A STATISTICAL APPROACH TO IBNR-RESERVES IN MARINE REINSURANCE*

By Joakim Hertig

Baltica-Nordisk Re, Copenhagen

Abstract

The run off-pattern of long-term reinsurance treaties is described by means and standard deviations of logarithmic increments of premiums and loss ratios in a normal distribution. From this description forecasts of ultimate claims and current IBNR-reserves are derived, with associated distributions and confidence limits. Aggregation from individual treaties to portfolio level is proposed by normal approximation. Security loading of IBNR-reserves is proposed by a contingency reserve at portfolio level

Keywords

Run off pattern, lognormal distribution, IBNR reserves, contingency reserves, marine reinsurance.

1. Introduction

The present work forms part of a project to improve rules for the establishment of technical reserves in the B-N Re. Particular problems arise in the area of long tail insurance, where claims occur years after expiration of the risk period. This problem of IBNR-reserving has been treated by several authors in recent years, with the common approach to estimate ultimate claims from which current reserves are derived. Taylor (1977) separates components of inflation and real development by calculational methods, and provides a deterministic forecast. Buhlmann, Schnieper and Straub (1980) introduces a probabilistic model, proposing a lognormal distribution of the percentage increment from one year to the next Kremer (1982) proposes an ANOVA-approach with future values of claims treated as missing values, also using a lognormal distribution. In the present work, a main objective was the establishment of an operational tool for underwriters without formal statistical background. Another objective was the establishment of confidence limits of reserves, at single treaty level as well as portfolio level. To meet these objectives, effective use of simple statistical methods, and simple identification of key variables, were emphasized rather than deep theoretical considerations. The resulting model applies univariate normal theory to the logarithmic increments from one development year to the next, sharing basic assumptions with the papers of Bühlmann, Schnieper and Straub, and of

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Kremer. Below, the approach is developed with statement of the IBNR-problem, definition of model variables, estimation procedure, testing parameter stability, forecasting ultimate premiums and claims, and thus establishment of the IBNR-reserve for a single reinsurance treaty. The method is demonstrated on an example treaty. From single treaty level aggregation to portfolio level is performed by use of normal approximation, and at portfolio level a further security loading or contingency reserve is provided through the confidence limit.

2. THE IBNR-PROBLEM

Long tail (non-life) insurance emanates from policies covering a period of usually one year, the claims being reported and settled during a longer period. The main areas of long tail insurance are marine insurance, where ships often sail with damages for several years until docked, and liability insurance where events covered may be discovered after several years and court negotiations add further to the duration. The insurer operating in these fields finds it difficult to quote adequate and competetive rates taking recent experience into account, and also

TABLE I

DEVELOPMENT OF A MARINE REINSURANCE TREATY

(Thousand DKK)

		Financial Year			
Underwriting Year		1975	1976	1977	1978
1975	Premium	310	288	31	−5
	Commission	85	80	8	−1
	Claims paid	31	239	147	34
	Claims outstanding	167	152	36	15
	Profit/Loss	26	−16	−8	−17
1976	Premium		310	289	30
	Commission		85	80	8
	Claims paid		39	262	170
	Claims outstanding		165	135	67
	Profit/Loss		21	−23	−80
1977	Premium			345	322
	Commission			95	88
	Claims paid			86	368
	Claims outstanding			154	136
	Profit/Loss			10	−116
Financial year total	Premium	310	598	665	347
	Commission	85	165	183	95
	Claims paid	31	278	495	572
	Claims outstanding	167	317	325	218
	Profit/Loss	26	5	−21	−213

meets difficulties in the establishment of loss reserves for past but still vaguely reported underwriting years.

To the reinsurer this problem is intensified, since he obtains no information on individual policies and claims, but usually receives brief quarterly statements on aggregated accounts for a treaty covering a whole portfolio, and a note on aggregate claims outstanding once a year.

Table I demonstrates a reinsurers difficulties.

This representative example is the Baltica share of a European marine reinsurance treaty, covering hull and cargo on a quota share basis. Premiums are received chiefly over two years and claims incur in the second and third year of development with still some considerable adjustments in the fourth and following years. The noted reserves do not suffice, and the reinsurer cannot just rely on reported results and obviously has to reinforce reserves not to carry hidden loses in his books. Experienced underwriters are able to propose reserve reinforcements, but their proposals tend to be individual. This is a problem of Incurred But Not (Enough) Reported = IBN(E)R reserves.

3. DEFINITION OF KEY VARIABLES

The basic tool in the analysis of development is the run off triangle, e.g., the triangle of accumulated premiums of the example treaty:

TABLE II
ACCUMULATED PREMIUMS
(Thousand DKK)

Underwriting Year	Development Year					
	1	2	3	4	5	6
1969						226
1970					261	261
1971				329	328	328
1972			434	436	435	435
1973		610	632	631	630	630
1974	420	704	739	738	736	736
1975	310	598	629	624	623	622
1976	310	599	629	631	629	
1977	345	667	680	679		
1978	491	731	737			
1979	581	815				
1980	577					

The earliest financial year still kept in the files was 1974 and in 1980 registration procedures were changed, such that the entire story of development was only recorded for the underwriting years 1974 and 1975. In line with the findings above, it is seen that the treaty more generally shows a substantial premium growth from the first to the second year of development, a moderate growth from the second to the third year and then only small adjustments.

These observations are more clearly exhibited by the increments between successive development years:

TABLE III

LOGARITHMIC INCREMENTS OF PREMIUM

Underwriting Years	Development Years				
	1→2	2→3	3→4	4→5	5→6
1970					0.001
1971				−0.001	−0.001
1972			0.003	−0.002	0.000
1973		0.036	−0.001	−0.002	0.000
1974	0.517	0.048	−0.001	−0.004	0.000
1975	0.659	0.051	−0.008	−0.002	−0.001
1976	0.657	0.050	0.002	−0.002	
1977	0.660	0.019	−0.001		
1978	0.398	0.009			
1979	0.338				
mean	0.538	0.035	−0.001	−0.002	−0.000
std deviation	0.144	0.018	0.004	0.001	0.001
dgs of freedom	5	5	5	5	5

In the project ideas were tested on a set of 40 marine treaties in order to assess their feasibility, and in all treaties similar stabilities of logarithmic premium increments were present. As the reinsurer does not get further information, only guesses of the causes of the stable patterns can be made. The phenomenon is well known by underwriters and commonly explained by the stability of underlying portfolios. Within these individual shipowners' dates of premium payment are thought to be stable, though rates and inflation may change the premium level from one underwriting year to another. But it appears that development patterns vary between treaties.

Turning toward the development of claims, the central point in relation to IBNR-reserving is the originally noted claims, i.e., accumulated paid claims plus originally noted loss reserves. With decent rating criteria the premium volume will reflect the expected volume of claims, and so the premium and loss developments will be dependent. In modelling key variables should be independent, and the loss quotient, i.e., originally noted claims in relation to accumulated premium, appears to be less dependent on premium than absolute volume of losses. Also, one may note that underwriters traditionally monitor loss developments by loss ratios rather than volume of losses, thus supporting loss ratio as a suitable key variable.

The loss quotient of the example treaty developed as shown:

TABLE IV

LOSS QUOTIENT

Underwriting Year	Development Year					
	1	2	3	4	5	6
1969						0 801
1970					0 781	0.791
1971				0 724	0 734	0 728
1972			0 762	0 756	0 749	0 753
1973		0 694	0 797	0 834	0 852	0 863
1974	0 748	0 798	0 914	0 954	0 972	0 981
1975	0 641	0 705	0 720	0 745	0 748	0 748
1976	0 657	0 727	0 855	0 866	0.865	
1977	0 695	0 884	0 972	0 967		
1978	0 698	0 822	0 871			
1979	0 746	0 823				
1980	0 758					

From this triangle a steady growth in loss ratios over development years is observed, but it is not assessed as easily as in the case of premiums. Again, the logarithmic increments describe the developments in a more easily intelligible manner:

TABLE V

LOGARITHMIC INCREMENTS OF LOSS QUOTIENTS

Underwriting Year	Development Year				
	1→2	2→3	3→4	4→5	5→6
1970					0 012
1971				0 014	−0 008
1972			−0 007	−0 010	0 006
1973		0 139	0 045	0 021	0 013
1974	0.065	0 135	0 044	0 018	0 009
1975	0 096	0 021	0.035	0 004	0 000
1976	0 102	0 162	0 013	−0 002	
1977	0 241	0 095	−0 006		
1978	0 164	0.057			
1979	0 098				
mean	0 128	0 102	0 021	0 007	0 005
std deviation	0 0643	0 0542	0 0238	0 0121	0 0082
dgs of freedom	5	5	5	5	5

It is seen that standard deviations of the loss quotient increments are larger than the ones of premium increments This means that loss quotient developments are subject to more fluctuations than premiums, as should be expected taking the ceding company's need to reinsure its portfolio into consideration

The study of loss quotient development patterns met some difficulties owing to the quality of our data, since registrations up to 1980 have been manual, not meeting the requirements of a computerized analysis. Apart from cases involving cumbersome data problems, stability of logarithmic increments turned not to be a general phenomenon of our sample treaties. No marked patterns of interdependence between increments of premiums and of loss ratios could be detected and only slight signs of a negative autocorrelation between increments of consecutive development years could be observed.

Description of the loss ratio developments by stable logarithmic increments implies that shifts in rate level will affect the level of loss ratios but not the development pattern. So a high loss quotient in an early development year indicates an underwriting year growing proportionately worse. To the extent that claims of a reinsurance treaty are made up by a considerable number of individual claims allowing for smoothing, this reasoning is a correct model of reality But large claims, as a total loss of a vessel, are in general readily reported and not affecting the subsequent smooth development of ordinary claims. For this reason large claims should be registered separately and not included in the loss ratio applied in establishment of IBNR-reserves.

Unfortunately large claims were not registered separately in the Baltica files, thus causing problems in the model fitting analysis.

4. ESTIMATION OF THE RUN OFF PATTERN

If we call the loss quotients Q, and number the underwriting years by i and the development years by j, we have

(4.1) $$Q_{ij} = \text{loss quotient of underwriting year } i \text{ at the end of development year } j$$

$$i = 69, \dots, 80, \dots$$

$$j = 1, \dots, 6$$

and we have the logarithmic increments

(4.2) $$dq_{ij} = \log(Q_{i,j+1}/Q_{i,j}), \qquad j = 1, \dots, 5.$$

Then the examination of data suggests use of the normal distribution

(4.3) $$dq_{ij} \sim N(\zeta_j, \sigma_j^2) \quad \text{independently}$$

the parameters ζ_j and σ_j^2 being estimated by

(4.4) $$\hat{\zeta}_j = (\Sigma_i\, dq_{ij})/N_j \sim N(\zeta_j, \sigma_j^2/N_j)$$

(4.5) $$\hat{\sigma}_j^2 = \Sigma_i\,(dq_{ij} - \hat{\zeta}_j)^2/(N_j - 1) \sim \sigma_j^2 \chi^2(f_j)/f_j, \qquad f_j = N_j - 1$$

where N_j is the number of observed increments from development year j to $j+1$. It follows from the theory that the $\hat{\zeta}_j$ and $\hat{\sigma}_j^2$ $j = 1, \dots, 5$ are mutually independent.

By these parameters we have obtained a description of the run-off pattern of the treaty. It should be noted that the parameters describe the one-step increments, thus allowing for estimation exploiting all data observed, including the latest observations. With more abundant data, an alternative description could be obtained by the logarithmic increment from present to ultimate stage. A such description would be advantageous in the forecasting procedure but is of little practical interest at present circumstances. Whichever description is applied, it can be used to test identity of run-off patterns by one-way analysis of variance. In the sample treaties only highly significant results were obtained, indicating individuality of treaties.

5. FORECASTING THE DEVELOPMENT OF A TREATY

Having observed underwriting year i at the P'th development year, the objective to forecast the ultimate loss ratio

$$Q_i = Q_{i,\infty} = Q_{i,6} \tag{5.1}$$

is obtained by application of the previous chapters.

From the normality of $dq_{i,j}$ follows that the conditional distribution of Q_i given $Q_{i,p}$,

$$Q_i | Q_{i,p} = Q_{i|p} = Q_{i,p} \exp(dq_{i,p} + \cdots + dq_{i,5}) | Q_{i,p} \tag{5.2}$$

is lognormal with logarithmic mean and variance

$$\begin{aligned} E[\log Q_{i|p}] &= E[\log Q_{i,p} + dq_{i,p} + \cdots + dq_{i,5} | Q_{i,p}] \\ &= \log Q_{i,p} + \zeta_p + \cdots + \zeta_5 \end{aligned} \tag{5.3}$$

$$\begin{aligned} \mathrm{Var}[\log Q_{i|p}] &= \mathrm{Var}[\log Q_{i,p} + dq_{i,p} + \cdots + dq_{i,5} | Q_{i,p}] \\ &= \sigma_p^2 + \cdots + \sigma_5^2. \end{aligned} \tag{5.4}$$

If the parameters $\zeta_p, \ldots, \zeta_5$ and $\sigma_p^2, \ldots, \sigma_5^2$ were known (5.3) might be applied as a forecast, to be evaluated in the normal distribution with variance (5.4). Now, the parameters are unknown and we have to substitute the estimates $\hat{\zeta}_p, \ldots, \hat{\zeta}_5$. So the individual logarithmic increments $dq_{i,j}$ are forecasted by the $\hat{\zeta}_j$

$$\begin{gathered} dq_{i,j} = \zeta_j + \varepsilon_{i,j} = \hat{\zeta}_j + (\zeta_j - \hat{\zeta}_j) + \varepsilon_{i,j} = \hat{\zeta}_j + \phi_{i,j} + \varepsilon_{i,j}, \\ \phi_{i,j} \sim N(0, \sigma_j^2 / N_j), \qquad \varepsilon_{i,j} \sim N(0, \sigma_j^2), \end{gathered} \tag{5.5}$$

thus introducing a forecasting error consisting of an estimation error $\phi_{i,j}$ and a pure forecasting error $\varepsilon_{i,j}$. We obtain a forecast of Q_i by

$$\tilde{Q}_{i|p} = Q_{i,p} \exp(\hat{\zeta}_p + \cdots + \hat{\zeta}_5) \tag{5.6}$$

which is lognormally distributed with logarithmic mean and variance

$$\begin{aligned} E[\log \tilde{Q}_{i|p}] &= \log Q_{i,p} + E[\hat{\zeta}_p + \cdots + \hat{\zeta}_5] \\ &= \log Q_{i,p} + \zeta_p + \cdots + \zeta_5, \end{aligned} \tag{5.7}$$

$$\text{(5.8)} \qquad \operatorname{Var}[\log \tilde{Q}_{i|p}] = \operatorname{Var}[\phi_{i,p} + \varepsilon_{i,p} + \cdots + \phi_{5,p} + \varepsilon_{5,p}]$$
$$= \sigma_p^2 \frac{N_p + 1}{N_p} + \cdots + \sigma_5^2 \frac{N_5 + 1}{N_5}.$$

Now, the lognormal distribution with logarithmic mean μ and variance σ^2 is right skew with

mean:	$\exp(\mu + \sigma^2/2)$
median.	$\exp(\mu)$
std. deviation:	$\exp(\mu + \sigma^2/2)\sqrt{\exp\sigma^2 - 1}$

and so a central forecast of Q_i is supplied by

$$\text{(5.9)} \qquad \hat{Q}_{i|p} = Q_{i,p} \exp(\hat{\zeta}_p + \cdots + \hat{\zeta}_5) \exp(\sigma_{(p)}^2/2)$$

with

$$\text{(5.10)} \qquad \sigma_{(p)}^2 = \hat{\sigma}_p^2 \frac{N_p + 1}{N_p} + \cdots + \hat{\sigma}_5^2 \frac{N_5 + 1}{N_5}$$

the variance of the forecast being estimated by

$$\text{(5.11)} \qquad \operatorname{Var}[\hat{Q}_{i|p}] = \hat{Q}_{i|p}\sqrt{\exp \sigma_{(p)}^2 - 1}$$

Applied to the example marine treaty the forecasted loss quotients may be presented by insertion in the run off triangle (2.4)

LOSS QUOTIENTS OBSERVED AND FORECASTS

Underwriting Year	Year of Development						Forecast	
	1	2	3	4	5	6	$\hat{Q}_{i,p}$	s
1969						0 801		
1970					0 781	0 791		
1971				0 724	0 734	0 728		
1972			0 762	0 756	0 749	0 753		
1973		0 694	0 797	0 834	0 852	0 863		
1974	0 748	0 798	0 914	0 954	0 972	0 981		
1975	0 641	0 705	0 720	0 754	0 748	0 748		
1976	0 657	0 727	0 855	0 866	0 865		0 869	0 008
1977	0 695	0 884	0 972	0 967			0 979	0 015
1978	0 698	0.822	0 871				0 901	0 027
1979	0 746	0 823					0 944	0 062
1980	0 758						0 991	0.096

s denotes the standard deviation of the forecast

These forecasted loss quotients offer a help to the assessment of rate levels of still developing underwriting years.

Applying an identical model to the premiums produces the forecasts:

PREMIUMS OBSERVED AND FORECASTS

Underwriting Year	Year of Development						Forecast	
	1	2	3	4	5	6	$\hat{p}_{i\|p}$	s
1969						226		
1970					261	261		
1971				329	328	328		
1972			434	436	435	435		
1973	•	610	632	631	630	630		
1974	420	704	739	738	736	736		
1975	310	598	629	624	623	622		
1976	310	599	629	631	629		629	0 1
1977	345	667	680	679			678	0 7
1978	491	731	737				734	3 2
1979	581	815					842	16 4
1980	577						1033	162 8

For a single underwriting year, the forecasts $\hat{P}_{i|p}$ and $\hat{Q}_{i|p}$ may be assumed independent

6. SETTING UP IBNR-RESERVES FOR A SINGLE TREATY

From the forecasts $\hat{P}_{i|p}$ of the ultimate premium and $\hat{Q}_{i|p}$ of the ultimate loss ratio, of underwriting year i observed at the Pth development year, a forecast of the ultimate financial result is derived.

With a fixed commission rate w the relation between ultimate premium P_i, loss ratio Q_i and financial result R_i is

$$R_i = P_i - wP_i - Q_iP_i = P_i(1 - w - Q_i). \tag{6.1}$$

Inserting estimates $\hat{P}_{i|p}$ and $\hat{Q}_{i|p}$ in (6.1) yields a forecast of R_i, whose distribution is of lognormal type, but translated and reversed on the real line. This distribution is easily studied, though involving some arithmetic complexity

The larger part of variation in this forecast of R_i is caused by $\hat{Q}_{i|p}$, as the variance of $P_{i|p}$ is in general much smaller than that of $\hat{Q}_{i|p}$. Further, extraneous information on the ultimate premium volume P_i is usually provided by cedants, allowing heuristic improvements of the premium forecast by combination of $\hat{P}_{i|p}$ and the cedant's information. Evaluation of R_i in the conditional distribution given P_i models this administrative procedure loosing only a small variance component, and so the conditional forecast of R_i is applied:

$$\hat{R}_{i|p} = P_i - wP_i - \hat{Q}_{i|p}P_i | P_i. \tag{6.2}$$

In this conditional distribution the stochastic element remaining is the volume of claims $\hat{Q}_{i|p}P_i$ which is lognormally distributed, and from this distribution confidence limits for $\hat{R}_{i|p}$ may be derived. Again the normal approximation with estimated standard deviation

$$\hat{Q}_{i|p} P_i \sqrt{\exp \sigma^2 - 1} \tag{6.3}$$

supplies an indication of the precision of $\hat{R}_{i|p}$, for the use of practitioners. Also, the normal approximation is useful when treaties are aggregated to a portfolio.

Applying this normal approximation we obtain forecasts of the sample treaty's open underwriting years

FORECASTED ULTIMATE RESULTS

Underwriting Year	Premium	Commission	Claims	Financial Result	Standard Deviation
1976	629	173	547	−91	4.4
1977	678	186	664	−172	10.0
1978	734	202	661	−129	17.9
1979	842	232	795	−185	49.3
1980	1033	284	1024	−275	98.3

The IBNR-reserve is the supplementary reserve needed to match the results on books with anticipated ultimate results, in the example:

Accounts and forecasted results of underwriting years	Underwriting year 75	76	77	78	79	80
	Booked at the end of 1980					
Premium	622	629	679	737	815	577
Commission	171	173	187	203	224	159
Claims paid	462	492	593	585	520	174
Claims outstanding	3	52	64	57	151	263
IBNR-reserve	0	3	7	21	105	256
Financial result	−14	−91	−172	−129	−185	−275
	Forecasted ultimate accounts					
Premium	622	629	678	734	842	1033
Commission	171	173	186	202	232	284
Claims	465	547	664	661	795	1024
Financial result	−14	−91	−172	−129	−185	−275
Commission rate	0.275	0.275	0.275	0.275	0.275	0.275
Loss quotient	0.748	0.869	0.979	0.901	0.944	0.991
Standard deviation of loss quotient	0.0	0.008	0.015	0.027	0.062	0.096

7. SETTING UP IBNR-RESERVES FOR A PORTFOLIO

Due to developments in portfolio composition from one underwriting year to the next, application of the procedure of the previous chapters on aggregate run-off triangles of a portfolio is less accurate than the summation of IBNR-reserves established at individual treaties. Also, use of individual IBNR-reserves facilitates the inclusion of explicit non-statistical information, and tracing the influence of important individual cases. Both facilities are essential to the transparency of the procedure to non-statisticians, and thus to the attainment of confidence by management.

In probabilistic terms the log normal distributions of the individual IBNR-reserves are smoothed in the summation to portfolio level, by the law of large numbers. So, the distributions of the portfolio reserves are not lognormal. Provided the portfolio consists of sufficiently many similar treaties, the normal approximation offers a reasonable assumption for evaluation of the portfolio reserves.

If the estimated IBNR-reserve of treaty x, underwriting year $\imath$ is

$$\hat{R}_\imath(x) \tag{7.1}$$

with standard deviation $s_\imath(x)$ obtained by (6.3), we obtain the portfolio IBNR-reserve of underwriting year $\imath$

$$\hat{R}_\imath(\cdot) = \Sigma_x \hat{R}_\imath(x) \tag{7.2}$$

with standard deviation

$$s_\imath(\cdot) = (\Sigma_x s_\imath^2(x))^{1/2}. \tag{7.3}$$

Correspondingly, the total IBNR-reserve over all underwriting years is

$$\hat{R}(\cdot) = \Sigma_\imath \Sigma_x \hat{R}_\imath(x) \tag{7.4}$$

$$s(\cdot) = (\Sigma_\imath \Sigma_x s_\imath^2(x))^{1/2}. \tag{7.5}$$

Some traditions of reserving procedures seem to argue for a security loading of reserves, the explicit meaning of which is not always clear. With a statistical approach, the security loading will be related to a confidence interval, and so the security loading should be proportional to the portfolio standard deviation (7.3) or (7.5) rather than the volume of reserves (7.2) or (7 4), or the premium. A proper security loading of individual treaties would turn out to be costly at a reasonable confidence level, and it should lead to systematic overreserving of the portfolio.

Security loading may conveniently take the shape of a contingency reserve established as a percentage point in the distribution of $R(\cdot)$. Exploiting the normal approximation the contingency reserve at security level 99% is 2.326 $s(\cdot)$ and at level 99.9% it is 3 090 $s(\cdot)$. If required for administrative reasons, the contingency reserve may be distributed on underwriting years, subfolios or individual treaties according to their standard deviations (7.3), thus ensuring a common security level of all components of the split.

8. TESTING PARAMETER STABILITY

Application of methods as described above require data input of high quality, and checking for instabilities. Ordinary statistical procedures are useful for this purpose.

Observing a new logarithmic increment $dq_{i,j}$ of loss quotients from the jth to the $j+1$st development year this can be evaluated against the previous parameter estimates $\hat{\zeta}_j$, $\hat{\sigma}_j^2$ by a Student's t-test:

(8.1) $$t_{i,j} = (dq_{i,j} - \hat{\zeta}_j)/\hat{\sigma}_j\sqrt{(N_j+1)/N_j} \sim t(f_j), \qquad N_j \geq 2$$

and it can be shown that the t-values

(8.2) $$t_{3,j}, t_{4,j}, \ldots, t_{N_j,j}$$

are mutually independent and independent of the new parameter estimates $\hat{\zeta}_j$, $\hat{\sigma}_j^2$. So these t-values supply a basis for testing parameter stability.

9. UNIFICATION OF STATISTICAL PROCEDURES AND UNDERWRITING INFORMATION

Application of the described statistical procedure may seem just straight-forward, but is not so. The very important aspect of including non-statistical information should be considered in any implementation of scientifically based procedures. In many cases underwriters will be able to explain a significant shift in parameters, in other cases some known changes must be taken into consideration though not yet evident in data, and in still other cases a t-test may draw the attention of underwriters to some unnoticed phenomenon. In any case, a careful study of the composition of portfolio results should be carried out and appropriate corrections accomplished, to obtain a unification of statistically based indications and general information.

10. OPEN ENDS

Though the log normal distribution supplies a useful tool, its validity in a strict sense may be doubted. In the sample data loosely referred to in this paper, the tail of the log normal appeared somewhat too thick, leading to use of a median forecast $\exp(\hat{\zeta})$ instead of the mean value forecast $\exp(\hat{\zeta}+\sigma^2/2)$ in practical applications.

A further study into the shape of distributions involved is desirable, in search of a distribution allowing aggregation from single risks to a reinsurance treaty and from treaties to a reinsurance portfolio, as well as a distribution fitting high quality data well.

Treaties with a firm run-off pattern, i.e., small variances σ_j^2 of logarithmic increments might be used as indicators of tendencies affecting each treaty in the portfolio. This may lead to the inclusion of credibility theory as described for instance by NORBERG (1979).

A last point is the explicit inclusion of interest and inflation into the procedure. This requires accountancy considerations and definitions well beyond the scope of the present paper, but still any insurer operating in long-tail insurance must pay attention to this aspect of business.

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