SHORT CONTRIBUTIONS

RUIN PROBABILITY FOR TRANSLATED COMBINATION OF EXPONENTIAL CLAIMS

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Abstract

An alternative expression for the coefficients in the ruin probability for the classical ruin model with translated combination of exponential claims is derived

Keywords

Probability of ruin; translated combination of exponentials

In a compound Poisson claim process with claim amounts distributed as a mixture of exponentials

$$p(x) = \sum_{i=1}^{n} A_i \beta_i e^{-\beta_i x}$$

for $x > 0$ where all $A_i > 0$ and $\sum_{i=1}^{n} A_i = 1$, it is well known that the ruin probability is also a linear combination of exponentials

$$\psi(u) = \sum_{i=1}^{n} C_i e^{-r_i u}$$

where $\{r_1, \ldots, r_n\}$ are solutions to the adjustment coefficient equation

$$(1+\theta)\, p_1 = \frac{M_X(r)-1}{r}$$

and $\{C_1, \ldots, C_n\}$ are determined by the partial fractions of

$$\sum_{i=1}^{n} \frac{C_i r_i}{r_i - r} = \frac{\theta}{1+\theta} \cdot \frac{\dfrac{M_X(r)-1}{r}}{(1+\theta)\, p_1 - \dfrac{M_X(r)-1}{r}}.$$

See Bowers et al. (1986), § 12 6 for details. This result was later extended by Dufresne and Gerber (1989) to the case when the claim distribution is a translated (density function moved by τ to the left) combination of exponentials. (Note that the A_i's need not be positive) They found that the coefficients C_i's are the solution to the system:

$$\sum_{k=1}^{n} \frac{\beta_i}{\beta_i - r_k} C_k = 1, \qquad i = 1, \ldots, n, \tag{1}$$

and gave C_k explicitly. In this note we give an alternative expression for the solution for (1):

$$C_k = \prod_{\substack{i \neq k \\ i=1}}^{n} \frac{r_i}{r_i - r_k} \prod_{i=1}^{n} \frac{\beta_i - r_k}{\beta_i}. \tag{2}$$

To verify (2), consider

$$\sum_{i=1}^{n} \frac{x}{x - r_i} C_i = 1 - \prod_{i=1}^{n} \frac{r_i(x - \beta_i)}{\beta_i(x - r_i)}$$

where the two sides are different expressions for the same rational function of (degree n/degree n) which has simple poles $\{r_1, \ldots, r_n\}$ and takes the value 1 at $x = \beta_1, \ldots, \beta_n$ and the value 0 at $x = 0$. Multiply by $x - r_k$ and let $x = r_k$ to obtain (2).

Two different expressions for C_k, (49) and (54) in DUFRESNE and GERBER (1989), arise naturally when a more detailed problem including the severity of ruin is studied These two expressions can be obtained from summing (9) and (22) in DUFRESNE and GERBER (1988) over j respectively

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