WORKSHOP
THE SOLVENCY OF A GENERAL INSURANCE COMPANY IN TERMS OF EMERGING COSTS

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ABSTRACT

The authors challenge the traditional balance sheet concept of the solvency of a general insurance company and put forward an emerging costs concept, which enables the true nature of the assets and liabilities to be taken into account, including their essential variability. Simulation is suggested as a powerful tool for use in examining the financial strength of a company. A simulation model is then used to explore the resilience of a company's financial position to a variety of possible outcomes and to assess the probability that the assets will prove adequate to meet the liabilities with or without an assumption of continuing new business. This suggests the need for an appropriate asset margin assessed individually for each company. The implications for the management and supervision of general insurance companies are explored. The suggestion is made that the effectiveness of supervision based on the balance sheet and a crude solvency margin requirement is limited. More responsibility should be placed on an actuary or other suitably qualified professional individual to report on the overall financial strength of the company, both to management and to the supervisory authorities.

KEYWORDS

Solvency; financial strength; asset margin; emerging costs; simulation; professional report.

1. THE NATURE OF SOLVENCY

1.1. The financial position of a general insurance company is normally disclosed through annual accounts for shareholders and through returns to relevant supervisory authorities. Solvency is demonstrated by showing that the assets exceed the liabilities. To a large degree the bases are chosen by the company. For supervisory purposes it is not just a question of the assets exceeding the liabilities. The assets must normally exceed the liabilities by a specified margin.

1.2. In life assurance there is a report by the actuary on the valuation of the liabilities. By contrast the basis on which general insurance liabilities have been assessed is not usually stated. Furthermore, whereas in life assurance actuaries take account of the assets and effectively advise on the total financial strength of the company, there is no one with this role in a general insurance company. It is frequently the case that no specific account is taken of the suitability of the

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assets to match the expected liabilities nor of the resilience of the balance sheet position disclosed to the inherent uncertainty in both assets and liabilities.

1.3. In principle, the balance sheet represents no more than the Directors' opinion about the financial position of the company. There is considerable uncertainty about the true amount of the liabilities and the realizable value of the assets. The auditors may place some restraints on how the Directors present the position but their role is largely confined to ensuring that what the Directors have done is reasonable.

1.4. There is in fact no single correct value that can be ascribed to either assets or liabilities. Different values may be appropriate according to one's perspective. Shareholders want a "true and fair" view, authorities want a cautious assessment of the position and tax authorities want as little as possible to be offset against taxable profits, to name but three interested parties. A balance sheet which shows a solvent position should reflect an expectation that the assets will be adequate, but it may, either deliberately or inadvertently, present a misleading picture. It certainly does not give any idea of the probability that the assets may prove to be inadequate to meet the liabilities.

1.5. In most countries a general insurance company is permitted by the supervisory authority to carry on writing business only if it has some specified excess of the value of the assets over the liabilities. This clearly increases the probability that the assets will prove sufficient to meet the liabilities, but most solvency margin requirements pay little or no attention to the differing degrees of uncertainty inherent in different types of business, nor do they distinguish adequately between the risks of running off the claims payments on the existing portfolio of business and the risks involved in continuing to write further business.

1.6. Reserving standards are frequently ill-defined or non-existent and do not require special provision to be made to cover the effects of changes in the value of assets on their adequacy to meet the liabilities. Problems may arise from some or all of the following:

adverse run-off of existing business;
poor underwriting experience;
failure to recover from reinsurers;
falls in asset values;
excessive expenses;
mismanagement, negligence or fraud.

A more extended description of the factors affecting solvency and a discussion of the interaction between solvency margin requirements and standards for technical provisions may be found in a paper by DAYKIN *et al.* (1984).

1.7. The object of a statutory solvency margin is two-fold. It reduces the probability that the assets will prove inadequate to meet the liabilities and it provides a buffer against further deterioration in a company's financial position which can occur in the period before its authorization to write new business can be withdrawn. The effect of a statutory minimum requirement is in practice also to set a somewhat higher formal standard in the market place.

1.8. A solvency margin is not required of other trading companies, but this can be said to reflect not only the nature of the business but also the extent of the insured's interest in the continued viability of the company. In many cases the insured can be exposed to quite serious liabilities in the event of the insurer failing to meet a claim. He cannot limit his liability in the way that he can with a trading company.

1.9. A company can carry on writing business only if the supervisory authority says that it meets the solvency requirements (cf. STEWART, 1971). The way in which they lay down requirements for this purpose will differ from the criteria which would be used by a Court in determining whether a company should be wound up. It is in fact relatively rare for insurance companies to be wound up by the Courts. It is more normal for the existing business to be run off to extinction or be transferred to another company. The latter procedure is more common in some countries than others.

2. BREAK UP OR GOING CONCERN?

2.1. The concern with safeguarding the position should the company cease trading is peculiarly the preserve of the supervisory authority. It arises because one of the main weapons available to the supervisory authority is the possibility of preventing a company from writing any further business. The supervisory authority will be subject to criticism if they stop a company from taking on any further business only when the position has been reached that the company cannot even meet its liabilities in respect of business already on its books. The consequence of this is that the supervisory authority will seek to close a company to new business when it can still be expected that the run-off of the existing liabilities will give rise to a surplus of assets, in other words the company is *de facto* solvent.

2.2. In order to achieve their objective of a "satisfactory" run-off, the supervisory authority is likely to take the view that outstanding claims provisions should be sufficient to enable all claims to be met with a reasonably high degree of probability. Failure to maintain an additional solvency margin over and above the outstanding claims provisions would not then imply that the company is unable to meet its existing liabilities, but that it does not have sufficient free resources to satisfy the supervisory authority that it should be permitted to continue writing business. Supervisory authorities using this approach are using what might be termed a "break-up" basis, i.e. it is assumed that no further business is written but existing business is run off to extinction.

2.3. In the EEC a two-stage solvency margin trigger has been adopted. The higher level is referred to as the required solvency margin and the lower is termed the guarantee fund. The origins of the EEC requirements have been described by DAYKIN (1984). If an insurer fails to maintain its required solvency margin it must provide to the supervisor a plan for the restoration of a sound financial position, which may include demonstration that on a properly drawn up business plan, and with realistic assumptions about profitability, the solvency margin will be restored within a reasonable short space of time. Only if the company fails to maintain the guarantee fund, set at one-third of the solvency margin, with a specified minimum in absolute terms, is immediate action to inject additional capital required in order to stave off withdrawal of authorization.

2.4. This approach is in practice not very different from one which assesses the company on a "going concern" basis. Theoretically, the main differences will be in relation to the provisions for outstanding claims, where a "going concern" basis might include less of a margin than a "break up" basis, and the provision for expenses, where a strict "break up" basis, would require a technical provision to be made to cover all the costs of running off the existing business. On a "going concern" basis these costs may be set against the continuing business of the company and it is probable that the past liabilities may be able to be run off for a lesser sum than on the break up basis. Yet another possible basis of assessment would be a "winding-up" basis, in which the assets are divided up and distributed on the basis of an estimate of the liabilities. This requires the assets to be realized at an early date, which in practice may have the effect of depressing market values. A summary of the main features of the different bases of assessment is given in Table 1.

2.5. A key objective of the management of an insurance company is to ensure that it does not have to cease trading. The accounts prepared for shareholders will reflect this by being prepared, as is the normal convention, on a going concern basis. Whilst continuing solvency is also a concern of the shareholders, in most cases this will be taken for granted, and the objective of the accounts should be to provide a true and fair view of the financial position of the company. For this purpose, technical provisions should not be overestimated or contain cautious margins and any adverse development of outstanding claims will emerge in due course and affect future profitability. In spite of the differences in the purposes for which the provisions are required, however, most companies adopt the same provisions for their accounts as they do for their statutory returns.

2.6. Whether seen from the viewpoint of the supervisory authority, from that of the shareholder or from that of an outside analyst, a common problem is the uncertainty as to the strength of the technical provisions. This might be helped

TABLE 1

COMPARISON OF ASSESSMENT BASES

	Assessment basis		
Assumption	Going concern	Break up	Winding-up
New business	Indefinite	None	None
Expenses	Claims settlement expenses only	All run-off expenses	All expenses of winding-up
Assets	Market or book values	Market value	Realization value
Liabilities	Best estimate	Cautious estimate	Best estimate of current value

by more clearly defined reserving standards and by more disclosure of the basis for the provisions, but there is still the fundamental weakness that the assets and the liabilities are not being valued on consistent bases and variability is not taken into account.

2.7. The division of a company's resources into technical provisions and "free" assets is not necessarily helpful from the point of view of establishing the true financial strength of the company. Most of the so-called surplus or "asset margin" (the excess of assets over liabilities) may in fact be needed to reduce the probability of being unable to meet the liabilities to an acceptably low level, particularly if the provisions are only "best estimates". To examine financial strength, all of the resources of the company need to be brought into consideration.

3 EMERGING COSTS

3.1. Although the current market value of the investments is increasingly coming to be disclosed in shareholders' accounts, at least in the UK, and is required by supervisory authorities for the statutory returns, its advantages are mainly in relation to its objectivity as a value to be placed on the company's investments, rather than in relation to its relevance to the ability of the company to meet its liabilities, even in the context of the break up basis.

3.2. The assets will not in practice have to be realized on a particular date and, in any case, by the time the accounts or returns have been prepared, the market value at the date to which those accounts relate is a matter of no more than historical interest. What is important is whether the proceeds of the assets, both capital and income, will prove sufficient to meet the liabilities as they emerge. This is true solvency.

3.3. The concept of projecting the emerging costs of the liabilities to which an enterprise is subject and placing them alongside the expected pattern of income is one which is familiar to actuaries in the life assurance and pensions contexts and is also fundamental to investment appraisal by economists in many other spheres of industry. However, little work seems to have been done on the application of the concept to general insurance companies.

3.4. There has been some theoretical consideration from the viewpoint of financial economics by KAHANE (1979) and KAHANE and BIGER (1977) which may not be widely known among actuaries. Actuarial concepts of looking at the company as a whole were applied to general insurance in a paper by BENJAMIN (1980) and the use of emerging costs was implicit in papers by RYAN (1980, 1984) on the use of simulation techniques in general insurance. COUTTS *et al.* (1984) set out more fully the fundamental concepts of the emerging costs of a general insurance company and a practical example was presented in a paper by DAYKIN and BERNSTEIN (1985) on run-off and asset risks.

3.5. The concept is a simple one. It involves analysing the inflows and outflows of actual cash in each successive year. The inflows may consist of some or all of the following:

premium income;
interest and dividends on assets;
maturity proceeds of assets;
reinsurance recoveries in respect of claims.

The outflows may consist of the following:

claims settled or amounts paid on account;
reinsurance premiums;
expenses;
tax;
dividends.

3.6. The effect of the various items in each year will be either a net amount available for investment or a shortfall. In the latter case assets need to be sold to meet the deficit. So long as there are sufficient assets available to enable all the outflows to be met as they arise, the company is solvent in an absolute sense, whatever the balance sheet may have shown. If all the assets have been realized but net liabilities still remain, the situation is one of *de facto* insolvency.

3.7. An emerging costs analysis should be carried out on the totality of the assets and liabilities of the company. For this purpose the dividing line between technical provisions and asset margin is of no real importance although estimates of future claims payments are necessary. The uncertainties of general insurance are such that it will not generally be sufficient to use deterministic values for the liabilities and the assets. Some measures of variability need to be introduced. However, this should not be allowed to detract from the essential simplicity of the concept. It only means that some or all of the items listed above should be treated as random variables. To handle this the emerging costs can be examined using simulation.

3.8. A single simulation is one realization of a random process in which each of the required quantities is assigned a value. By examining a large number of simulations a picture can be obtained of the likely pattern of development resulting from the interaction of the various variables. Simulation permits the use of stochastic models for the investment processes and allows the uncertainty in the outstanding claims and in the profitability of new business to be taken into account. The approach has much in common with the ideas developed by the Finnish Solvency Working Party (PENTIKAINEN and RANTALA, 1982) and extended to cover run-off risk by PENTIKAINEN and RANTALA (1986), although they did not use a stochastic approach for the investments.

3.9. In practice the various elements may be modelled in a variety of different ways. For some purposes very complex models may be desirable; for others a simpler model may suffice, although any model which is going to give a reasonably realistic representation of the real world is bound to be fairly complex. The important principle is that the totality of the company's operations is being considered.

3.10. The procedure is very flexible. It might enable, for example, questions to be asked about the impact of alternative strategies for premium rating or investment and about the effect of possible adverse claims development or failure to recover from reinsurers. It provides a management tool and also seems to offer a way forward for more rational supervision. This would involve the submission to the authorities of a report on total financial strength by an actuary or other suitably qualified expert, as a supplement to minimum balance sheet requirements. The result would be a system much better able to take account of the true position of each company, having regard to the specific risks to which it is subject and the inherent uncertainties of both assets and liabilities.

4. THE SIMULATION MODEL

4.1. *General Structure*

4.1.1. In order to demonstrate the potential of the emerging costs approach we present here a model which provides a representation of the dynamics of a general insurance operation. In order to be reasonably realistic the model is quite complex but, however complicated the model, it is essential that the concepts should be capable of being put across in a straightforward way and the results must be capable of being presented in ways that can be directly related to management concerns such as corporate strategy and decision-making.

4.1.2. At its most basic, the model is a projection of cash flow, bringing together income from premiums and from assets and outgo in respect of expenses, tax, dividends and claims, determining the net balance for each year, investing or disinvesting as the case may be and proceeding similarly for as many years into the future as one wishes. It may be considered more fully in terms of three separate components:

liabilities arising from existing business;
future premiums and the liabilities resulting from the risks underwritten;
asset returns and asset value movements.

4.1.3. A mathematical formulation of the model is given in Appendices 1 and 2 and a description of the computer program in Appendix 3.

4.2. *Existing Liabilities*

4.2.1. The existing liabilities, as shown in the balance sheet, consist of estimates of outstanding claims, including IBNR, and unearned premium reserves (including any additional amount for unexpired risks). Unearned premiums can be dealt with along similar lines to new written premiums (see Section 4.3) since the uncertainty includes uncertainty about the adequacy of premium rates in relation to events which have not yet occurred.

4.2.2. As far as outstanding claims are concerned, there is uncertainty about the amounts of claims and about when they will be settled. The model needs to provide an adequate representation of this uncertainty. We make the simplifying assumption that the variability in rates of settlement can be subsumed into a variation in the amount of claims settled in each period.

4.2.3. The first stage is to estimate the expected claims payments in each successive year for each year of origin. In order to do this, fixed settlement patterns have to be specified in constant money terms. The model permits different run-off patterns to be assumed for different types of business. Inflation then has to be allowed for. Future inflation is generated by a stochastic model and this is combined with the expected settlements in constant money terms to give the expected development of claim amounts. The inflation model is an integral part of the models used for the assets (see Paragraph 4.4.4).

4.2.4. The variability of claim amounts payable in each period can be dealt with in a variety of ways. In an earlier paper describing the application of a similar model to a run-off situation, DAYKIN and BERNSTEIN (1985) proposed that the actual outstanding claims settled in each year in respect of each year of origin should be varied. They assumed that each separate entry in the run-off triangle was distributed about the mean estimate of claims settled at that particular duration for that year of origin in accordance with a log normal distribution. This was attractive as a means of simulating the interaction between different years of origin and different classes of business, but it resulted in a somewhat lengthy simulation process.

4.2.5. In order to simplify the model and allow account to be taken of different sizes of company, the model presented here uses an aggregate approach, whereby the amount that is varied is the total amount of claims settled in a particular period, for all years of origin combined. This aggregate figure is assumed to vary according to a normal distribution with a standard deviation of the type:

$$aX + b\sqrt{X}$$

where X is the mean estimate of total claim payments in the year and a and b are suitably chosen constants. We understand that a similar formula is used by the Finnish supervisory authority for their statutory minimum solvency margin (see Appendix 5 for discussion of this formula which can be considered to be an approximation to the formula derived by BUCHANAN and TAYLOR, 1986).

4.2.6. The amounts payable in future years in respect of risks arising from future written premiums and from unearned premium reserves are included with the amounts payable in respect of existing liabilities before applying the overall variability formula. The extent of the assumed variability can be adjusted by varying the constants a and b in the formula above. For a standard basis we have assumed that they take the values 0.15 and 75 respectively, with claims amounts being expressed in £ sterling. The amounts payable in successive years are assumed to vary independently of each other. The variability is intended to cover not only stochastic variability of claim amounts, but also uncertainty about the expected run-off model in constant money terms. Uncertainty about future inflation is dealt with separately.

4.2.7. Two typical run-off patterns have been assumed, characterized as short and long-tailed. Details are given in Appendix 3. In order to place a value on the technical provisions which would be established at the outset in respect of the outstanding claims, it has been assumed that inflation would be allowed for at 5% a year and that the resulting outstanding claims would not be discounted. (For further discussion on the interaction between the reserving basis and the solvency margin, see Paragraph 5.2.)

4.2.8 In practice an actual outstanding claims portfolio could be used as the basis for the input to the model in respect of existing liabilities. It would need to be expressed as an expected run-off in constant money terms. For illustrative purposes, however, we have assumed that the outstanding claims have been generated in a similar way to the liabilities in respect of future written premiums, by specifying a rate of real premium growth and claim ratios. For the purpose of generating the outstanding claims at the base date no variability was assumed in the historic claim ratios, in contrast to the process described in Section 4.3. In conjunction with the specified run-off patterns and the inflation model, the liabilities generated in this way give rise to estimates of outstanding claims payable in each future year in respect of each past year of origin.

4 3. *Future Written Premiums*

4.3.1. Future premiums are generated from an assumed initial premium level and an assumed real annual growth rate. The effects of inflation are then built in explicitly. Although the existing portfolio of business is generated by assuming a past pattern of premium growth, as described in Paragraph 4.2.8 above, a different growth rate assumption may be made for the future. The proportions of written premiums which are assumed to relate to different types of business can be specified. The written premiums are taken to be net of commission and initial expenses.

4.3.2. For each year for which additional premiums are assumed to be written, a ratio of claims to premiums net of commission and expenses is generated for each type of business. The ratio is assumed to be normally distributed with mean and standard deviation to be specified. The resulting ratio is applied to the assumed net written premiums to produce an initial estimate of total claims in respect of that business, without any allowance for future inflation or for discounting. This ratio is such that a value of 100% implies break-even if future investment income exactly balances inflation. The assumed proportions of claims settled in each future year are then applied to obtain uninflated estimates of expected claims payments. Future inflation, as generated by the model described below (Paragraph 4.4.5), is incorporated when the expected claim payments in terms of constant money have been aggregated with the corresponding estimates in respect of the existing liabilities. The combined estimates are then varied as described in Paragraph 4.2.5 above.

4.3.3. Since the claim ratios generated are ratios of claims to written premiums net of commission and expenses, no explicit allowance needs to be made for these items of outgo. Expenses of claims settlement are assumed to be included in the costs of settled claims.

4.3.4. This relatively simple approach has been used as a practical expedient in view of the complexity of the underlying risk process. An alternative approach, described by BEARD *et al.* (1984) and developed in the Report to the Finnish Solvency Working Party (PENTIKAINEN and RANTALA, 1982), would be to treat the basic claims process as a Poisson process and then build on a series of "structure variables" to take account of:

trends of claims frequency;
long term variations in premium rate adequacy;
year to year fluctuations in mean claims frequency.

Further assumptions then have to be made about the claims size distribution.

4.3.5. Whilst it is clearly possible to specify a model which takes explicit account of each of these, the added complexity can only be justified if the parameters of the model can be satisfactorily determined. We have not as yet been able to assemble data in a suitable form for calibrating such a model. The problem of calibration still arises with the simpler model, but it is intuitively more accessible and enables judgement to be applied in the area which is probably of the greatest importance, i.e. changes in the relationship between premium levels prevailing in the market and the underlying risk premium. This is the factor described as "long-term cycles" by PENTIKAINEN and RANTALA (1982).

4.3.6. Although the adequacy of premium rates does exhibit the characteristics of a business cycle, experience seems to show that the variation does not have a regular periodicity or a constant amplitude. A considerable degree of judgement is needed to decide where in the "cycle" the industry finds itself at any particular moment. Our model allows for the user to give explicit consideration to this and requires the mean claim ratio for the next couple of years to be estimated. If the model were to be used to examine the effects of future written premiums over a longer period than 2 years, further consideration would need to be given to modelling this component. The assumption of a normal distribution of claim ratios about the mean is a not unreasonable approximation, bearing in mind the large numbers of claims involved.

4.4. *Asset Variability*

4.4.1. The variability inherent in the asset portfolio of a company depends on the nature and distribution of the assets. The realizable value of many assets will vary from day to day as market conditions change. In our model, the initial distribution of the assets by category has to be specified and the various components of the asset distribution are then analysed separately, simulating the income generated and the capital value of each type of asset for each future year. Rules need to be specified for investment and disinvestment.

4.4.2. Three different types of asset are assumed: cash, irredeemable Government securities and ordinary shares. A more realistic model would replace investment in irredeemable Government securities with short, medium and long-dated securities. In practice, however, short-dated securities behave somewhat like cash and long-dated securities like irredeemables, so the model can be regarded as a tolerable proxy and gets round the problem of the reducing life of dated stocks over time. Allowance has not been made for a proportion of the assets being effectively non-interest-bearing (e.g. agents' balances) but this could easily be done.

4.4.3. The development of the various components of the asset distribution has been represented by a series of interrelated stochastic processes, suggested by WILKIE (1984, 1986) which generate future scenarios for the values of different types of asset and the income from them. Although Wilkie's models were not originally intended to be used for relatively short-term simulations such as those with which we are concerned, we have adopted them as a readily available and coherent model of asset movements and inflation. Further work is needed on suitable asset models and the sensitivity of the results to the particular models used. Our results show that this is a most important aspect of the whole simulation model.

4.4.4. The models are described in detail in Appendix 2. In addition to the models of asset returns and asset values, the Wilkie models include a model for inflation and this has been used where it is needed in the simulation of the liabilities.

4 4.5. The initial asset mix is based on assets covering the technical provisions and assets representing the asset margin. Different distributions may be specified for each. A variety of different investment and disinvestment strategies may be applied to the total funds.

4.5. *Results of the Simulations*

4.5.1. The number of potential combinations of variables is vast, even allowing each variable to take only three or four different values. We have limited our considerations by adopting a standard set of parameters and normally varying only one parameter at a time.

4.5.2. The simulation process involves sampling scenarios from an infinite set and the results are necessarily subject to statistical error. For any particular case which is of interest more simulations can be carried out in order to improve the accuracy of the estimate. In order to illustrate the results on a large number of scenarios, we have limited our considerations to 1000 simulations for each. For each parameter combination the same 1000 sets of random numbers are used, so that the comparisons are not significantly affected by any bias in the particular sets of random numbers chosen.

4.5.3. Figure 1 shows, for illustrative purposes, the results of 100 simulations, assuming no new business. This demonstrates the general shape of the results, which is common to all the scenarios, although the variability differs greatly. The

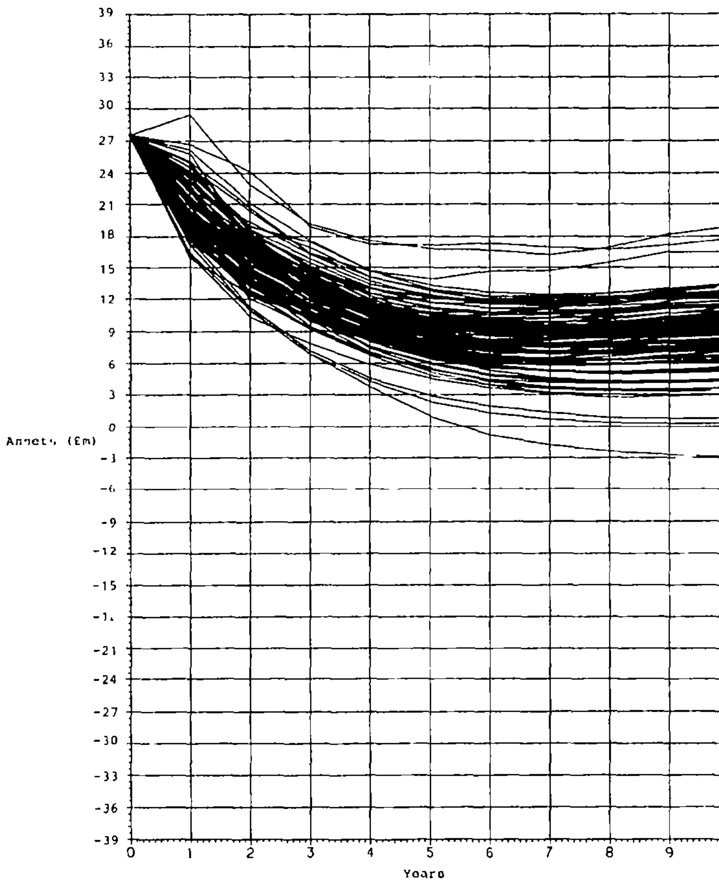

FIGURE 1 Run-off of assets assuming no new business (100 simulations)

graph shows the assets of the model company year by year throughout the run-off of the business.

4.5.4. When a line goes below the x-axis, this implies that all the assets have been exhausted on that particular simulation. If that should occur before the end of the run-off, true insolvency has occurred. In describing the results of the simulations, insolvency is used in this sense, without regard to the way in which the financial position of the company might be presented in the accounts or statutory returns at the base date or at any later date.

4.5.5. We thus define:

> an insolvency occurs when the assets run out before all the liabilities have been met (on an emerging costs basis).

In the simulations a realization which runs into insolvency is allowed to continue

TABLE 2
DISTRIBUTION OF ASSETS AT END OF RUN-OFF FROM 1000 SIMULATIONS ON STANDARD BASIS

Remaining assets[1] as % of net written premiums[2]	Number of cases: Pure run-off	Number of cases: 2 years' new business
Less than 0	8	50
0–20	31	34
20–40	67	62
40–60	128	65
60–80	124	81
80–100	136	78
100–120	128	93
120–140	104	91
140–160	69	84
160 and over	205	362
Mean	112	144

1 Deflated to the date of assessment using the retail prices index
2 Premiums net of commission and initial expenses

by borrowing (at the rate of interest on cash plus a margin of 3%); this permits one to see how insolvent it becomes.

4.5.6. On the standard basis, insolvency in this sense occurred in 8 cases out of 1000 with no new business and 50 cases with 2 years' new business. The distribution of assets at the end of the run-off, deflated to the date of assessment using the retail prices index, is summarized in Table 2. The written premiums in question are those in the year before the base date. It should be recalled that the written premiums are net of commission and expenses. Results expressed as percentages of net written premiums can be rated down (say, by applying a factor of 75% or 80%, depending on the type of business) to obtain comparable results in terms of gross written premiums. The mean level of remaining assets for the 1000 simulations was 112% of net written premiums with no new business and 144% with 2 years' new business, with standard deviation of 70% and 109% respectively of net written premiums.

4.5.7. Full details of the assumptions underlying the standard basis are given in Appendix 4. However, we will return to the results after commenting on the application of the model.

4.6. *Application of the Model*

4.6.1. A simulation model of the insurance company, based on the emerging costs concept, provides a powerful and flexible tool for examining the dynamics of an insurer's operation, for exploring the effects of uncertainty and for developing the financial aspects of corporate strategy within a logical framework. This should be of value both to management and to the supervisory authorities.

Crucial to this process would be the presence of a suitably qualified actuary or other expert within the company, or acting as consultant, who could develop a suitable model and apply the necessary judgement to the use of the model in the circumstances of the particular company. The responsible expert would report to management on the financial strength of the company, taking all relevant factors into account.

4.6.2. The simulation approach would also enable the actuary to advise management on the potential effects of different new business and investment strategies, the risks involved and the return on capital which might be expected if additional capital is injected to enable a particular strategy to be adopted.

4.6.3. A report on the financial strength of the company could accompany the statutory returns to the supervisory authorities. The actuary would be answerable to the supervisor on the details of this report. One could envisage this leading to an informed dialogue between the supervisory authority and the company under scrutiny on the nature of the proposed corporate strategy, whether in relation to investment policy, growth or premium levels. The supervisor could then ask for an assessment of the effect of alternative strategies and seek agreement with the company on appropriate changes to its strategy as a condition for being permitted to continue writing business.

5. SOLVENCY CONSIDERATIONS

5.1. The results of the simulations can be presented in terms of numbers of insolvencies out of a given number of simulations. This is an estimate of the probability of ruin. Each result derives from an assumption about the excess of assets over technical provisions (the "asset margin") and a specified basis for calculating the latter. Given a basis for the technical provisions, the process can be used to derive the required initial asset margin in order to achieve a specified probability of ruin in a particular case.

5.2. The required asset margin will clearly differ according to differing definitions of the technical provisions. Table 3 illustrates this point. The table shows the technical provisions on the standard basis described above and the technical provisons on alternative bases as to inflation and discounting, but for the same set of outstanding claims. The table shows what asset margins would be necessary, expressed both as a percentage of technical provisions and as a percentage of net written premiums, in order to achieve the same degree of overall security as the technical provisions on the standard basis. Technical provisions on the standard basis are calculated assuming 5% inflation and no discounting. Thus if the reserves do not allow for any future inflation, or have been discounted using a rate of interest equal to the assumed rate of inflation, an asset margin of 21% of net written premiums or 9% of technical provisions would be needed to produce the same level of total assets as the technical provisions alone on the standard basis. The figures in this table underline the arbitrary nature of a statutory solvency requirement unless standards of technical provisions can be adequately specified.

TABLE 3

TECHNICAL PROVISIONS AND ASSET MARGINS

Reserving basis (net inflation assumed) %	Technical provisions[1]	Asset margin to achieve same security as standard: % of net written premiums[2]	% of technical provisions
−5	20450	39	19
0	22264	21	9
5	24364	0	0
10	26805	−24	−9
15	29656	−53	−18

1 Based on 40% long tail business and 60% short tail
2 Premiums net of commission and initial expenses

5.3. First we give some results for a pure run-off, i.e. with no future premiums assumed to be written. The outstanding claims and unexpired risks are allowed to emerge and the adequacy of the total assets (technical provisions and asset margin) is examined. Table 4 shows the number of insolvencies and the mean assets remaining at the end of the run-off and the standard deviation of the assets remaining on some alternative bases. Table 5 gives a similar set of results with the inclusion of 2 further years' written premiums. Appendix 4 gives details of all the assumptions and a full set of results.

5.4. Tables 6 and 7 show the asset margins required to achieve a probability of ruin of 1 in 100 for each of the combinations of assumptions in Tables 4 and 5 respectively, assuming that the technical provisions are established on the standard basis of 5% inflation and no discounting. The asset margins are shown in terms of both net written premiums in the year before the base date and as a percentage of technical provisions at the base date. The results can be expressed in terms of net written premiums even for the pure run-off case, since these are the premiums in the year before the base date when premiums are assumed to cease. As described in Paragraph 4.2.8, we have in fact generated the outstanding claims from past premiums. The difference between Tables 6 and 7 provides a measure of the additional capital needed in order to go on writing business for two more years.

5.5. It is clear that the results obtained depend critically on the models used and the parameters assumed. More work is needed on a number of different aspects. However, the results presented do appear consistent and sensible and variations in relation to changing parameter values conform with general reasoning.

5.6. It is difficult from these results to draw conclusions about an appropriate level of a minimum statutory solvency margin. In fact we have avoided using the term solvency margin in this section because of its special significance in statutory terms and have referred to the necessary margin as the asset margin. Our asset margins relate to particular assumptions about the basis for the technical provisions and provide a defined degree of security in relation to specified scenarios

TABLE 4

SUMMARY OF RESULTS FOR PURE RUN-OFF OF BUSINESS (WITH 1000 SIMULATIONS)

	Assumptions				No of insolvencies	Mean assets remaining[1] (%)	Standard deviation of assets remaining[1] (%)
	Standard basis				8	112	70
1	Net written premiums [2]						
	(a) £1m a year				20	113	75
	(b) £10m a year (s)				8	112	70
	(c) £100m a year				6	112	69
2	Proportion of long-tailed business						
	(a) 20% of net written premiums[2]				3	94	55
	(b) 40% of net written premiums[2] (s)				8	112	70
	(c) 60% of net written premiums[2]				13	130	85
3	Initial asset distribution						
		Cash	Gilts	Equities			
	(a)	TP + AM	—	—	3	95	53
	(b)	—	TP + AM	—	20	120	98
	(c)	—	—	TP + AM	49	136	115
	(d)	½TP	½TP	AM (s)	8	112	70
4	Initial asset margin.						
	(a) 0% of net written premiums[2]				134	52	55
	(b) 20% of net written premiums[2]				36	83	62
	(c) 40% of net written premiums[2] (s)				8	112	70
	(d) 60% of net written premiums[2]				2	147	80
	(e) 80% of net written premiums[2]				0	180	90
5	Asset selling rules						
	(a) Equities, gilts; cash				9	102	66
	(b) Cash; gilts; equities				7	123	79
	(c) In proportion to holdings (s)				8	112	70
	(d) Sell best performer first				14	108	70

1 Deflated to the date of assessment using the retail prices index and expressed as a percentage of net written premiums[2] in the year before the date of assessment (see Appendix 3 6 8)

2 Premiums net of commission and expenses

(s) indicates the assumption made for the standard basis

on the basis of our model. A statutory solvency margin, in the sense in which it is usually used, provides a general level of security, independent of the particular circumstances of the company, against all possible future scenarios, including the effect of unquantifiable risks such as fraud, mismanagement and the failure of reinsurers.

5.7. A starting point for consideration of an appropriate level of statutory solvency margin might be to look at the asset margin for a company with a fairly standard distribution of business, a moderate growth rate and investment entirely in cash. In our view the resulting margin ought to be in two parts:

a percentage of the technical provisions at the assessment date;
a percentage of written premiums.

The former represents the margin required in respect of the run-off risks and the latter the margin required in respect of writing up to two years' further new

TABLE 5

SUMMARY OF RESULTS WITH 2 FURTHER YEARS' BUSINESS (WITH 1000 SIMULATIONS)

Assumptions				No of insolvencies	Mean assets remaining[1] (%)	Standard deviation of assets remaining[1] (%)
Standard basis				50	144	109
1 Net written premiums[2]						
(a) £1m a year				61	144	117
(b) £10m a year (s)				50	144	109
(c) £100m a year				43	144	107
2 Proportion of long-tailed business						
(a) 20% of net written premiums[2]				48	120	91
(b) 40% of net written premiums[2] (s)				50	144	109
(c) 60% of net written premiums[2]				52	168	130
3 Real growth rate (past and future)						
(a) −20% a year				53	171	134
(b) No growth (s)				50	144	109
(c) +50% a year				83	144	121
4 Mean claim ratio[3] (short-tailed)						
(a) 80% of net written premiums[2]				7	187	117
(b) 100% of net written premiums[2] (s)				50	144	109
(c) 125% of net written premiums[2]				165	90	103
5 Variability of claim ratio (short-tailed)						
(a) Standard deviation 5% NWP[2]				49	144	109
(b) Standard deviation 10% NWP[2] (s)				50	144	109
(c) Standard deviation 15% NWP[2]				48	144	111
6 Mean claim ratio[3] (long-tailed)						
(a) 80% of net written premiums[2]				17	163	106
(b) 100% of net written premiums[2] (s)				50	144	109
(c) 125% of net written premiums[2]				105	120	114
7 Variability of claim ratio (long-tailed).						
(a) Standard deviation 10% NWP[2]				50	144	109
(b) Standard deviation 15% NWP[2] (s)				50	144	109
(c) Standard deviation 20% NWP[2]				49	144	110
8 Initial asset distribution						
	Cash	Gilts	Equities			
(a)	TP + AM	—	—	46	118	86
(b)	—	TP + AM	—	79	155	152
(c)	—	—	TP + AM	86	181	172
(d)	½TP	½TP	AM (s)	50	144	109
9 Initial asset margin						
(a) 0% of net written premiums[2]				196	74	98
(b) 40% of net written premiums[2] (s)				50	144	109
(c) 80% of net written premiums[2]				11	216	133

1 Deflated to the date of assessment using the retail prices index and expressed as a percentage of net written premiums[2] in the year before the date of assessment (see Appendix 3 6 8)

2 Premiums net of commission and expenses

3 Ratio of claims (including claims settlement expenses), without allowance for future inflation or for discounting, to premiums net of commission and expenses (see Paragraph 4 3 2).

(s) indicates the assumption made for the standard basis

TABLE 6

ASSET MARGINS REQUIRED TO ACHIEVE 1/100 PROBABILITY OF RUIN — NO FUTURE NEW BUSINESS

Assumptions				Asset margin as % of NWP[1]	Asset margin as % of technical provisions
Standard basis				40	15
1 Net written premiums [1]					
(a) £1m a year				55	25
(b) £10m a year (s)				40	15
(c) £100m a year				35	15
2 Proportion of long-tailed business·					
(a) 20% of net written premiums[1]				30	15
(b) 40% of net written premiums[1] (s)				40	15
(c) 60% of net written premiums[1]				45	15
3 Initial asset distribution					
	Cash	Gilts	Equities		
(a)	TP + AM	—	—	30	10
(b)	—	TP + AM	—	60	25
(c)	—	—	TP + AM	80	35
(d)	½TP	½TP	AM (s)	40	15
4 Asset selling rules					
(a) Equities; gilts, cash				40	15
(b) Cash, gilts, equities				35	15
(c) In proportion to holdings (s)				40	15
(d) Sell best performer first				55	25

1 Premiums net of commission and expenses
(s) indicates the assumption made for the standard basis

business. To the new business margin might be added a contingency loading to cover other unquantifiable risks.

5.8. This would provide a basic safety net for an average company, assuming that technical provisions were at least up to the standard envisaged. Statutory reserving standards might be necessary to achieve this, since it has to be recognized that a solvency margin requirement based on technical provisions has a similar weakness to one based on written premiums. If the provisions are understated the requirement is reduced, whereas it should in fact be higher.

5.9. Alongside such a basic solvency requirement would be a requirement for a report by an actuary or other expert on the overall financial strength of the company. This would transcend the arbitrary dividing line between technical provisions and solvency margin and would take specific account of the nature of the business written by the company, the proportions of different types of business, the assets held, and all other relevant factors, including the nature of and the security of the reinsurance programme.

5.10. If a requirement for an actuarial report is not introduced, then further consideration would need to be given to whether the solvency margin requirement should include components relating to the assets held and the reinsurance recoveries expected. Regard should also be had to the nature of the outstanding claims portfolio and the type of business being written. However, such a solution would be far from ideal.

TABLE 7

ASSET MARGINS REQUIRED TO ACHIEVE 1/100 PROBABILITY OF RUIN — TWO YEARS' NEW BUSINESS

Assumptions	Asset margin as % of NWP[1]	Excess asset margin as compared to run-off (as % of NWP[1])
Standard basis	90	50
1 Net written premiums[1]		
(a) £1 a year	100	45
(b) £10m a year (s)	90	50
(c) £100m a year	80	45
2 Proportion of long-tailed business		
(a) 20% of net written premiums[1]	80	50
(b) 40% of net written premiums[1] (s)	90	50
(c) 60% of net written premiums[1]	95	50
3 Real growth rate (past and future).		
(a) −20% a year	100	35
(b) No growth (s)	90	50
(c) +50% a year	115	85
4 Mean claim ratio[2] (short-tailed)		
(a) 80% of net written premiums[1]	30	0
(b) 100% of net written premiums[1] (s)	90	50
(c) 125% of net written premiums[1]	125	75
5 Variability of claim ratio (short-tailed)		
(a) Standard deviation 5% NWP[1]	80	45
(b) Standard deviation 10% NWP[1] (s)	90	50
(c) Standard deviation 15% NWP[1]	85	45
6 Mean claim ratio[2] (long-tailed)		
(a) 80% of net written premiums[1]	50	20
(b) 100% of net written premiums[1] (s)	90	50
(c) 125% of net written premiums[1]	115	60
7 Variability of claim ratio (long-tailed)·		
(a) Standard deviation 10% NWP[1]	85	50
(b) Standard deviation 15% NWP[1] (s)	90	50
(c) Standard deviation 20% NWP[1]	90	50
8 Initial asset distribution: Cash / Gilts / Equities		
(a) TP + AM / — / —	85	55
(b) — / TP + AM / —	110	50
(c) — / — / TP + AM	135	55
(d) $\frac{1}{2}$TP / $\frac{1}{2}$TP / AM (s)	90	50

1 Premiums net of commission and expenses
2 Ratio of claims (including claims settlement expenses), without allowance for future inflation or for discounting, to premiums net of commission and expenses (see Paragraph 4 3 2)
(s) indicates the assumption made for the standard basis

6. REINSURANCE

6.1. Reinsurance business accepted may be regarded as another class of business, which is often particularly volatile and unpredictable. Appropriate reserving levels for casualty reinsurance business are likely to present particular problems, since it can take many years for the liabilities (including IBNR) to develop fully. Solvency margins certainly ought to have regard to this uncertainty. In principle there seems no reason why the simulation approach should not also provide some insights in this area of an insurers' portfolio.

6.2. Much more difficult to handle in the context of the assessment of financial strength is the security of reinsurance cessions. Many insurers are critically dependent on their ability to recover from reinsurers, since the size of the risks they write is such as to bankrupt or cripple them if they had to bear the liability alone. One safeguard against reinsurance failure is to spread reinsurance cessions widely, so that there is not any great dependence on particular reinsurers. However, this does not remove the need to look carefully at the security of individual reinsurers chosen for the programme.

6.3. From the reserving point of view, a decision has to be made on the extent to which reinsurance recoveries can be relied on. Extreme caution might point towards reserving for the full gross liability but this is not a practical commercial possibility in most cases. Clearly recoveries from reinsurance companies already known to be in trouble should be ignored or heavily discounted, but it is more difficult to know what should be done when there are no specific known problems. In accounting terms it may be difficult to set up a provision against an unseen and unquantifiable possibility of reinsurance failure. On the other hand the accountancy concept of prudence would preclude taking credit in advance for receipts which are uncertain, so it would be possible to justify taking only partial credit for reinsurance recoveries, depending on an assessment of the viability of the reinsurers.

6.4. The issue is of particular importance in considering the overall financial strength of the company. This would be one aspect which the actuary would need to cover in his report. Different approaches may be acceptable in different circumstances but simulation does seem to offer a promising way forward. Further work is clearly needed in this area to develop ways of modelling reinsurance recoveries. It has been assumed in our model that all claims are net of reinsurance. This may be good enough for many companies, with relatively little dependence on reinsurance. However, it will be far from adequate for other companies for which the possibility of failure to recover from reinsurers is a significant one and the potential impact disastrous in solvency terms. Some tentative ideas of a possible way of tackling this are set out in Appendix 6.

6.5. A detailed examination of the reinsurance programme can hardly be practicable for the supervisory authorities and here again it seems that an actuary's report would help. No general solvency requirement can be a substitute for this. The practice adopted for the EEC solvency margins of reducing the solvency margin requirement calculated on the basis of gross written premiums to allow for reinsurance based on actual recoveries in the past three years, but with a maximum reduction of 50%, is a very rough and ready solution and does not have any regard to the actual dependence on reinsurers for future recoveries. With non-proportional reinsurance the premium can be very small in relation to the potential liability, so no simple percentage of premium is likely to make sense as a solvency margin. A percentage of anticipated recoveries from reinsurers would have a stronger rationale, but it would be difficult to find a logical basis for any particular percentage.

7. CONCLUSIONS

7.1. We have outlined the weaknesses in the traditional balance sheet concept for describing the true financial strength of a general insurance company. Assets and liabilities should not be treated as independent aspects and much more attention needs to be focused on the uncertainties and on the company's resilience in the face of such uncertainties. Appropriate techniques have been developed by actuaries for dealing with these problems in the life and pensions areas and similar principles can be used to begin to tackle the general insurance problem. The parallels are drawn out in a paper by COUTTS and DEVITT (1986).
7.2. However, there are also differences, arising mainly from the greater volatility of claim amounts in general insurance. The problem of variability can be explored by means of simulation. A simulation model of a general insurance company provides a powerful tool for analysing the impact of all types of uncertainty and assessing the true financial strength of the company.
7.3. A solvency margin requirement expressed in terms of a simple percentage of written premiums (or in terms of a percentage of technical provisions, which might be more appropriate to cover the run-off risk) cannot have proper regard to the risk to which each company is subject, whether as regards the assets or liabilities. It must, therefore, be seen as a general underlying safety net, providing a margin against the effects not only of stochastic variations but also of mismanagement, fraud or simply error, and permitting the statutory authority to operate a satisfactory control system.
7.4. Despite our strong belief that the solvency margin should relate to the various risks affecting the financial position of an insurance company, we acknowledge that there will be interest in the use of our model to provide a rationale for a minimum statutory solvency margin. Time has so far prevented us from carrying out sufficient simulations to explore the full implications of the assumptions made and, in particular, the response of the Wilkie model to changes in parameters. We have also only shown the results for a probability of ruin of 1% and this level is of course crucial to the resulting asset margins.
7.5. Nevertheless at this level of security Table 6 shows that, for a moderate sized company, writing £100m of net premiums but otherwise on our standard basis, the margin necessary to cover the run-off risks would be 15% of technical provisons, assuming that the provisions for outstanding claims are set up on an undiscounted basis with allowance for inflation at 5% (the mean value used in the Wilkie model). The margin might be reduced to 10% if all the investments are assumed to be held in cash. Such a margin may be over-stringent as a minimum for larger companies but Table 6 indicates that it should be higher for small companies. A similar standard to a 10% margin would be obtained for the

mix of business considered here by setting up provisions for outstanding claims allowing for inflation at 10% with no discounting.

7.6. Care has to be taken in interpreting the extra margin implied to be necessary to allow for the risks contingent on writing new business for two years. The margins to cover the run-off risk have been expressed for this purpose in terms of net written premium and these margins (for the respective sets of assumptions) have then been subtracted from the margins obtained assuming two further years in business. It could be argued that if the risks of new business and run-off are to be provided for independently, then the model should be run with no past business in order to assess the appropriate margin for new business risks. We have not done this as we do not believe that the two issues are independent, there being interactions in regard to both assets and the variability of the run-off of claims. Assuming that the margins expressed as a percentage of net written premiums are additive, Table 7 indicates a margin of a 50% premium net of commission and expenses for a £100m company, otherwise on our standard basis apart from investment being entirely in cash. This might be equivalent to 35–40% of actual gross written premiums.

7.7. Such a solvency margin requirement appears rather high and it is worth considering briefly some of the major factors which give rise to it. A significant part arises from the effect of simulated future inflation and the possibility that returns on cash will not be adequate to compensate for it. This suggests that the risks might be reduced with greater use of index-linked stocks.

7.8. Much of it also arises from the assumption on the standard basis of a mean claim ratio of 100% of net written premiums. As described in Paragraph 4.3.2, this implies break-even if future investment income exactly balances inflation. Thus the assumption is that business is written on a basis where the only profit on an expected value basis is to the extent that a positive real rate of return can be obtained. This might be perceived as too stringent for a minimum solvency margin requirement, although it is not unrealistic in current conditions. The requirement could be reduced by about 1% of actual written premiums for every percentage point by which the expected claim ratios are reduced below 100%.

7.9. Any general solvency requirement will have its limitations. Apart from the points mentioned in Paragraph 7.3, there is also the problem of relating the requirement to written premiums or to technical provisions, which may themselves be more adequate for some companies than for others. The adequacy of the technical provisions is of particular importance (cf. Paragraph 5.2), since they determine what assets are apparently available as a margin. There is, therefore, a need for consistent standards to be applied in setting technical provisions, suggesting that there would be considerable advantages in requiring the provisions to be established on the basis of advice from an actuary or other claims reserving expert, acting within the framework of an appropriate professional standard. However, it has to be acknowledged that there is always likely to be some uncertainty about the strength of technical provisions.

7.10. We have also argued that a crude minimum solvency margin requirement cannot adequately have regard to the true level of risk for a particular company.

The supervisory authority is not well-placed to assess each company's risk situation in detail on an individual basis and the answer would seem to be to rely on an appointed actuary or other similarly qualified person within the company (or acting as a consultant to the company). The actuary would be responsible for reporting both to management and to the supervisory authority on the financial strength of the company, taking all relevant factors into account. A summary of the actuary's report could appear in the statutory returns, with full details being available to the supervisory authority on request. The supervisory authority would be able to question the actuary on the effects of alternative assumptions and could then discuss with management an appropriate strategy for reducing the risk profile to an acceptable level.

7.11. The actuary would need to use simulation techniques in performing his duties. There is plenty of scope for developing appropriate simulation models for this task and one such model is presented here as an example of what can be done. Apart from providing a framework for analysing the existing position of the company, such models could be powerful tools for answering a wide variey of "what if?" questions, such as:

what changes do there need to be to premium rates to make a particular line of business worth writing?
is the investment strategy too risky with the present asset margin?
what additional capital would be needed to pursue a particular strategy?
will the strategy give a reasonable expected return on the additional capital?

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APPENDIX 1

DESCRIPTION OF SIMULATION MODEL OF GENERAL INSURANCE COMPANY

A1.1. In standard risk theory the year to year transition formula is of the form:

$$\Delta U = B + I - X - C - T$$

where

ΔU is the change in the solvency margin U;
B is the earned premium income, including safety and expense loadings;
I is the net income from investments;
X is incurred claims;
C is the cost of administration, reinsurance etc.;
T is dividends, tax, etc.

By implication, incurred claims includes changes to estimates of outstanding claims generated in previous years and included in the technical provisions at the start of the year in question. This formulation is also deficient in that changes in the values of investments are ignored.

A1.2. *General Formula*

More generally, we define:

$$\begin{aligned}\Delta A(j) &= A(j) - A(j-1)\\ &= \sum_k A_k(j) - \sum_k A_k(j-1)\\ &= \sum_k A_k(j-1)[\{1 + y_k(j-1)\}\{1 + g_k(j-1)\} - 1]\\ &\quad + \left\{B(j) - C(j) - T(j) - \sum_{i \leqslant j} x(i;j)\right\}\end{aligned}$$

where

$A(j)$ is the total value of the assets at the end of year j; $A_k(j)$ is the total value of component k of the asset portfolio at the end of year j (in our model $k = 1$ for cash, 2 for irredeemable government securities, 3 for ordinary shares);

$y_k(j)$ is the yield on asset component k at the end of the year j. In particular, in our model:

$y_1(j) = c(j) - 0.01$
$y_2(j) = c(j)$
$y_3(j) = y(j)$
where $c(j)$ is the yield on 2.5% Consols;

$y(j)$ is the dividend yield on the Financial Times Actuaries All-Share Index;

$g_k(j)$ is the proportionate change in capital values between the end of years j and $(j+1)$. In particular, in our model:

$$g_1(j) = 0$$

$$g_2(j) = \frac{c(j)}{c(j+1)}$$

$$g_3(j) = \frac{d(j+1)y(j)}{d(j)y(j+1)}$$

where $d(j)$ is an index of share dividends (= dividend yield × price index) corresponding to $y(j)$;

$B(j)$ is the *written* premium income in year j including safety and expense loadings;

$C(j)$ is the cost in year j of administration, commission, reinsurance etc.;

$T(j)$ is the amount paid out in dividends and tax in year j;

$X(i; j)$ is the amount settled in year j in respect of claims arising in year i.

We now define $B'(j)(=B(j)-C(j))$ as the written premiums in year j net of commission and all expenses other than claims settlement expenses and $X(i; j)$ as including claims settlement expenses.

A1.3. *Asset and Inflation Models*

The asset components $A_k(j)$ can be defined in a variety of ways relative to the total $\sum_k A_k(j)$. For example, if investment or disinvestment is proportional to the value of assets brought forward to the end of the year from the previous year-end,

$$A_k(j) = \frac{A(j)}{A(j-1)} A_k(j-1).$$

If proportions $p_k (\sum_k p_k = 1)$ are specified such that p_k of any new investment is invested in component k:

$$A_k(j) = A_k(j-1)\{1 + y_k(j-1)\}\{1 + g_k(j-1)\} + p_k\left\{B(j) - C(j) - T(j) - \sum_{i \leqslant j} X(i; j)\right\}.$$

We also define $q(j)$ as the retail price index at the end of year j and $r(j)$ as the price growth in year j:

$$r(j) = \frac{q(j)}{q(j-1)} - 1.$$

The variables $q(j)$, $d(j)$, $y(j)$ and $c(j)$ are defined by an interrelated set of autoregressive models, described in detail in Appendix 2.

A1.4. *Tax and Dividends Model*

The dividends and tax term is expressed in terms of the investment income and an input parameter t, representing the proportion of investment income absorbed by tax and dividends paid to shareholders, by the following:

$$T(j) = t \sum_k A_k(j-1) y_k(j-1).$$

A1.5. *Model of Claims Generation Process*

We define written premiums in the year prior to the date of assessment (taken as the time $j = 0$) as $B(0)$ and the rate of growth of written premiums before and after that date as e_1 and e_2. Then:

$$\begin{aligned} B'(j) &= B'(0)(1 + e_1)^j \quad (j < 0) \\ &= B'(0)(1 + e_2)^j \quad (j \geqslant 0) \end{aligned}$$

and

$$B_k'(j) = f_k B'(j) \text{ for } k = 1, 2, 3$$

where f_k is the proportion of written premiums in respect of type of business k ($k = 1$ for short-tailed, 2 for long-tailed and 3 for very long-tailed).

Claims are assumed to be generated from written premiums by means of a variable claims ratio and specified proportions settled in each year of the run-off. Thus the estimated payment in year j in respect of premiums written in year i is given by:

$$X(i; j) = \sum_k s^k(j) R_k(i) B_k'(i) \prod_{l=i+1}^{j} \{1 + r(l)\}$$

where

$R_k(i)$ is the uninflated, undiscounted claims ratio in year i assumed to be normally distributed with mean R_k and standard deviation σ_r^k. For $i \leqslant 0$, $R_k(i) = R_k$.

$s^k(j)$ is the proportion of uninflated, undiscounted claims from type of business k that are assumed to be settled in development year j.

A1.6. *Model of Claims Settlement*

Claims settled in each year of development are aggregated from all the separate years of origin, whether before or after the date of assessment. The total amount of claims settled in year j, $X(j)$, is assumed to be normally distributed with mean $\bar{X}(j)$ and standard deviaton $a\bar{X}(j) + b\sqrt{\bar{X}(j)}$ where a and b are specified constants and $\bar{X}(j)$ is defined as:

$$\bar{X}(j) = \sum_{i \leqslant j} \sum_k s^k(j - i) R_k(i) B_k'(i) \prod_{l=i+1}^{j} \{1 + r(l)\}.$$

A1.7. *Technical Reserves*

The technical reserves, TR(0), at the date of assessment are calculated from the estimates of claims to be settled in future years arising from premiums earned prior to the date in question. They allow for inflation at a specified rate, r, and discounting at a specified rate, d. They can be expressed as follows:

$$\mathrm{TR}(0) = \tfrac{1}{2} \sum_k B'_k(0) R_k \sum_{j=0}^{20} s^k(j)(1+r)^j(1+d)^{-j}$$

$$+ \sum_k R_k \sum_{i=-20}^{-1} B'_k(i) \sum_{j=-i}^{20} s^k(j)(1+r)^j(1+d)^{-j} + \tfrac{1}{2} B'_k(0).$$

The initial solvency margin, SM(0), is defined as a function of written premiums in the year before the date of assessment:

$$\mathrm{SM}(0) = \alpha B'(0).$$

The initial assets are thus given by:

$$A(0) = \mathrm{TR}(0) + \mathrm{SM}(0).$$

APPENDIX 2

DESCRIPTION OF STOCHASTIC MODELS USED FOR ASSETS AND INFLATION

A2.1. The investment and inflation models used are those proposed by WILKIE (1984, 1986). A summary of the specification of the model is given below. The variables used are:

- $q(t)$ The UK retail prices index.
- $d(t)$ An index of share dividends.
- $y(t)$ The dividend yield on these same share indices, that is, the dividend index at the specified date divided by the share price index at that date.
- $c(t)$ The yield on 2.5% Consols (irredeemable), which is taken as a measure of the general level of fixed interest yields in the market.

A2.2. The model used for $q(t)$ is:

$$\nabla \ln\{q(t)\} = \mu_q + \alpha_q[\nabla \ln\{q(t-1)\} - \mu_q] + \sigma_q z_q(t)$$

where the backwards difference operator ∇ is defined by

$$\nabla x(t) = x(t) - x(t-1)$$

and $z_q(t)$ is a sequence of independent identically distributed unit normal variates. The values adopted for the parameters are:

$$\mu_q = 0.05,\ \alpha_q = 0.6,\ \sigma_q = 0.05.$$

A2.3. The model for $y(t)$ is:

$$\ln\{y(t)\} = \omega_y \nabla \ln\{q(t)\} + y_n(t)$$

where

$$y_n(t) = \ln(\mu_y) + \alpha_y[y_n(t-1) - \ln(\mu_y)] + \sigma_y z_y(t)$$

and $z_y(t)$ is a sequence of independent identically distributed unit normal variates. The values adopted for the parameters are:

$$\mu_y = 0.04,\ \alpha_y = 0.6,\ \omega_y = 1.35,\ \sigma_y = 0.175.$$

A2.4. The model for $d(t)$ is:

$$\nabla \ln\{d(t)\} = \omega_d\left(\frac{\delta_d}{1-(1-\delta_d)\bar{B}}\right)\nabla \ln\{q(t)\} + \alpha_d\ \nabla \ln\{q(t)\} + \beta_d\sigma_y z_y(t-1) + \sigma_d z_d(t) + \gamma_d\ \sigma_d z_d(t-1)$$

where the backwards step operator $\bar{B}$ is defined by

$$\bar{B}x(t) = x(t-1)$$

and hence

$$\bar{B}^n x(t) = x(t-n)$$

and $z_d(t)$ is a sequence of independent identically distributed unit normal variates.

The term in parentheses above involving δ_d represents an infinite series of lag effects, with exponentially declining coefficients:

δ_d,
$\delta_d(1-\delta_d)$,
$\delta_d(1-\delta_d)^2$, etc.

The sum of these coefficients is unity, so this part of the formula represents the lagged effect of inflation, with unit gain. This means that if retail prices rise by 1% this term will also, eventually, rise by 1%. We can alternatively describe it as the "carried forward" effect of inflation $m(t)$, where

$$m(t) = \delta_d\ \nabla \ln\{q(t)\} + (1-\delta_d)m(t-1),$$

from which we see that the amount that enters the dividend model each year is δ_d times the current inflation rate, plus $(1-\delta_d)$ times the amount brought forward from the previous year, and that this total is then carried forward to the next year. The values adopted for the parameters are:

$$\omega_d = 0.8,\ \delta_d = 0.2,\ \alpha_d = 0.2,\ \beta_d = -0.2,$$
$$\gamma_d = 0.375,\ \sigma_d = 0.075.$$

A2.5. The model for $c(t)$ is:

$$c(t) = \omega_c\left(\frac{\delta_c}{1-(1-\delta_c)\bar{B}}\right)\ \nabla \ln\{q(t)\} + n(t),$$

where

$$\ln\{n(t)\} = \ln(\mu_c) + (\alpha_c\bar{B} + \beta_c\bar{B}^2 + \gamma_c\bar{B}^3)[\ln\{n(t)\} - \ln(\mu_c)] + \phi_c\sigma_y z_y(t) + \sigma_c z_c(t),$$

where $z_c(t)$ is a sequence of independent identically distributed unit normal variates.

The term in parentheses in δ_c has a similar form to the δ_d term in the dividend model, though the parameter value is different. It represents the current value of expected future inflation as an exponentially weighted moving average of past rates of inflation. The values adopted for the parameters are:

$$\omega_c = 1.0,\ \delta_c = 0.045,\ \mu_c = 0.035,\ \alpha_c = 1.20,$$
$$\beta_c = -0.48,\ \gamma_c = 0.20,\ \phi_c = 0.06,\ \sigma_c = 0.14.$$

A2.6. Interested readers are referred to WILKIE (1986) for interpretation of what the model implies and how it can be used. A fuller description of the derivation of the model is given in WILKIE (1984).

A2.7. There is no specific provision in Professor Wilkie's model for cash as an investment. We have assumed that the return on cash for any year is the Consols yield at the start of the year less one percentage point.

APPENDIX 3
THE SIMULATION PROGRAM

A3.1.1. In order to simulate the run-off of an insurance company it is necessary to make decisions in regard to many parameters. Furthermore it is not easy, *ab initio*, to select sensible values for many of them. The program is written in such a way as to allow for a range of values of each of the parameters. As there are at least 20 parameters or values that may vary, and several may have up to 8 or 9 values, it is impossible to provide for every possible combination of values of the parameters. Not only would the program take too long to run but the volume of output would be too great to comprehend.

A3.1.2. The program is written, therefore, to allow each of the parameters to vary in turn over its whole range, whilst the others are kept constant at a "normal" or standard level. It also permits an analysis by two parameters at a time, for every possible combination of the various levels of those two parameters.

A3.1.3. For each parameter combination the same 1000 sets of random numbers are used, so that the comparisons are not significantly affected by any bias in the particular sets of random numbers chosen.

The Basis of the Simulations

A3.2.1. The program works from a series of written premiums, going back sufficiently far into the past to include every year for which claims are still to be run off. Provision is made for three alternative bases for the future.

1. A wind-up — an assumed return of the unearned premium reserve (UPR) as the policyholders claim on the liquidator for the unearned part of their premiums.
2. A run-off — the UPR is translated into a pattern of future claims payments and included with payments in respect of the outstanding claims and IBNR.

3. A continuing business — the future period of writing premiums can be selected and after that there is a run-off as in 2 above.

A3.2.2. It is necessary to generate claim ratios for each type of business and for provision to be made for the claims ratios to vary stochastically. The classes of business are characterized by the length of run-off period and settlement pattern and the proportions of business written in each category of tail are set by three parameters.

A3.2.3. The investment model is that given by WILKIE (1986). The investment mix may be varied according to the nature of the business and the initial investment mix is specified separately for the technical provisions and asset margin. The rules for selling and buying investments may be selected. Buying is likely to occur where there is a continuing business and written premiums are growing but it can also arise in the later years of a run-off where the income from the assets is large, particularly in the case of larger initial asset margins.

A3.2.4. The volume of written premiums may be allowed to grow or diminish over the years since this affects the ratio of outstanding claims to the latest year's written premiums and also the relative importance of income and outgo in respect of future business where this is assumed.

A3.2.5. Corporation tax is payable by a general insurance company in the UK on its profits, which include capital gains as well as income and exclude any allowance for indexation of the purchase price of securities. However, such "income" is not subject to tax if it is used to pay claims and expenses, and it seems likely that with a company that is in any danger of becoming insolvent there will be past losses carried forward, as well as future claims outgo, that will probably absorb most, if not all, of the income. This will mean that the effective rate of tax on interest will be very low. Provision is made for notional rates of tax for the first five years, at rates well below the current rates of corporation tax. The "tax" is assumed also to include the payment of dividends to shareholders. This will result in an overstatement of the outcome in scenarios where the company remains solvent, but this is not the main feature of the results with which we are concerned. The tax treatment in the model could clearly be made more sophisticated.

The Investment Model

A3.3. The Wilkie model has been used, notwithstanding the author's warning that it was not developed for short-term forecasting. Wilkie's own view is that its use in these simulations can be justified. We have examined the output from the model over several hundred simulations of 30 years and have satisfied ourselves that the variations in the values do not appear unreasonable in the light of experience over recent years. However, this includes the possibility of a collapse in the market such as occurred in 1974 and it might be thought that such a collapse would require special dispensations allow the majority of insurers to continue to write business. Care should, therefore, be exercised in interpreting the results in so far as they depend upon the impact of temporary abrupt falls in market values.

Future Statutory Solvency

A3.4.1. For a continuing company it is necessary to examine the financial position at the end of each year, if not more often. Accounts and returns have to be presented and a simulation of the future development of the company for management purposes would need to have regard to how the position might appear in presentational terms at each future reporting date.

A3.4.2. For a company that is already being run-off or to test what would happen in such circumstances, the reporting constraint is less relevant and our aim has been to look at "true" solvency, rather than the position as constrained by reporting conventions. The model simply looks at the adequacy of the assets to meet the liabilities as they are simulated to arise during the run-off. It does not check the solvency position as it might be reported to shareholders or to the supervisory authority at points during the run-off. Such a factor could be introduced if a procedure for deciding on appropriate bases for the technical provisions in future years were to be defined.

The Choice of Parameters and Their Values

A3.5. Every parameter is allowed to have at most 9 values, but need not be given more than 1. The parameters are numbered 1 to 13 and their levels 1 to 9. The value for level 5 is the standard and a value must be inserted for this parameter in every case, even if it is not included in the list of parameters to be analysed, since the program requires a value to be assigned for every parameter. A detailed list of the parameters and the factors underlying their choice is given below:

1. *Written premiums.* The values used are £1,000,000, £10,000,000 and £100,000,000 a year. For a larger amount of business the purely stochastic variation would be negligible in comparison with other variability, so that the results would be unlikely to differ significantly from those for £100,000,000 a year. The written premiums are taken as being net of initial expenses and commission.
2. *Claim ratio — very long.* This is the claim ratio for future business of a very long-tailed nature, i.e. with a run-off period of 20 years. So far no such business has been included in the simulations. Claim ratios are assumed to include the expenses of claim settlement with actual claim costs but they are related to written premiums net of commission and expenses (cf. Paragraph 4.3.2).
3. *Standard deviation — very long.* This is the standard deviation of the above claim ratio.
4. *Claim ratio — long* This is the claim ratio for future business of a long-

tailed nature, i.e. with a run-off period of 10 years. We have used values of 80%, 100%, 125% and 150%.

5. *Standard deviation — long.* We have used 5%, 10%, 15%, 20% and 25% of written premiums.

6. *Claim ratio — short.* This is the claim ratio for future business of a short-tailed nature. We have used 80%, 100%, 125% and 150% here also.

7. *Standard deviation — short.* We have used 5%, 10%, 15% and 20% of written premiums.

8. *Growth rates.* Separate real growth rates may be assumed before and after the assessment date. Rates varying from 0.8 to 1.5 have been used and the effect of zero real growth up to the date of assessment and positive or negative growth thereafter, and vice versa, have been examined. Inflation is automatically allowed for in the program so that the growth assumptions relate to growth in real terms.

9. *Proportions of business.* These are the proportions of written premiums represented by very long-tailed business, long-tailed business and short-tailed business. Only the first two are given: the program calculates the short-tailed and checks that it is not negative.

10. *Asset mix — solvency margin.* The proportions of equities and gilts are given separately. The proportion of "cash" is calculated and checked to see that it is not negative.

11. *Asset mix — technical provisions.* As above (10).

12. *Asset margin.* This is expressed as a percentage of the net written premiums in the last year before the date of assessment. This margin is allowed to range from nil to 120%. The normal value has been taken as 40%. Different reserving strength, arising from the assumptions made in calculating the outstanding claims, allowing for inflation and for discounting, can be studied by looking at different asset margins (cf. Paragraph 5.2). On the standard basis the technical provisions are established using 5% inflation and no discounting.

13. *Selling rules.* There are 8 alternative rules, namely:
   (*a*) Sell equities until they are exhausted, then gilts and finally cash.
   (*b*) Equities, cash, gilts.
   (*c*) Gilts, equities, cash.
   (*d*) Gilts, cash, equities.
   (*e*) Cash, gilts, equities.
   (*f*) Cash, equities, gilts.
   (*g*) Sell rateably (i.e. in proportion to the current value of holdings).
   (*h*) Sell each year whatever has performed best since the start of the run-off.

Investment (where there is a surplus of income over outgo) is always done rateably. It is also necessary to specifiy:

1. The number of future years. This is limited to the range 1 to 10 but the parameter can take the values 0 or -1 meaning that we are assuming no new business written and that we have either a run-off (0) or a wind-up (-1).

2. The number of simulations.
3. The number of parameters to be analysed, that is 1 or 2.
4. The existence of the very long-tailed class. This was included as an option to avoid having very long loops which are not needed where there is no such class. It was merely a program-writing device.

The Program Plan

A3.6. The program has been written to permit it to be run on FORTRAN IV (otherwise known as FORTRAN 66). In particular we have avoided the use of negative values in arrays. For this purpose we have assumed that the past is represented by years 1 to 20 and the future by years 21 to 46. Whilst this means that some arrays have to be larger than they would otherwise need to be, the simplification is worthwhile. The program is divided into sections:
1. *Initialization.* This sets out the values of the parameters, dimensions the arrays and sets some initial values. The values of the parameters could be inserted by lead cards if preferred. This section also includes the values of: number of future years, number of parameters, very long-tailed option and the number of simulations. This section also contains some data manipulation and checking to avoid time-consuming operations later in the program.
2. *The random number generator.* This generates the necessary number of random normal variates and stores them in an array for use by the later stages of the program. This ensures that the same numbers are used for every variation within a single simulation. They are recalculated for each further simulation. The random number generator of the machine has been used to generate uniform random variates in the range 0 to 2. After subtracting 1 these are used in Marsaglia's polar method to generate the corresponding random normal variates. This method requires pairs of uniform randoms and produces normal variates if, and only if, the sum of the squares of the two variates is less than 1. The program counts the number of useful pairs and stops when it has enough to fill the array.

We have tested this process and found that a distribution of 3 million variates was very closely normal, using 9-figure tables of the normal integral for the test. This however does not test that they come in a random order and we have further tested them to count the number of cases where there is a run of 1 increase or 2 increases and so on up to 7 increases. It is not difficult to calculate theoretically the expected number of such runs both upwards and downwards and their expected size. The results are within expected limits. The methods will be described in detail in a paper to be written by two of the authors of this report, together with notes on the times taken to make the calculations. These seem to vary considerably from one method to another. It is perhaps worth mentioning that what we require are representative sequences rather than purely random ones. Kendall and Babington Smith noted in 1938 that a sequence of $10^{10^{10}}$ random numbers is almost certain to contain a sequence of a million zeros (or, for that matter any other sequence you care to specify). This might be a random sequence but it is not very useful in practice.
3. *Investment values.* The program now calculates the investment values for up

to 26 future years, depending on the particular run-off period involved. The values are of:

1. The retail price index.
2. Equity dividends.
3. Equity yield.
4. Gilt yield.
5. Equity price.
6. Gilt price.
7. Cash yield.
8. Borrowing rate.
9. A net income multiplier (see below).
10. An equity price ratio.
11. A gilt price ratio.
12. A mean retail price index.

The reason for a "cash" yield is that gilts are assumed to relate to medium or long term, whilst cash is either cash on deposit or very short term gilts. It is assumed that the cash yield is 1% below that of gilts and that when cash becomes negative and we have to borrow, it is at a rate 2% higher than the gilt rate. The gilt and equity yields have a minimum of 0.5%. The equity price ratio is the square root of the ratio of the equity price at the end of the relative year to its value at the start of the year. Its purpose is to revalue equities from the year-end value, on which the income is based, to the mid-year value at which it is assumed that sales take place or purchases are made. After the mid-year transactions the remaining values of gilts and equities are updated to the year-end by a further multiplication by the equity (or gilt) price ratio. Although interest is calculated on the values at the start of the year, allowance is made for the loss of income on selling during the year by multiplying the net outgo by a factor of 1 plus half the average yearly yield on the investments. Whilst this assumes that the values of all three classes are equal, the effect of differences is likely to be too small to be of any consequence in practice.

4. *Best investment.* The next section is really a continuation of section 3 in that it calculates which of the three classes of investment has performed best since the start of year 21 and stores this information for use later in the program.

5. *Outstanding claims.* The program now calculates the outstanding claims at the end of year 20. For each earlier year the program calculates the claims according to the mean claim ratios and then, using the run-off rates shown in Table A3.1, calculates the amounts, in constant money terms, which it expects to pay out in each future year.

These are stored in an array by year of expected payment and the total is accumulated, allowing for 5% future inflation, in a variable TOTOS which is the total provision for claim amounts outstanding at the end of year 20. By using these run-offs we have automatically taken into account the IBNR claims. If we have a wind-up situation then TOTOS is the technical provision. If we are considering a run-off or a continuing business then we must add 50% of the written

TABLE A3 1

From Abbott, *et al*, 1981

Duration from year of origin	Proportion of claims settled (%) Short-tail	Long-tail
0	61 2	5 6
1	24 1	25 3
2	5 2	18 7
3	3 7	13 2
4	2 7	10.4
5	2 2	7 9
6	0 9	6 4
7	—	4 6
8	—	3 8
9	—	3 0
10	—	1 1
	100 0	100 0

premium for year 20 into TOTOS as the unearned premium reserve. This figure for technical provisions, together with the asset margin obtained from the product of the assumed asset margin percentage and the written premiums for year 20, enable us to calculate the initial amounts of each type of asset using the specified proportions. This is the initial investment portfolio.

6. *Future premiums.* We next add into the arrays of future payments the expected contribution to claims outgo arising from future written premiums and from the unearned premium reserve for the last year, to give the expected claims outgo in constant money terms.

7. *Emerging costs.* The program now has the information to enable it to calculate the expected payments in each future year. The claims outgo is adjusted for inflation according to the Wilkie model and is allowed to vary stochastically. We assume a normal distribution and a formula of $0.15X = 75\sqrt{X}$ as the standard deviation for the total claim outgo in any year. The square root factor is dominant for the smaller amounts and the smaller companies but for the larger companies the stochastic variation is negligible and it is only realistic to assume some sort of overall secular variation (see Appendix 5).

We take the values of assets at the beginning of the year and calculate the income on each type of asset, reducing the total income for the year by the tax factor where appropriate. We then have the outgo, adjusted to allow for inflation and stochastic variation, less the income and less any written premiums for a continuing business. As mentioned in A3.6.3 we adjust for the loss of part of the year's investment income as a result of net selling during the year (or vice versa in a net buying situation). Investment or disinvestment is assumed to take place at mid-year values. If there is net investment, it is assumed to be made proportionately to the existing values of the three classes of investment. Where there is net outgo, the specified selling rule is applied.

8. *Final assets.* This process continues until the last year's claims outgo has been paid. The final assets are in the currency of the final year as a result of the application of the investment model which revalues the assets, combined with the models

for income and outgo in each year which allow implicitly for future inflation. In order to bring the final asset value into the currency of the start of the run-off, it is divided by the ratio of the retail price index in the final year to that at the date of assessment. The result is then expressed as a percentage of the written premiums in year 20 (the year before the date of assessment). These values from the 1000 simulations are grouped into ranges and output as a distribution, together with their mean and standard deviation.

APPENDIX 4
RESULTS OF SIMULATION

Full details of the results of 1000 simulations on a variety of different bases are set out in Tables A4.1 to A4.4. Tables A4.1 and A4.2 show summary distributions of the simulations by the assets remaining at the end, as well as the number of insolvencies and the mean and standard deviation of the distributions. Results are also given for a few additional variants not tabulated in Tables 4 and 5. Tables A4.3 and A4.4 also include a number of additional variants and Table A4.4 shows the additional asset margin required in the case of 2 years' new business as compared to the pure run-off with the same assumptions (in so far as these are applicable).

The tables show the standard basis at the top and also in each of the groups of alternative assumptions (marked (s)). The variants examine the effect of varying the one assumption referred to, whilst leaving all the other assumptions the same as in the standard basis.

The assumptions underlying the standard basis are as follows:

Net written premiums[1]	£10m a year
Proportion of long-tailed business	40% of net written premiums
Past growth	In line with inflation
Future growth	In line with inflation
Mean claim ratio[2] (short-tailed)	100% of net written premiums[1]
Standard deviation of CR[2] (short-tailed)	10% of net written premiums[1]
Mean claim ratio[2] (long-tailed)	100% of net written premiums[1]
Standard deviation of CR[2] (long-tailed)	15% of net written premiums[1]
Initial asset distribution	Technical provisions: 50% cash; 50% gilts Asset margin: 100% equities
Asset selling rule	Proportionate to holdings
Asset margin (for Tables A4.1 and A4.2)	40% of net written premiums

1 Premiums net of commission and expenses
2 Ratio of claims (including claims settlement expenses), without allowance for future inflation or for discounting, to premiums net of commission and expenses (see Paragraph 4 3 2)

TABLE A4 1

SUMMARY OF RESULTS FOR PURE RUN-OFF OF BUSINESS (WITH 1000 SIMULATIONS)

Assumptions	No of insolvencies	No of simulations with remaining assets[1] of. 0%–40%	40%–80%	80%–120%	120%–160%	Over 160%	Mean assets remaining[1] %	Standard deviation of assets remaining[1] %
Standard basis	8	98	252	264	173	205	112	70
1 Net written premiums.[2]								
(a) £1m a year	20	119	227	250	171	213	113	75
(b) £10m a year (s)	8	98	252	264	173	205	112	70
(c) £100m a year	6	93	258	275	164	204	112	69
2 Proportion of long-tailed business·								
(a) 20% of net written premiums[2]	3	126	321	293	153	104	94	55
(b) 40% of net written premiums[2]	8	98	252	264	173	205	112	70
(c) 60% of net written premiums[2]	13	88	197	230	184	288	130	85
3 Initial asset distribution								
Cash / Gilts / Equities								
(a) TP + AM / — / —	3	128	303	295	162	109	95	53
(b) — / TP + AM / —	20	159	220	208	142	251	120	98
(c) — / — / TP + AM	49	124	182	181	149	315	136	115
(d) ½TP / ½TP / AM (s)	8	98	252	264	173	205	112	70
(e) ½TP + ½AM / ½TP + ½AM / —	14	127	260	255	150	194	107	72
(f) ½TP + ½AM / — / ½TP + ½AM	12	85	222	278	184	219	118	72
(g) TP / — / AM	3	93	283	312	176	133	102	54

TABLE A4 1 (*Continued*)

Assumptions	No of insolvencies	No. of simulations with remaining assets[1] of					Mean assets remaining[1] %	Standard deviation of assets remaining[1] %
		0%–40%	40%–80%	80%–120%	120%–160%	Over 160%		
Standard basis	8	98	252	264	173	205	112	70
4. Initial asset margin								
(a) 0% of net written premiums[2]	134	336	292	138	58	44	52	55
(b) 20% of net written premiums[2]	36	210	312	222	117	102	83	62
(c) 40% of net written premiums[2] (s)	8	98	252	264	173	205	112	70
(d) 60% of net written premiums[2]	2	34	149	244	212	360	147	80
(e) 80% of net written premiums[2]	0	12	74	184	211	519	180	90
(f) 100% of net written premiums[2]	0	4	34	117	186	659	212	100
5 Asset selling rules:								
(a) Equities, gilts, cash	9	133	285	262	152	159	102	66
(b) Equities, cash, gilts	16	142	233	241	143	225	113	81
(c) Gilts, equities, cash	5	123	246	285	181	160	105	61
(d) Gilts, cash, equities	3	67	249	314	201	166	111	58
(e) Cash; gilts, equities	11	134	212	237	168	238	120	83
(f) Cash, equities, gilts	7	78	245	244	185	241	123	79
(g) In proportion to holdings (s)	8	98	252	264	173	205	112	70
(h) Sell best performer first	14	131	239	268	158	190	108	70

1 Deflated to the date of assessment and expressed as a percentage of net written premiums[2] in the year before the date of assessment (see Appendix 3 6 8)
2 Premiums net of commission and expenses
(s) indicates the assumption made for the standard basis

TABLE A4 2

SUMMARY OF RESULTS WITH 2 FURTHER YEARS' BUSINESS (WITH 1000 SIMULATIONS)

Assumptions	No of insolvencies	No of simulations with remaining assets[1] of: 0%–40%	40%–80%	80%–120%	120%–160%	Over 160%	Mean assets remaining[1] %	Standard deviation of assets remaining[1] %
Standard basis	50	96	146	171	175	362	144	109
1. Net written premiums [2]								
(a) £1m a year	61	106	144	155	161	373	144	117
(b) £10m a year (s)	50	96	146	171	175	362	144	109
(c) £100m a year	43	96	144	181	172	364	144	107
2 Proportion of long-tailed business:								
(a) 10% of net written premiums[2]	52	136	217	220	164	211	108	83
(b) 20% of net written premiums[2]	48	116	190	212	159	275	120	91
(c) 40% of net written premiums[2] (s)	50	96	146	171	175	362	144	109
(d) 60% of net written premiums[2]	52	83	120	141	159	445	168	130
(e) 80% of net written premiums[2]	53	77	99	117	126	528	191	151
(f) 90% of net written premiums[2]	58	71	93	106	118	554	203	162
3 Future real growth rate (in constant money terms).								
(a) −20% a year (real past growth −20% p a)	53	78	118	143	144	464	171	134
(b) No growth (no real past growth) (s)	50	96	146	171	175	362	144	109
(c) +50% a year (real past growth +50% p a)	83	94	139	149	155	380	144	121
(d) +30% a year (no real past growth)	66	93	131	144	151	415	155	127
(e) +50% a year (no real past growth)	86	80	122	130	137	445	164	141
4 Mean claim ratio[3] (short-tailed)								
(a) 80% of net written premiums[2]	7	47	105	129	177	535	187	117
(b) 100% of net written premiums[2] (s)	50	96	146	171	175	362	144	109
(c) 125% of net written premiums[2]	165	167	177	168	120	203	90	103
(d) 150% of net written premiums[2]	380	158	183	110	67	102	35	102

TABLE A4 2 (*Continued*)

SUMMARY OF RESULTS WITH 2 FURTHER YEARS' BUSINESS (WITH 1000 SIMULATIONS)

Assumptions	No of insolvencies	No of simulations with remaining assets[1] of: 0%–40%	40%–80%	80%–120%	120%–160%	Over 160%	Mean assets remaining[1] %	Standard deviation of assets remaining[1] %
Standard basis	50	96	146	171	175	362	144	109
5. Variability of claim ratio (short-tailed)								
(a) Standard deviation 5% NWP[2]	49	97	142	178	170	364	144	109
(b) Standard deviation 10% NWP[2] (s)	50	96	146	171	175	362	144	109
(c) Standard deviation 15% NWP[2]	48	96	148	168	173	367	144	111
(d) Standard deviation 20% NWP[2]	52	94	144	170	172	368	145	112
6 Mean claim ratio[3] (long-tailed)								
(a) 80% of net written premiums[2]	17	62	136	161	195	429	163	106
(b) 100% of net written premiums[2] (s)	50	96	146	171	175	362	144	109
(c) 125% of net written premiums[2]	105	139	155	160	138	303	120	114
(d) 150% of net written premiums[2]	195	139	164	143	119	240	97	120
7 Variability of claim ratio (long-tailed)								
(a) Standard deviation 5% NWP[2]	49	84	159	172	171	365	144	109
(b) Standard deviation 10% NWP[2]	50	89	156	172	168	365	144	109
(c) Standard deviation 15% NWP[2] (s)	50	96	146	171	175	362	144	109
(d) Standard deviation 20% NWP[2]	49	101	137	176	176	361	144	110
(e) Standard deviation 25% NWP[2]	50	104	135	174	172	365	144	111
8 Initial asset distribution								
Cash / Gilts / Equities								
(a) TP + AM / — / —	46	130	186	204	173	261	118	86
(b) — / TP + AM / —	79	121	141	147	124	388	155	152
(c) — / — / TP + AM	86	91	112	131	113	467	181	172
(d) $\frac{1}{2}$TP / $\frac{1}{2}$TP / AM (s)	50	96	146	171	175	362	144	109
(e) $\frac{1}{2}$TP + $\frac{1}{2}$AM / $\frac{1}{2}$TP + $\frac{1}{2}$AM / —	61	120	162	159	163	335	136	113
(f) $\frac{1}{2}$TP + $\frac{1}{2}$AM / — / $\frac{1}{2}$TP + $\frac{1}{2}$AM	43	77	141	174	159	406	152	112
(g) TP / — / AM	41	97	166	208	182	306	128	87

9 Initial asset margin								
(a) 0% of net written premiums[2]	196	204	197	154	97	151	74	98
(b) 20% of net written premiums[2]	103	148	197	167	140	244	109	105
(c) 40% of net written premiums[2] (s)	50	96	146	171	175	362	144	109
(d) 60% of net written premiums[2]	24	54	115	151	158	497	180	123
(e) 80% of net written premiums[2]	11	29	75	127	140	619	216	133
(f) 100% of net written premiums[2]	3	18	45	88	128	718	252	144
10 Asset selling rules								
(a) Equities, gilts, cash	56	120	161	175	150	338	136	113
(b) Equities, cash, gilts	63	103	156	169	138	371	144	120
(c) Gilts; equities; cash	51	106	154	165	181	343	137	103
(d) Gilts, cash, equities	35	87	141	192	197	348	142	99
(e) Cash, gilts, equities	41	76	143	173	173	394	154	116
(f) Cash; equities, gilts	52	90	144	153	168	393	151	119
(g) In proportion to holding (s)	50	96	146	171	175	362	144	109
(h) Sell best performer first	58	101	154	164	159	364	142	114

1 Deflated to the date of assessment and expressed as a percentage of net written premiums[2] in the year before the date of assessment (see Appendix 3 6 8)
2 Premiums net of commission and expenses
3 Ratio of claims (including claims settlement expenses), without allowance for future inflation or for discounting, to premiums net of commission and expenses (see Paragraph 4.3.2)
(s) indicates the assumption made for the standard basis

TABLE A4 3

ASSET MARGINS REQUIRED TO ACHIEVE 1/100 PROBABILITY OF RUIN — NO FUTURE NEW BUSINESS

Assumptions				Asset margin as % of NWP[1]	Asset margin as % of technical provisions
Standard basis				40	15
1 Net written premiums[1]					
(a) £1m a year				55	25
(b) £10m a year (s)				40	15
(c) £100m a year				35	15
2 Proportion of long-tailed business					
(a) 20% of net written premiums[1]				30	15
(b) 40% of net written premiums[1] (s)				40	15
(c) 60% of net written premiums[1]				45	15
3. Initial asset distribution					
	Cash	Gilts	Equities		
(a)	TP + AM	—	—	30	10
(b)	—	TP + AM	—	60	25
(c)	—	—	TP + AM	80	35
(d)	$\frac{1}{2}$TP	$\frac{1}{2}$TP	AM (s)	40	15
(e)	$\frac{1}{2}$TP + $\frac{1}{2}$AM	$\frac{1}{2}$TP + $\frac{1}{2}$AM	—	50	20
(f)	$\frac{1}{2}$TP + $\frac{1}{2}$AM	—	$\frac{1}{2}$TP + $\frac{1}{2}$AM	45	20
(g)	TP	—	AM	30	10
4. Asset selling rules:					
(a) Equities; gilts; cash				40	15
(b) Equities, cash; gilts				50	20
(c) Gilts, equities; cash				35	15
(d) Gilts, cash, equities				30	10
(e) Cash; gilts, equities				35	15
(f) Cash, equities; gilts				45	20
(g) In proportion to holdings (s)				40	15
(h) Sell best performer first				55	25

1. Premiums net of commission and expenses
(s) indicates the assumption made for the standard basis

TABLE A4 4

ASSET MARGINS REQUIRED TO ACHIEVE 1/100 PROBABILITY OF RUIN — TWO YEARS NEW BUSINESS

Assumptions	Asset margin as % of NWP[1]	Excess asset margin as compared to pure run-off (as % of NWP[1])
Standard basis	90	50
1 Net written premiums[1]		
(a) £1m a year	100	45
(b) £10m a year (s)	90	50
(c) £100m a year	80	45
2 Proportion of long-tailed business		
(a) 10% of net written premiums[1]	75	45
(b) 20% of net written premiums[1]	80	50
(c) 40% of net written premiums[1] (s)	90	50
(d) 60% of net written premiums[1]	95	50
(e) 80% of net written premiums[1]	100	40
(f) 90% of net written premiums[1]	105	40

TABLE A4 4 (*Continued*)

ASSET MARGINS REQUIRED TO ACHIEVE 1/100 PROBABILITY OF RUIN — TWO YEARS' NEW BUSINESS

Assumptions				Asset margin as % of NWP[1]	Excess asset margin as compared to pure run-off (as % of NWP[1])
Standard basis				90	50
3 Future growth rate (in constant money terms)					
(a) −20% a year (real past growth −20% p.a)				100	35
(b) No growth (no real past growth) (s)				90	50
(c) +50% a year (real past growth +50% p a.)				115	85
(d) +30% a year (no real past growth)				100	65
(e) +50% a year (no real past growth)				120	85
4 Mean claim ratio (short-tailed)					
(a) 80% of net written premiums[1]				30	0
(b) 100% of net written premiums[1] (s)				90	50
(c) 125% of net written premiums[1]				125	75
(d) 150% of net written premiums[1]				180	115
5 Variability of claim ratio (short-tailed)					
(a) Standard deviation 5% NWP[1]				80	45
(b) Standard deviation 10% NWP[1] (s)				90	50
(c) Standard deviation 15% NWP[1]				85	45
(d) Standard deviation 20% NWP[1]				90	50
6 Mean claim ratio[2] (long-tailed)					
(a) 80% of net written premiums[1]				50	20
(b) 100% of net written premiums[1] (s)				90	50
(c) 125% of net written premiums[1]				115	60
(d) 150% of net written premiums[1]				150	85
7 Variability of claim ratio (long-tailed)					
(a) Standard deviation 5% NWP[1]				80	45
(b) Standard deviation 10% NWP[1]				85	50
(c) Standard deviation 15% NWP[1] (s)				90	50
(d) Standard deviation 20% NWP[1]				90	50
(e) Standard deviation 25% NWP[1]				90	50
8 Initial asset distribution					
	Cash	Gilts	Equities		
(a)	TP + AM	—	—	85	55
(b)	—	TP + AM	—	110	50
(c)	—	—	TP + AM	135	55
(d)	$\frac{1}{2}$TP	$\frac{1}{2}$TP	AM (s)	90	50
(e)	$\frac{1}{2}$TP + $\frac{1}{2}$AM	$\frac{1}{2}$TP + $\frac{1}{2}$AM	—	90	55
(f)	$\frac{1}{2}$TP + $\frac{1}{2}$AM	—	$\frac{1}{2}$TP + $\frac{1}{2}$AM	90	45
(g)	TP	—	AM	75	45
9 Asset selling rules					
(a) Equities, gilts, cash				95	55
(b) Equities; cash; gilts				95	45
(c) Gilts; equities; cash				90	55
(d) Gilts, cash, equities				70	40
(e) Cash, gilts, equities				85	50
(f) Cash, equities, gilts				95	50
(g) In proportion to holdings (s)				90	50
(h) Sell best performer first				95	40

1 Premiums net of commission and expenses

2 Ratio of claims (including claims settlement expenses), without allowance for future inflation or for discounting, to premiums net of commission and expenses (see Paragraph 4 3 2)

(s) indicates the assumption made for the standard basis

APPENDIX 5
VARIABILITY OF CLAIMS OUTGO

A5.1. In DAYKIN and BERNSTEIN (1985) it was assumed that the amount of the payments made in each development year for each year of origin varied log-normally. This meant that a payment amount that was to be varied stochastically was multiplied by $\exp(RS+M)$ where R is a random normal variate, S the standard deviation and M the mean. In order that the overall mean should be correct the value of M has to be equal to minus half the square of the standard deviation. This formula is suitable for a single payment, but in most cases the payment amounts considered were the totals of several or many individual amounts. Furthermore different values would need to be adopted for funds of different sizes if account was to be taken of the fact that variation is not the same for a small fund as for a large one.

A5.2. This was cumbersome and not entirely satisfactory, so an alternative approach was sought. The formula should reflect the number of payments involved and, if possible, the ratio of the standard deviation to the mean (the coefficient of variation). Consideration was given to the estimation of the numbers of claims (or claim payments) in each year's totals. We were unable to obtain any figures from actual portfolios but information from returns to the supervisory authority and from other sources suggested that for short-tailed business an average payment rising from £500 in the year of occurrence by multiples of 2 to £16,000 in the last year of development was not unreasonable. For long-tailed business the average payments rose over 10 years from £800 to £15,000.

A5.3. We assumed that coefficients of variation were in the range of 2 to 10, increasing at later durations as fewer, larger claims are settled. We were then able to estimate both the numbers of claims and their average amounts for different mixes of business by year of development. For this purpose it was assumed that claims were identical with payments, and whilst this is clearly not the case, it is not thought that it would make much difference if we were able to make more detailed assumptions. These calculations suggested that the formula for standard deviation should be a multiple of the square root of the number of claims, or its deemed equivalent, the total amount of payment. For convenience we used the amount of money, even though inflation would involve a change in the multiplier over time.

A5.4. It must be realized that precision was out of the question since we could not take into account all the possible variations in the make-up of a portfolio. It was also necessary to have regard to the fact that the bulk of the outstanding claims are paid in the first two or three years of run-off and relate primarily to the latest two or three years' business. Calculations showed that out of total outstandings of £1 million about one-half was paid in the first year and a quarter in the next year. By year 7 the payments were under £20,000, so that variation in these later years was less significant in the overall context. What is more, for many insurers the later payments, if they turn out to be large, may well be recoverable from reinsurers and so not form part of the problem for net run-off patterns. It simply moves the problem to another area. Further consideration would need to be given to the variability of the tail in the case of a company with a lot of long-tailed business and relatively high retentions.

A5.5. Experiment suggested that a multiplier of about 50 to 100 times the square root of the amount (in pounds sterling in 1986) was of the right order of magnitude. However, it was clear that whilst this gave a reasonable amount of variation for the smaller insurer it was wholly inadequate for a large one. In present conditions most of the variation for the larger fund arises from secular change and this is more likely to be proportional to the actual amount to be paid than to its square root. The problem is to choose a multiplier to give a realistic variation. Experience over recent years suggests that it must be at least 0.1, to give a variation of 20% in 95% of all cases. We finally adopted the formula

$$\mathrm{SD} = aX + b\sqrt{X}$$

using values of 0.15 for a and 75 for b.

A5.6. This formula is similar to one which we understand was introduced by the Finnish supervisory authority in 1952. Whilst we are well aware of the approximations and assumptions involved in its derivation, we think it is adequate for the purpose, although it can be considered as simply one of a class of possible formulae. It also greatly simplifies the calculations. As indicated above, the earlier paper calculated the outgo for each future year for each year of occurrence and for each length of tail separately and applied the stochastic factor to each such amount. The main effect of this was to reduce the overall variation compared with applying the same formula to the total and this effect can be achieved by adjusting the overall level of the variation. It was decided, therefore, to calculate the total outgo in each year, including that from future business where appropriate, and apply the variability factor to the total.

A5.7. It is interesting to compare the values produced by the formula with those from the exponential basis. The comparisons, with values of R corresponding to the 5%, 25% and 50% points, are shown in Table A5.1. The correspondence

TABLE A5 1

STOCHASTIC MULTIPLIER ($1 + RS$) FOR DIFFERENT VALUES OF R AND STANDARD DEVIATION (S)

	Random normal variate (R)				
	−1 96	−0 675	0	0 675	1 96
log-normal					
$S = 0$ 3	0 53	0 78	0 96	1 17	1.72
$S = 0$ 5	0 33	0 63	0 88	1.24	2 35
square root formula ($S = 0$ 15$X + 75\sqrt{X}$)					
$X = 100,000$	0 24	0 74	1 0	1 26	1.76
$X = 1,000,000$	0.56	0.85	1 0	1 15	1.44
$X = 10,000,000$	0 66	0 88	1 0	1.12	1 34

between the two formulae, coupled with the size of variation by insurer, suggests that the new formula is in line with the old but more realistic in its relation to the actual amounts of payments.

APPENDIX 6

POSSIBLE APPROACH TO SIMULATING REINSURANCE RECOVERIES

A6.1. It is not possible to simulate reinsurance recoveries in our model in any very precise way, firstly because it is too complicated and secondly because the model simulates claims only in aggregate. It would in principle be possible to think in terms of a specified number of reinsurers, each bearing a share of the anticipated reinsurance recoveries, and find a way to model the failure of reinsurers. Rather easier, and probably no less realistic, would be to go directly to the proportion recovered. One way of approaching the problem is set out below.

A6.2. Reinsurers would be allocated to say, three categories — strong, average and weak. For any class of business the proportion of reinsurance recoveries anticipated from each of the three categories of reinsurer would be input as data. The model would then be to apply a process, defined separately for each category, to determine the proportion not recovered in respect of any particular year's estimated gross claim payments. There remains, of course, the problem of estimating gross claims payments and simulating their out-turn, so that there would be considerable practical problems in implementing an approach of this sort.

A6.3. The probability of recovery would be related to the gross claims out-turn. This could be done by taking the estimate of gross claims paid in the year in question to be the mean estimate of claims paid, based on proportions expected to be settled in the year, the rate of inflation assumed in setting the technical provisions and, in the case of claims arising from future business, the mean claim ratio. There would then be a set of formulae, one for each category of reinsurer, to define the proportion of gross claims paid in the year which is assumed not to be recovered, based on the ratio of gross claims out-turn to estimated gross claims for the year. For year j we might, for example, define the proportion not recovered, $Y(j)$, by:

Weak

$$Y(j) = \frac{k(j)}{200} \qquad 0 < k(j) < 200$$

Average

$$Y(j) = \frac{k(j) - 50}{500} \qquad 50 < k(j) < 550$$

Strong

$$Y(j) = \frac{k(j) - 100}{800} \qquad 100 < k(j) < 900$$

where

$$1+\frac{k(j)}{100}=\frac{X(j)}{\sum_{i\leq j}\hat{X}(i:j)}$$

$X(j)$ = actual total gross claims settled in year j

and

$\sum_{i\leq j}\hat{X}(i:j)$ = expected gross claims settlement in year j in respect of year of origin i on basis of mean claims ratio, assumed settlement pattern and expected inflation.

In terms of the notation of Appendix 1:

$$\hat{X}(i;j)=\sum_k s^k(j-1)R_kB_k^j(i)(1+r)^{j-i}.$$

A6.4. The formulae can obviously be adapted to reflect one's ideas of a plausible model for reinsurance recoveries. The general principle of these illustrative formulae is that one would expect higher proportions not recovered for weaker reinsurers and that, above a certain threshold, higher claims relative to the expected level of claims imply a higher proportion not recovered. These formulae do not attempt to distinguish between high claims as a result of high initial loss ratios, high inflation and adverse development. In principle one could also develop some form of cumulative trigger so that failure to recover increased with a series of high claims payments rather than simply on the basis of a single year.

A6.5. Consideration would also need to be given to whether to apply the formulae to all classes together or each class separately. Possibly the most realistic would be to apply it to the total claims on those classes of business where significant amounts are reinsured.

A6.6. The simple approach suggested here may not be sufficiently realistic for some companies for whom reinsurance recovery is a major issue. Further development of these ideas is clearly needed. However, it is suggested that it may be possible to obtain a useful indication of the role of reinsurance in a particular case by the use of straightforward models.

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