aligned} \tag{1.2}$$

$$p_{n_1, 0} = \left(d_0 + \frac{d_1}{n_1} \right) p_{n_1-1, 0}, \quad n_1 \geq 1, \tag{1.3}$$

$$p_{0, n_2} = \left(e_0 + \frac{e_2}{n_2} \right) p_{0, n_2-1}, \quad n_2 \geq 1. \tag{1.4}$$

If (1.2)-(1.4) together with the initial value $p_{0,0}$ should determine the p.f., then at least two of the probabilities on the right-hand side of (1.2) should have a non-zero coefficient.

In section 3 we present some examples of distributions in class (1.2)-(1.4): Trinomial distribution, bivariate Negative Binomial and bivariate Poisson.

2. RECURSION FOR THE BIVARIATE COMPOUND DISTRIBUTION

In this section, we first derive a recursion for g when (1.2) is satisfied, then for the initial values of g when (1.3) and (1.4) are verified. Finally, a link with Panjer’s formula is established.

The following theorem gives the recursion for g .

Theorem 1. If (1.2) is satisfied, the compound distribution $g(x_1, x_2)$ can be evaluated by the following recursive method:

$$\begin{aligned}
 g(x_1, x_2) = & \sum_{y_1=1}^{x_1} \sum_{y_2=1}^{x_2} \left(a_0 + a_1 \frac{y_1}{x_1} + a_2 \frac{y_2}{x_2} + a_{12} \frac{y_1 y_2}{x_1 x_2} \right) f_1(y_1) f_2(y_2) \\
 & \times g(x_1 - y_1, x_2 - y_2) + \sum_{y_1=1}^{x_1} \left(b_0 + b_1 \frac{y_1}{x_1} \right) f_1(y_1) g(x_1 - y_1, x_2) \\
 & + \sum_{y_2=1}^{x_2} \left(c_0 + c_2 \frac{y_2}{x_2} \right) f_2(y_2) g(x_1, x_2 - y_2), \quad x_1, x_2 \geq 1.
 \end{aligned} \tag{2.1}$$

Proof: Using (1.2) in (1.1) and the fact that

$$f_i^{*n_i}(0) = \begin{cases} 1, & n_i = 0 \\ 0, & n_i \neq 0 \end{cases}, \quad (2.2)$$

we have for $x_1 \geq 1, x_2 \geq 1$

$$g(x_1, x_2) = \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \left[\left(a_0 + \frac{a_1}{n_1} + \frac{a_2}{n_2} + \frac{a_{12}}{n_1 n_2} \right) p_{n_1-1, n_2-1} + \left(b_0 + \frac{b_1}{n_1} \right) p_{n_1-1, n_2} + \left(c_0 + \frac{c_2}{n_2} \right) p_{n_1, n_2-1} \right] f_1^{*n_1}(x_1) f_2^{*n_2}(x_2). \quad (2.3)$$

We will now introduce conditional expectations like e.g. in the proof of Panjer's formula in Sundt (1993). Splitting the sum in (2.3) into three terms, we obtain for the first one

$$\begin{aligned} & \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \left(a_0 + \frac{a_1}{n_1} + \frac{a_2}{n_2} + \frac{a_{12}}{n_1 n_2} \right) p_{n_1-1, n_2-1} f_1^{*n_1}(x_1) f_2^{*n_2}(x_2) = \\ & \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} E \left[a_0 + a_1 \frac{Y_{11}}{x_1} + a_2 \frac{Y_{21}}{x_2} + a_{12} \frac{Y_{11} Y_{21}}{x_1 x_2} \mid \sum_{j=1}^{n_i} Y_{ij} = x_i, i = 1, 2 \right] p_{n_1-1, n_2-1} \\ & \times f_1^{*n_1}(x_1) f_2^{*n_2}(x_2) = \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \sum_{y_1=1}^{x_1} \sum_{y_2=1}^{x_2} \left(a_0 + a_1 \frac{y_1}{x_1} + a_2 \frac{y_2}{x_2} + a_{12} \frac{y_1 y_2}{x_1 x_2} \right) p_{n_1-1, n_2-1} \\ & \times f_1(y_1) f_2(y_2) f_1^{*(n_1-1)}(x_1 - y_1) f_2^{*(n_2-1)}(x_2 - y_2) = \\ & \sum_{y_1=1}^{x_1} \sum_{y_2=1}^{x_2} \left(a_0 + a_1 \frac{y_1}{x_1} + a_2 \frac{y_2}{x_2} + a_{12} \frac{y_1 y_2}{x_1 x_2} \right) f_1(y_1) f_2(y_2) \\ & \times \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} p_{n_1-1, n_2-1} f_1^{*(n_1-1)}(x_1 - y_1) f_2^{*(n_2-1)}(x_2 - y_2) = \\ & \sum_{y_1=1}^{x_1} \sum_{y_2=1}^{x_2} \left(a_0 + a_1 \frac{y_1}{x_1} + a_2 \frac{y_2}{x_2} + a_{12} \frac{y_1 y_2}{x_1 x_2} \right) f_1(y_1) f_2(y_2) g(x_1 - y_1, x_2 - y_2). \end{aligned}$$

For the second term we obtain

$$\begin{aligned} & \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \left(b_0 + \frac{b_1}{n_1} \right) p_{n_1-1, n_2} f_1^{*n_1}(x_1) f_2^{*n_2}(x_2) = \\ & \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} E \left[b_0 + b_1 \frac{Y_{11}}{x_1} \mid \sum_{j=1}^{n_1} Y_{1j} = x_1 \right] p_{n_1-1, n_2} f_1^{*n_1}(x_1) f_2^{*n_2}(x_2) = \\ & \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \sum_{y_1=1}^{x_1} \left(b_0 + b_1 \frac{y_1}{x_1} \right) p_{n_1-1, n_2} f_1(y_1) f_1^{*(n_1-1)}(x_1 - y_1) f_2^{*n_2}(x_2) = \\ & \sum_{y_1=1}^{x_1} \left(b_0 + b_1 \frac{y_1}{x_1} \right) f_1(y_1) \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} p_{n_1-1, n_2} f_1^{*(n_1-1)}(x_1 - y_1) f_2^{*n_2}(x_2) = \\ & \sum_{y_1=1}^{x_1} \left(b_0 + b_1 \frac{y_1}{x_1} \right) f_1(y_1) g(x_1 - y_1, x_2). \end{aligned}$$

Analogously, for the third term we get

$$\begin{aligned} & \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \left(c_0 + \frac{c_2}{n_2} \right) p_{n_1, n_2-1} f_1^{*n_1}(x_1) f_2^{*n_2}(x_2) = \\ & \sum_{y_2=1}^{x_2} \left(c_0 + c_2 \frac{y_2}{x_2} \right) f_2(y_2) g(x_1, x_2 - y_2). \end{aligned}$$

By summing all these terms we obtain the recursive formula (2.1). □

In conclusion, $g(x_1, x_2)$ can be calculated when all $g(z_1, z_2)$ are known for $z_1 < x_1, z_2 < x_2$.

Remark 1. In particular, Theorem 2.2 in Hesselager (1996) is a special case of Theorem 1 in the present paper.

Let us now consider recursions for the initial values:

Theorem 2. Considering the starting value $g(0, 0) = p_{0,0}$, then
 (a) if (1.3) holds, we have the following recursive formula

$$g(x_1, 0) = \sum_{y_1=1}^{x_1} \left(d_0 + d_1 \frac{y_1}{x_1} \right) f_1(y_1) g(x_1 - y_1, 0), \quad x_1 \geq 1; \tag{2.4}$$

(b) if (1.4) holds, we also have

$$g(0, x_2) = \sum_{y_2=1}^{x_2} \left(e_0 + e_2 \frac{y_2}{x_2} \right) f_2(y_2) g(0, x_2 - y_2), \quad x_2 \geq 1. \tag{2.5}$$

Proof: For $x_2 = 0$ and $x_1 \geq 1$, using (2.2) and (1.3) in (1.1), we have

$$\begin{aligned} g(x_1, 0) &= \sum_{n_1=0}^{\infty} p_{n_1,0} f_1^{*n_1}(x_1) \\ &= \sum_{n_1=1}^{\infty} \left(d_0 + \frac{d_1}{n_1} \right) p_{n_1-1,0} f_1^{*n_1}(x_1). \end{aligned} \quad (2.6)$$

Introducing conditional expectations like in the proof of Theorem 1, we obtain

$$\begin{aligned} g(x_1, 0) &= \sum_{n_1=1}^{\infty} E \left[d_0 + d_1 \frac{Y_{11}}{x_1} \mid \sum_{j=1}^{n_1} Y_{1j} = x_1 \right] p_{n_1-1,0} f_1^{*n_1}(x_1) \\ &= \sum_{n_1=1}^{\infty} \sum_{y_1=1}^{x_1} \left(d_0 + d_1 \frac{y_1}{x_1} \right) p_{n_1-1,0} f_1(y_1) f_1^{*(n_1-1)}(x_1 - y_1) \\ &= \sum_{y_1=1}^{x_1} \left(d_0 + d_1 \frac{y_1}{x_1} \right) f_1(y_1) \sum_{n_1=1}^{\infty} p_{n_1-1,0} f_1^{*(n_1-1)}(x_1 - y_1) \\ &= \sum_{y_1=1}^{x_1} \left(d_0 + d_1 \frac{y_1}{x_1} \right) f_1(y_1) g(x_1 - y_1, 0). \end{aligned}$$

We used (2.6) to get the last equality.

Similarly, we obtain the recursive formula for $g(0, x_2)$, based on (1.4). \square

Assume now that N_1 and N_2 are independent with marginal probability functions q_{n_1} and r_{n_2} that satisfy Panjer's recursions

$$\begin{cases} q_{n_1} = \left(\alpha + \frac{\beta}{n_1} \right) q_{n_1-1}, & n_1 = 1, 2, \dots \\ r_{n_2} = \left(\gamma + \frac{\delta}{n_2} \right) r_{n_2-1}, & n_2 = 1, 2, \dots \end{cases} \quad (2.7)$$

We introduce the marginal probability functions g_1 and g_2 of X_1 and X_2 , which are now independent so that $g(x_1, x_2) = g_1(x_1)g_2(x_2)$.

In the following, we shall show that under assumptions (2.7) the conditions of Theorems 1 and 2 are satisfied, and that using these theorems is equivalent to applying Panjer's recursion on g_1 and g_2 .

For $n_1, n_2 > 0$ we have

$$p_{n_1, n_2} = q_{n_1} r_{n_2} = \left(\alpha\gamma + \frac{\beta\gamma}{n_1} + \frac{\alpha\delta}{n_2} + \frac{\beta\delta}{n_1 n_2} \right) p_{n_1-1, n_2-1},$$

that is, (1.2) is satisfied with

$$a_0 = \alpha\gamma, \quad a_1 = \beta\gamma, \quad a_2 = \alpha\delta, \quad a_{12} = \beta\delta; \quad b_0 = b_1 = c_0 = c_2 = 0.$$

Insertion in (2.1) gives

$$\begin{aligned}
 g(x_1, x_2) &= \sum_{y_1=1}^{x_1} \sum_{y_2=1}^{x_2} \left(\alpha\gamma + \beta\gamma\frac{y_1}{x_1} + \alpha\delta\frac{y_2}{x_2} + \beta\delta\frac{y_1y_2}{x_1x_2} \right) \\
 &\quad \times f_1(y_1)f_2(y_2)g(x_1 - y_1, x_2 - y_2) \\
 &= \left(\sum_{y_1=1}^{x_1} \left(\alpha + \beta\frac{y_1}{x_1} \right) f_1(y_1)g_1(x_1 - y_1) \right) \\
 &\quad \times \left(\sum_{y_2=1}^{x_2} \left(\gamma + \delta\frac{y_2}{x_2} \right) f_2(y_2)g_2(x_2 - y_2) \right) \\
 &= g_1(x_1)g_2(x_2).
 \end{aligned}$$

The last equality follows from Panjer’s formula.

We also have $p_{n_1,0} = q_{n_1}r_0 = \left(\alpha + \frac{\beta}{n_1} \right) q_{n_1-1}r_0 = \left(\alpha + \frac{\beta}{n_1} \right) p_{n_1-1,0}$, that is, (1.3) satisfied with $d_0 = \alpha, d_1 = \beta$. Insertion in (2.4) gives

$$\begin{aligned}
 g(x_1, 0) &= \sum_{y_1=1}^{x_1} \left(\alpha + \beta\frac{y_1}{x_1} \right) f_1(y_1)g(x_1 - y_1, 0) = \\
 &= \left(\sum_{y_1=1}^{x_1} \left(\alpha + \beta\frac{y_1}{x_1} \right) f_1(y_1)g_1(x_1 - y_1) \right) g_2(0) = g_1(x_1)g_2(0).
 \end{aligned}$$

The last equality follows from Panjer’s formula. Analogous for (2.5).

By these calculations we have shown that:

- (a) When N_1 and N_2 are independent with marginal distributions satisfying Panjer’s condition, then their joint distribution satisfies (1.2)-(1.4).
- (b) Using the algorithms of Theorems 1 and 2 with this counting distribution is equivalent with evaluating the joint p.f. of X_1 and X_2 as the product of their marginal probability functions which are evaluated separately by Panjer’s recursion.

Therefore, we have a close relation between Theorems 1, 2 and Panjer’s recursion.

3. EXAMPLES OF BIVARIATE DISTRIBUTIONS SATISFYING (1.2)-(1.4)

The following bivariate distributions are taken from Kocherlakota and Kocherlakota (1992). As an extension of the univariate case, these bivariate distributions are natural candidates for modelling the claim frequency for different risk portfolios. For example, the bivariate Poisson distribution was already used by Picard (1976), Lemaire (1985), Partrat (1993) etc.

Example 1: Let us consider the following model (Hesselager, 1996): The distribution of $K = N_1 + N_2$ satisfies Panjer's recursion

$$P(K = k) = \left(a + \frac{b}{k}\right)P(K = k - 1), \quad k = 1, 2, \dots,$$

for some constants a and b , and the conditional distribution of N_1 given $N_1 + N_2$ is binomial with

$$P(N_1 = n_1 | N_1 + N_2 = n_1 + n_2) = \binom{n_1 + n_2}{n_1} \rho_1^{n_1} \rho_2^{n_2},$$

with $\rho_1 + \rho_2 = 1$. In Theorem 2.1, Hesselager (1996) proved that for this model we have the recursions

$$p_{n_1, n_2} = \rho_1 \left(a + \frac{b}{n_1}\right) p_{n_1-1, n_2} + a \rho_2 p_{n_1, n_2-1}, \quad n_1 \geq 1, \quad (3.1)$$

$$p_{n_1, n_2} = \rho_2 \left(a + \frac{b}{n_2}\right) p_{n_1, n_2-1} + a \rho_1 p_{n_1-1, n_2}, \quad n_2 \geq 1, \quad (3.2)$$

with $p_{n_1, -1} = p_{-1, n_2} = 0$. These are particular cases of (1.2)-(1.4), so that Hesselager's (1996) Theorem 2.2 is a particular case of Theorems 1 and 2.

Thus, for (3.1) we have

$$\begin{aligned} a_0 = a_1 = a_2 = a_{12} = 0; \quad b_0 = d_0 = a \rho_1; \quad b_1 = d_1 = b \rho_1; \\ c_0 = e_0 = a \rho_2; \quad c_2 = e_2 = 0, \end{aligned}$$

and for (3.2) we have

$$\begin{aligned} a_0 = a_1 = a_2 = a_{12} = 0; \quad b_0 = d_0 = a \rho_1; \quad b_1 = d_1 = 0; \\ c_0 = e_0 = a \rho_2 \text{ and } c_2 = e_2 = b \rho_2. \end{aligned}$$

Particular case 1. The Trinomial distribution with parameters (m, r_1, r_2) , m positive integer, $r_1, r_2 \in (0, 1)$, has the p.f.

$$p_{n_1, n_2} = \frac{m!}{n_1! n_2! (m - n_1 - n_2)!} r_1^{n_1} r_2^{n_2} (1 - r_1 - r_2)^{m - n_1 - n_2}. \quad (3.3)$$

We make the convention that $0! = 1$.

It is easy to verify that (3.3) satisfies the recursive relations

$$p_{n_1, n_2} = \frac{r_1}{1 - r_1 - r_2} \left(\frac{m+1}{n_1} - 1\right) p_{n_1-1, n_2} - \frac{r_2}{1 - r_1 - r_2} p_{n_1, n_2-1}, \quad n_1 \geq 1,$$

$$p_{n_1, n_2} = -\frac{r_1}{1 - r_1 - r_2} p_{n_1-1, n_2} + \frac{r_2}{1 - r_1 - r_2} \left(\frac{m+1}{n_2} - 1\right) p_{n_1, n_2-1}, \quad n_2 \geq 1,$$

that is (3.1)-(3.2) and the condition $\rho_1 + \rho_2 = 1$ for

$$\rho_1 = \frac{r_1}{r_1 + r_2}, \rho_2 = \frac{r_2}{r_1 + r_2}, a = -\frac{r_1 + r_2}{1 - r_1 - r_2} \text{ and } b = (m + 1)\frac{r_1 + r_2}{1 - r_1 - r_2}.$$

Particular case 2. The bivariate Negative Binomial distribution with parameters $(\lambda, \theta_1, \theta_2)$, $\lambda > 0$, $\theta_1, \theta_2 \in (0, 1)$ has the p.f.

$$p_{n_1, n_2} = \frac{\Gamma(\lambda + n_1 + n_2)}{\Gamma(\lambda)n_1!n_2!} \theta_1^{n_1} \theta_2^{n_2} (1 - \theta_1 - \theta_2)^\lambda. \tag{3.4}$$

It is easy to verify that (3.4) satisfies the recursions

$$\begin{aligned} p_{n_1, n_2} &= \theta_1 \left(1 + \frac{\lambda - 1}{n_1} \right) p_{n_1 - 1, n_2} + \theta_2 p_{n_1, n_2 - 1} \\ &= \theta_1 p_{n_1 - 1, n_2} + \theta_2 \left(1 + \frac{\lambda - 1}{n_2} \right) p_{n_1, n_2 - 1}. \end{aligned}$$

We have (3.1)-(3.2) under the assumption $\rho_1 + \rho_2 = 1$ for

$$\rho_1 = \frac{\theta_1}{\theta_1 + \theta_2}, \rho_2 = \frac{\theta_2}{\theta_1 + \theta_2}, a = \theta_1 + \theta_2, b = (\lambda - 1)(\theta_1 + \theta_2).$$

Example 2: The bivariate Poisson distribution, which depends on three positive parameters $\lambda_1, \lambda_2, \lambda_3$, has the p.f.

$$p_{n_1, n_2} = e^{-(\lambda_1 + \lambda_2 + \lambda_3)} \sum_{k=0}^{\min\{n_1, n_2\}} \frac{\lambda_1^{n_1 - k} \lambda_2^{n_2 - k} \lambda_3^k}{(n_1 - k)!(n_2 - k)!k!}, n_1, n_2 \geq 0.$$

It satisfies the recursive relations (Partrat, 1993; Hesselager, 1996)

$$\begin{aligned} p_{n_1, n_2} &= \frac{\lambda_3}{n_1} p_{n_1 - 1, n_2 - 1} + \frac{\lambda_1}{n_1} p_{n_1 - 1, n_2} \\ &= \frac{\lambda_3}{n_2} p_{n_1 - 1, n_2 - 1} + \frac{\lambda_2}{n_2} p_{n_1, n_2 - 1}, \quad n_1, n_2 \geq 1 \\ p_{0, 0} &= e^{-(\lambda_1 + \lambda_2 + \lambda_3)}, \\ p_{n_1, 0} &= \frac{\lambda_1}{n_1} p_{n_1 - 1, 0}, \quad n_1 \geq 1, \\ p_{0, n_2} &= \frac{\lambda_2}{n_2} p_{0, n_2 - 1}, \quad n_2 \geq 1. \end{aligned} \tag{3.5}$$

Corresponding to (1.2)-(1.4), we have for example for (3.5)

$$a_0 = a_2 = a_{12} = 0, \quad a_1 = \lambda_3; \quad b_0 = 0, \quad b_1 = \lambda_1; \quad c_0 = c_2 = 0; \\ d_0 = 0, \quad d_1 = \lambda_1; \quad e_0 = 0, \quad e_1 = \lambda_2.$$

When applying these recursions in Theorem 1, we obtain the same recursions as Hesselager (1996) for the compound distribution.

Remark 2. These examples show that the parameters in (1.2) are not always uniquely determined. Thus one has some freedom to adjust the parametrisation to make the recursions of Theorem 1 more convenient.

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